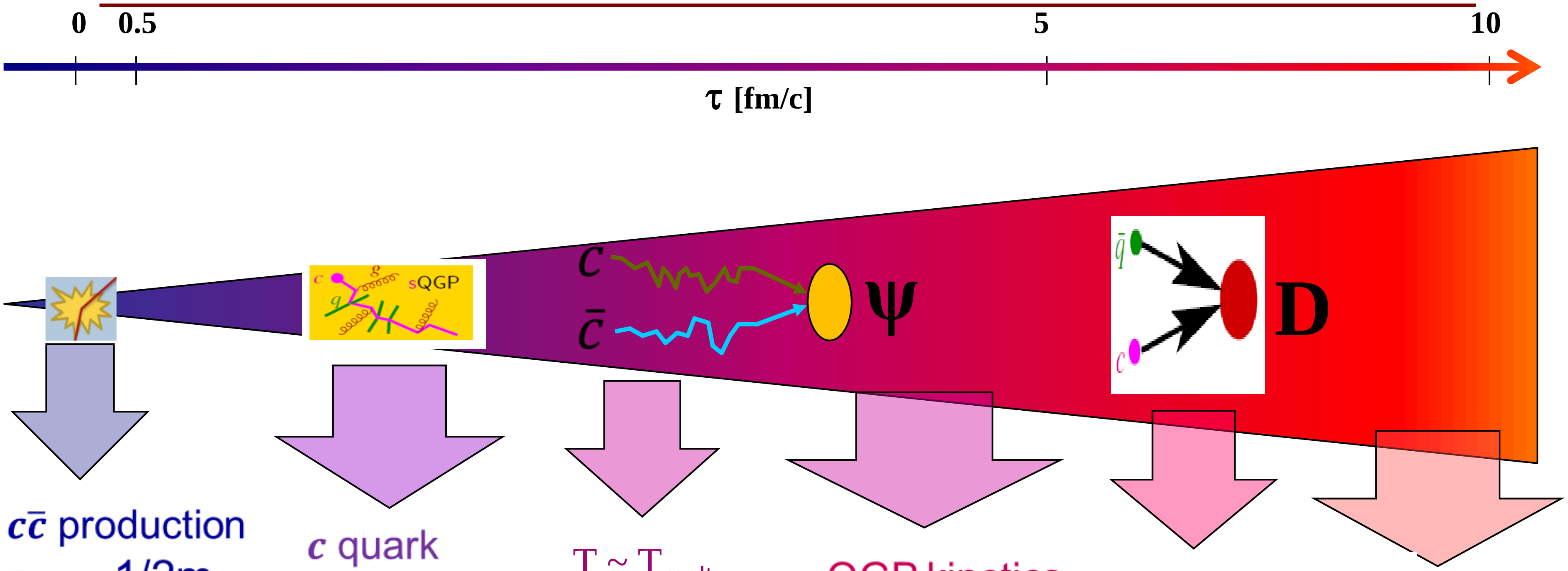
SDU Qingdao lectures

Lectures on Heavy-Flavor Probes of Quark-Gluon Plasma

Min He

Nanjing Univ. of Sci. & Tech., Nanjing, China

Heavy flavor transport as probes of QGP



$c\bar{c}$ production

τ_{hard}~1/2m$_c$, buildup of ψ wave packet

τ_ψ~1 fm/c

c quark Brownian diffusion in QGP

τ_c^{eq}~5 fm/c

T ~ T_{melt} onset of quarkonium regeneration

QGP kinetics

$c+\bar{c}\leftrightarrow\psi$

$\tau_\psi^{eq}\sim 1/\Gamma_\psi$

T ~ T_c hadronization:

c+q → D

c+q+q → Λ_c

hadronic diffusion

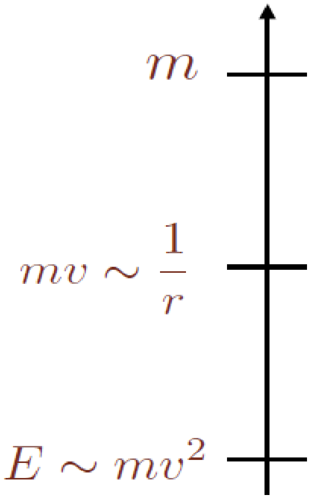
Lecture III

heavy quarkonium production in AA collisions

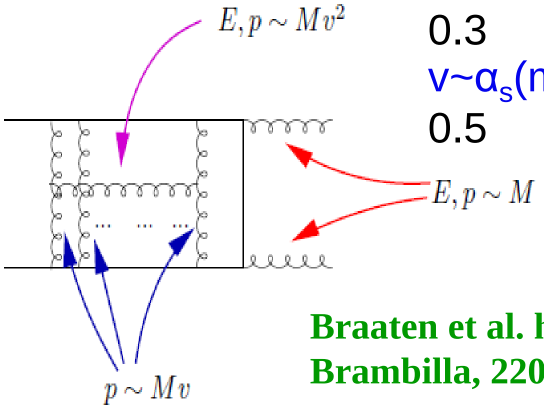
- vacuum properties: spectroscopy, static energy & pNRQCD
- HQ potential vs free energy at finite T
- Equilibrium properties: potential models & T-matrix, reaction rates
- Phenomenology: semi-classical modelling of transport
- Phenomenology: Open quantum system approach

Heavy quarkonium: multi-scale system

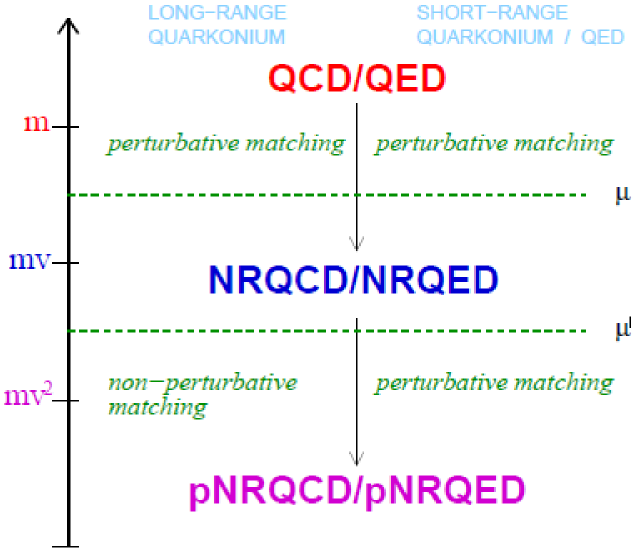
Quarkonium non relativistic scales in QCD



	charm	bottom
production (perturbative)	1.5 GeV	4.7 GeV
binding/relative mom.	0.9 GeV	1.5 GeV
level-spacing/E_B	0.5 GeV	0.5 GeV
v^2	0.3	0.1
$v \sim \alpha_s(mv)$	0.5	0.3



Braaten et al. hep-ph/9602374
Brambilla, 2204.11295



- NRQCD factorization: short-distance part producing ccbar in singlet/octet with small relative p; long-distance part: pointlike ccbar to bind & form chi_c

$$d\sigma(\chi_{cJ}+X) = d\widehat{\sigma}(c\bar{c}(\underline{1},{}^3P_J)+X)\,|R'_{\chi_c}(0)|^2 + (2J+1)\,d\widehat{\sigma}(c\bar{c}(\underline{8},{}^3S_1)+X)\,\langle\mathcal{O}_8^{\chi_c}\rangle$$

- pNRQCD: further integrating out the scale mv~1/r d.o.f. = QQbar singlet & octet

$$\mathcal{L} = \int d^3r\, \mathrm{Tr}\left\{ \mathrm{S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s + \dots\right)\mathrm{S} + \mathrm{O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o + \dots\right)\mathrm{O}\right\}$$
$$\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{H.c.}\right\} + \frac{1}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{c.c.}\right\} + \cdots$$

Non-relativistic potential model

- use phenomenological static potentials, e.g., Cornell potential

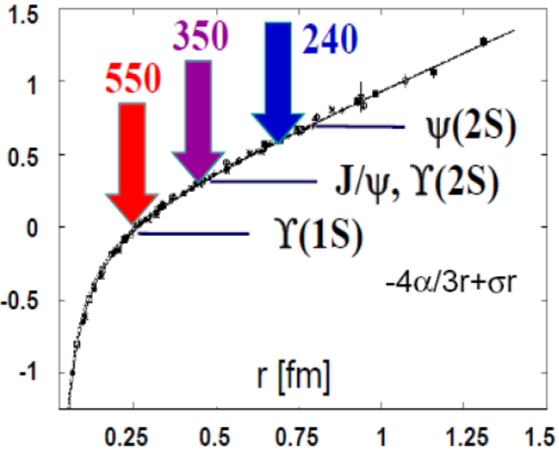
$$V(r) = \sigma r - \frac{\alpha}{r}$$

- long-range scale: confining (non-perturbative QCD), string tension, $\sigma \simeq 0.2\ \mathrm{GeV}^2$
- short-range scale: Coulomb-like (pQCD), $\alpha \simeq \pi/12$
- heavy-quarkonium states from non-relativistic Schrödinger equation

$$\left[2m_Q - \frac{1}{m_Q}\Delta + V(r)\right]\Phi_i(\vec{r}) = M_i\phi_i(r)$$

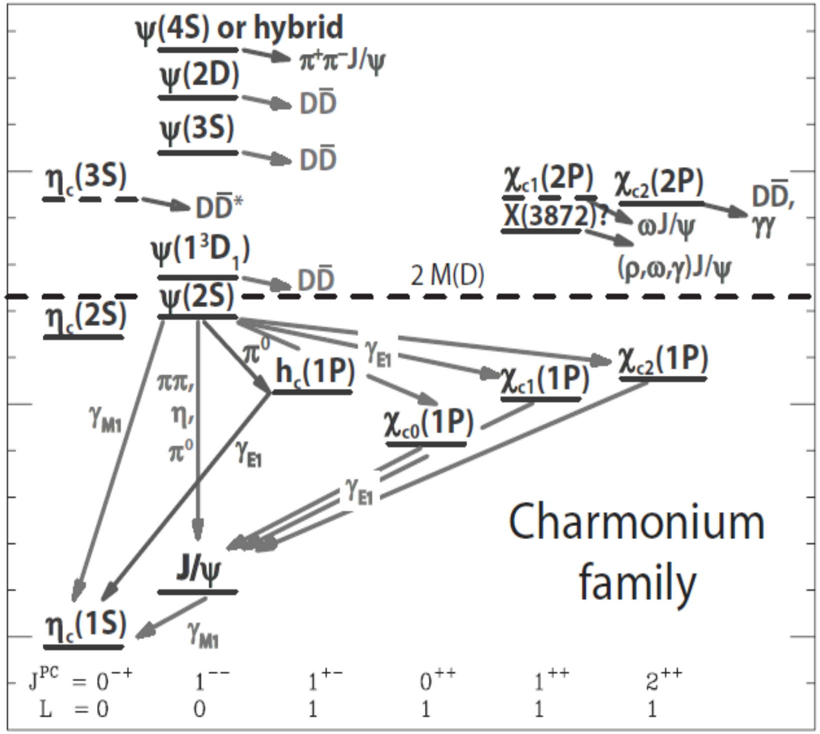
- fit to spin-averaged heavy-quarkonium spectra $\Rightarrow m_c = 1.25\ \mathrm{GeV},\ m_b = 4.65\ \mathrm{GeV},\ \sqrt{\sigma} = 0.445\ \mathrm{GeV},\ \alpha = \pi/12$
- from wave function $\langle r_i^2 \rangle = \langle \Phi_i\, |\mathbf{r}|\, \Phi_i \rangle$

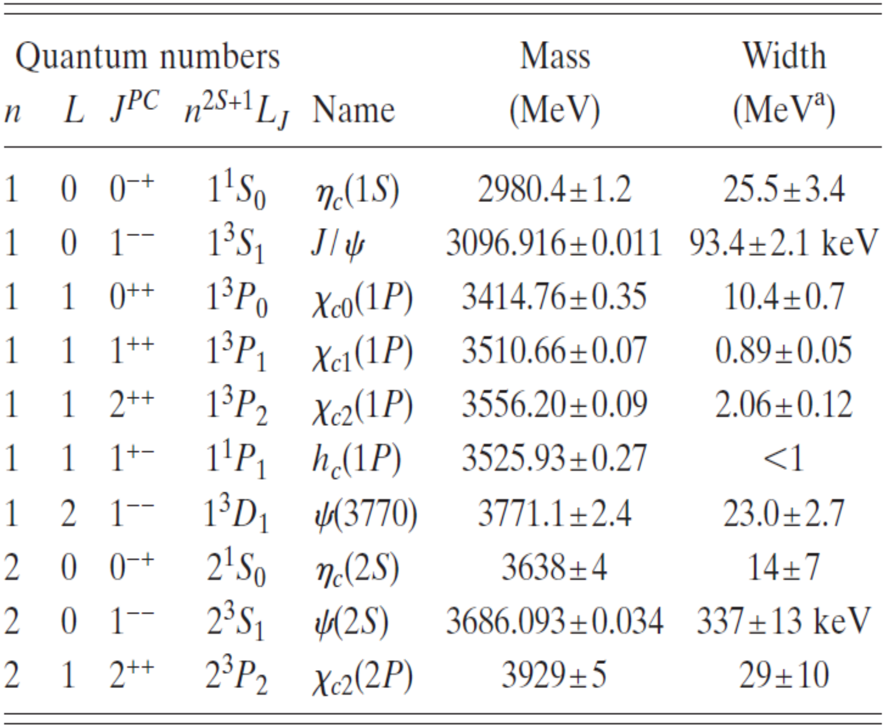
state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
ΔE [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
ΔM [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
r_0 [fm]	0.50	0.72	0.90	0.28	0.44	0.56	0.68	0.78



- Only Υ(1S) is color-Coulomb
- J/ψ & Υ(2S) are bound by confining force

Charmonium spectroscopy





Quantum numbers					Mass	Width
n	L	J^{PC}	$n^{2S+1}L_J$	Name	(MeV)	(MeV^a)
1	0	0^{-+}	1^1S_0	$\eta_c(1S)$	2980.4±1.2	25.5±3.4
1	0	1^{--}	1^3S_1	J/ψ	3096.916±0.011	93.4±2.1 keV
1	1	0^{++}	1^3P_0	$\chi_{c0}(1P)$	3414.76±0.35	10.4±0.7
1	1	1^{++}	1^3P_1	$\chi_{c1}(1P)$	3510.66±0.07	0.89±0.05
1	1	2^{++}	1^3P_2	$\chi_{c2}(1P)$	3556.20±0.09	2.06±0.12
1	1	1^{+-}	1^1P_1	$h_c(1P)$	3525.93±0.27	<1
1	2	1^{--}	1^3D_1	$\psi(3770)$	3771.1±2.4	23.0±2.7
2	0	0^{-+}	2^1S_0	$\eta_c(2S)$	3638±4	14±7
2	0	1^{--}	2^3S_1	$\psi(2S)$	3686.093±0.034	337±13 keV
2	1	2^{++}	2^3P_2	$\chi_{c2}(2P)$	3929±5	29±10

Eichten et al., RMP80,1161(2008)

- Schrodinger equation with spin-dependent potential: $V(r)=V_{Cornell}(r) + V_{SD}(r)$
  ➔ fine and hyperfine splitting

$$V_{SS}(r) = \frac{1}{3m_Q m_{\bar{Q}}}\nabla^2 V_V(r) = \frac{16\pi\alpha_s}{9m_Q m_{\bar{Q}}}\delta^3(\mathbf{r})$$

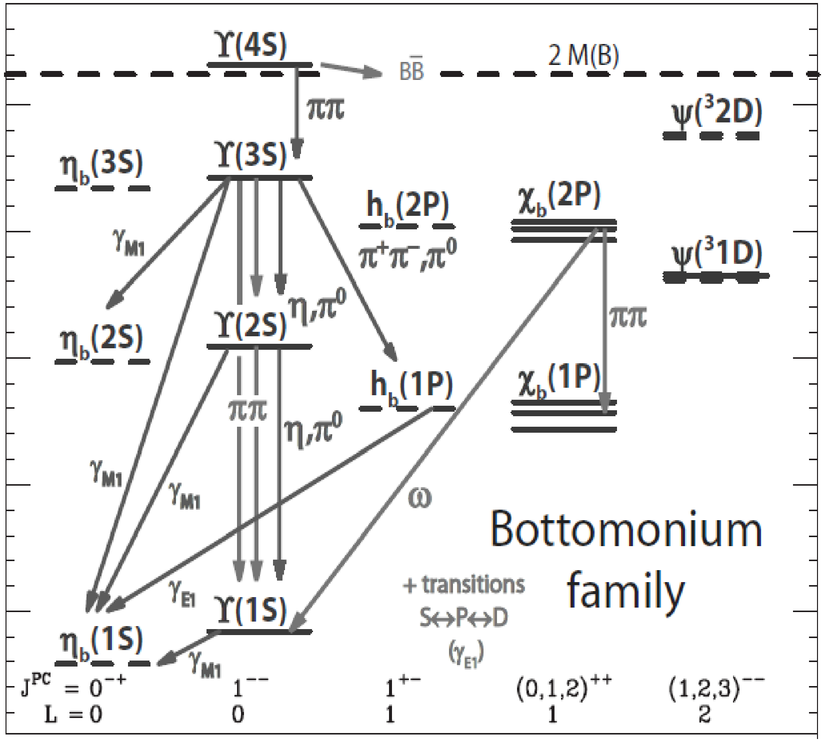
$$V_{LS}(r) = \frac{1}{2m_Q m_{\bar{Q}} r}\left(3\frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr}\right)$$

