

# A novel Parton shower algorithm based on the small-x evolution equation

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- **YS**, Shu-yi Wei and Jian Zhou, [Phys.Rev.D 107, 016017 \(2023\)](#).
- **YS**, Shu-yi Wei and Jian Zhou, arXiv: [2307.04185/hep-ph](#).

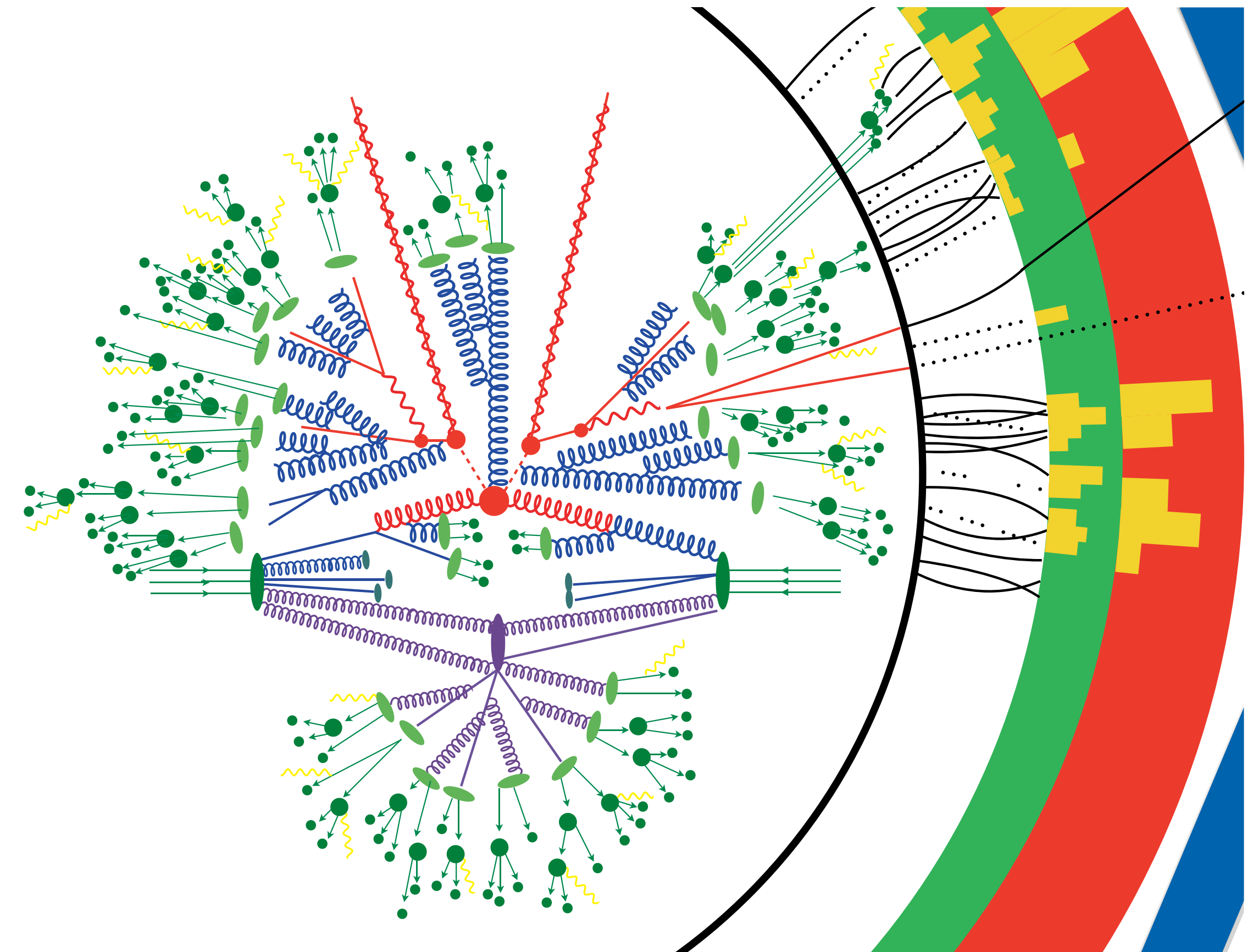
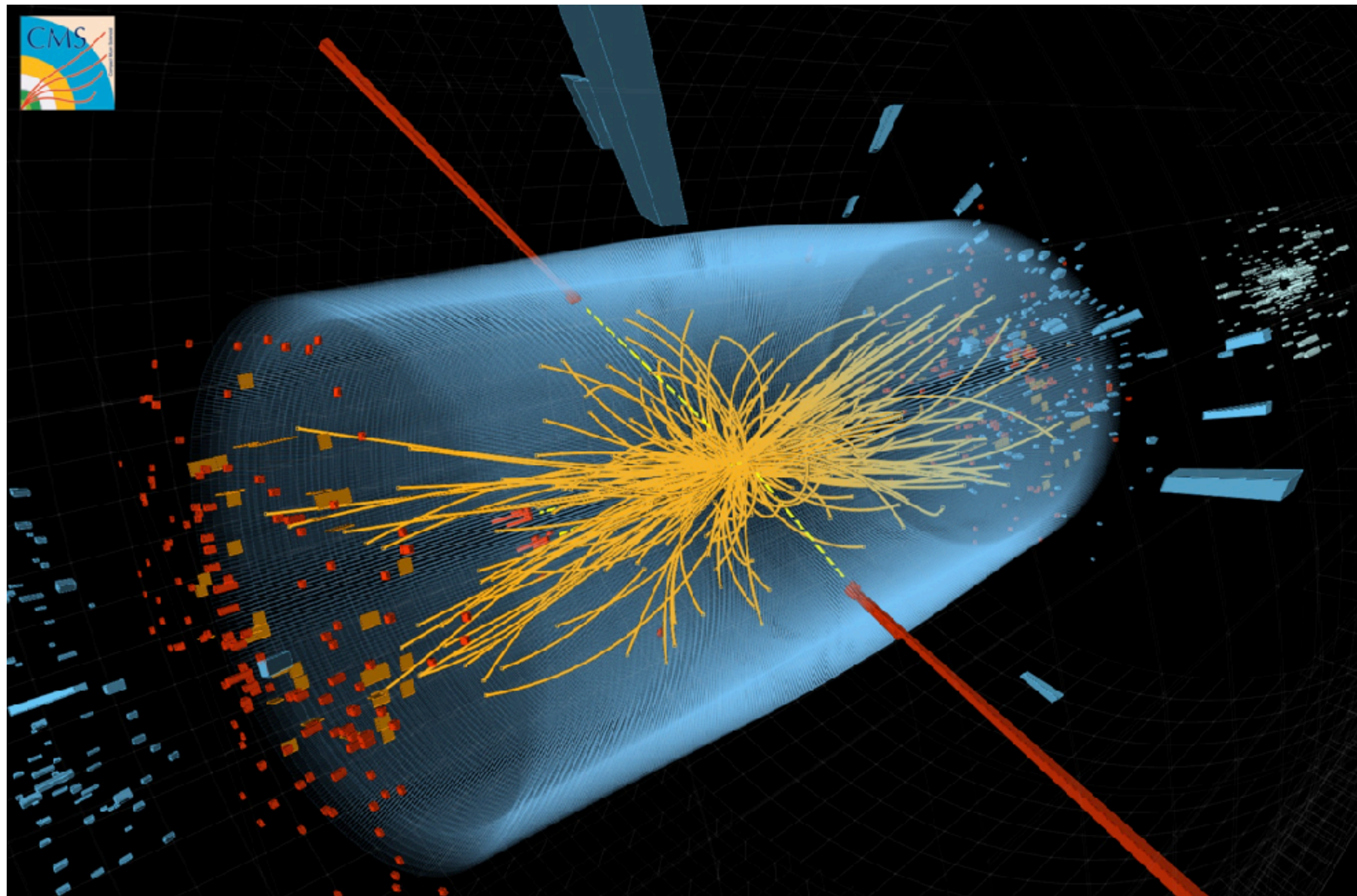
QCD与中高能核物理暑期学校

2023.07.12

# Outline

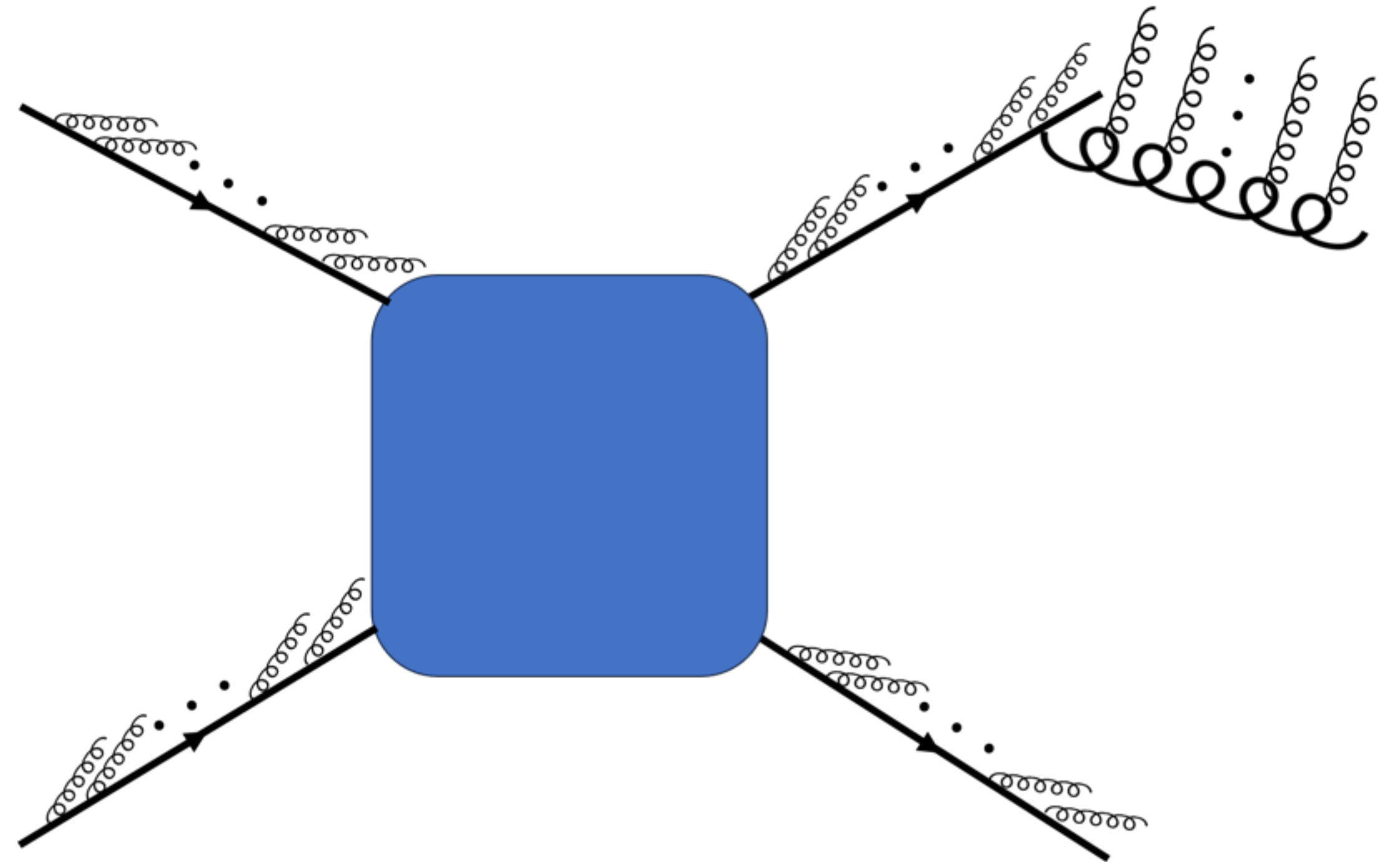
- 1) Introduction
- 2) Parton shower algorithm based on the Gribov-Levin-Ryskin evolution equation
- 3) Summary

# Monte Carlo event generator in high energy collisions



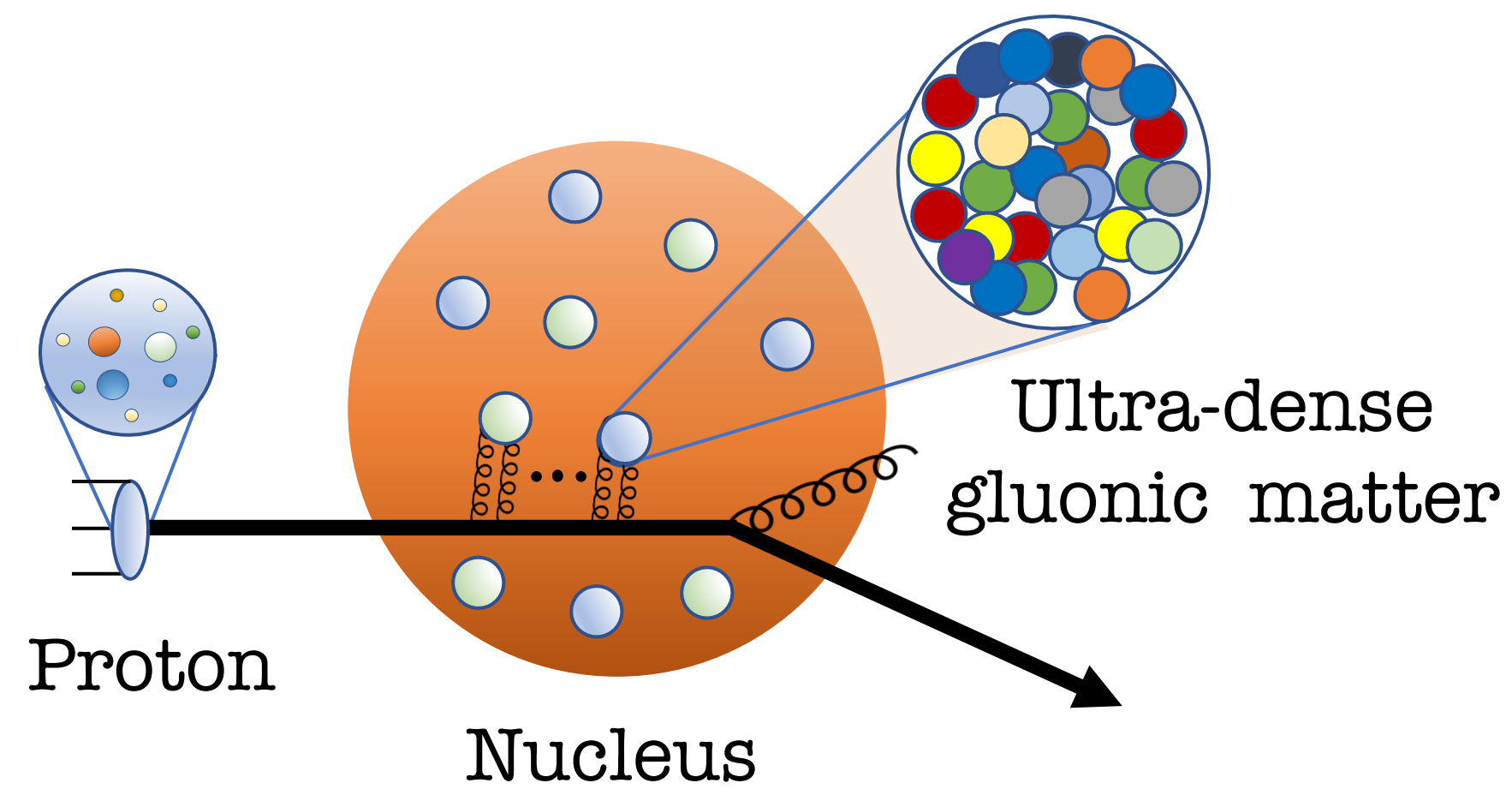
Credit: Benjamin Nachman

# Parton shower algorithms in M.C. event generator



Parton shower algorithms are dedicated to simulating and describing the **radiation behavior** of **quarks** and **gluons**.

# Forward physics at pA collisions and Future EIC



Electron-Ion Collider

GOALS THE MACHINE BENEFITS SCIENCE NEWS IMAGES

quarks account for only a small fraction of their mass.

The Electron-Ion Collider will keep America at the forefront of the international nuclear physics and particle accelerator communities.

ENERGY

### 3D structure of protons and nuclei

The EIC will bring high-energy electrons into head-on collisions with high-energy protons or atomic nuclei to produce "freeze-frame" snapshots of those particles' inner structure, creating the first-ever tomographic 3D images of the "ocean" of gluons within. These images will tell scientists how gluons and quarks bind each other to form the particles within and around us.

More

### Gluon saturation and the color glass condensate

Recent experiments and advances in theory suggest that protons, neutrons, and nuclei appear as dense "walls" of gluons at high energies, creating what may be among the strongest force fields in nature. Discovering and studying this form of matter, the "color glass condensate," will provide deeper insight into why matter in this subatomic realm is stable.

More

### Solving the mystery of proton spin

The EIC will be the world's first polarized electron-proton collider—meaning the "spins" of both colliding particles can be aligned in a controlled way. This will make it possible to experimentally solve the outstanding mystery of how the teeming quarks and gluons inside the proton combine their spins to generate the overall spin carried by the proton.

