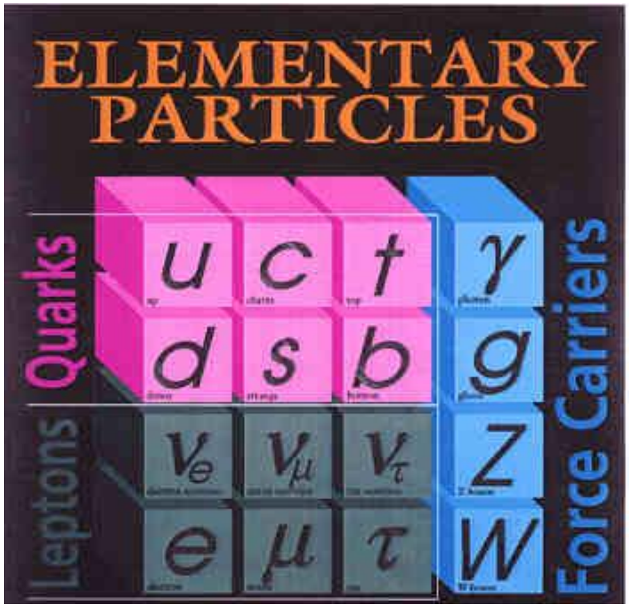
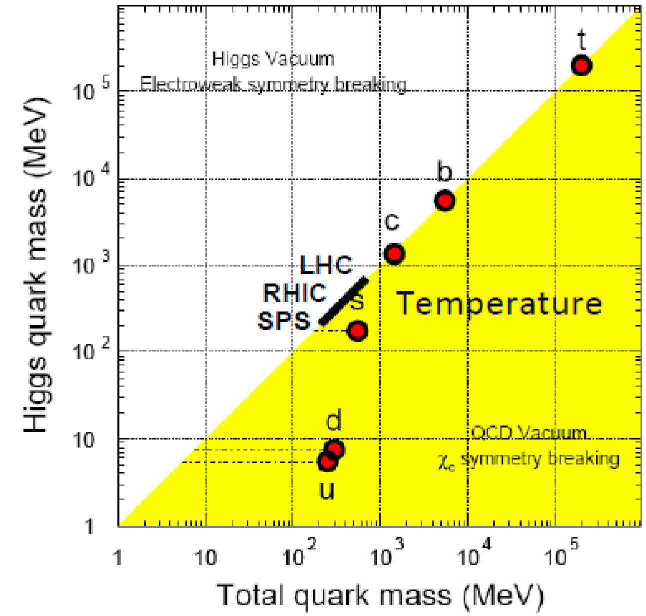
Lectures on Heavy-Flavor Probes of Quark-Gluon Plasma

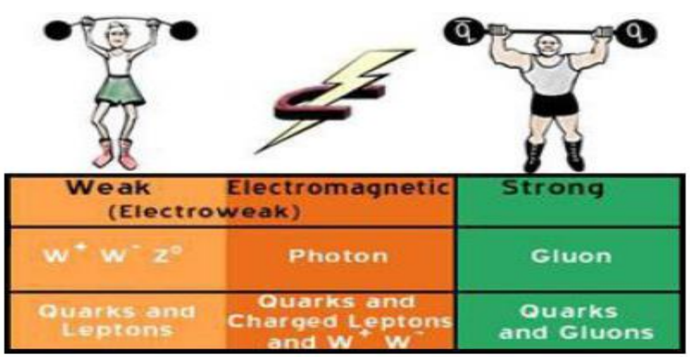
Min He

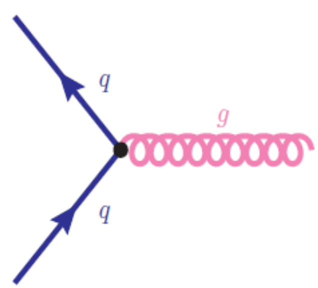
Nanjing Univ. of Sci. & Tech., Nanjing, China

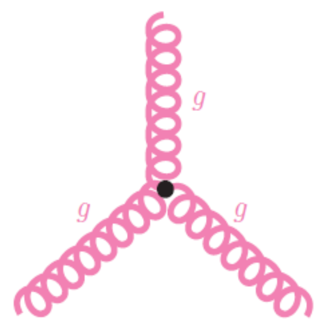
Quarks & Forces











QCD & Symmetries

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathcal{F}^a_{\mu\nu}\mathcal{F}_a^{\mu\nu} + \bar{\psi}(i\not{D} - M)\psi = \mathcal{L}_{chiral} - (m_u\bar{u}u + m_d\bar{d}d) + \mathcal{L}_{scbt}$$

❖ 2-flavor light sector: chiral symmetry

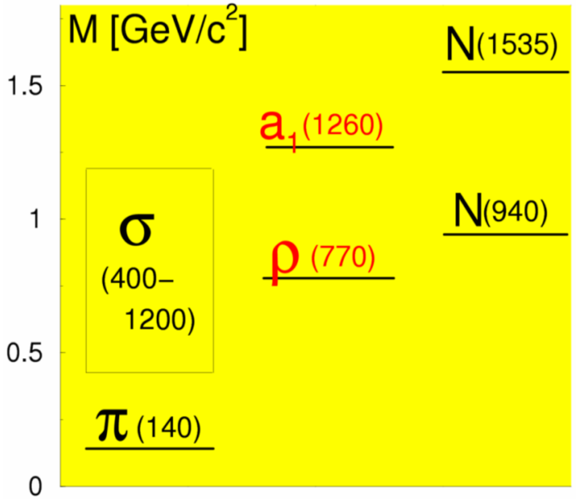
$$\mathcal{L}_{QCD} = (\bar{u}_L, \bar{d}_L)\, i\not{D} \begin{pmatrix} u_L \\ d_L \end{pmatrix} + (\bar{u}_R, \bar{d}_R)\, i\not{D} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + O(m_q) - \frac{1}{4}G^2_{a\mu\nu}$$

Symmetry	Transformation	Current	Name	Manifestation
$SU_V(2)$	$\psi \to e^{-i\tau\cdot\omega/2}\psi$	$J^k_\mu = \bar{\psi}\gamma_\mu\tau^k\psi$	isospin	approx. conserved
$U_V(1)$	$\psi \to e^{-i\alpha}\psi$	$j_\mu = \bar{\psi}\gamma_\mu\psi$	baryonic	always conserved
$SU_A(2)$	$\psi \to e^{-i\tau\cdot\theta\gamma_5/2}\psi$	$J^k_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\tau^k\psi$	chiral	CSB; Goldstone mode
$U_A(1)$	$\psi \to e^{-i\beta\gamma_5}\psi$	$j_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi$	axial	$U_A(1)$ "puzzle"

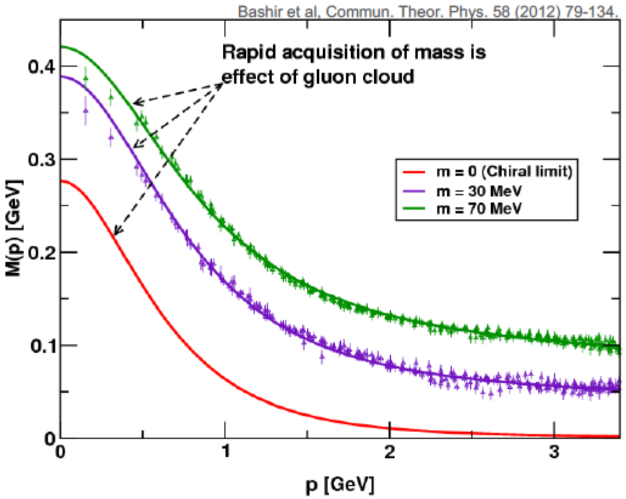
❖ DCSB: condensate vs dynamical breaking

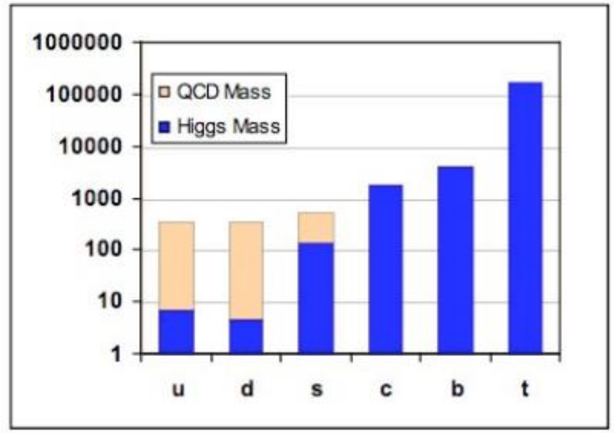
$$\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle \approx 5\ fm^{-3}$$

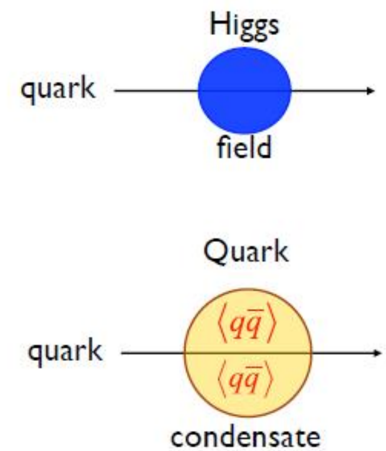
$$SU(2)_L \otimes SU(2)_R \to SU(2)_V$$



Higgs vs QCD masses







❖ Nucleon mass decomposition $M = \frac{1}{2M}\langle N(p)|T^{\mu}_{\ \mu}|N(p)\rangle$

$$M = \frac{1}{2M}\langle N(p)|\frac{\beta(\alpha_s)}{2\alpha_s}G^{a\mu\eta}G^{a}_{\ \mu\eta}$$

$$+\sum_{q=u,d,s}\gamma_{m_q} m_q\bar{q}q|N(p)\rangle + \frac{1}{2M}\langle N(p)|\sum_q m_q\bar{q}q|N(p)\rangle$$

- dynamically generated via mainly gluon self-interaction
- Higgs (current quark) mass

❖ Heavy flavor hadrons: heavy quark symmetry ➔ HQEF

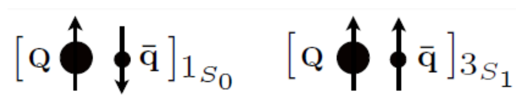
$$M_H = m_Q + \Lambda_H + O(1/m_Q) + \cdots$$

$$\Lambda_H = \frac{1}{2M_H}\left\langle H\left|\frac{\beta(\alpha_s)}{4\alpha_s}G^2\right|H\right\rangle$$

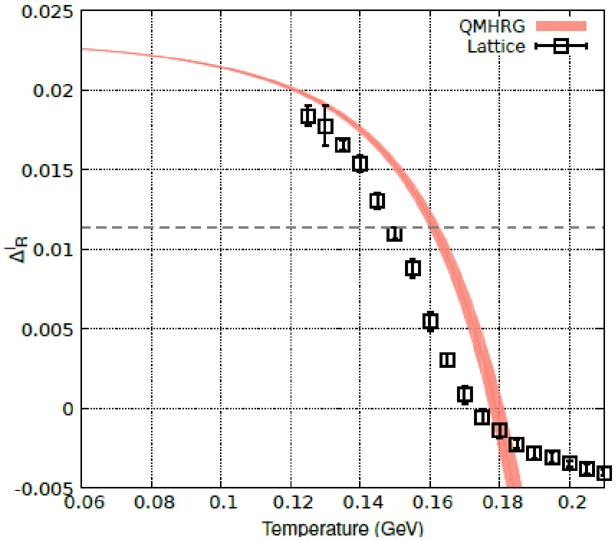
- Hyperfine/spin $\propto 1/m_Q$ splitting small

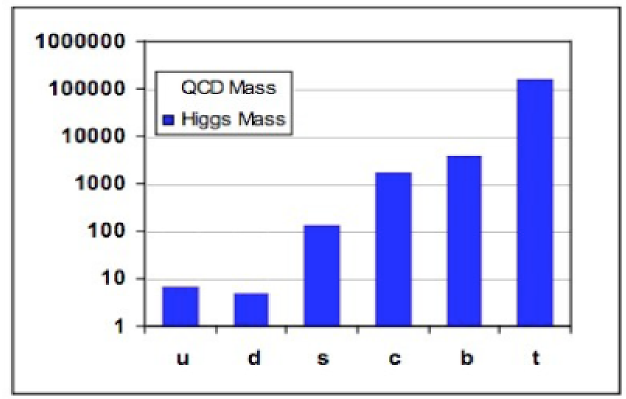
Experimental mass splittings between 1^- and 0^- mesons

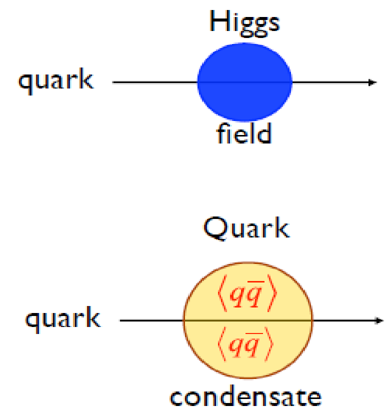
	Δ (MeV)
$M_{D^{*+}} - M_{D^+}$	140.64 ± 0.09
$M_{D^{*0}} - M_{D^0}$	142.12 ± 0.07
$M_{D_s^{*\pm}} - M_{D_s^{\pm}}$	141.6 ± 1.8
$M_{B^*} - M_B$	46.0 ± 0.6
$M_{B_s^*} - M_{B_s}$	47.0 ± 2.6



At high temperatures …





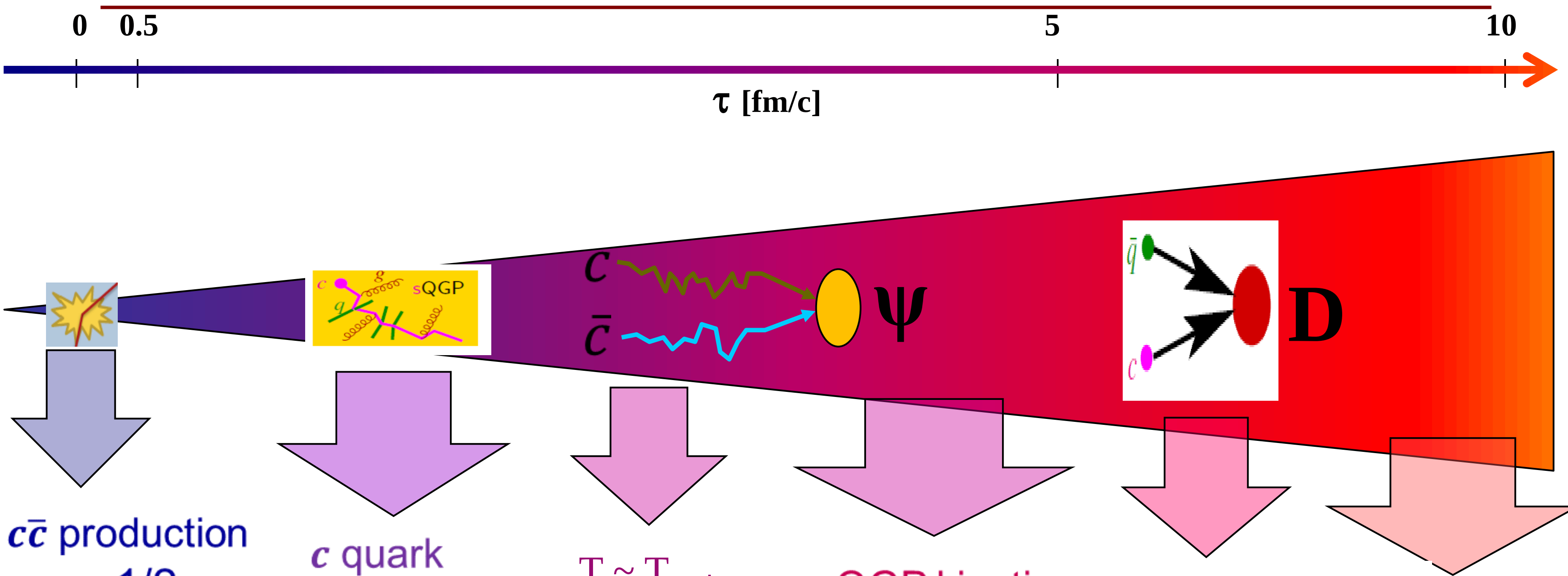


Heat "melts" the quark condensate:
QCD mass disappears above T_c.
(Partial) chiral symmetry restoration

❖ Unique role and advantage of HQs enabled by its large lass

- $m_{c,b} >> \Lambda_{QCD}$ produced by pQCD processes (out of equil.)
- $\tau_0 << \tau_{QGP}$ they go through all the QGP lifetime
- $m_{c,b} >> T_0$ *no thermal production*
- $\tau_{eq} > \tau_{QGP} >> \tau_{q,g}$ carry more information
- $m>>T \rightarrow q^2<<m^2$ transport reduced to Brownian motion
- $q_0 << |\vec{q}|$ Concept of potential V(r) <-> lQCD

Heavy flavor transport as probes of QGP



$c\bar{c}$ production
τ_{hard}~1/2m_c,
buildup of ψ
wave packet
τ_ψ~1 fm/c

c quark
Brownian
diffusion
in QGP
τ_c^{eq}~5 fm/c

$T \sim T_{melt}$
onset of
quarkonium
regeneration

QGP kinetics
$c + \bar{c} \leftrightarrow \psi$
$\tau_\psi^{eq} \sim 1/\Gamma_\psi$

$T \sim T_c$
hadronization:
c+q → D
c+q+q → Λ_c

hadronic
diffusion

Outline

Lecture I: Open heavy flavor production in pp

- pQCD production
- hadronization:empirical fragmentation functions
- SHM approach

Lecture II: Open heavy flavor probes of QGP

- HQ interactions in QGP: pQCD vs T-matrix
- diffusion: Langevin vs Boltzmann
- Hadronization: recombination vs fragmentation
- Phenomenology: R_{AA} & v_2 vs hadro-chemistry
- extracting $\mathcal{D}_s(2\pi T)$

Lecture III: Heavy quarkonium probes of QGP

- Vacuum bound states, potential models
- HQ potential: remnants of confining force
- Reaction rates, transport approaches
- open quantum approach to Υ states

Lecture I

Open heavy flavor production in pp collisions

- pQCD production
- hadronization:empirical fragmentation functions
- SHM approach

Heavy quark production: pQCD

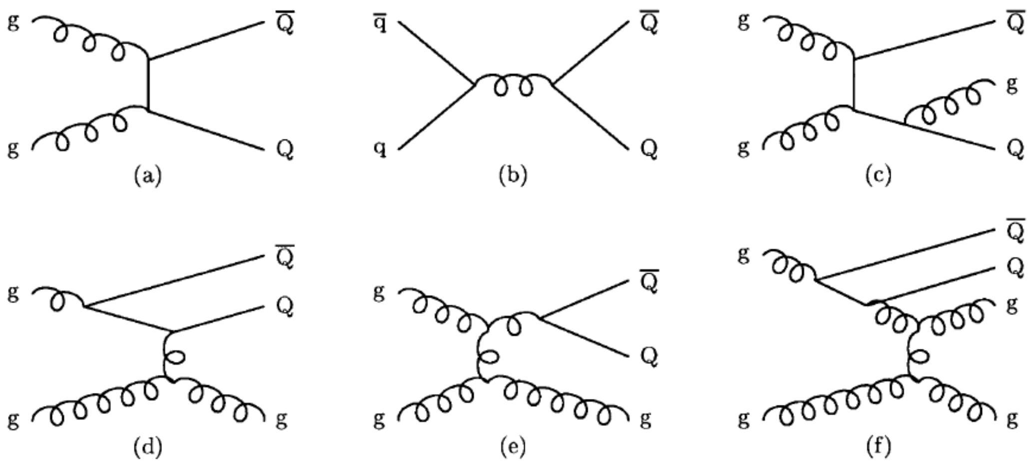
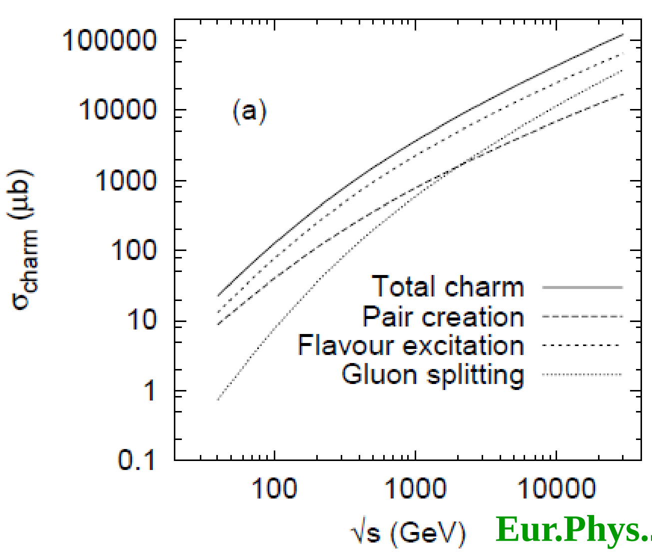