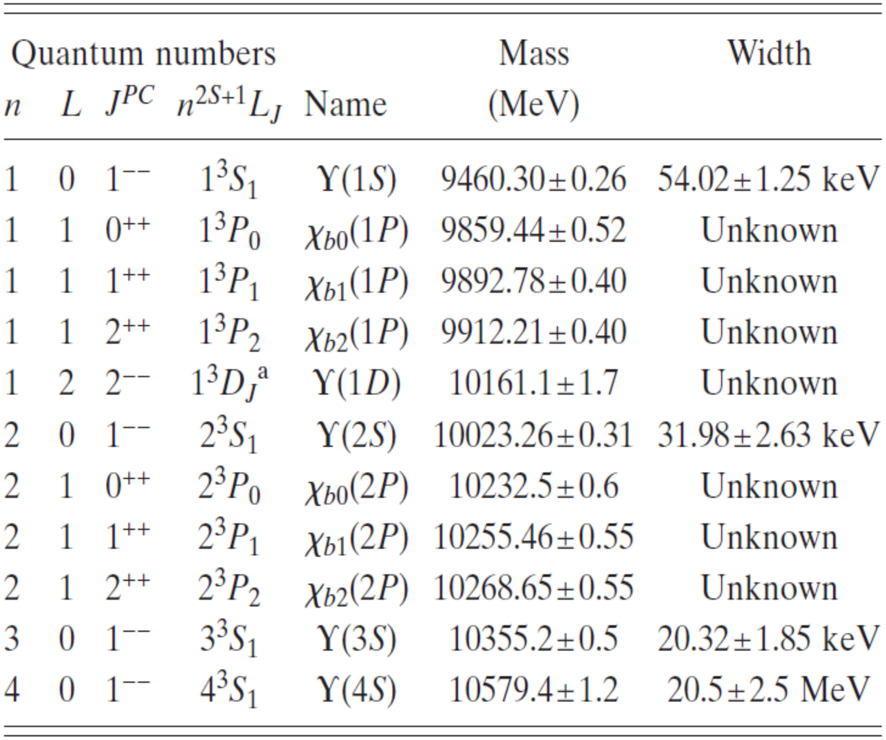
$$V_T(r) = \frac{1}{6m_Q m_{\bar{Q}}}\left(3\frac{dV_V^2(r)}{dr^2} - \frac{1}{r}\frac{dV_V(r)}{dr}\right)$$

$$V_{SD}(r) = V_{SS}(r)\left[S(S+1) - \frac{3}{2}\right] + V_{LS}(r)(\mathbf{L}\cdot\mathbf{S})$$
$$+V_T(r)\left[S(S+1) - 3(S\cdot\hat{r})(S\cdot\hat{r})\right]$$

Eichten & Quigg, PRD49,5845(1995); Soni et al., Eur.Phys. J. C(2018)78:592

Bottomonium spectroscopy





Quantum numbers					Mass	Width
n	L	J^{PC}	$n^{2S+1}L_J$	Name	(MeV)	
1	0	1^{--}	1^3S_1	$\Upsilon(1S)$	9460.30 ± 0.26	54.02 ± 1.25 keV
1	1	0^{++}	1^3P_0	$\chi_{b0}(1P)$	9859.44 ± 0.52	Unknown
1	1	1^{++}	1^3P_1	$\chi_{b1}(1P)$	9892.78 ± 0.40	Unknown
1	1	2^{++}	1^3P_2	$\chi_{b2}(1P)$	9912.21 ± 0.40	Unknown
1	2	2^{--}	$1^3D_J{}^a$	$\Upsilon(1D)$	10161.1 ± 1.7	Unknown
2	0	1^{--}	2^3S_1	$\Upsilon(2S)$	10023.26 ± 0.31	31.98 ± 2.63 keV
2	1	0^{++}	2^3P_0	$\chi_{b0}(2P)$	10232.5 ± 0.6	Unknown
2	1	1^{++}	2^3P_1	$\chi_{b1}(2P)$	10255.46 ± 0.55	Unknown
2	1	2^{++}	2^3P_2	$\chi_{b2}(2P)$	10268.65 ± 0.55	Unknown
3	0	1^{--}	3^3S_1	$\Upsilon(3S)$	10355.2 ± 0.5	20.32 ± 1.85 keV
4	0	1^{--}	4^3S_1	$\Upsilon(4S)$	10579.4 ± 1.2	20.5 ± 2.5 MeV

Eichten et al., RMP80,1161(2008)

- $n=n_r + 1$, n_r = number of nodes of radial wave functions
- $P=(-1)^{L+1}$, $C=(-1)^{L+S}$

Vacuum static Q-Qbar potential

- A quark-antiquark color singlet state with gauge link/Wilson line

$$|\phi^{lj}_{\alpha\beta}\rangle \equiv \frac{\delta_{lj}}{\sqrt{3}}\bar{\psi}^i_\alpha(x)U^{ik}(x,y,C)\psi^k_\beta(y)|0\rangle$$

$$U(x,y;C) = P\exp\left\{ig\int_y^x A_\mu(z)\,dz^\mu\right\}$$

$$U'(x,y;C) = \exp\{i\theta(x)\}U(x,y;C)\exp\{-i\theta(y)\}$$

- Q-Qbar meson correlator

$$G(T) = \langle\phi(\mathbf{x},0)|\phi(\mathbf{y},T)\rangle = \langle\phi(\mathbf{x},0)|\exp(-iHT)|\phi(\mathbf{y},0)\rangle$$

➔ in large Euclidean time limit only ground state survives

$$G(-iT) = \sum_n \langle\phi(\mathbf{x},0)|\psi_n\rangle\langle\psi_n|\phi(\mathbf{y},0)\rangle\exp(-E_nT)$$

$$\to \langle\phi(\mathbf{x},0)|\psi_0\rangle\langle\psi_0|\phi(\mathbf{y},0)\rangle\exp(-E_0T) \quad \text{for} \quad T\to\infty$$

- ground state energy:

$$E_0 = -\lim_{T\to\infty}\frac{\log G(-iT)}{T}$$

- consider statice Q-Qbar pair (separation r) created at t=0, and annihilated at t=T

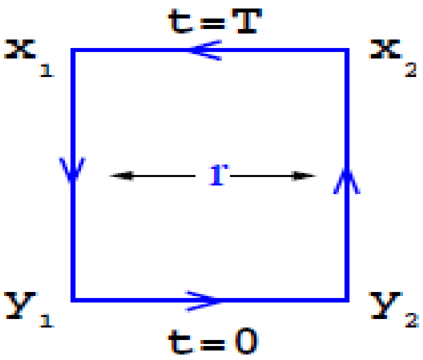
$$G_{\beta_1\beta_2\alpha_1\alpha_2}(T) \xrightarrow{m\to\infty} \delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)(P_+)_{\beta_1\alpha_1}(P_-)_{\alpha_2\beta_2} \times e^{-2mT}\langle \mathrm{Tr\,P}e^{ig\oint_{\Gamma_0}dz_\mu A_\mu(z)}\rangle$$

Wilson loop along a rectangular loop Γ_0

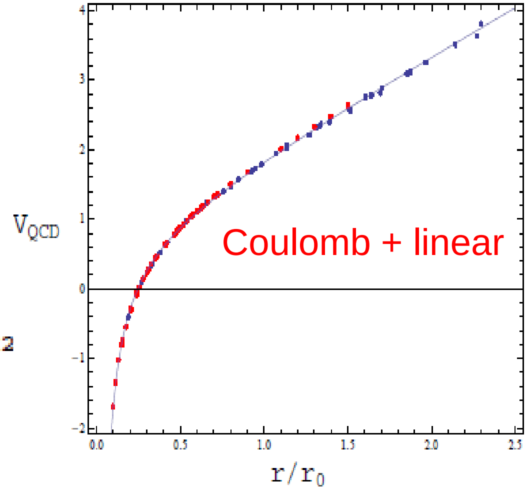
$$W(\Gamma_0) = \mathrm{Tr\,P}e^{ig\oint_{\Gamma_0}dz_\mu A_\mu(z)}.$$

➔ Static Q-Qbar energy/potential:

$$V_0(r) \equiv E_0(r) = -\lim_{T\to\infty}\frac{1}{T}\log\langle W(\Gamma_0)\rangle$$



lattice Q-Qbar potential



Sumino, 1411.7853

Brambilla, 2204.11295

Finite-T Q-Qbar potential (I)

- The Wilson loop obeys the Schrodinger equation

$$i\partial_t W_\square(t,r) = \Phi(t,r) W_\square(t,r).$$

Q-Qbar potential defined at late times $V(r) = \lim_{t \to \infty} \Phi(r,t).$

- Spectral function of real-time vs imaginary time Wilson loop

$$W_\square(r,t) = \int d\omega e^{-i\omega t} \rho_\square(r,\omega), \qquad W_\square(r,\tau) = \int d\omega e^{-\omega\tau} \rho_\square(r,\omega).$$

➔ Q-Qbar potential

$$V(r) = \lim_{t \to \infty} \frac{\int d\omega\, \omega e^{-i\omega t} \rho_\square(r,\omega)}{\int d\omega e^{-i\omega t} \rho_\square(r,\omega)} = \lim_{t \to \infty} i \frac{\partial \log W(r, \tau = it, T)}{\partial t}$$

- It is challenging to reconstruct the spectral function from lattice data of Euclidean Wilson loop

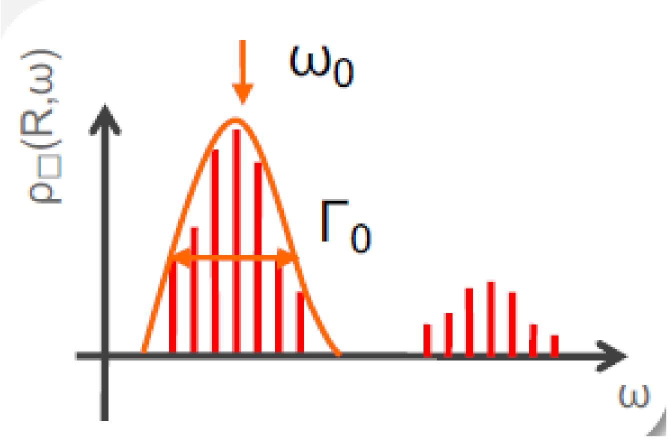
If time-independent potential description holds for all times: $\Phi(t,r) = V(r)$ ➔ W(t,r)=exp(-iV(r,T)t)

➔ Breit-Wigner peak position & width associated with ReV & ImV

$$\rho_r(\omega, T) = \int dt W_\square(r,t,T) e^{-i\omega t}.$$

$$\rightarrow \rho(\omega, r) = \frac{\mathrm{Im}[V](r)}{\mathrm{Im}[V](r)^2 + (\mathrm{Re}[V](r) - \omega)^2}$$

Burnier & Rothkopf, PRD87,114019(2013)



Finite-T Q-Qbar potential (II)

- Perturbative evaluation from HTL (static) gluon propagator **Laine et al.,JHEP03(2007)054**

$$D_{00}(\omega=0,\boldsymbol{q}) = \frac{-1}{\boldsymbol{q}^2+m_D^2} + i\,\frac{\pi m_D^2 T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}$$

one-loop self-energy of the space-like gluon exchanged between Q-Qbar
has an imaginary part ➔ cutting it corresponds to Q scattering off medium q/g: Landau damping

- Fourier transform ➔ ReV + ImV

$$V_\infty(\boldsymbol{r}_1-\boldsymbol{r}_2) \equiv g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\left[\frac{1}{\boldsymbol{q}^2+m_D^2} - i\,\frac{\pi m_D^2 T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}\right]$$

$$= -\frac{g^2}{4\pi}\left[m_D + \frac{e^{-m_D r}}{r}\right] - i\,\frac{g^2 T}{4\pi}\phi(m_D r)$$

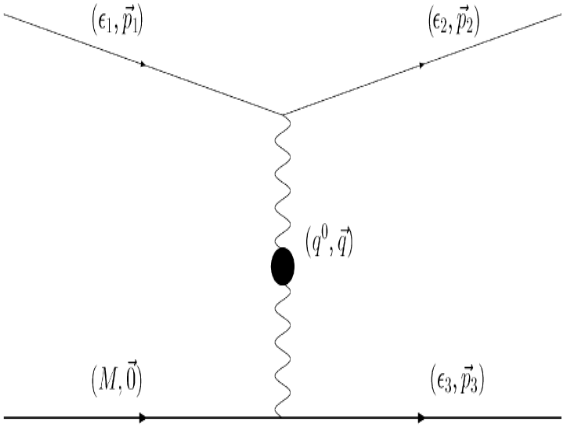
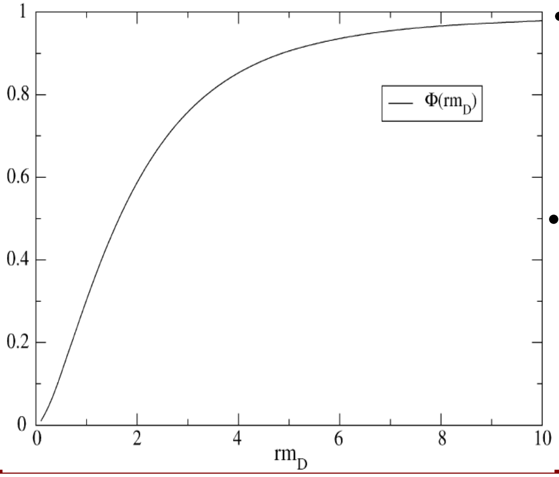


Fig. C.1. The collision process responsible for the imaginary part of the $Q\bar{Q}$ correlator.

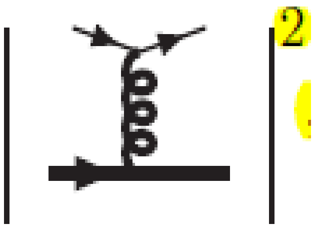


- HTL ReV identical to free energy (screened Coulomb only) at the same order

- ImV →2*Γ_Q when r→∞

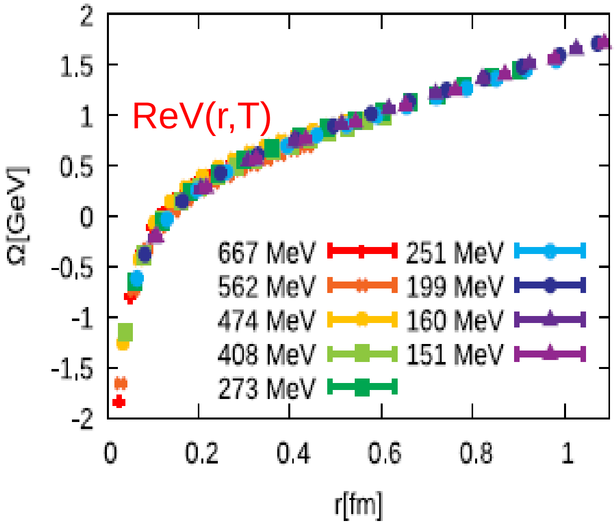
Blazoit et al., NPA806(2008)312-338

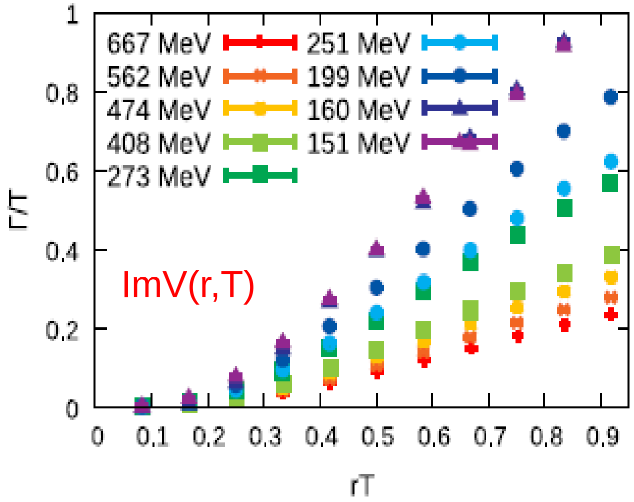
$$\phi(x) \equiv 2\int_0^\infty dz\,\frac{z}{(z^2+1)^2}\left[1-\frac{\sin(zx)}{zx}\right]$$



Finite-T Q-Qbar potential (III)

- ReV & ImV extracted from lattice data of Euclieean Wilson loop **Petreczky et al., PRD105, 054513 (2022)**





- ReV little screening up to 3-4*Tc; ImV large & increases toward high T

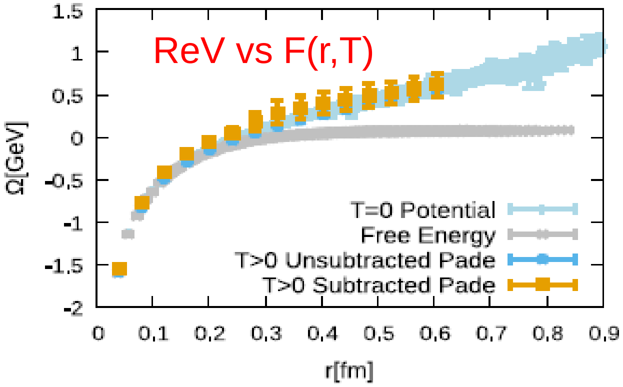


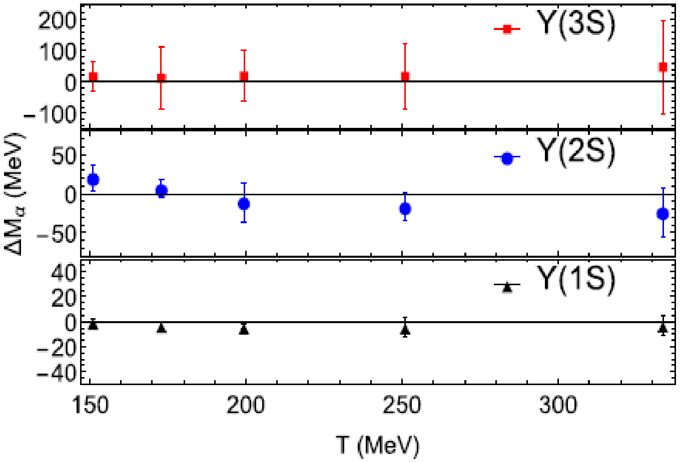
FIG. 21. Comparison of extracted Ω using subtracted and unsubtracted correlators using Padé pole analysis with the $T = 0$ effective mass and color singlet free energy at $T = 408$ MeV

