An introduction to TMDs.

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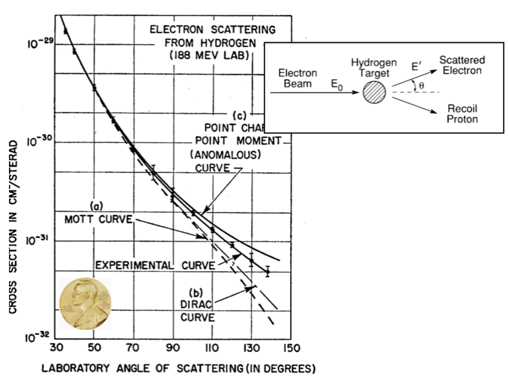
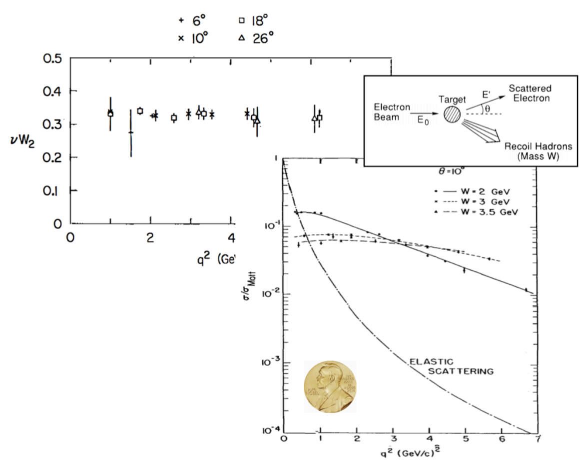
Goal of this lecture:

- gauge link in TMD
- The Idea of scale dependence
- Quantum effects in TMD

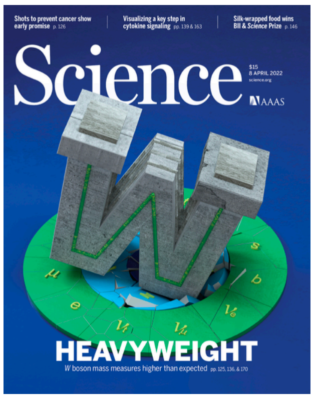
Why hadron structures?

- A fruitful field w/ a long history
  - * Proton is not fundamental   1950's
  - * DIS   1960's – 1970's
    - ⇒ Parton Model
    - ⇒ discovery of asymptotic freedom ←
    - ⇒ discovery of QCD ←   50 years today.





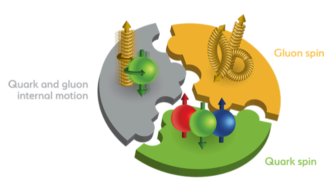
* Crucial input for the LHC physics.



⇒ Major uncertainties

e.g. Higgs. W mass, ...

* A lot yet to answer



⇒ Spin puzzle

⇒ mass decomposition

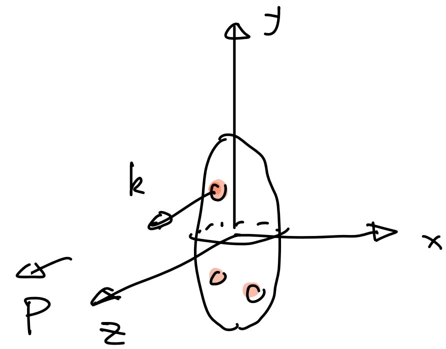
⇒ gluon saturation

} major focus of EIC & EicC

See Liang's talk ×2

What is TMD?

— Collinear PDF



$f(\xi)$, $\quad \xi \equiv k^+/p^+$

$\simeq$ Prob. to find a parton w/ longitudinal momentum fraction ξ

* The information is incomplete.

$\Longrightarrow$ transverse? spin? ...

* observed large polarization in contrast with (naive) collinear predictions

$\propto \frac{4\alpha_s}{3} \frac{m_q}{Q^2}$ (Kane, et al, PRL 41, 1978)

VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

Transverse Quark Polarization in Large-p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

and

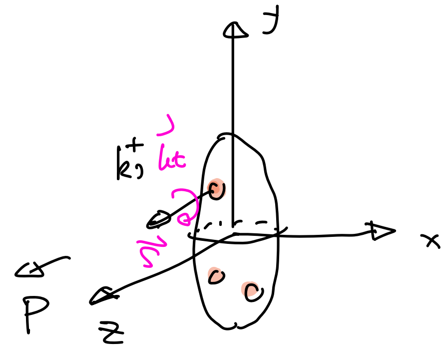
J. Pumplin and W. Repko
Physics Department, Michigan State University, East Lansing, Michigan 48823

(Received 5 July 1978)

We point out that the polarization P of a scattered or produced quark is calculable perturbatively in quantum chromodynamics for $e^+e^- \to q\bar{q}$, large-p_T hadron reactions, and large-Q^2 leptoproduction, and is infrared finite. The quantum-chromodynamics prediction is that $P = 0$ in the scaling limit. Experimental tests are or will soon be possible in $pp \to \Lambda X$ [where presently $P(\Lambda) \simeq 25\%$ for $p_T > 2$ GeV/c] and in $e^+e^- \to$ quark jets.