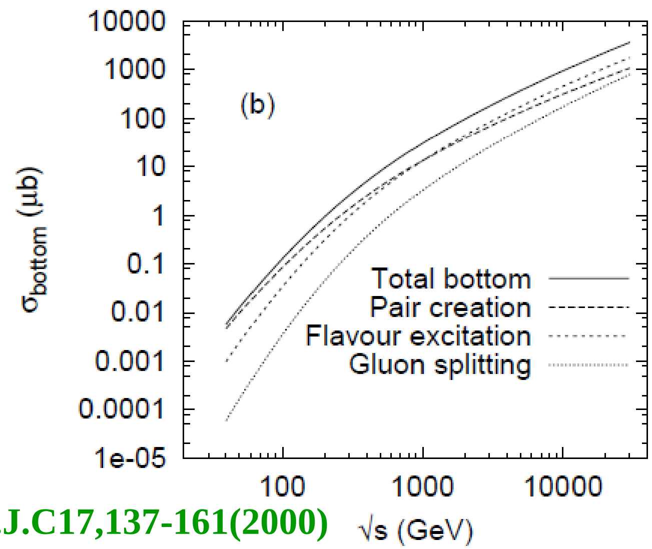
Fig. 1a–f. Examples of heavy-flavor production diagrams. **a,b** Leading order. **c** Pair creation (with gluon emission). **d** Flavor excitation. **e** Gluon splitting. **f** Events classified as gluon splitting but of flavor-excitation character

Norrbin and Sjostrand, Eur.Phys.J.C17,137-161 (2000)

Combridge, NPB151 (1979) 429-456

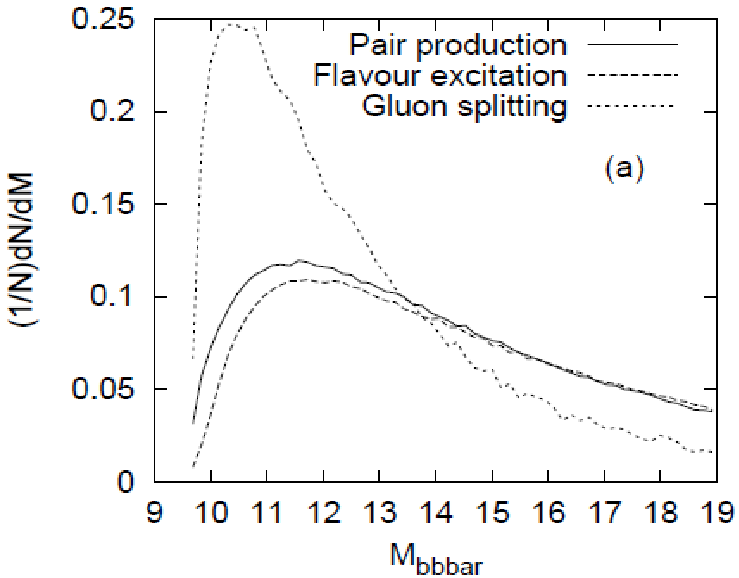


Eur.Phys.J.C17,137-161(2000)



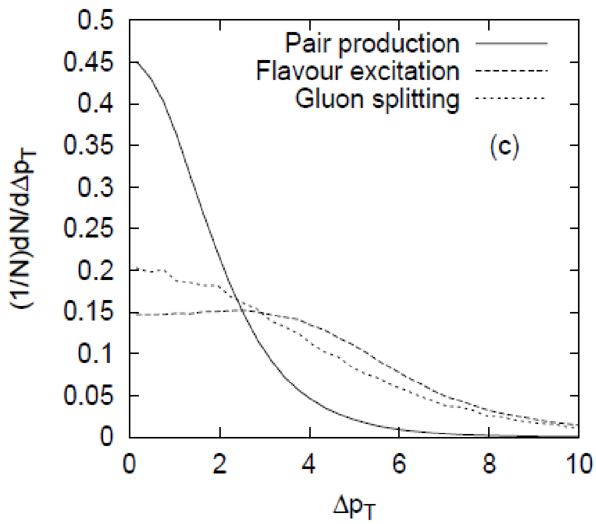
- total cross sections in pp collisions: pair creation dominant at low root(s), flavor excitation important, gluon splitting comparable at high root(s)
- no non-perturbative effects contributing to the total cross section

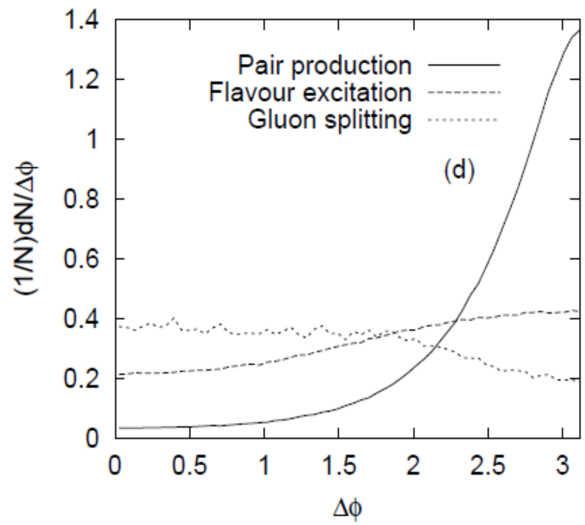
b-bbar correlations @ Tevatron



- pair invariant mass distribution: gluon splitting ~ s-channel gluon exchange ➔ σ~1/s suppressed at large s~M_{bbbar}
- pair creation & flavor excitation ~ t-channel contributions

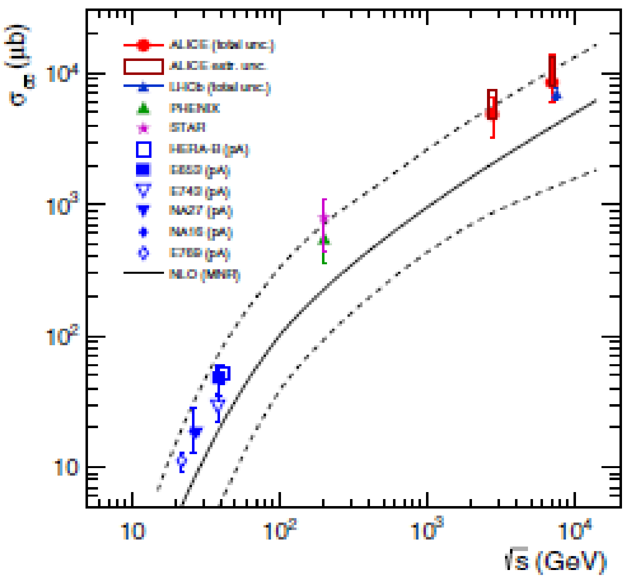
Norrbin and Sjostrand, Eur.Phys.J.C17,137-161 (2000)

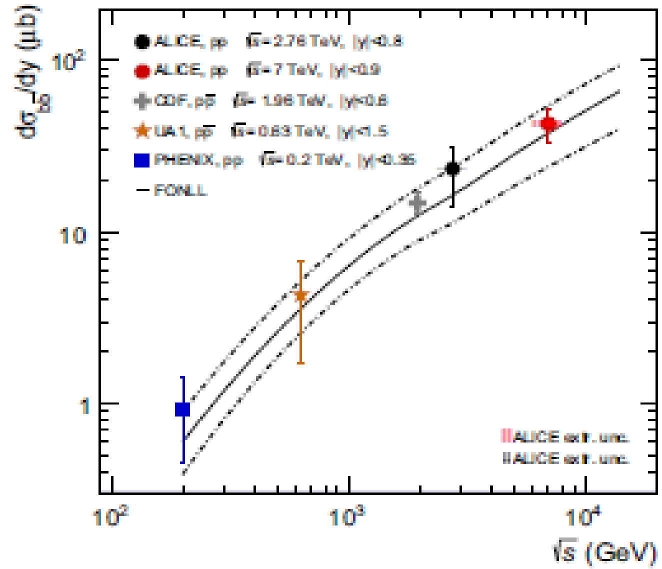




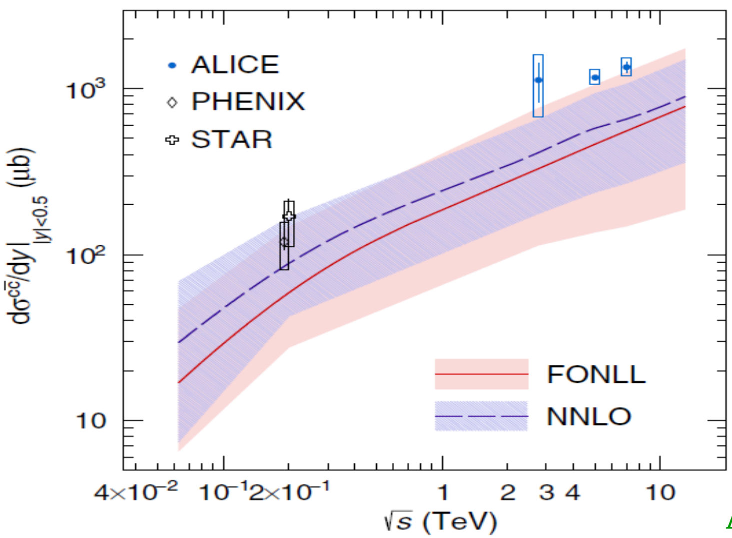
- LO pair creation ~ opposite & compensating p_T ➔ peaked at Δp_T=0
- gluon-splitting b-bbar ~ collinear ➔ peaked at Δφ=0

Measurements of Q-Qbar cross sections





Andronic et al., Eur.Phys.J.C (2016) 76:107



- ALICE, 5.02 TeV pp, $d\sigma^{ccbar}/dy$~1mb at y=0, significantly higher than before ➔ impact on modelling of charmonium transport in QGP [direct measurement of all charm hadrons]
- ALICE, 7 TeV pp, $d\sigma^{bbbar}/dy$~57 mub at y=0 [indirect measurements of HF muons]
- LHCb, 7 TeV pp, $d\sigma^{bbbar}/dy$~27 mub at y=2-2.5 [indirect measurements of HF muons]

ALICE, PRD105, L011103 (2022)

Heavy quark fragmentation

❖ From HQ to heavy hadron (pp): non-perturbative fragmentation

$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T;\mu_F,\mu_R) = \text{PDF}(x_1,\mu_F)\cdot\text{PDF}(x_2,\mu_F) \otimes \frac{d\sigma^c}{dp_T^c}(x_1,x_2,\mu_R,\mu_F) \otimes D_{c\to H_c}(z=p_{H_c}/p_c,\mu_F)$$

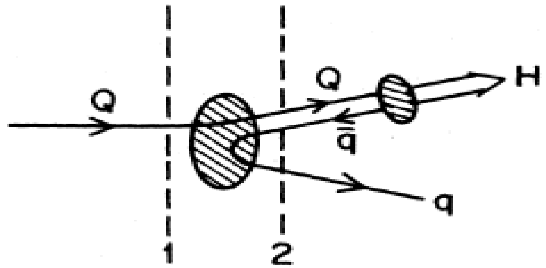
Parton distribution functions (PDFs)

Hard scattering cross section (pQCD)

Fragmentation function (hadronization)

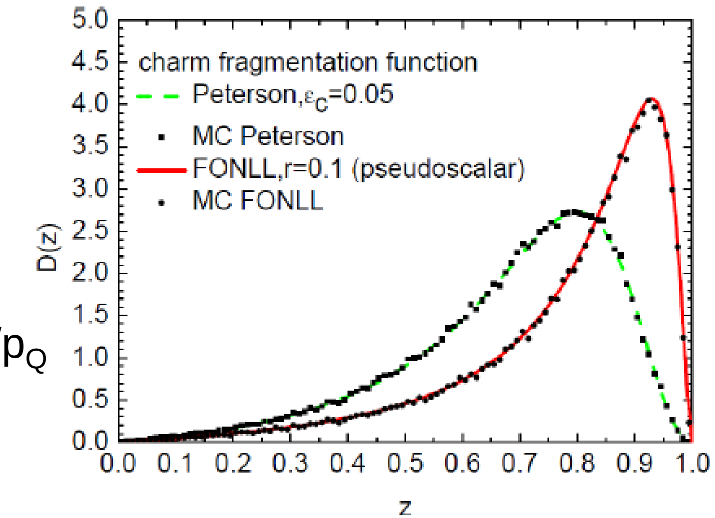
- HQ large mass >>Λ_{QCD} ➔ hadroniztion & production well separated ➔ factorization

❖ Fragmentation functions, e.g. Peterson FF **Peterson et al., PRD27,105 (1983)**



$$D_Q^H(z)=\frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2}$$ with $z=p_H/p_Q$

$$\sum\int dz\, D_Q^H(z)=1$$



Heavy quark FF fitted to e^+e^-

❖ In e^+e^- collisions, there's no initial pdf. E.g. for $e^+e^- \to (\gamma, Z) \to D^{*\pm} + X$

$$\frac{1}{\sigma_{\rm tot}} \frac{d\sigma(e^+e^- \to D^{*\pm} + X)}{dx} = \frac{1}{\sigma_{\rm tot}} \sum_a \int_x^1 \frac{dz}{z} D_a\left(\frac{x}{z}, M_f^2\right) \frac{d\sigma_a}{dz}\left(z, \mu^2, M_f^2\right)$$

- x=E(D*)/(root(s)/2)=2E(D*)/root(s) **Kniehl et al., Z.Phys.C677(1997); PRD71, 094013 (2005)**
- Parameterizing the FF at starting scale M_f=μ, then evolving the FF with DGLAP eq. to M_f=Mz, such that it fits the e^+e^- data when convoluted with hard-scattering $d\sigma_a/dz$

X_c	Order	Q	N	α	β	ϵ
D^0	LO	c	0.998	...	...	0.163
		b	71.8	1.65	5.19	...
	NLO	c	1.16	...	...	0.203
		b	97.5	1.71	5.88	...
D^+	LO	c	0.340	...	...	0.148
		b	48.5	2.16	5.38	...
	NLO	c	0.398	...	...	0.187
		b	64.9	2.20	6.04	...
D_s^+	LO	c	0.0704	...	...	0.0578
		b	40.0	2.05	4.93	...
	NLO	c	0.0888	...	...	0.0854
		b	21.8	1.64	4.71	...
Λ_c^+	LO	c	0.0118	...	...	0.0115
		b	44.1	1.97	6.33	...
	NLO	c	0.0175	...	...	0.0218
		b	27.3	1.66	6.24	...

Heavy quark FF fitted to e^+e^- (continued)

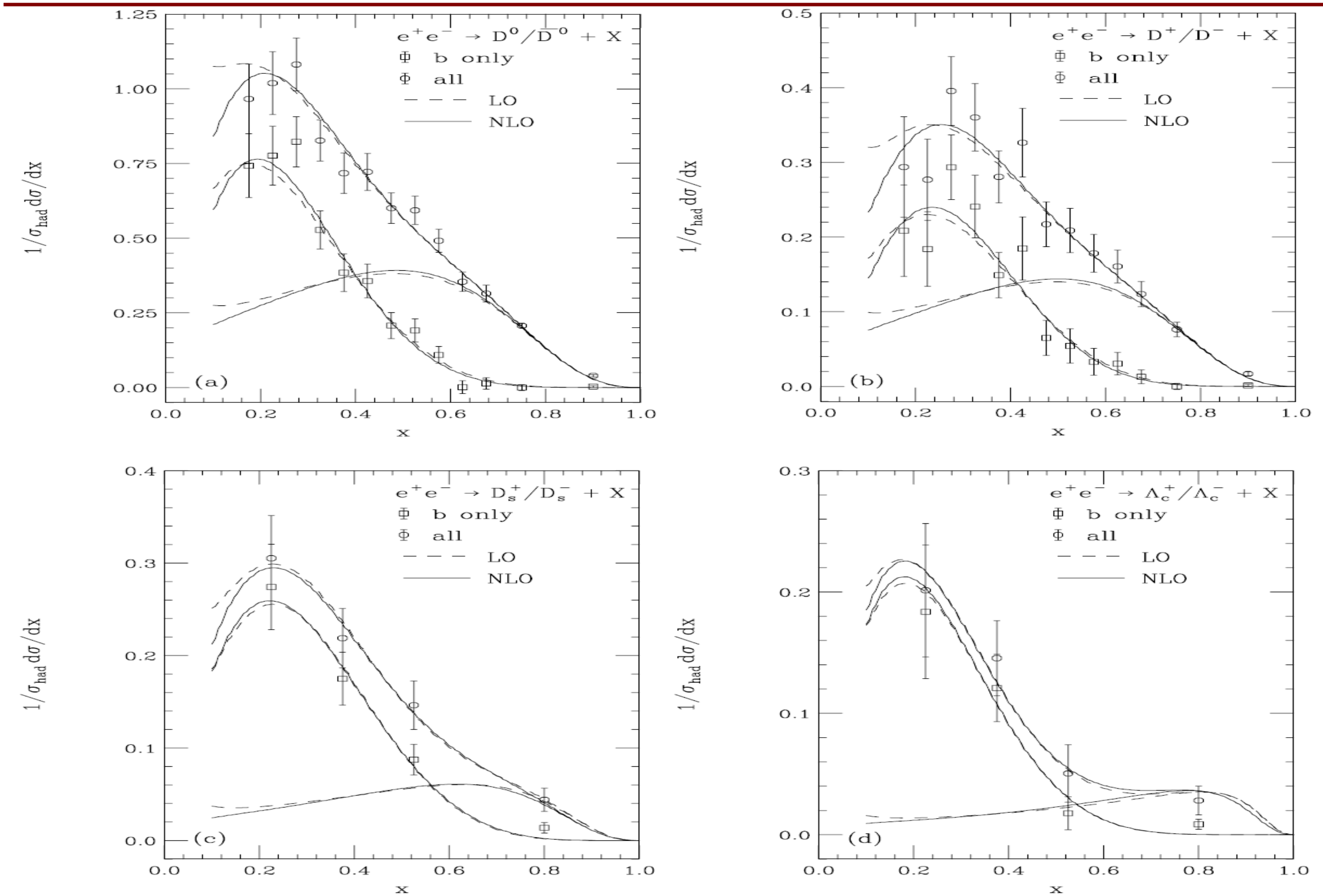
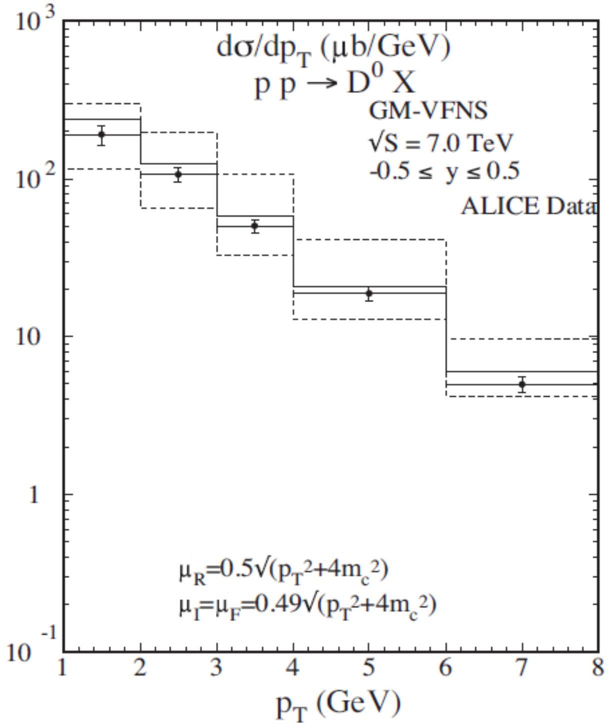
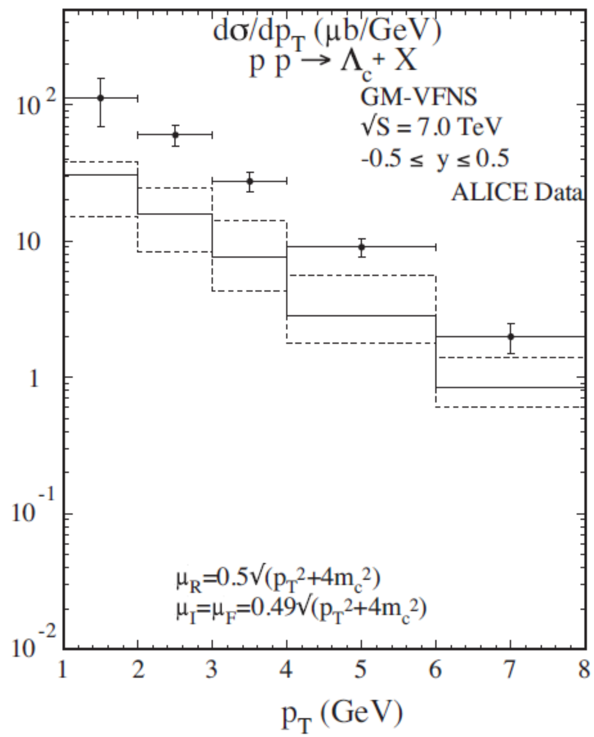


FIG. 1. The normalized differential cross sections $(1/\sigma_{tot})d\sigma/dx$ of inclusive (a) $D^0/\overline{D}^0$, (b) $D^{\pm}$, (c) $D_s^{\pm}$, and (d) $\Lambda_c^{\pm}$ production in e^+e^- annihilation on the Z-boson resonance evaluated at LO (dashed lines) and NLO (solid lines) with our respective FF sets are compared with the OPAL data [8] renormalized as explained in the text (circles). The same is also done for the $Z \longrightarrow b\overline{b}$ subsamples (squares). In addition, our LO and NLO fit results for the $Z \longrightarrow c\overline{c}$ contributions are shown. In each case, the X_c hadron and its charge-conjugate counterpart are summed over.