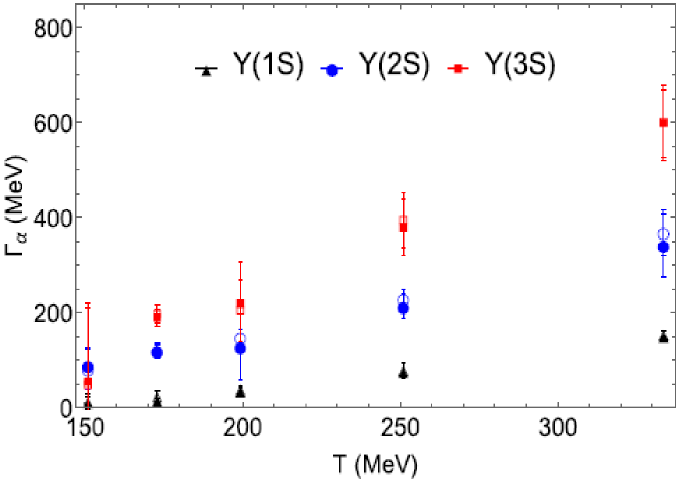
- Q-Qbar potential represent interactions on time-scale $1/E_B$ or $1/\Gamma_{diss}$,shorter than 1/T➔ insignificant screening

- Q-Qbar free energy F on long time-scale >>1/T ➔ significant screening

Finite-T Q-Qbar potential (IV)

- lattice data of bottomonia:
  little mass shift & large width





Petreczky et al., PLB800(2020)135119

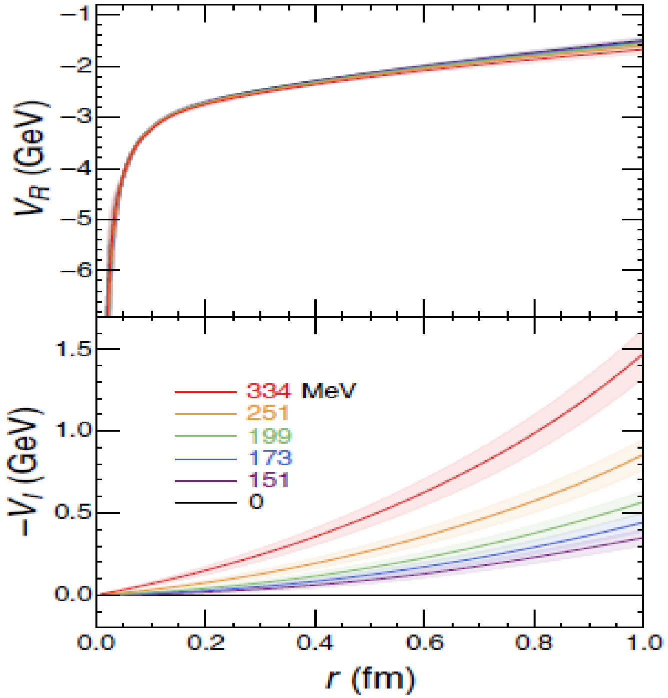
- DNN extraction of Q-Qbar potential by fitting to lattice data within Schrodinger eq.

Shi et al., PRD105, 014017 (2022)

$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + iV_I(T,r)]\psi_n = E_n\psi_n$$

$$\mathrm{Re}[E_n] = m_n - 2m_b \qquad \mathrm{Im}[E_n] = -\Gamma_n$$

ReE ~ ReV, ImE~ Γ ~ ImV: little mass shift ➔ ReV little screening; large width ➔ large ImV

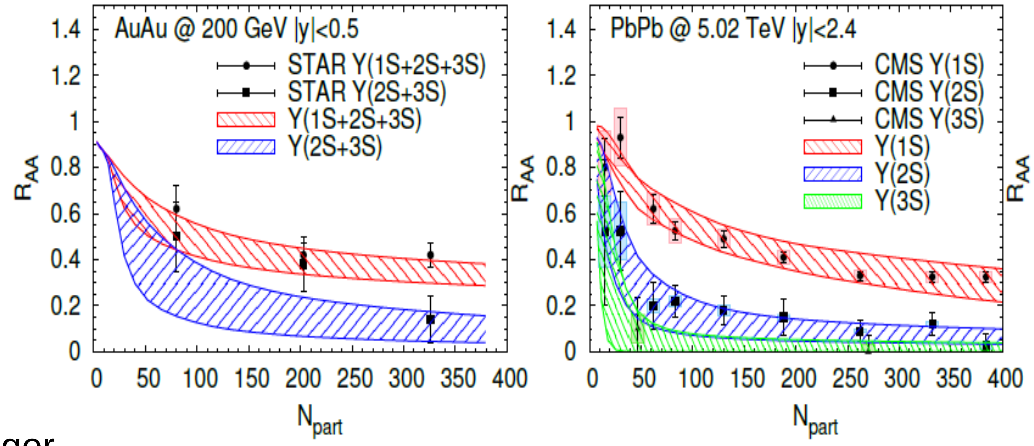


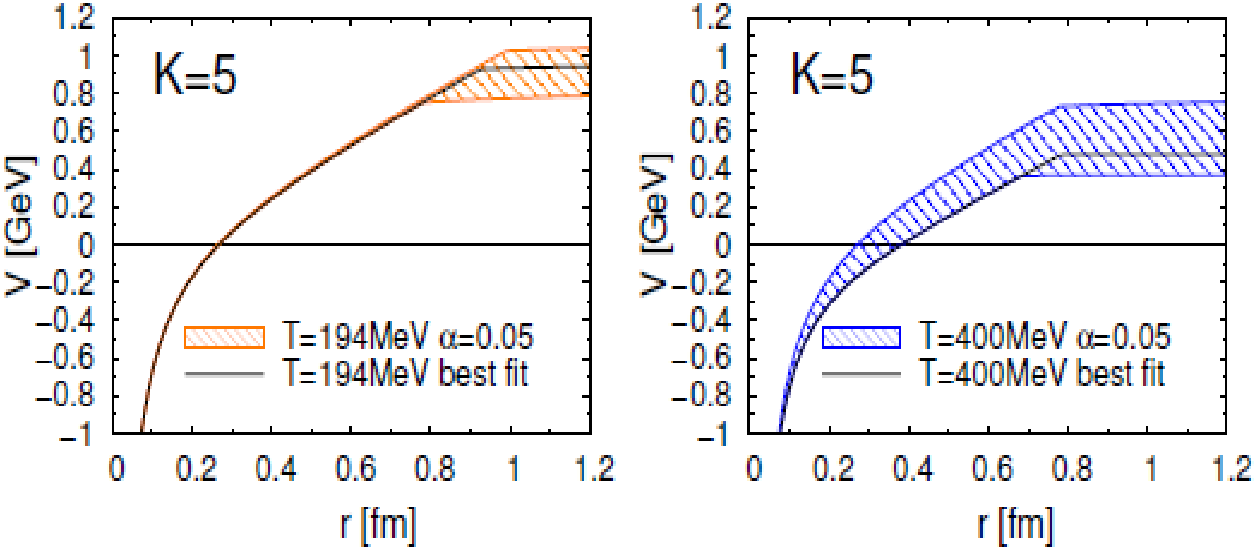
Extracting HQ potential from bottomoium R_{AA}

- statistical transport analysis of Υ data with trial input potential **Du et al. ’19**

- trial screened Cornell potential with screening parameters → T-matrix to get Υ(nS) binding energies → quasifree reaction rates (E_B larger →interference stronger) → rate equation → R_{AA}

- K on single-Q collisional rate Γ_Q larger → stronger potential → larger E_B → stronger interference to render Γ_{diss} amenable to R_{AA}





- **extracted potential little screening at T<250 MeV**

- **even at T~ 400 MeV, still significant residual confining force**

- **consistent with T-matrix solution of potential in the strongly coupled scenario, & qualitatively consistent with latest lattice results**

Free energy of static Q-Qbar

McLerran, Svetitsky, PRD 24 (81) 450

$\psi_a^\dagger(\tau, x)$, $\psi_a(\tau, x)$-creation annihilation operators for static quarks at time τ and position x

$\psi_a^{\dagger c}(\tau, x)$, $\psi_a^c(\tau, x)$-creation annihilation operators for static anti-quarks at time τ and position x

$$[\psi_a(\tau, x), \psi_b^\dagger(\tau, y)]_+ = \delta(x-y)\delta_{ab}$$

$$(-i\partial_\tau - gA_0(\tau, x))\,\psi(\tau, x) = 0$$

formal solution $\psi(\tau, x) = \mathcal{P}\exp\left(ig\int_0^\tau d\tau' A_0(\tau', x)\right)\psi(0, x) = W(x)\psi(0, x)$

lattice : $W(x) = \prod_{x_0=0}^{N_\tau - 1} U_0(x, \tau)$

Free energy of static quark anti-quark pair

$$Z(\beta)e^{-\beta F(x,y)} = \sum_s < s|e^{-\beta H}|s >$$

$|s>$ denotes any state with a static quark at position x and static anti-quark at position y;

Free energy of static Q-Qbar

Let us denote by $|s' >$ states with no static quarks

$$e^{-\beta F(x,y)} = \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s'|\psi_a(0,x)\psi_b^c(0,y)e^{-\beta H}\psi_{a'}^\dagger(0,x)\psi_{b'}^{\dagger c}(0,y)|s'>(1)$$

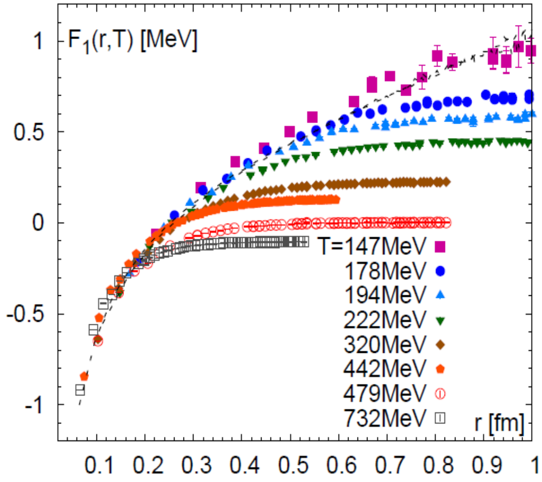
$$\boxed{e^{-\beta H}e^{\beta H}}$$

$$e^{-\beta H}O(\tau)e^{\beta H} = O(\tau+\beta)$$

$$= \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s'|e^{-\beta H}\psi_a(\beta,x)\psi_b^c(\beta,y)\psi_{a'}^\dagger(0,x)\psi_{b'}^{\dagger c}(0,y)|s'$$

$$= Z(\beta)\frac{1}{N_c^2} < \mathrm{Tr}W(x)\mathrm{Tr}W^\dagger(y) >= Z(\beta)G(r,T),\ r = |x-y|$$

$L(x) = \mathrm{Tr}W(x)$- Polyakov loop



➔ Singlet Q-Qbar free energy ~ correlator of two Polyakov loops:

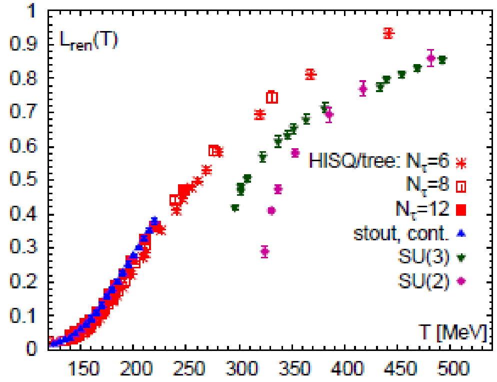
$$\exp(-F_1(r,T)/T) = \frac{1}{N}\langle \mathrm{Tr}[L^\dagger(x)L(y)]\rangle,$$

- LO HTL: screened Coulomb only F<0

$$F_1(r,T) = -\frac{N^2-1}{2N}\frac{\alpha_s}{r}\exp(-m_D r) - \frac{(N^2-1)\alpha_s m_D}{2N}$$

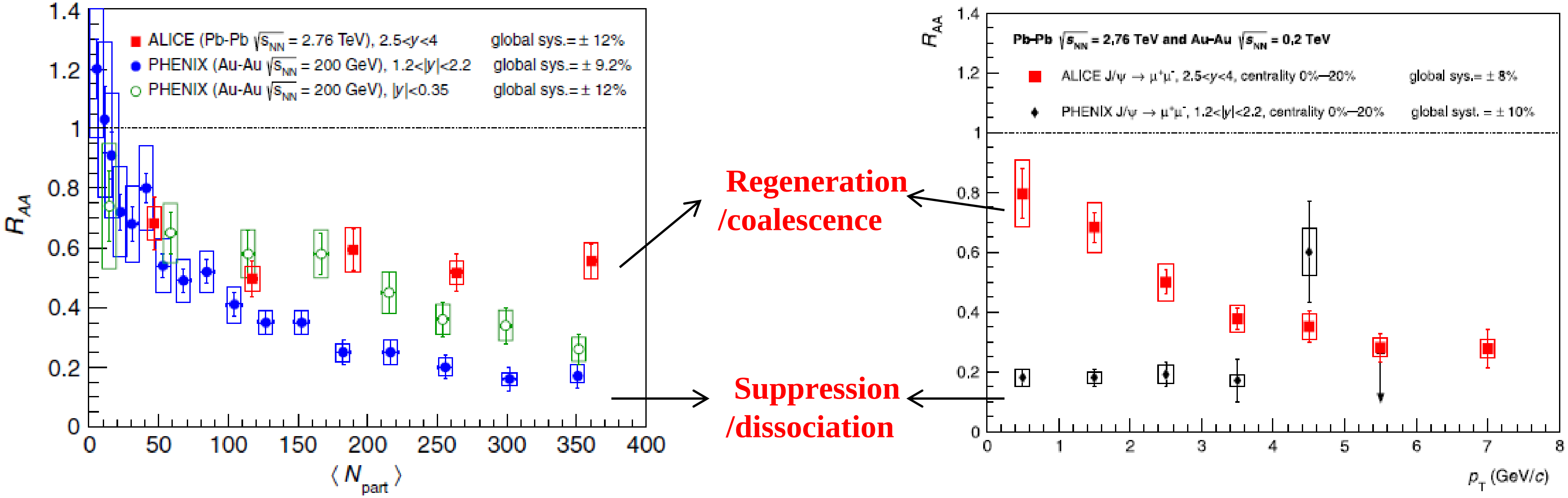
- Lattice: F>0 for T<2*T$_c$, remnant of linear confining term
- Singe-Q F=deconfinement order parameter for pure YM without dynamical light quarks

$$P = \frac{1}{N}\mathrm{tr}L,\ \langle P\rangle = \exp(-F_Q(T)/T)$$



- At low T, P➔0➔infinite energy for a free Q ➔ confinement
- At high T, P increases ➔ finite F_Q ➔deconfinement

Heavy quarkonium in QGP



Regeneration /coalescence

Suppression /dissociation

How does the suppression of heavy quarkonium in QGP come about?

- static color screening? Classic paradigm, but probably insignificant from state-of-the-art lattice studies
- dynamical inelastic collisional dissociation: inelastic reaction rate
- regeneration of charmonia from abundant near-thermalized charm & anticharm quarks, in particular at the LHC energies

Melting by static screening

- Karsch-Satz seminar work **Karsch-Satz, Z.Phys.C37,617-622(1988)**
  vacuum ornell potential → screening of both Coulomb and confining part

$$V(r,0) = \sigma r - \frac{\alpha}{r}, \qquad V(r,T) = (\sigma/\mu(T))(1-e^{-\mu(T)r}) - (\alpha/r)\,e^{-\mu(T)r}$$

➔ $V(r\rightarrow\infty,T) = \sigma/\mu(T) = 2*\triangle m_Q(T)$ & $m_Q^{eff}(T)=m_Q^0 + \triangle m_Q(T)$

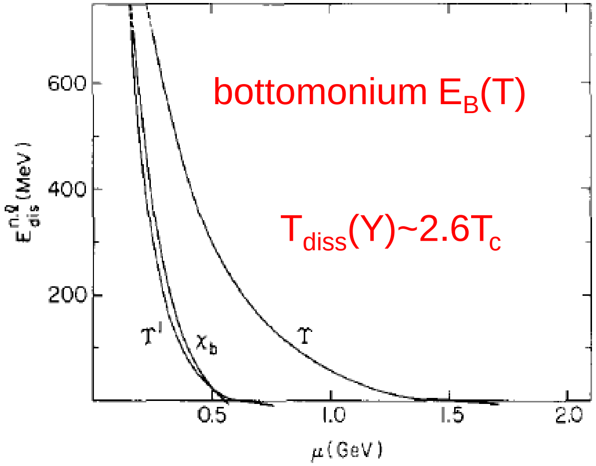
- Schrodinger equation for Q-Qbar with T-dependent potential

$$H(r,T) = 2m - \frac{1}{m}\nabla^2 + V(r,T) \qquad [H(r,\mu) - E_{n,l}(\mu)]\,\Phi_{n,l}(r,\mu) = 0,$$

