A novel Parton shower algorithm based on the small-x evolution equation

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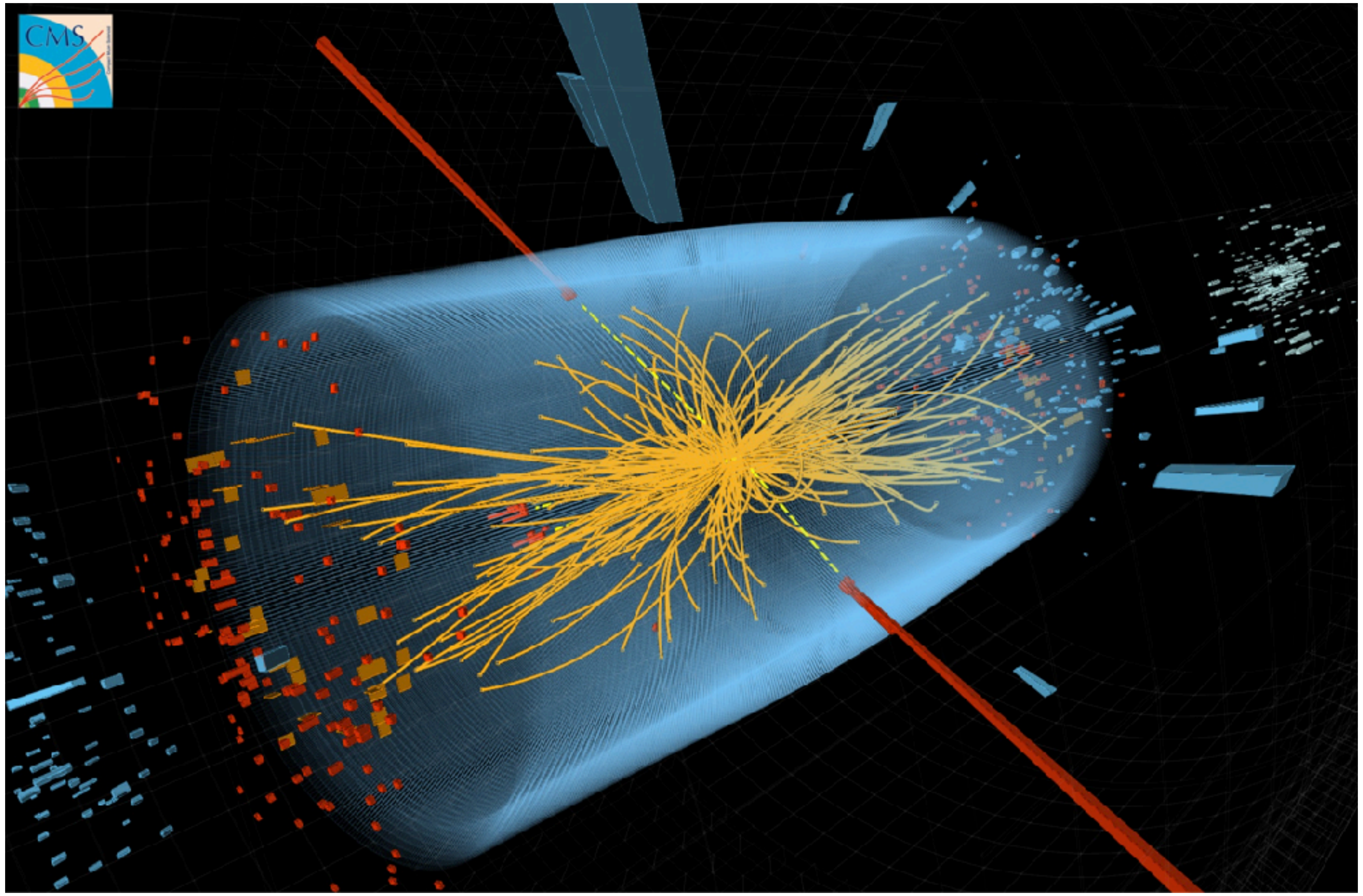
- YS, Shu-yi Wei and Jian Zhou, Phys.Rev.D 107, 016017 (2023).
- YS, Shu-yi Wei and Jian Zhou, arXiv: 2307.04185/hep-ph.

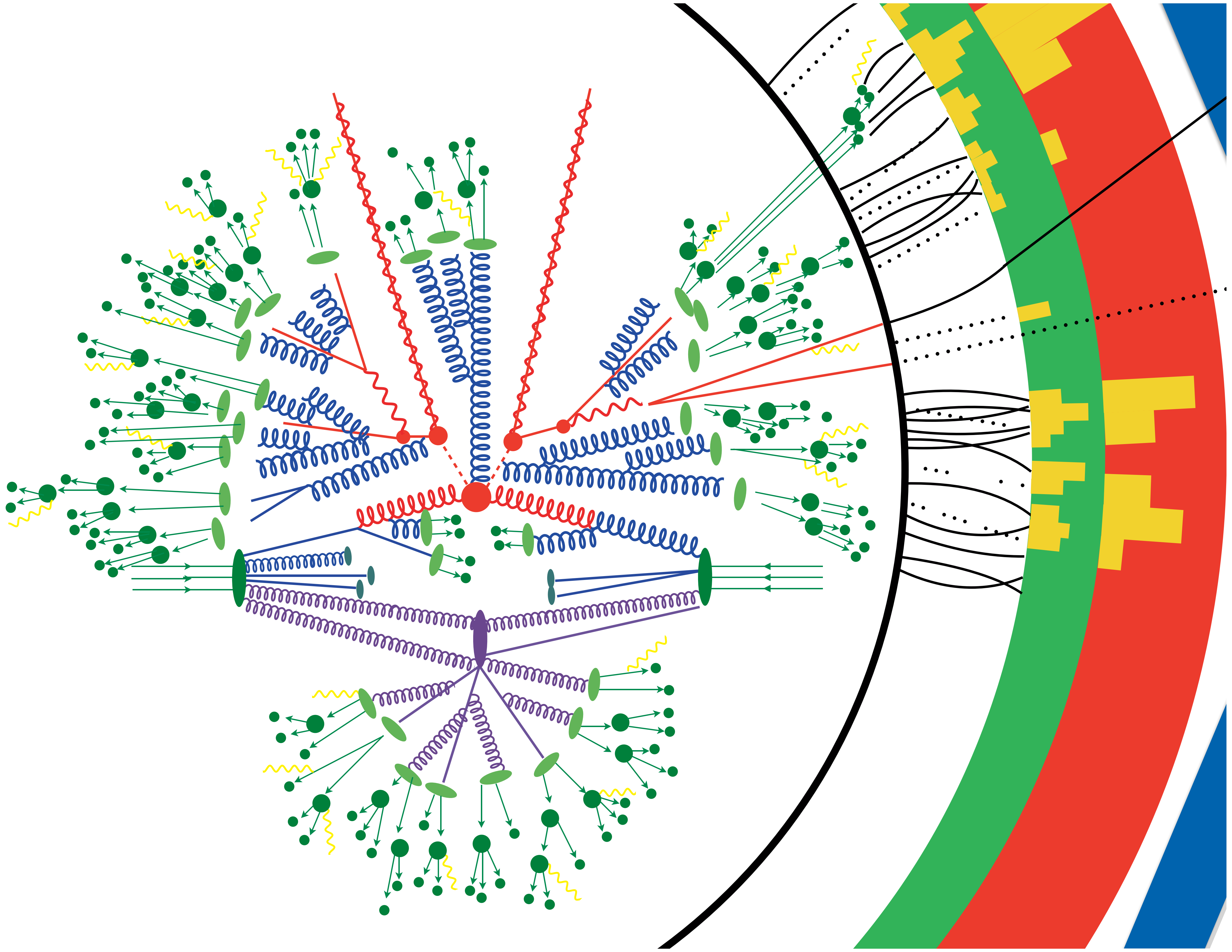
QCD与中高能核物理暑期学校
2023.07.12

Outline

1) Introduction
2) Parton shower algorithm based on the Gribov-Levin-Ryskin evolution equation
3) Summary

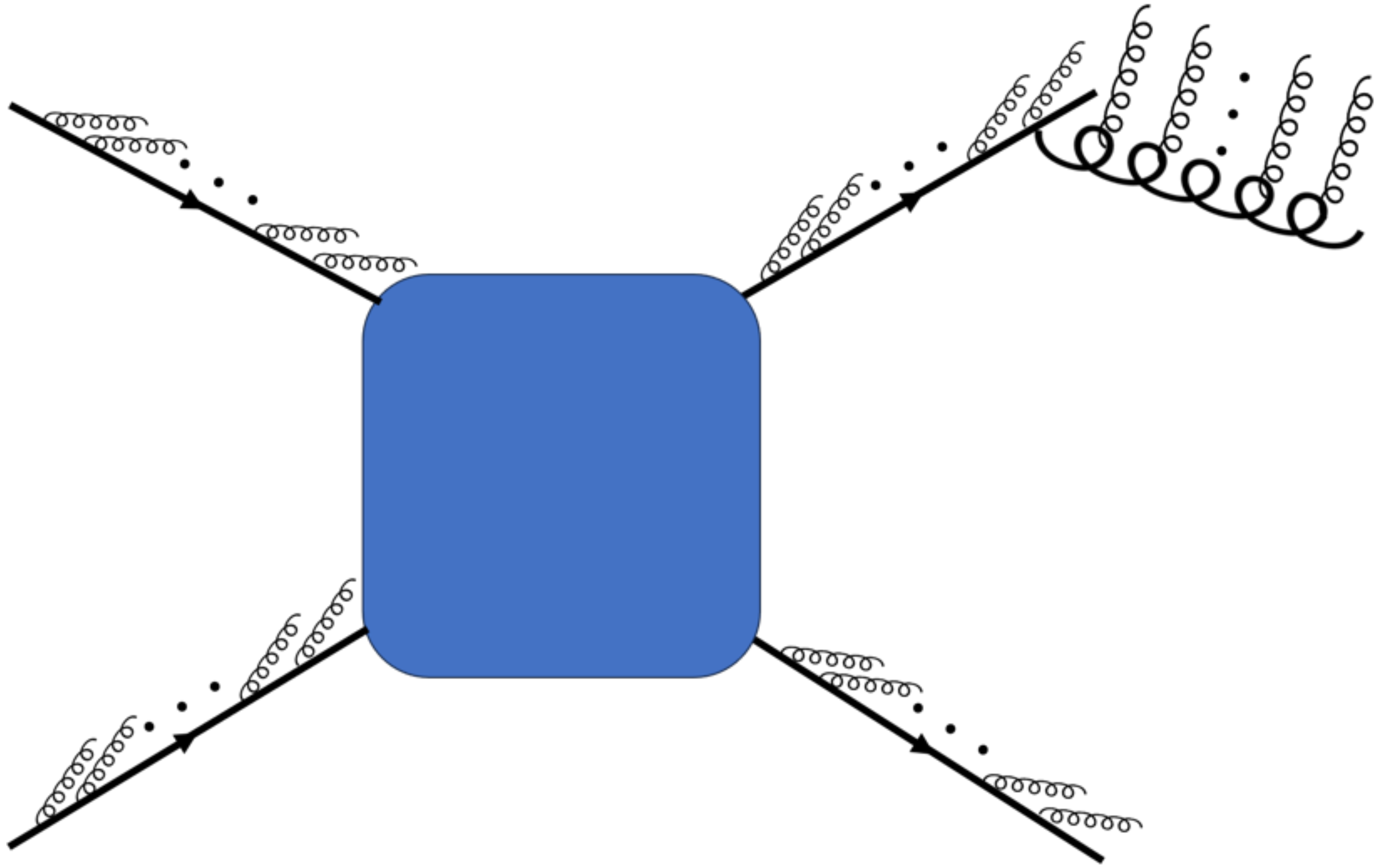
Monte Carlo event generator in high energy collisions





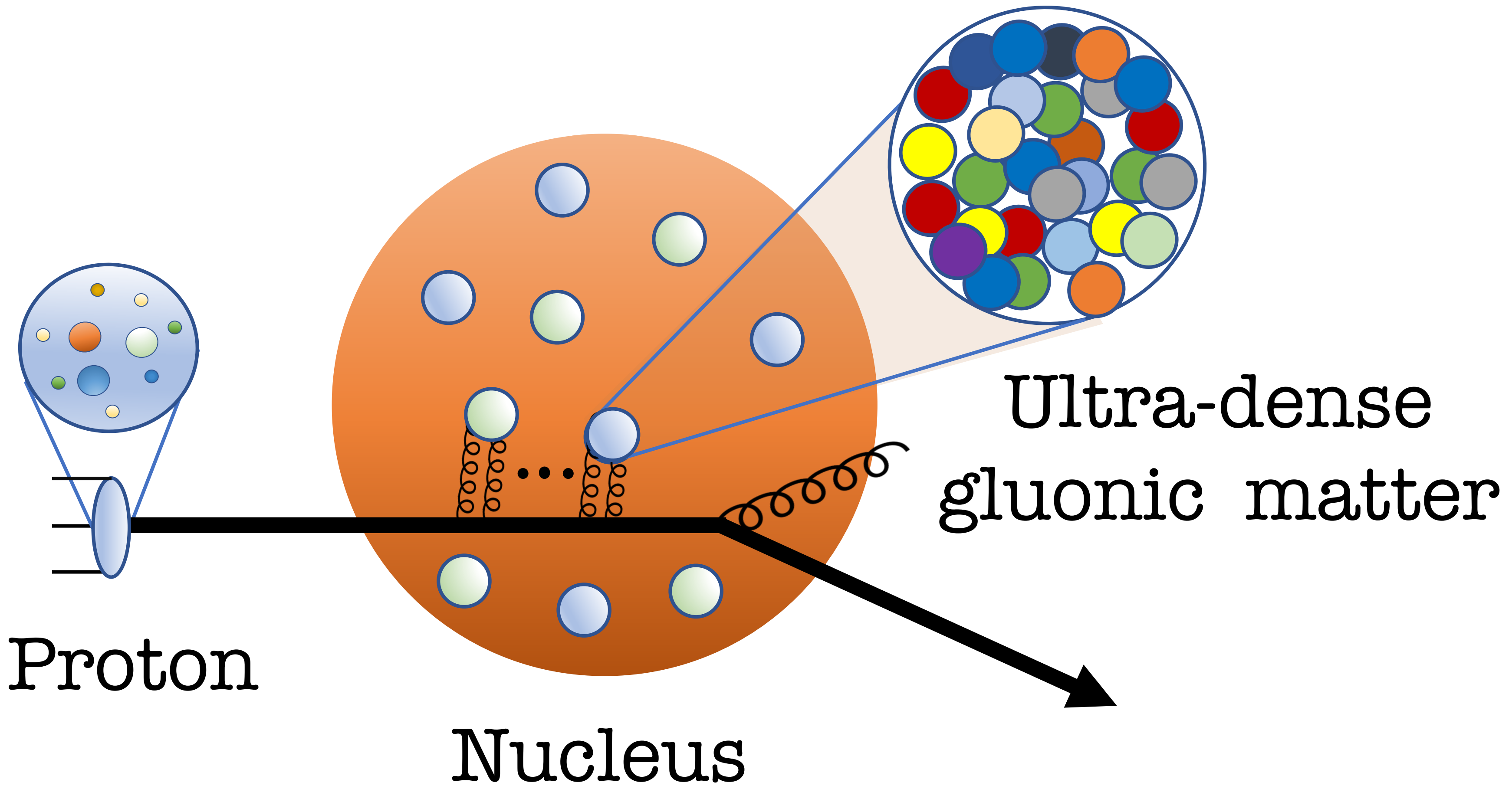
Credit: Benjamin Nachman

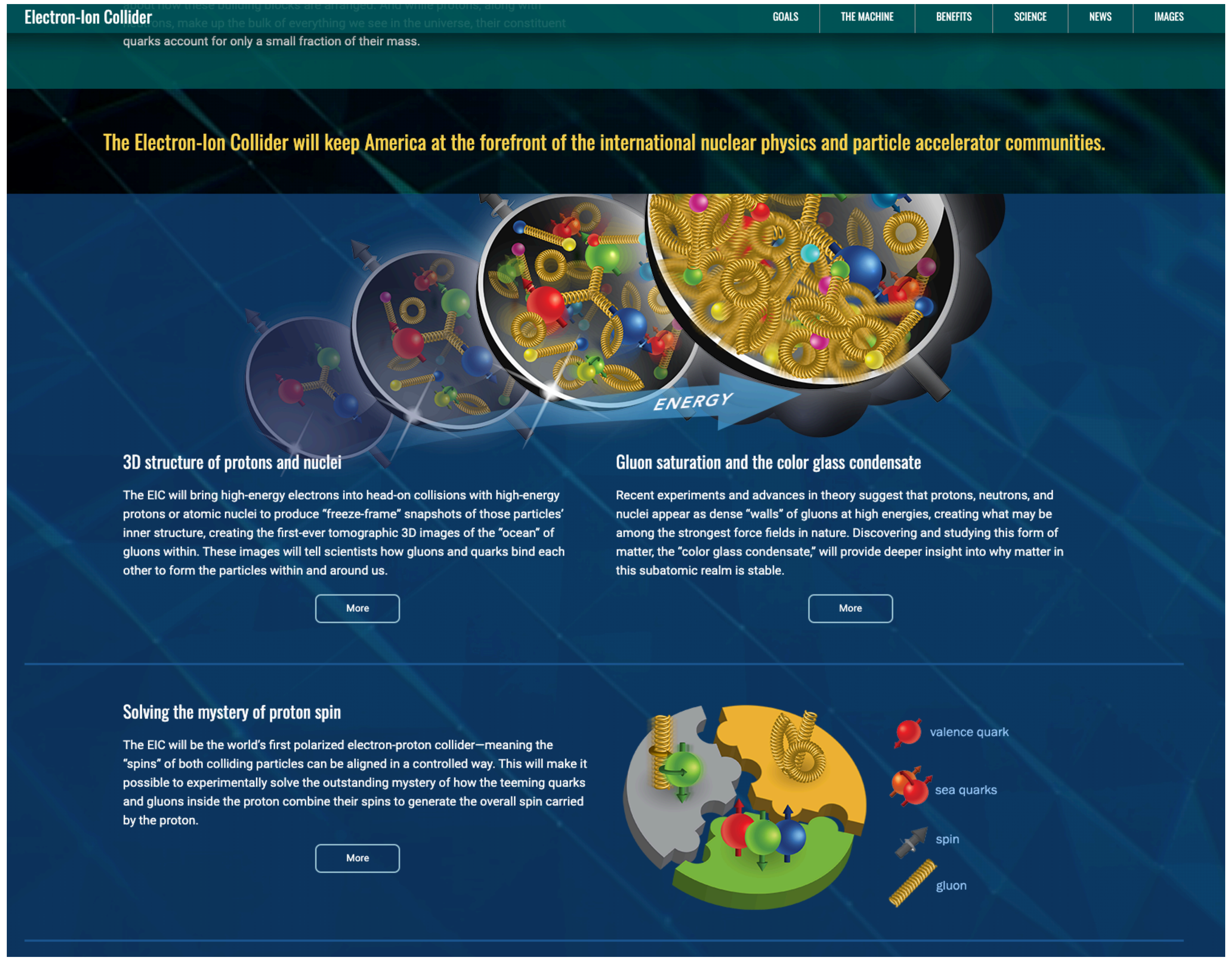
Parton shower algorithms in M.C. event generator



Parton shower algorithms are dedicated to simulating and describing the radiation behavior of quarks and gluons.