— TMD (Transverse momentum dependent) PDF

$f(\xi, \vec{k}_t)$



$\simeq$ Prob. to find a parton

w/ ξ and transverse momentum $\vec{k}_t$,

PDF

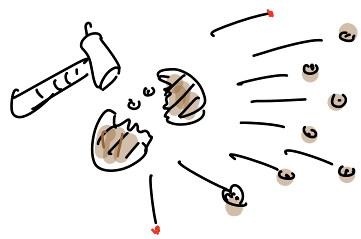
		Parton Polarization		
		Unpol.	longitudinal	transverse
Nucleon. Polar.	U	$f_1 =$		$h_1 =$ Boer-Mulders
	L		$g_{1L} =$ helicity	
	T	$f_{1T}^{\perp} =$ Sivers		

F.F.

	U	T
U	$D_1 =$	$H_1^{\perp} =$ Collins

Some Comments

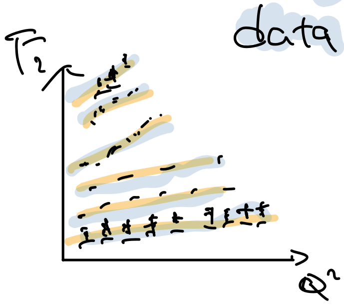
- How to probe ?



Smash the proton

$$\sigma = \hat{\sigma} \otimes f$$ — global fit

data

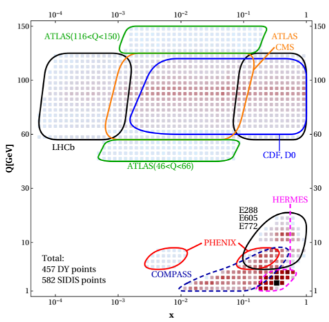


Perturbative Calculation to high precision - fixed order
- resummation

SV19

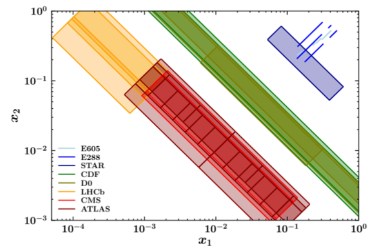
Drell-Yan (457 bins)

SIDIS (582 bins)



Pavia19

Drell-Yan (353 bins)

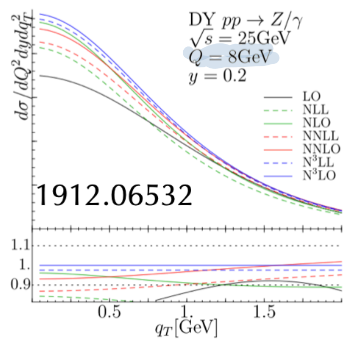


$x_1 = Qe^{y}/\sqrt{s}, \quad x_2 = Qe^{-y}/\sqrt{s}$

⇐ G data choice

SV19

G precision ⇒



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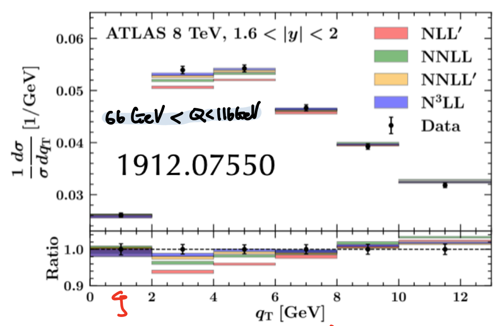
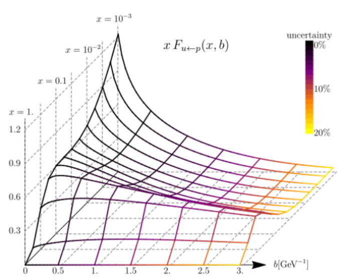
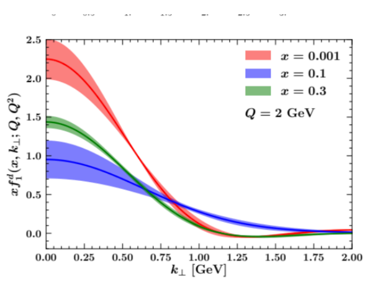


Suppression due to $e^{-\frac{Q}{q_T}}$

Sudakov suppression





⇐ fit to extract TMDs.

not bad precision, if assumed fitting form

* for TMD

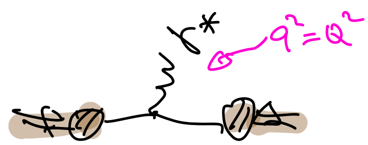
⇒ Drell-Yan, usually low Q^2

⇒ SIDIS,

} Quite limited data sets

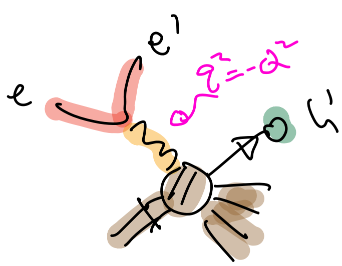
Drell-Yan $\quad h(P_1,S_1) + h(P_2,S_2) \to \gamma^*/Z(q)$