- Def: binding energy at screening mass $\mu(T) \propto gT$

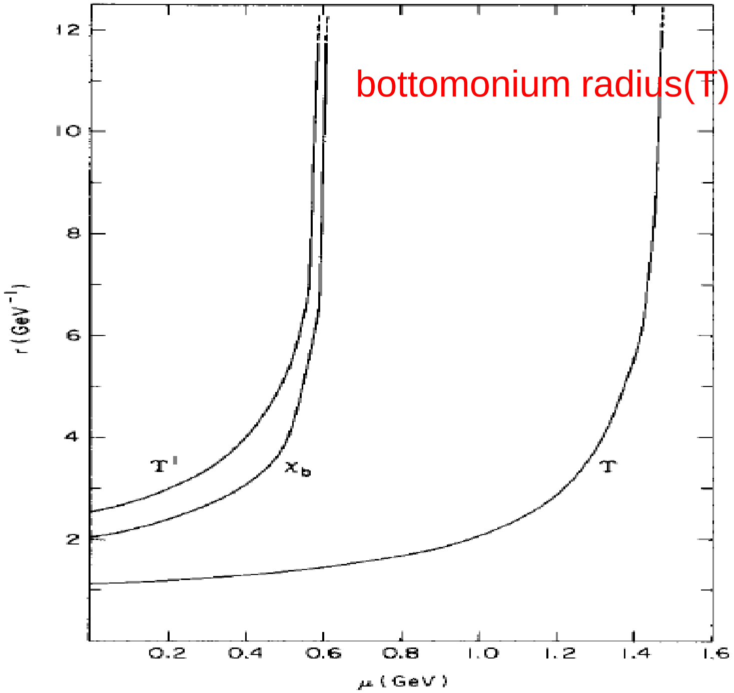
$$E_{\rm dis}^{n,l}(\mu) \equiv 2m + \sigma/\mu - E_{n,l}(\mu)$$

➔ dissociation point: $E_{\rm dis}^{n,l}(\mu_c) = 0$



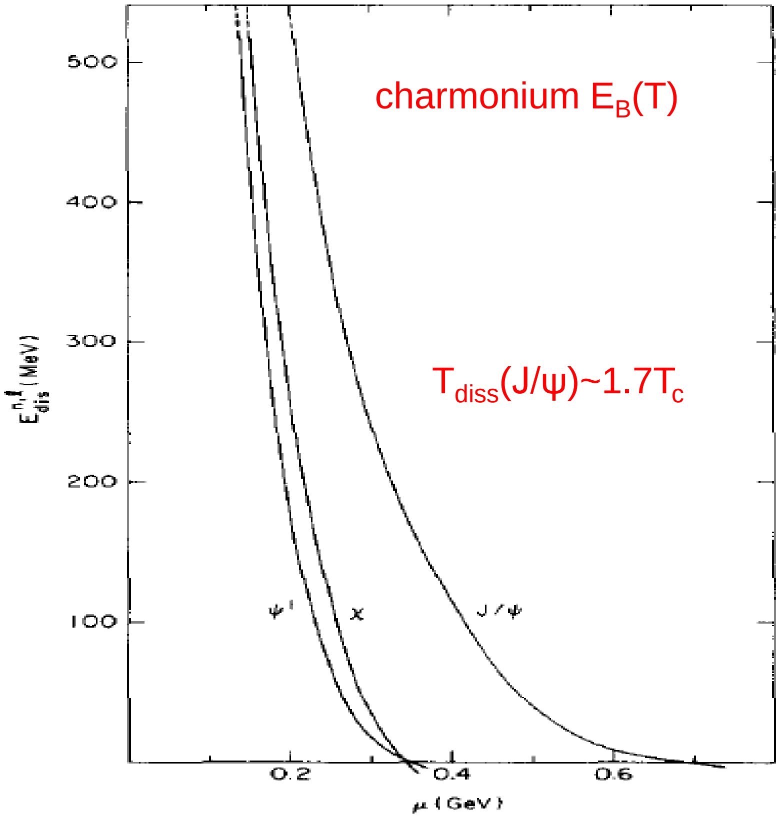
bottomonium $E_B(T)$

$T_{diss}(Y)$~$2.6T_c$



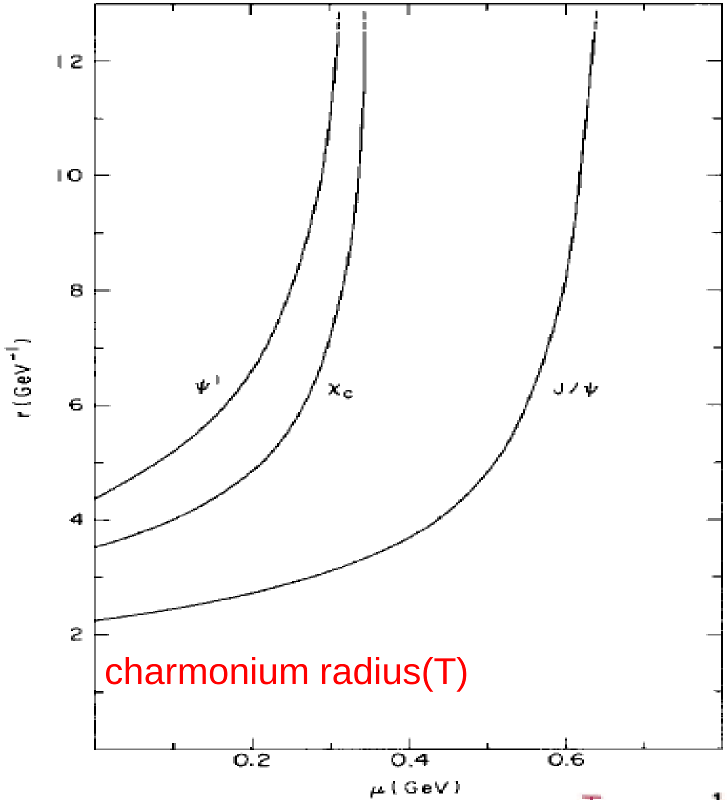
bottomonium radius(T)

Melting by static screening



charmonium $E_B(T)$

$T_{diss}(J/\psi)\sim1.7T_c$

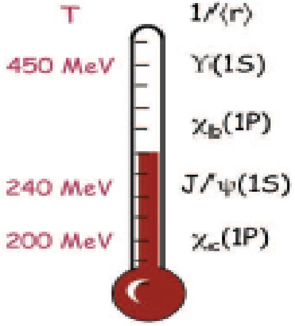


charmonium radius(T)

Karsch-Satz, Z.Phys.C37,617-622(1988)

- sequential melting of heavy quarkonia in the order of vacuum binding energies
- a thermometer to measure QGP temperatures

➔ But need to go beyond this too simple, purely static, old scenario

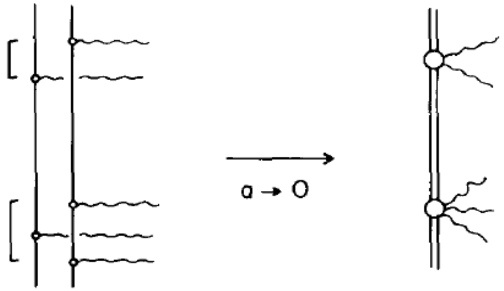


Dynamical dissociation: LO

- Peskin's OPE analysis in vacuum: Coupling of heavy quarkonium to external gluons is a short-distance process **Peskin, Nucl. Phys. B 156, 365 (1979)**

color-octet QQbar can only persist over short space-time range $\Delta t \sim \frac{1}{V_8 - V_1} \sim \frac{a}{g_s^2} \sim \frac{1}{\epsilon_B},$

➔gluon emissions assemble into small singlet clusters: OPE local operators



- This leads to gauge-invariant multipole expansion of the external soft gluons around the small-size Q-Qbar bound state ➔ NR Hamiltonian **Yan, PRD22,1652(1980)**

$$H_0 = \frac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \frac{\lambda^a}{2}\frac{\bar{\lambda}^a}{2} V_2(|\vec{r}|),$$

$$H_I = Q^a A_0^a(t,\vec{0}) - \vec{d}^a \cdot \vec{E}^a(t,\vec{0}) - \vec{m}^a \cdot \vec{B}^a(t,\vec{0}) + \ldots,$$

with

$$\vec{d}^a = \frac{1}{2} g_s \vec{r} \left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right),$$

$$\vec{m}^a = \frac{g_s}{2m_Q}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\left(\frac{\vec{\sigma}}{2} - \frac{\vec{\sigma}\,'}{2}\right),$$

color-electric dipole coupling (E1)

color-magnetic dipole coupling (M1)

Deriving the gluo-dissociation cross section: g + ψ → c + cbar

- **Color E1 transition**

--- **Weyl gauge**

$$H_{E1} = -\vec{d}^a \cdot \vec{E}^a(t,\vec{0}) = \frac{g_s}{2}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\vec{r} \cdot \frac{\partial \vec{A}^a}{\partial t}$$

with

$$\vec{A}^a(t,\vec{x}) = \sum_{\vec{k},\lambda} N_{\vec{k}}\vec{\epsilon}_{\vec{k}\lambda}[a^a_{\vec{k}\lambda}e^{i\vec{k}\cdot\vec{x}-i\omega_{\vec{k}}t} + h.c.]$$

Peskin'Coulomb reproduced

---- **Fermi's golden rule: 1st order perturbation**

$$\sigma^{g+J/\psi\to c+\bar{c}}_{E_1}(E_g) = \frac{g_s^2\pi}{2\cdot 9}E_g\frac{V}{(2\pi)^3}\int d^3\vec{p} \times |<(c\bar{c})_8,\vec{p}|\vec{r}|J/\psi>|^2\delta(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}),$$ **Coulomb appro.**

$$\sigma^{g+J/\psi\to c+\bar{c}}_{E_1,\mathrm{Coulomb}}(E_g) = \frac{2^7}{9}g_s^2\frac{\epsilon_B^{5/2}}{m_Q}\frac{(E_g-\epsilon_B)^{3/2}}{E_g^5}$$

- **Color M1 transition**

$$H_{M1} = -\vec{m}^a \cdot \vec{B}^a(t,\vec{0}) = -\frac{g_s}{2m_Q}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\left(\frac{\vec{\sigma}}{2} - \frac{\vec{\sigma}\,'}{2}\right)\cdot\nabla\times\vec{A}^a(t,\vec{0})$$

A novel contribution

--- **1st order perturbation**

$$\sigma^{g+J/\psi\to c+\bar{c}}_{M_1}(E_g) = \frac{g_s^2\pi}{2\cdot 3}\frac{E_g}{m_Q^2}\frac{V}{(2\pi)^3}\int d^3\vec{p} \times |<(c\bar{c})_8|J/\psi>|^2\delta(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}).$$ **Coulomb appro.**

$$\sigma^{g+J/\psi\to c+\bar{c}}_{M_1,\mathrm{Coulomb}}(E_g) = \frac{2^3}{3}g_s^2\frac{\epsilon_B^{5/2}}{m_Q^2}\frac{(E_g-\epsilon_B)^{1/2}}{E_g^3}$$

- **Selection rules**

--- **E1: ΔL = 1, ΔS = 0; M1: ΔL = 0, ΔS = 1 from singlet to octet transition**

--- **P-wave states χ_c & χ_b also derived**

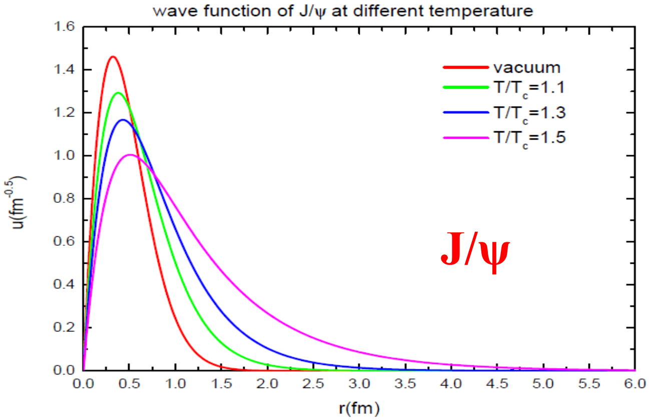
In-medium potential Schrodinger

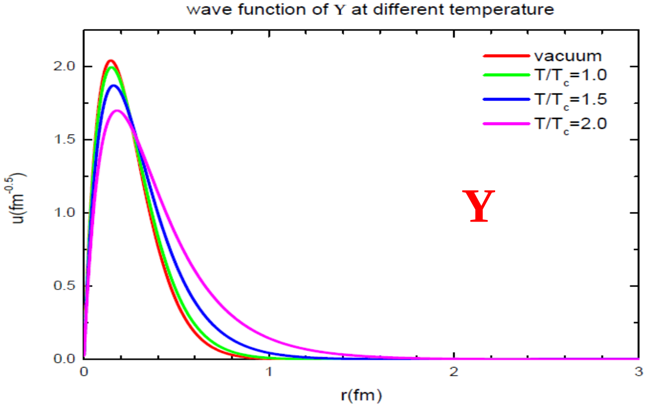
- **In-medium potential model** Satz 88

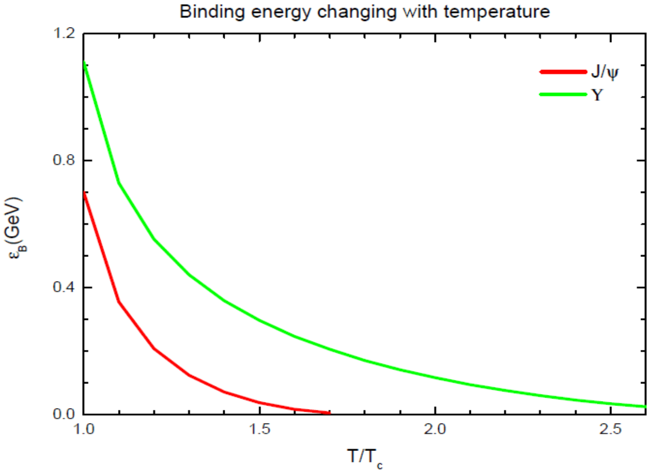
$$V(r,T) = -\frac{\alpha}{r}e^{-m_D(T)r} + \frac{\sigma}{m_D(T)}(1-e^{-m_D(T)r})$$

$$[H(r,T) - E_{n,l}(T)]\psi_{n,l}(r,\theta,\phi) = 0$$

- **Vacuum/in-medium quarkonium wave func., binding $\varepsilon_B(T)$**


wave function of J/ψ at different temperature


wave function of Υ at different temperature


Binding energy changing with temperature

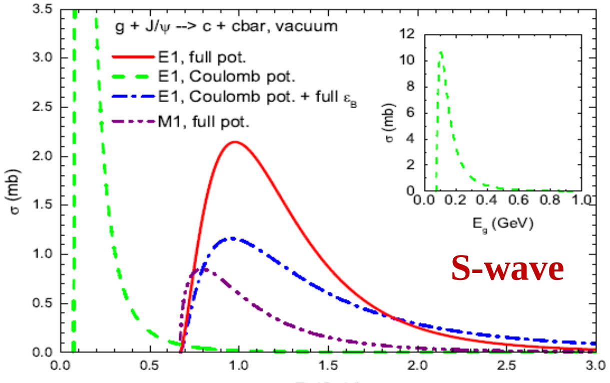
--- higher T, stronger screening

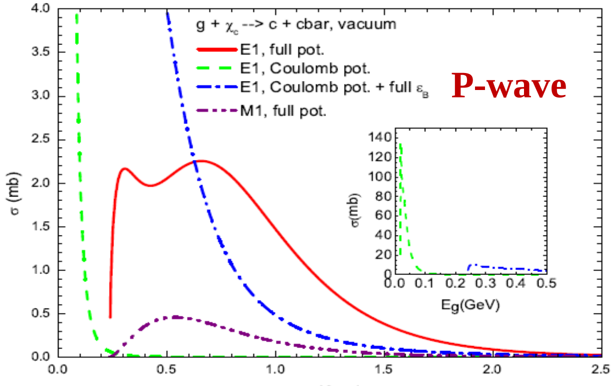
--- bound state size grows, binding energy decreases

--- $T_d(J/\psi)\sim 1.7T_c$, $T_d(\Upsilon)\sim 2.6T_c$

Gluo-diss. g + ψ → c + cbar cross section

- **Vacuum cross sections**

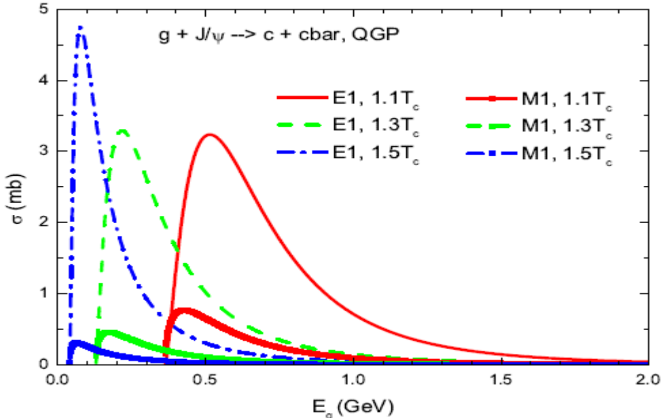


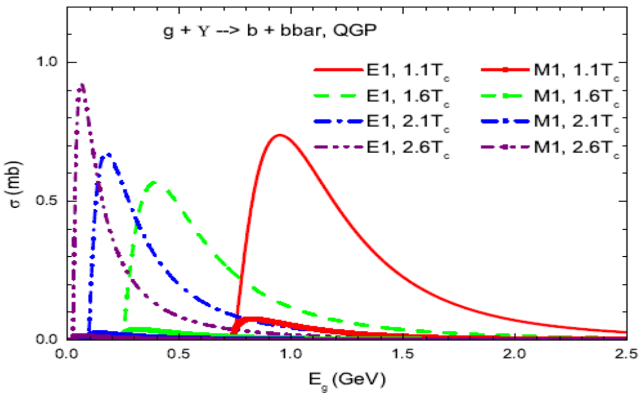


--- Coulomb wave func. + full ε_B differs from the full potential result by ~50%

--- M1 overtakes E1 at low energies ← ΔL= 0 s-wave isotropic scattering dominant

- **In-medium cross sections**





--- as T increases, dipole size grows → E1 increases, M1 most prominent at low T

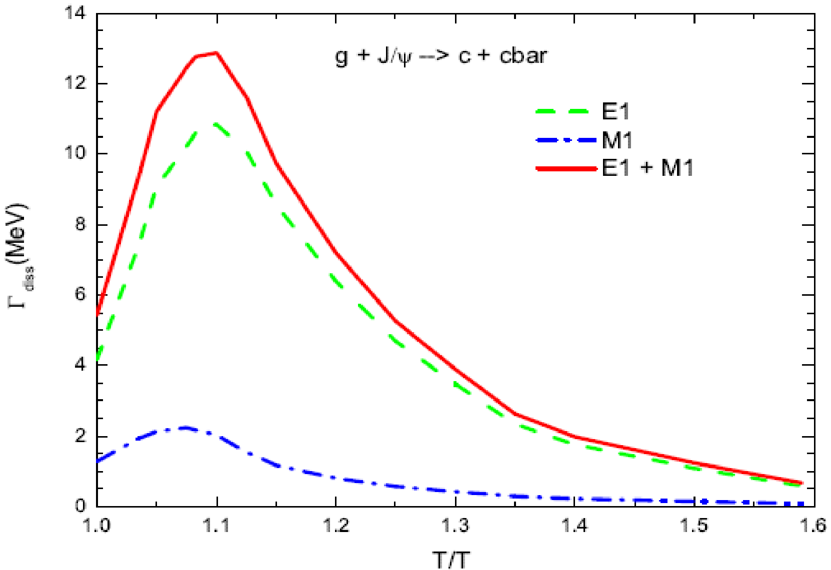
--- more tightly bound Y(1S) much smaller cross section