Forward physics at pA collisions and Future EIC

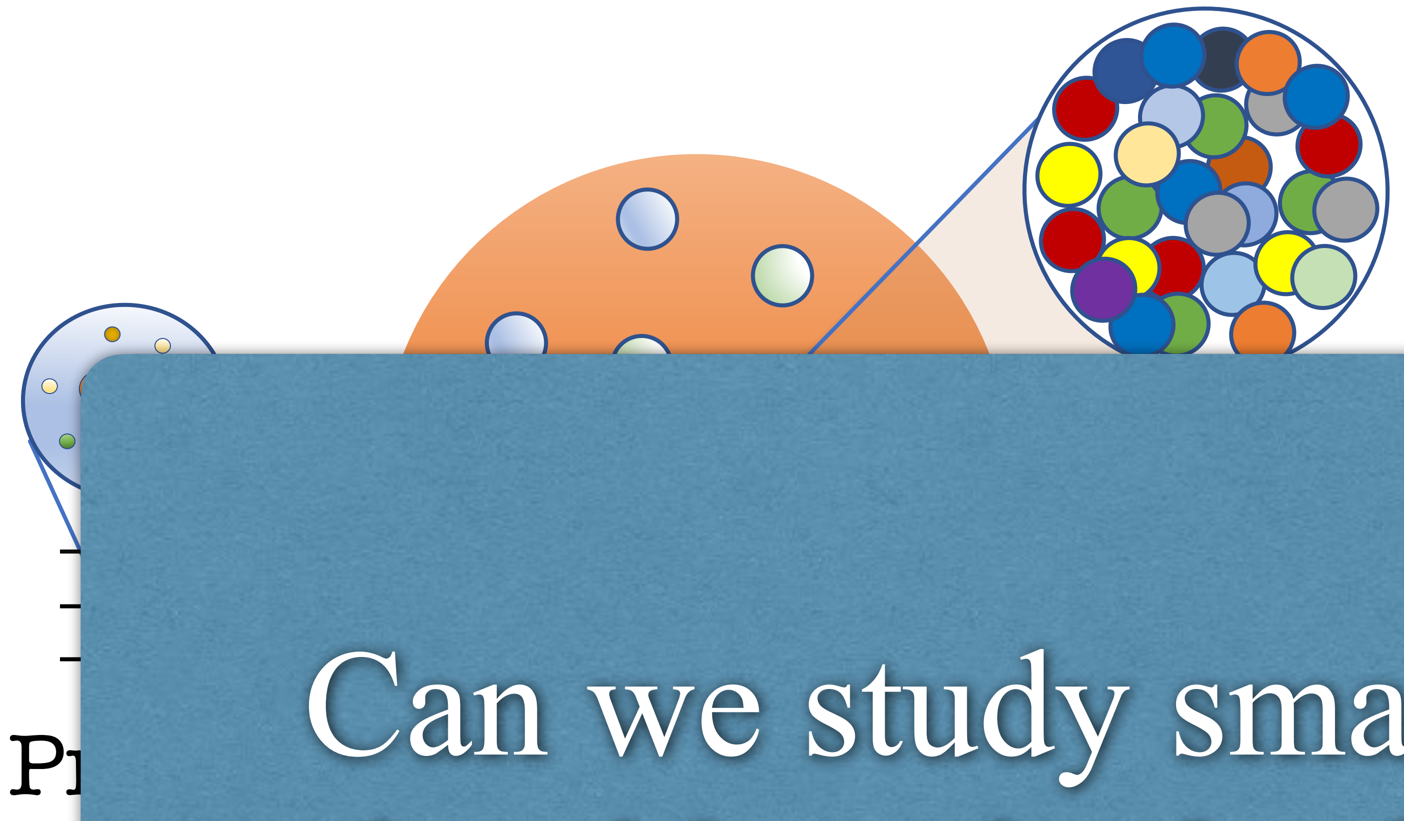


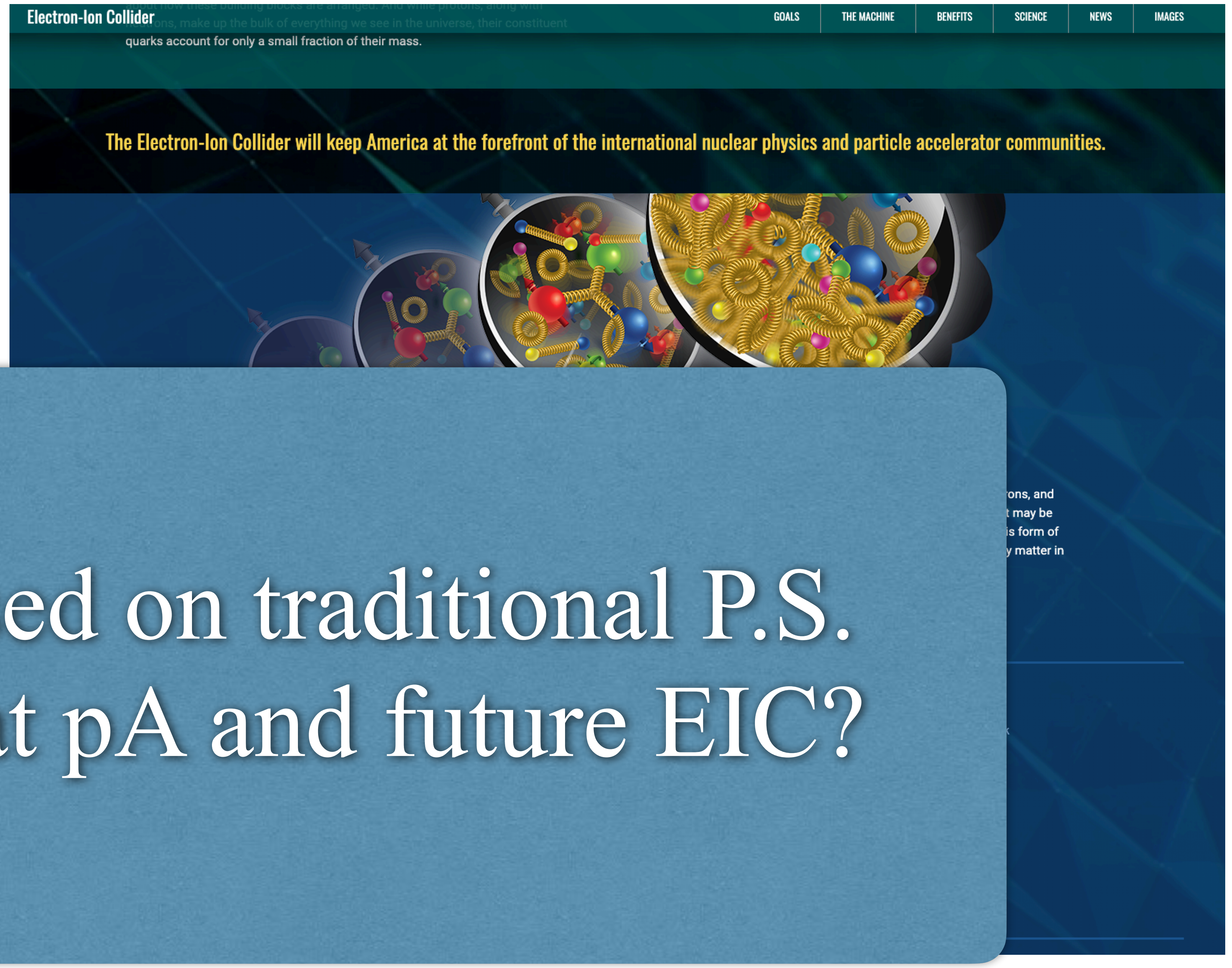


EIC in Brookhaven. High-luminosity and high-precision.

In the forward region at pA and EIC, we can study parton non-linear evolutions and explore gluon saturation in the small-x.

Forward physics at pA collisions and Future EIC



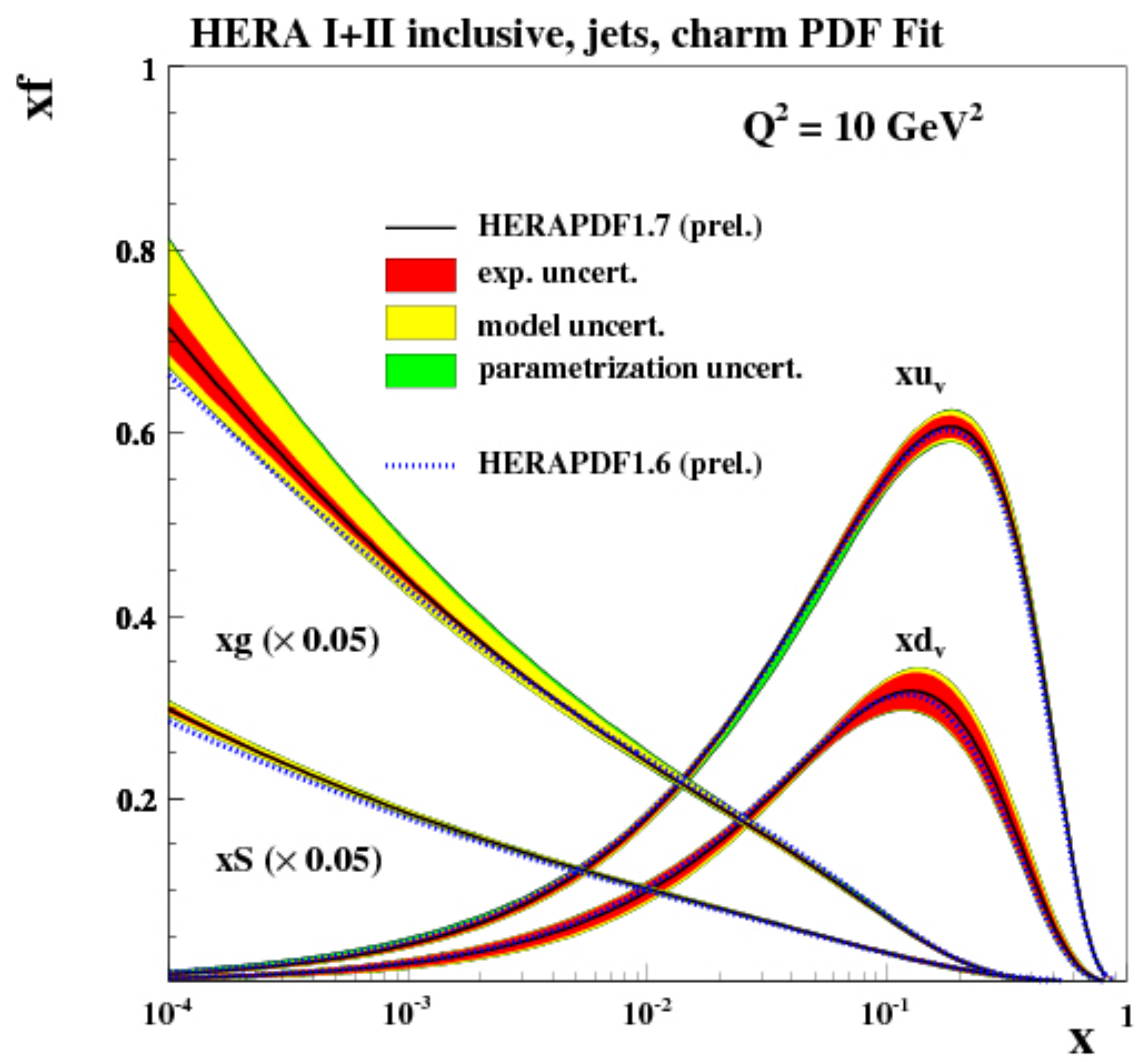


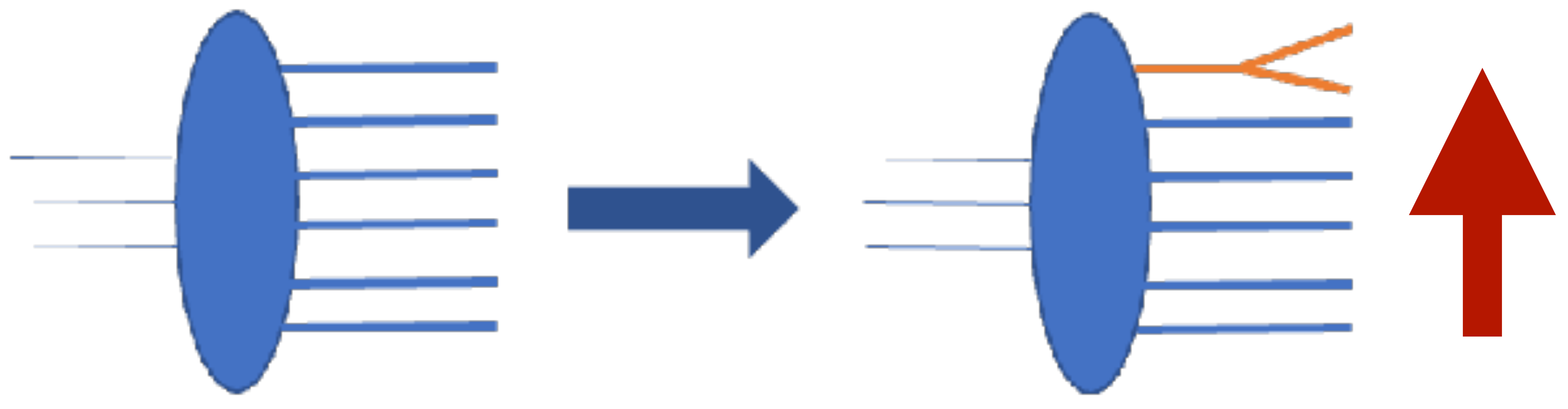
Can we study small-x physics based on traditional P.S. algorithms in the forward region at pA and future EIC?

EIC in Brookhaven. High-luminosity and high-precision.

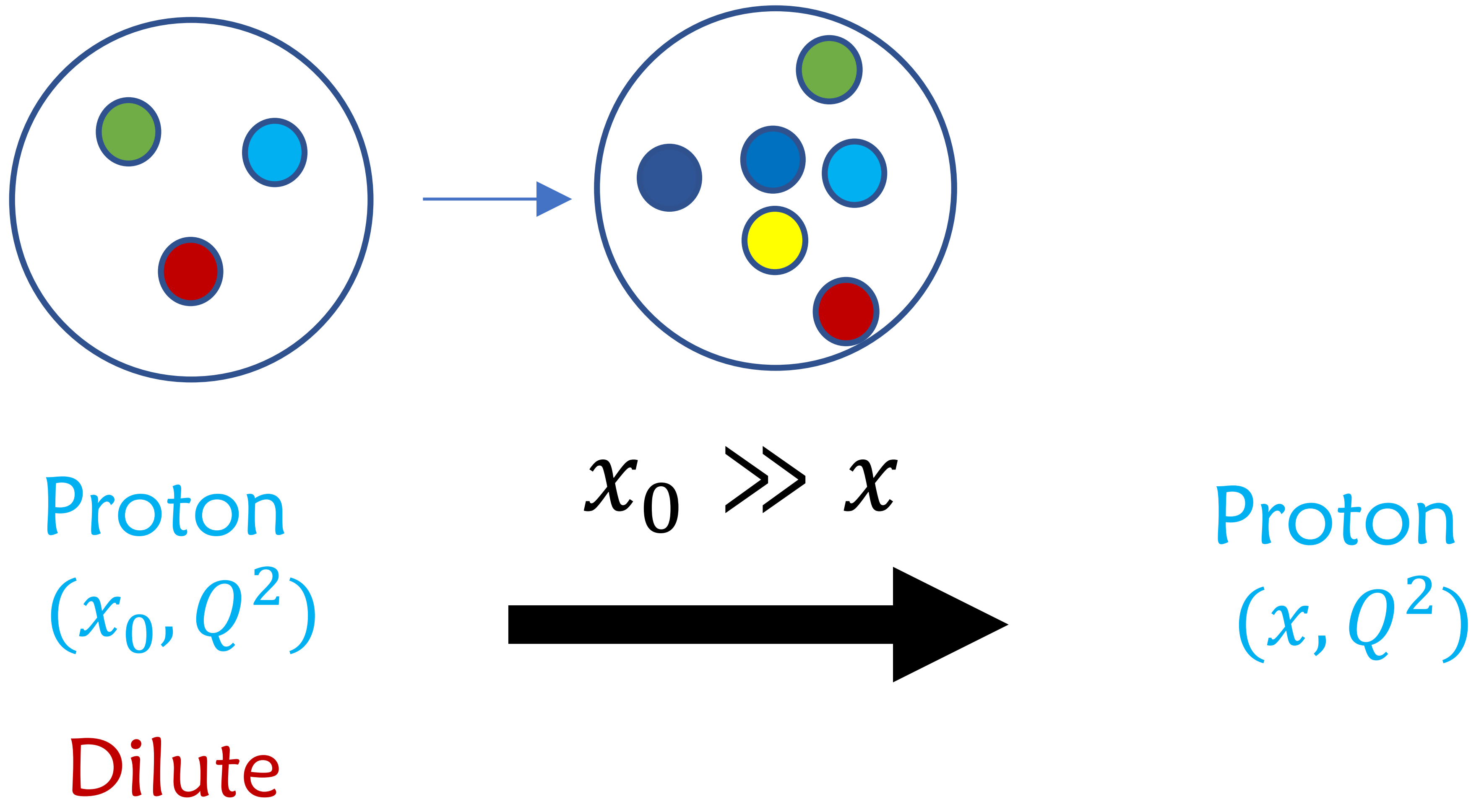
In the forward region at pA and EIC, we can study parton non-linear evolutions and explore gluon saturation in the small-x.