$q_t \lll Q$



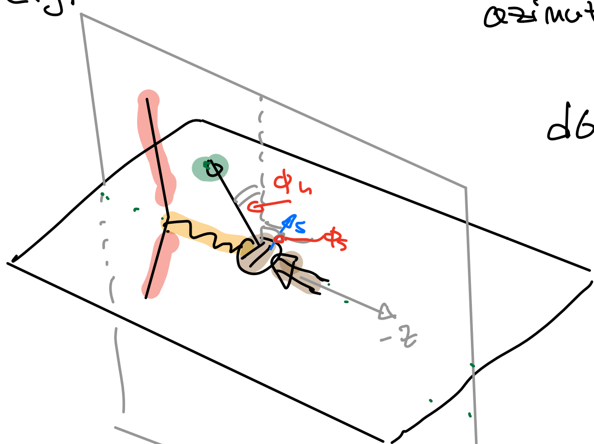
SIDIS. $\quad \ell(\ell,h) + p(P,S) \to \ell(\ell') + h(P_h) + X$

helicity $\quad$ spin $\quad$ $P_{ht} \lll Q$



many different frames to use

e.g.



all angles defined w.r.t lep-γ-proton plane

Internal structure translates to azimuthal asymmetries

$$d\sigma \sim F_{UU,T} + \cos(2\phi_h) F_{UU}^{\cos 2\phi} + S_T \sin(\phi_h - \phi_S) F_{UT}^{\sin(\phi_h-\phi_S)} + \cdots$$

Boer-Mulders

Sivers

$\Rightarrow$ 8 structures at leading power

vs limited data sets

Challenging !!

$\Rightarrow$ Possible soft contamination

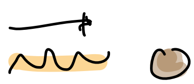
- What proton the PDFs describe?

  * Factorization Theorem

$$\sigma = \hat{\sigma} \otimes F$$

$\Rightarrow$ assume the proton is inf. boosted

$\Rightarrow$ σ is boost invariant

 v.s. 

fixed target | lab. frame

$f(x, k_t)$ for an inf. boosted proton

- Definition of the PDFs.

$f(x, \vec{k}_t)$

$$\sim \int_{-\infty}^{+\infty} dy^- d^2\vec{y}_t \; e^{-ixp^+y^-} e^{-i\vec{k}_t\cdot\vec{y}_t}$$

$$\langle P | \bar{\xi}(y^-, \vec{y}_t) \frac{\gamma^+}{2} W^\dagger[\gamma'] W[\underset{\text{process dependent path.}}{\gamma}] \xi(0) | P\rangle$$

* ξ : (Collinear) quark field $\Rightarrow$ boosted

See Dingyu's lecture

* gauge Link

$\Rightarrow$ gauge inv.

$\Rightarrow$ final/initial state interaction

$\Rightarrow$ fact. breaking effects, Sivers

$\Rightarrow$ originated from the inf. boost picture

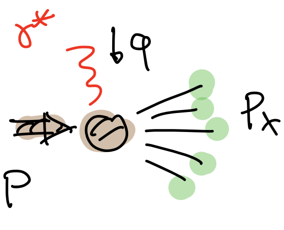
DIS $q^2 = -Q^2$, $x_B \equiv \frac{Q^2}{2P\cdot q}$

Breit Frame:

$q \sim (0,0,0,-Q)$

$P \sim \frac{Q}{2x_B}(1,0,0,1)$

$x_B P + q \sim (\frac{Q}{2}, 0, 0, -Q/2)$

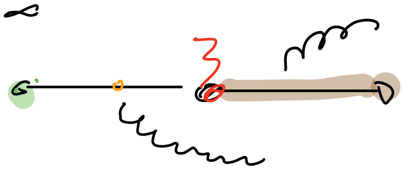


$Q \to \infty$

inf. boost

$n\cdot P_X \equiv P_X^- \to \infty$

$\bar{n}\cdot P \equiv P^+ \to \infty$

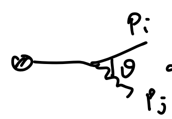


$\bar{n} = (1,0,0,-1)$

$n = (1,0,0,1)$

Possible dominant radiations :

n-collinear radiation $\Leftarrow$ the reason for PDF.