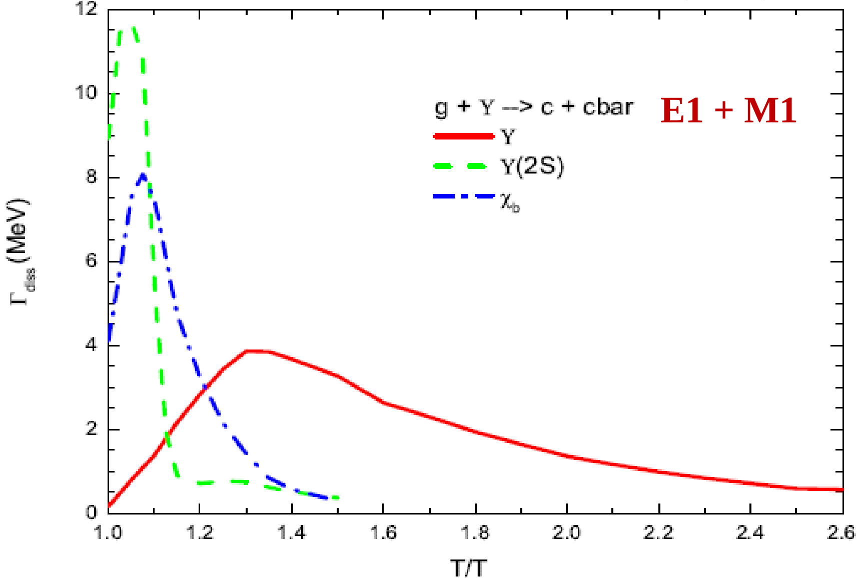
Gluo-dissociation rates

- **Heavy quarkonium (at rest) gluo-dissociation rate**

$$\Gamma_{\rm diss}(T) = d_g \int \frac{d^3k}{(2\pi)^3} f_g(E(\vec{k}))v_{\rm rel}\sigma(|\vec{k}|,T)$$

Chen & MH, PRC96, 034901 (2017)





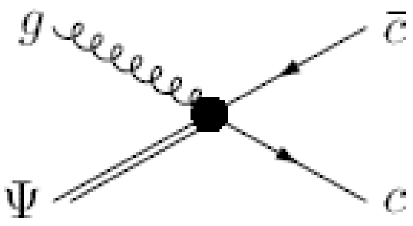
--- M1 most prominent for J/ψ at low T, accounting for ~10-25% of the total (E1+M1) rate in T_c --1.2T_c ➔ could be significant as system stays long

--- at low T, ε_B large, tightly bound J/ψ or Υ ➔ gluon sees the bound state as a whole ➔ LO gluo-dissociation sensible ➔ rate increases with T

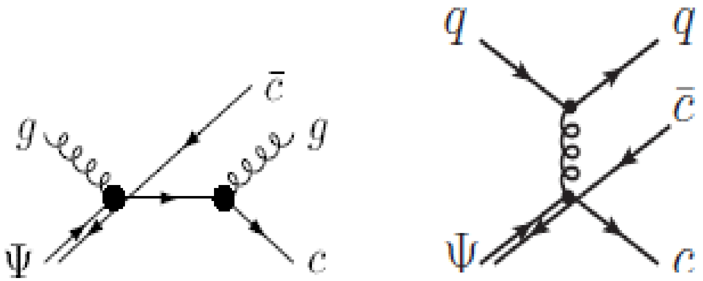
--- at high T, ε_B decreases, σ [$k^2 * f_g(k)$] shifts toward lower [higher] energy ➔ phase space mismatched ➔ rate drops off fast ➔ calling for NLO

Dynamical dissociation: NLO

- LO, i.e. gluo-dissociation --- the incoming gluon sees the Ψ as a whole, which is reasonable for a tightly-bound state (and relatively low-energy, long-wavelength gluon)
  ➔ applicable at low temperatures, efficient when gluon wave-length ~ Ψ size
- At high temperatures, Ψ binding energy drops off and radius grows, the incoming gluon should see individual Q or Qbar in Ψ, opening up new phase space **Rapp et al., 2001 & 2011**

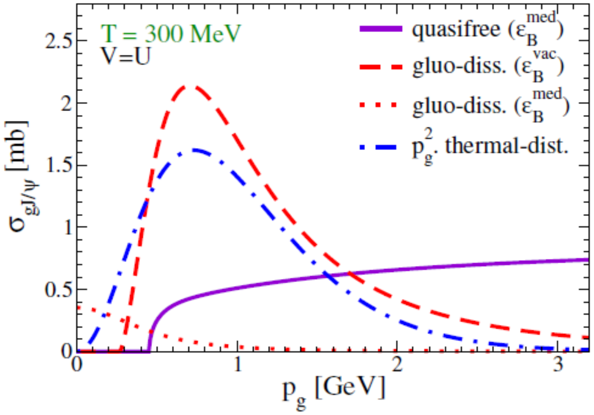


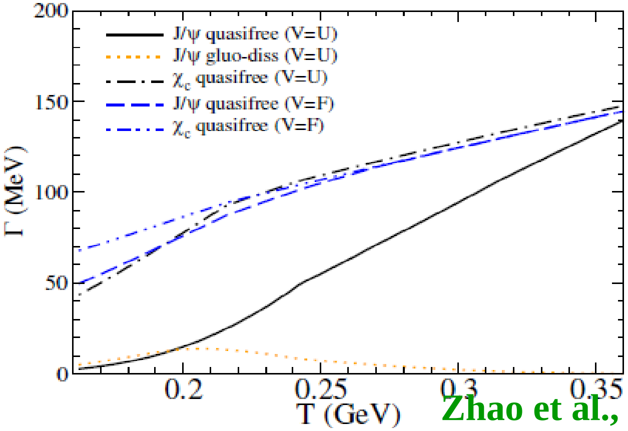
LO: g + Ψ → c + cbar



NLO: g/q/qbar + Ψ → g/q/qbar + c + cbar

- Quasi-free: σ(g+Ψ→g+c+cbar)=2*σ(g+c→g+c), with a spectator cbar:
  off-shellness of Q & Qbar within Ψ & bound state wave function effects are neglected

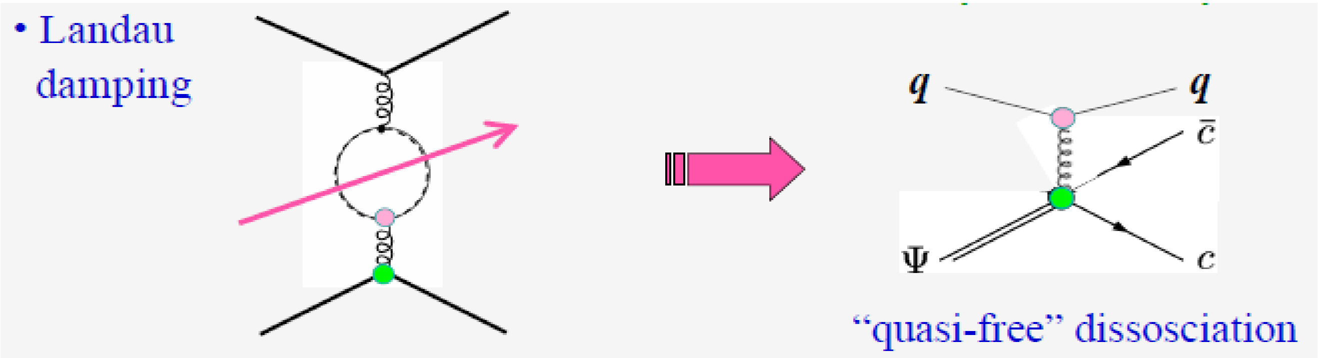




Zhao et al., 2010

NLO dissociation: going beyond quasi-free

- **Quasi-free: effects of bound state wave func. completely ignored**



“quasi-free” dissosciation

- **Starting from the QCD Hamiltonian in Weyl gauge $A^{0a}=0$**

$$
\begin{aligned}
H_{\rm QCD} &= \int d^3\vec{x}\,\psi^\dagger(t,\vec{x})[\beta m_q - i\vec{\alpha}\cdot(\vec{\partial} - i g_s \vec{A}^a(t,\vec{x})\frac{\lambda^a}{2})]\psi(t,\vec{x}) \\
&+ \int d^3\vec{x}[\frac{1}{2}\vec{E}^a(t,\vec{x}) + \frac{1}{2}\vec{B}^a(t,\vec{x})] \\
&= H_{\rm kin} + V_{q\bar{q}g} + V_{Q\bar{Q}g} + V_{3g} + V_{4g}
\end{aligned}
$$

with

$$
\vec{E}^a = -\frac{\partial \vec{A}^a}{\partial t} - \nabla A^{0a} + g_s f^{abc}\vec{A}^b A^{0c} = -\frac{\partial \vec{A}^a}{\partial t}
$$

$$
\vec{B}^a = \nabla \times \vec{A}^a - \frac{1}{2} g_s f^{abc}\vec{A}^b \times \vec{A}^c
$$

Effective Hamiltonian

Yan 80, Sumino 14
Brambilla 05 pNRQCD

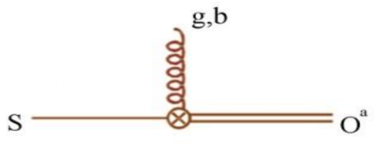
- **QCD multipole expansion for heavy quarkonium system**

$$V_{Q\bar{Q}g} \rightarrow H_0 + V_{SO} + V_{OO}$$

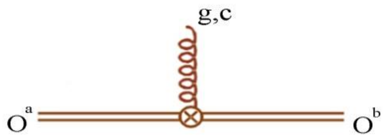
for the QQbar system bound by internal gluons

$$H_0 = \frac{\vec{p}^2}{m_Q} + V_0(r) + V_8(r)$$

- **One is left with 4 interaction vertices at the order of O(g_s)**



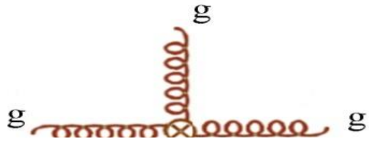
$$< O,a|V_{SO}|S >=< O,a|\frac{1}{2}g_s\vec{r}(\frac{\lambda^b}{2} - \frac{\bar{\lambda}^b}{2}) \cdot \vec{E}^b|S >= \frac{g_s}{\sqrt{2N_c}}\vec{E}^b \cdot < O|\vec{r}|S > \delta^{ab}$$



$$< O,a|V_{OO}|O,b >= \frac{ig_s}{2}d^{abc}\vec{E}^c \cdot < O|\vec{r}|O >$$

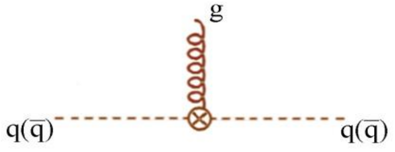
With

$$d^{abc} = 2tr[\lambda^a/2\{\lambda^b/2, \lambda^c/2\}]$$



$$V_{3g} \sim \int d^3\vec{x}\frac{1}{2}\vec{B}^a \cdot \vec{B}^a = (-\frac{g_s}{2})f^{abc}\int d^3\vec{x}(\nabla \times \vec{A}^a) \cdot (\vec{A}^b \times \vec{A}^c)$$

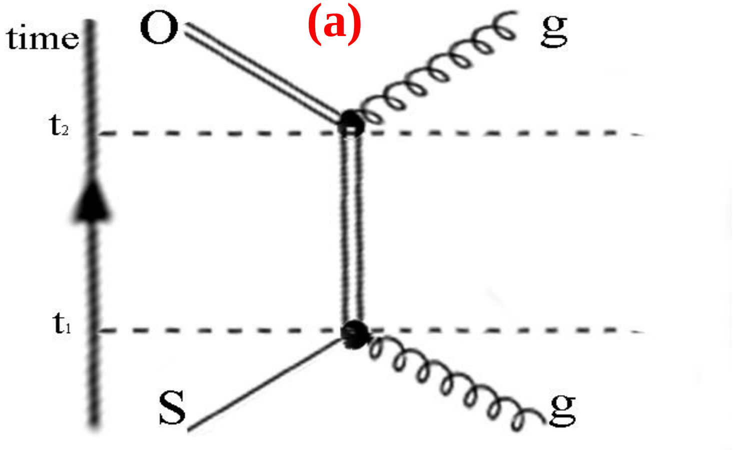
V_{4g} is at the order of O(g_s^2) & thus neglected

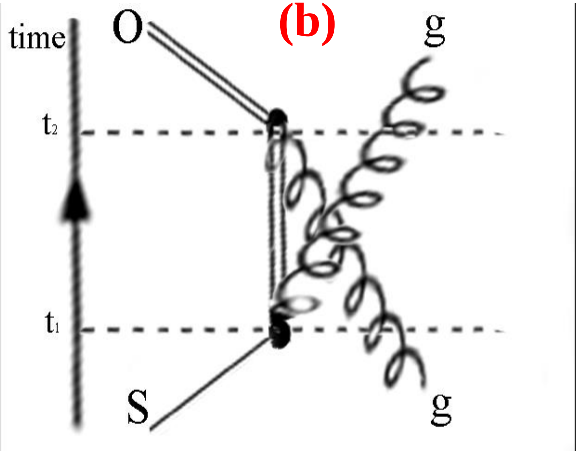


$$V_{q\bar{q}g} = g_s \int d^3\vec{x}q^\dagger\vec{\alpha} \cdot \vec{A}^a\frac{\lambda^a}{2}q$$

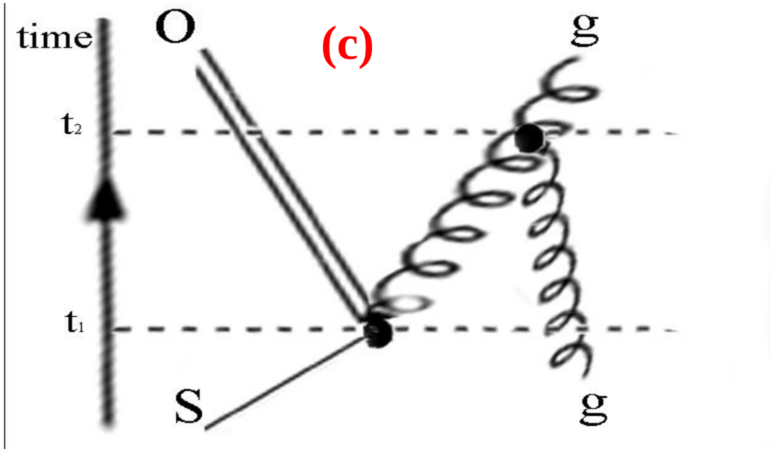
NLO: g + Ψ → g + c + cbar diagrams

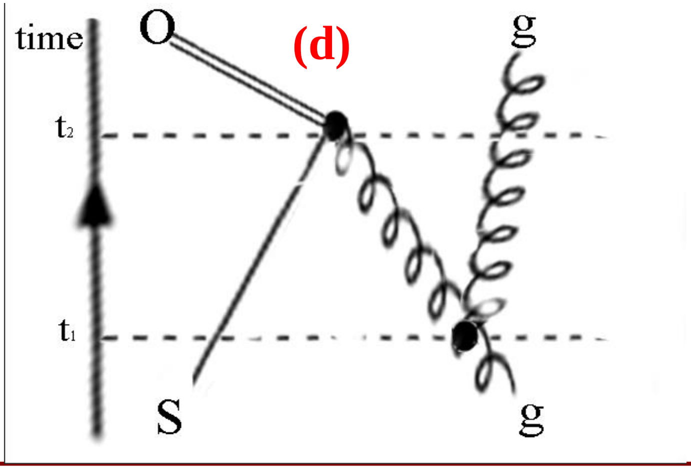
- s- & u-channel diagrams formed out of V_{SO} & V_{OO} analogous to Compton





- t-channel diagrams formed out of V_{SO} & V_{3g} looks like quasi-free diagram





NLO: 2nd order QM perturbation

- **Transition amplitude for g + Ψ → g + c + cbar**

$$T_{fi}=\sum_m \frac{<f|H_I|m><m|H_I|i>}{E_i-E_m+i\epsilon},$$

Chen & MH, PLB786 (2018) 260–267

initial/final state: $|i>=|J/\psi,g(\vec{k},\lambda,a)>$ $|f>=|(c\bar{c})_8(\vec{p},b),g(\vec{\kappa},\sigma,c)>$