small-x gluon evolution equations



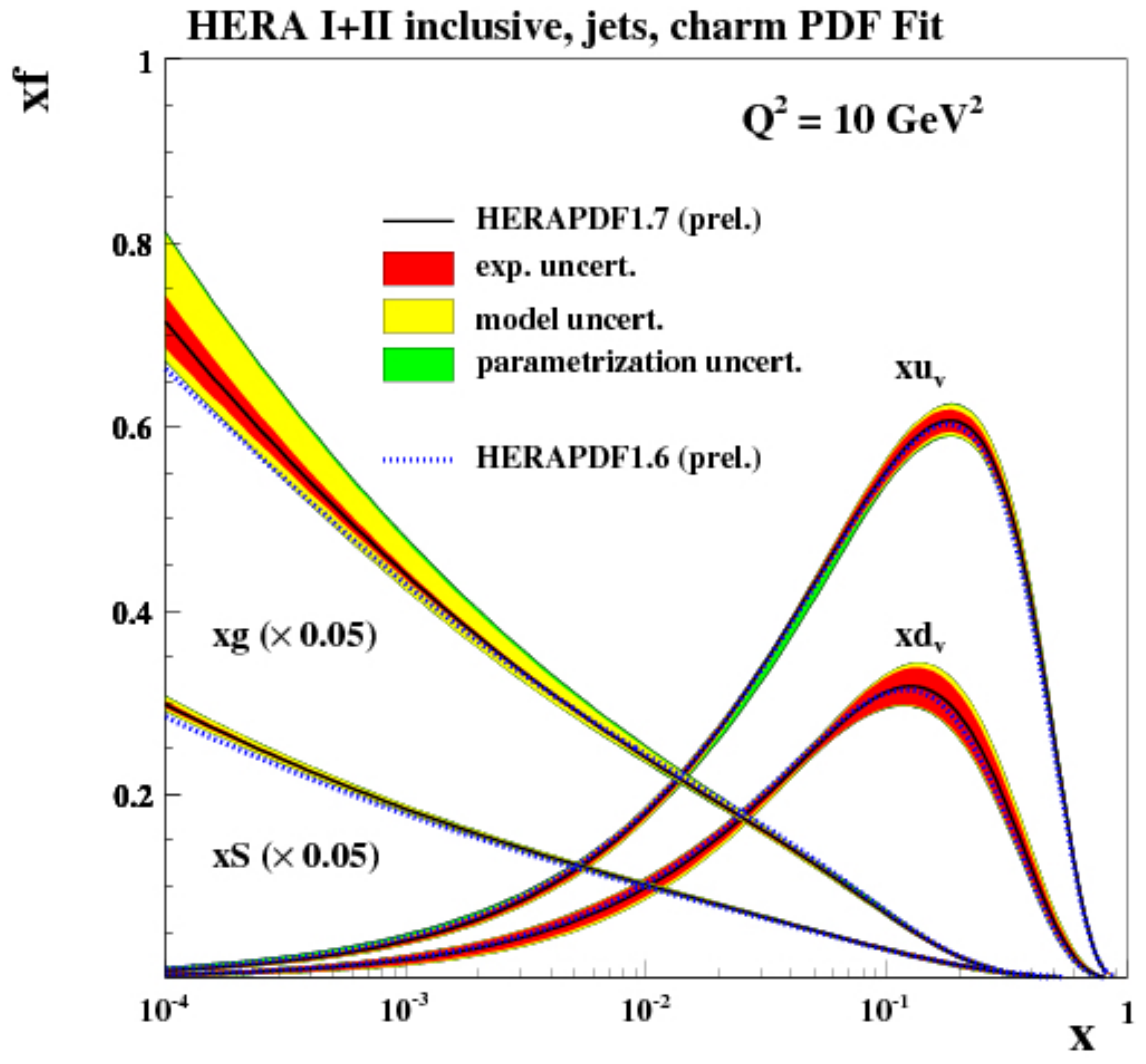


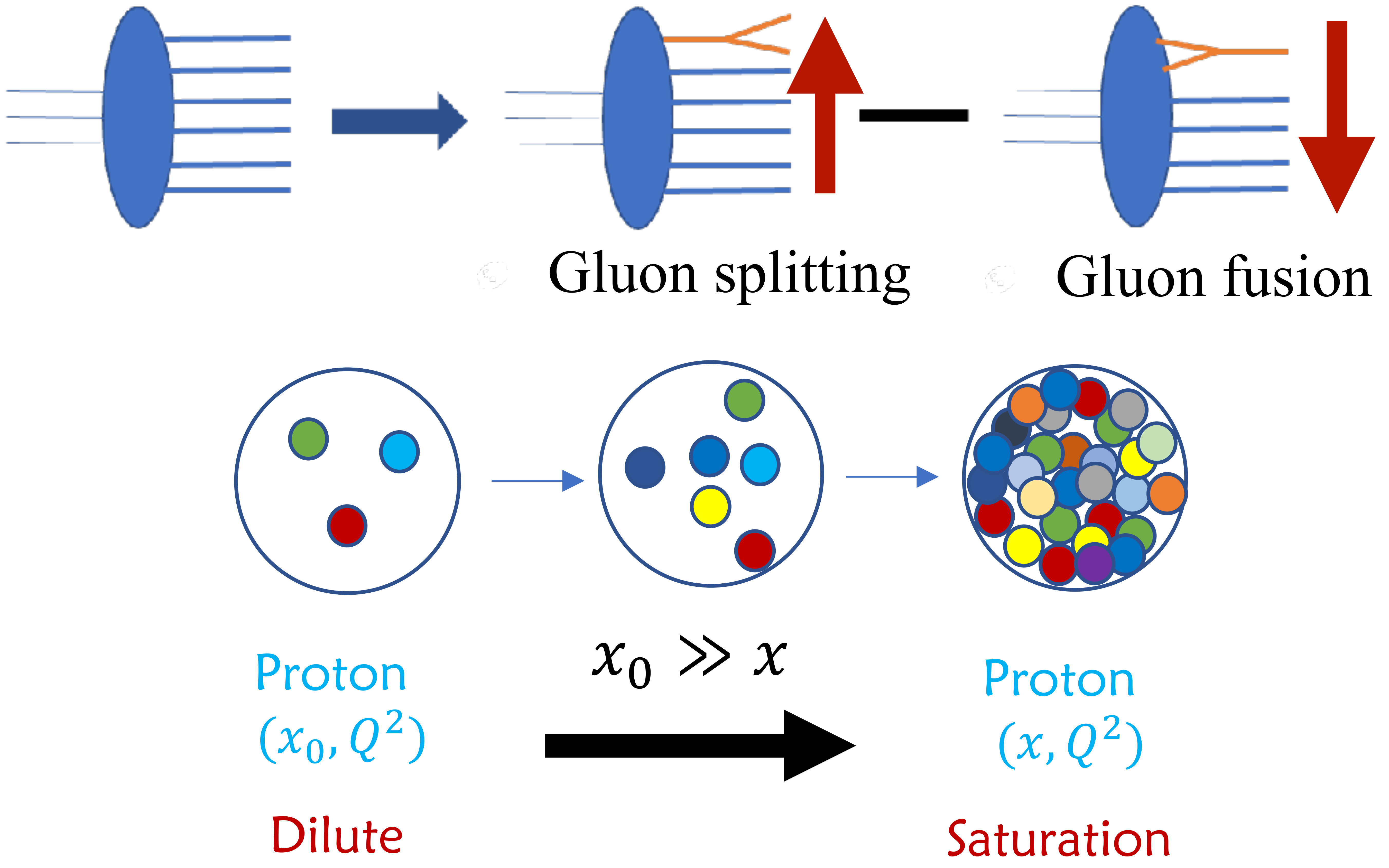
Gluon splitting



- Gluons rapidly increase as x decreases, gluons dominate in small x region

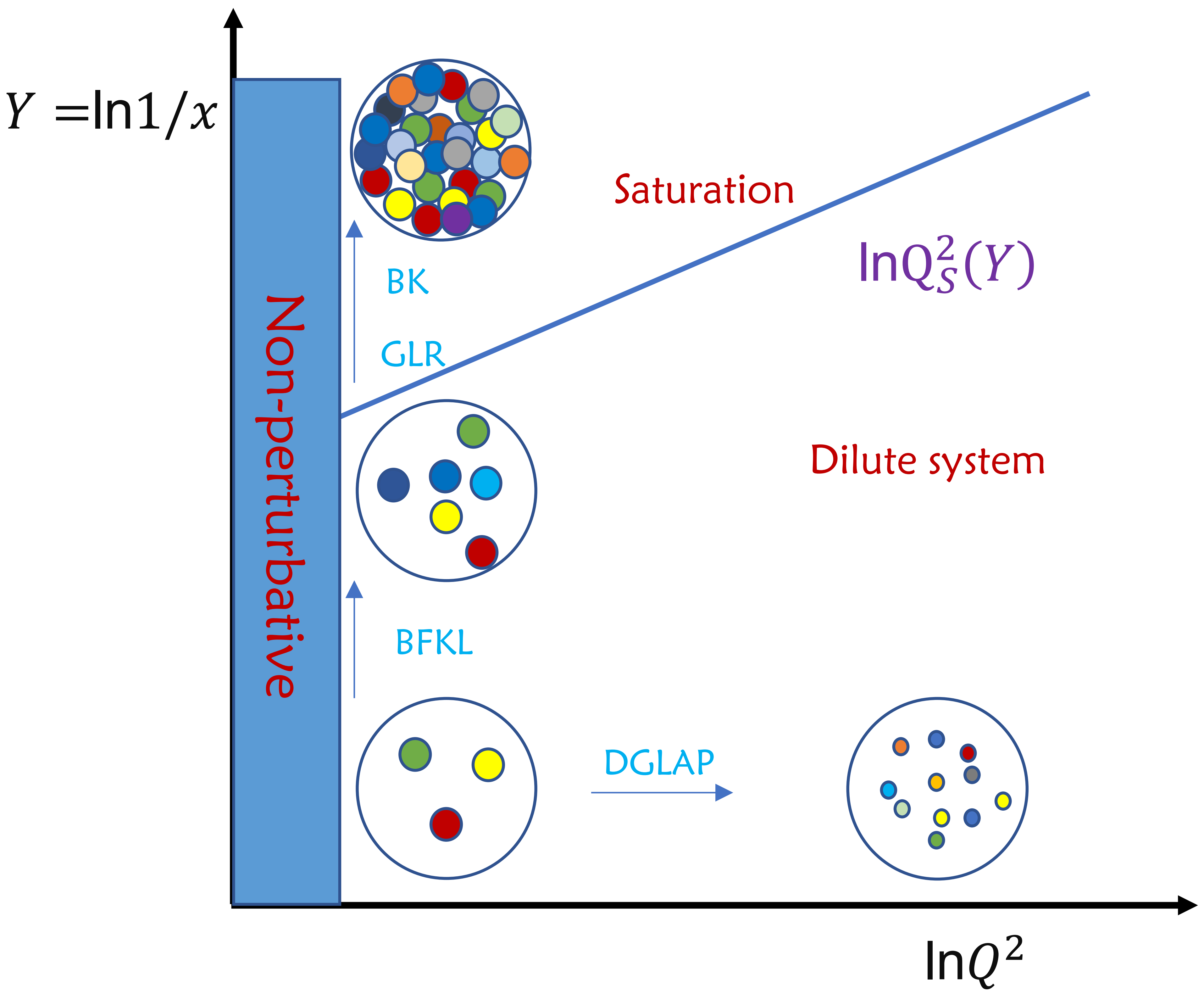
small-x gluon evolution equations

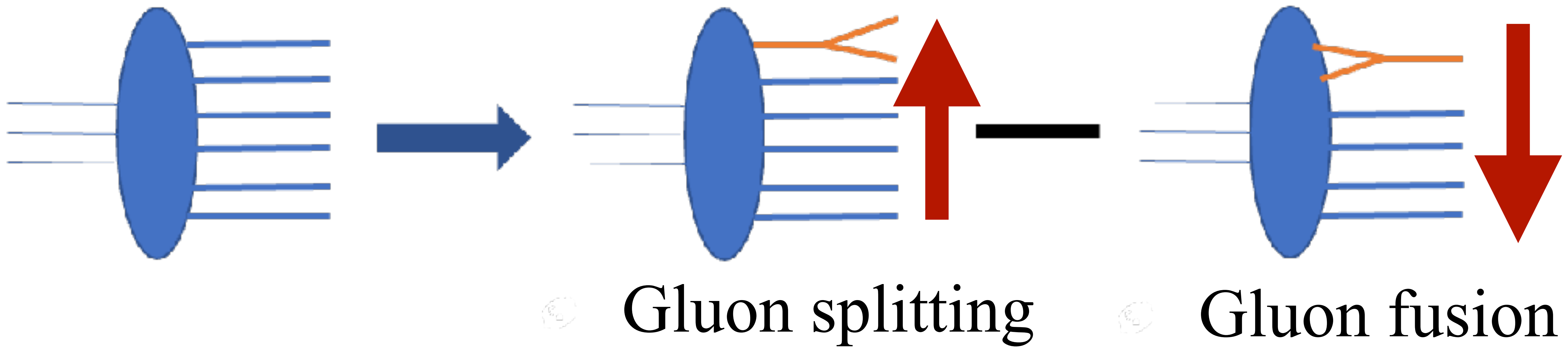


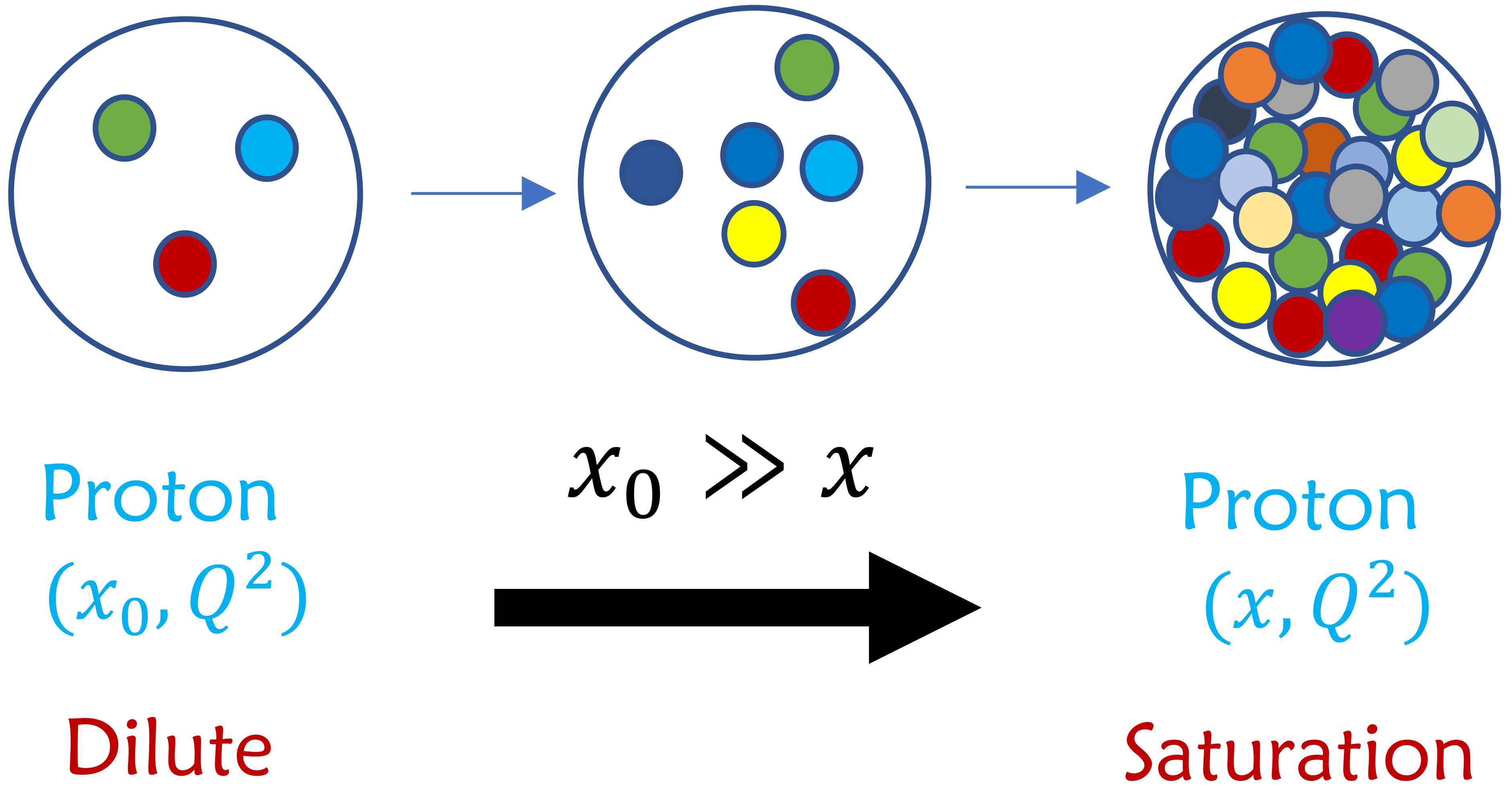


- Gluons rapidly increase as x decreases, gluons dominate in small x region
- Using BFKL, GLR and BK equation instead of DGLAP equation.
- GLR/BK equations are the non-linear evolution equations which describe partons non-linear evolution in small-x region.

small-x gluon evolution equations







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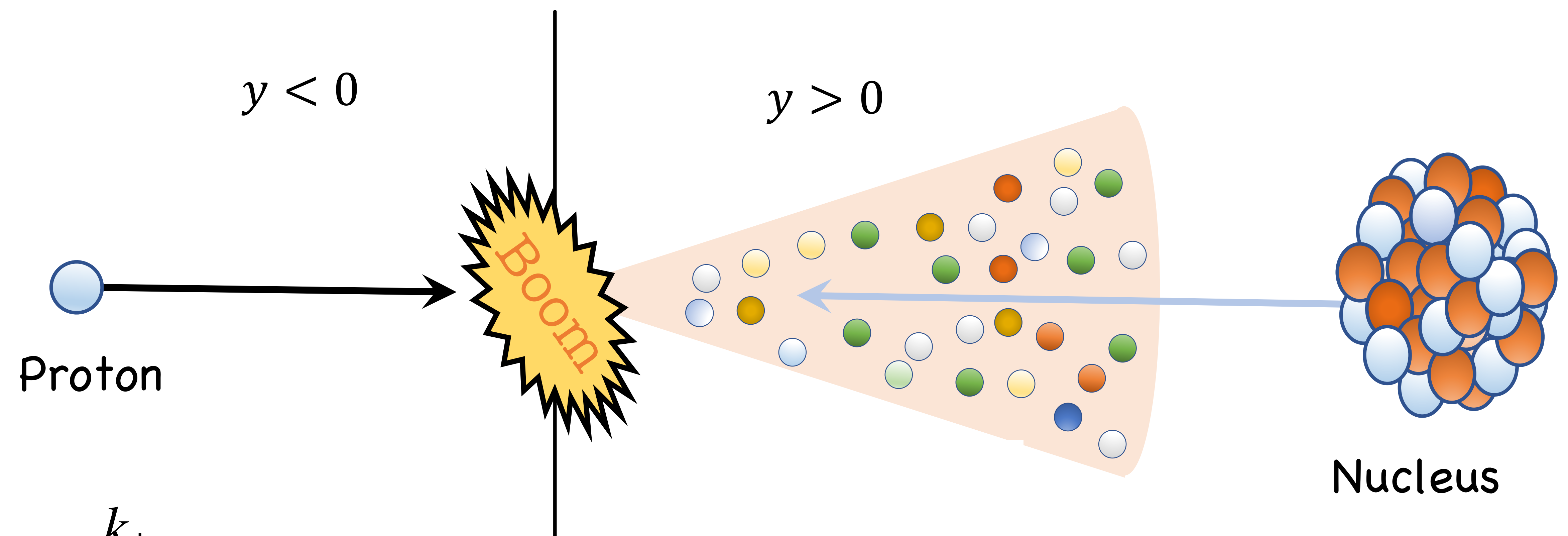
Color Glass condensate