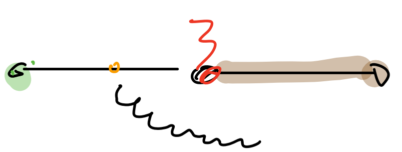


$$\propto \frac{1}{2 p_i \cdot p_j} \sim \frac{1}{E_i E_j (1-\cos\theta_{ij})} \xrightarrow[\theta_{ij} \to 0]{} \infty$$

$\bar{n}$-collinear radiation } For SIDIS

soft radiation } See Shao's lecture

$\Rightarrow$ We are particularly interested in the radiation emitted from $P_x \parallel \bar{n}$ but propagate along n.



emitted from $\bar{n}$
propagate along n

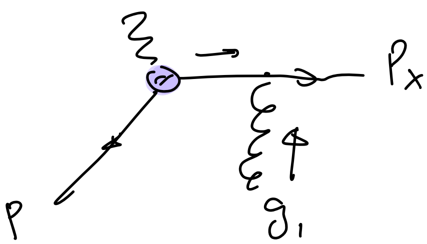
See Belitsky, Ji, Yuan, 0208038

Brodsky, Hwang, Schmidt, 0206259

Rothstein, Stewart, 1601.04695

+ ...

Consider one emission:



$p_X^- \to \infty$

$g_1 \parallel p$ then take $g_1^+ \to 0$ limit

For simplicity, we require $p_X^\mu = p_X^- \frac{\bar{n}^\mu}{2}$

also

$$\bar{u}_X \not{p}_X = 0 = \bar{u}_X p_X^- \frac{\not{\bar{n}}}{2} \Rightarrow \bar{u}_X \not{\bar{n}} = 0$$

and

$$g^\mu = \underset{O(\lambda)}{g^+ \frac{n^\mu}{2}} + \underset{O(\lambda)}{g^- \frac{\bar{n}^\mu}{2}} + \underset{O(\lambda)}{g_t^\mu} \qquad \lambda \sim \frac{g_t}{Q} \ll 1$$

$$\Rightarrow \quad \bar{u}_X \, i g_s \not{\epsilon}(g_1) \, i \frac{\not{p}_X - \not{g}_1}{(p_X - g_1)^2 + i\epsilon}$$

$$= \bar{u}_X \, i g_s \left(\underbrace{\cancel{\epsilon_1^+ \frac{\not{\bar{n}}}{2}}}_{0} + \epsilon_1^- \frac{\not{n}}{2} + \not{\epsilon}_{1t} \right) i \frac{\not{p}_X - \not{g}_1}{-p_X^- g_1^+ - g_{1t}^2 + i\epsilon}$$

$$= \bar{u}_x \, i g_s \, \epsilon_1^{-} \frac{\alpha}{2} \, i \left(\frac{p_x^- - g_1^-}{-p_x^- g_1^+ - g_{1t}^2 + i\epsilon} \right)$$

longitudinal

$$+ \bar{u}_x \, i g_s \, \underbrace{\not{\epsilon}_{1t}}_{\text{transverse}} \, i \, \frac{\cancel{p_x^-} - g_1^-}{-p_x^- g_1^+ - g_{1t}^2 + i\epsilon}$$

$$\simeq \bar{u}_x \, i g_s \, (-i) \frac{\epsilon^-(g_1)}{g_1^+ - i\epsilon} \sim \mathcal{O}\left(\frac{1}{\lambda}\right)$$

$$+ \bar{u}_x \, i g_s \, \not{\epsilon}_{1t} \, \frac{i\left(-g_1^+ \frac{\not{\alpha}}{2} - \not{g}_{1t}\right)}{-p_x^- g_1^+ - g_{1t}^2 + i\epsilon} \sim$$

naively vanishes as $p_x^- \to \infty$

however, could be dominant if $g_1^+ \to \infty$ faster than p_x^-

Glauber potential !!

$$\simeq \bar{u}_x \, i g_s \, (-i) \frac{\bar{n} \cdot \epsilon(g_1)}{g_1^+ - i\epsilon}$$

$\Leftarrow$ eikonal approx. gives rise to Wilson line

$$+ \bar{u}_x \, i g_s \, i \, \frac{\vec{\not{\epsilon}}_t(g_1) \, \vec{g}_{1t}}{\vec{g}_{1t} - i\epsilon}$$

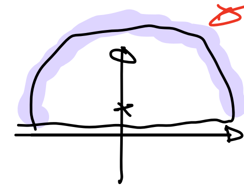
$\Leftarrow$ Glauber Potential gives rise to a phase that breaks the fact. Theorem

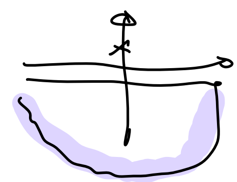
$g_1^+ \to \infty \Rightarrow y \to \infty$

Now let us take the Fourier transformation to go to the position space, by noting ~~that~~

integrand vanishes on the boundary

$$\int \frac{dq_1^+}{(2\pi)} \frac{1}{q_1^+ - i\epsilon} e^{i q_1^+ y^-/2} = \begin{cases} \quad & y^- > 0 \\ = 0 & y^- < 0 \end{cases}$$





$$= i\,\theta(y^-)$$

and

$$\int \frac{d^2\vec{q}_t}{(2\pi)^2} \frac{\vec{q}}{\vec{q}_{1t}^{\,2} - i\epsilon} e^{-i\vec{q}_t\cdot\vec{y}_t}$$

$$= i\vec{\partial} \int \frac{d^2\vec{q}_t}{(2\pi)^2} \frac{1}{\vec{q}_{1t}^{\,2} - i\epsilon} e^{-i\vec{q}_t\cdot\vec{y}_t}$$

$$= -\frac{i}{2\pi}\vec{\partial} \ln|\vec{y}_t| = \frac{-i}{2\pi}\frac{\vec{y}_t}{\vec{y}_t^{\,2}}$$