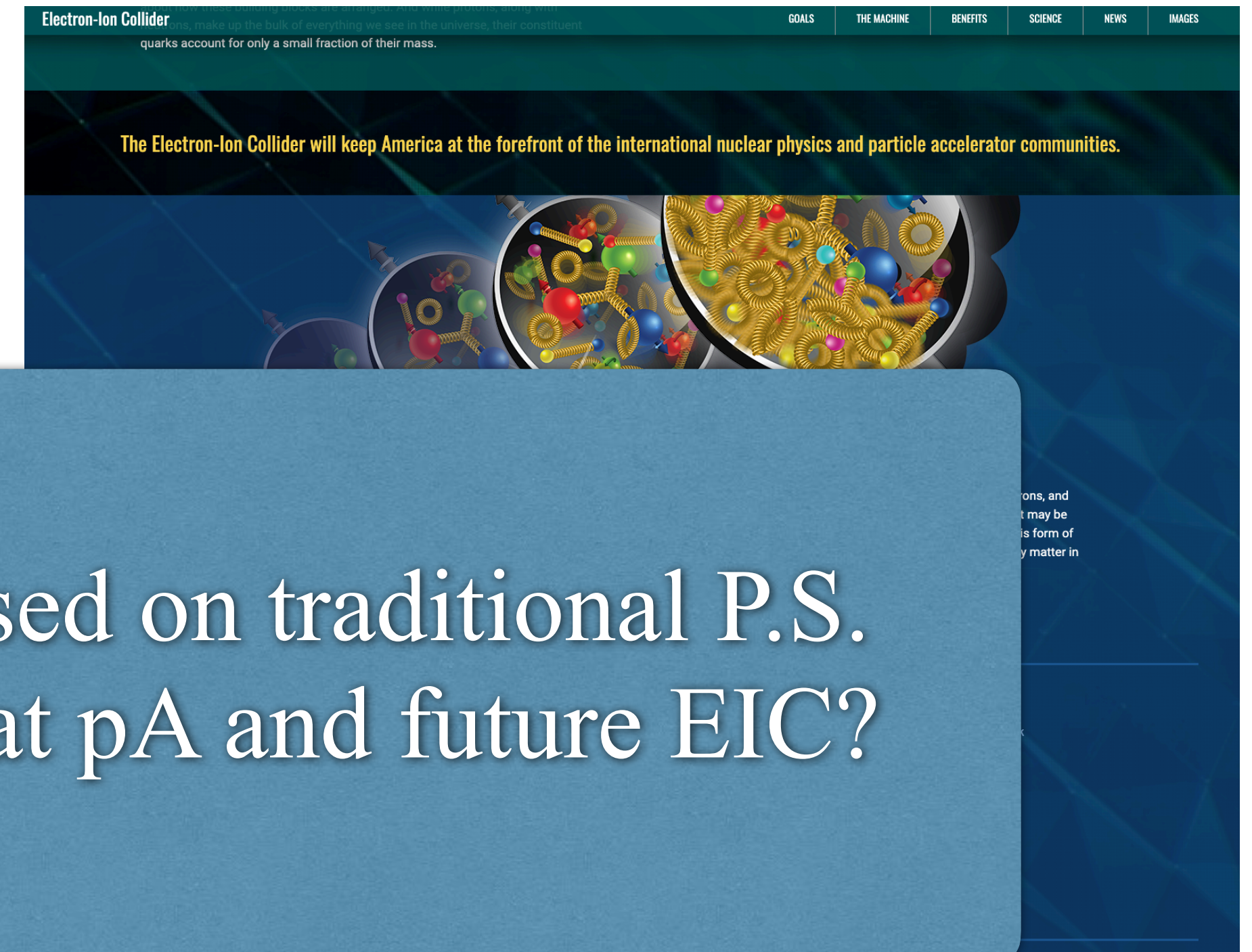
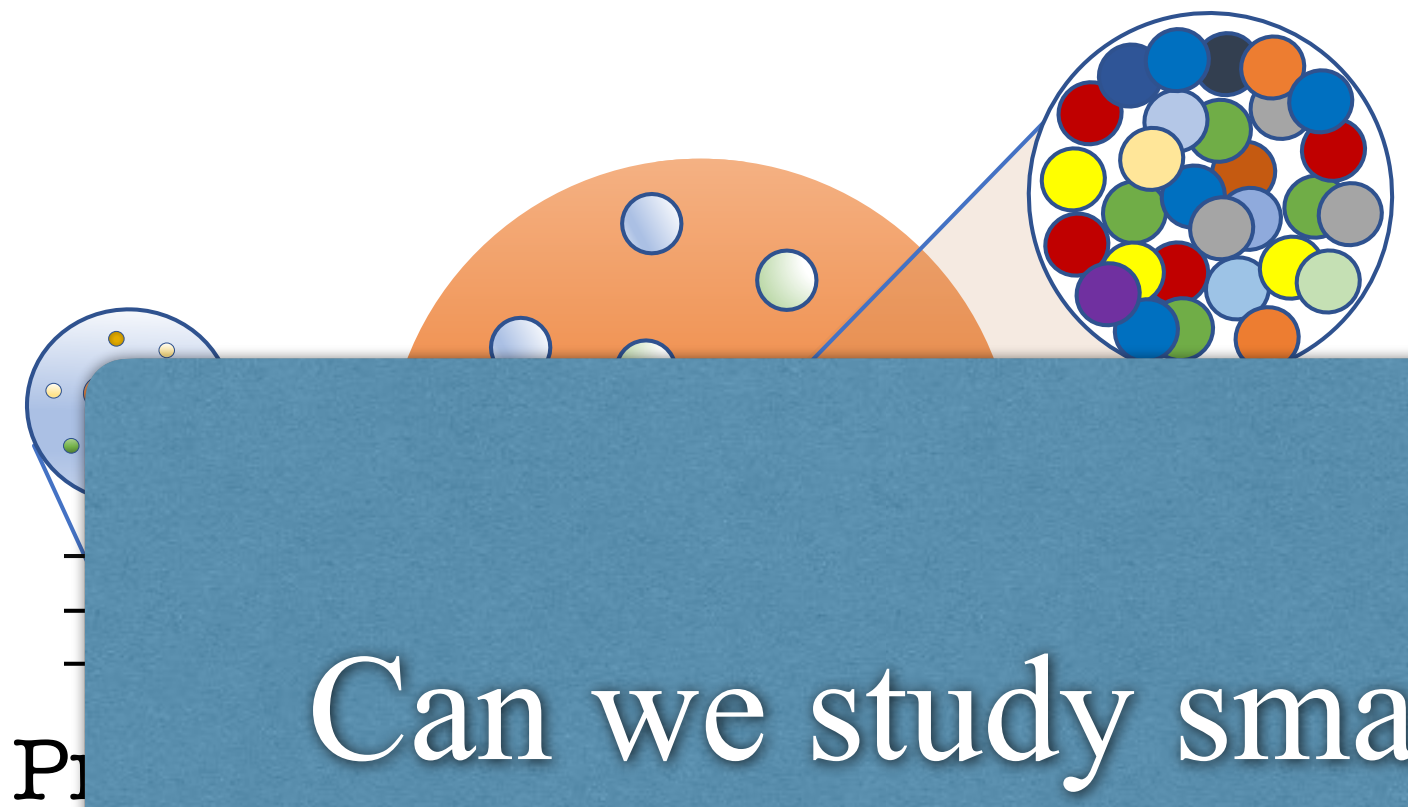
More

- valence quark
- sea quarks
- spin
- gluon

EIC in Brookhaven. **High-luminosity** and **high-precision**.

In the forward region at pA and EIC, we can study parton **non-linear** evolutions and explore **gluon saturation** in the **small-x**.

# Forward physics at pA collisions and Future EIC

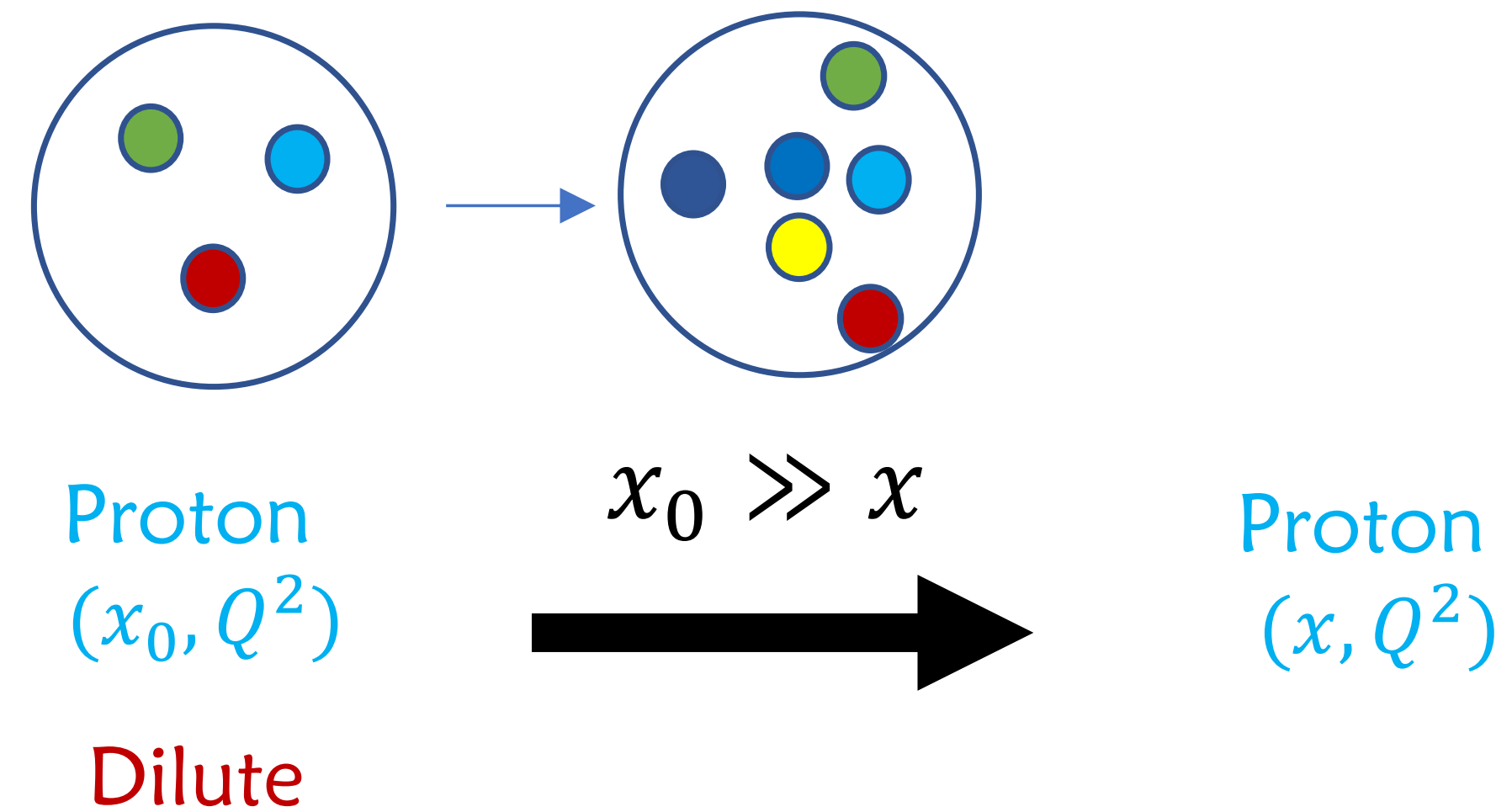
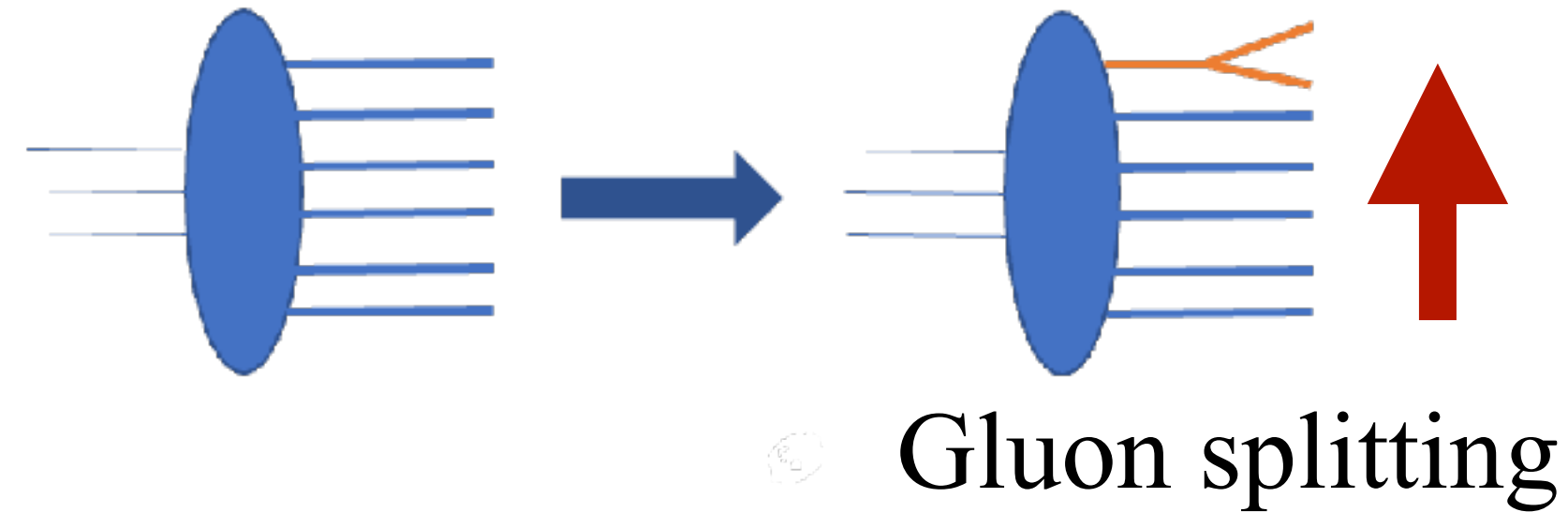
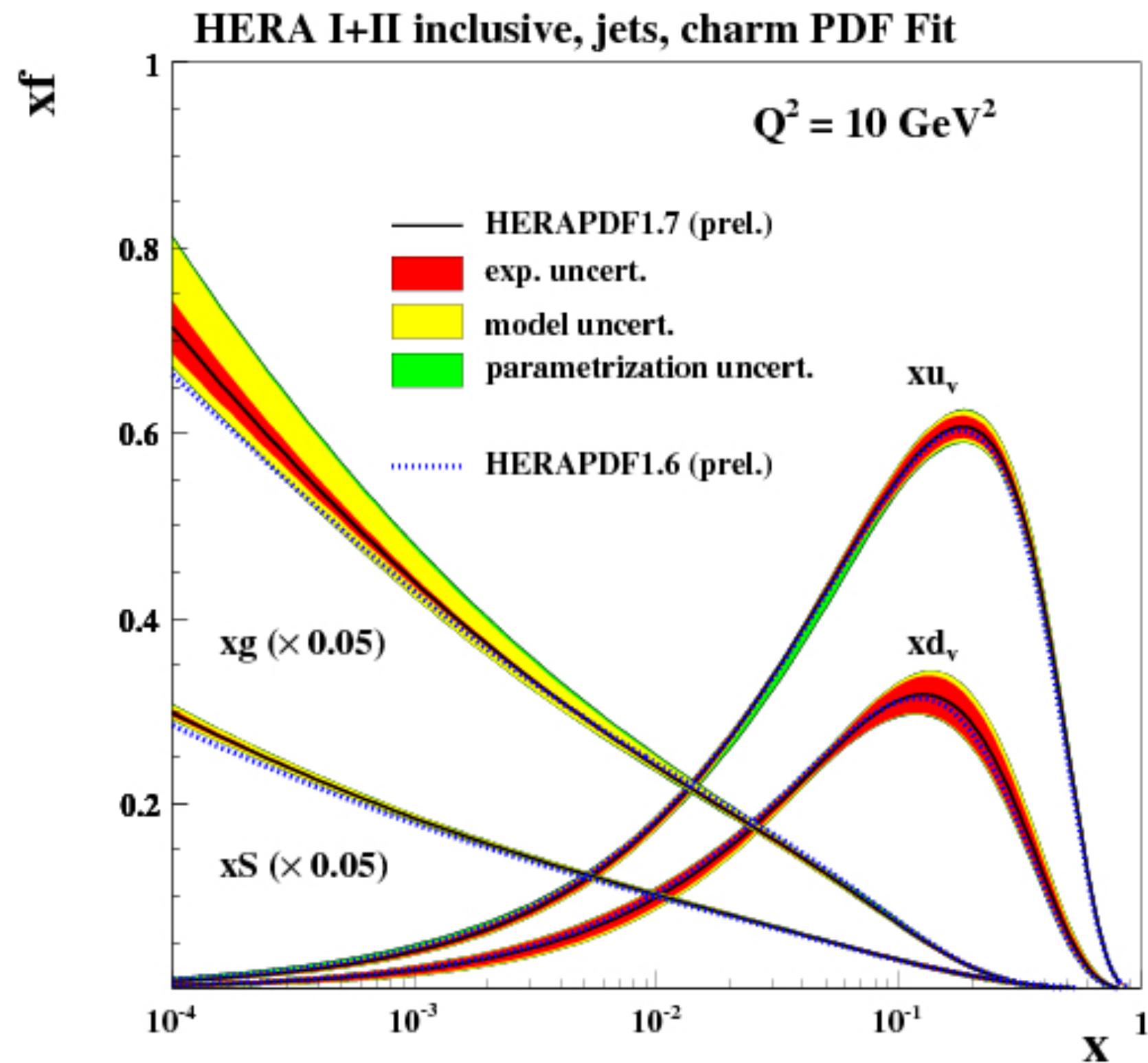


Can we study small- $x$  physics based on traditional P.S. algorithms in the forward region at pA and future EIC?

EIC in Brookhaven. **High-luminosity** and **high-precision**.