Forward physics at pA collisions

For the single hadron production, LO cross-section likes [Dumitru, Jalilian-Marian, PRL, 02]

$$\frac{d\sigma^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z)\right]$$

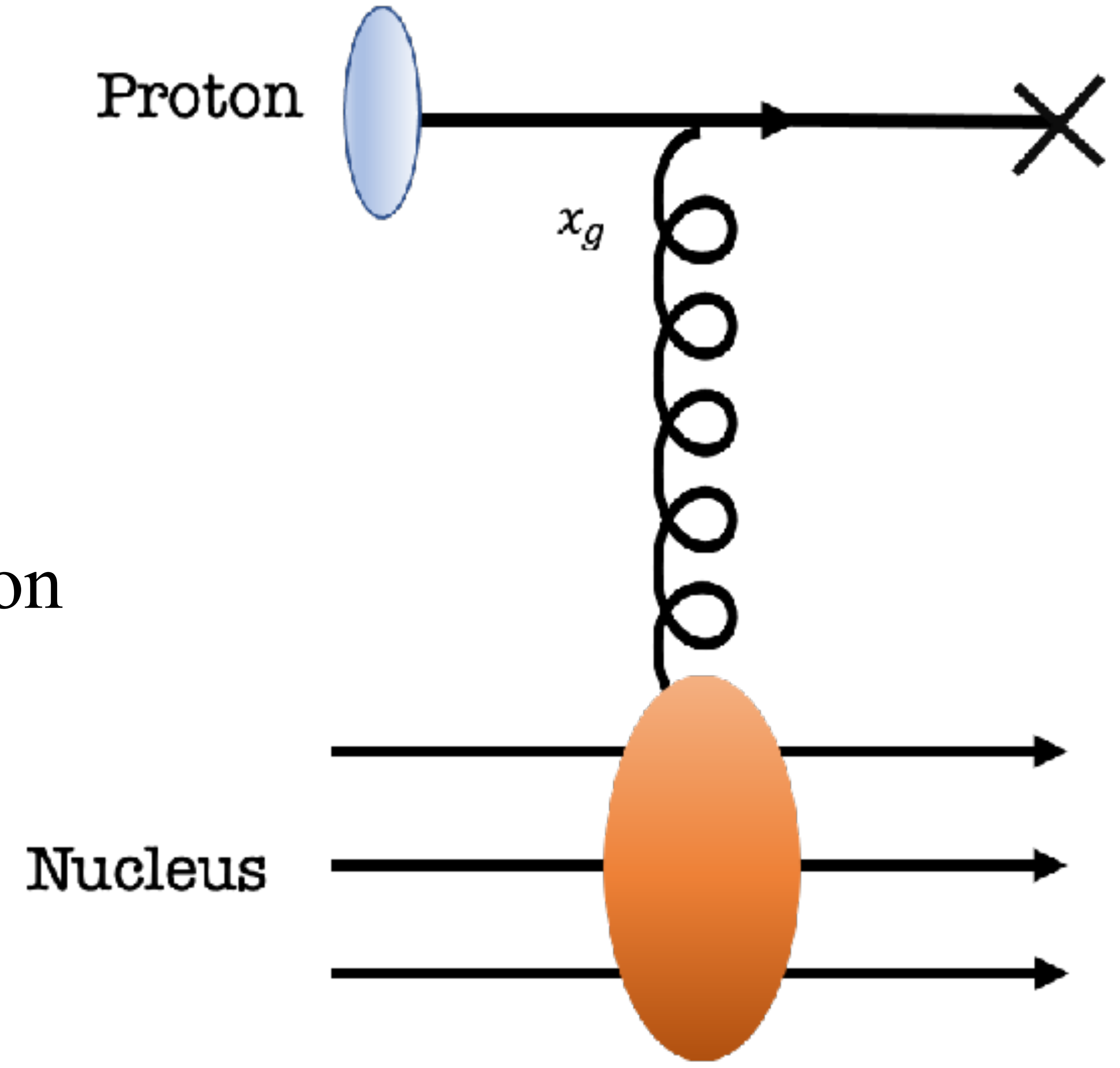


$x_p = \frac{k_\perp}{\sqrt{s}}e^{y} \sim 1$

$x_g = \frac{k_\perp}{\sqrt{s}}e^{-y} \ll 1$

PDFs & FFs
DGLAP evolution

$\mathcal{F}(k_\perp)$ UGD
GLR/BK evolution



(Dilute)

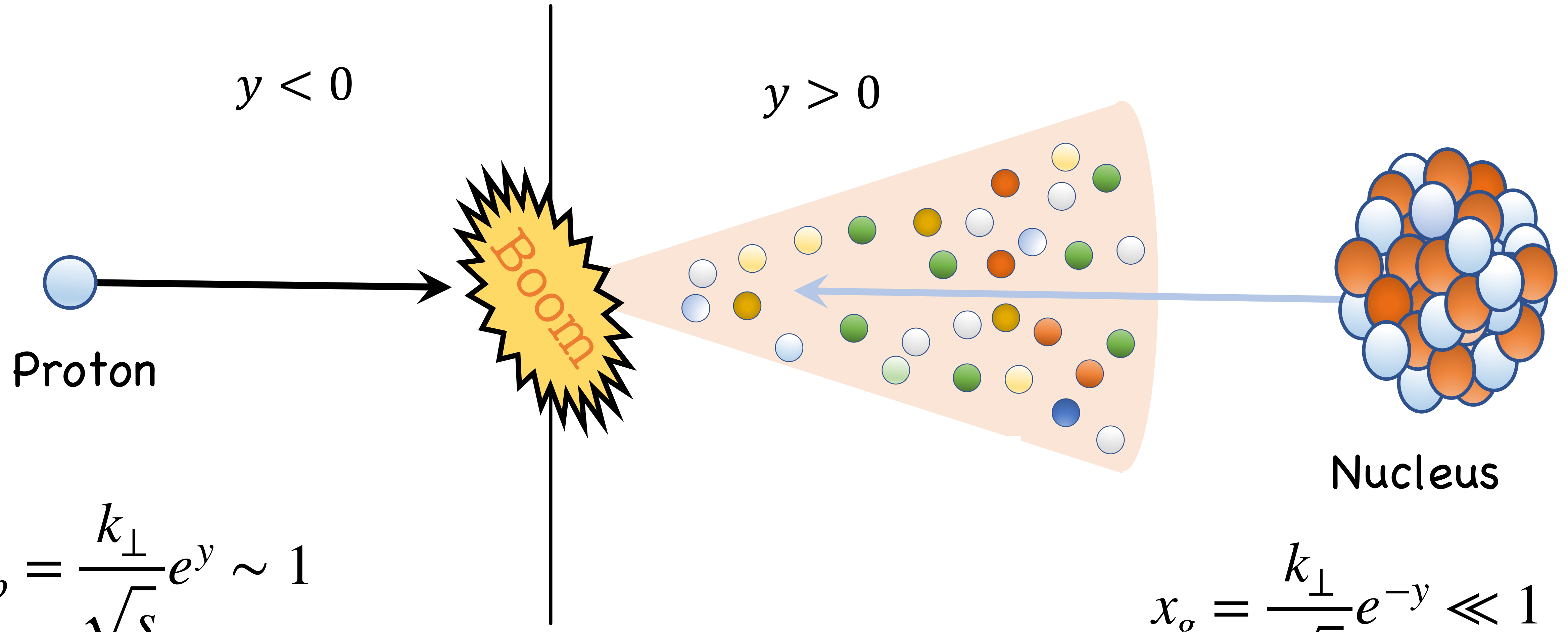
Small-x gluon! (Dense)

We can study parton non-linear evolution and gluon saturation in the Forward region at pA.

Forward physics at pA collisions

For the single hadron production, LO cross-section likes [Dumitru, Jalilian-Marian, PRL, 02]

$$\frac{d\sigma^{pA\to hX}}{d^2p_\perp dy_h}\Big|_{y=0} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z)\right]$$



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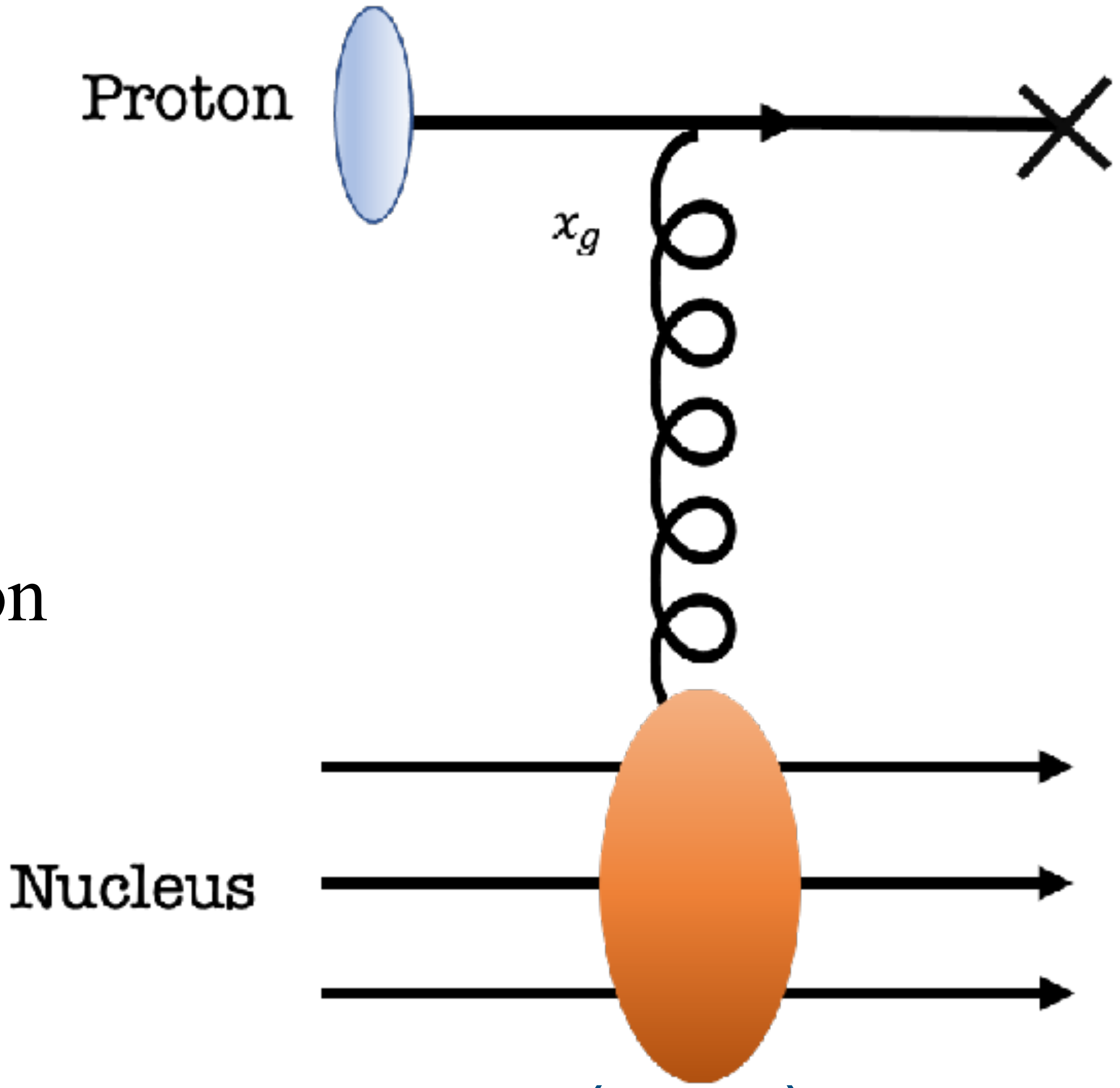
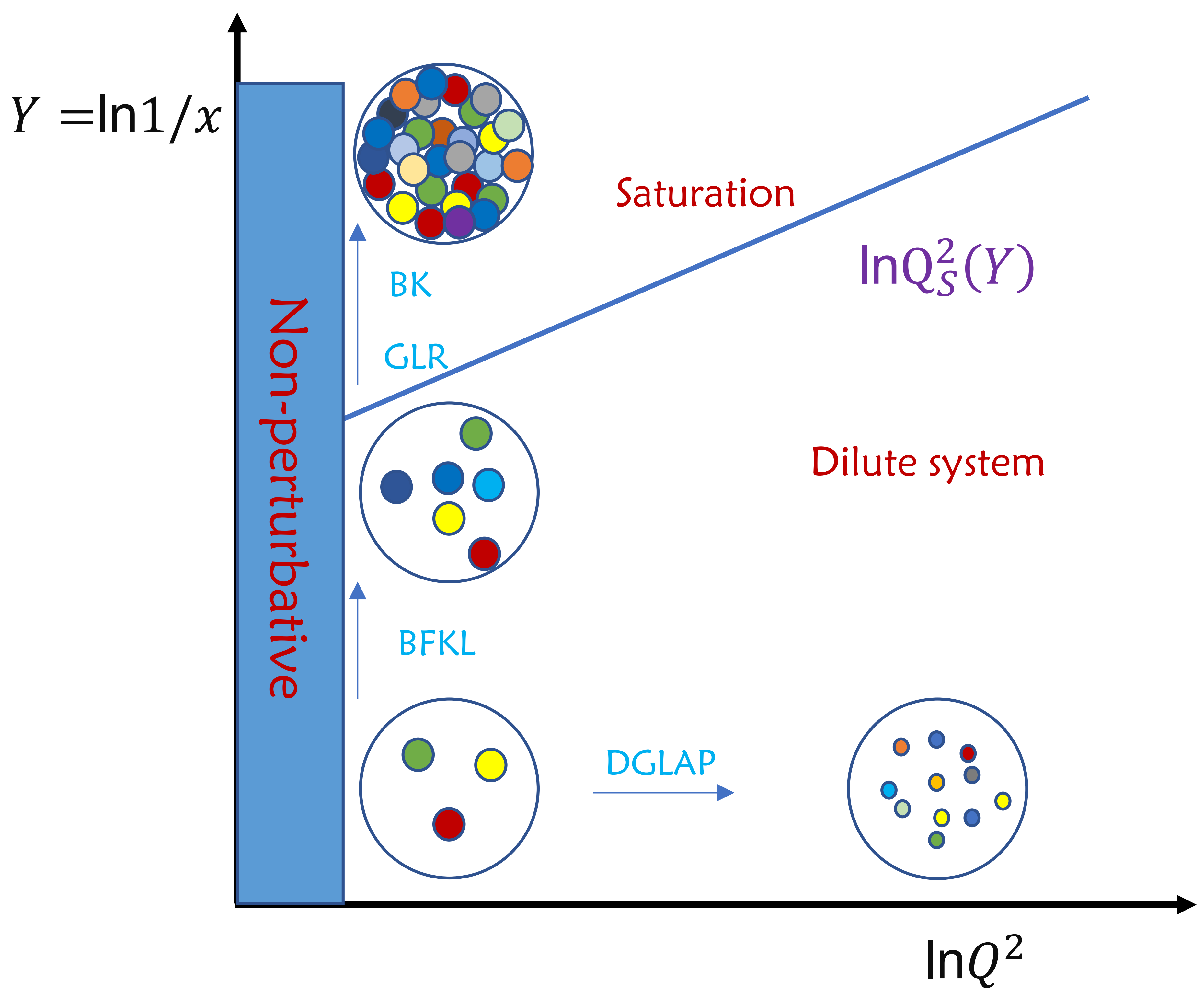


YS, Wang, Wei, Xiao, Phys. Rev. Lett. 128 (2022) 20, 202302.

In the forward hadron production, we can describe all the pp, dAu, and pPb data from RHIC to LHC.