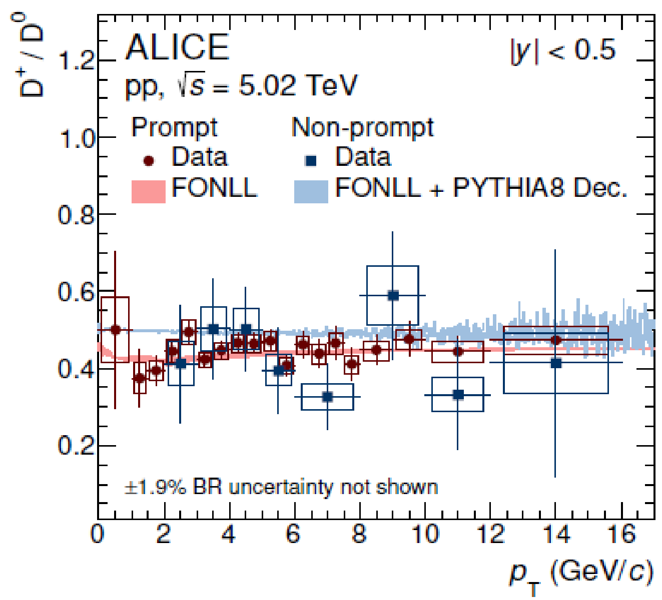
FFs universal from e^+e^- to pp?

❖ phenomenological FF: constrained by e^+e^- & usually assumed universal





- FF extracted from e^+e^- applied to pp: FONLL scheme, D^+/D^0 reproduced

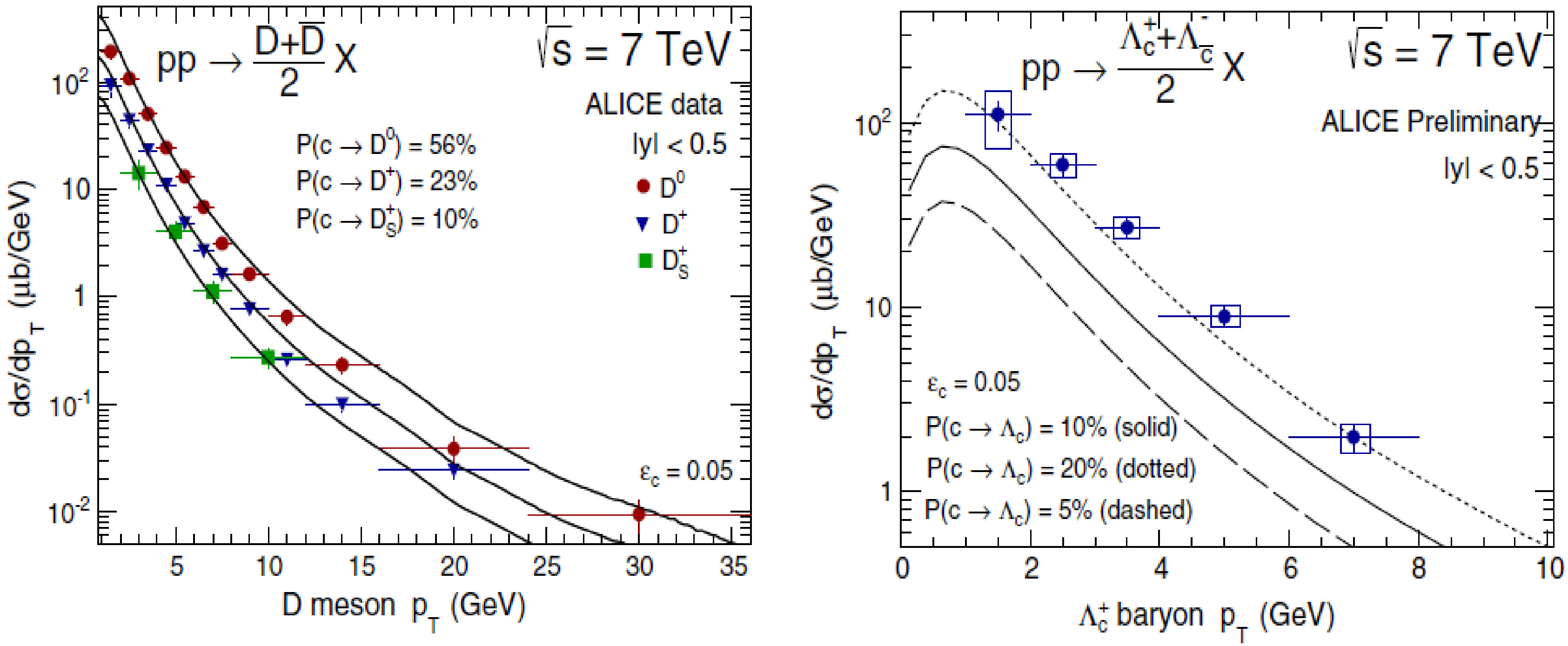


- FF extracted from e^+e^- applied to pp: GM-VFNS scheme D^0-meson well reproduced but Λ_c much underestimated

Kniehl et al., PRD101, 114021 (2020)

FFs universal from e^+e^- to pp?

❖ phenomenological FF: constrained by e^+e^- & usually assumed universal

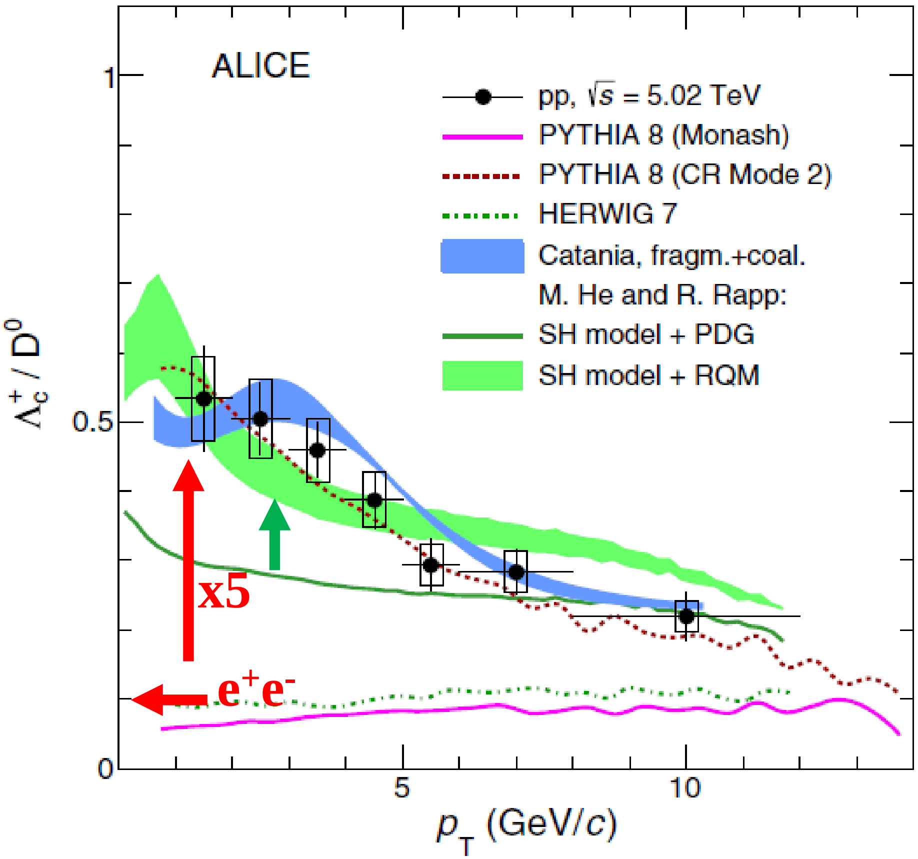


- FF extracted from e^+e^- applied to pp: k_T-factorization scheme
  D-mesons well reproduced but Λ_c much underestimated

  Maciula and Szczurek., PRD98, 014016 (2018)

- new production channel for Λ_c baryons in high-energy pp collisions ?

Λ_c^+/D^0 @ 5 TeV pp collisions



ALICE, PRL 127, 202301(2021)
ALICE, PRC 104, 054905(2021)

- PYTHIA8: Color-reconnection with junctions frag. into baryons

Bierlich et al. '15

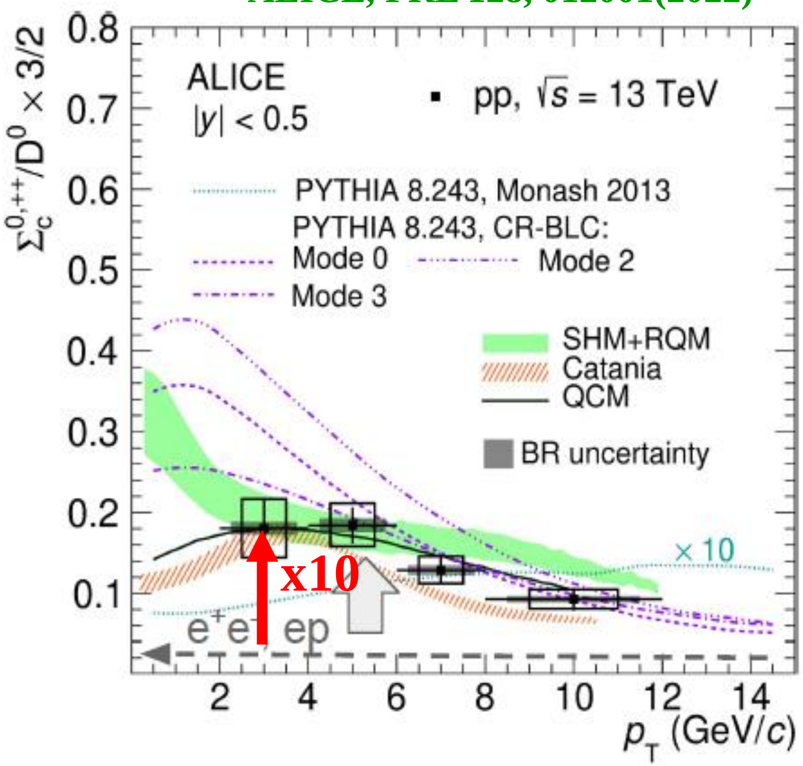
- Catania: c-q(-q) coalescence in a small QGP fireball Minissale et al. '21

- Statistical hadronization in q-rich environment (unlike e^+e^-)
  - augmented by "missing" charm-baryons assuming *relative* chemical equilibrium

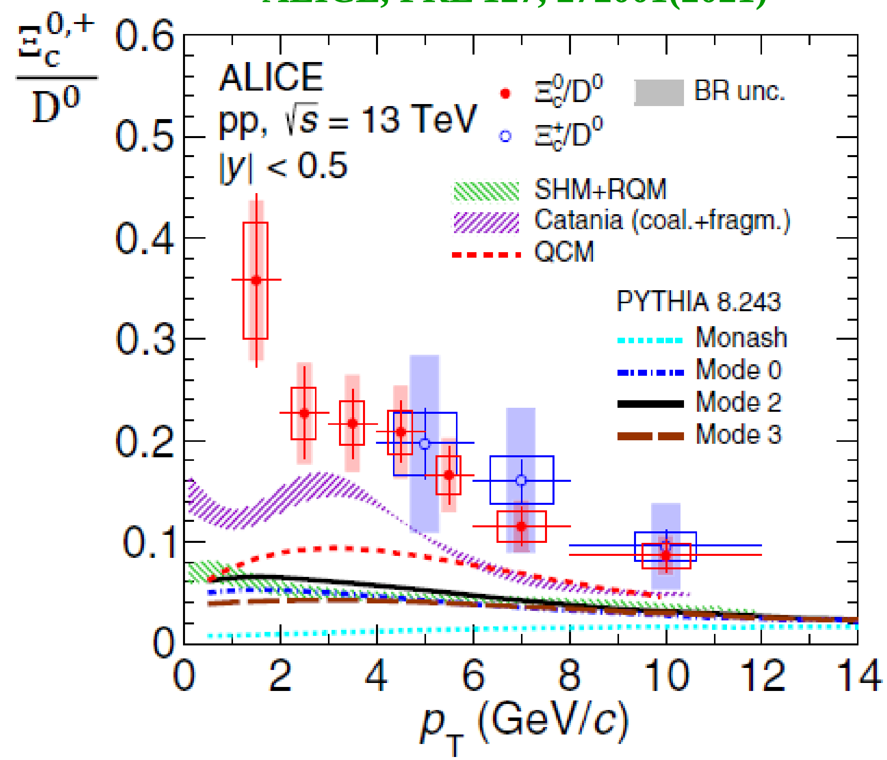
MH & Rapp '19

Σ_c/D^0 & Ξ_c/D^0

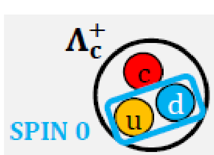
ALICE, PRL 128, 012001(2022)

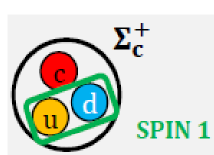


ALICE, PRL 127, 272001(2021)



- ❖ Σ_c/D^0 x10 enhanced despite more massive spin-1 diquark
- ❖ Ξ_c/D^0 ratio underestimated by all models





Statistical Hadronization of HQs

❖ Statistical hadronization model

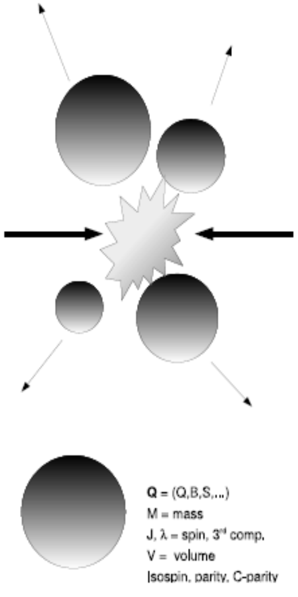


Figure 1: High energy collisions are assumed to give rise to multiple clusters at the hadronization stage [top]. Each cluster [bottom] is a colorless extended massive object endowed with abelian charges (electric, strange, baryonic etc.), intrinsic angular momentum and other quantum numbers such as parity, *C*-parity and isospin.

- Thermodynamic equilibrium = filling accessible phase space uniformly

  P. B.-Munzinger et al., nucl-th/0304013
  F. Becattini, 0901.3643

- High-energy collisions ➔ clusters/fireballs at hadronization ➔ collectively equivalent global cluster (EGC): colorless & endowed with conserved Abelian charges
- Occurring at a critical energy density [microscopic canonical ensemble] ➔ replaced by a hadronization temperature T_H~160-170 MeV

- SHM applies also to elementary collisions vs heavy-ion collisions (where multiparton scatterings leading to kinetic thermalization)

  F. Becattini, Eur. Phys. J. C (2008) 56: 493–510

Hadrons born into equilibrium?!

Eur. Phys. J. C 52, 187–201 (2007)
DOI 10.1140/epjc/s10052-007-0368-6

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Thermal hadronization and Hawking–Unruh radiation in QCD

P. Castorina[1], D. Kharzeev[2,a], H. Satz[3]

[1] Dipartimento di Fisica, Università di Catania, and INFN Sezione di Catania, Via Santa Sofia 64, 95123 Catania, Italy
[2] Physics Department, Brookhaven National Laboratory, Upton, NY 11973-5000, USA
[3] Fakultät für Physik, Universität Bielefeld, 33501 Bielefeld, Germany

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Abstract. We conjecture that, because of color confinement, the physical vacuum forms an event horizon for quarks and gluons, which can be crossed only by quantum tunneling, i.e., through the QCD counterpart of Hawking radiation at black holes. Since such radiation cannot transmit information to the outside, it must be thermal, of a temperature determined by the chromodynamic force at the confinement surface, and it must maintain color neutrality. We explore the possibility that the resulting process provides a common mechanism for thermal hadron production in high energy interactions, from e^+e^- annihilation to heavy ion collisions.

PACS. 04.70.Dy; 12.38.Aw; 12.38.Mh; 12.40.Ee; 25.75.Nq; 97.60.Lf

- Hadron thermodynamic state is not reached by dynamical equilibration among constituents (partons or hadrons), ➔
  but rather a generic fingerprint of hadronization or a feature of QCD vacuum

Grand-canonical SHM in HIC

❖ CG-SHM: charges are not exactly conserved but only on average by fugacities/chemical potentials

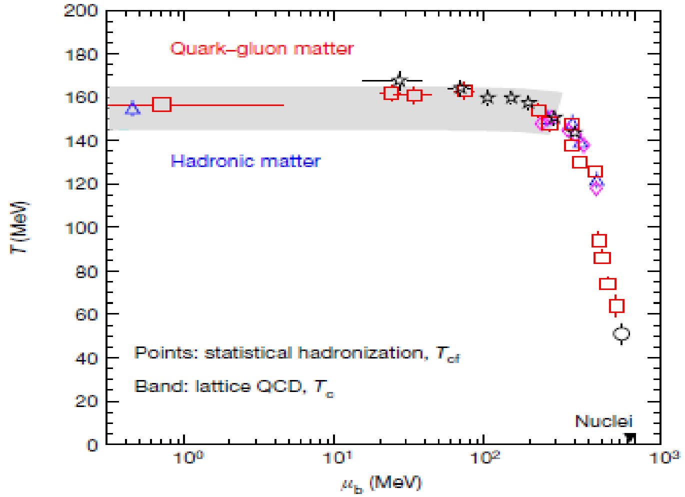
$$Z^{GC}(T,V,\mu_Q) = \mathrm{Tr}[e^{-\beta(H-\sum_i \mu_{Q_i} Q_i)}], \quad \ln Z(T,V,\vec{\mu}) = \sum_i \ln Z_i(T,V,\vec{\mu})$$