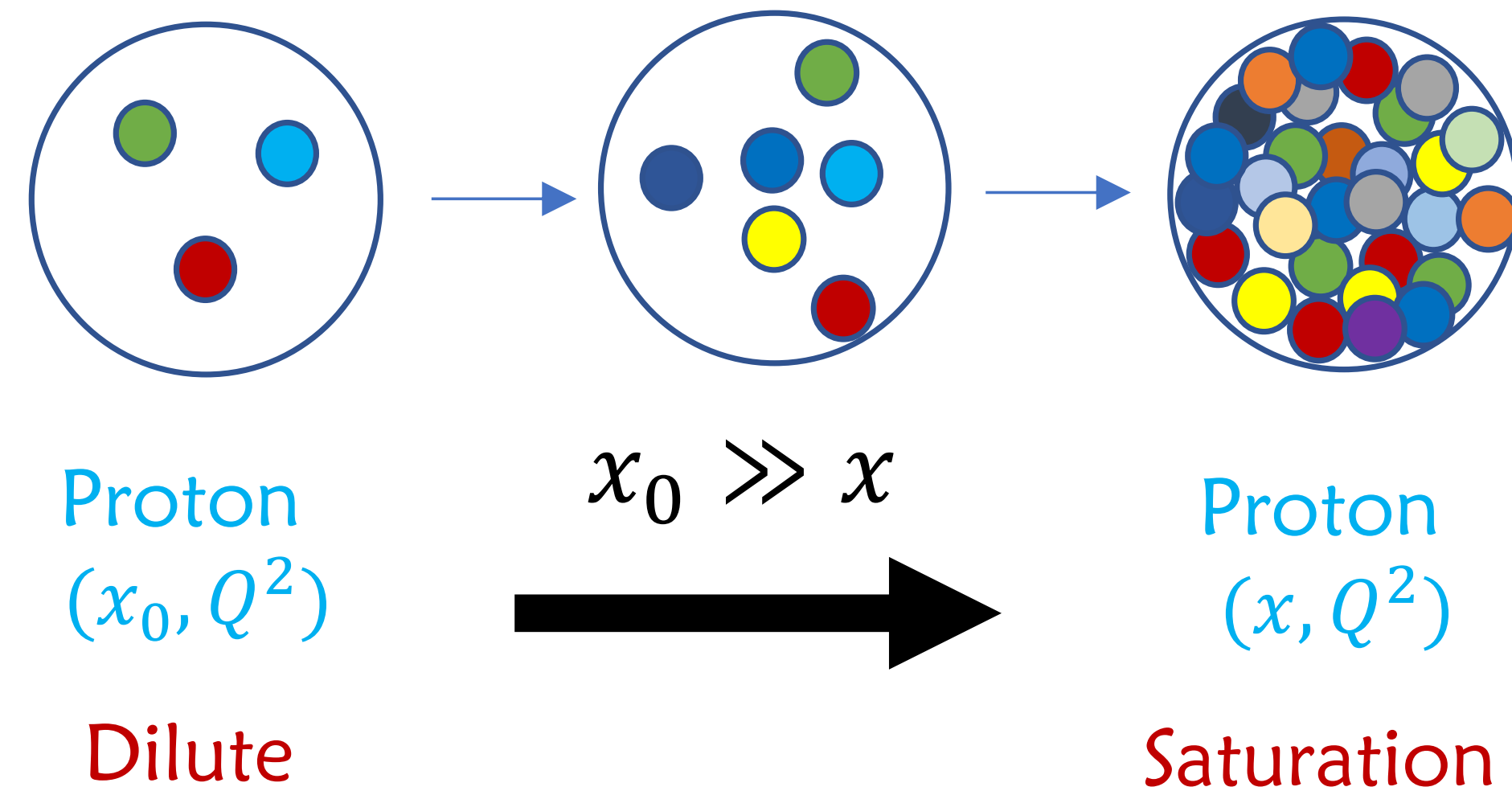
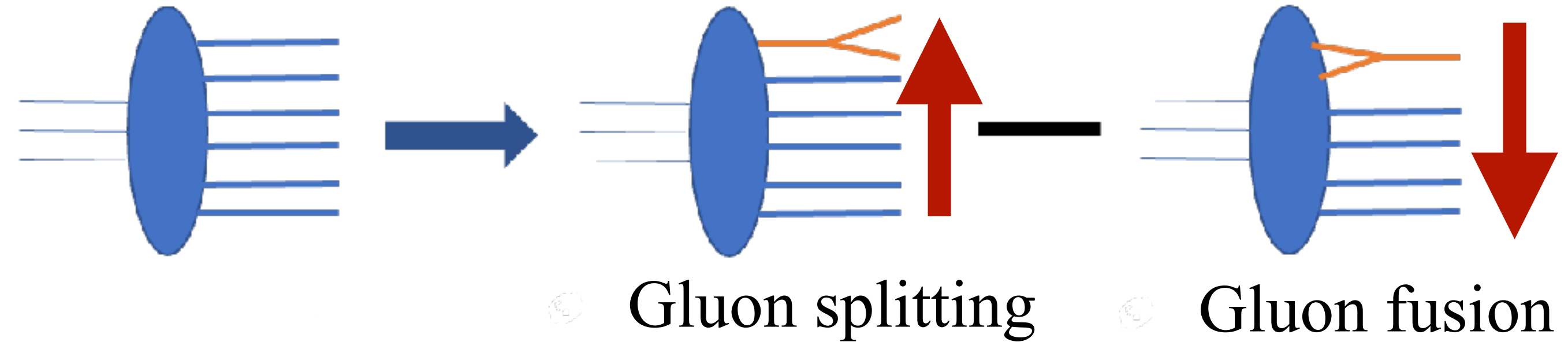
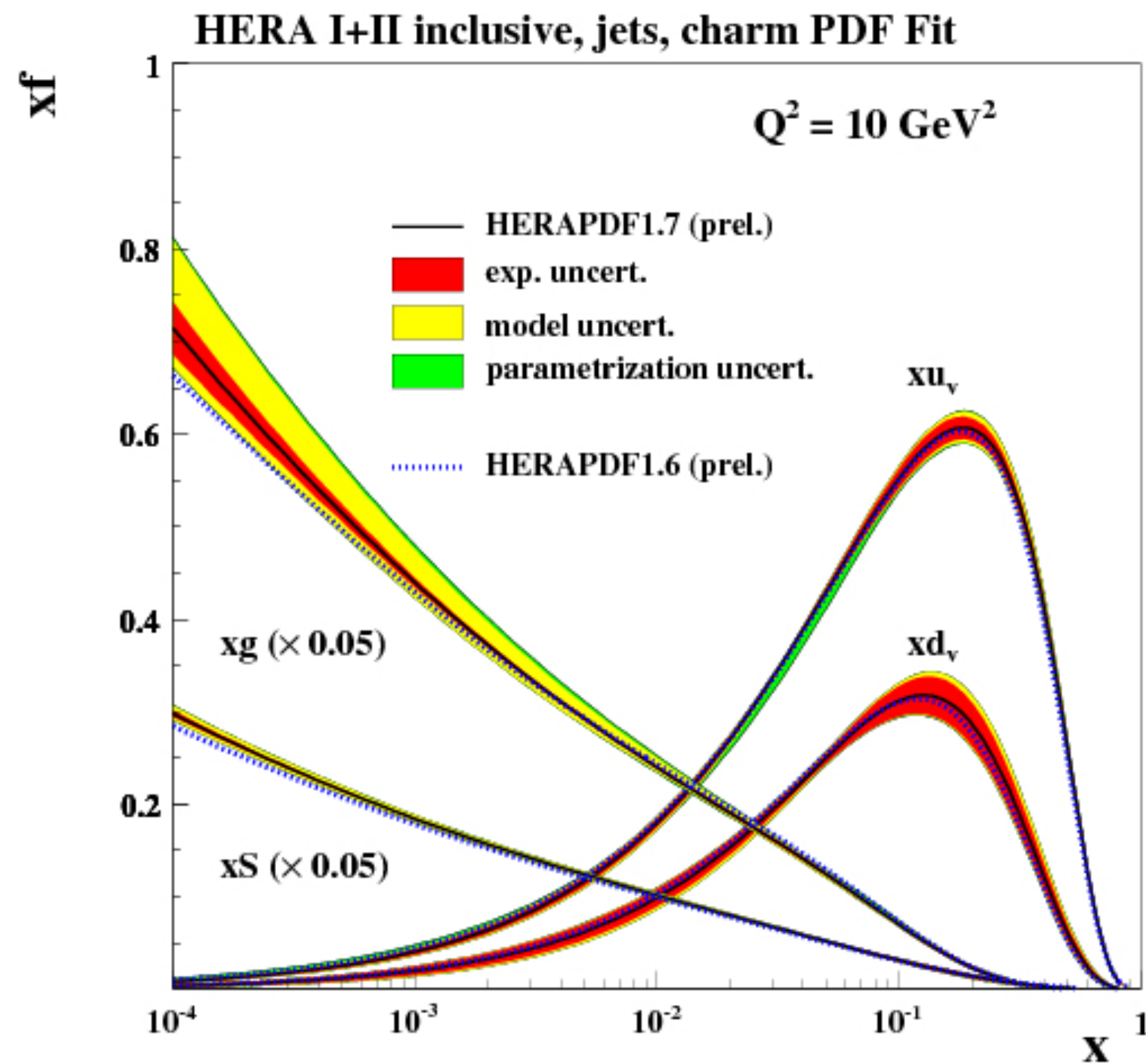
In the forward region at pA and EIC, we can study parton **non-linear** evolutions and explore **gluon saturation** in the **small- $x$** .

# small-x gluon evolution equations



■ Gluons rapidly increase as  $x$  decreases, gluons dominate in small  $x$  region

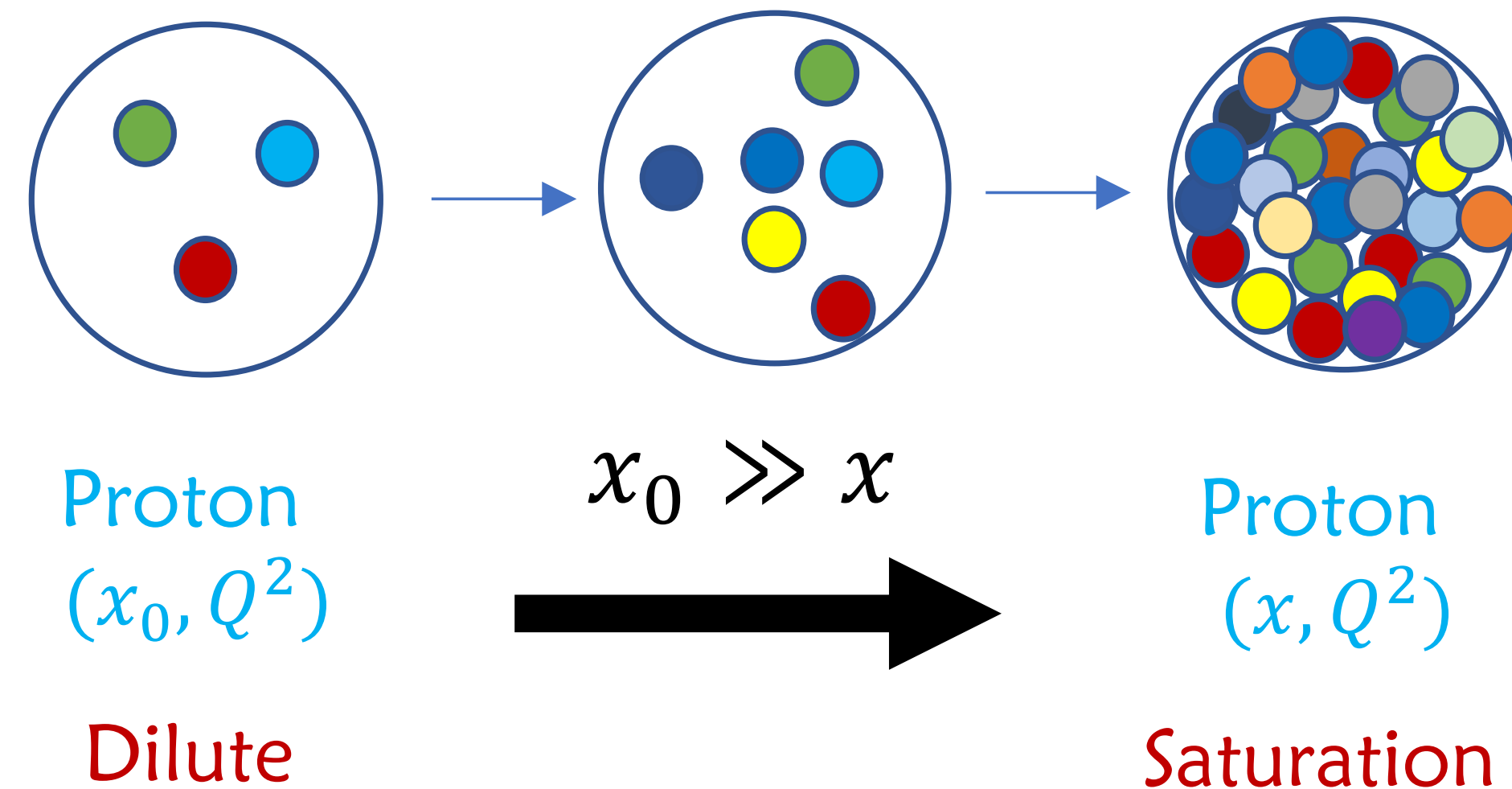
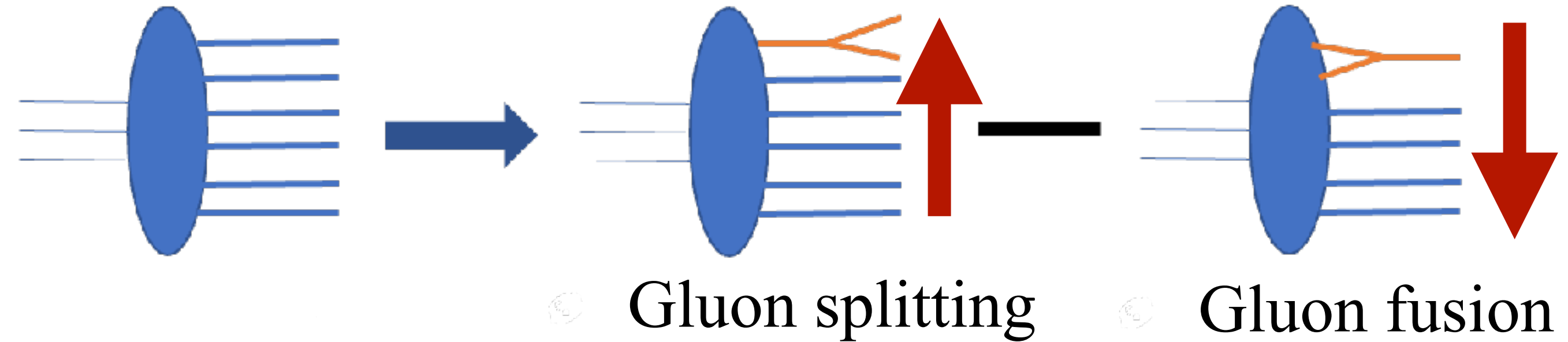
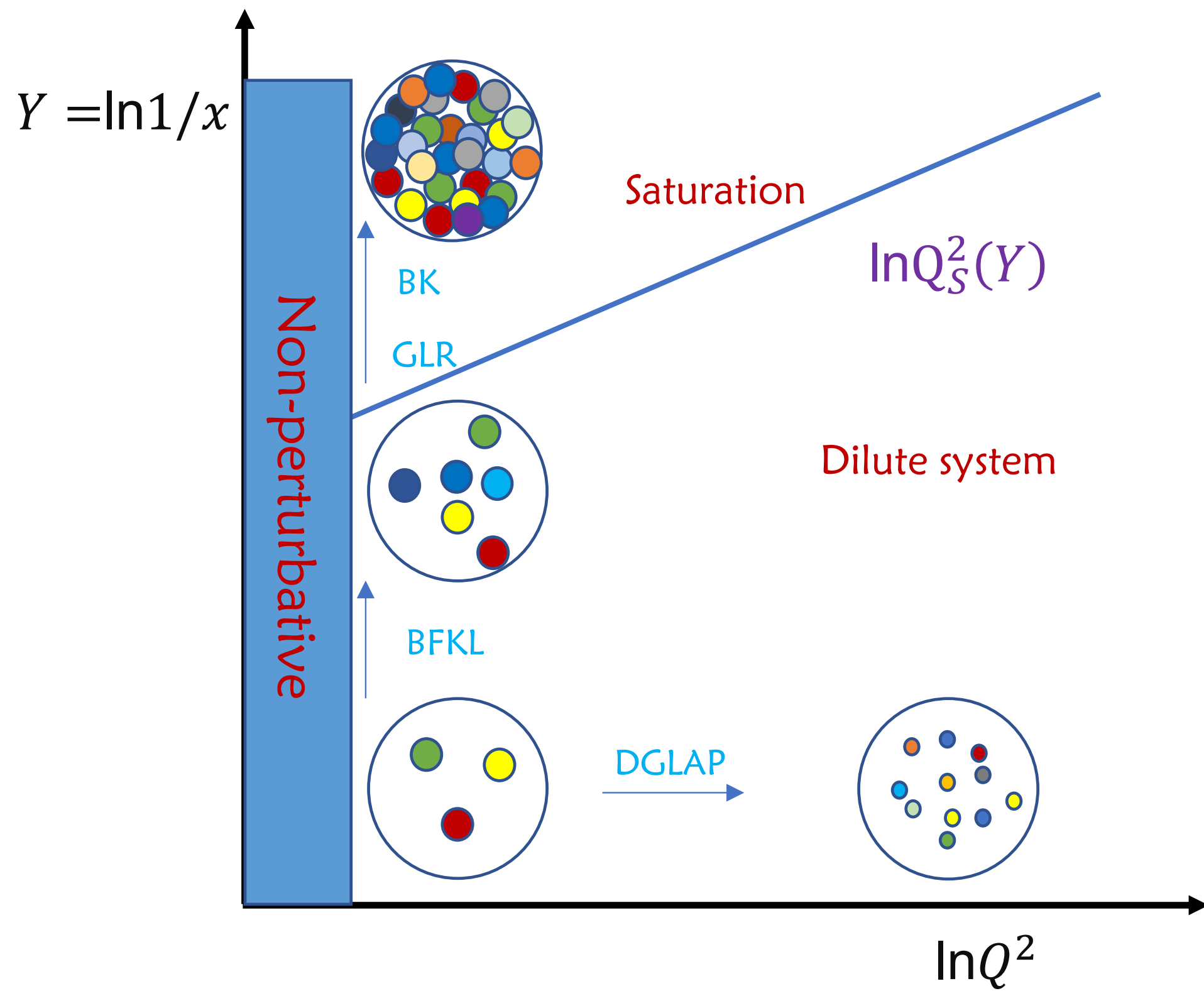
# small-x gluon evolution equations



- Gluons rapidly increase as  $x$  decreases, gluons dominate in small  $x$  region
- Using BFKL, GLR and BK equation instead of DGLAP equation.
- **GLR/BK** equations are the **non-linear** evolution equations which describe **partons non-linear** evolution in **small- $x$**  region.