Forward physics at pA collisions



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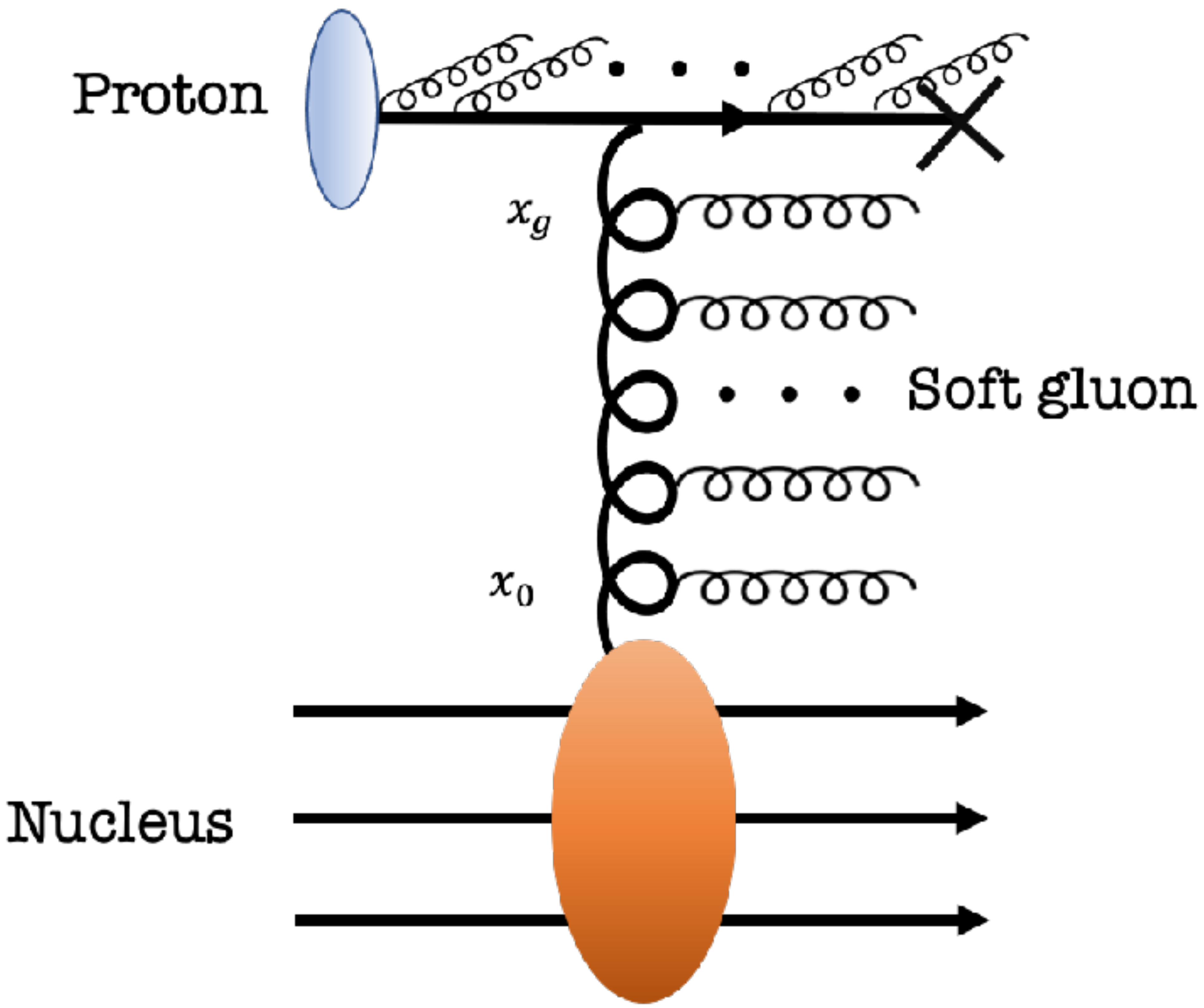
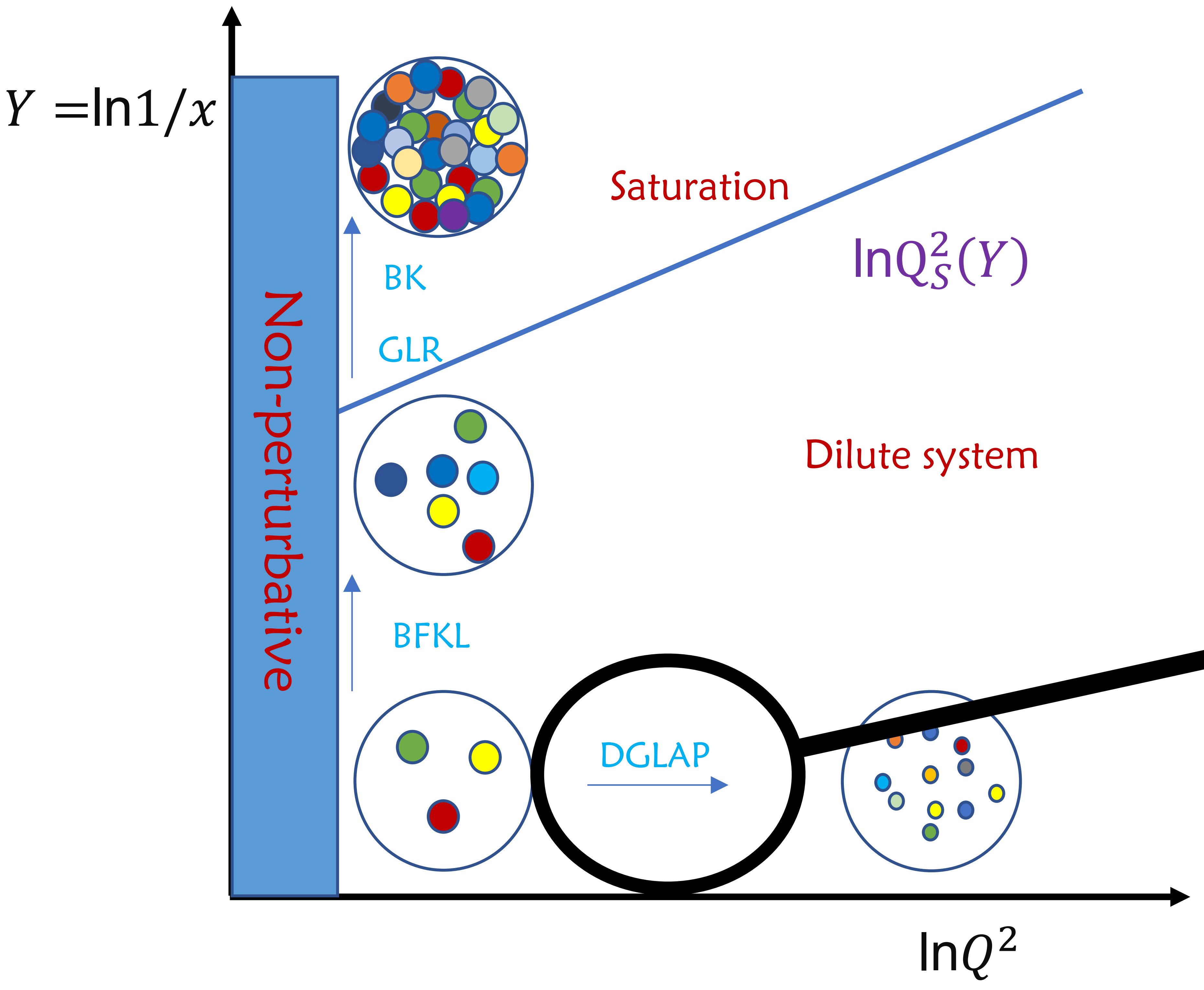


Forward physics at pA collisions



PDFs & FFs
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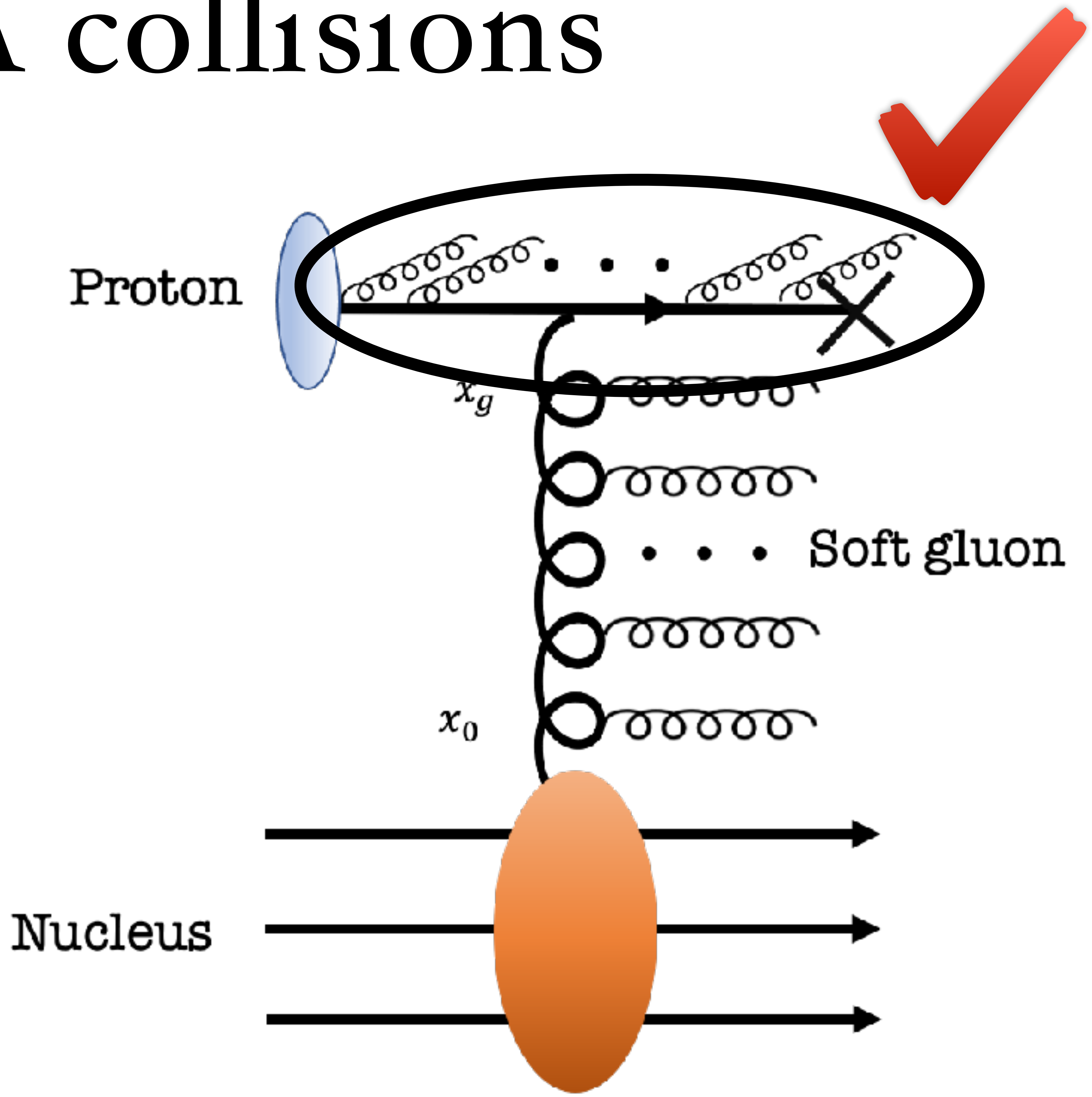
GLR/BK evolution

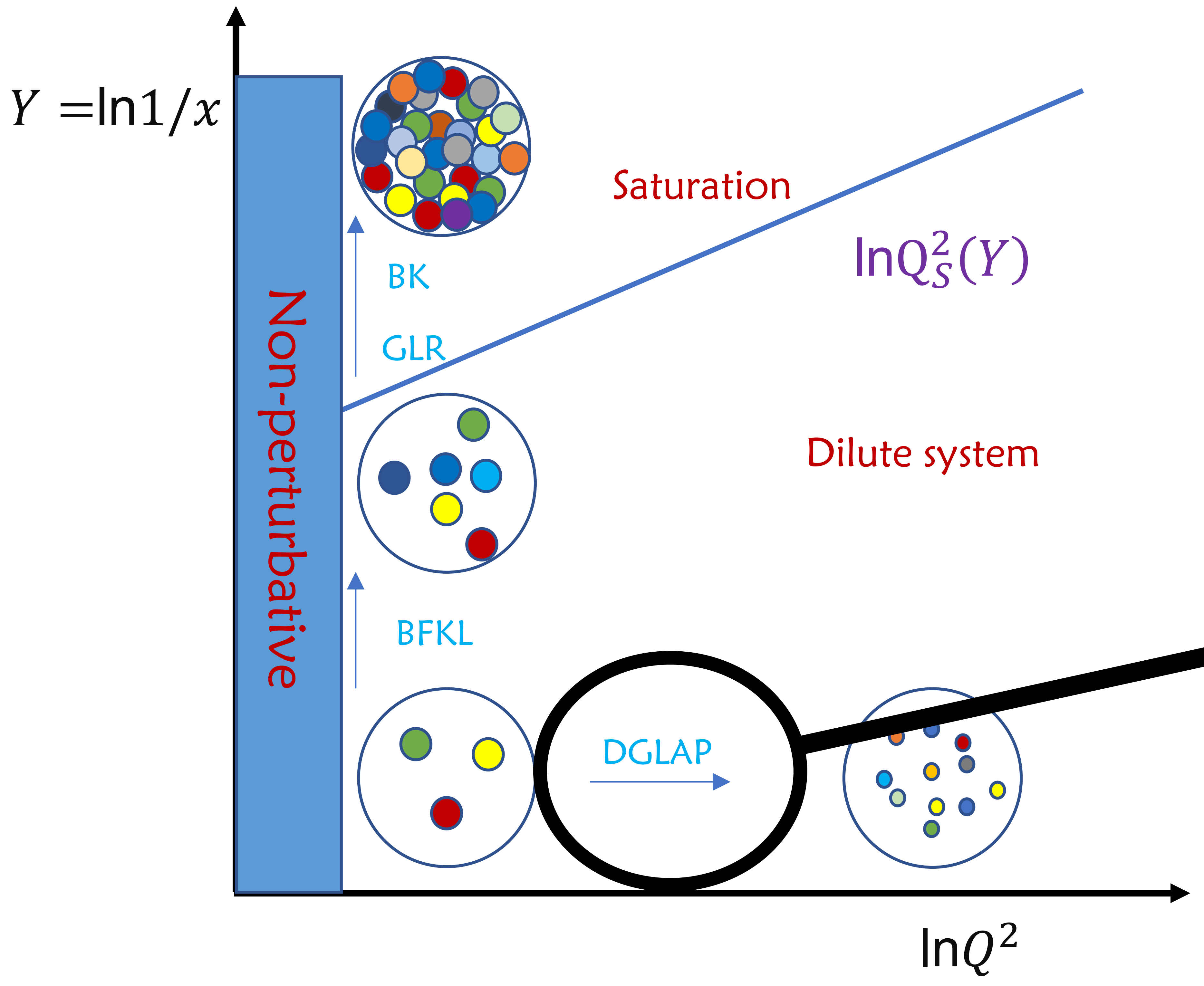
CAT

Cascade

PYSHOW



Forward physics at pA collisions



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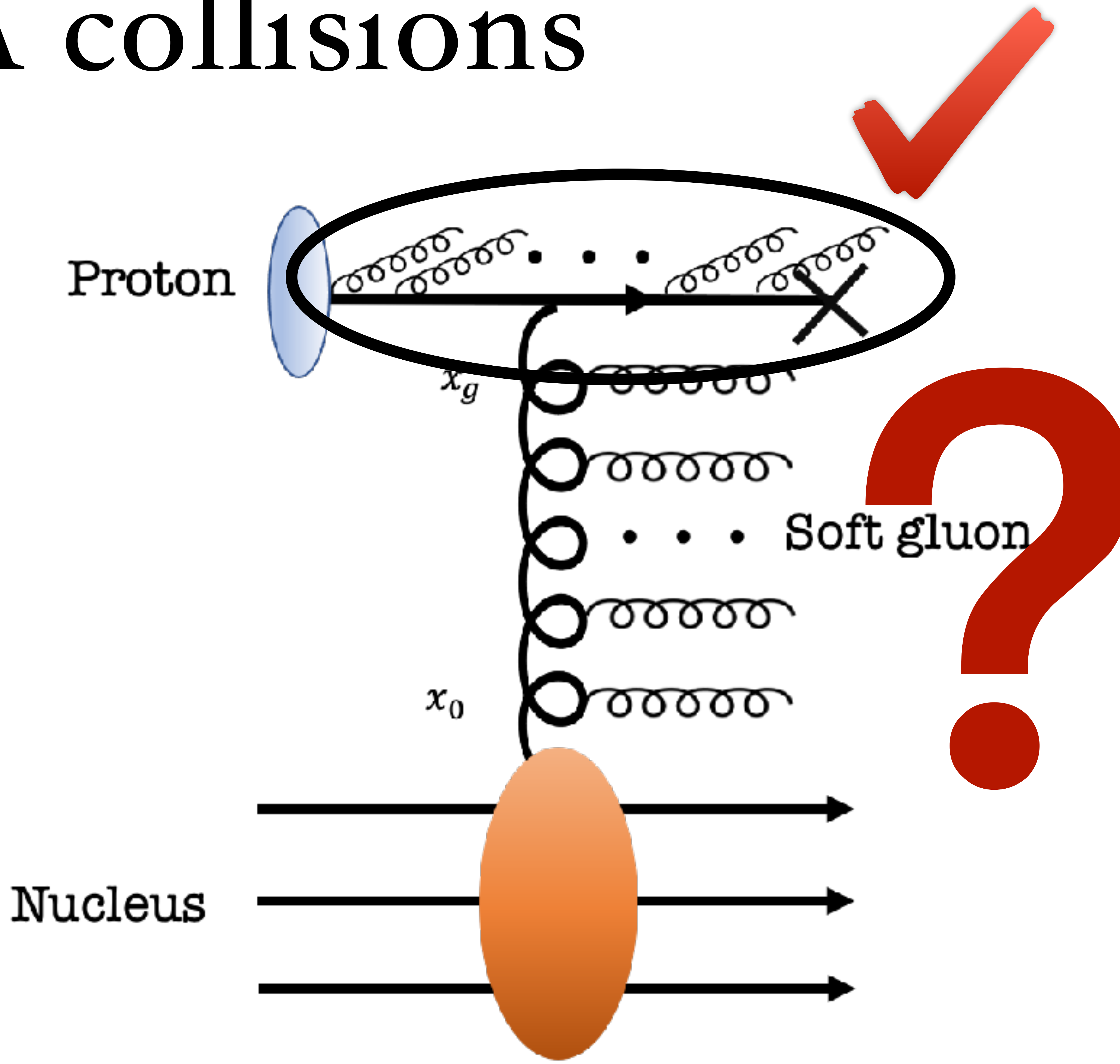
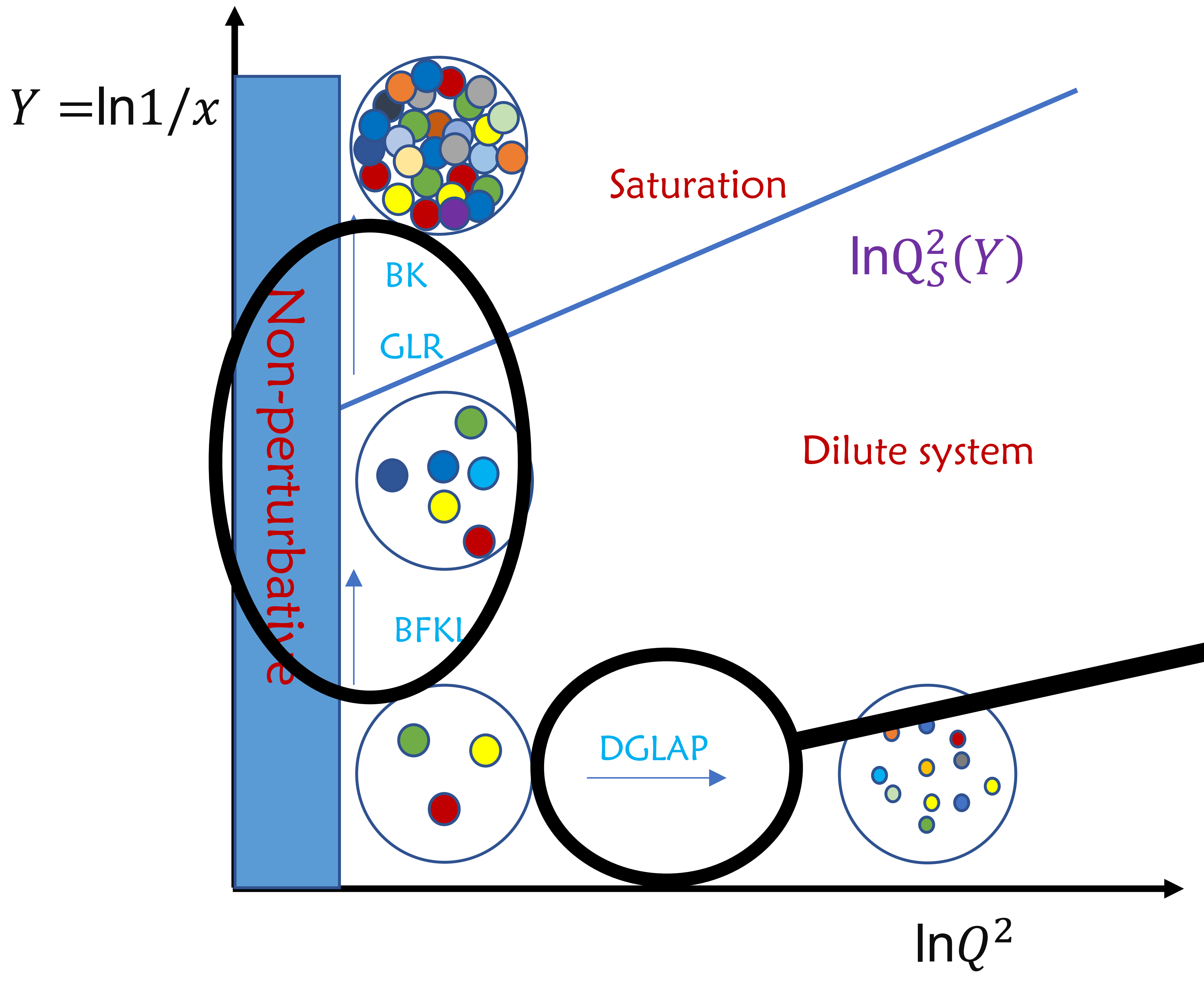