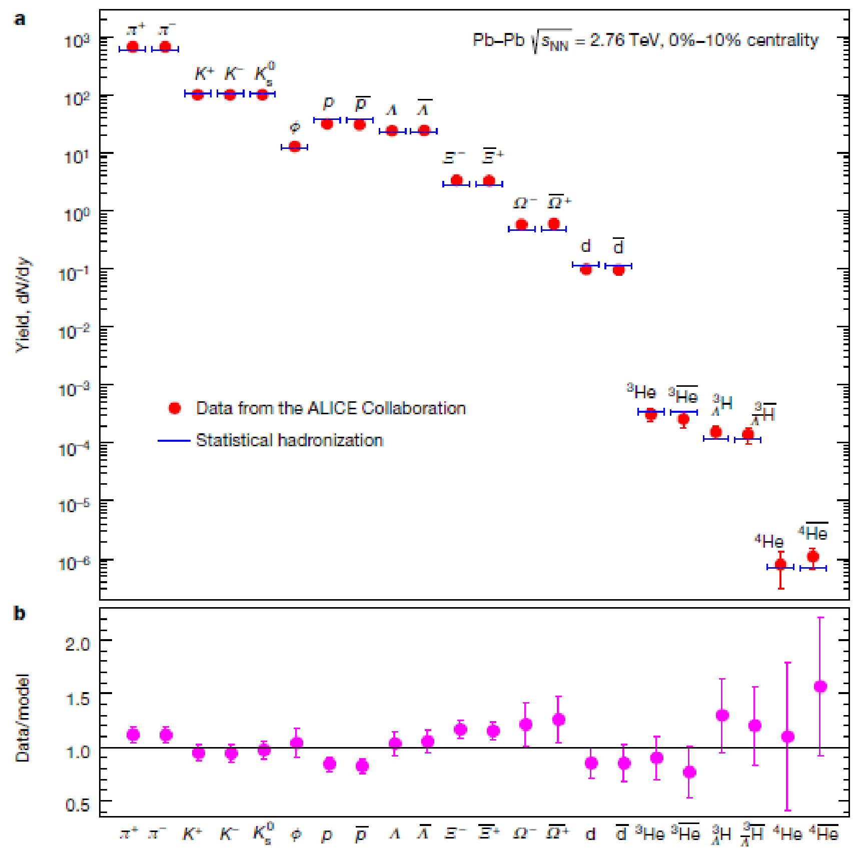
A. Andronic et al., Nature, 561, 321 (2018)

$$\ln Z_i(T,V,\vec{\mu}) = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 dp \ln[1\pm\lambda_i \exp(-\beta\epsilon_i)],$$
$$= \frac{VT g_i}{2\pi^2}\sum_{k=1}^{\infty}\frac{(\pm1)^{k+1}}{k^2}\lambda_i^k m_i^2 K_2(\frac{k m_i}{T})$$

$$\lambda_i(T,\vec{\mu}) = \exp(\frac{B_i\mu_B + S_i\mu_S + Q_i\mu_Q}{T})$$

$$n_i(T,\vec{\mu}) = \frac{\langle N_i\rangle}{V} = \frac{T g_i}{2\pi^2}\sum_{k=1}^{\infty}\frac{(\pm1)^{k+1}}{k}\lambda_i^k m_i^2 K_2(\frac{k m_i}{T}).$$





Augmented SHM for charm-hadrons in pp

- PDG: 5 Λ_C (I=0) ,3 Σ_C (I=1),8 Ξ_C (I=1/2),2 Ω_C (I=0) missing baryons?!
  RQM: 18 extra Λ_C , 42 extra Σ_C, 62 extra Ξ_C, 34 extra Ω_C up to 3.5 GeV ➔ supported by lattice PRD 84 (2011) 014025; PoS LAT. 2014 (2015) 084; PLB 737 (2014) 210

- Grand-canonical SHM density T_H=170 MeV

$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2(\frac{m_i}{T_H})$$

n_i ($\cdot 10^{-4}$ fm^{-3})	D^0	D^+	D^{*+}	D_s^+	Λ_c^+	$\Xi_c^{+,0}$	Ω_c^0
PDG(170)	1.161	0.5098	0.5010	0.3165	0.3310	0.0874	0.0064
RQM(170)	1.161	0.5098	0.5010	0.3165	0.6613	0.1173	0.0144

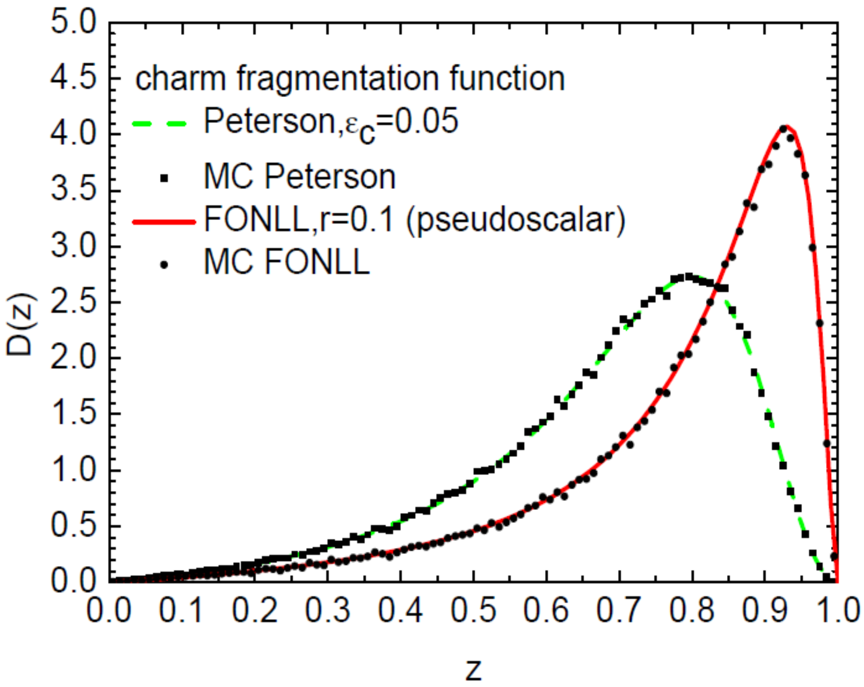
r_i	D^+/D^0	D^{*+}/D^0	D_s^+/D^0	Λ_c^+/D^0
PDG(170)	0.4391	0.4315	0.2736	0.2851
RQM(170)	0.4391	0.4315	0.2726	0.5696

MH, Rapp, PLB795 (2019) 117–121

- Here, strong feeddowns of excited states all included: BR=100% to Λ_C^+ for all Λ_C & Σ_C even above DN (2805 MeV) threshold
- Strangeness supp. γ_s=0.6 & charm fugacity γ_c=1

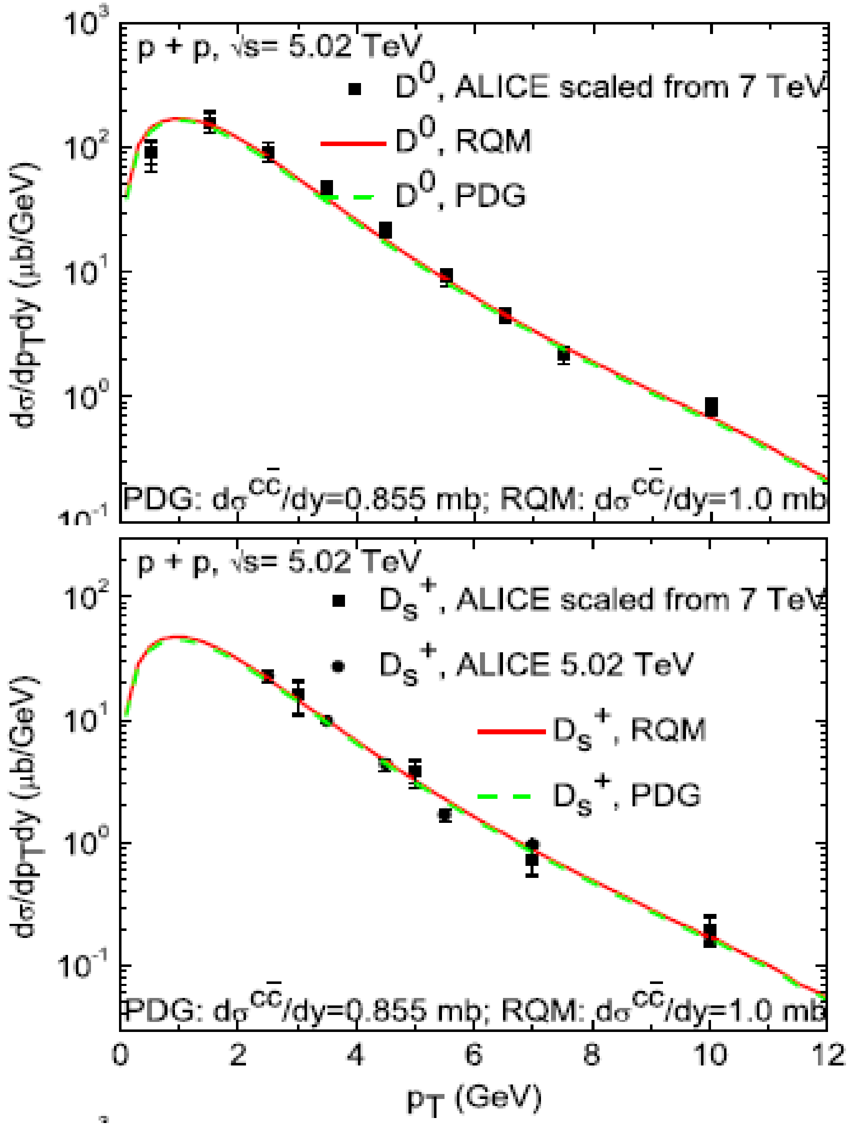
Fragmentation & decay simulations

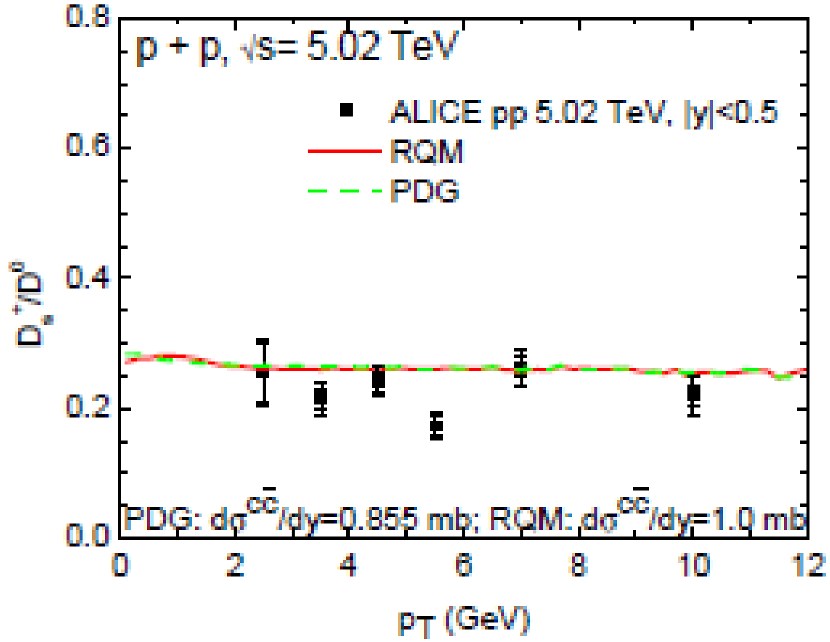
- FONLL fragmentation of charm quarks into all kinds of charm-hadrons: relative weight <-->SHM thermal densities



- Decay simulations of all excited states to ground states D^0, D^+, D_s^+, Λ_c^+, Ξ_c & Ω_c

Results: charm-mesons

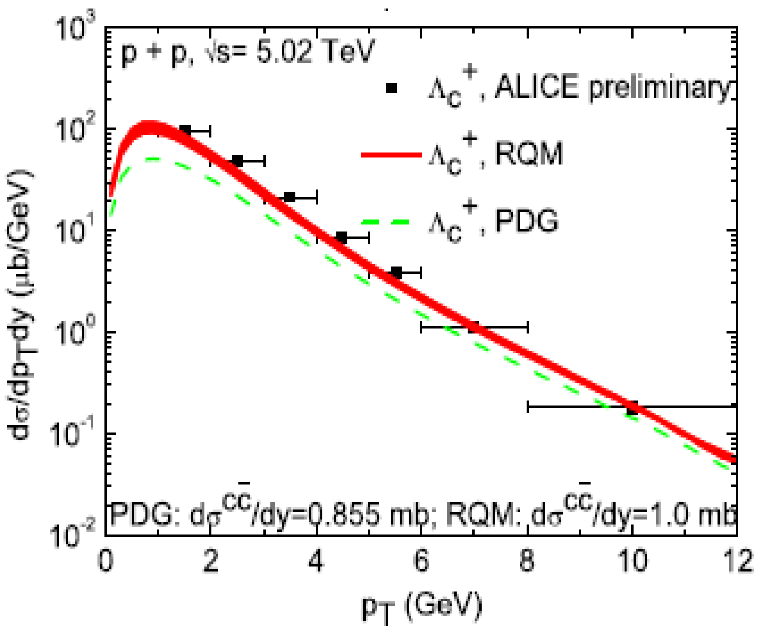




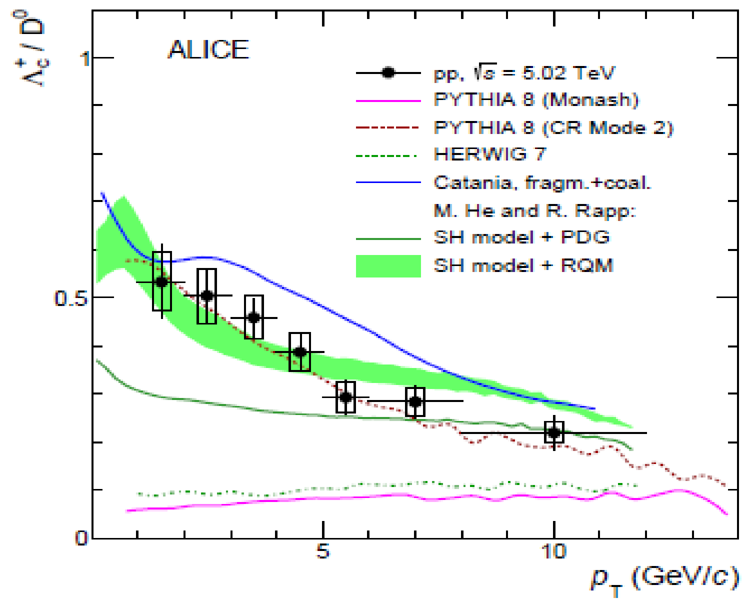
- both PDG & RQM work for charm-mesons
- but with different fitted charm dσ/dy=0.855 vs 1.0 mb

MH, Rapp, PLB795 (2019) 117–121

Results: charm-baryons

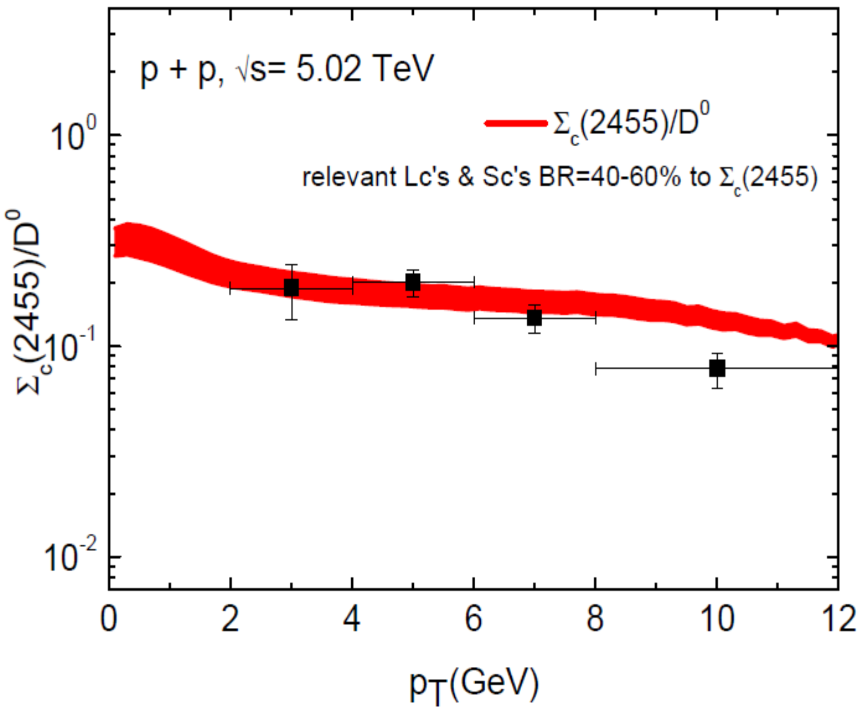


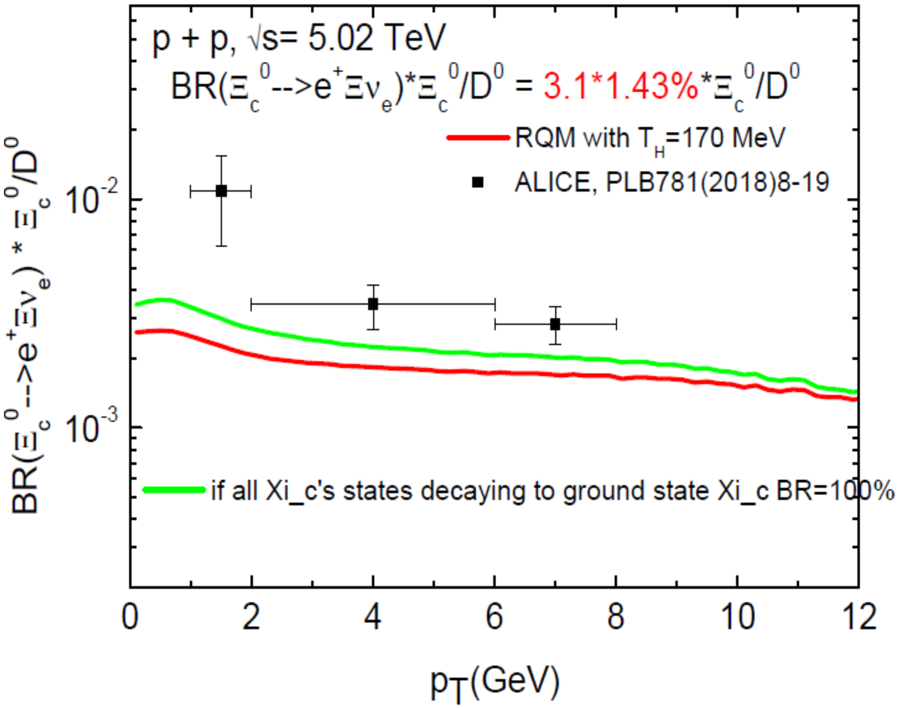
ALICE: PRL127 (2021) 20, 202301



- Λ_C^+ favors RQM with dσ/dy=1.0 mb: low p_T enhancement from feeddowns of RQM augmented baryons
- uncertainty band: BR=50%-100% to ground state Λ_C^+ for Λ_C's & Σ_C's above DN (2805 MeV) threshold

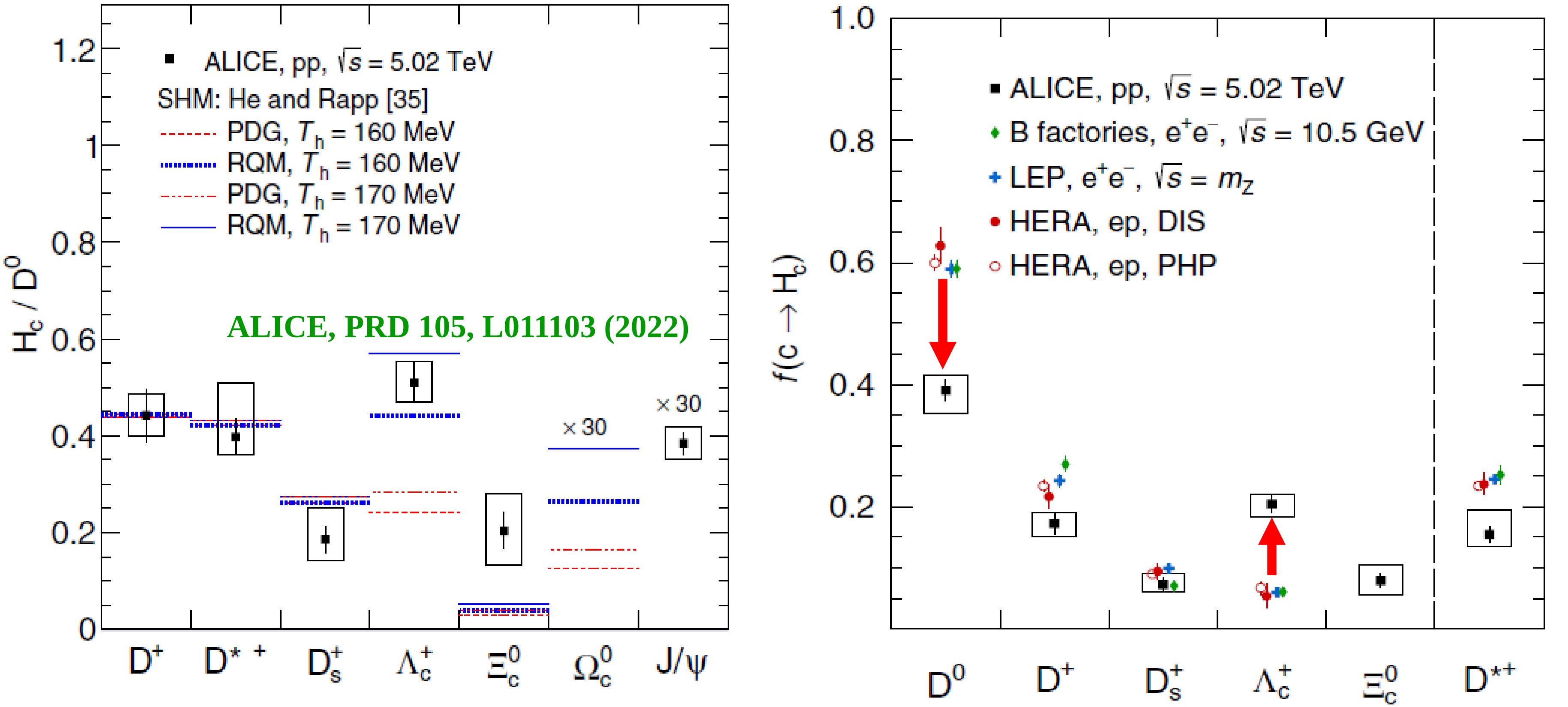
Results: charm-baryons (cont.)