# small-x gluon evolution equations



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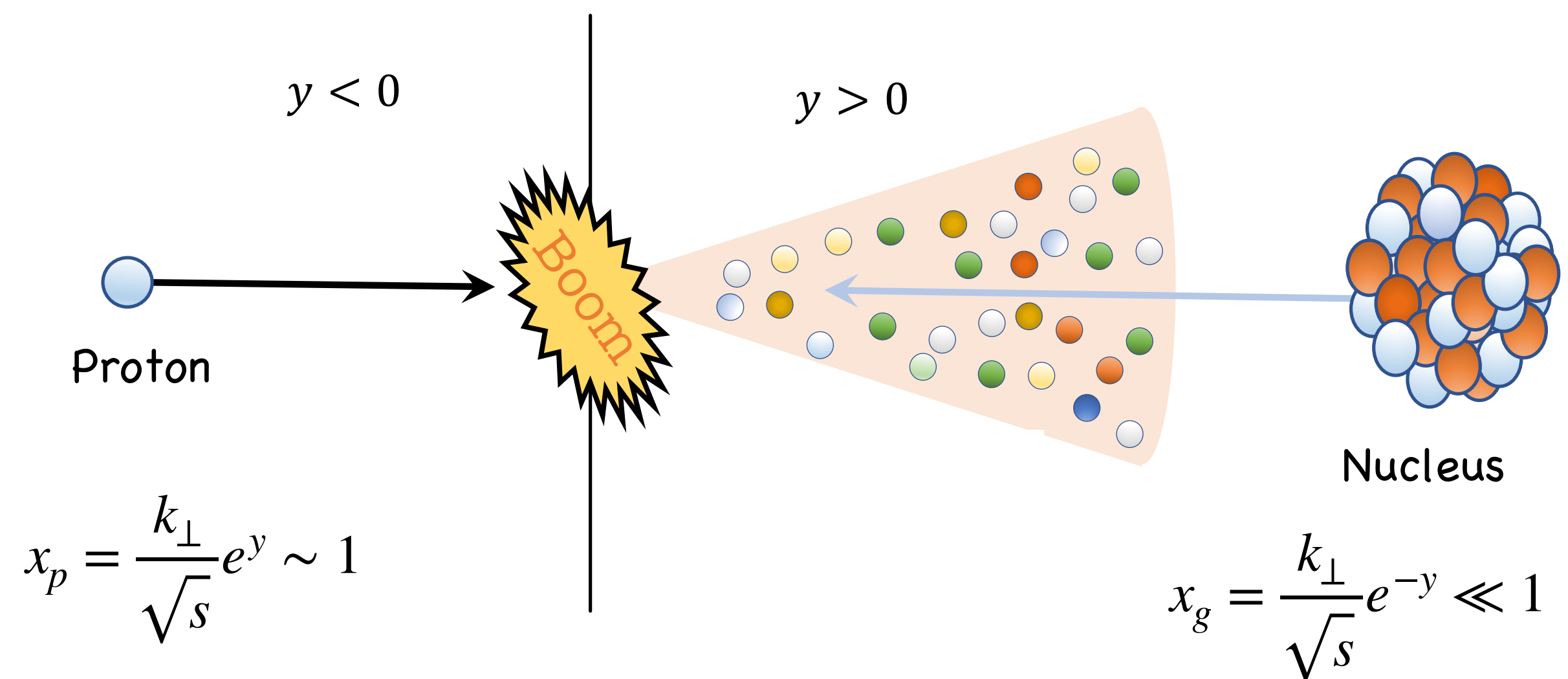
● Color Glass condensate

# Forward physics at pA collisions

For the single hadron production, LO cross-section likes

[Dumitru, Jalilian-Marian, PRL, 02]

$$\frac{d\sigma^{pA \rightarrow hX}}{d^2p_\perp dy_h} = \int_0^1 \frac{dz}{z^2} \left[ \sum_f x_p q_f(x_p) \mathcal{F}(k_\perp) D_{h/q}(z) + x_p g(x_p) \tilde{\mathcal{F}}(k_\perp) D_{h/g}(z) \right]$$

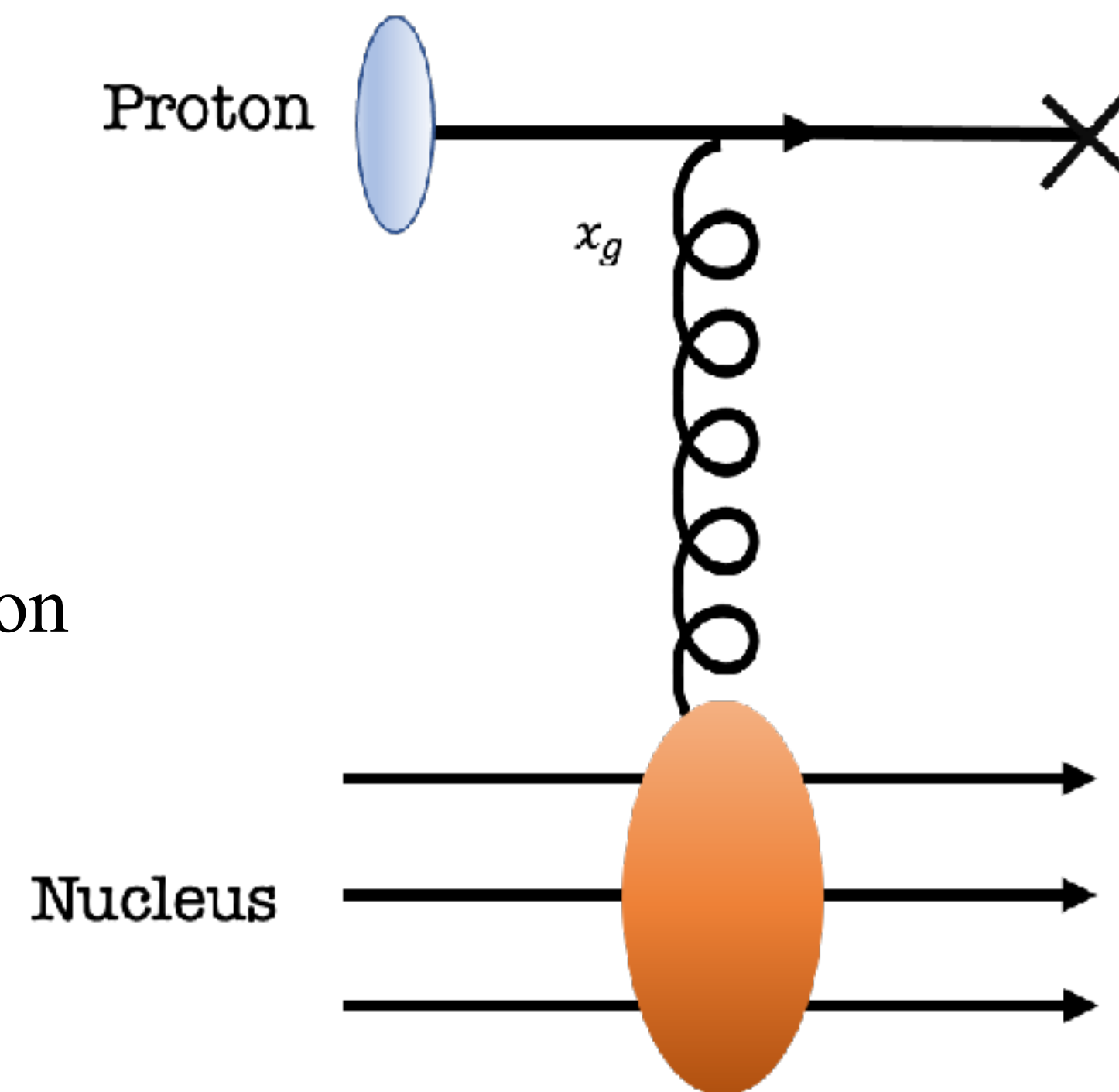


(Dilute)

Small- $x$  gluon! (Dense)

PDFs & FFs  
DGLAP evolution

$\mathcal{F}(k_\perp)$  UGD  
GLR/BK evolution



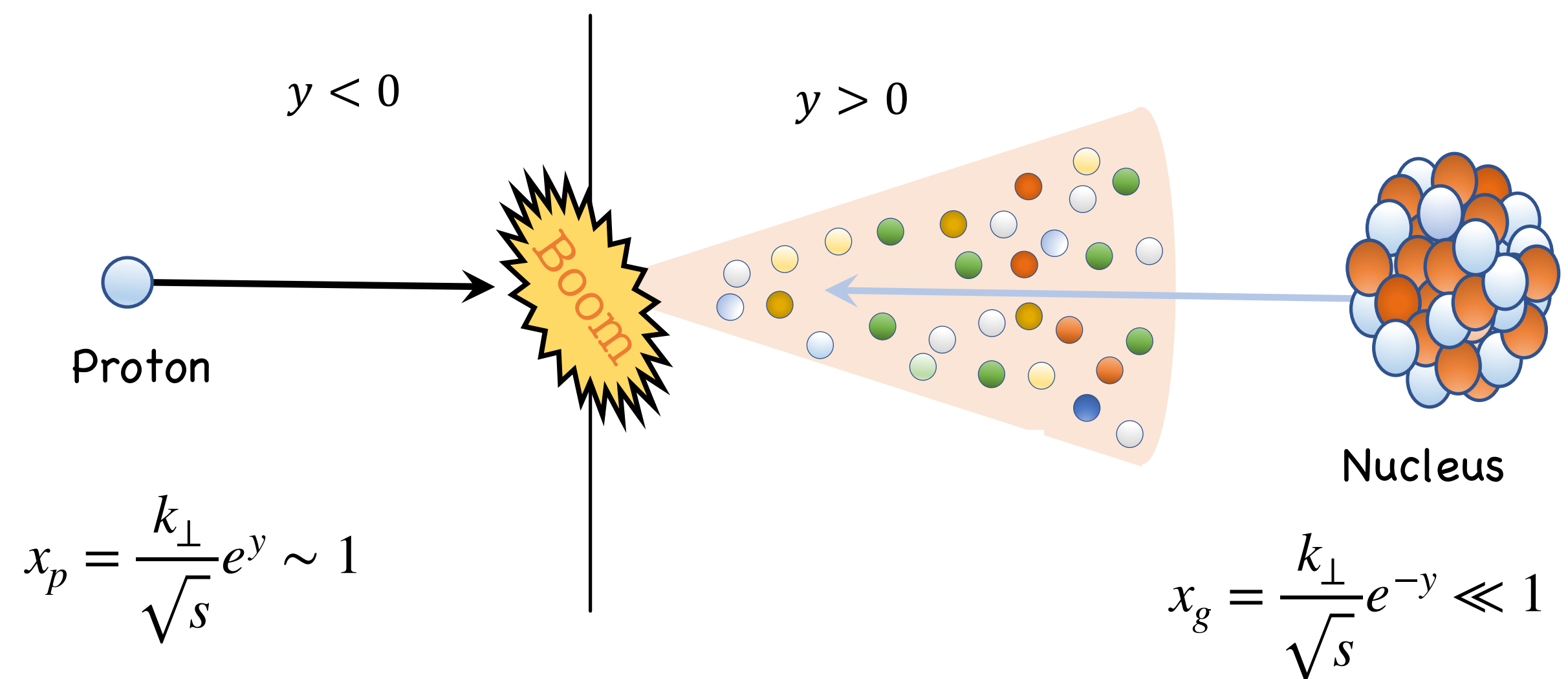
We can study parton non-linear evolution and gluon saturation in the Forward region at pA.

# Forward physics at pA collisions

For the single hadron production, LO cross-section likes

[Dumitru, Jalilian-Marian, PRL, 02]

$$\frac{d\sigma^{pA \rightarrow hX}}{d^2p_{\perp} dy_h} = \int_0^1 \frac{dz}{z^2} \left[ \sum_f x_p q_f(x_p) \mathcal{F}(k_{\perp}) D_{h/q}(z) + x_p g(x_p) \tilde{\mathcal{F}}(k_{\perp}) D_{h/g}(z) \right]$$

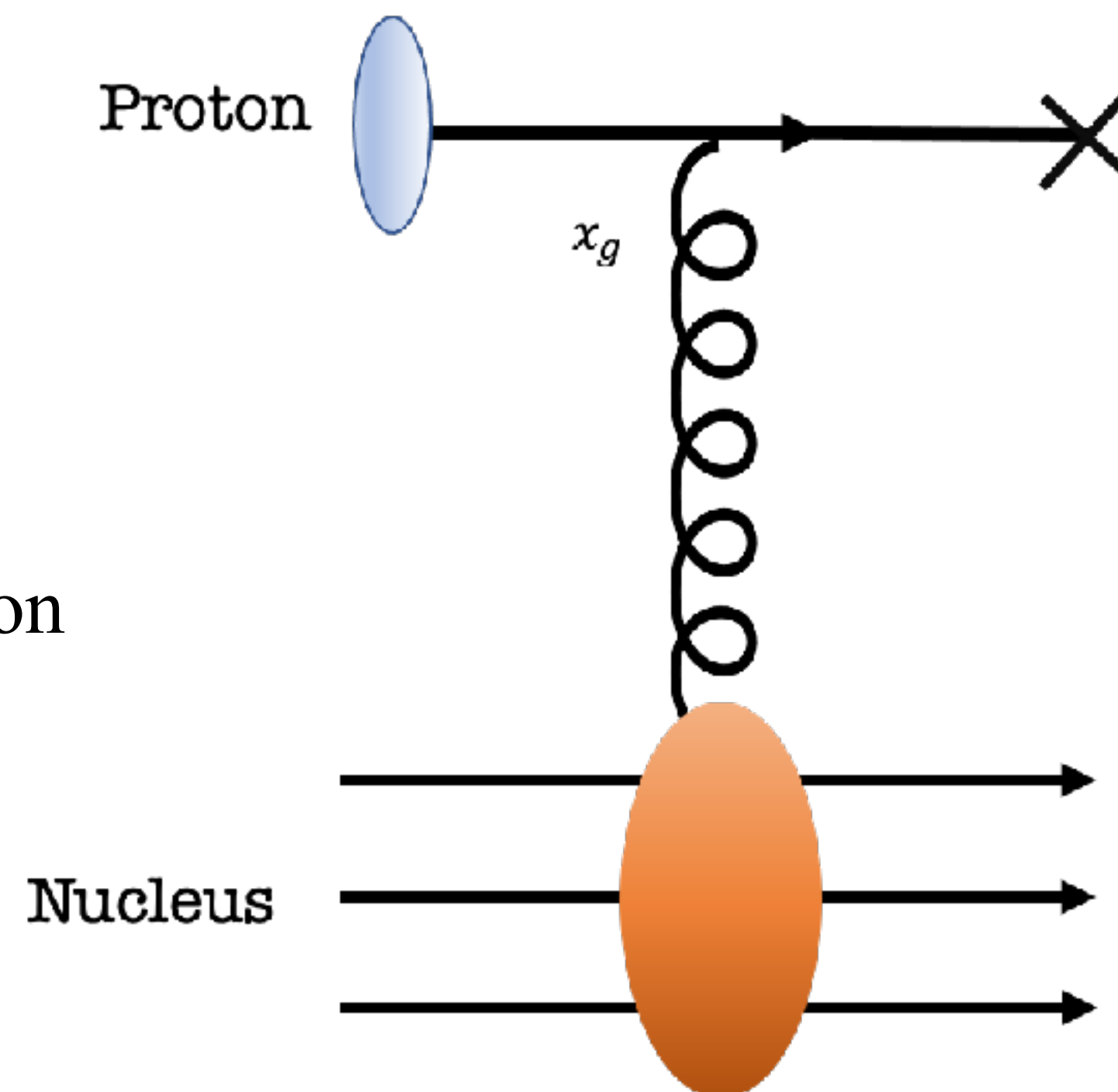


(Dilute)

Small- $x$  gluon! (Dense)

PDFs & FFs  
DGLAP evolution

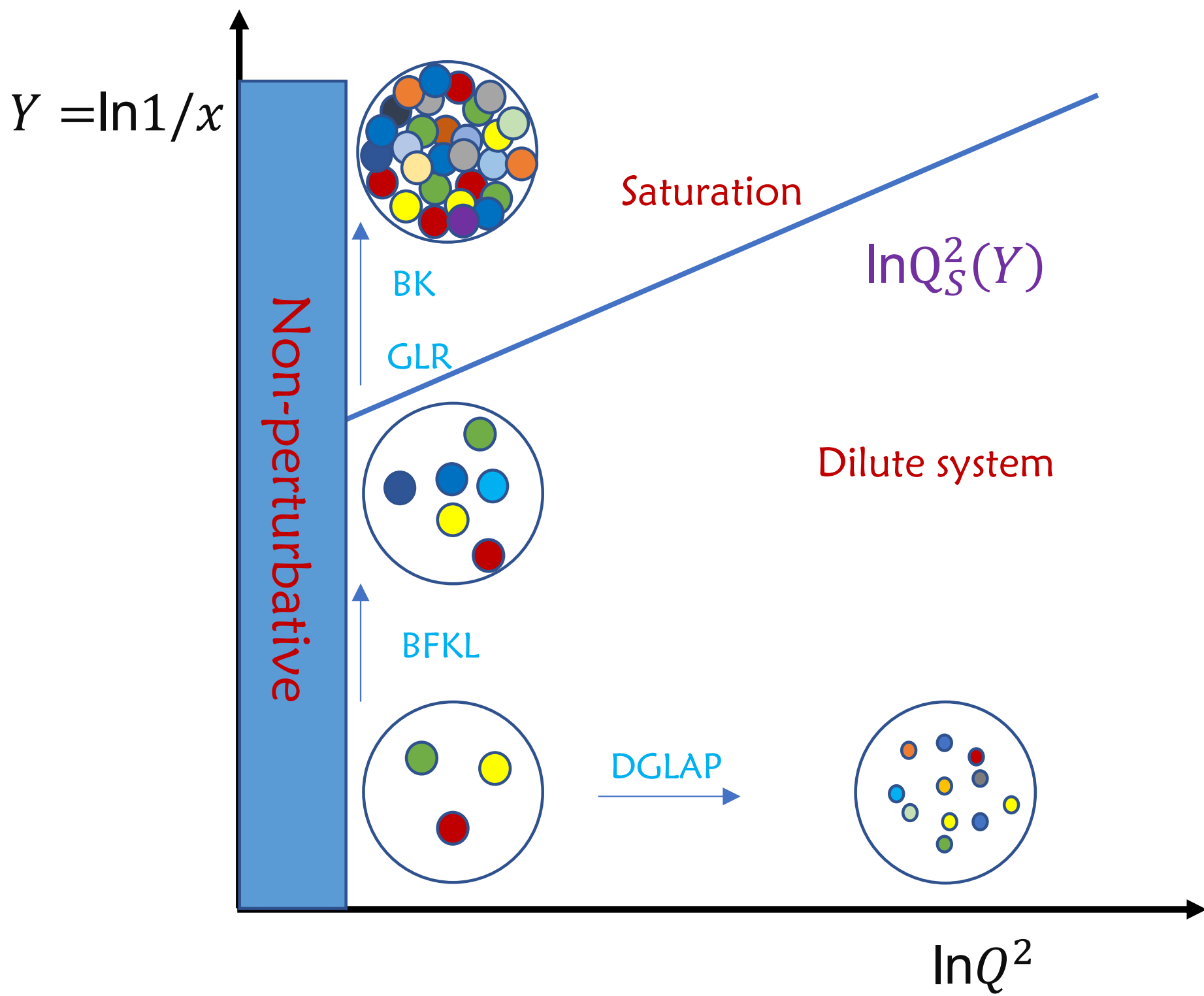
$\mathcal{F}(k_{\perp})$  UGD  
GLR/BK evolution



YS, Wang, Wei, Xiao, Phys. Rev. Lett. 128 (2022) 20, 202302.