- **Intermediate states & amplitudes for diagrams (a) & (b)**

--- (a) $|m>=|(c\bar{c})_8(\vec{q},d)>$

$$\begin{aligned}<m|V_{SO}|i>&=\delta^{ad}\frac{ig_s}{2}\sqrt{\frac{\omega_{\vec{k}}}{3V}}<(c\bar{c})_8(\vec{q})|\vec{r}\cdot\vec{\epsilon}_{\vec{k}\lambda}|J/\psi>\\&=\delta^{ad}\frac{(-g_s)}{V}\sqrt{\frac{\pi\omega_{\vec{k}}}{3}}\,\vec{\epsilon}_{\vec{k}\lambda}\cdot\frac{\vec{q}}{q}\int r^3drj_1(qr)R_{10}(r)\end{aligned}$$

$$\begin{aligned}<f|V_{OO}|m>&=\frac{g_s}{2}\sqrt{\frac{\omega_{\vec{\kappa}}}{2V}}\delta^{cf}d^{bdf}\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot<(c\bar{c})_8(\vec{p})|\vec{r}|(c\bar{c})_8(\vec{q})>\\&=\frac{(-ig_s)}{2V}(2\pi)^3d^{bdc}\sqrt{\frac{\omega_{\vec{\kappa}}}{2V}}\epsilon_{\vec{\kappa}\sigma}\cdot\nabla_{\vec{q}}\delta^3(\vec{q}-\vec{p}),\qquad(8)\end{aligned}$$

$$A(p,k)=\frac{\int r^3drj_1(pr)R_{10}(r)}{p(-\epsilon_B+\omega_{\vec{k}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)},$$

$$B(p,k)=\frac{\frac{2p}{m_Q}\int r^3drj_1(pr)R_{10}(r)}{(-\epsilon_B+\omega_{\vec{k}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)^2}-\frac{\int r^4drj_2(pr)R_{10}(r)}{(-\epsilon_B+\omega_{\vec{k}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)}$$

$$\begin{aligned}T_{fi}^{(a)}&=\frac{(-ig_s^2)}{2V^2}(2\pi)^3\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}\sum_d\delta^{ad}d^{bdc}\frac{V}{(2\pi)^3}\int d^3\vec{q}\vec{\epsilon}_{\vec{k}\lambda}\cdot\frac{\vec{q}}{q}\\&\times\int r^3drj_1(qr)R_{10}(r)\frac{\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\nabla_{\vec{q}}\delta^3(\vec{q}-\vec{p})}{-\epsilon_B+\omega_{\vec{k}}-\frac{\vec{q}^2}{m_Q}+i\epsilon}\\&=d^{bac}\frac{ig_s^2}{2V}\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}\vec{\epsilon}_{\vec{k}\lambda}\cdot[A(p,k)\vec{\epsilon}_{\vec{\kappa}\sigma}+(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})\frac{\vec{p}}{p^2}B(p,k)].\end{aligned}$$

$\propto$ **d^{abc}**

wave func., ΔL=2, dipole transition twice

NLO: 2nd order QM perturbation (cont.)

--- (b) $|m>=|(c\bar{c})_8(\vec{q},d),g(\vec{k}_1,\lambda_1,d_1),g(\vec{k}_2,\lambda_2,d_2)>$

$$C(p,\kappa)=\frac{\int r^3drj_1(pr)R_{10}(r)}{p(-\epsilon_B-\omega_{\vec{\kappa}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)}$$

$$D(p,\kappa)=\frac{\frac{2p}{m_Q}\int r^3drj_1(pr)R_{10}(r)}{(-\epsilon_B-\omega_{\vec{\kappa}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)^2}-\frac{\int r^4drj_2(pr)R_{10}(r)}{(-\epsilon_B-\omega_{\vec{\kappa}}-\frac{\vec{p}^2}{m_Q}+i\epsilon)}$$

$$T_{fi}^{(b)}=-d^{bac}\frac{ig_s^2}{2V}\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}$$ ∝ $\mathbf{d^{abc}}$

$$\times\,\vec{\epsilon}_{\vec{k}\lambda}\cdot[C(p,\kappa)\vec{\epsilon}_{\vec{\kappa}\sigma}+(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})\frac{\vec{p}}{p^2}D(p,\kappa)]$$

- **Intermediate states & amplitudes for diagrams (c) & (d)**

--- (c) $|m\quad>=\quad|(c\bar{c})_8(\vec{q},d),g(\vec{k}_1,\lambda_1,d_1),g(\vec{k}_2,\lambda_2,d_2)>$

$$T_{fi}^{(c)}=f^{abc}\frac{(-ig_s^2)}{V}\sqrt{\frac{\pi}{6V\omega_{\vec{k}}\omega_{\vec{\kappa}}}}\{(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\kappa})$$

$$+(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{k})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{p})-(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\epsilon}_{\vec{\kappa}\sigma})\frac{(\vec{k}^2-\vec{k}\cdot\vec{\kappa})\vec{\kappa}\cdot\vec{p}+(\vec{\kappa}^2-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p}}{(\vec{k}-\vec{\kappa})^2}\}$$

$$\times\frac{1}{p}\int r^3drj_1(pr)R_{10}(r)\frac{1}{-\epsilon_B^{J/\psi}-\frac{\vec{p}^2}{m_Q}-\omega(\vec{k}-\vec{\kappa})+i\epsilon}.$$ ∝ $\mathbf{f^{abc}}$

--- (d) $|m>=|J/\psi,g(\vec{k}_1,\lambda_1,d_1),g(\vec{k}_2,\lambda_2,d_2)\ >$

$$T_{fi}^{(d)}=f^{abc}\frac{ig_s^2}{V}\sqrt{\frac{\pi}{6V\omega_{\vec{k}}\omega_{\vec{\kappa}}}}\{(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\kappa})+(\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{k})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{p})$$

$$-(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\epsilon}_{\vec{\kappa}\sigma})\frac{(\vec{k}^2-\vec{k}\cdot\vec{\kappa})\vec{\kappa}\cdot\vec{p}+(\vec{\kappa}^2-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p}}{(\vec{k}-\vec{\kappa})^2}\}$$ ∝ $\mathbf{f^{abc}}$

$$\times\frac{1}{p}\int r^3drj_1(pr)R_{10}(r)\frac{1}{\omega(\vec{k})-\omega(\vec{\kappa})-\omega(\vec{k}-\vec{\kappa})+i\epsilon}.$$

- **d^{abc}*f^{abc}=0 => no interference between** $T_{fi}^{(a)}+T_{fi}^{(b)}$ **&** $T_{fi}^{(c)}+T_{fi}^{(d)}$

NLO cross section: (a) + (b)

- 2$^{\text{nd}}$ transition rate

$$\Gamma^{(a+b)}_{i\to f}=\frac{2\pi}{\hbar}\sum_f|T^{(a)}_{fi}+T^{(b)}_{fi}|^2\delta(E_i-E_f)$$

- Cross section= rate/flux, using $\frac{1}{4\pi}\int d\Omega_{\vec{k}}\frac{1}{2}\sum_{\lambda=1,2}|\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\rho}|^2=\frac{1}{3}|\vec{\rho}|^2$

$$\sigma^{(a+b)}(E_g)=2\pi V\frac{V}{(2\pi)^3}\int d^3\vec{p}\sum_b\frac{V}{(2\pi)^3}\int d^3\vec{\kappa}\sum_\sigma\sum_c\frac{1}{4\pi}\int d\Omega_{\vec{k}}$$

$$\times\frac{1}{2}\sum_\lambda\frac{1}{8}\sum_a|T^{(a)}_{fi}+T^{(b)}_{fi}|^2\delta(-\epsilon_B+\omega_{\vec{k}}-\frac{\vec{p}^2}{m_Q}-\omega_{\vec{\kappa}}),$$

$$=\frac{5}{216\pi^2}g_s^4E_g\int p^2dp\int\kappa^2d\kappa\omega_{\vec{\kappa}}\times\{\cdots\}\delta(-\epsilon_B+E_g-\frac{\vec{p}^2}{m_Q}-\omega_{\vec{\kappa}})$$

where

$$\begin{aligned}\{\cdots\}=&[A^2(p,k)+\frac{1}{3}B^2(p,k)+\frac{2}{3}A(p,k)B(p,k)]\\&+[C^2(p,\kappa)+\frac{1}{3}D^2(p,\kappa)+\frac{2}{3}C(p,\kappa)D(p,\kappa)]\\&-2[A(p,k)C(p,\kappa)+\frac{1}{3}(A(p,k)D(p,\kappa)\\&+B(p,k)C(p,\kappa)+B(p,k)D(p,\kappa))].\qquad(21)\end{aligned}$$

--- Infrared divergence from A(p, κ) & B(p, κ) ∝ 1/ κ if gluons are massless, to be regularized by finite thermal gluon mass

NLO cross section: (c) + (d)

- Cross section= rate/flux, using $\sum_{\sigma} \epsilon^{(i)}_{\vec{\kappa}\sigma} \epsilon^{(j)}_{\vec{\kappa}\sigma} = \delta^{ij} - \frac{\kappa^i \kappa^i}{\vec{\kappa}^2}$

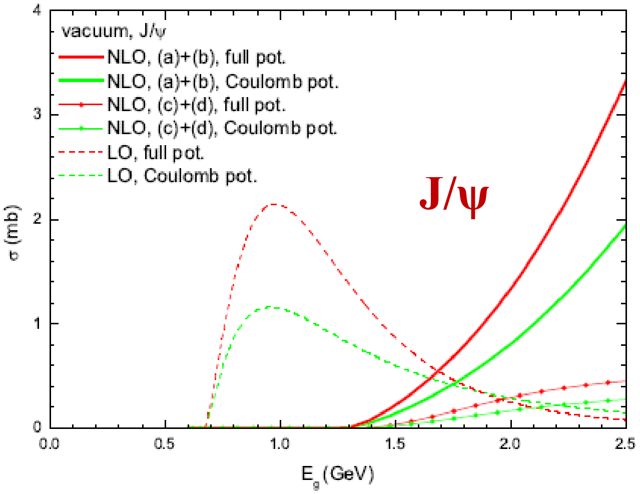
$$\sigma^{(c+d)}(E_g) = \frac{g_s^4}{8\pi^4} \frac{1}{E_g} \int dp \int d\kappa \frac{\kappa^2}{\omega(\vec{\kappa})} \int d\Omega_{\vec{\kappa}} \int d\Omega_{\vec{k}}$$

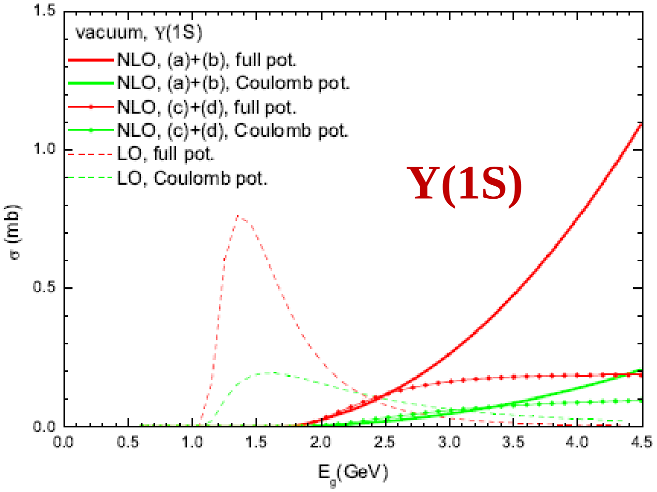
$$[\int r^3 dr j_1(pr) R_{10}(r)]^2 \times g(\vec{\kappa}, \vec{p}, \vec{k})$$

$$\times [\frac{\omega(\vec{\kappa}) - \omega(\vec{k})}{\omega^2(\vec{k} - \vec{\kappa}) - (\omega(\vec{k}) - \omega(\vec{\kappa}))^2}]^2 \delta(-\epsilon_B + E_g - \frac{\vec{p}^2}{m_Q} - \omega_{\vec{\kappa}})$$

--- soft-collinear divergence (the square bracket) for massless-k / / κ gluons

- Vacuum cross section: assuming a "constituent" gluon mass m_g=600 MeV





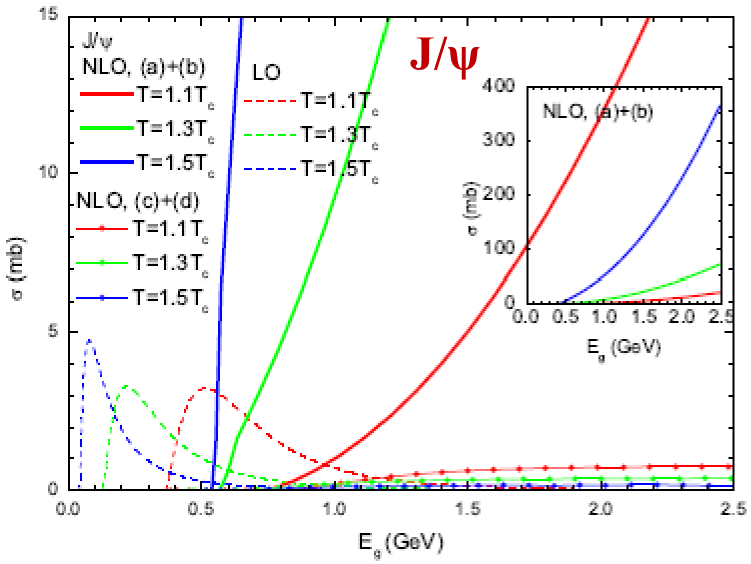
---- NLO quickly takes over from LO; no fall-off

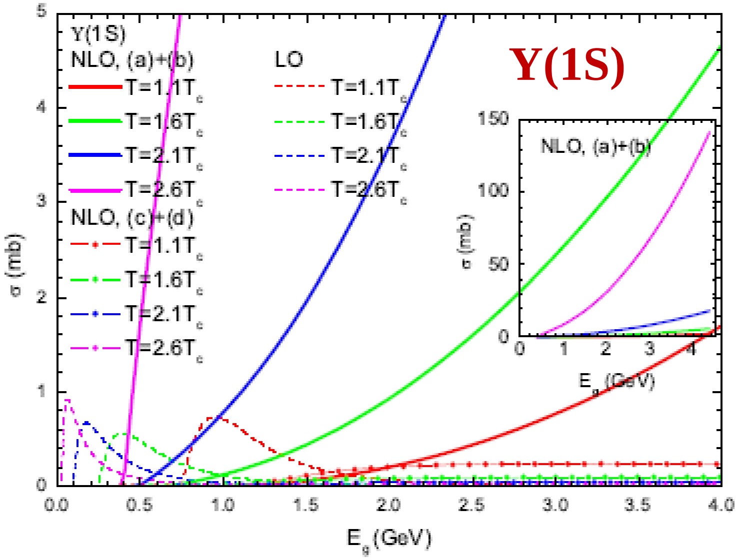
--- NLO (a)+(b) increasing with E_g, while (c)+(d) leveling off

NLO cross section: finite temperature

- **In-medium cross section**

--- divergences regularized by gluons' thermal mass $m_g(T) = \sqrt{3/4}\, g_s T$





--- NLO quickly takes over from LO; no fall-off

-- NLO (c)+(d): decreasing with T, expanding wave function overcome by decreasing ε_B

--- NLO (a)+(b): increases fast with T, due to expanding wave function & decreasing ε_B

-- near T_d, dipole size blows up > gluon wave-length, dipole transition to be invalidated

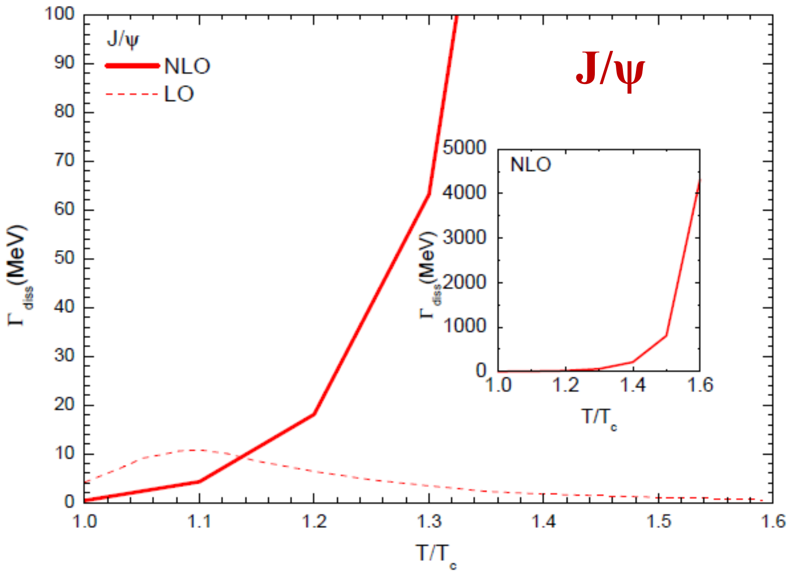
➔ near T_d, cross sections may not be quantitatively reliable

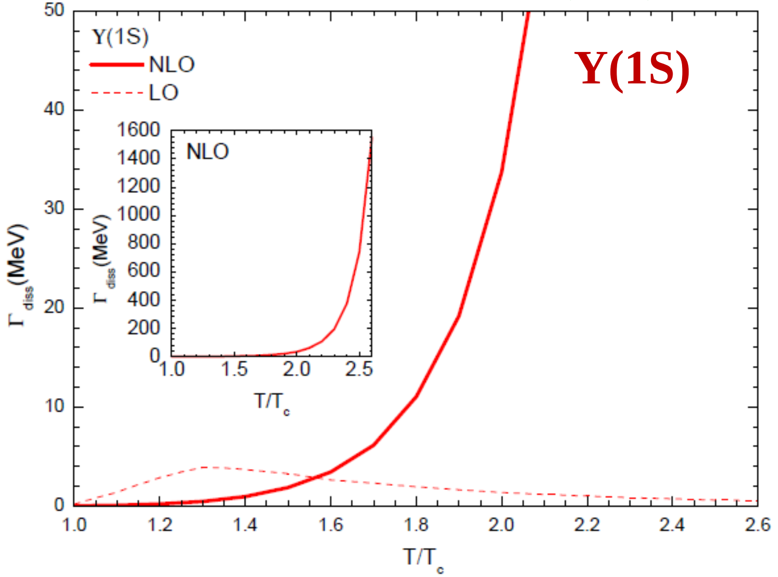
NLO dissociation rate

- **Heavy quarkonium (at rest) NLO-dissociation rate**

$$\Gamma_{\rm diss}(T) = d_g \int \frac{d^3k}{(2\pi)^3} f_g(E(\vec{k})) v_{\rm rel}\sigma(|\vec{k}|,T)$$