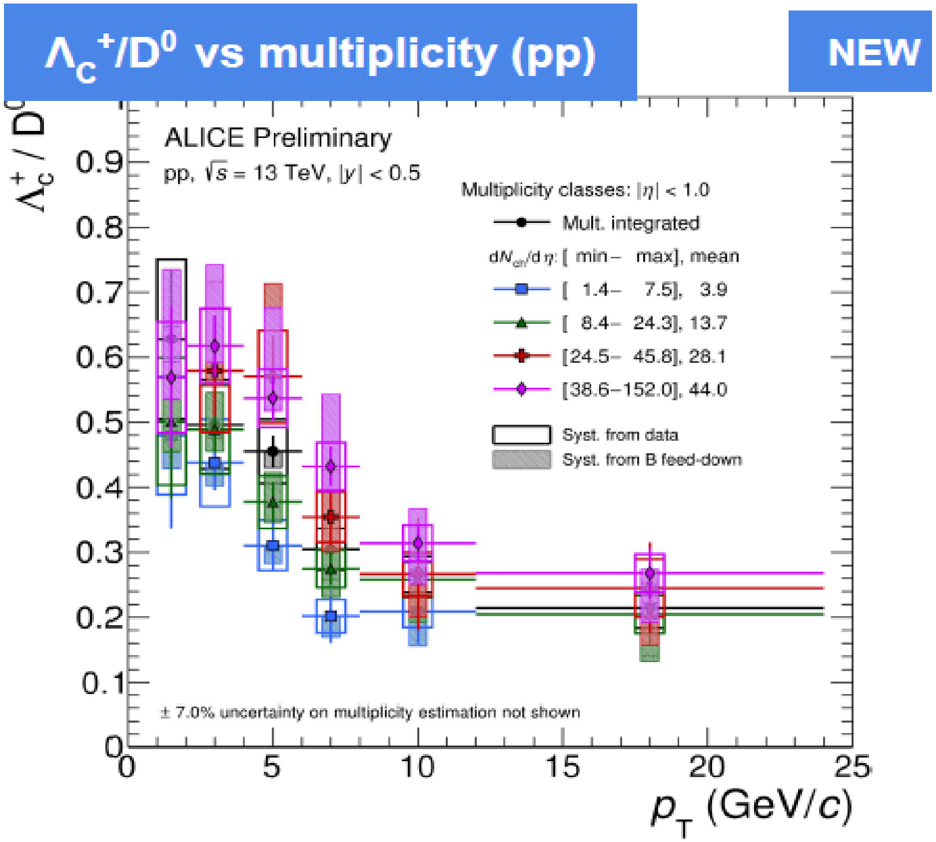
- $\Sigma_c(2455)/D^0$ can also be accounted for within uncertainties
- But Ξ_c^0/D^0 much underestimated, although already twice as large as PYTHIA8(CR) ~ 0.001

Charm-hadron fractions in 5 TeV pp



- charm quark fragmentation is non-universal from e^+e^- to pp
- charm content shuffled from meson (D^0) to baryon (Λ_c) sector
- full charm-hadrons measured, $d\sigma^{cc}/dy$~1.16 mb at mid-y
  - ➔ significant impact on charmonia production in HIC

Λ_c^+/D^0 : $dN_{ch}/d\eta$ dependence



- Significant enhancement with increasing charged-particle multiplicity
- Might be straightforwardly consistent with statistical coalescence: Λ_C^+~cqq, D^0~cq ➔ Λ_C^+/D^0 ~ q ~ $dn_{ch}/d\eta$

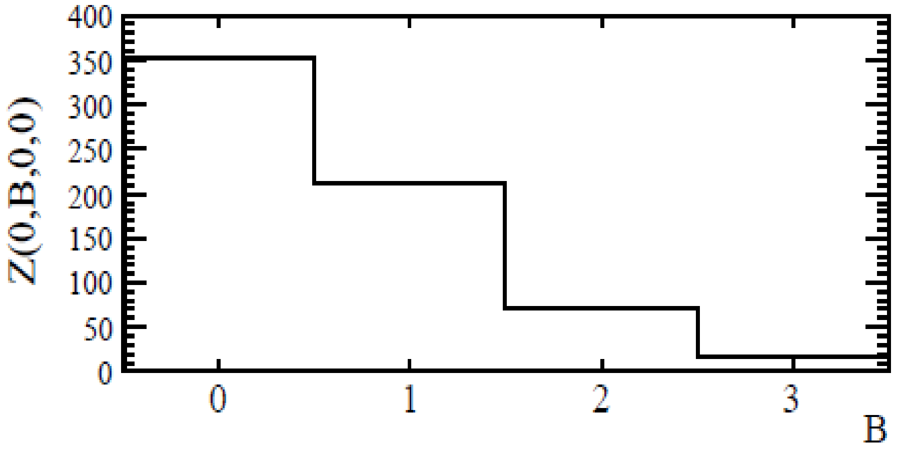
Canonical-ensemble SHM

➢ Canonical partition function conserving charge $\vec{Q} = (Q, B, S, C)$

$$Z(\vec{Q}) = \frac{1}{(2\pi)^4}\int_0^{2\pi} d^4\phi e^{i\vec{Q}\cdot\vec{\phi}} \exp[\sum_j \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} e^{-i\vec{q}_j\cdot\vec{\phi}} z_j]$$

where $z_j = (2J_j + 1)\frac{VT_H}{2\pi^2} m_j^2 K_2(\frac{m_j}{T_H})$

- Pair creation➔ energy-expensive: once a baryon created, simultaneous creation of an antibaryon is required to fulfill baryon-number conservation



➢ Hadron mean number

$$\langle N_j \rangle^{CE} = \langle N_j \rangle^{GCE} \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

chemical factor characterizing canonical suppression for charged hadron with $\vec{q}_j \neq 0$

Y. Chen and MH, PLB 815 (2021) 136144

Chemical factors: canonical suppression

CF	V=10 fm^3	20	50	100	200
D^0	0.025877	0.066239	0.190294	0.373107	0.627886
D^+	0.025439	0.065891	0.190002	0.372841	0.627669
D_s^+	0.015805	0.053178	0.178586	0.362376	0.619125
Λ_c^+	0.016956	0.055485	0.182039	0.365923	0.622147
Ξ_c^{+0}	0.009884	0.042956	0.167943	0.352535	0.611073
Ω_c	0.003495	0.022604	0.130312	0.312514	0.576383
Λ_c^+/D^0	0.655254	0.837649	0.956620	0.980745	0.990860
D_s^+/D^0	0.610774	0.802820	0.938474	0.971239	0.986047

pp mid-rapidity neutral (Q,B,S,C)=(0,0,0,0)

$T_H = 170$ MeV, $\gamma_s = 0.6$, $\gamma_c = 15$

- Column at given V: CF progressively smaller for hadrons containing more charges (S, B, BS, BSS)
- Row: CF increases with volume, tending to the common residual canonical charm supp. (canonical B/S supp. diminishing)
- Relative CF: Λ_C^+/D^0 & D_s/D^0 increases with volume toward unity

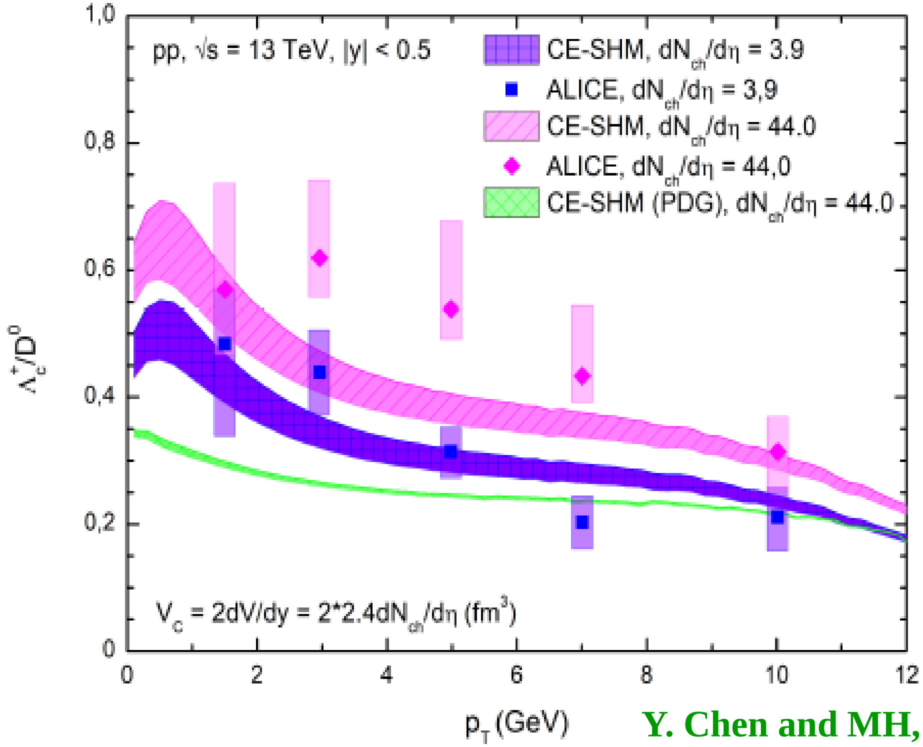
CE-SHM densities with feeddowns

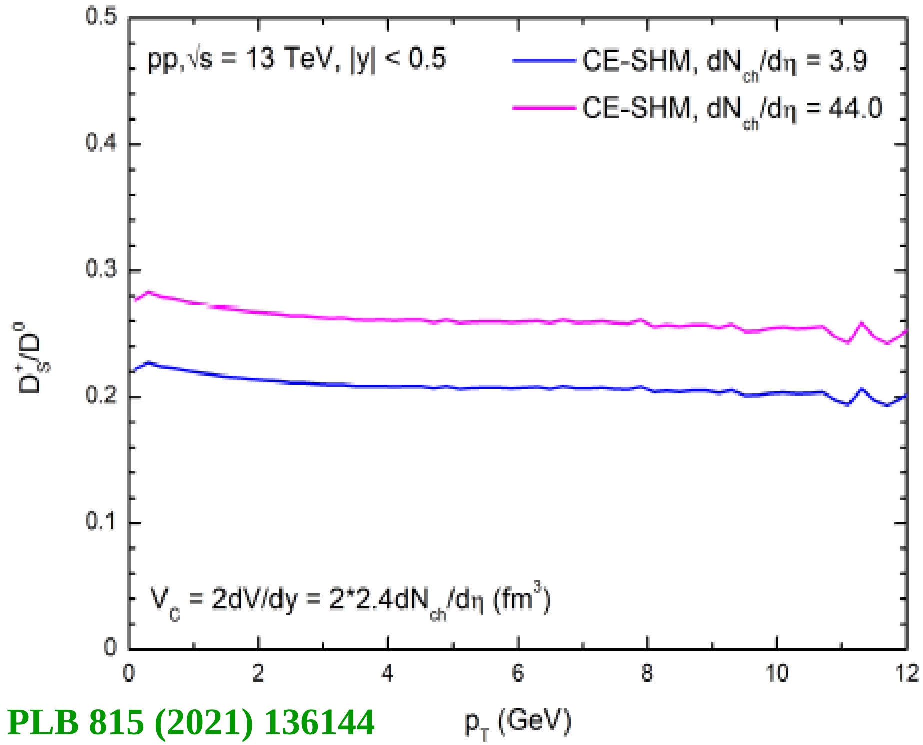
n_j $(\cdot 10^{-4}\text{fm}^{-3})$	V=10 fm^3	20	50	100	200	GCE
D^0	0.445553	1.148287	3.310131	6.495330	10.934662	17.420949
D^+	0.194705	0.503847	1.453016	2.851351	4.800262	7.647869
D_s^+	0.075040	0.252484	0.847910	1.720531	2.939551	4.747914
Λ_c^+(BR50%)	0.126963	0.439135	1.497132	3.045487	5.207572	8.415360
Λ_c^+(BR100%)	0.149573	0.519555	1.776775	3.617118	6.187127	10.001702
Ξ_c^{+0}	0.016539	0.071955	0.281624	0.591389	1.025276	1.678110
Ω_c	0.000756	0.004889	0.028184	0.067592	0.124662	0.216283
Λ_c^+/D^0(BR50%)	0.284956	0.382426	0.452288	0.468873	0.476244	0.483060
Λ_c^+/D^0(BR100%)	0.335702	0.452461	0.536769	0.556880	0.565827	0.574119
D_s^+/D^0	0.168420	0.219879	0.256156	0.264887	0.268829	0.272540

pp mid-rapidity neutral (Q,B,S,C)=(0,0,0,0) $T_H = 170\,\text{MeV}$, $\gamma_s = 0.6$, $\gamma_c = 15$

- Row: density of each charm-hadron increases with volume
- Λ_C^+/D^0 & D_s/D^0: marked system-size dependence: a ~40% reduction from V=200 (~GCE-SHM) to V=10 fm^3

Fragmentation & decay: p_T-dependent ratios





Y. Chen and MH, PLB 815 (2021) 136144

- Splitting of Λ_C^+/D^0 between $dN_{ch}/d\eta$=3.9 vs 44.0 roughly reproduced, due to additional canonical baryon supp. on charm-baryons, which becomes stronger toward smaller system-size

- Similar splitting of D_s^+/D^0 by additional canonical strangeness supp.

Augmented SHM for bottom-hadrons in pp

- "Missing" bottom baryons: Ebert et al., PRD 84 (2011) 014025

  PDG: 5 B, 4 B_s, 3 Λ_b, 2 Σ_b, 4 Ξ_b, 1 Ω_b

  RQM: 25 B, 20 B_s, 30 Λ_b, 46 Σ_b, 75 Ξ_b, 42 Ω_b

- Relativistic quark model (RQM): Q-light diquark bound states

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_{d,B}(\mathbf{q})$$

Ebert et al., PRD 84 (2011) 014025

TABLE I. Masses M and form factor parameters of diquarks. S and A denote scalar and axial-vector diquarks which are antisymmetric $[\cdots]$ and symmetric $\{\cdots\}$ in flavor, respectively [4].

	Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV^2)
Λ_b →	$[u,d]$	S	710	1.09	0.185
Σ_b →	$\{u,d\}$	A	909	1.185	0.365
Ξ_b →	$[u,s]$	S	948	1.23	0.225
	$\{u,s\}$	A	1069	1.15	0.325
Ω_b →	$\{s,s\}$	A	1203	1.13	0.280

PDG vs RQM heavy baryons (Λ_Q & Σ_Q)

TABLE II. Masses of the Λ_Q ($Q = c, b$) heavy baryons (in MeV).

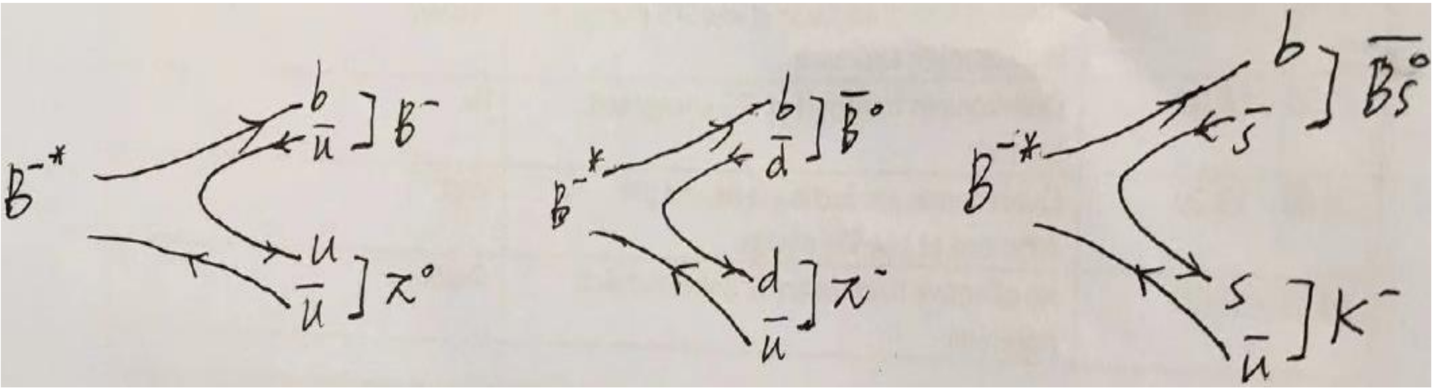
		$Q=c$		$Q=b$	
$I(J^P)$	Qd state	M	M^{exp} [1]	M	M^{exp} [1]
$0(\frac{1}{2}^+)$	1S	2286	2286.46(14)	5620	5620.2(1.6)
$0(\frac{1}{2}^+)$	2S	2769	2766.6(2.4)?	6089	
$0(\frac{1}{2}^+)$	3S	3130		6455	
$0(\frac{1}{2}^+)$	4S	3437		6756	
$0(\frac{1}{2}^+)$	5S	3715		7015	
$0(\frac{1}{2}^+)$	6S	3973		7256	
$0(\frac{1}{2}^-)$	1P	2598	2595.4(6)	5930	
$0(\frac{1}{2}^-)$	2P	2983	$2939.3(^{1.4}_{1.5})$?	6326	
$0(\frac{1}{2}^-)$	3P	3303		6645	
$0(\frac{1}{2}^-)$	4P	3588		6917	
$0(\frac{1}{2}^-)$	5P	3852		7157	
$0(\frac{3}{2}^-)$	1P	2627	2628.1(6)	5942	
$0(\frac{3}{2}^-)$	2P	3005		6333	
$0(\frac{3}{2}^-)$	3P	3322		6651	
$0(\frac{3}{2}^-)$	4P	3606		6922	
$0(\frac{3}{2}^-)$	5P	3869		7171	
$0(\frac{3}{2}^+)$	1D	2874		6190	
$0(\frac{3}{2}^+)$	2D	3189		6526	
$0(\frac{3}{2}^+)$	3D	3480		6811	
$0(\frac{3}{2}^+)$	4D	3747		7060	
$0(\frac{5}{2}^+)$	1D	2880	2881.53(35)	6196	
$0(\frac{5}{2}^+)$	2D	3209		6531	
$0(\frac{5}{2}^+)$	3D	3500		6814	

TABLE III. Masses of the Σ_Q ($Q = c, b$) heavy baryons (in MeV).