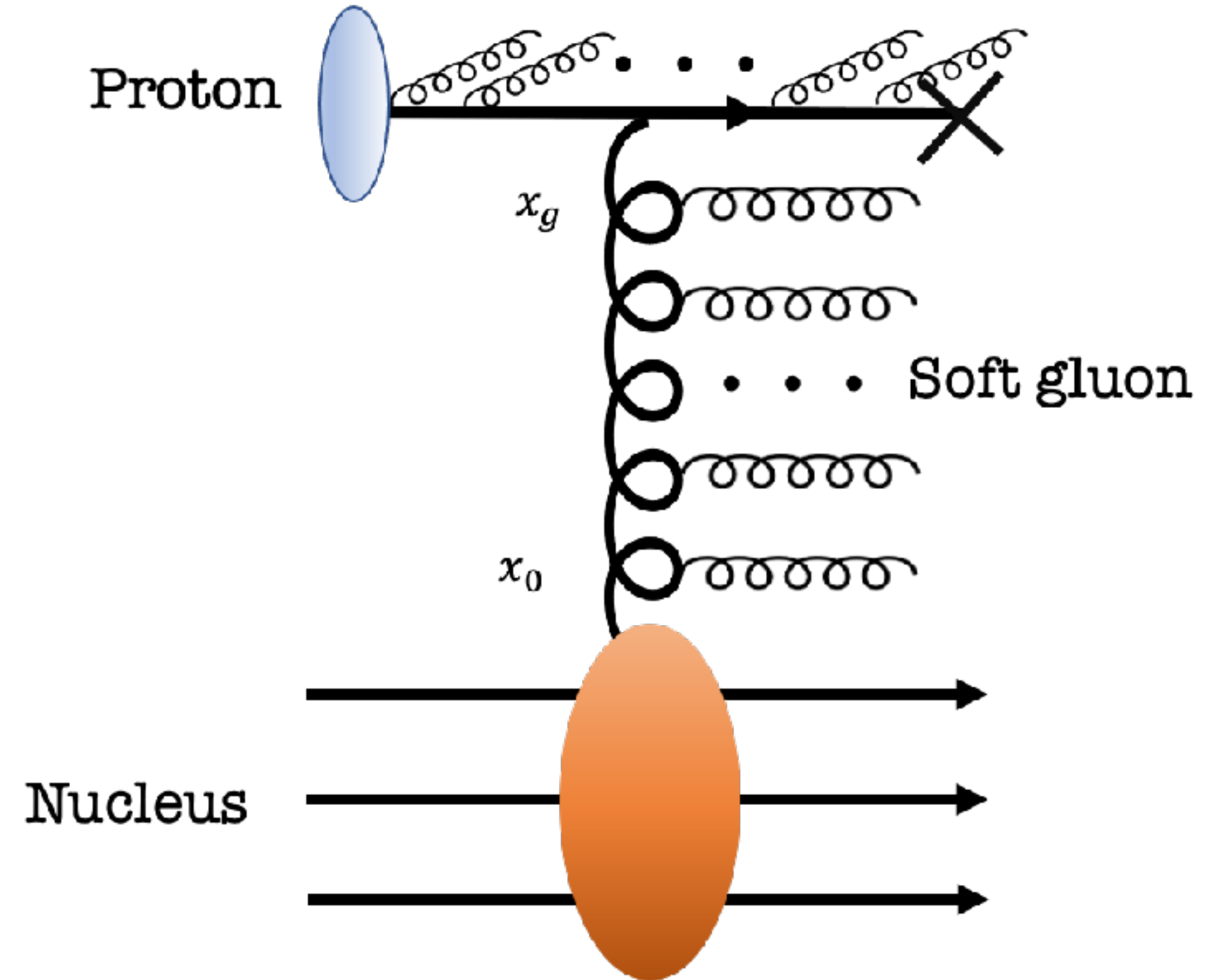
In the forward hadron production, we can describe all the pp, dAu, and pPb data from RHIC to LHC.

# Forward physics at pA collisions

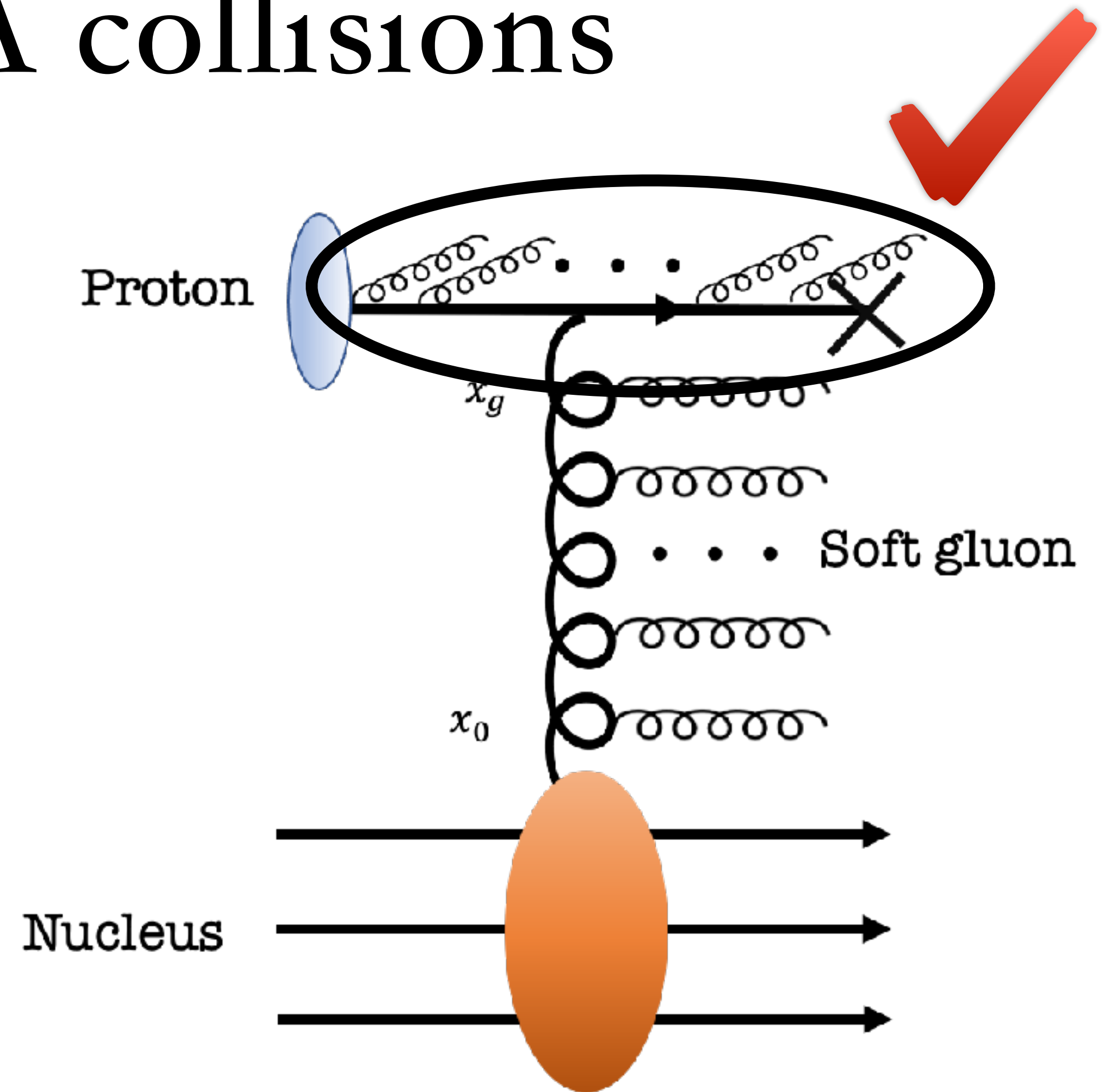
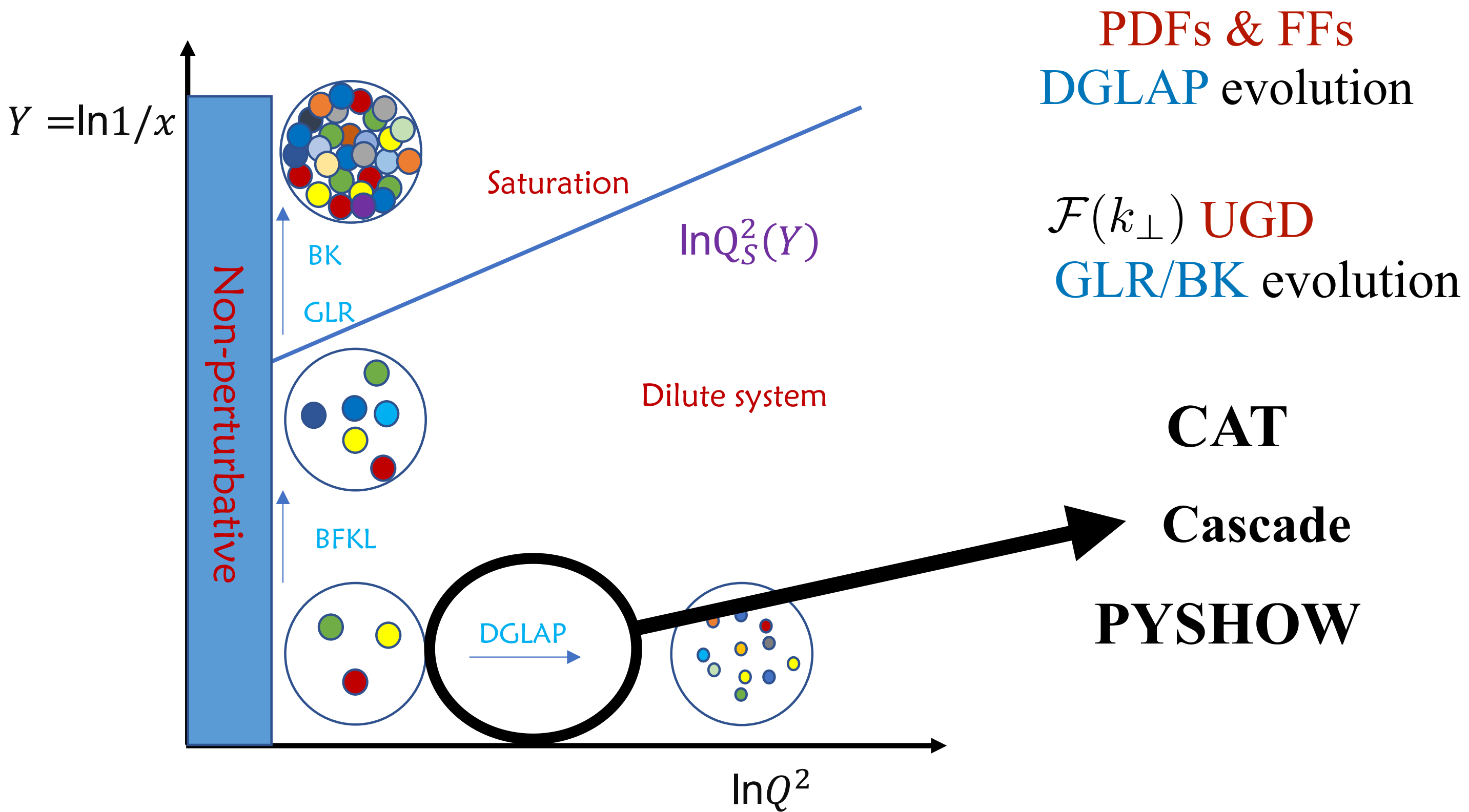


PDFs & FFs  
DGLAP evolution

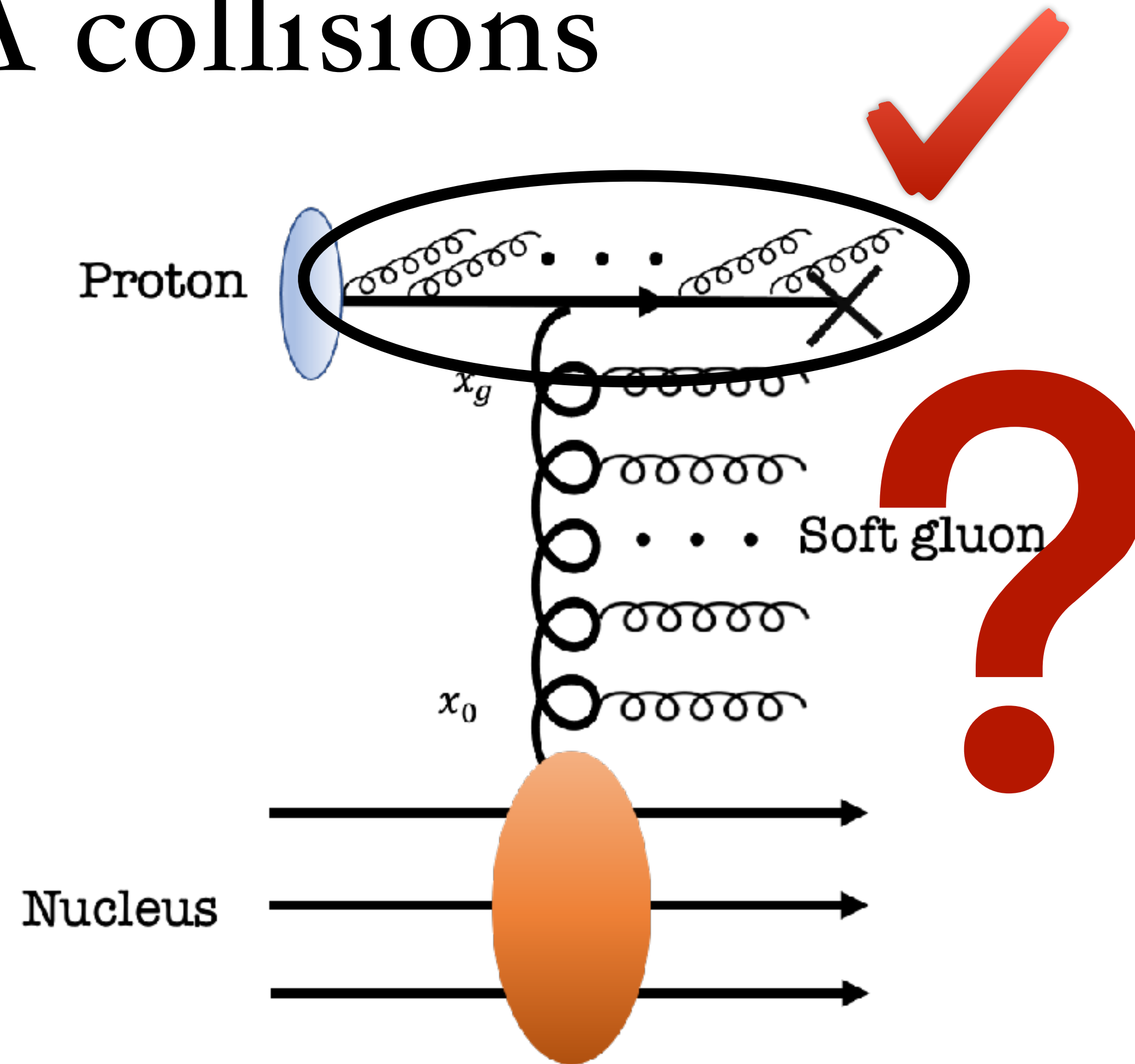
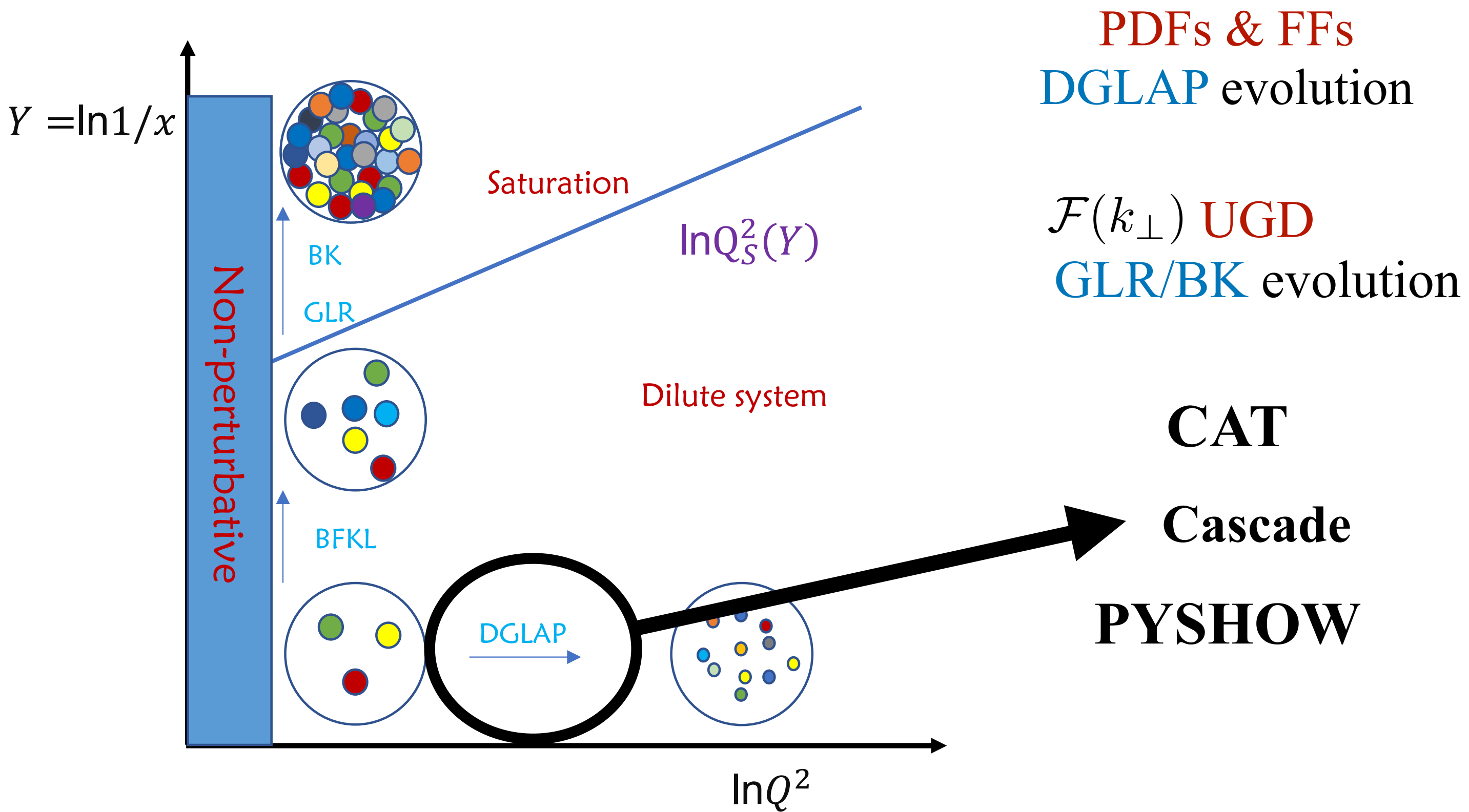
$\mathcal{F}(k_\perp)$  UGD  
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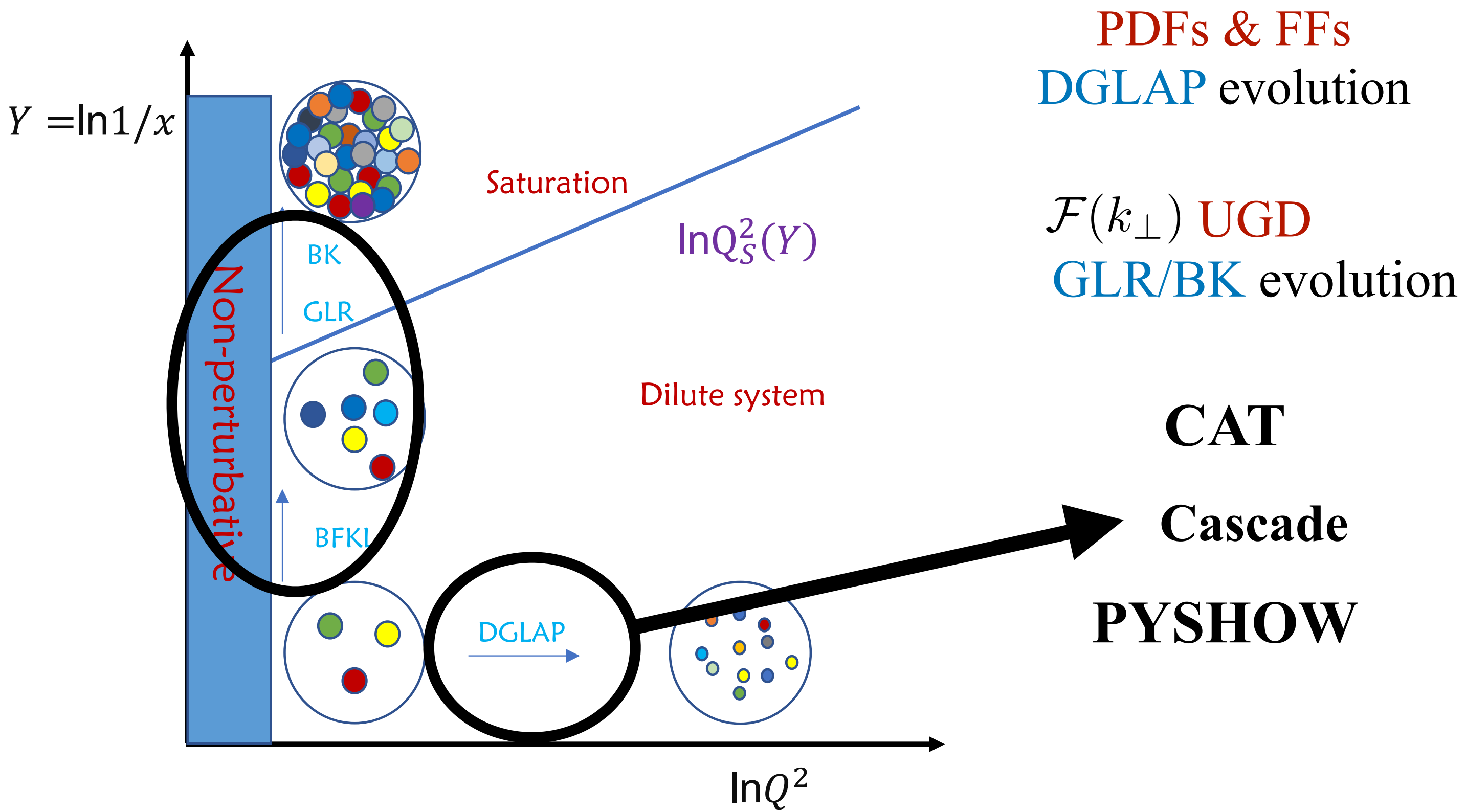
# Forward physics at pA collisions