Forward physics at pA collisions



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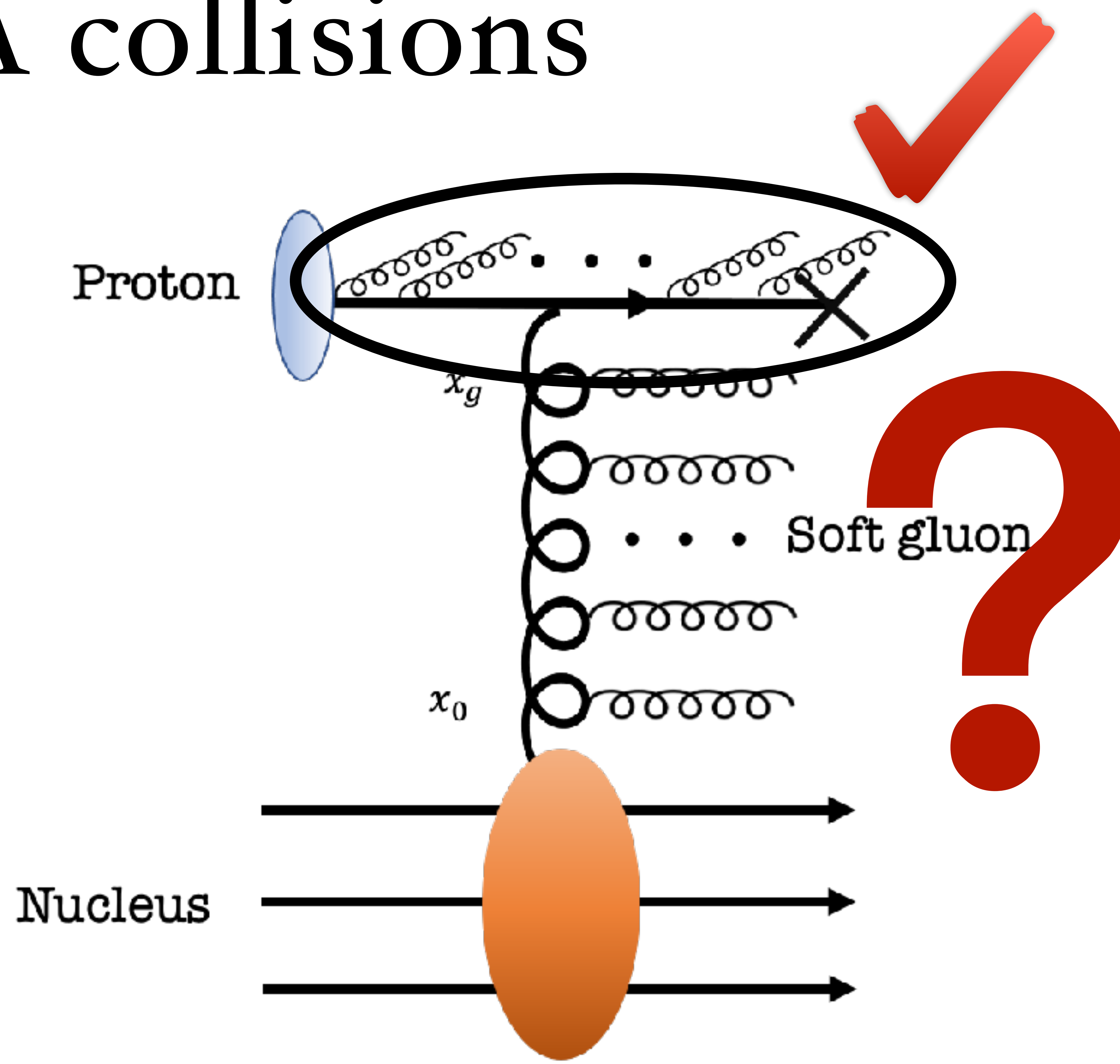
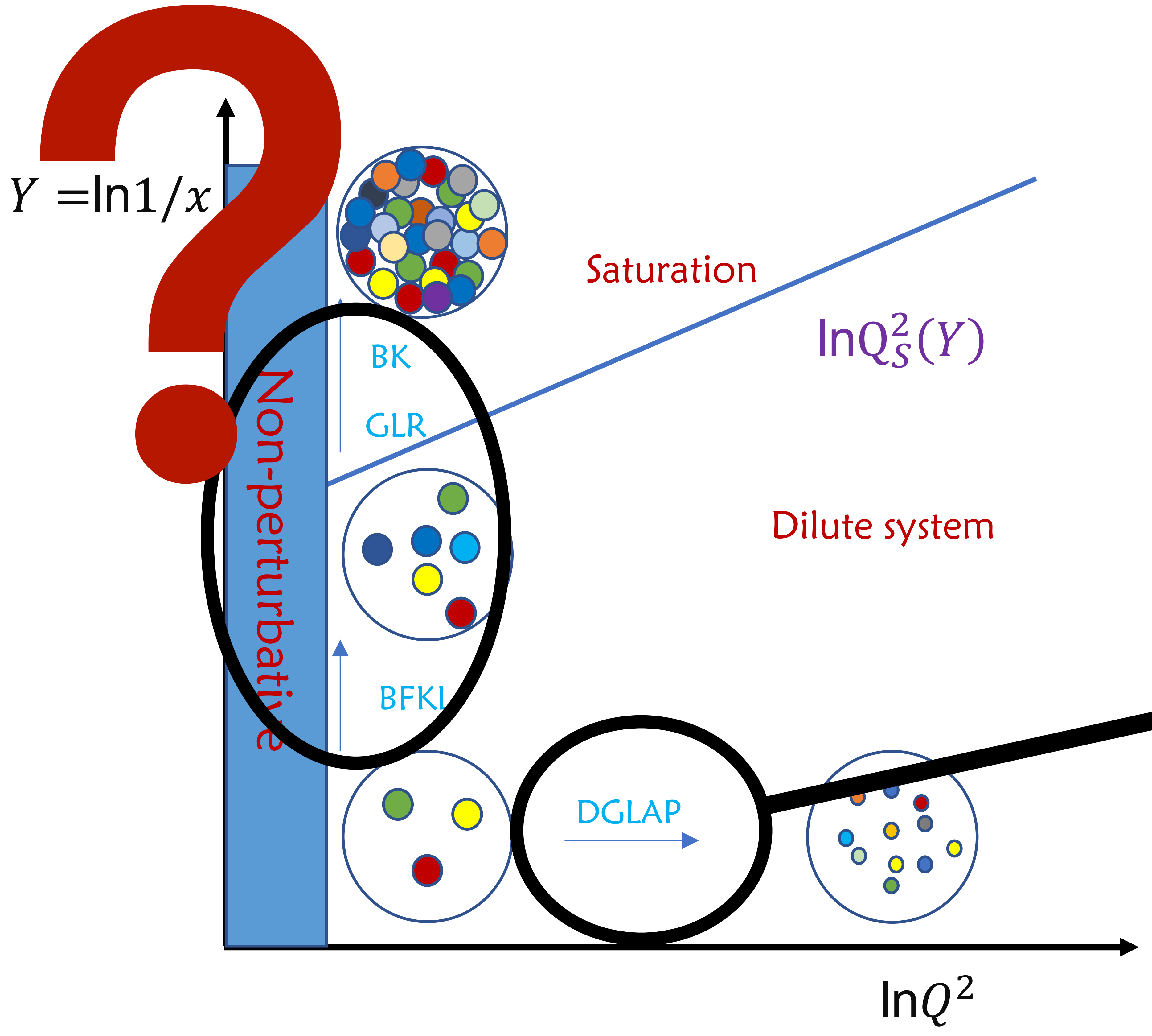


Forward physics at pA collisions



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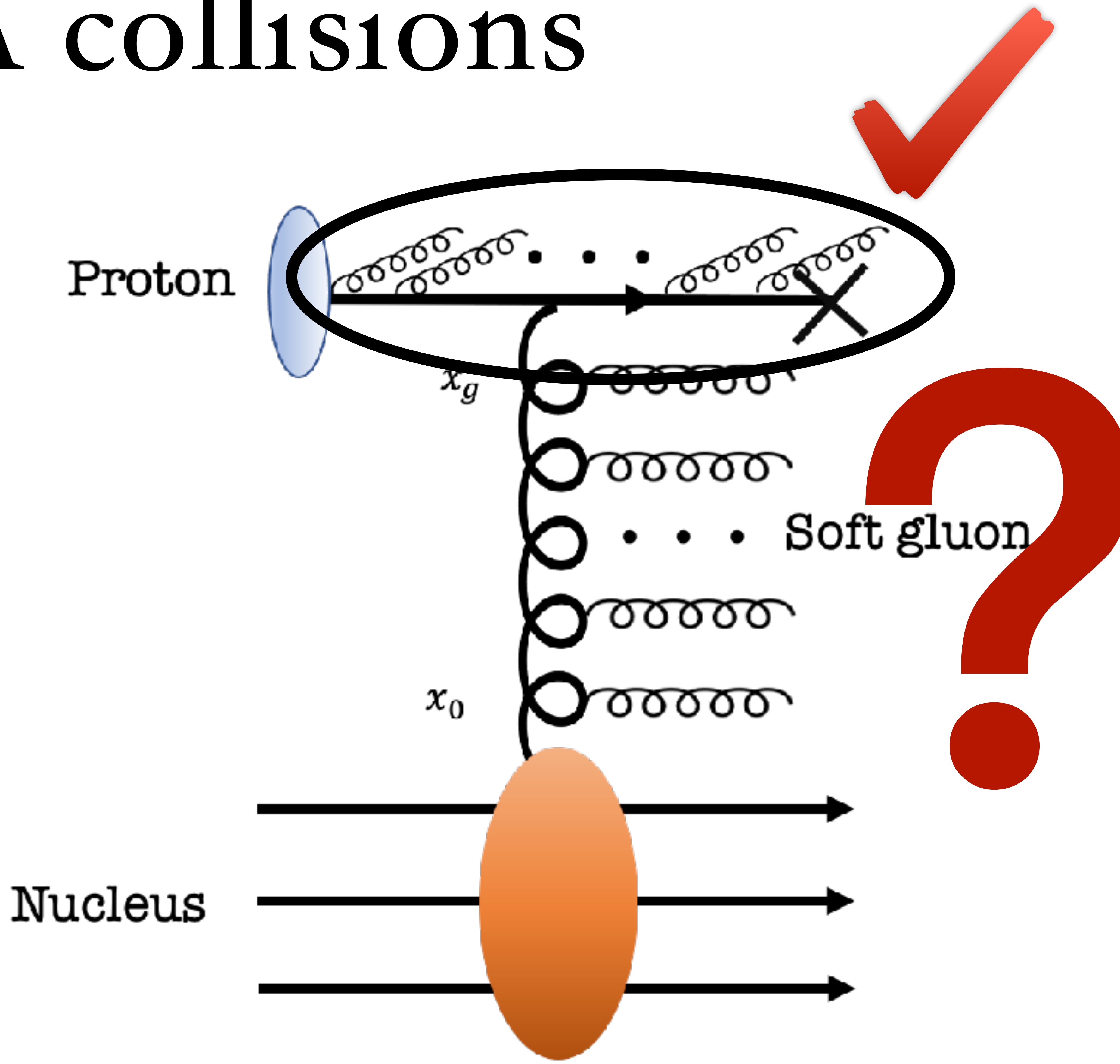
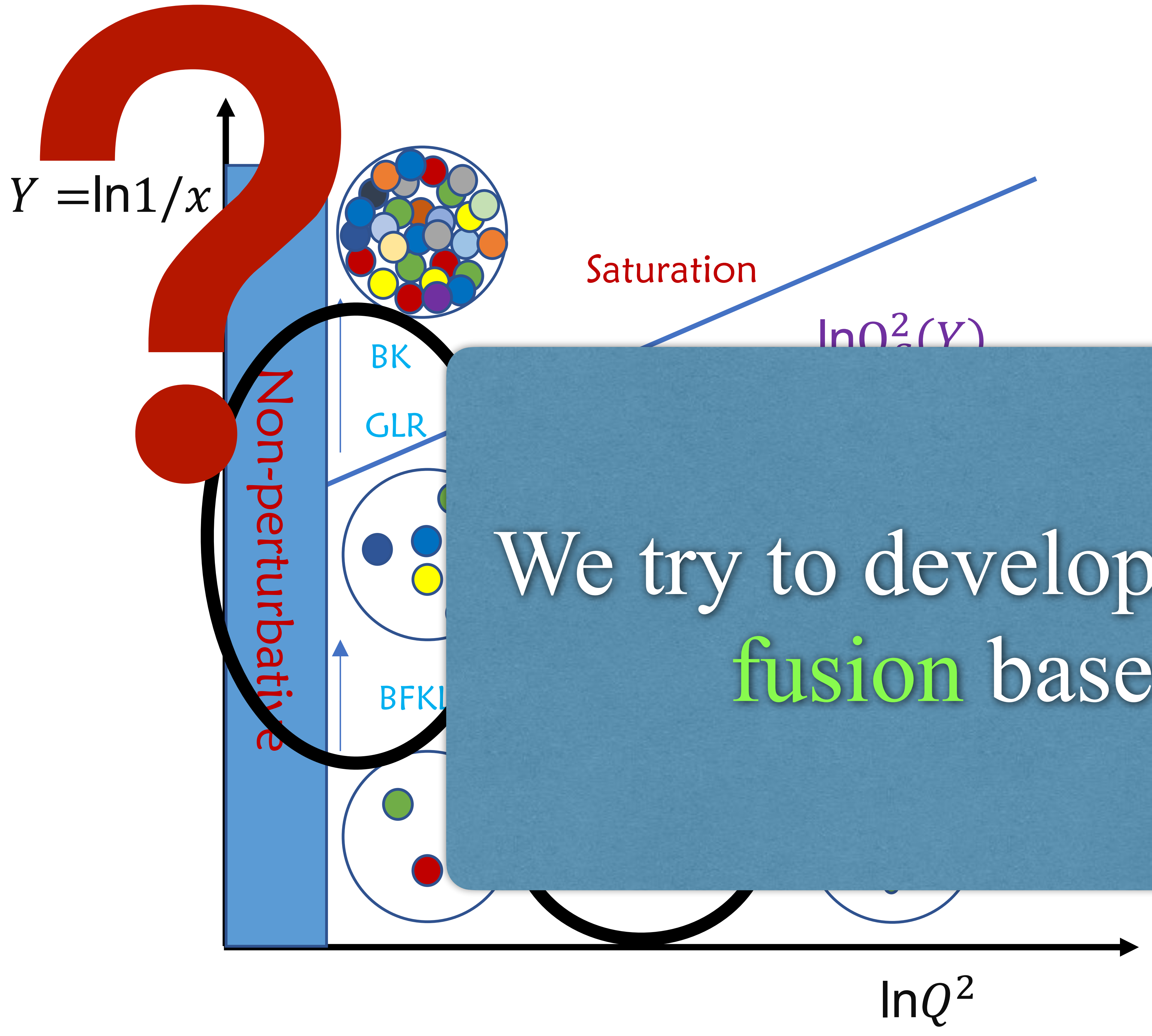


- gluon fusion effect is absent in all existing generators.
- Developing a P.S. algorithm based on the small-x evolution equation is important.

Forward physics at pA collisions



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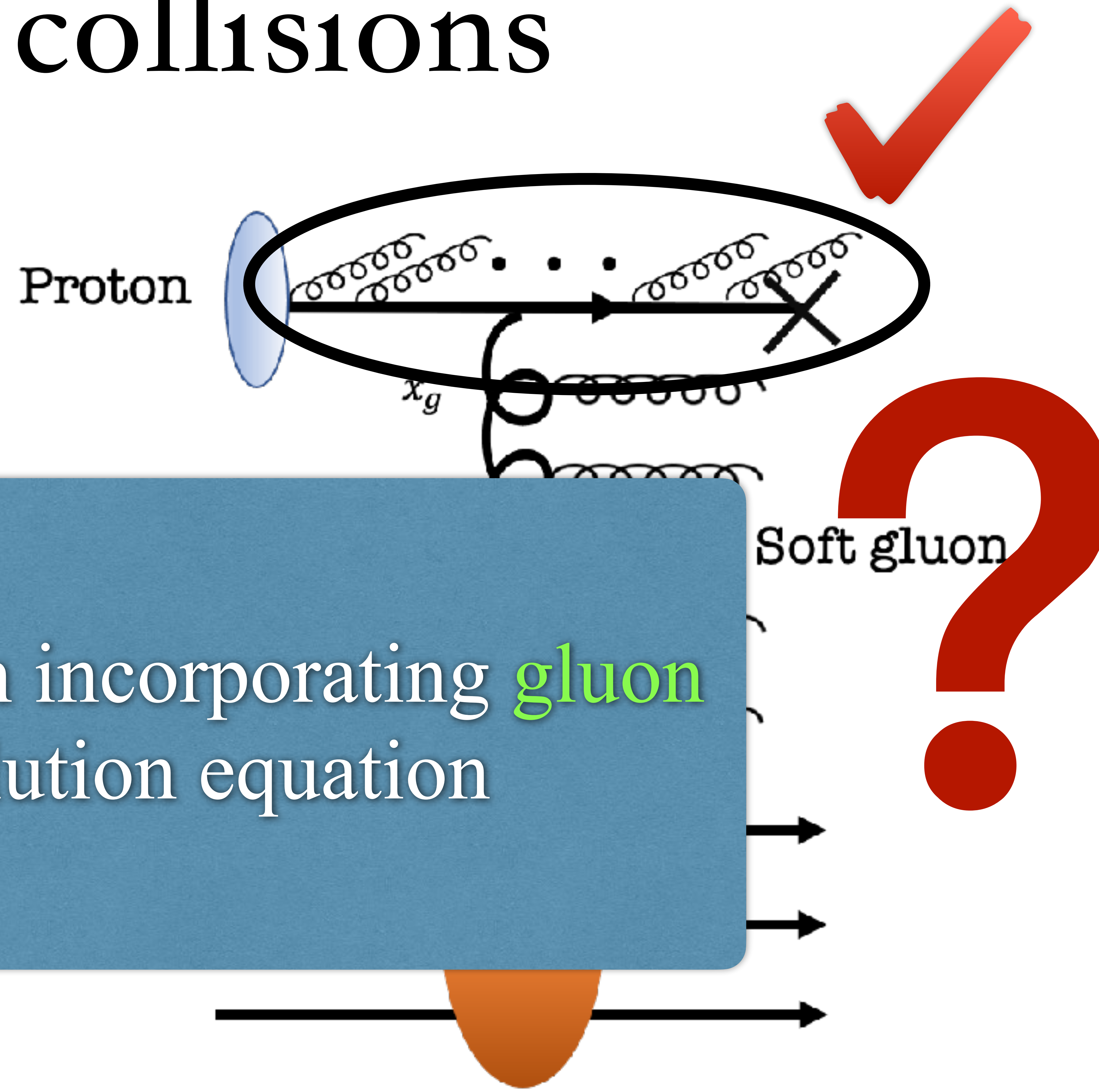


We try to develop a new P.S. algorithm incorporating gluon fusion based on the small-x evolution equation

- gluon fusion effect is absent in all existing generators.
- Developing a P.S. algorithm based on the small-x evolution equation is important.