To write

$$\simeq \bar{u}_x \, i\frac{g_s}{2}\int dy_1^- \, \bar{n}\cdot A(y_1^-, \vec{y}_t)\, \theta(y_1^- - y)$$

$$+ \bar{u}_x \, i\frac{g_s}{2\pi}\int d\vec{y}_{1t} \, \vec{A}_t(\infty, \vec{y}_{1t})\, \vec{\nabla} \ln|\vec{y}_t - \vec{y}_{1t}|$$

$y_1^+ \to 0 \Rightarrow y^- \to \infty$

at the boundary $A \to U\cancel{A}U^{-1} - \frac{i}{g_s}U\nabla U^{-1}$

$0 \quad A \to 0$

$\Rightarrow$ pure phase $A = -\frac{i}{g_s}U\nabla U^{-1}$ at boundary.

$$\simeq \bar{u}_x \, i g_s/_2 \int dy_1^- \, \bar{n}\cdot A(y_1^-, \vec{y}_t)\, \theta(y_1^- - y^-)$$

$$+ \bar{u}_x \, i\frac{g_s}{2\pi}\int d\vec{y}_{1t} \, \cancel{\nabla} \, \Phi(\infty, \vec{y}_{1t})\, \cancel{\nabla} \ln|\vec{y}_t - \vec{y}_{1t}|$$

$$\simeq \bar{u}_x \, i g_s/_2 \int dy_1^- \, \bar{n}\cdot A(y_1^-, \vec{y}_t)\, \theta(y_1^- - y^-)$$

$$+ \bar{u}_x \, -i\frac{g_s}{2\pi}\int d\vec{y}_{1t} \, \Phi(\infty, \vec{y}_{1t})\, \underbrace{\vec{\nabla}^2 \ln|\vec{y}_t - \vec{y}_{1t}|}_{(2\pi)\,\delta(\vec{y}_t - \vec{y}_{1t})}$$

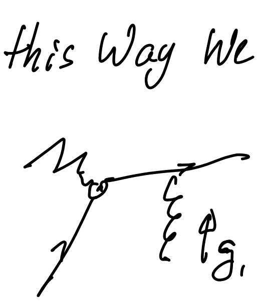
$$\simeq \bar{u}_x \, i g_s/_2 \int dy_1^- \, \bar{n}\cdot A(y_1^-, \vec{y}_t)\, \theta(y_1^- - y^-)$$

$$\bar{u}_x \, -i g_s \, \Phi(\infty, \vec{y}_t)$$

Note that $\vec{\nabla}\Phi(\infty, \vec{y}_t) = \vec{A}_t(\infty, \vec{y}_t)$

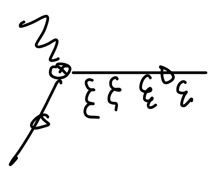
$$\Rightarrow \quad \Phi(\infty, \vec{y}_t) = -\int_{\vec{y}_t}^{\infty} \vec{A}_t(\infty, \vec{y}_t)\, d\vec{y}_{1t}$$

In this way we have



$$= \left\{ ig_s \int_{y^-}^{\infty} dy_1^{\mu} A_{\mu}(y_1^-, \vec{y}_t) - ig_s \int_{\vec{y}_t}^{\infty} dy_1^{\mu} A_{\mu}(\infty, \vec{y}_{1t}) \right] \xi(y^-, \vec{y}_t)$$

$$\Rightarrow \quad \sum_{\substack{i=0 \\ \text{all perm.}}}^{\infty}$$

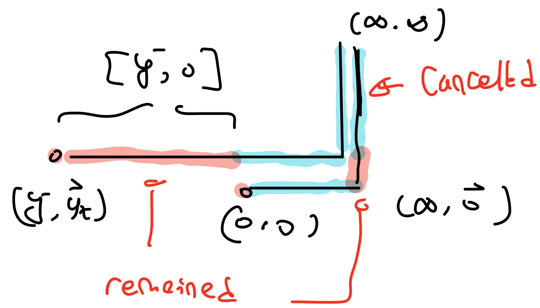


$$= P\exp\left[ig_s \int_{y^-}^{\infty} dy_1^{\mu} A_{\mu}(y_1^-, \vec{y}_t)\right]$$
$$P\exp\left[-ig_s \int_{\vec{y}_t}^{\infty} dy_1^{\mu} A_{\mu}(\infty, \vec{y}_{1t})\right] \quad \xi(y^-, \vec{y}_t)$$