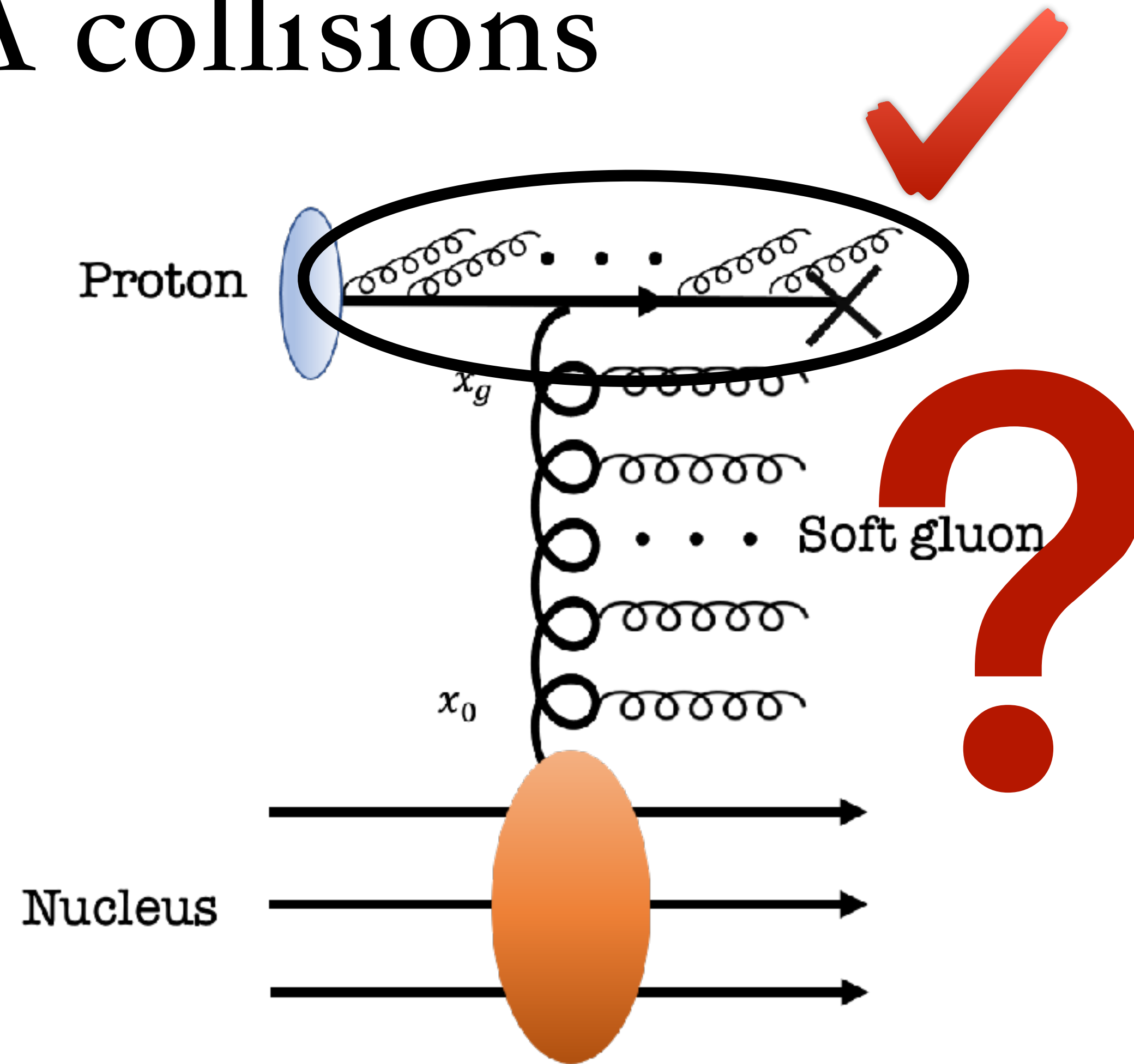
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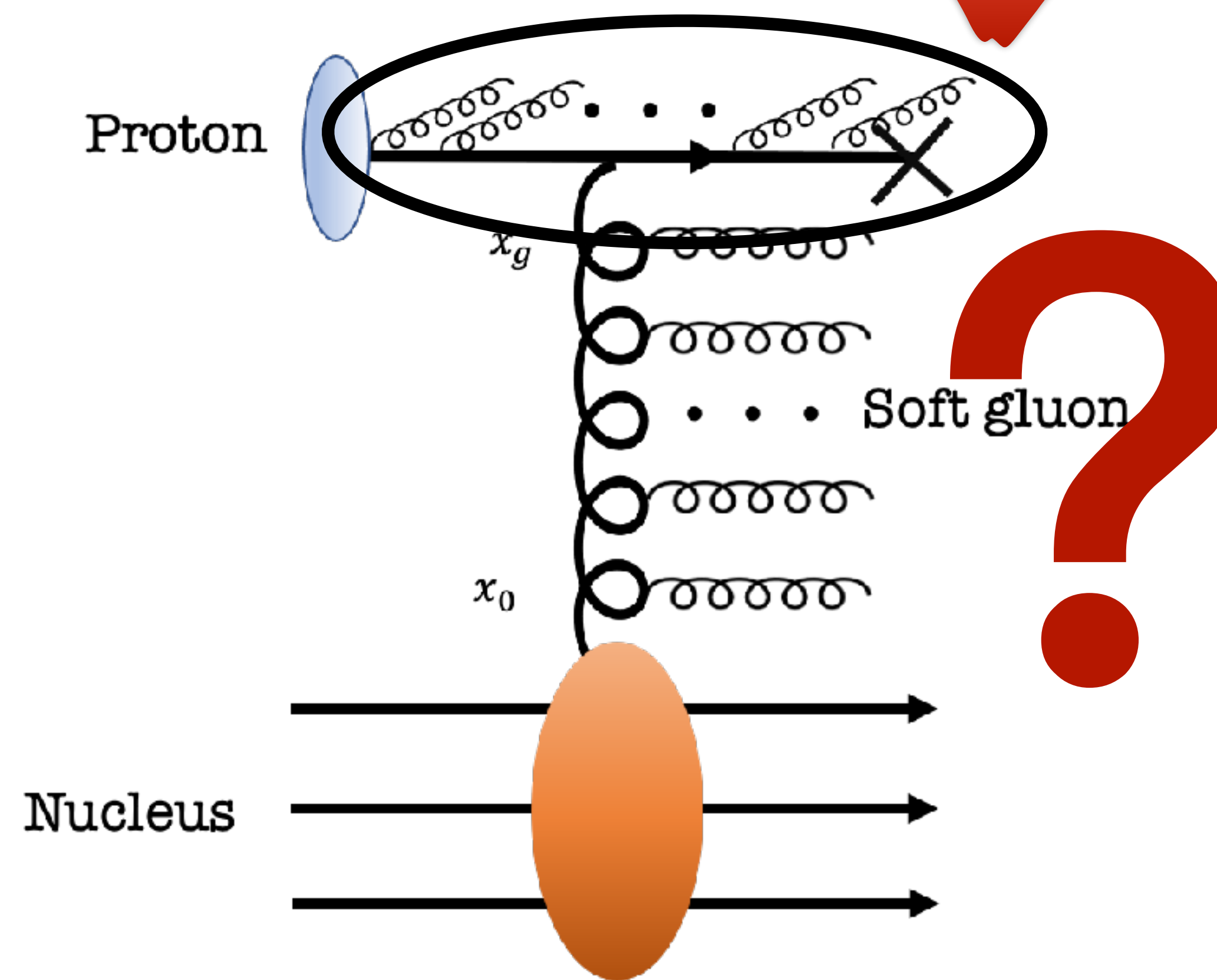
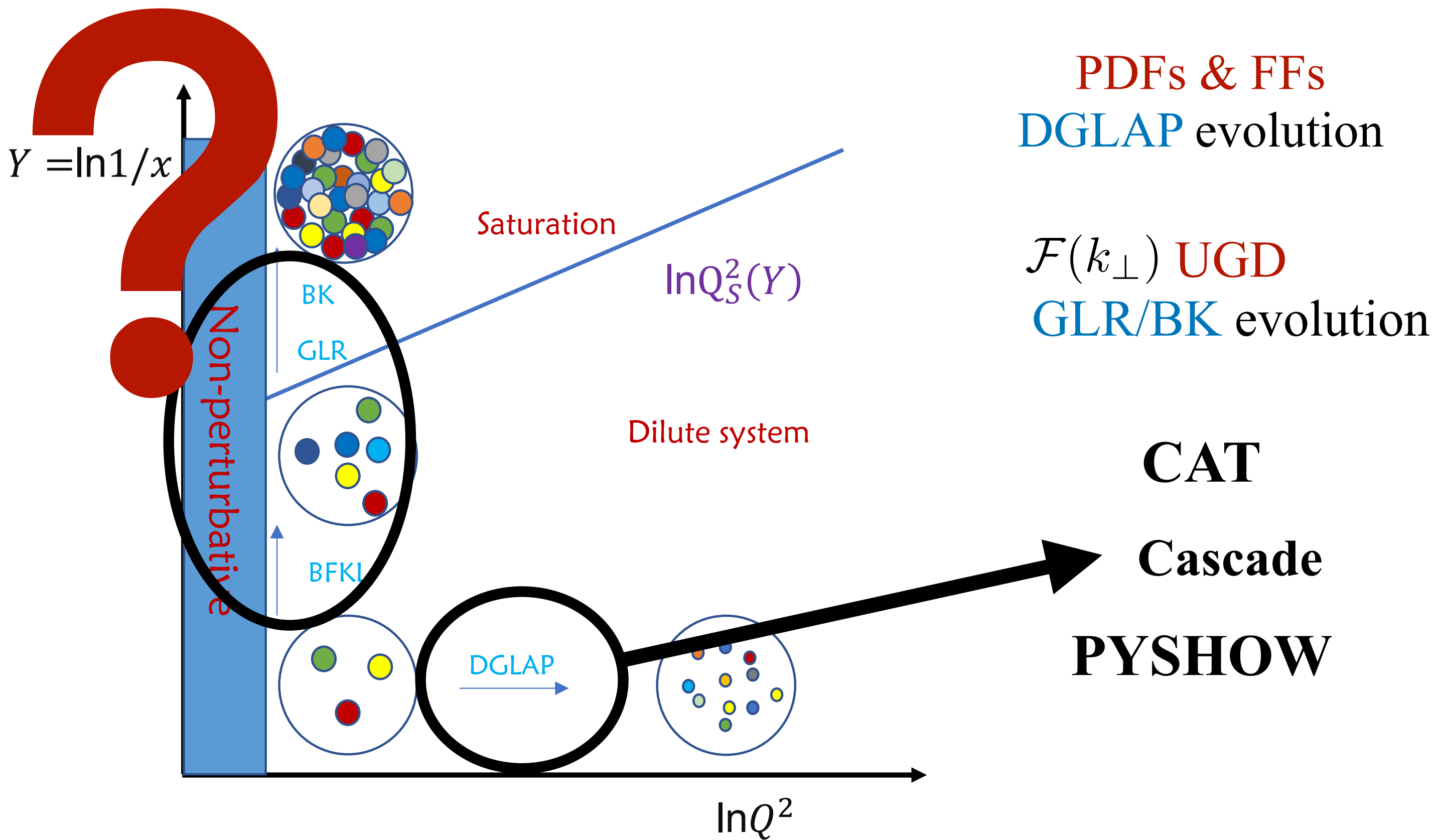
# Forward physics at pA collisions



CAT  
 Cascade  
 PYSHOW



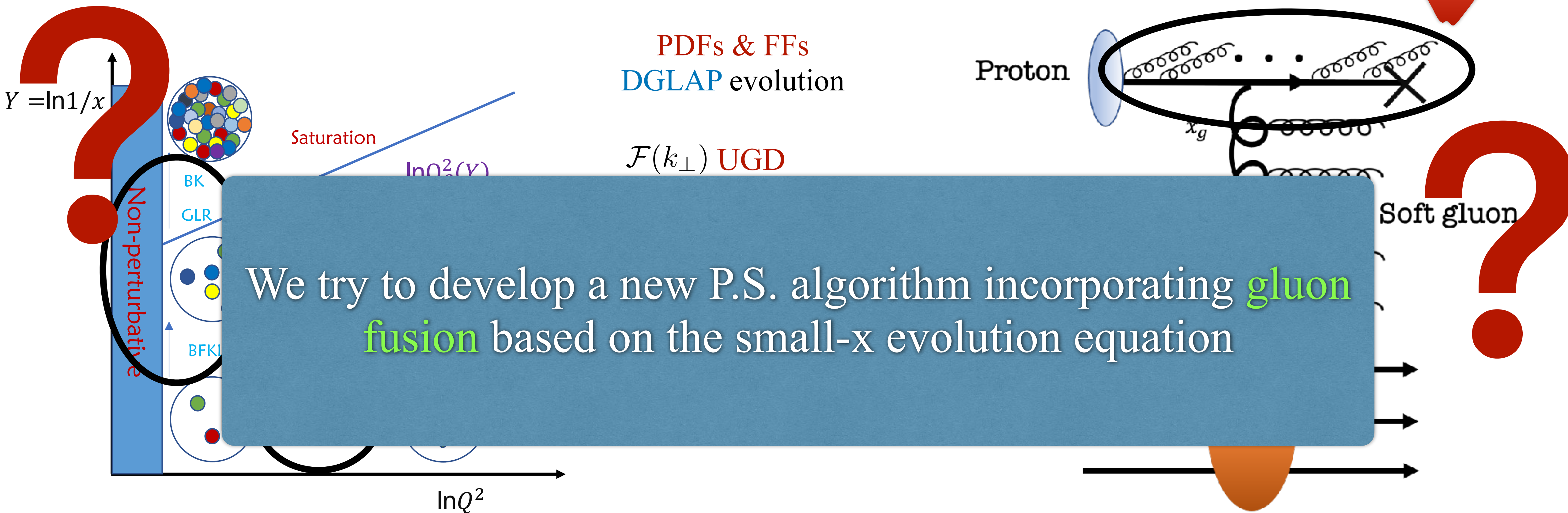
# Forward physics at pA collisions



- gluon fusion effect is absent in all existing generators.
- Developing a P.S. algorithm based on the small-x evolution equation is important.