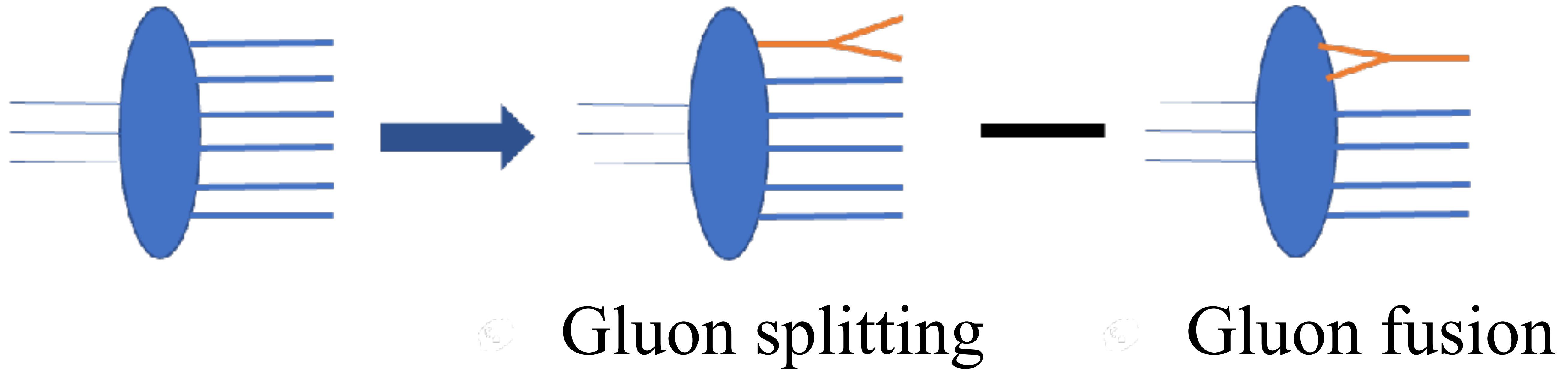
2) Novel Parton shower algorithm based on the Gribov-Levin-Ryskin (GLR) equation

YS, S. Y. Wei and J. Zhou, Phys.Rev.D 107, 016017 (2023).

YS, S. Y. Wei and J. Zhou, ArXiv: 2307.04185/hep-ph.

GLR evolution Equation

$$\eta = \ln \frac{1}{x}$$



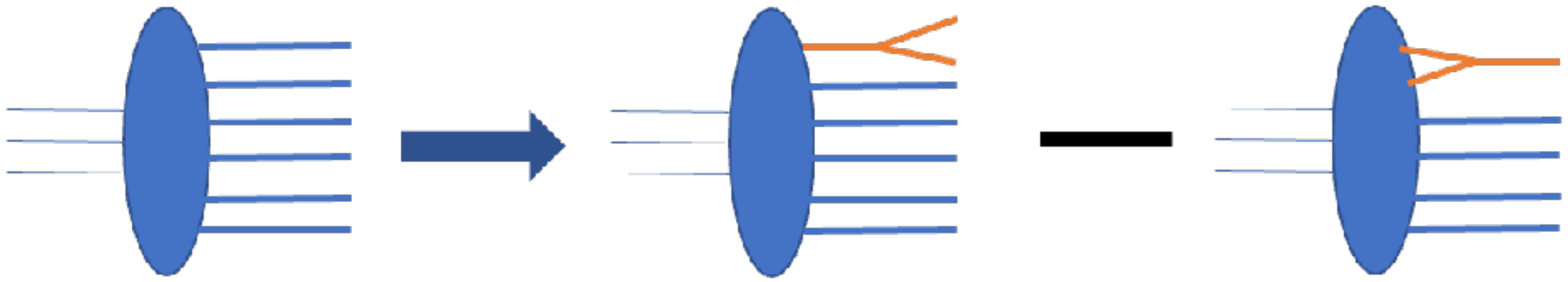
- The standard GLR equation (unfolded one) [Gribov, Levin, Ryskin, PR, 83]

$$\frac{\partial N(\eta, k_\perp)}{\partial \eta} = \frac{\bar{\alpha}_s}{\pi}\left[\int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} N(\eta, k_\perp + l_\perp) - \int_0^{k_\perp} \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} N(\eta, k_\perp)\right] - \bar{\alpha}_s N^2(\eta, k_\perp)$$

- GLR equation is the non-linear evolution equation that describes the gluon diffusion process.

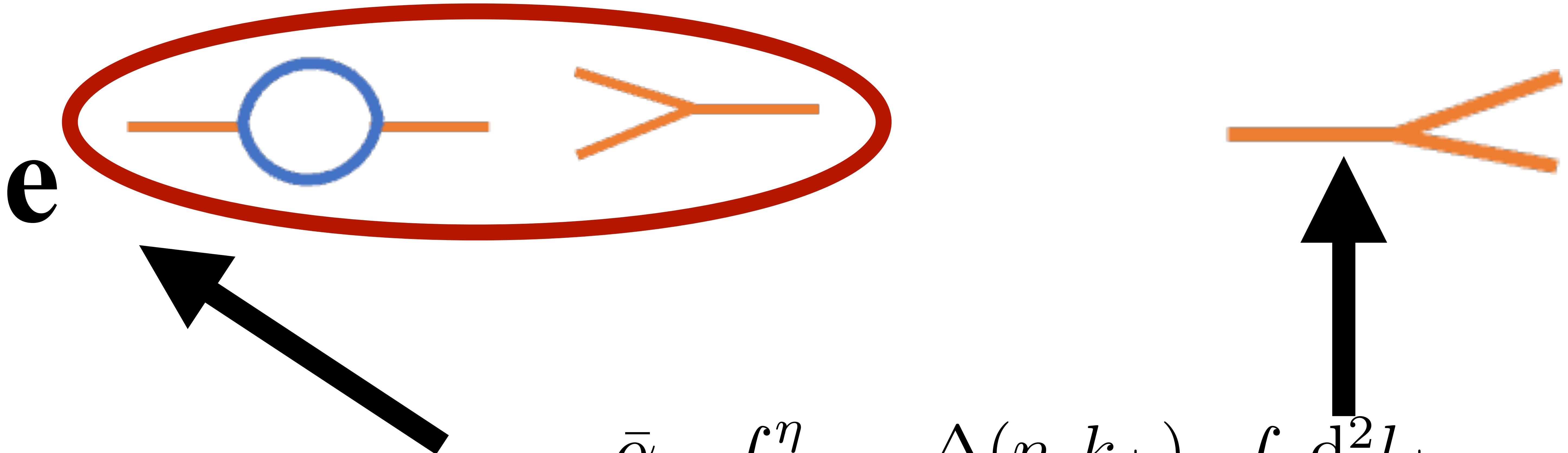
GLR evolution Equation

$$\eta = \ln \frac{1}{x}$$



Gluon splitting   Gluon fusion

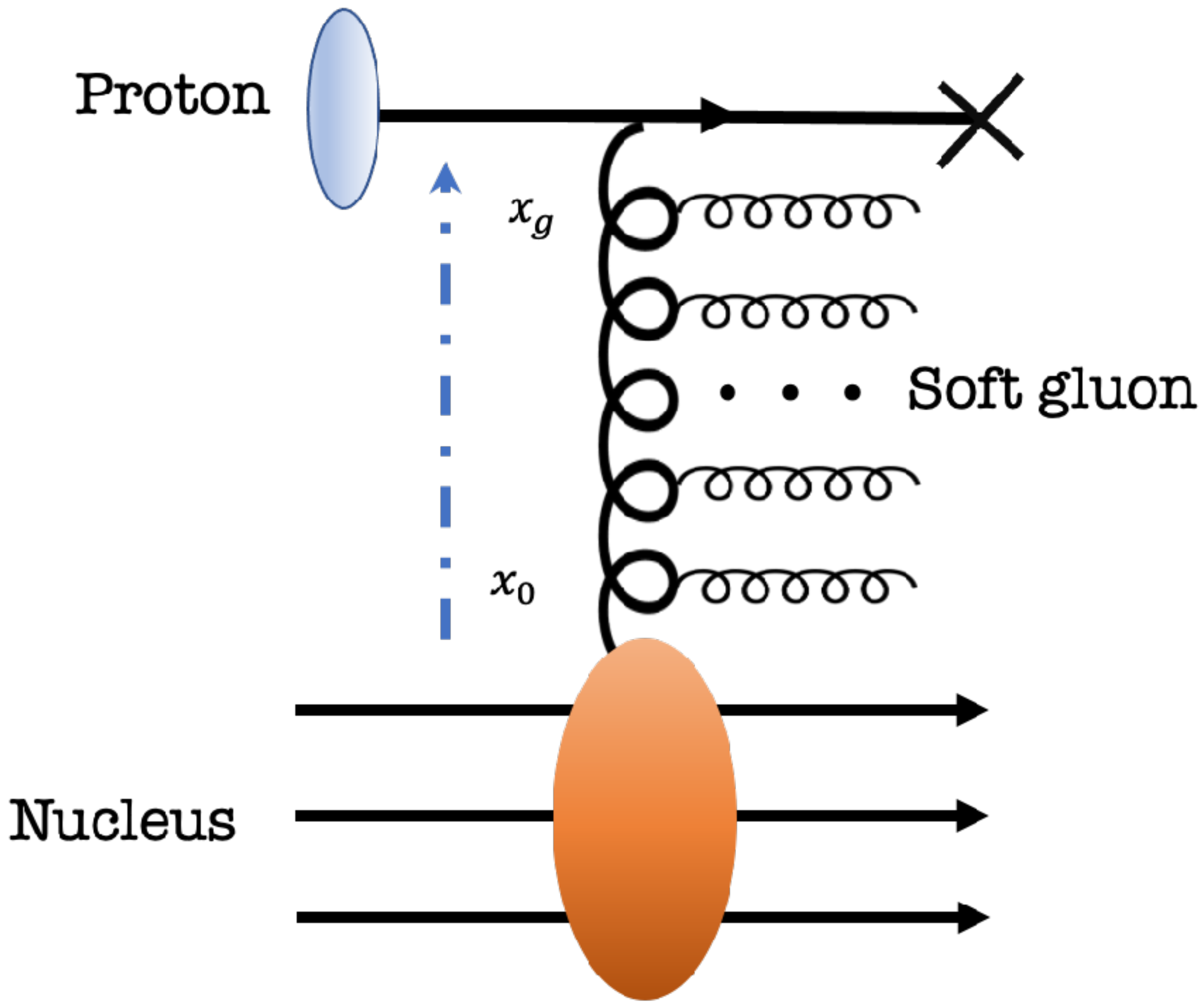
- Non-Sudakov form factor $\Delta(\eta, k_\perp) = \exp\left\{-\bar{\alpha}_s \int_{\eta_0}^{\eta} d\eta' \left[\ln \frac{k_\perp^2}{\mu^2} + N(\eta', k_\perp)\right]\right\}$
- The integral GLR equation (folded one)



$$N(\eta, k_\perp) = N(\eta_0, k_\perp)\Delta(\eta, k_\perp) + \frac{\bar{\alpha}_s}{\pi} \int_{\eta_0}^{\eta} \mathrm{d}\eta' \frac{\Delta(\eta, k_\perp)}{\Delta(\eta', k_\perp)} \int_\mu \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} N(\eta', l_\perp + k_\perp)$$

- GLR equation is the non-linear evolution equation that describes the gluon diffusion process.

The forward evolution



The forward evolution

- First step: non-Sudakov form factor

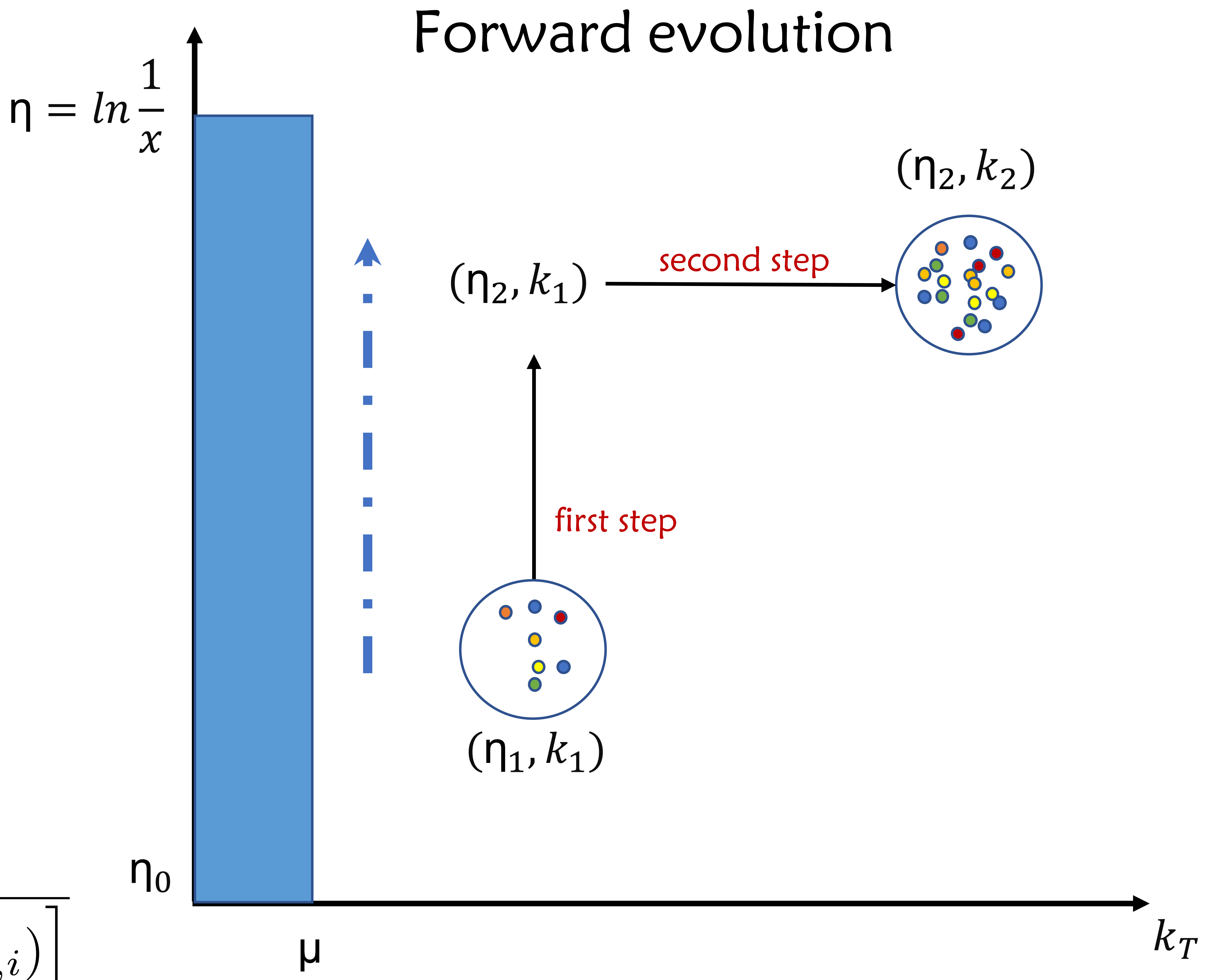
$$\mathcal{R} = \exp\left[-\bar{\alpha}_s \int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta' \left(\ln \frac{k_\perp^2}{\mu^2} + N(\eta', k_\perp)\right)\right]$$

- Second step: Real splitting kernel

$$\mathcal{R}_2 \int_{\mu}^{P_\perp} \frac{\mathrm{d}^2 l'_\perp}{l'^2_\perp} = \int_{\mu}^{|l_\perp|} \frac{\mathrm{d}^2 l'_\perp}{l'^2_\perp}$$

- The generated event has to be re-weighted

$$\mathcal{W}(\eta_i, \eta_{i+1}; k_{\perp,i}) = \frac{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \ln(P_\perp^2/\mu^2)}{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \left[\ln(k_{\perp,i}^2/\mu^2) + N(\eta, k_{\perp,i})\right]}$$