Chen & MH, PLB786 (2018) 260–267





--- **The artifact of LO dropping off toward high T: replaced by NLO increase**

--- **Near T_d, $\Gamma_{\rm diss}$ ~ GeV: very fast break-up, conceptually consistent with static dissociation by color screening**

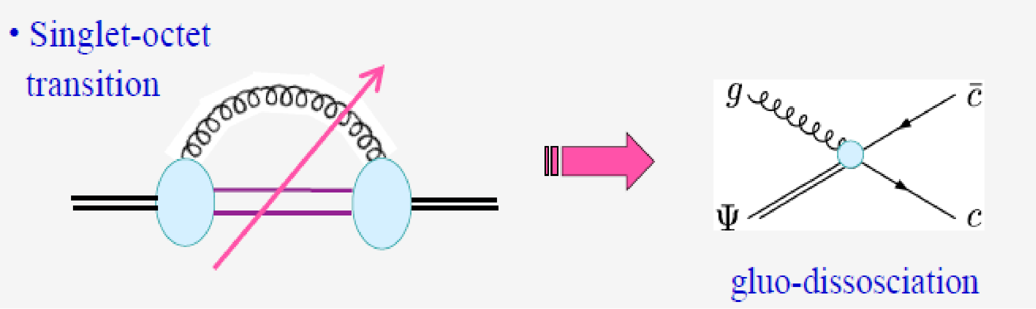
--- **Quantitatively might be questionable as dipole approximation becomes invalidated, but empirically supported/needed by phenomenological transport study, e.g. Strickland '15, $\Gamma_{\rm diss}$ > 2 GeV needed for T>=T_d**

pNRQCD point of view: Ψ dissociation

- pNRQCD: d.o.f. = singlet & octet QQbar states + light quarks & gluons

$$\mathcal{L}_{\mathrm{pNRQCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i + \int d^3r\ \mathrm{Tr}\left\{\mathrm{S}^\dagger\left[i\partial_0 - h_s\right]\mathrm{S} + \mathrm{O}^\dagger\left[iD_0 - h_o\right]\mathrm{O}\right.$$
$$\left. + V_A\left(\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{H.c.}\right) + \frac{V_B}{2}\mathrm{O}^\dagger\left\{\mathbf{r}\cdot g\mathbf{E}\,,\mathrm{O}\right\} + \ldots\right\}.$$

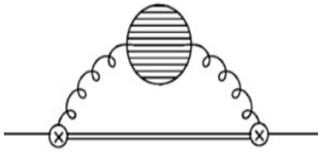
- Singlet-to-octet transition ~ gluo-dissociation, singlet 1-loop selfenergy➔cutting: g+Ψ→c-cbar octet



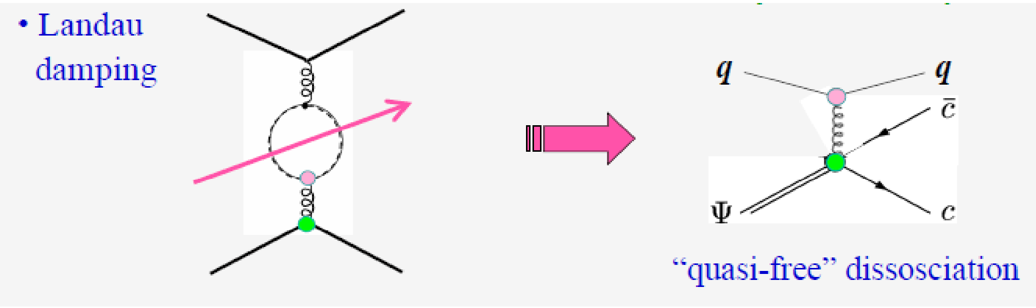
Brambilla et al., 2008,2011

- Landau damping (ImV) ~ parton inelastic collisional dissociation ~ quasi-free **Brambilla et al., 2013**

$$\Gamma_{nl} = -\langle n,l|2\,\mathrm{Im}\,V_s^{(0)}(r)|n,l\rangle \qquad -2\,\mathrm{Im}\,V_s^{(0)}(r) = \frac{g^2TC_Fm_D^2}{\pi}\int_0^\infty \frac{dt\,t}{(t^2+m_D^2)^2}\left(1-\frac{\sin(tr)}{tr}\right)$$



Singlet 2-loop selfenergy➔cutting: g+Ψ→g+c-cbar octet



interference term →0 when r →0;
→ 2*Γ_Q when r→∞

Rapp et al., 2011

Dissociation rate from Schrodinger with ImV

- Solving the eigenstate Schrodinger equation with Q-Qbar complex potential

Strickland et al., PRD97,016017(2018)

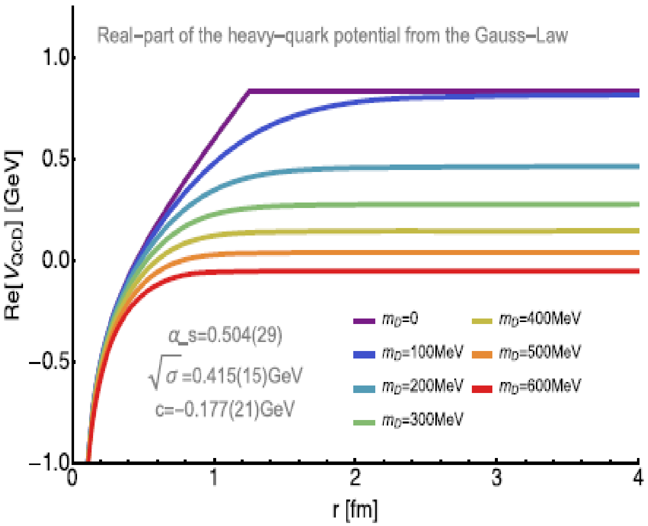
$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + iV_I(T,r)]\psi_n = E_n\psi_n \quad \Rightarrow \quad \Gamma_i = -2\mathrm{Im}[E_i]$$

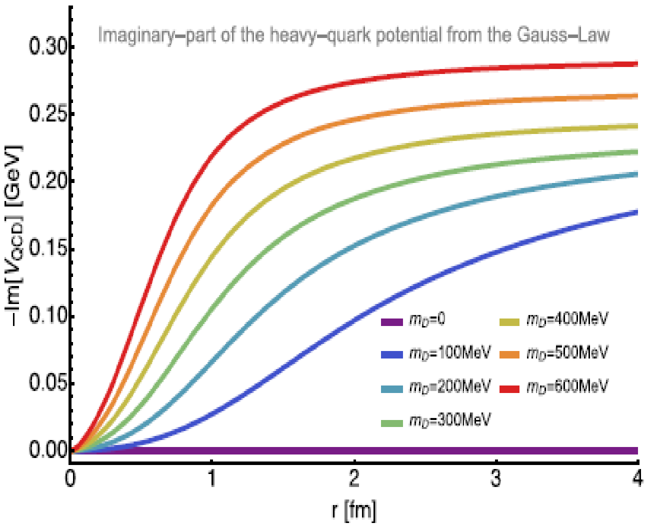
complex potential taken from fitting lattice data: Coulomb + string ReV + ImV

$$V_c(r) = -\alpha_s\left[m_\mathrm{D} + \frac{e^{-m_\mathrm{D}r}}{r} + iT\phi(m_\mathrm{D}r)\right]$$

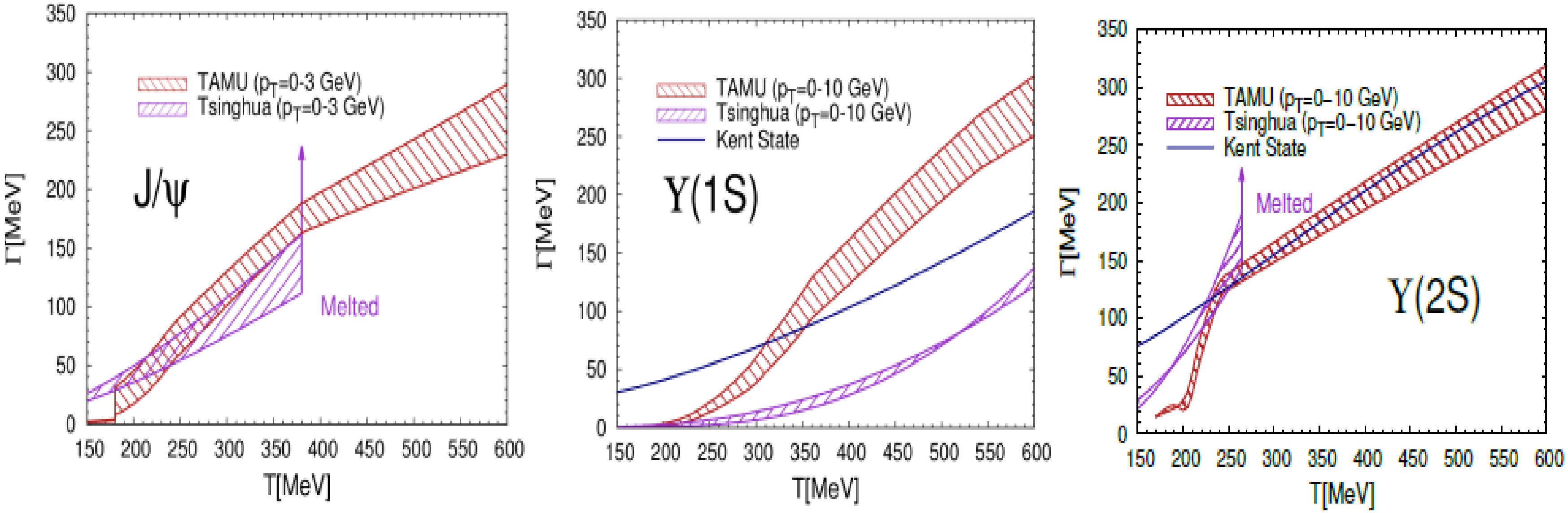
$$\mathrm{Re}V_s(r) = -\frac{\Gamma[\frac{1}{4}]}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma[\frac{1}{4}]}{2\Gamma[\frac{3}{4}]}\frac{\sigma}{\mu}$$

$$\mathrm{Im}V_s(r) = -i\frac{\sigma m_D^2 T}{\mu^4}\psi(\mu r) = -i\alpha_s T\psi(\mu r)$$





Summary of dissociation rates



MH, van Hees, Rapp, arXiv:2204.09299

- TAMU: quasi-free with T-matrix binding energies; interference effect (1-sin(tr)/tr) implemented for Υ states
- Tsinghua: gluo-dissociation with geometric scaling with in-medium radius of bound states
- .Kent: computed from Schrodinger eigen-energy with ImV_{QQbar}

➔ Reasonable agreement in values for J/ψ and Υ(2S) but differing considerably for Υ(1S) between different groups (although very different underlying assumptions)

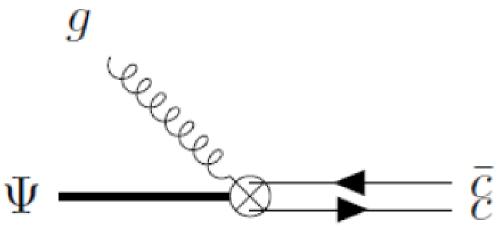
Semi-classical transport approaches

- Boltzmann equation for Ψ transport in QGP

$$\partial f_\Psi/\partial t + \vec{v}_\Psi \cdot \vec{\nabla} f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi \qquad f_\Psi(x,p,t) = \frac{(2\pi)^3 dN_\Psi}{d^3x\, d^3p}$$

loss term gain term

- For 2 ←>2 process: gluo-dissociation $\Psi + g \leftrightarrows c + \bar{c},$



$$\alpha_\Psi(p,x) = \frac{1}{2E_\Psi}\int d\Phi_1(p_g)\; d\Phi_2(p_c,p_{\bar{c}})$$

$$\times\,(2\pi)^4\delta^{(4)}(p_c+p_{\bar{c}}-p-p_g)\, d_g\, f_g(p_g)\, |\overline{\mathcal{M}_{\Psi g\to c\bar{c}}(s,t)}|^2$$

$$= \int \frac{d^3p_g}{(2\pi)^3}\, d_g\, f_g(p_g)\, v_{rel}\, \sigma_\Psi(s) = \Gamma(p,T)$$ → dissociation rate

$$\beta_\Psi(p,x) = \frac{1}{2E_\Psi}\int d\Phi_1(p_g) d\Phi_2(p_c,p_{\bar{c}})(2\pi)^4\delta^{(4)}(p_c+p_{\bar{c}}-p-p_g)$$ → regeneration via c-cbar recombination

$$\times|\overline{\mathcal{M}_{c\bar{c}\to\Psi g}(s,t)}|^2\, d_c\, f_c(p_c)\, d_{\bar{c}}\, f_{\bar{c}}(p_{\bar{c}})\,(1+f_g(p_g))\;,$$

Semi-classical Boltzmann

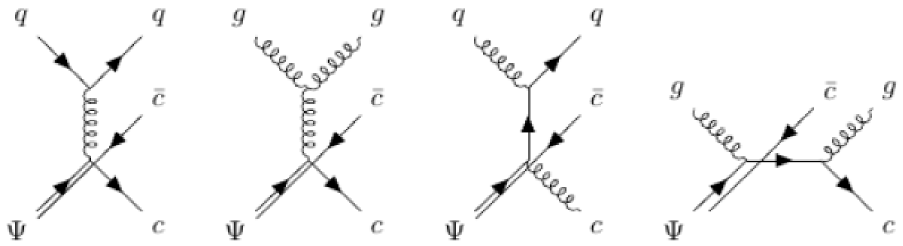
- Boltzmann equation for Ψ transport in QGP

$$\partial f_\Psi/\partial t + \vec{v}_\Psi \cdot \vec{\nabla} f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi \qquad f_\Psi(x,p,t) = \frac{(2\pi)^3 dN_\Psi}{d^3x\, d^3p}$$

loss term

gain term

- For 2 ←>3 process: quasi-free/Landau damping dissociation $i+\Psi \to i+c+\bar{c}\ (i{=}g,q,\bar{q})$



$$\alpha_\Psi(p,x) = \frac{1}{2E_\Psi}\sum_i \int d\Phi_1(p_i)\Phi_3(p_c,p_{\bar{c}},\bar{p}_i)(2\pi)^4\delta^{(4)}(p_c+p_{\bar{c}}+\bar{p}_i-p-p_i)$$

$$\times\, |\overline{\mathcal{M}_{\Psi i\to c\bar{c}i}(s,t)}|^2\, d_i\, f_i(p_i)\,(1\pm f_i(\bar{p}_i))\ ,$$

$$= \int \frac{d^3p_g}{(2\pi)^3}\, d_g\, f_g(p_g)\, v_{rel}\, \sigma_\Psi(s) = \Gamma(p,T)$$

→ dissociation rate

$$\beta_\Psi(p,x) = \frac{1}{2E_\Psi}\sum_i \int d\Phi_1(p_i) d\Phi_3(p_c,p_{\bar{c}},\bar{p}_i)(2\pi)^4\delta^{(4)}(p_c+p_{\bar{c}}+\bar{p}_i-p-p_i)$$

$$\times\, |\overline{\mathcal{M}_{c\bar{c}i\to\Psi i}(s,t)}|^2\, d_c\, f_c(p_c)\, d_{\bar{c}}\, f_{\bar{c}}(p_{\bar{c}})\, d_i\, f_i(\bar{p}_i)\,(1\pm f_i(p_i))\ .$$

→ regeneration via
c-cbar recombination

Semi-classical Boltzmann: formal solution

- Boltzmann equation for Ψ transport in QGP

$$\partial f_\Psi/\partial t + \vec{v}_\Psi \cdot \vec{\nabla} f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi \qquad f_\Psi(x,p,t) = \frac{(2\pi)^3 dN_\Psi}{d^3x\, d^3p}$$

loss term gain term

- Formation solution **A.Polleri, arXiv:nucl-th/0303065 (2003); Yan & Zhuang, PRL97, 232301 (2006)**

$$f_\Psi(p,x,t) = f_\Psi(p, x - v_\Psi(t-t_0), t_0) e^{-\int_{t_0}^{t} dt' \alpha_\Psi(p, x - v_\Psi(t-t'), t')}$$
$$+ \int_{t_0}^{t} dt' \beta_\Psi(p, x - v_\Psi(t-t'), t') e^{-\int_{t'}^{t} dt'' \alpha_\Psi(p, x - v_\Psi(t-t''), t'')}$$

- 1st term: describing the dissociation/loss of the initially produced heavy quarkonia, at t_0, known as primordial component

- 2nd term: increasing with time, describing the regeneration process of heavy quarkonia from recombination of charm and anticharm quarks from t_0 to t’, and their subsequent dissociation from t’ to t

Reduction of Boltzmann to rate equation

- When c & cbar (as well as light gluons/quarks) are in full thermal equilibrium,

  For 2 ←>2 process: gluo-dissociation $\Psi + g \leftrightarrows c + \bar{c},$

  $$E_g + E_\Psi = E_c + E_{\bar{c}} \quad \Rightarrow \quad e^{-E_g/T}\cdot\gamma_c^2 e^{-E_\Psi/T} = \gamma_c e^{-E_c/T}\cdot\gamma_c e^{-E_{\bar{c}}/T}$$