# Forward physics at pA collisions



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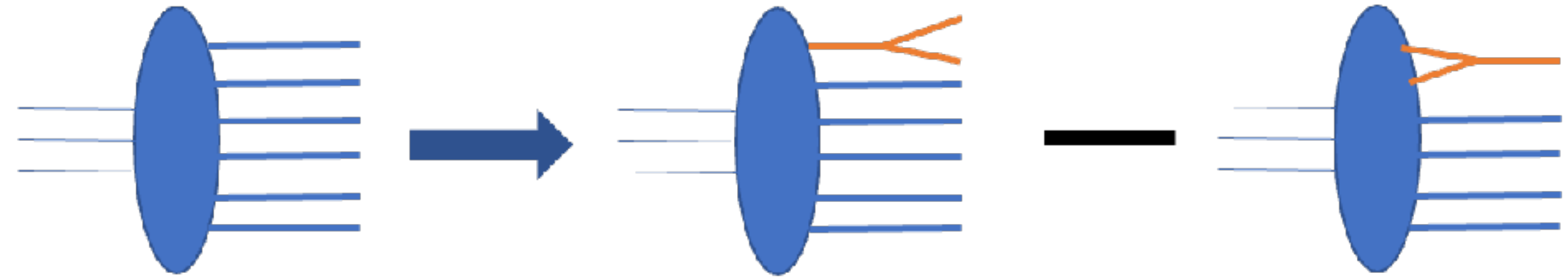
## 2) Novel Parton shower algorithm based on the Gribov-Levin-Ryskin (GLR) equation

YS, S. Y. Wei and J. Zhou, Phys.Rev.D 107, 016017 (2023).

YS, S. Y. Wei and J. Zhou, ArXiv: 2307.04185/hep-ph.

# GLR evolution Equation

$$\eta = \ln \frac{1}{x}$$



● Gluon splitting

● Gluon fusion

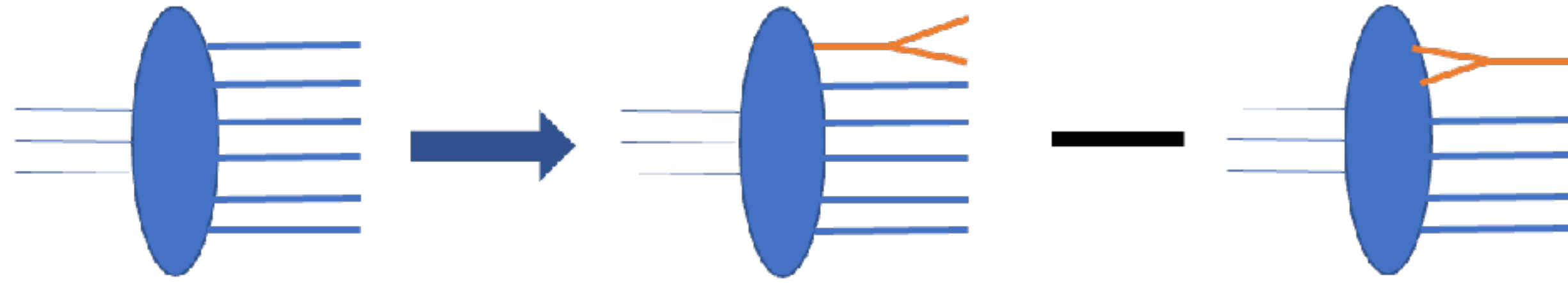
- The standard GLR equation (unfolded one) [Gribov, Levin, Ryskin, PR, 83]

$$\frac{\partial N(\eta, k_{\perp})}{\partial \eta} = \frac{\bar{\alpha}_s}{\pi} \left[ \int \frac{d^2 l_{\perp}}{l_{\perp}^2} N(\eta, k_{\perp} + l_{\perp}) - \int_0^{k_{\perp}} \frac{d^2 l_{\perp}}{l_{\perp}^2} N(\eta, k_{\perp}) \right] - \bar{\alpha}_s N^2(\eta, k_{\perp})$$

- GLR equation is the non-linear evolution equation that describes the gluon diffusion process.

# GLR evolution Equation

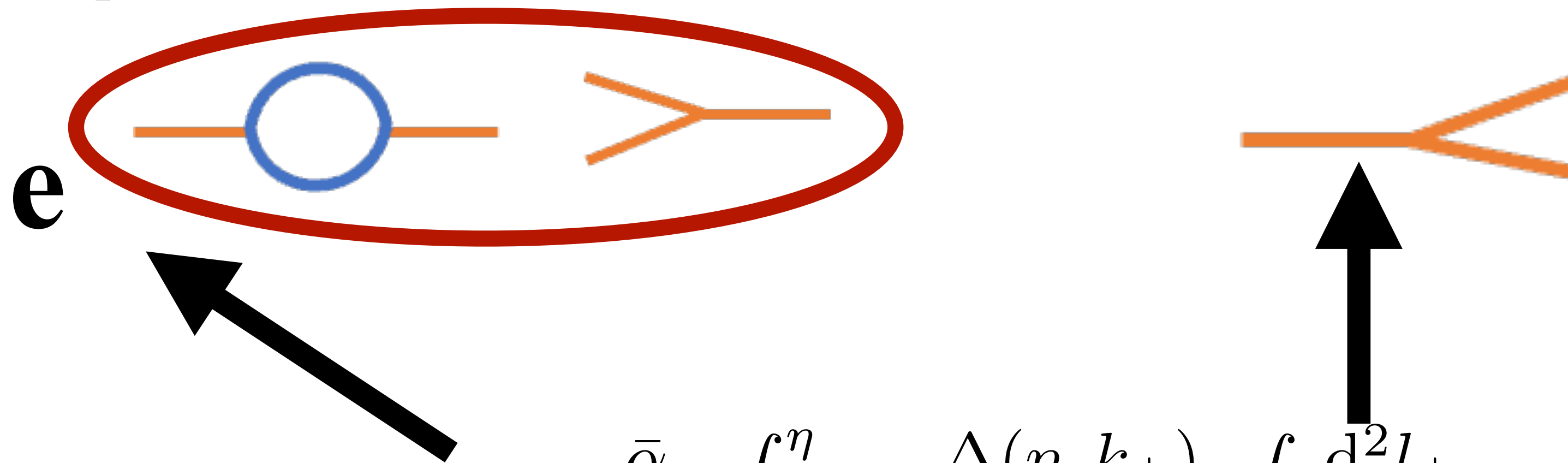
$$\eta = \ln \frac{1}{x}$$



● Gluon splitting

● Gluon fusion

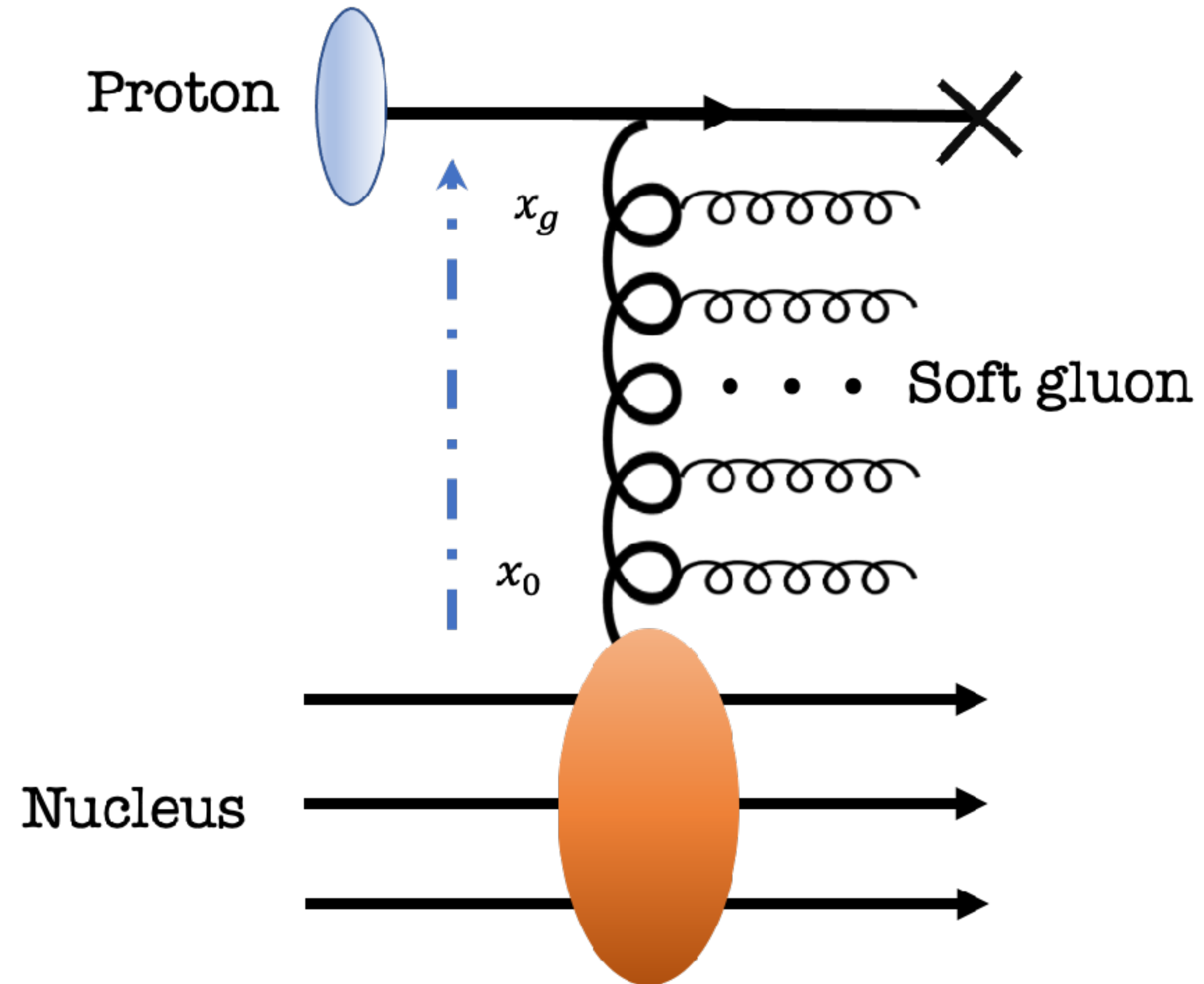
- Non-Sudakov form factor  $\Delta(\eta, k_{\perp}) = \exp \left\{ -\bar{\alpha}_s \int_{\eta_0}^{\eta} d\eta' \left[ \ln \frac{k_{\perp}^2}{\mu^2} + N(\eta', k_{\perp}) \right] \right\}$
- The integral GLR equation (folded one)



$$N(\eta, k_{\perp}) = N(\eta_0, k_{\perp}) \Delta(\eta, k_{\perp}) + \frac{\bar{\alpha}_s}{\pi} \int_{\eta_0}^{\eta} d\eta' \frac{\Delta(\eta, k_{\perp})}{\Delta(\eta', k_{\perp})} \int_{\mu} \frac{d^2 l_{\perp}}{l_{\perp}^2} N(\eta', l_{\perp} + k_{\perp})$$

- GLR equation is the non-linear evolution equation that describes the gluon diffusion process.

# The forward evolution



# The forward evolution

First step: non-Sudakov form factor

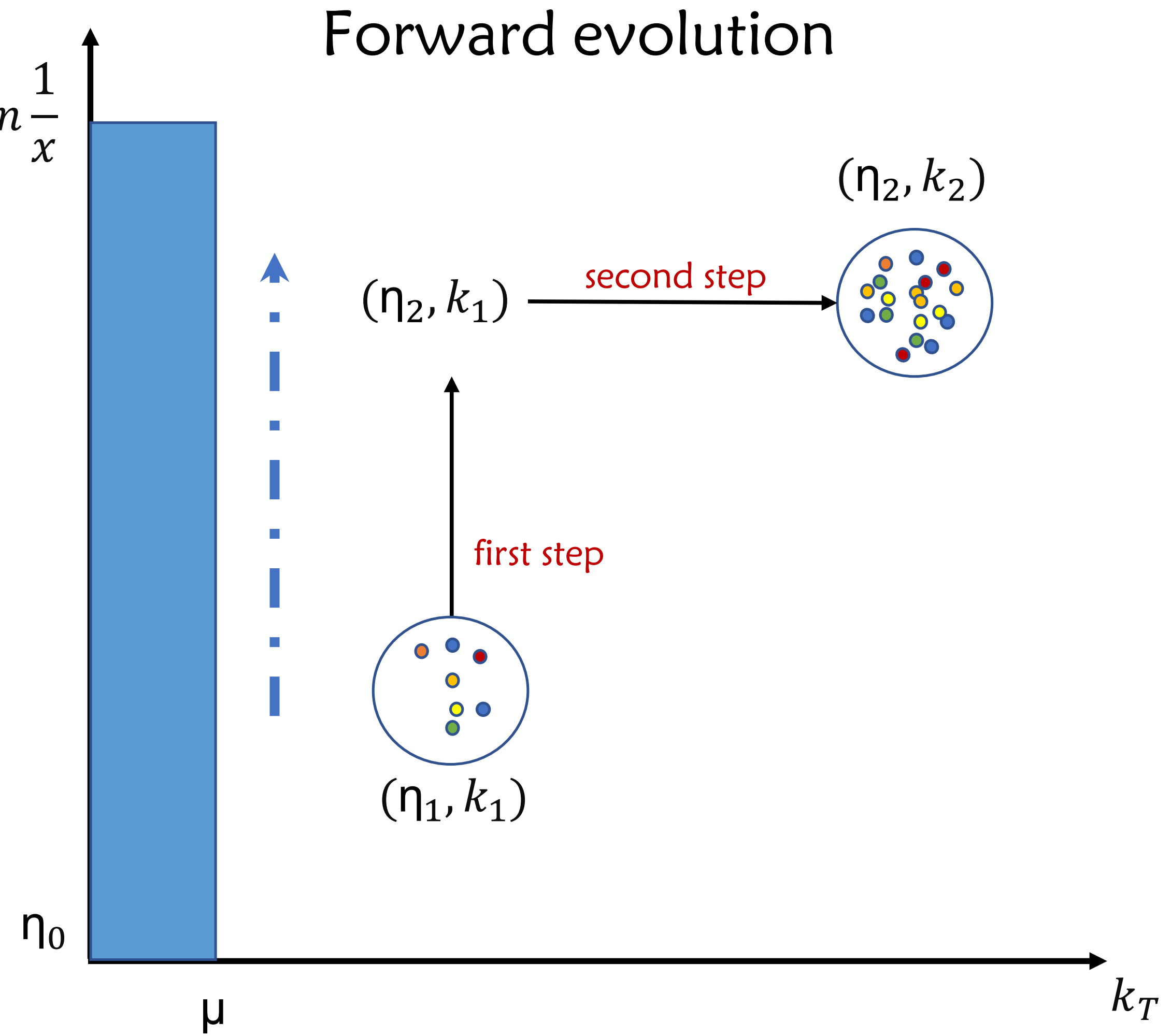
$$\mathcal{R} = \exp \left[ -\bar{\alpha}_s \int_{\eta_i}^{\eta_{i+1}} d\eta' \left( \ln \frac{k_{\perp}^2}{\mu^2} + N(\eta', k_{\perp}) \right) \right] \quad \eta = \ln \frac{1}{x}$$

Second step: Real splitting kernel

$$\mathcal{R}_2 \int_{\mu}^{P_{\perp}} \frac{d^2 l'_{\perp}}{l'_{\perp}{}^2} = \int_{\mu}^{|l_{\perp}|} \frac{d^2 l'_{\perp}}{l'_{\perp}{}^2}$$

The generated event has to be re-weighted

$$\mathcal{W}(\eta_i, \eta_{i+1}; k_{\perp, i}) = \frac{\int_{\eta_i}^{\eta_{i+1}} d\eta \ln(P_{\perp}^2 / \mu^2)}{\int_{\eta_i}^{\eta_{i+1}} d\eta \left[ \ln(k_{\perp, i}^2 / \mu^2) + N(\eta, k_{\perp, i}) \right]}$$



# The forward evolution

First step: non-Sudakov form factor

$$\mathcal{R} = \exp \left[ -\bar{\alpha}_s \int_{\eta_i}^{\eta_{i+1}} d\eta' \left( \ln \frac{k_{\perp}^2}{\mu^2} + N(\eta', k_{\perp}) \right) \right]$$