		$Q=c$		$Q=b$	
$I(J^P)$	Qd state	M	M^{exp} [1]	M	M^{exp} [1]
$1(\frac{1}{2}^+)$	1S	2443	2453.76(18)	5808	5807.8(2.7)
$1(\frac{1}{2}^+)$	2S	2901		6213	
$1(\frac{1}{2}^+)$	3S	3271		6575	
$1(\frac{1}{2}^+)$	4S	3581		6869	
$1(\frac{1}{2}^+)$	5S	3861		7124	
$1(\frac{3}{2}^+)$	1S	2519	2518.0(5)	5834	5829.0(3.4)
$1(\frac{3}{2}^+)$	2S	2936	$2939.3(^{1.4}_{1.5})$?	6226	
$1(\frac{3}{2}^+)$	3S	3293		6583	
$1(\frac{3}{2}^+)$	4S	3598		6876	
$1(\frac{3}{2}^+)$	5S	3873		7129	
$1(\frac{1}{2}^-)$	1P	2799	$2802(^{4}_{7})$	6101	
$1(\frac{1}{2}^-)$	2P	3172		6440	
$1(\frac{1}{2}^-)$	3P	3488		6756	
$1(\frac{1}{2}^-)$	4P	3770		7024	
$1(\frac{1}{2}^-)$	1P	2713		6095	
$1(\frac{1}{2}^-)$	2P	3125		6430	
$1(\frac{1}{2}^-)$	3P	3455		6742	
$1(\frac{1}{2}^-)$	4P	3743		7008	
$1(\frac{3}{2}^-)$	1P	2798	$2802(^{4}_{7})$	6096	
$1(\frac{3}{2}^-)$	2P	3172		6430	
$1(\frac{3}{2}^-)$	3P	3486		6742	
$1(\frac{3}{2}^-)$	4P	3768		7009	
$1(\frac{3}{2}^-)$	1P	2773	2766.6(2.4)?	6087	
$1(\frac{3}{2}^-)$	2P	3151		6423	
$1(\frac{3}{2}^-)$	3P	3469		6736	
$1(\frac{3}{2}^-)$	4P	3753		7003	
$1(\frac{5}{2}^-)$	1P	2789		6084	

Strong decay systematics: BR's estimation

- Counting all possible diagrams once above the threshold

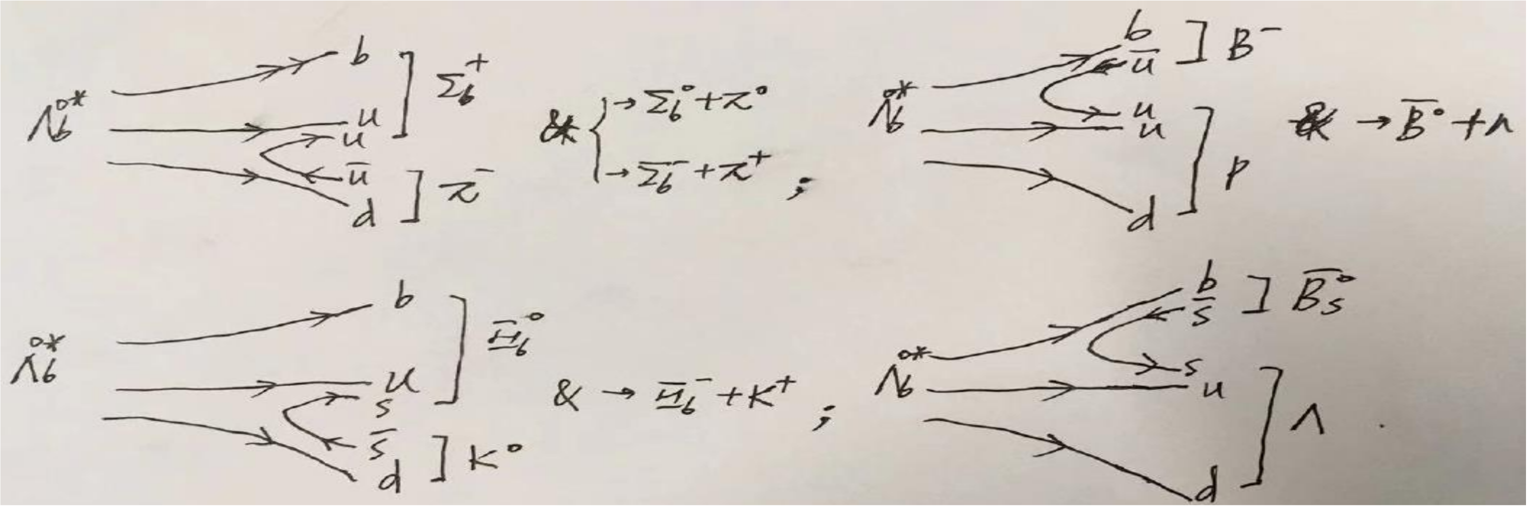


- Probability of producing a q-qbar pair ∝ $\exp(-2m/T_H)$
  ➔ $\exp(-2m_q/T_H)$: $\exp(-2m_s/T_H)$ = 1 : 1/3 [m_q~8, m_s~100 MeV]
  ➔ diagrams involving s-sbar counted as 1/3

- E.g. BR(B^{-*}→B^-+π^0)=1/(1+1+1/3)=43%
  BR(B^{-*}→B^0bar+π^-)=1/(1+1+1/3)=43%
  BR(B^{-*}→B^0_sbar+π^0)=1/3/(1+1+1/3)=14%

Strong decay systematics: BR's estimation

- Counting all possible diagrams once above the mass threshold



- E.g. BR(Λ_b^{0*}→ Σ_b+ π^- →Λ_b^0 +2π)=3/(3+2+2*1/3+1/3)=54%
  BR(Λ_b^{0*}→ B^- + p) =1/(3+2+1/3+2*1/3)=16%
  BR(Λ_b^{0*}→ Ξ_b + K) =2/3/(3+2+1/3+2*1/3)=11%
  BR(Λ_b^{0*}→ B^0_sbar + Λ) =1/3/(3+2+1/3+2*1/3)=6%

- Results comparable to (limited) results computed in $^3P_0$ model

Ferretti et al., PRD 97114020 (2018); Yu et al., 2206.08128

Augmented SHM for bottom hadrons in pp

- Grand-canonical SHM densities $n_i^{\text{primary}} = \frac{d_i}{2\pi^2}\gamma_s^{N_s^i} m_i^2 T_H K_2(\frac{m_i}{T_H})$
  - ➔ Weakly decaying b-hadrons (ground states)

$$n_\alpha = n_\alpha^{\text{primary}} + \sum n_i^{\text{primary}} \cdot BR(i \to \alpha)$$

- strangeness supp. γ_s=0.6
  bottom fugacity γ_b=1

n_α $(\cdot 10^{-12}\ \text{fm}^{-3})$	B^-	$\bar{B}^0$	$\bar{B}_s^0$	Λ_b^0	$\Xi_b^{0,-}$	Ω_b^-
PDG(170)	1.0094	1.0089	0.29308	0.31591	0.10097	0.002341
PDG(160)	0.12655	0.12649	0.036622	0.034241	0.010520	0.00023076
RQM(170)	1.2045	1.2041	0.32513	0.61702	0.19548	0.0063204
RQM(160)	0.14567	0.14561	0.039664	0.061914	0.018819	0.00061087

MH & Rapp, arXiv:2209.13419

f_α	B^-	$\bar{B}^0$	$\bar{B}_s^0$	Λ_b^0	$\Xi_b^{0,-}$
PDG(170)	0.3697	0.3695	0.1073	0.1157	0.03698
PDG(160)	0.3782	0.3780	0.1094	0.1023	0.03144
RQM(170)	0.3391	0.3389	0.09152	0.1737	0.05503
RQM(160)	0.3533	0.3532	0.09620	0.1502	0.04565

r_α	$\bar{B}^0/B^-$	$\bar{B}_s^0/B^-$	Λ_b^0/B^-	$\Xi_b^{0,-}/B^-$
PDG(170)	0.9995	0.2904	0.3129	0.1000
PDG(160)	0.9995	0.2894	0.2706	0.08313
RQM(170)	0.9994	0.2699	0.5122	0.1623
RQM(160)	0.9996	0.2723	0.4250	0.1292

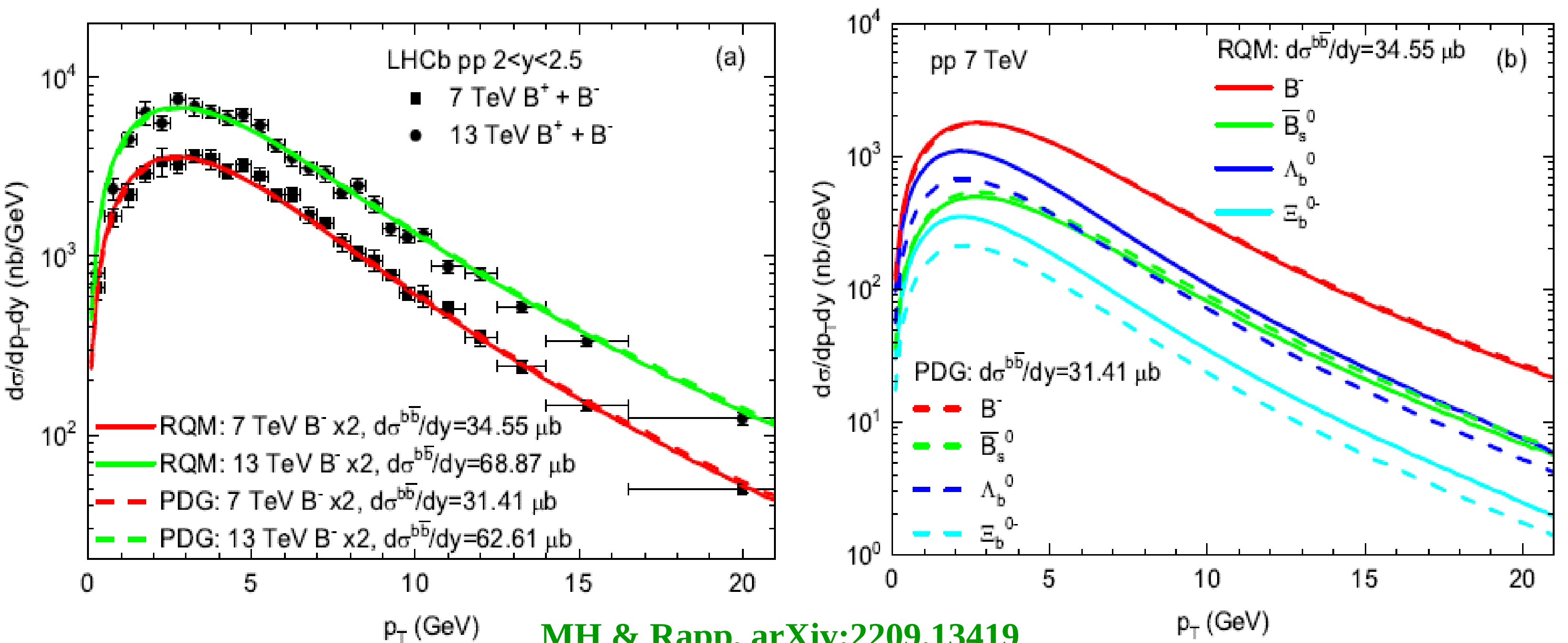
- RQM vs PDG: Λ_b & Ξ_b fraction both enhanced by ~50%
- RQM f_α very comparable to p-pbar data by Tevatron

$$f_u = f_d = 0.340 \pm 0.021,\ f_s = 0.101 \pm 0.015$$
$$f_{\text{baryon}} = 0.220 \pm 0.048$$

HFLAG,Eur. Phys. J. C (2021) 81:226

Fit of p_T-spectra & cross sections

- FONLL b-quark spectrum + FF $D_{b \to H_b}(z) \propto z^{\alpha}(1-z)$ to all states (weight ∝ density) + decay simulations ➔ ground states p_T-spectra

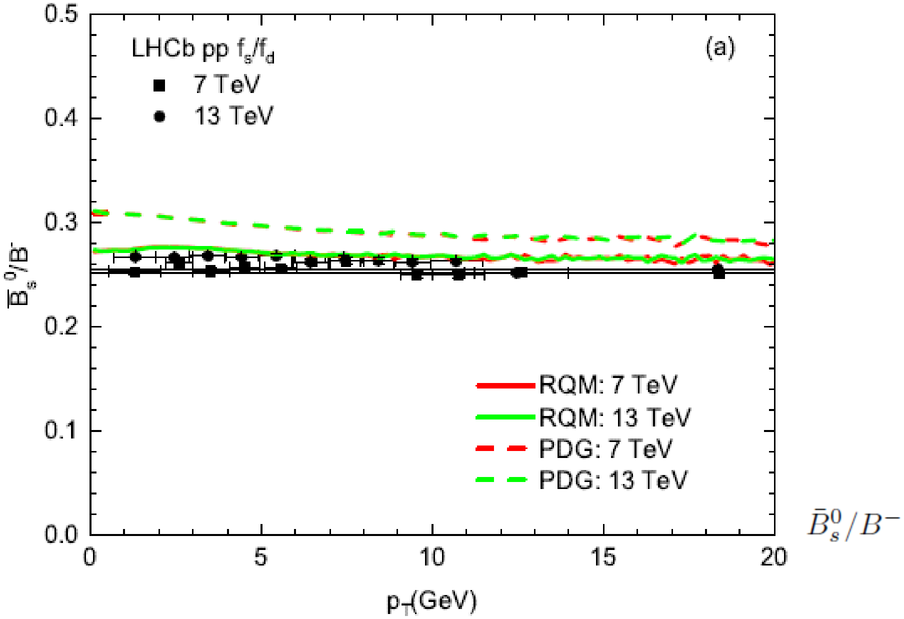


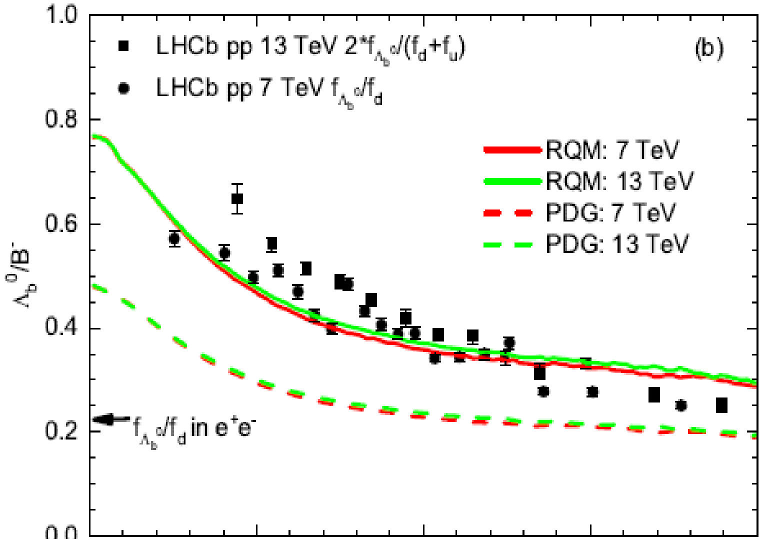
MH & Rapp, arXiv:2209.13419

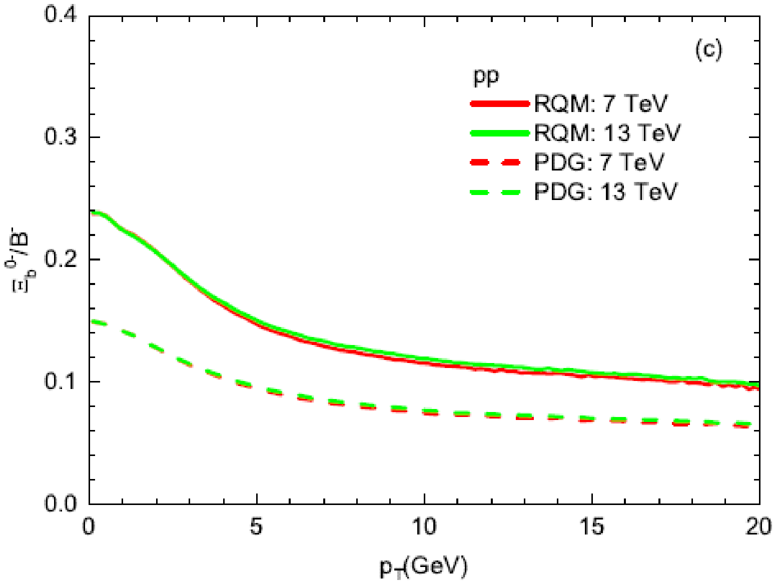
- B^++B^- data equally well described by RQM & PDG, but ~10% smaller $d\sigma^{bbar}/dy$ for PDG

- The decrease is due to reduction of bottom content in the baryon sector

Bottom hadro-chemistry: ratios







- **B^0_sbar/B^-** & **Λ_b^0/B^-** almost unchanged from 7 to 13 TeV
- PDG-only curves far off; RQM states needed
- $f_{\Lambda b}/f_d$ in pp significantly larger than e^+e^- **MH & Rapp, arXiv:2209.13419**

Summary: HQ hadronization in pp

- Statistical hadronization of HQs in hadronic collisions works well
  ➔ heavy hadron fragmentation fractions & ratios well explained, especially for bottom for the first time, by augmenting PDG states with many more “missing” states/baryons predicted by RQM/lattice QCD

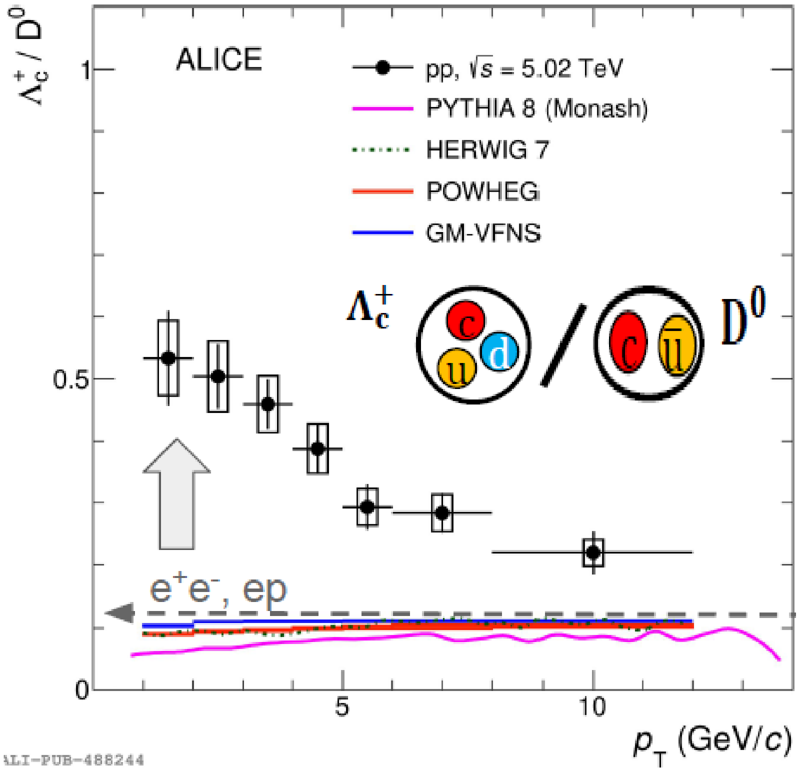
- Baryon/meson ratio significantly enhanced in hadronic (pp, pp-bar) than e^+e^-
  ➔ shuffling of charm/bottom content from meson to baryon sector
  ➔ universaility assumption of HQ fragmentation is not true

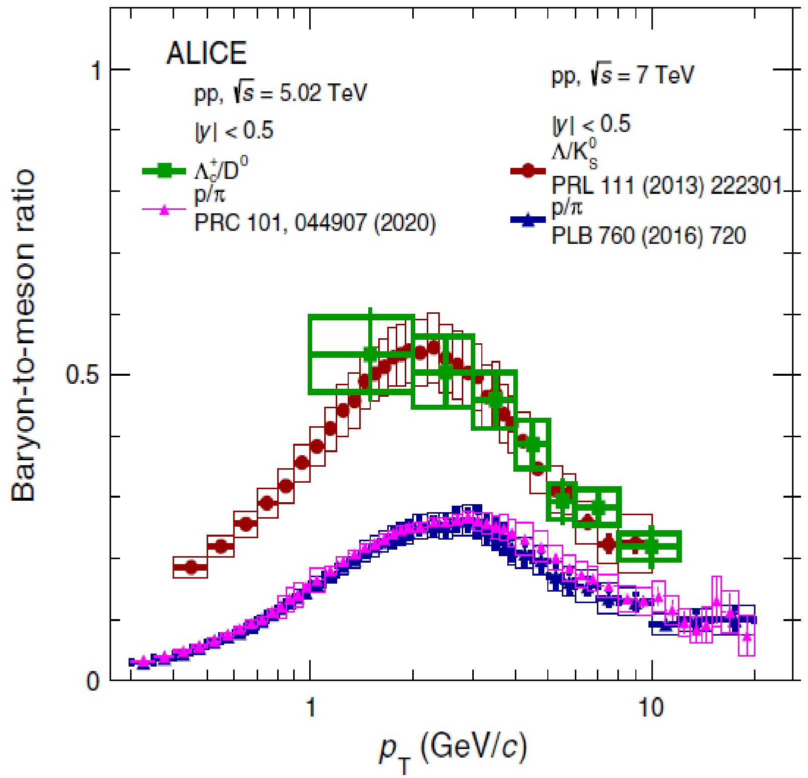
- More intuitively, high-energy pp, pp-bar provide a quark-rich environment conducive to (statistical) recombination of HQ with light quarks, especially to the formation of heavy baryons, VS e^+e^- with less phase space