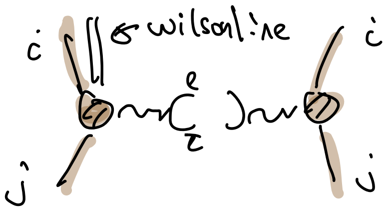
$W[\gamma] =$

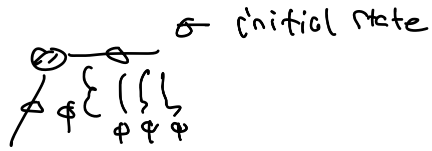
TMD in SIDIS

$$f_{SIDIS} \propto \bar{\xi}(y^-; \vec{y}_t)\, W^\dagger[\gamma']\, W[\gamma]\, \xi(0,\vec{0})$$

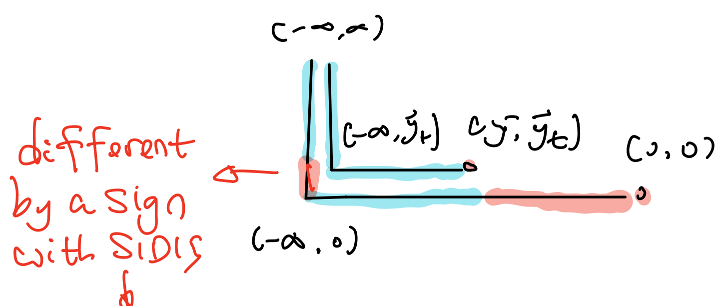


TMD in Drell-Yan





in view of j



$\Rightarrow$ leads to a sign flip in Sivers.

$$f_{1T}^\perp\big|_{SIDIS} = -f_{1T}^\perp\big|_{\text{Drell-Yan}}$$

* "Bare definition" $\int d^2k_t\, f(x, \vec{k}_t) = f(x)$

holds for the "bare" PDF.

but breaks down for

"renormalized RGE" PDF.

$f(x, \vec{k}_t)$

$$\sim \int_{-\infty}^{+\infty} dy^- d^2 y_t \, e^{-ixPy^-} e^{-i \vec{k}_t \vec{y}_t}$$

$$\sum_X \frac{\gamma^+_{\alpha\beta}}{2} \langle P | T^\dagger T \bar{\xi}_\alpha(y^-, y_t) T^\dagger T |X\rangle\langle X| \xi_\beta(0) | P \rangle$$

translational operation

$\sum_X |X\rangle\langle X| = 1$ complete set

$$= \sum_X \int_{-\infty}^{+\infty} dy^- d^2 \vec{y}_t \, e^{-ixPy^-} e^{-i\vec{k}_t \vec{y}_t} \; e^{iPy^-} e^{i\vec{P}_t \vec{y}_t} \, e^{-iP_X y^-} e^{-i\vec{P}_{Xt}\vec{y}_t}$$

$$\frac{\gamma^+_{\alpha\beta}}{2} \langle P | \bar{\xi}_\alpha(0) |X\rangle\langle X| \xi_\beta(0) |P\rangle$$

$$= \underbrace{\delta\left((1-x)P - P_X^+\right) \delta^{(2)}\left(\vec{P}_t - \vec{k}_t - \vec{P}_{Xt}\right)}_{\text{Conservation law of the proton system}} \frac{\gamma^+_{\alpha\beta}}{2} \left| \langle P | \bar{\xi}_\alpha(0) | X \rangle \right|^2$$

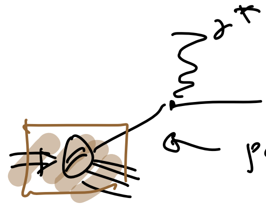
* Matrix element squared for

← Parton being detected by the "hard" interaction. "not well defined"

} X, inclusive over the rest

* ambiguity, running of the PDFs

LO

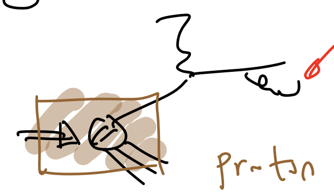


parton being detected by the "hard interaction"

Proton system

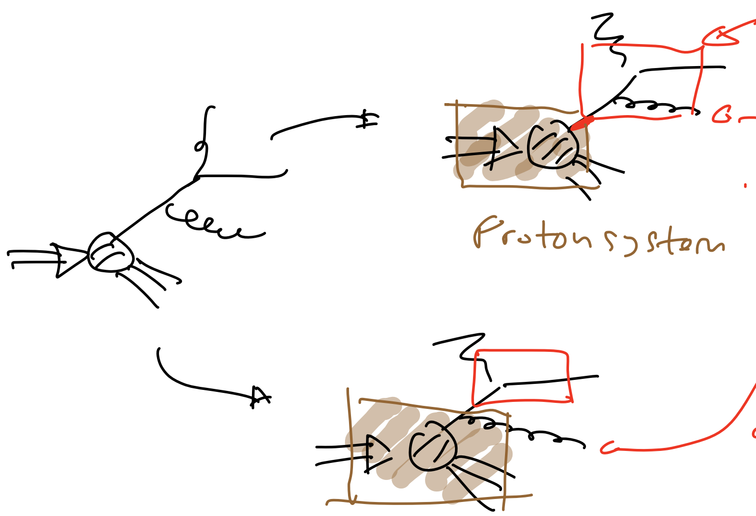
"well-defined" at LO, naive parton model

NLO



final radiation, okay ✓

proton system



hard

excluded ?