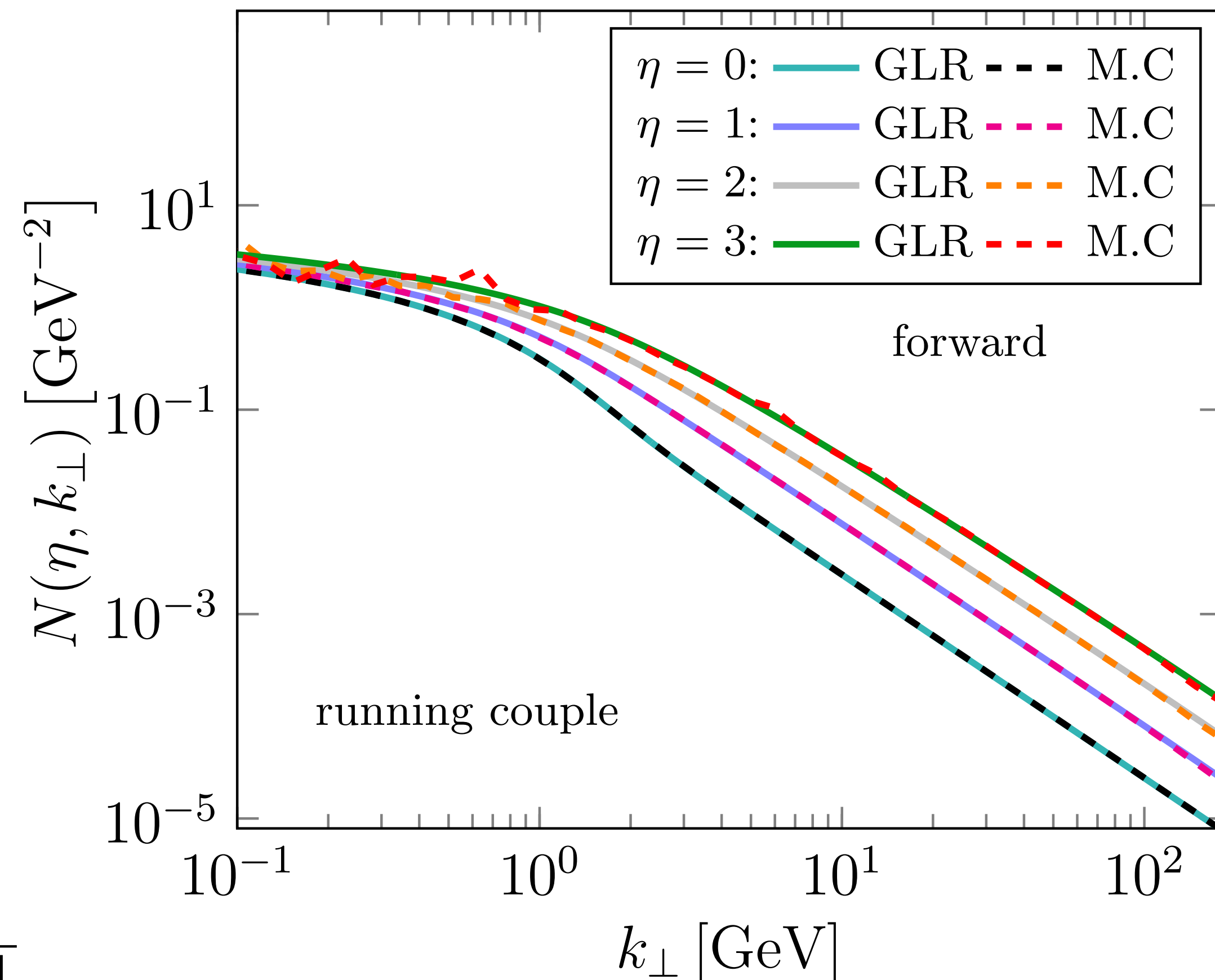
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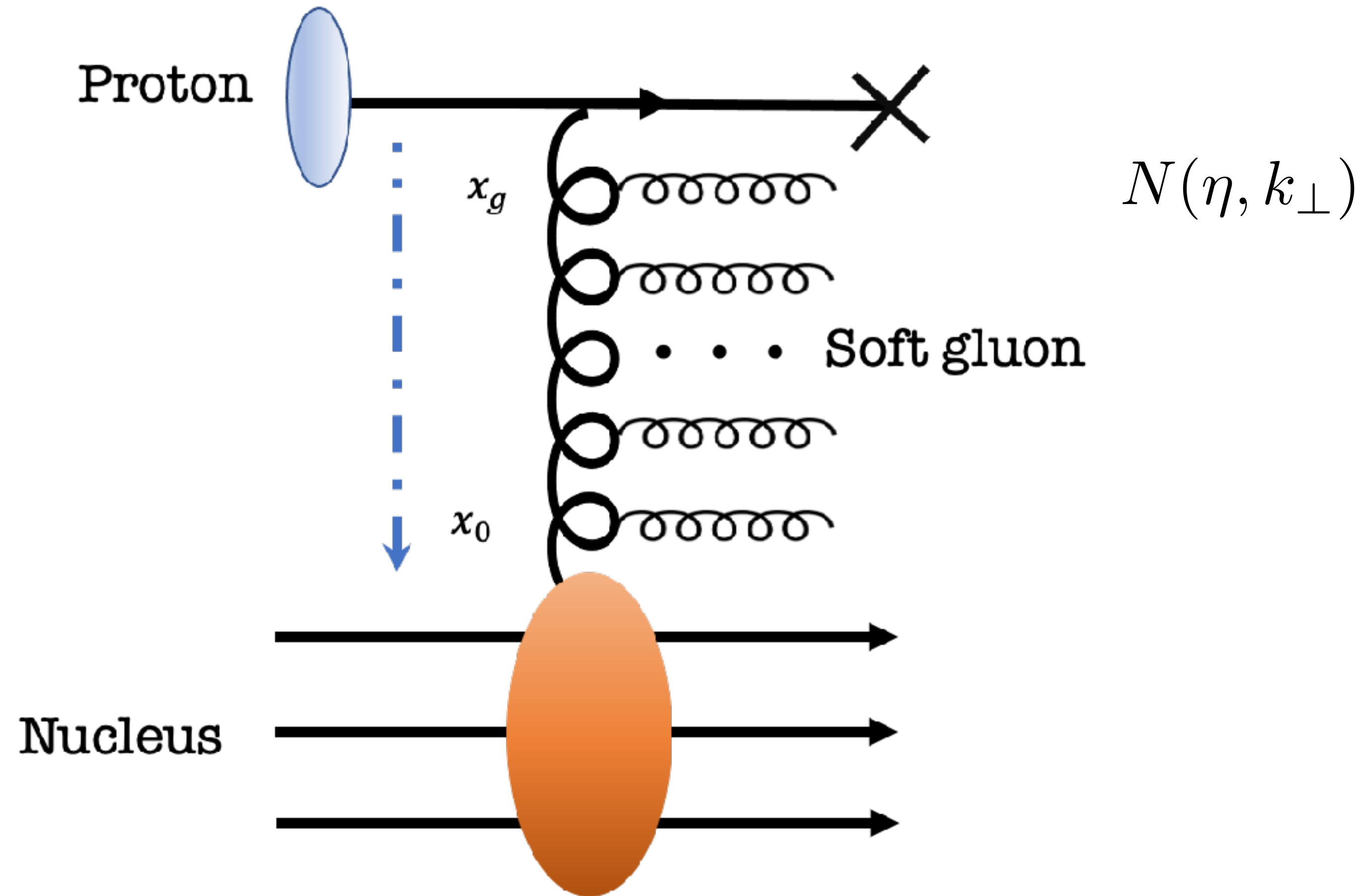
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- Agree with the numerical solutions of the GLR equation.



# The backward evolution



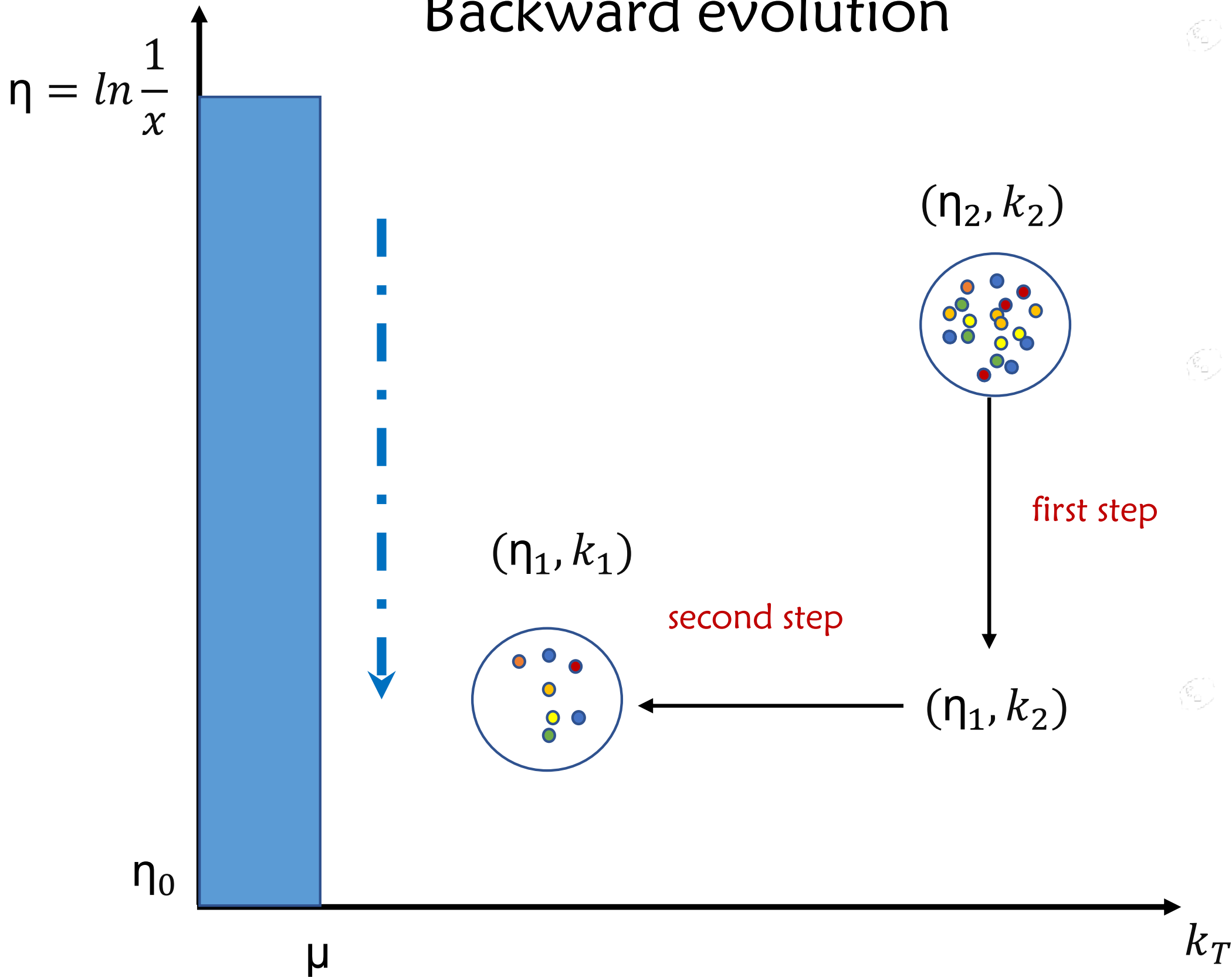
- As a more efficient procedure, the backward evolution approach is also presented.



# The backward evolution



Backward evolution



- First step: backward non-Sudakov form factor

$$\mathcal{R} = \exp \left[ -\frac{\bar{\alpha}_s}{\pi} \int_{\eta_i}^{\eta_{i+1}} d\eta \int_{\mu} \frac{d^2 l_{\perp}}{l_{\perp}^2} \frac{N(\eta, k_{\perp, i+1} + l_{\perp})}{N(\eta, k_{\perp, i+1})} \right]$$

- Second step: Real splitting

$$\frac{\bar{\alpha}_s}{\pi} \int_{\mu}^{l_{\perp}} \frac{d^2 l'_{\perp}}{l'_{\perp}{}^2} N(\eta_i, k_{\perp, i+1} + l'_{\perp}) = \mathcal{R}_2 \frac{\bar{\alpha}_s}{\pi} \int_{\mu}^{P_{\perp}} \frac{d^2 l'_{\perp}}{l'_{\perp}{}^2} N(\eta_i, k_{\perp, i+1} + l'_{\perp})$$

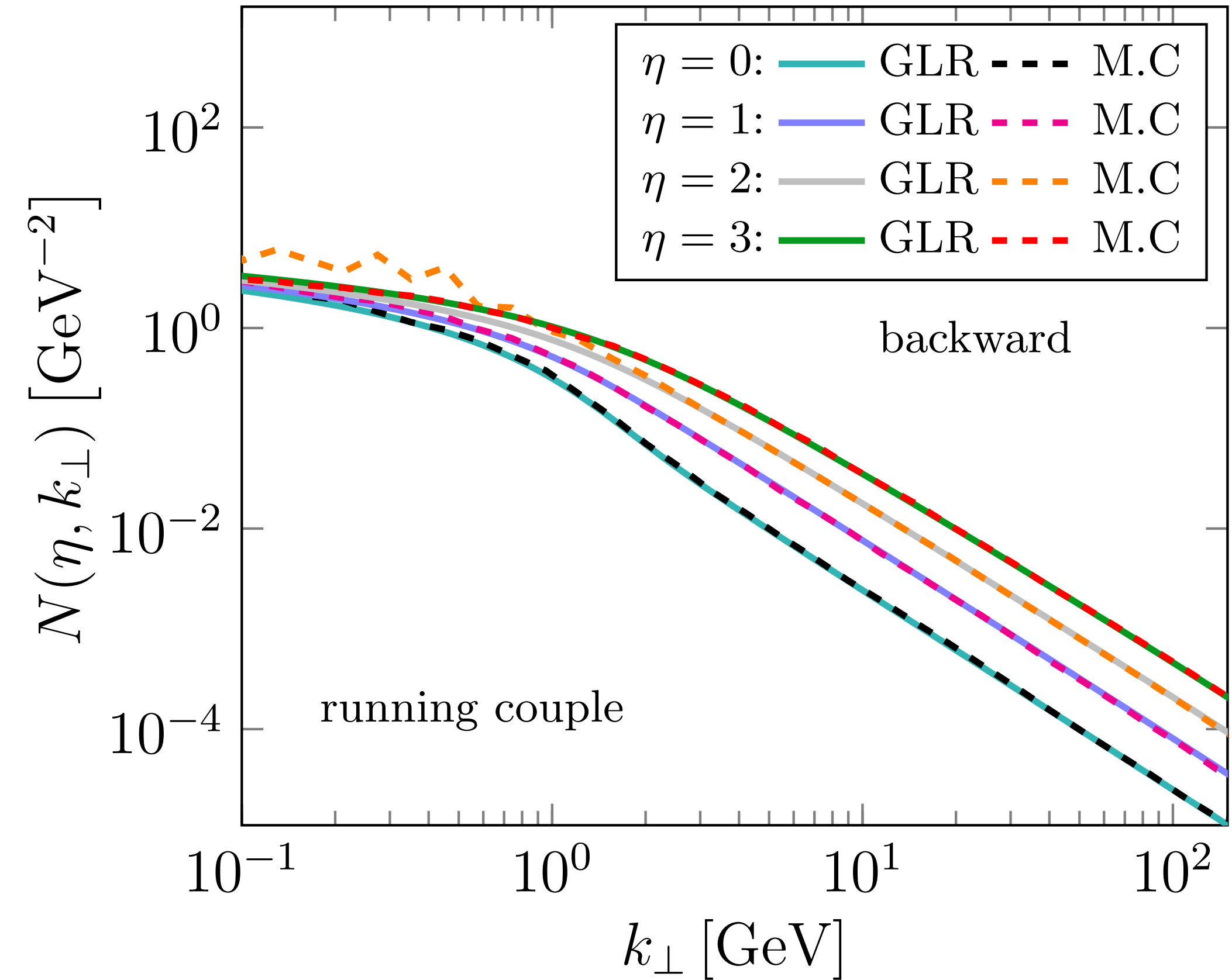
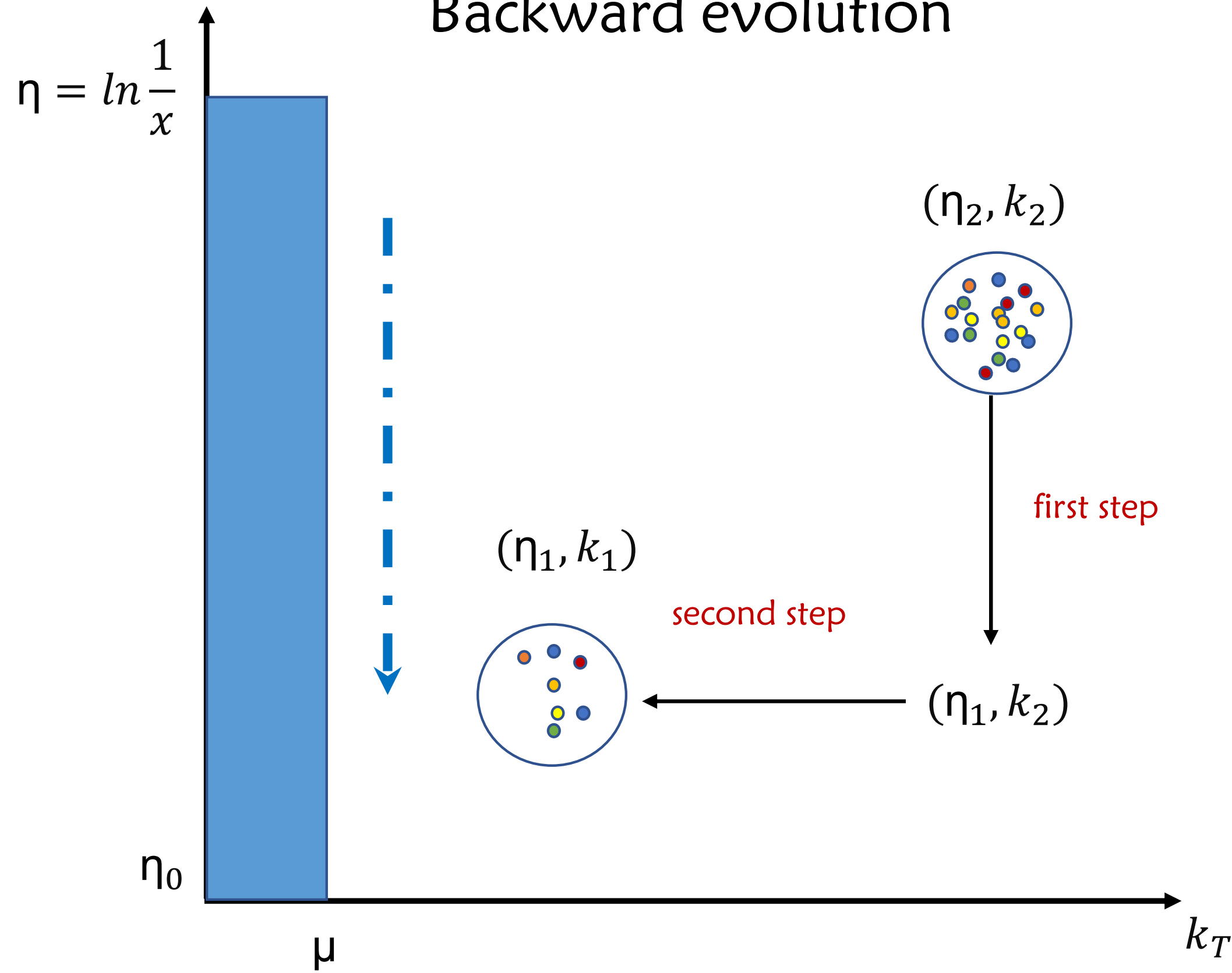
- The generated event has to be re-weighted

$$\mathcal{W}_{\text{back}}(\eta_{i+1}, \eta_i; k_{\perp, i+1}, k_{\perp, i}) = \frac{\int_{\eta_i}^{\eta_{i+1}} d\eta \left[ \ln(k_{\perp, i}^2 / \mu^2) + N(\eta, k_{\perp, i}) \right]}{\int_{\eta_i}^{\eta_{i+1}} d\eta \ln(P_{\perp}^2 / \mu^2)}$$

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# The backward evolution

Backward evolution



- As a more efficient procedure, the backward evolution approach is also presented.
- Agree with the numerical solutions of the GLR equation.

# Parton shower

GLR

v.s.

DGLAP

gluon splitting  
gluon fusion

parton splitting

The evolution variable:

$$\eta = \ln(1/x)$$

$Q$

The generated event:

reweight

Unitary

# Summary and outlook

- The first parton shower algorithm incorporating gluon fusion is based on the GLR evolution equation.
- Di-jet in EIC

Thank you !