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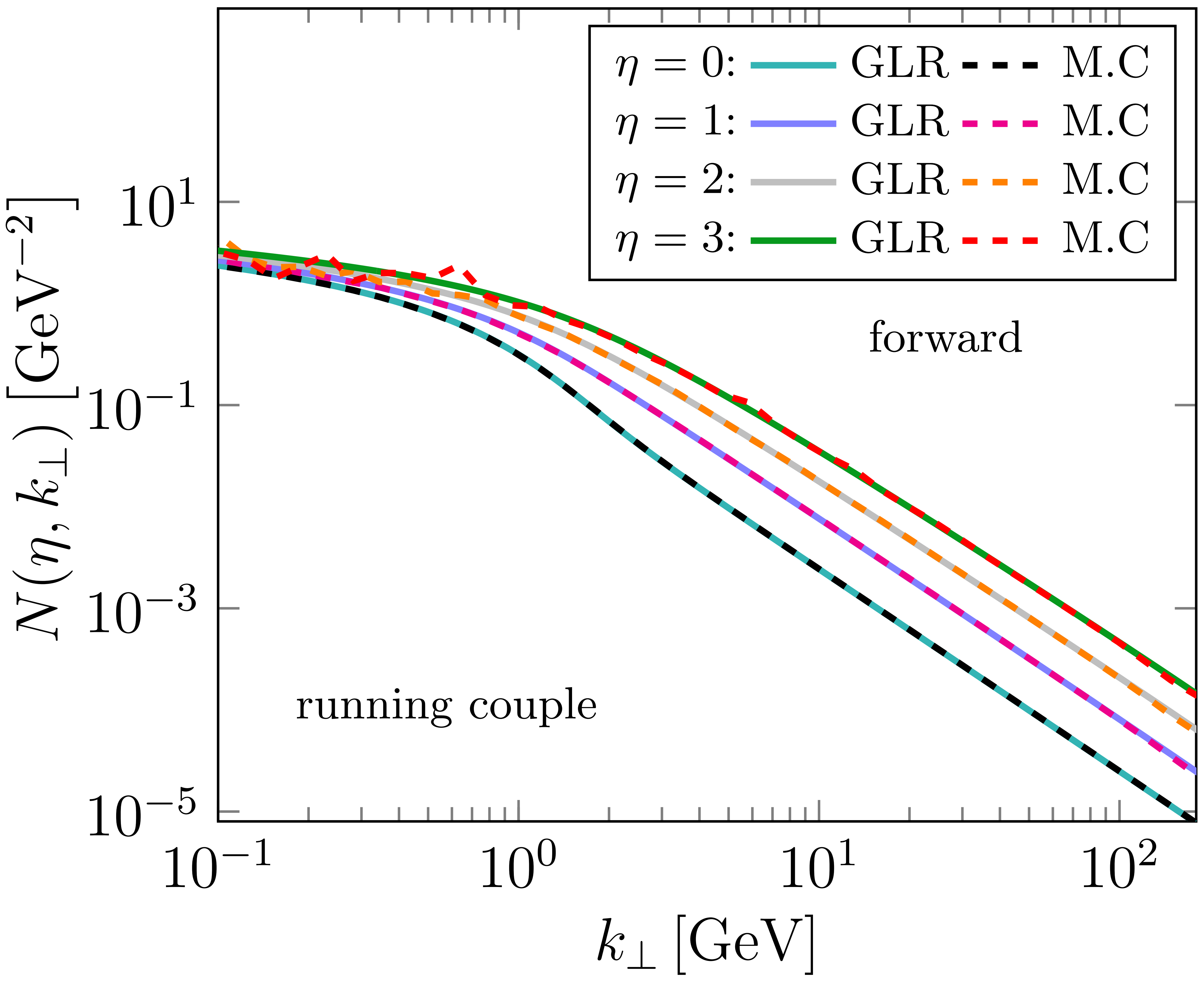
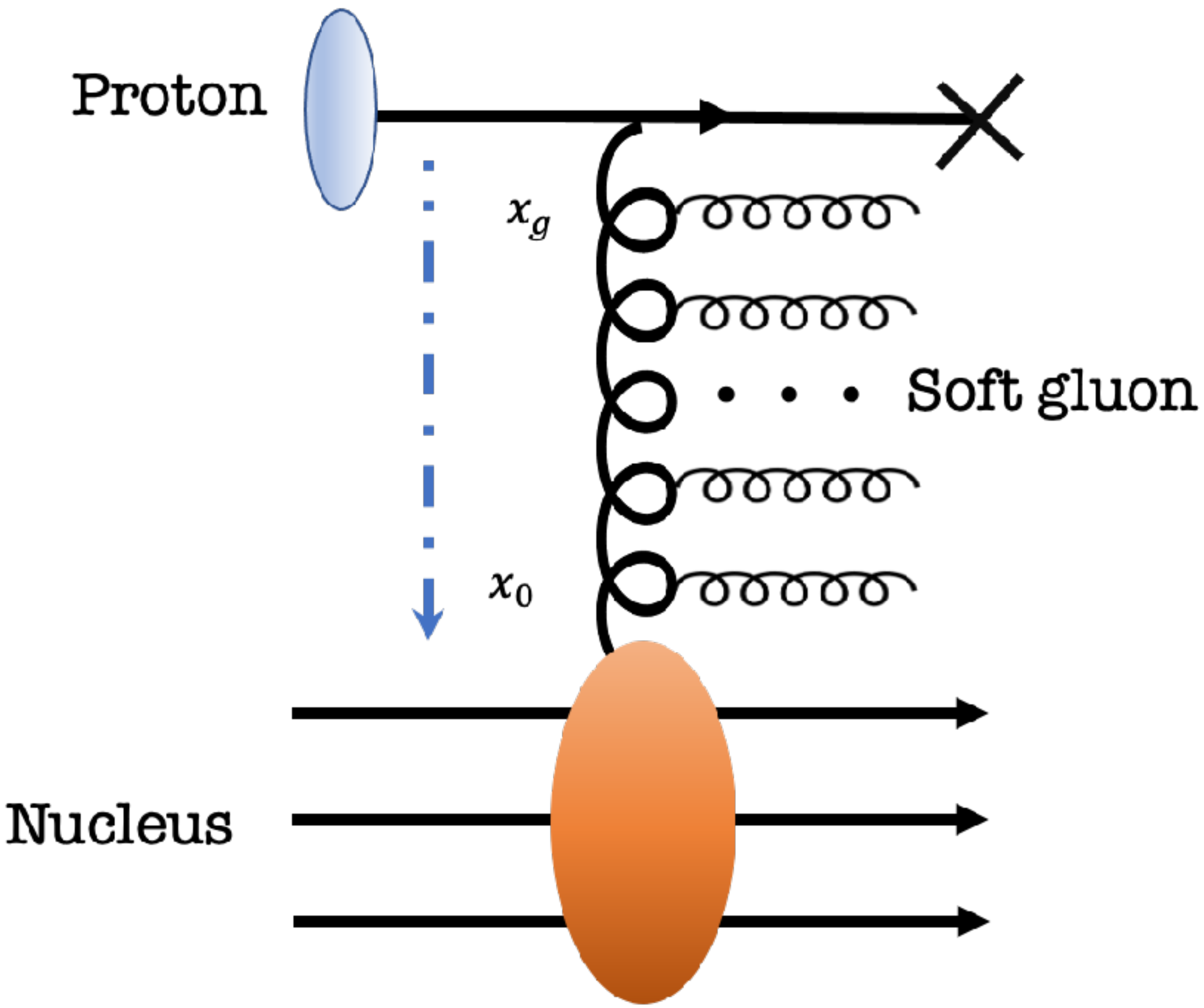


- Agree with the numerical solutions of the GLR equation.

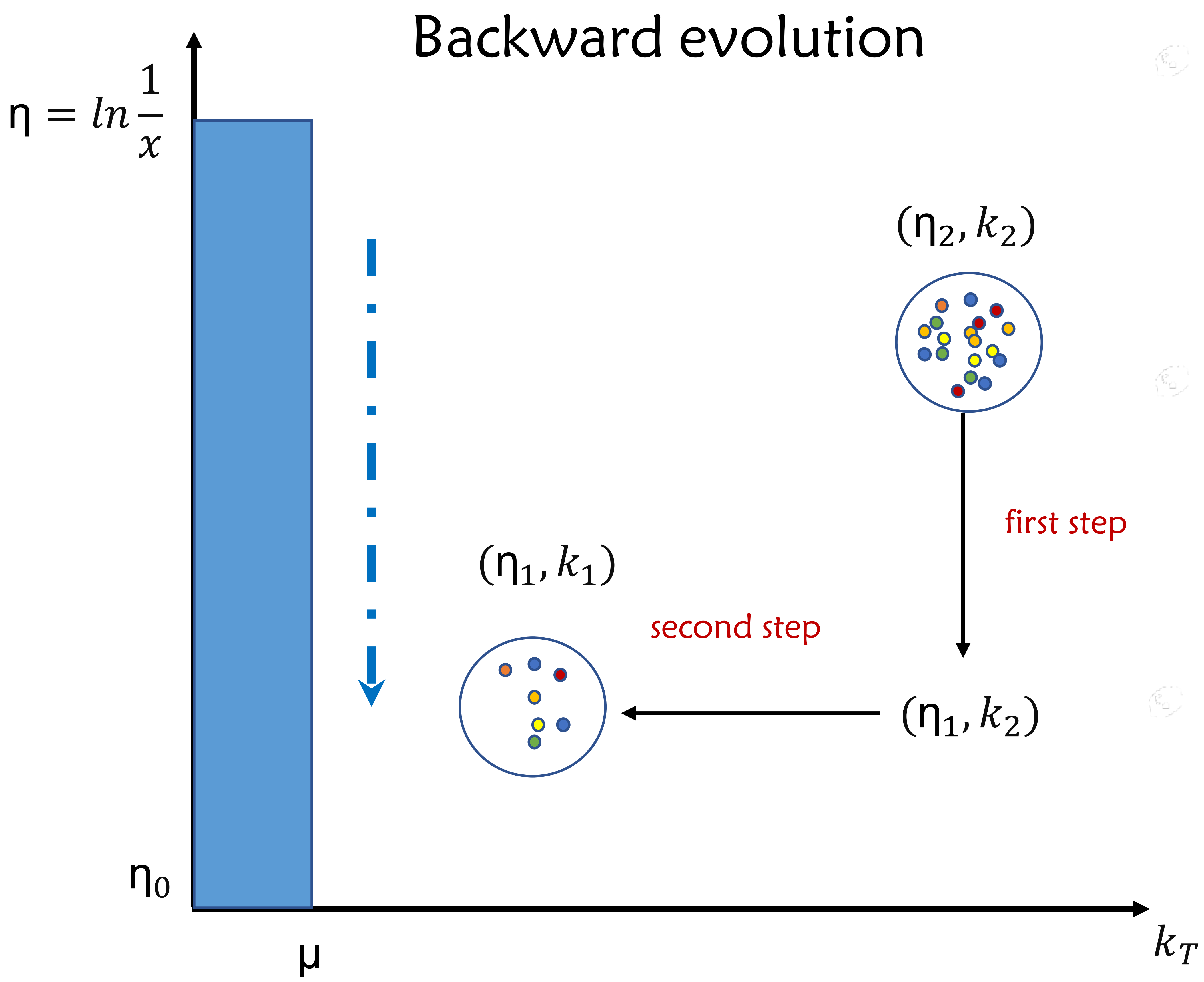
The backward evolution



$N(\eta, k_\perp)$

- As a more efficient procedure, the backward evolution approach is also presented.

The backward evolution



- First step: backward non-Sudakov form factor

$$\mathcal{R} = \exp\left[-\frac{\bar{\alpha}_s}{\pi}\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \int_{\mu} \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \frac{N(\eta, k_{\perp,i+1}+l_\perp)}{N(\eta, k_{\perp,i+1})}\right]$$

- Second step: Real splitting

$$\frac{\bar{\alpha}_s}{\pi}\int_{\mu}^{l_\perp} \frac{\mathrm{d}^2 l'_\perp}{l'^2_\perp} N(\eta_i, k_{\perp,i+1}+l'_\perp) = \mathcal{R}_2 \frac{\bar{\alpha}_s}{\pi}\int_{\mu}^{P_\perp} \frac{\mathrm{d}^2 l'_\perp}{l'^2_\perp} N(\eta_i, k_{\perp,i+1}+l'_\perp)$$

- The generated event has to be re-weighted

$$\mathcal{W}_{\mathrm{back}}(\eta_{i+1}, \eta_i; k_{\perp,i+1}, k_{\perp,i}) = \frac{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \left[\ln(k_{\perp,i}^2/\mu^2) + N(\eta, k_{\perp,i})\right]}{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \ln(P_\perp^2/\mu^2)}$$

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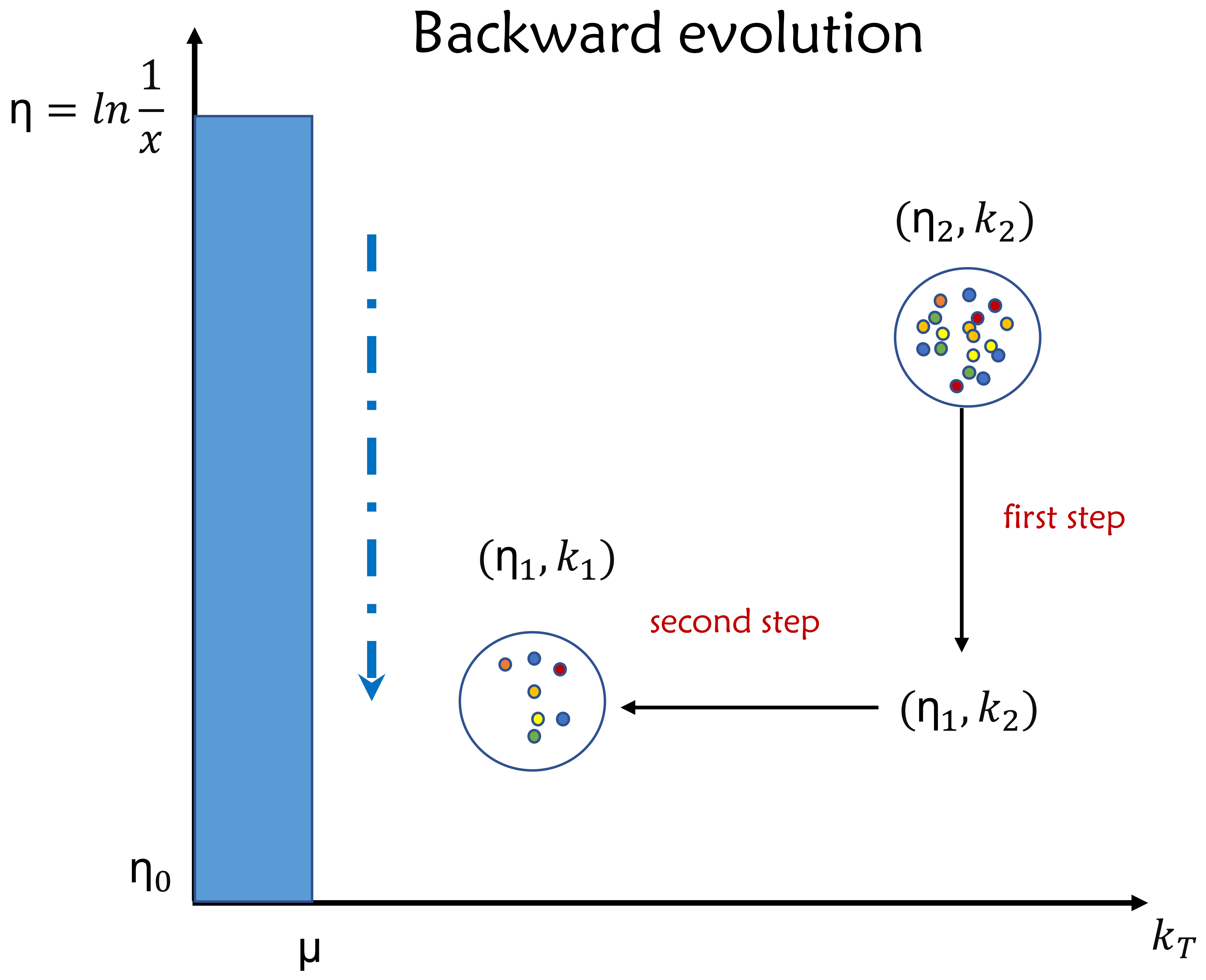
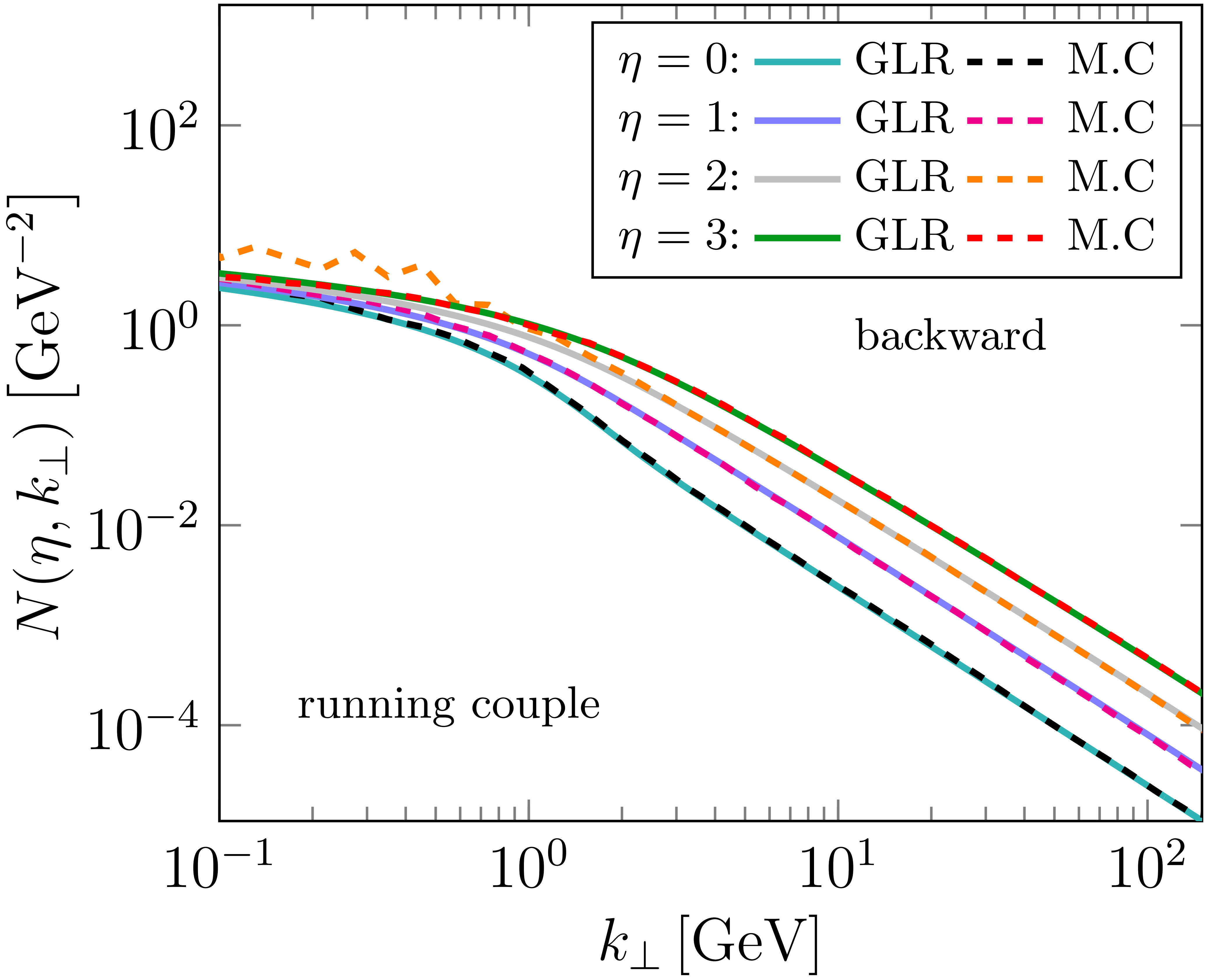
The backward evolution





- As a more efficient procedure, the backward evolution approach is also presented.
- Agree with the numerical solutions of the GLR equation.

Parton shower

GLR	v.s.	DGLAP
gluon splitting gluon fusion		parton splitting

The evolution variable:

$\eta = \ln(1/x)$	Q

The generated event:

reweight	Unitary

Summary and outlook

- The first parton shower algorithm incorporating gluon fusion is based on the GLR evolution equation.

- Di-jet in EIC

Thank you !