Proton system

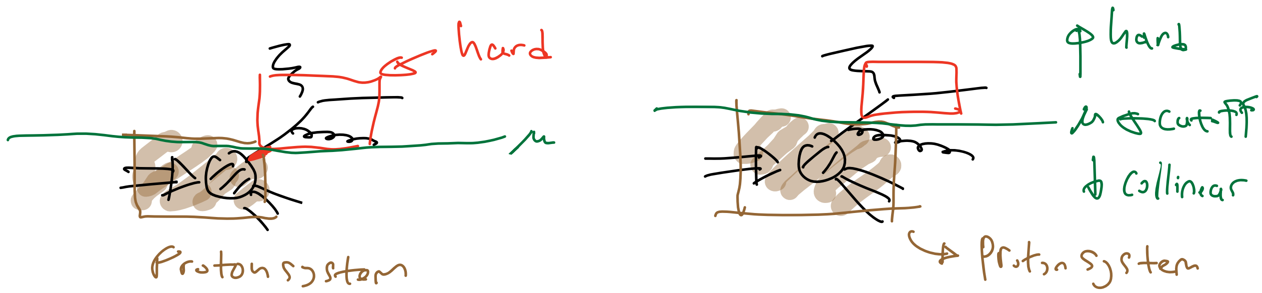
$f(x, \Lambda_{cut})$

included ?

Proton system

+ interference ? ⟹ breaks naive factorization ?

In some sense "artificial" a parameter of the cross-section.



both are good as long as a cutoff is specified to define the PDFs.

part of the "def" of a pdf.

$$\frac{d\sigma}{d\ln\mu} = 0 \Rightarrow \frac{d\hat{\sigma}}{d\ln\mu} = -\frac{df}{d\ln\mu}$$

$$\frac{df}{d\ln\mu} = D \otimes f, \quad \frac{d\hat{\sigma}}{d\ln\mu} = -D \otimes f.$$

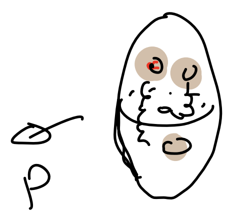
evolution kernel, determined by "poles"

≠ DGLAP for Coll. PDF

≠ Sudakov for TMD

* Theoretical Fundament for the Factorization scale-choice

- More than Classic Prob.



: quantum system

$F(x,t)$ — classic Prob.

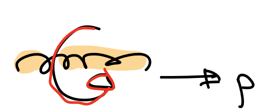
— evolution & Quantum effect.

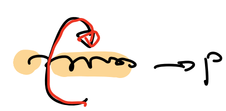
Other Quantum effects

* Interference

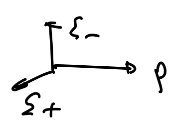
Parton has helicities (inf. boost + Massless limit)

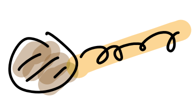
gluon





$\varepsilon_+^\mu = (0,1,i,0)$ $\qquad$ $\varepsilon_-^\mu = (0,1,-i,0)$

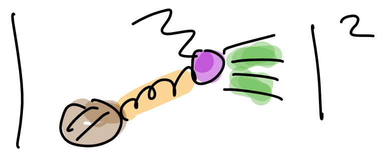




unpolarized proton

$|g\rangle = f_+|+\rangle + f_-|-\rangle$

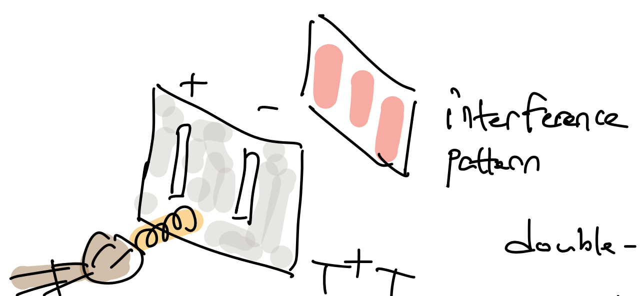
superposition



$$\propto \left| T|g\rangle \right|^2 = \langle g|T^\dagger T|g\rangle$$

$$= \underbrace{|g_+|^2\langle +|T^\dagger T|+\rangle + |g_-|^2\langle -|T^\dagger T|-\rangle}_{\text{classic}}$$

$$+ \underbrace{g_+^* g_- \langle +|T^\dagger T|-\rangle + g_-^* g_+ \langle -|T^\dagger T|+\rangle}_{\text{interference}}$$



double-slit experiment in the helicity space

if both "+" and "−" are open

$\Rightarrow$ "full" description of proton structure by density matrix

$$\hat{\rho} \equiv \rho_i^* |i\rangle\langle j| \rho_j = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$

$$\rho_{ij} \equiv \rho_i^* \rho_j$$

$\Rightarrow$ Unpolarized PDFs.