Back-up

The following are back-up pages

Λ_c^+/D^0 enhancement surprise

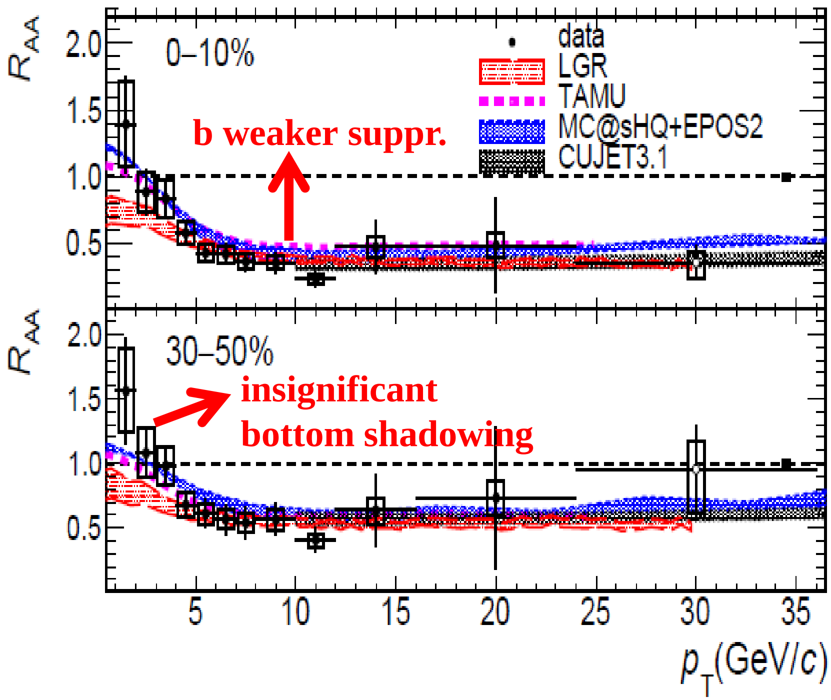




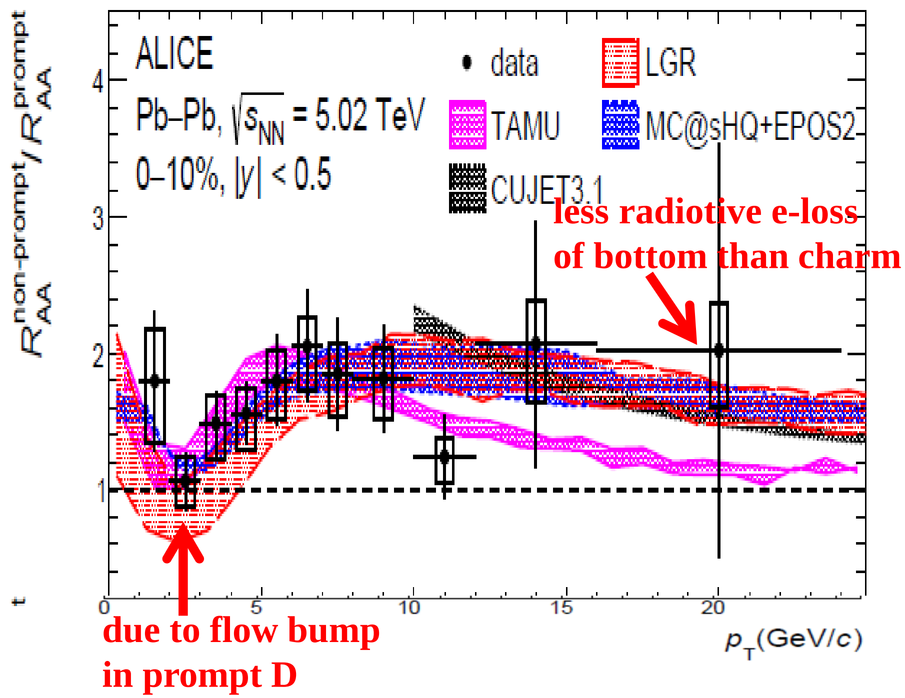
- ❖ a factor ~5 enhancement w.r.t. e^+e^- at low p_T, much underestimated by FFs tuned to e^+e^-
- ❖ decreasing toward high p_T, trend similar to Λ/K and p/pi

Flavor dependence: charm vs bottom

nonprompt D (← B) R_{AA} , ALICE:2202.00815



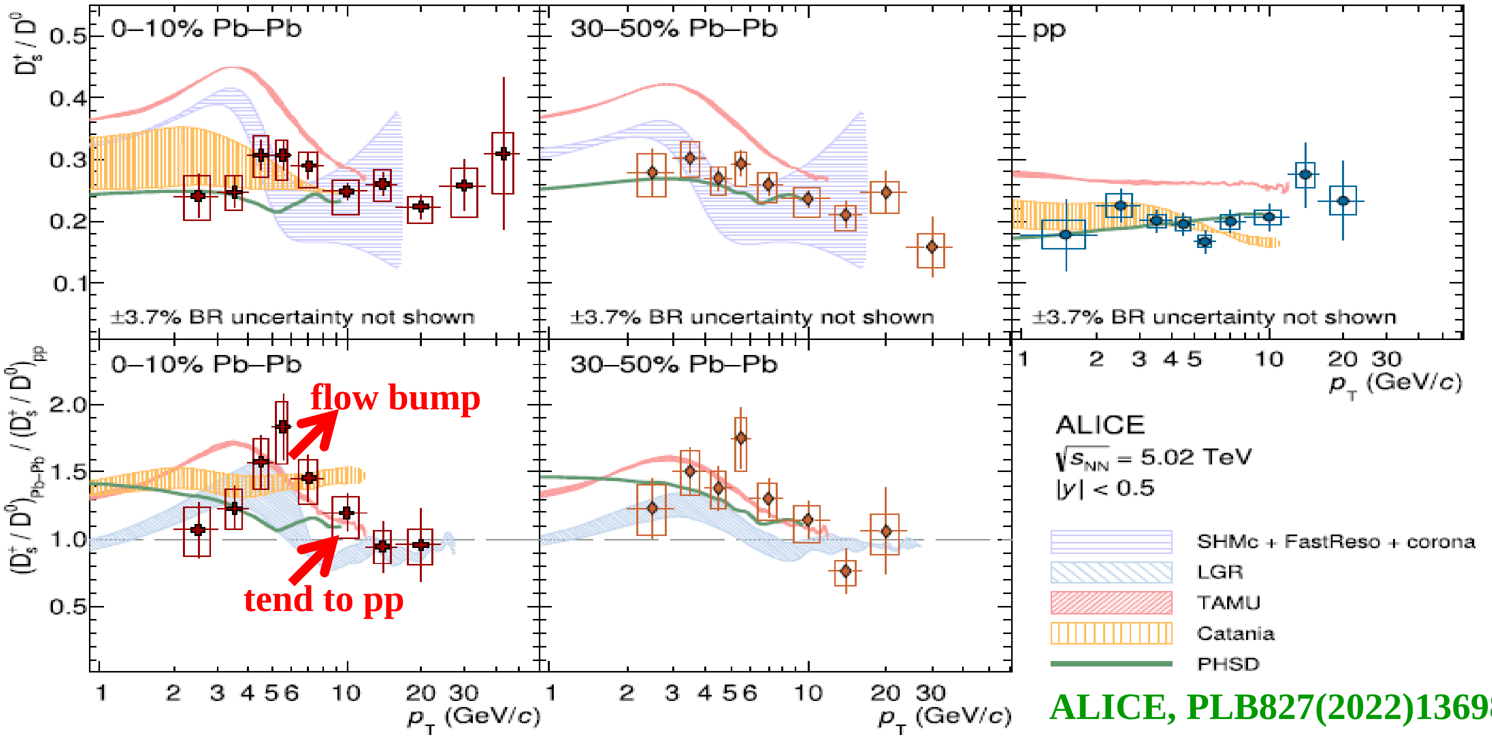
nonprompt-to-prompt D-meson R_{AA} ratio



❖ x3 mass: b-quark longer thermalization time at low p_T than charm
less flow added to b from recombination with u/d/s

❖ high p_T>15 GeV: b-quark less radiative e-loss← stronger “dead cone”

Charm hadro-chemistry: D_s/D^0

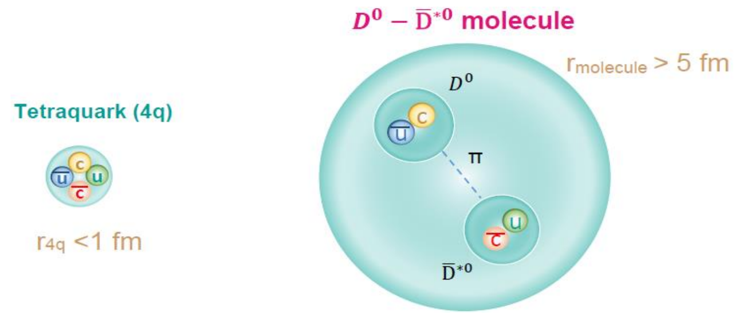


ALICE, PLB827(2022)136986

- low p_T: enhancement due to charm recombination in a strangeness-equilibrated QGP reproduced by Cantania & PHSD; overestimated by TAMU in both pp and PbPb
- high p_T: tending to pp value as fragmentation takes over
- flow bump due to recombination with flowing s-quark heavier than u/d, predicted by TAMU (RRM w/ SMCs) & SHMc (hydro blastwave spectrum)

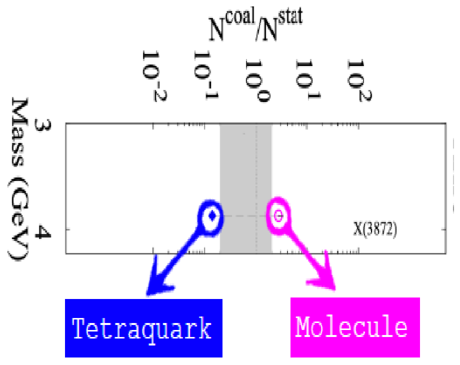
X(3872) production in HIC

❖ inner structure: compact tetraquark vs loosely bound molecule

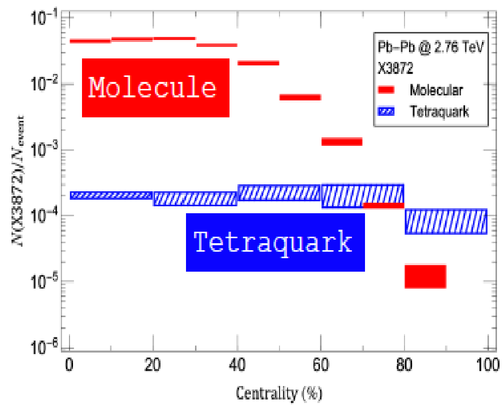


coalescence model

Cho et al. '11

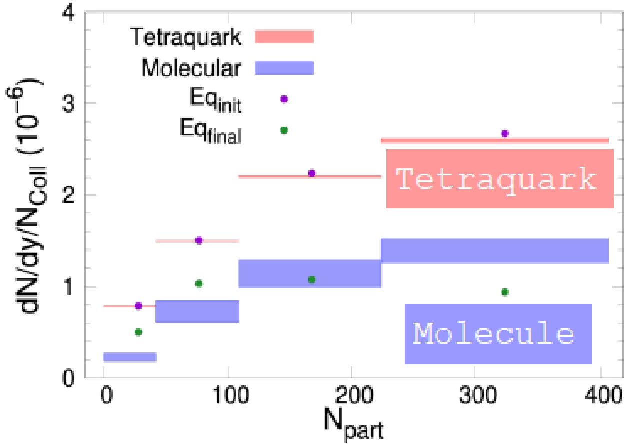


coalescence within AMPT zhang et al. '21



transport model

Wu et al. '21



- $N_{molecule} > N_{tetraquark}$ by x10 or 100, yet no account of hadron phase reactions πX <-->DD*
  ➔to be better constrained
- $N_{tetraquark} > N_{molecule}$ by x2, molecule regenerated in late hadronic phase, tetraquark chem. freezeout at T_c

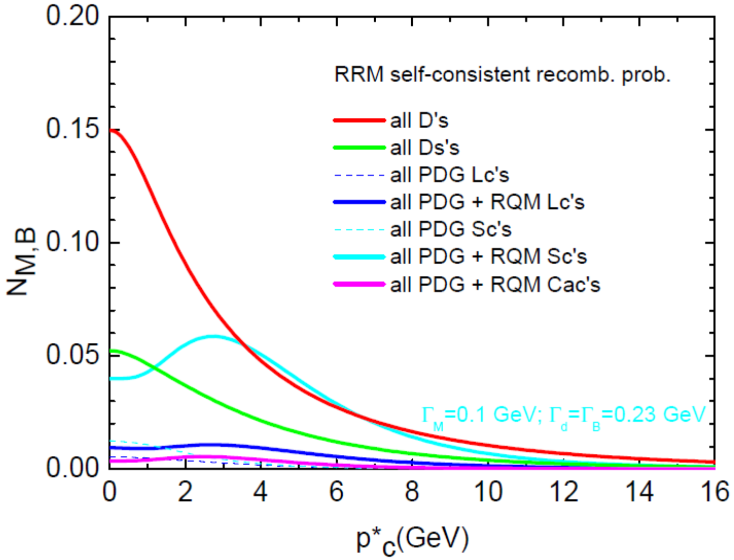
Charm quark recombination probability

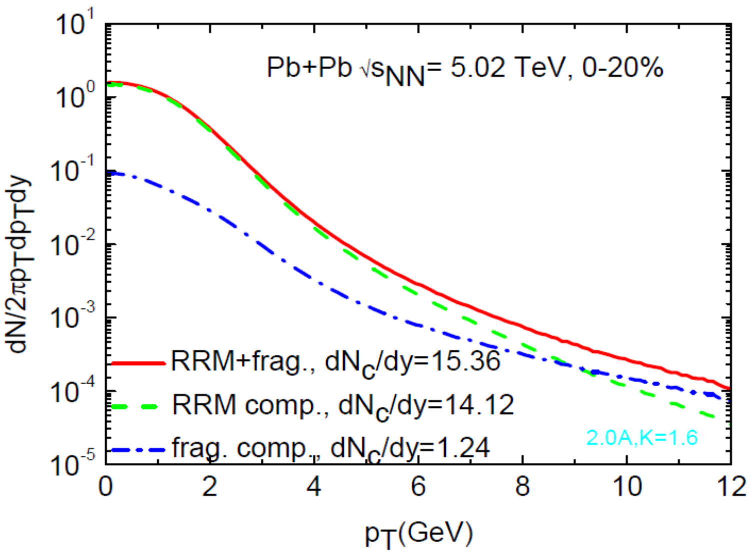
- No. of mesons/baryons formed from a single c-quark of rest frame p_c^*

$$N_M(p_c^*) = \int \frac{d^3\vec{p}_1^*}{(2\pi)^3} g_q e^{-E(\vec{p}_1^*)/T_{pc}} \frac{E_M(\vec{p}^*)}{m_M \Gamma_M} \sigma(s) v_{\rm rel},$$

$$N_B(p_c^*) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} g_1 e^{-E(\vec{p}_1)/T_c} g_2 e^{-E(\vec{p}_2)/T_c} \frac{E_d(\vec{p}_{12})}{m_d \Gamma_d} \sigma(s_{12}) v_{\rm rel}^{12}(\vec{p}_1, \vec{p}_2) \frac{E_B(\vec{p})}{m_B \Gamma_B} \sigma(s_{d3}) v_{rel}^{d3}(\vec{p}_{12}, \vec{p}_{30}),$$

- Renormalizing $N_M(p_c^*)$ and $N_B(p_c^*)$ by a common factor ~4 for all charmed mesons/baryons such that $\sum_M P_{{\rm coal},M}(p_c^* = 0) + \sum_B P_{{\rm coal},B}(p_c^* = 0) = 1$





➔ charm conservation consistently built in, in an (e-by-e) way without spoiling the relative chemical equilibrium realized by RRM

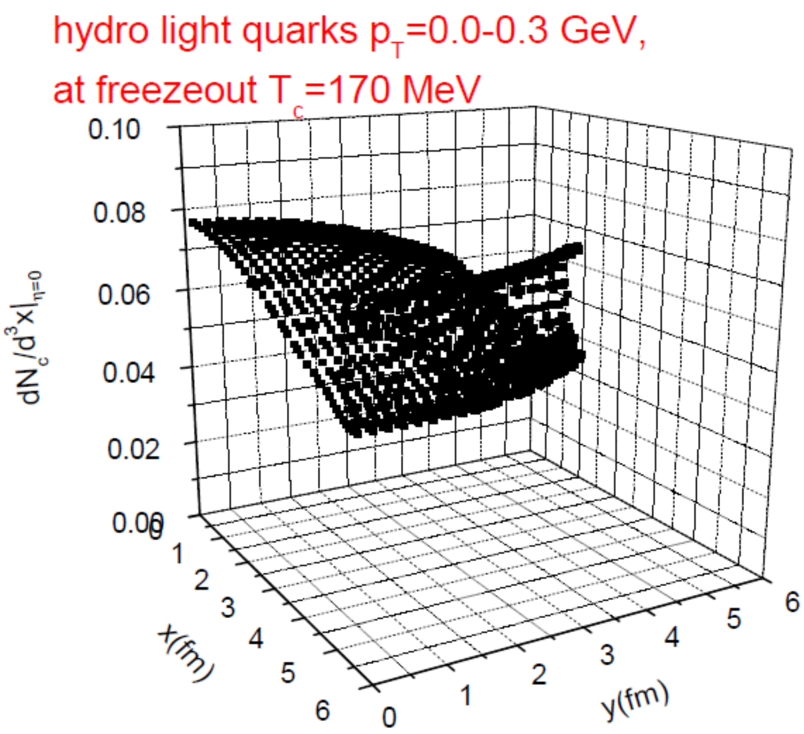
Space-momentum correlations: light-q

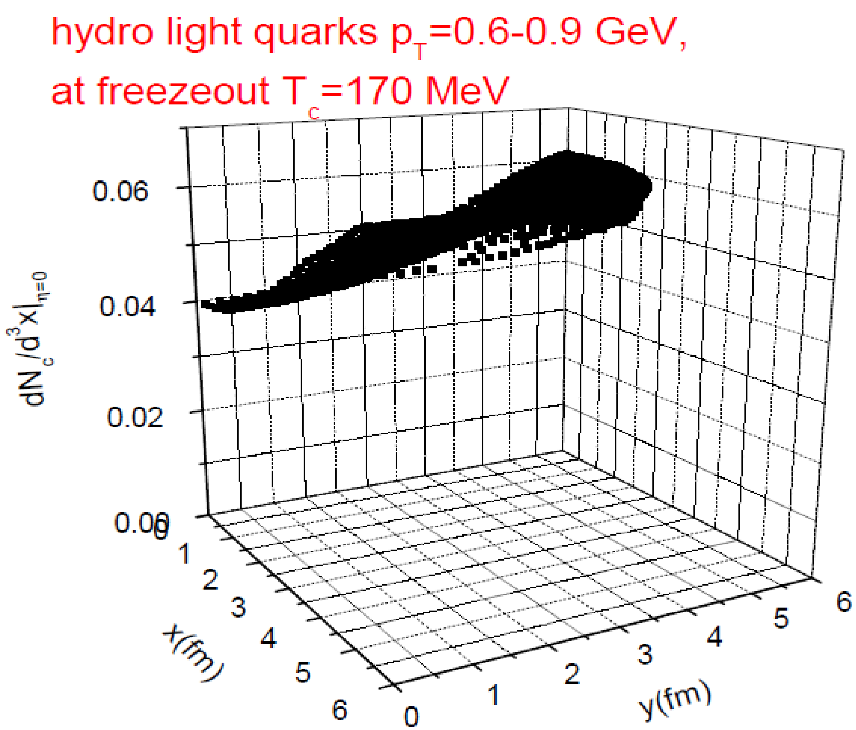
- hydro: a manifestation of SMCs

$$f_q^{eq}(\vec{x},\vec{p}) = g_q e^{-p\cdot u(x)/T(x)} = g_q e^{-\gamma_T(x)[m_T \cosh(y-\eta)-\vec{p}_T\cdot\vec{v}_T(x)]/T(x}$$

longitudinal boost invariance: y- η   transverse SMCs pT▪vT

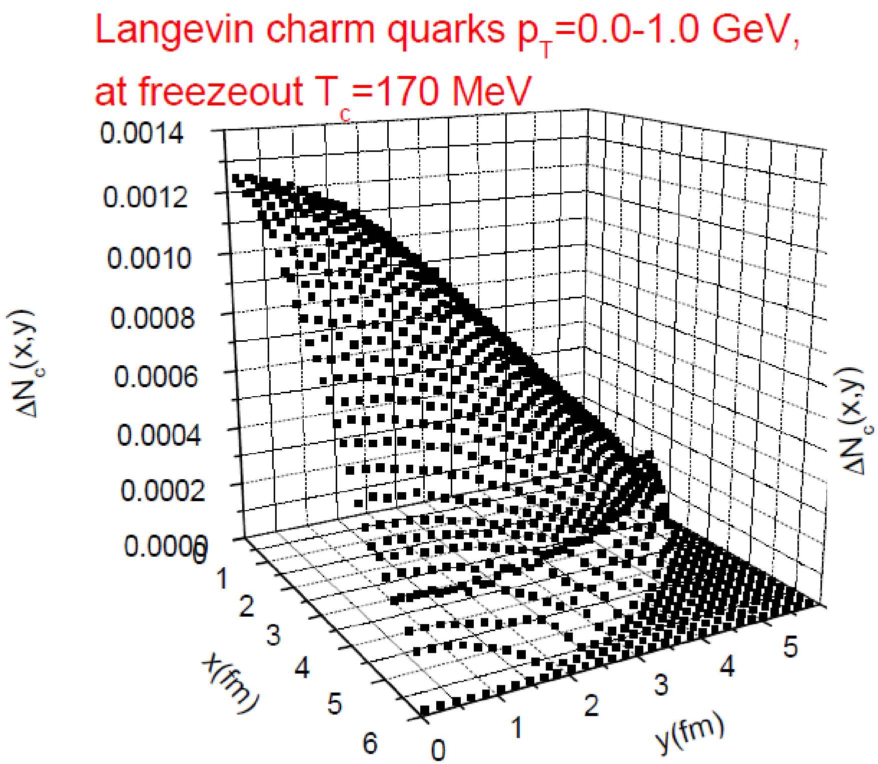
- hydro-q: low (high) p_T more concentrated in center (boundary)