$$f \propto \frac{1}{2} \mathrm{Tr}(\hat{\rho}) = \frac{1}{2}(\rho_{++} + \rho_{--})$$

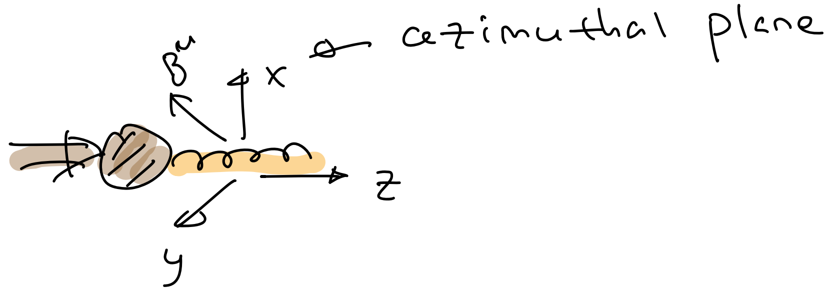
averaged over all helicities

$\Rightarrow$ off diagonal elements encode the quantum interference information

$\Rightarrow$ Boer-Mulders Function in the context of TMDs.

$\Rightarrow$ e.g. linearly polarized gluon

$$f_g^{\mu\nu}(x,k_t) \sim \int e^{-ixPy^-} e^{i\vec{k}_t\cdot\vec{y}_t} \langle P|\, B^{\mu}(y^-,\vec{y}_t)\, B^{\nu}(0)\,|P\rangle$$



$$\sim g_t^{\mu\nu} f_g(x,k_t) + \left(\frac{k_t^\mu k_t^\nu}{k_t^2} + \frac{1}{2} g_t^{\mu\nu}\right) h_g(x,k_t)$$

$$g_t^{\mu\nu} = g^{\mu\nu} - \frac{P^\mu \bar{n}^\nu + P^\nu \bar{n}^\mu}{\bar{n}\cdot P}, \qquad P = Q(1,0,0,1), \quad \bar{n} = (1,0,0,-1)$$

- only tensor structure allowed by the rotational symmetry + ...

$$g_t^{\mu\nu}, \quad \cancel{\epsilon_t^{\mu\nu}}, \quad k_t^\mu k_t^\nu$$

- When sum over the Polarization, reduced to the unpolarized TMD

$$\sum_{Pol.} \epsilon_\mu^{+} \epsilon_\nu^{*+} + \epsilon_\mu^{-} \epsilon_\nu^{*-} = -g_{\mu\nu,t}$$

$$\Rightarrow \quad -g_{\mu\nu,t} f_g^{\mu\nu} \sim f_g$$

- h_g is from interference between "+" & "−" helicities.

$$\epsilon_+^\mu = \frac{1}{\sqrt{2}}(0, 1, i, 0)$$

$$\epsilon_-^\nu = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$

$$\epsilon_+ \cdot \epsilon_+^* = -1 - (i)(-i) = -1$$

$$\epsilon_+ \cdot \epsilon_-^* = 0$$

$$\Rightarrow \quad \epsilon_+^\mu \epsilon_-^{*\nu} f_{\mu\nu} = \epsilon_+^\mu \epsilon_-^{*\nu} \frac{k_t^\mu k_t^\nu}{k_t^2} h_g$$

$$= \frac{1}{2} \frac{(k_x + i k_y)^2}{k_t^2} h_g = \frac{1}{2} e^{2i\phi} h_g$$

phase leads to $\cos 2\phi$ interference

$$\mathcal{M}^{\mu\nu} f_{\mu\nu} \simeq \mathcal{M}^{\mu\nu}\, \epsilon^{+}_{\mu}\epsilon^{*+}_{\mu'}\, \epsilon^{*-}_{\nu}\epsilon^{-}_{\nu'}\, F^{\mu'\nu'} + c.c.$$

$$\simeq \mathcal{M}^{\mu\nu}\, \epsilon^{+}_{\mu}\epsilon^{*-}_{\nu}\, \epsilon^{*+}_{\mu'}\epsilon^{-}_{\nu'}\, F^{\mu'\nu'} + c.c.$$

$$\simeq \mathcal{M}^{+-} e^{-2i\phi_h}\, e^{2i\phi}\, h_g + c.c.$$

$$\simeq \cos 2(\phi_h - \phi)\, h_g \quad \cdots$$

- has been observed for linearly Pol. Photon by STAR,

limited progress for quark & gluon.

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RESEARCH ARTICLE | PHYSICS

Tomography of ultrarelativistic nuclei with polarized photon-gluon collisions

STAR COLLABORATION Authors Info & Affiliations

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