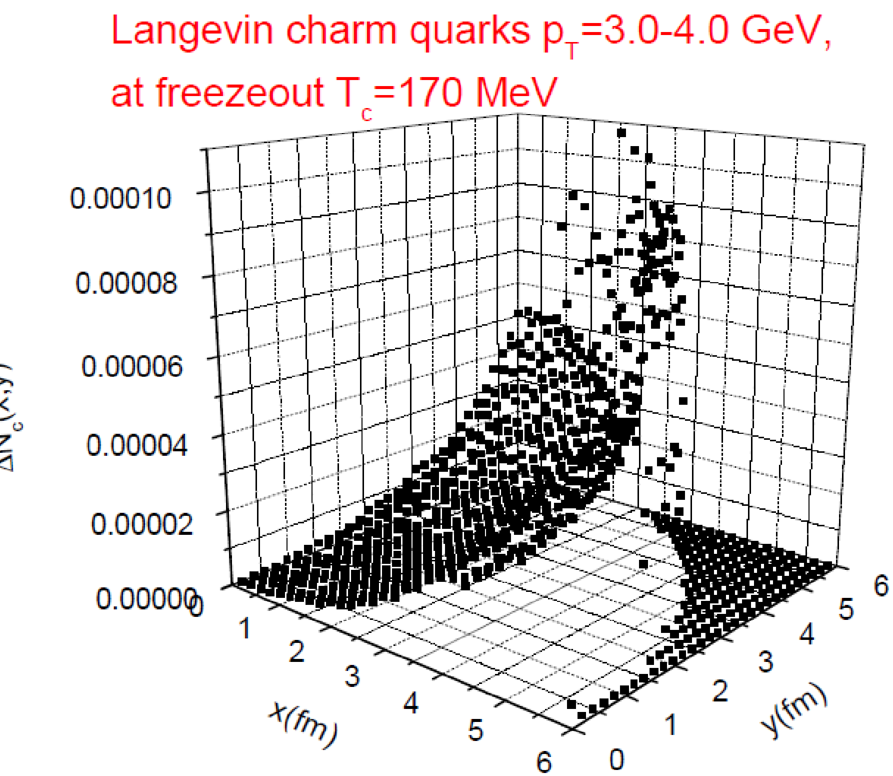




SMCs: Langevin charm quarks

- Langevin-c: low (high) p_T more populated in central (outer)





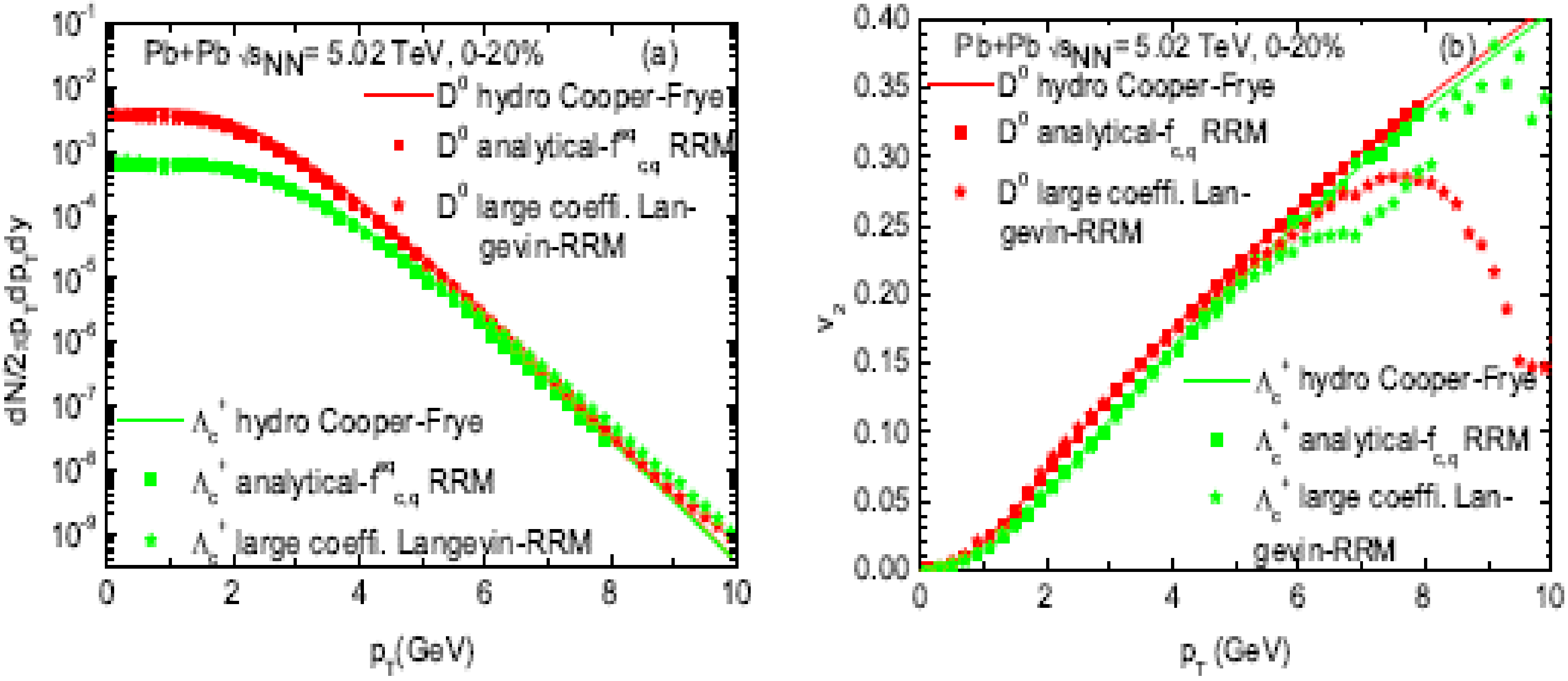
- SMCs usually neglected in ICMs: uniformly distributed independent of p_T

$$f_{c,q}(\vec{x},\vec{p}) = (2\pi)^3 \frac{dN_{c,q}}{d^3\vec{x}d^3\vec{p}} = \frac{(2\pi)^3}{VE(\vec{p})} \frac{dN_{c,q}}{p_T dp_T d\phi_q dy}$$

RRM equilibrium mapping

- Event-by-event Langevin-RRM simulation with very large trans. coeffi. & with SMCs properly incorporated

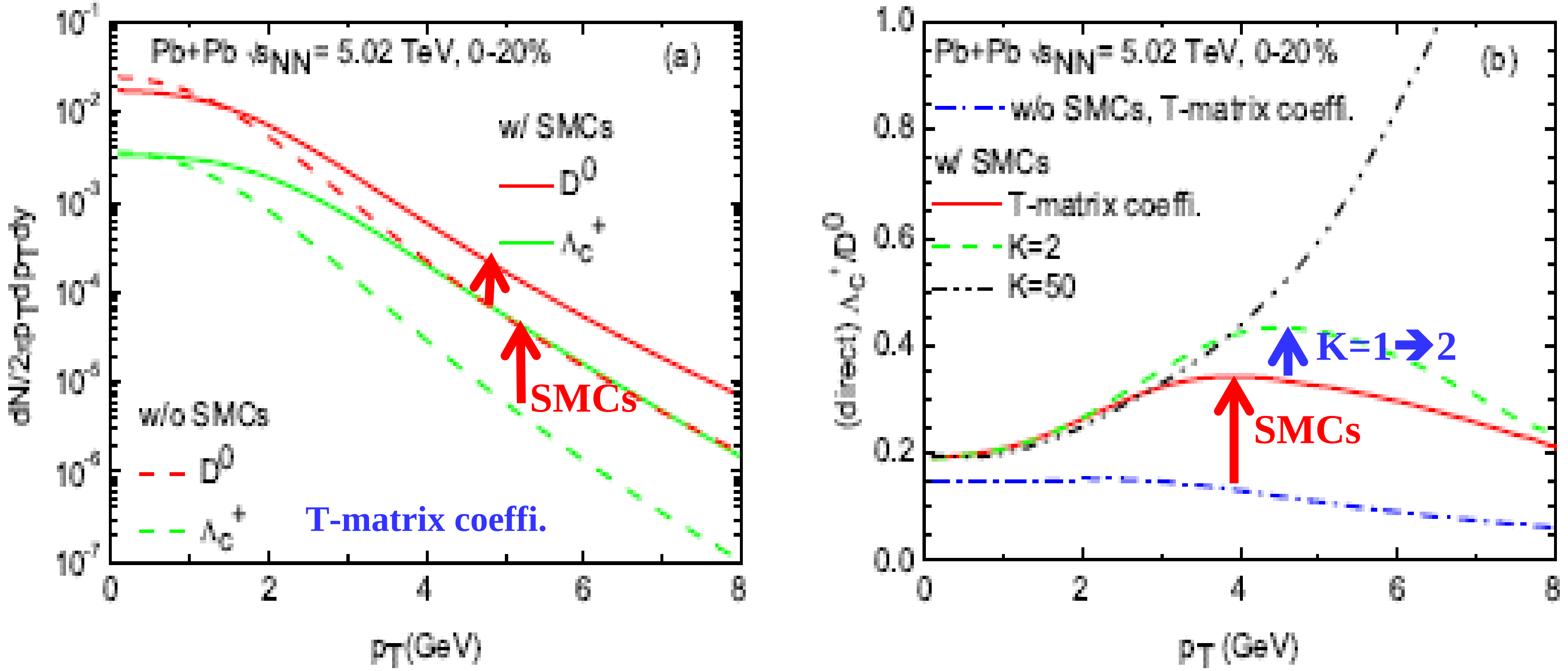
  ➔ kinetic & chemical equil. mapping



➔ Observables come out as RRM predictions with realistic T-matrix coeffi.

Direct D^0 & Λ_c^+ production via RRM

- Including SMCs makes spectra harder & enhances the Λ_c^+/D^0

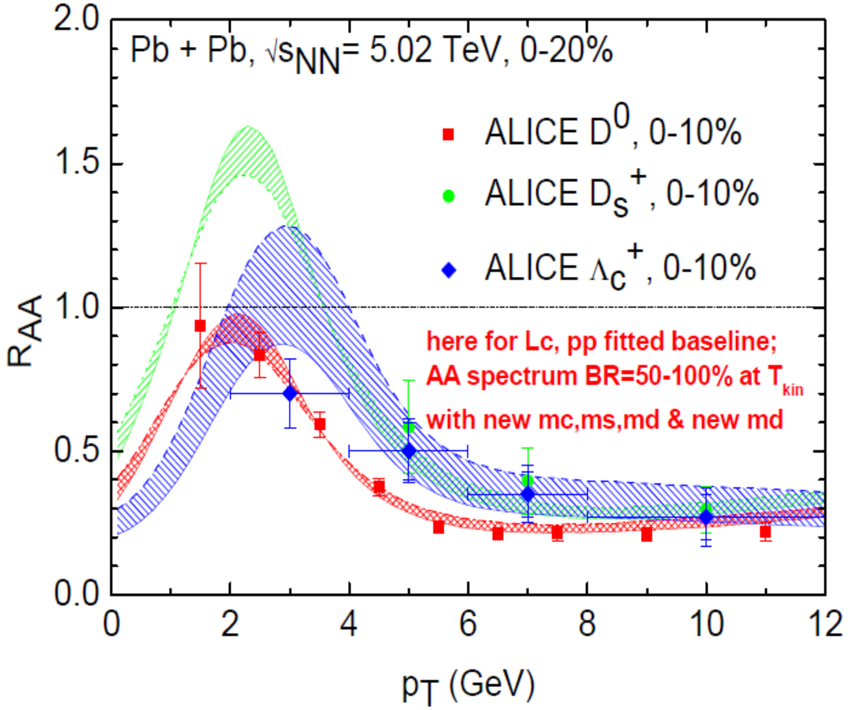


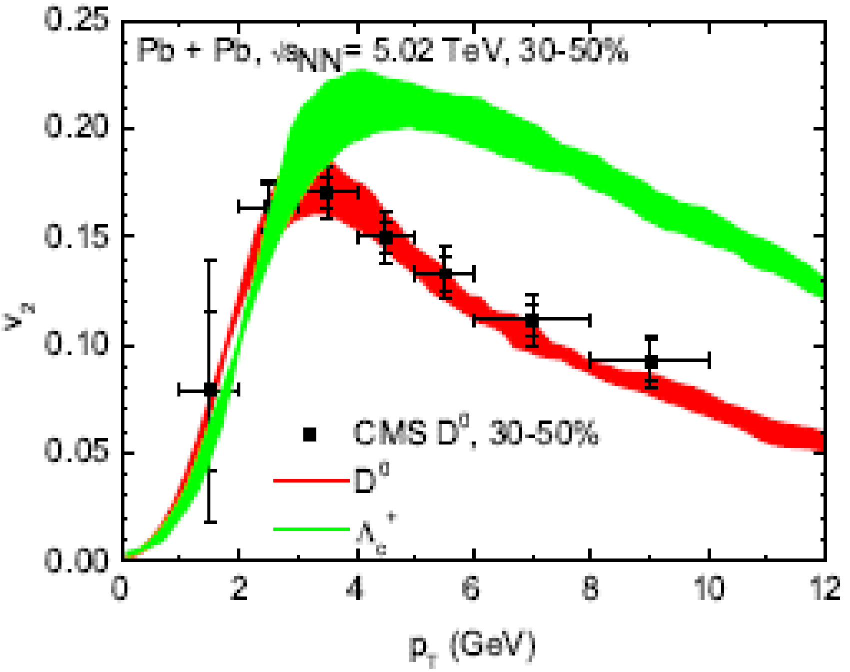
- Fast-moving c-quarks [p_T~ 3-4 GeV] moving to outer part of fireball find higher-density of harder [p_T~ 0.6-0.9 GeV] light quarks for recombination

- An effect entering squared for the recombination production of Λ_c^+
➔ larger enhancement for Λ_c^+ ➔ Λ_c^+/D^0 ratio enhanced!

D^0, D_s^+ & Λ_c^+ suppression & elliptic flow

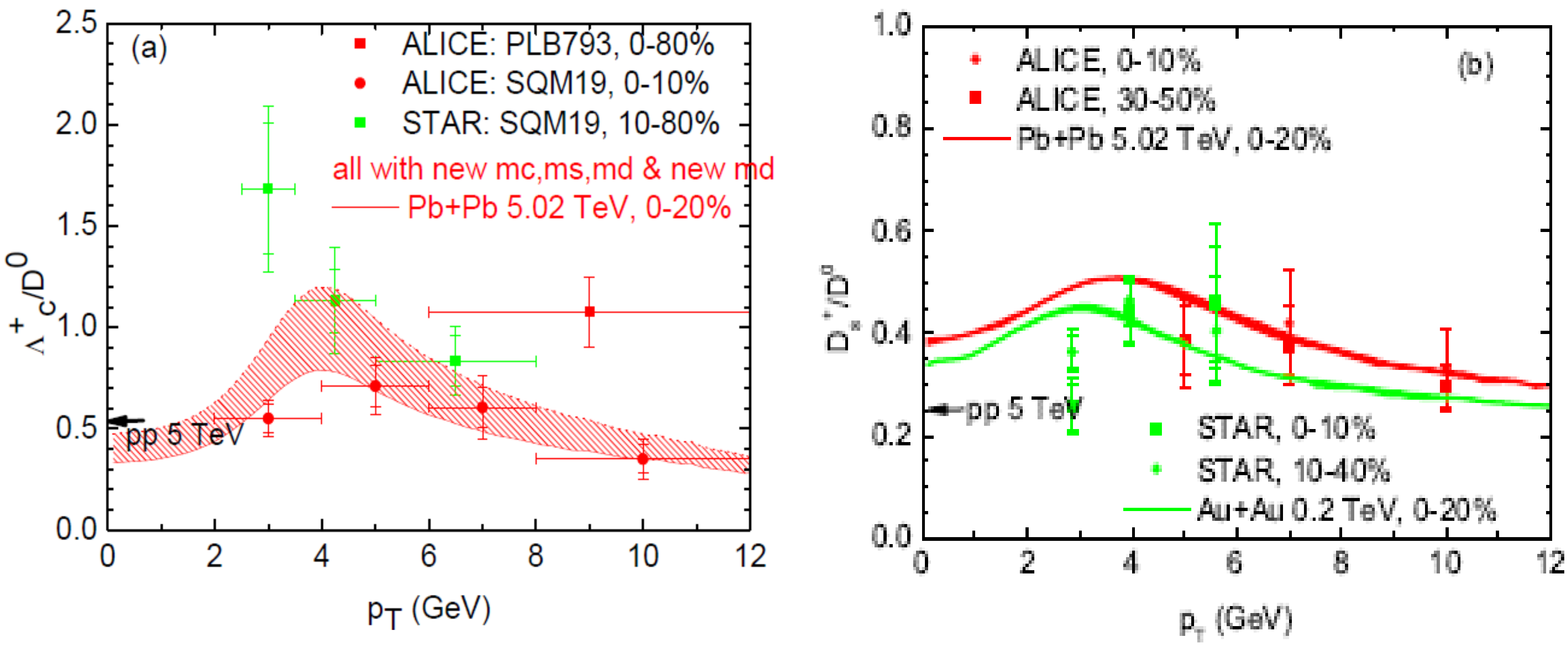
➢ Final D^0, D_s^+ & Λ_c^+, including feeddowns from all RQM baryons





- T-matrix coefficient*K-factor(=1.6), to compensate for radiative e-loss; uncertainty: BR=50-100% to Λ_c^+ for Λ_C's & Σ_C's above DN (2805 MeV)
- Hadronic phase diffusion also included: seamlessly connected to hadronization (RRM+frag), increasing D-meson v_2 by ~15%

Charm-hadron ratios: Λ_c^+/D^0 & D_s^+/D^0

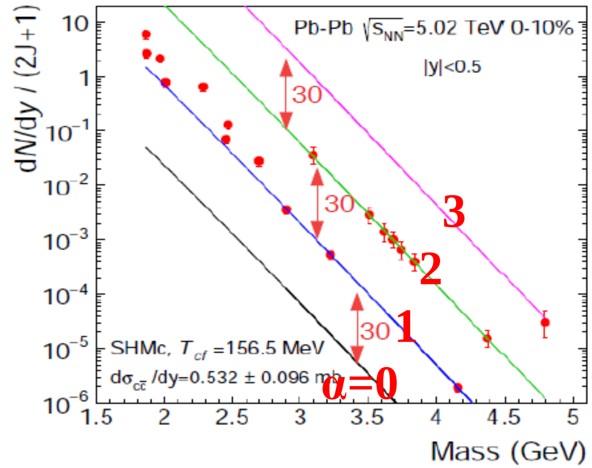


- Λ_c^+/D^0 : low p_T approaching RRM equil. limit = SHM pp;
  intermediate p_T enhancement from RRM with SMCs;
  high p_T fragmentation tending to pp value

- D_s^+/D^0 enhancement: recombination of charm in a strangeness-equilibrated QGP

Hadronization: SHMc Andronic, PBM et al. 2104.12754

- SHMc: open-charm statistical hadronization at T_c $\frac{dN(h^i_{oc,\alpha})}{dy} = g^\alpha_c \, V \, n^{th}_i \frac{I_\alpha(N^{tot}_c)}{I_0(N^{tot}_c)}$
  - multicharm baryons α=1,2,3 emerging pattern
  - yields enhanced by $g_c^\alpha \sim 30^\alpha$ than pure thermal →strong signal of deconfinement



- SHMc yields + blast wave→p_T spectra