  For 2 ←>3 process: quasi-free/Landau damping dissociation $i + \Psi \rightarrow i + c + \bar{c}\ (i{=}g,q,\bar{q})$

  $$E_g + E_\Psi = E'_g + E_c + E_{\bar{c}} \quad \Rightarrow \quad e^{-E_g/T}\cdot\gamma_c^2 e^{-E_\Psi/T} = e^{-E'_g/T}\cdot\gamma_c e^{-E_c/T}\cdot\gamma_c e^{-E_{\bar{c}}/T}$$

  detailed balance condition at the level of scattering matrix element squared:

  $$\overline{|\mathcal{M}_{i\to f}|^2} = (g_f/g_i)\overline{|\mathcal{M}_{f\to i}|^2}$$

  ➔ gain term : $\beta = \Gamma_\Psi(p,T)\cdot f_\Psi^{\rm eq}(x,p)$

- This leads to (true even when including charm fugacity) $\dfrac{df_\Psi(x,p,t)}{dt} = -\Gamma_\Psi(p,T)[f_\Psi(x,p,t) - f_\Psi^{\rm eq}(x,p)]$

- Assume a momentum average dissociation rate $\Gamma_\Psi(T)$, and upon integration over x and p,
  ➔ kinetic rate equation for the integrated yield

$$\frac{\mathrm{d}N_\Psi(t)}{\mathrm{d}t} = -\Gamma_\Psi(t)[N_\Psi(t) - N_\Psi^{\rm eq}(t)]$$

reaction rate Γ_ψ

regeneration toward equilibrium

Kinetic rate equation

- can be decomposed into two equations for primordial and regenerated component

$$N_\Psi(\tau) = N_\Psi^{\rm prim}(\tau) + N_\Psi^{\rm reg}(\tau)$$

for primordial component: $\frac{dN_\Psi^{\rm prim}}{d\tau} = -\Gamma_\Psi^{\rm diss} N_\Psi^{\rm prim}$ with initial condition: $N_\Psi^{\rm prim}(0) = N_\Psi(0)$

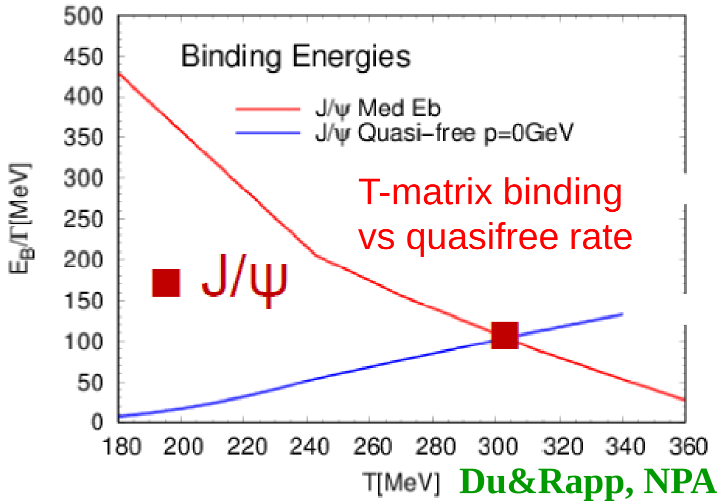
for regeneration component: $\frac{dN_\Psi^{\rm reg}}{d\tau} = -\Gamma_\Psi^{\rm diss}\left(N_\Psi^{\rm reg} - N_\Psi^{\rm eq}\right)$ with initial condition:

$$N_\Psi^{\rm reg}(\tau < \tau_0^\Psi) = 0 \qquad T(\tau_0^\Psi) = T_\Psi^{\rm diss}$$

- Two transport parameters: reaction rate & equilibrium limit

$$N_{q\bar q} = \frac{1}{2}\gamma_q n_{\rm op} V_{\rm FB}\frac{I_1(\gamma_q n_{\rm op} V_{\rm FB})}{I_0(\gamma_q n_{\rm op} V_{\rm FB})} + \gamma_q^2 n_{\rm hid} V_{\rm FB} \qquad N_Q^{\rm eq}(T,\gamma_Q) = V_{\rm FB}\, d_Q\, \gamma_Q(T)^2 \int \frac{d^3p}{(2\pi)^3} f^B(m_Q, T)$$

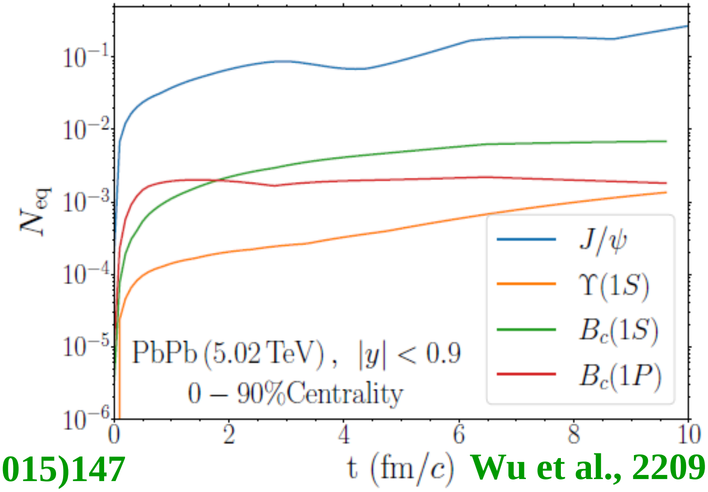
  - Correction to the equilibrium limit, due to off-equilibrium distribution of c & cbar quarks with $\tau_c^{\rm eq}$=3-5 fm/c: $N_\Psi^{\rm eq} = \mathcal{R}(\tau) N_\Psi^{\rm stat}, \quad \mathcal{R}(\tau) = 1 - \exp\left(-\tau/\tau_c^{\rm eq}\right)$



T-matrix binding vs quasifree rate

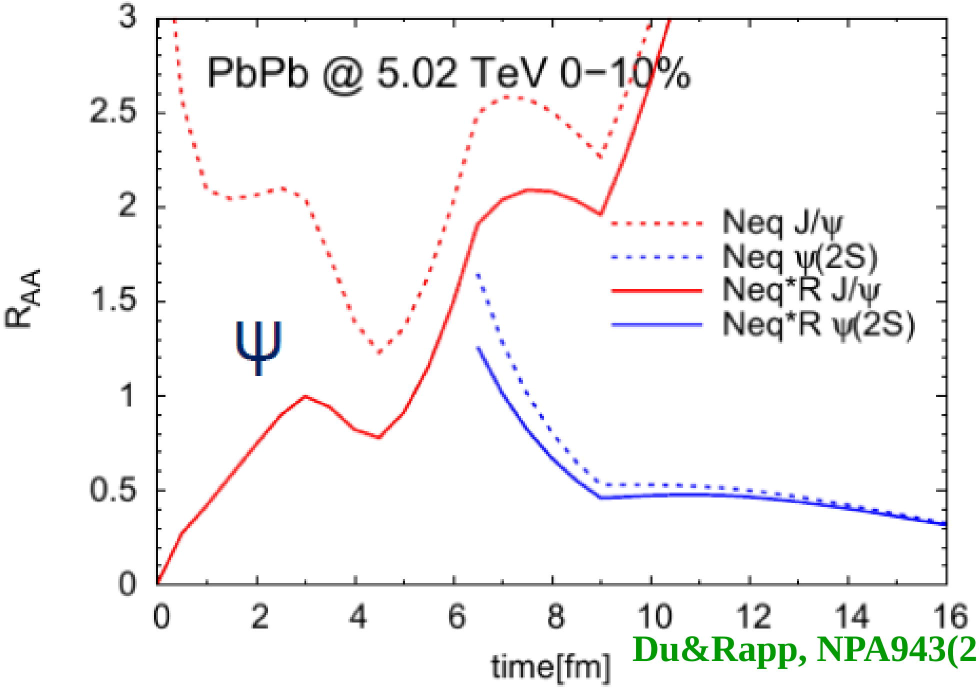
■ J/ψ

Du&Rapp, NPA943(2015)147



Wu et al., 2209.13795

Equilibrium limit: J/ψ vs ψ(2S)



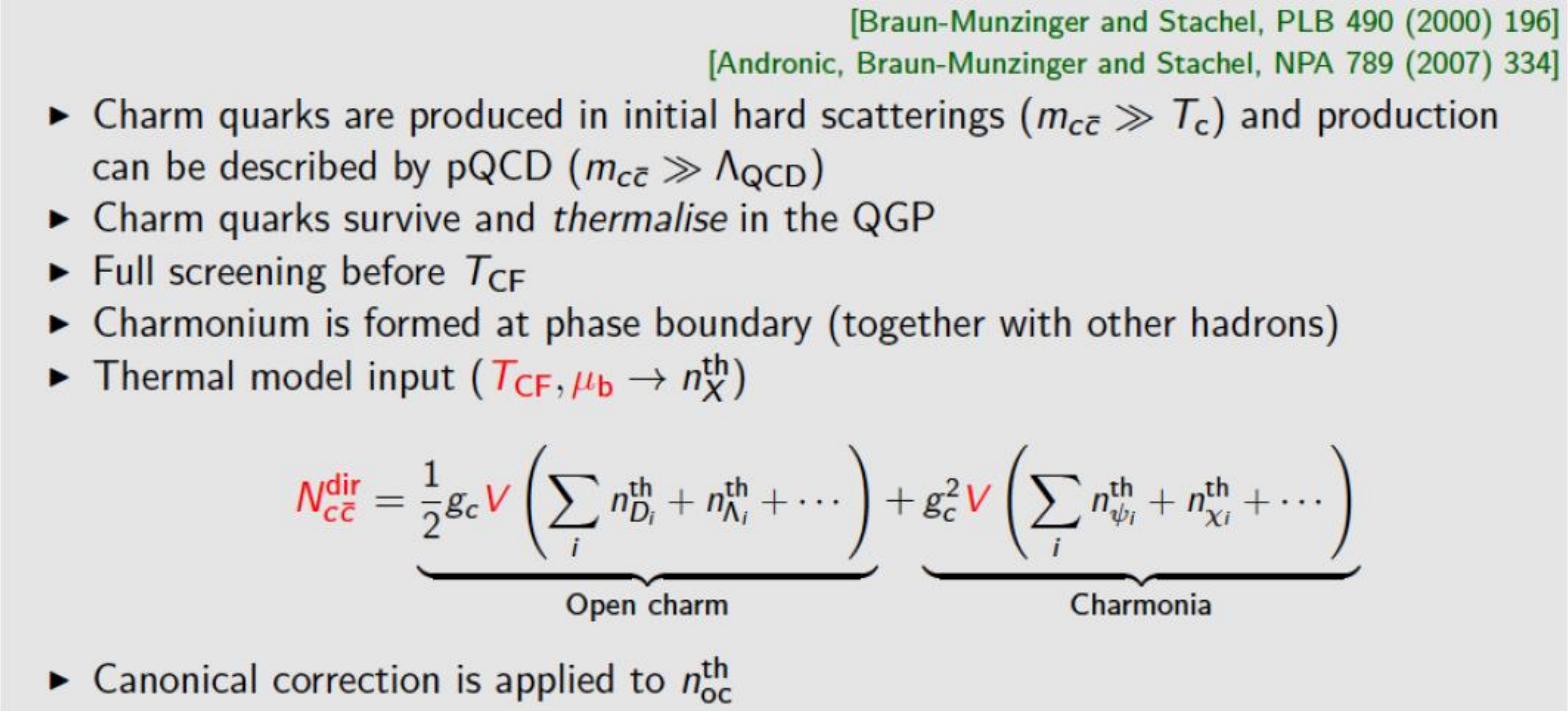
Du&Rapp, NPA943(2015)147

- For large-mass particle m>>T: $n \propto (mT)^{3/2} e^{-m/T}$
- But $N_\Psi << N_D$ $N_{c\bar{c}} \simeq \gamma_c n_{\rm op}^{\rm th} V \quad \Rightarrow \gamma_c \propto (m_D T)^{-3/2} \cdot e^{m_D/T}/V$
- The charmonium equilibrium number

$$N_\Psi^{\rm eq} = V\gamma_c^2 \cdot n_\Psi^{\rm th} = V\gamma_c^2 (m_\Psi T)^{3/2} e^{-m_\Psi/T} \propto m_\Psi^{3/2}/[m_D^3 T^{3/2} V] \cdot e^{E_B/T}$$

where binding energy $E_B = 2m_D - m_\Psi$

Statistical production of charmonia



[Braun-Munzinger and Stachel, PLB 490 (2000) 196]
[Andronic, Braun-Munzinger and Stachel, NPA 789 (2007) 334]

- Charm quarks are produced in initial hard scatterings ($m_{c\bar{c}} \gg T_c$) and production can be described by pQCD ($m_{c\bar{c}} \gg \Lambda_{\text{QCD}}$)
- Charm quarks survive and *thermalise* in the QGP
- Full screening before T_{CF}
- Charmonium is formed at phase boundary (together with other hadrons)
- Thermal model input ($T_{\text{CF}}, \mu_b \rightarrow n_X^{\text{th}}$)

$$N_{c\bar{c}}^{\text{dir}} = \underbrace{\frac{1}{2} g_c V \left(\sum_i n_{D_i}^{\text{th}} + n_{\Lambda_i}^{\text{th}} + \cdots \right)}_{\text{Open charm}} + \underbrace{g_c^2 V \left(\sum_i n_{\psi_i}^{\text{th}} + n_{\chi_i}^{\text{th}} + \cdots \right)}_{\text{Charmonia}}$$

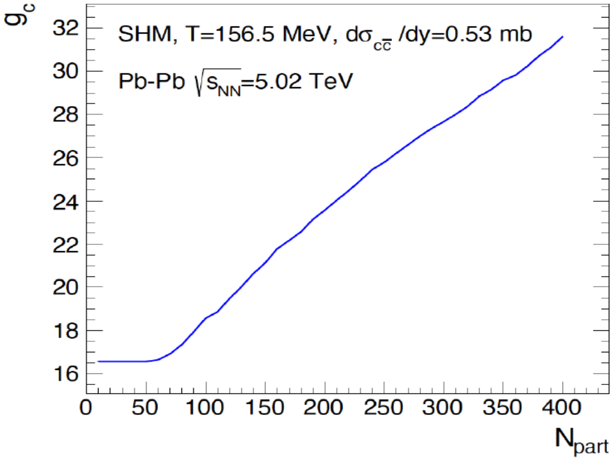
- Canonical correction is applied to $n_{\text{oc}}^{\text{th}}$

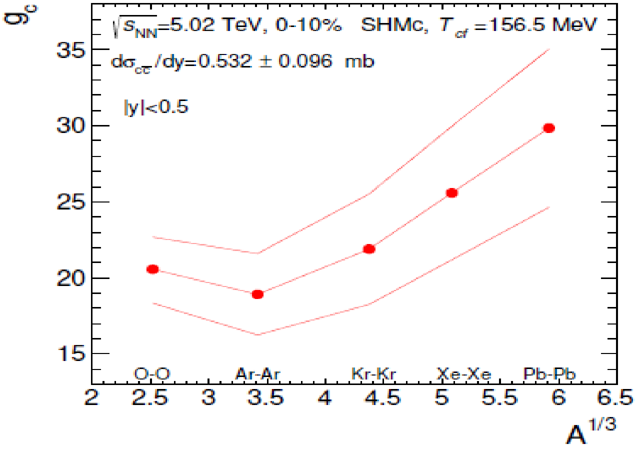
- With exact charm conservation, charm balance equation: $N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \quad \rightarrow g_c$
- Production yields $N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$
- Charm fugacity determined by the input charm cross section (subject to CNM/shadowing):
  $N_{ccbar}=\langle T_{AA}\rangle d\sigma^{ccbar}/dy$

$$\sigma_{c\bar{c}} = 1/2 \ [\sigma_{D^+} + \sigma_{D^-} + \sigma_{D^0} + \sigma_{\bar{D}^0} + \sigma_{\Lambda_c} + \sigma_{\bar{\Lambda}_c} \cdots]$$

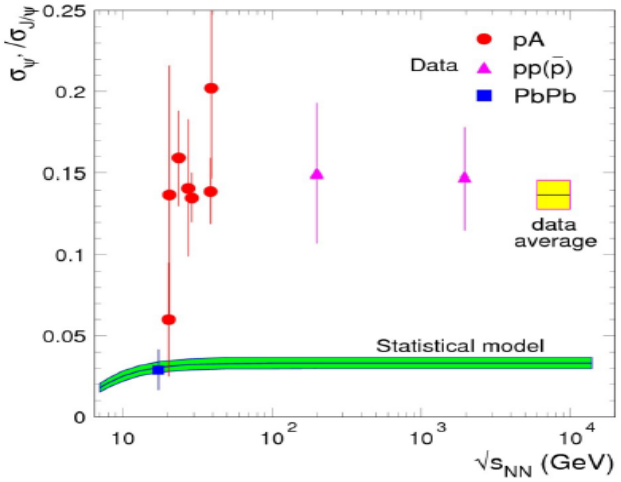
SHM of charmonia

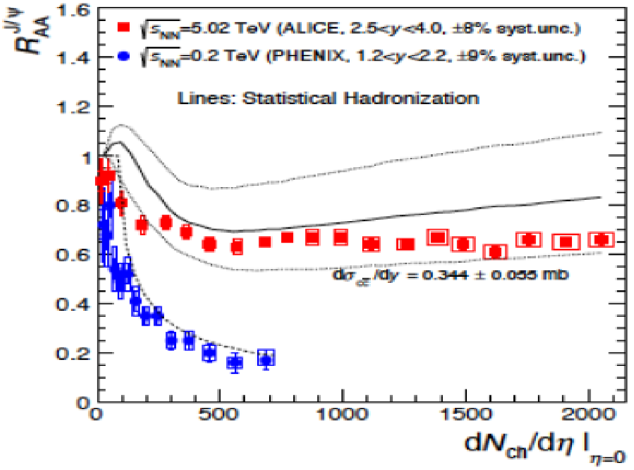
- Typical charm fugacity at LHC energies



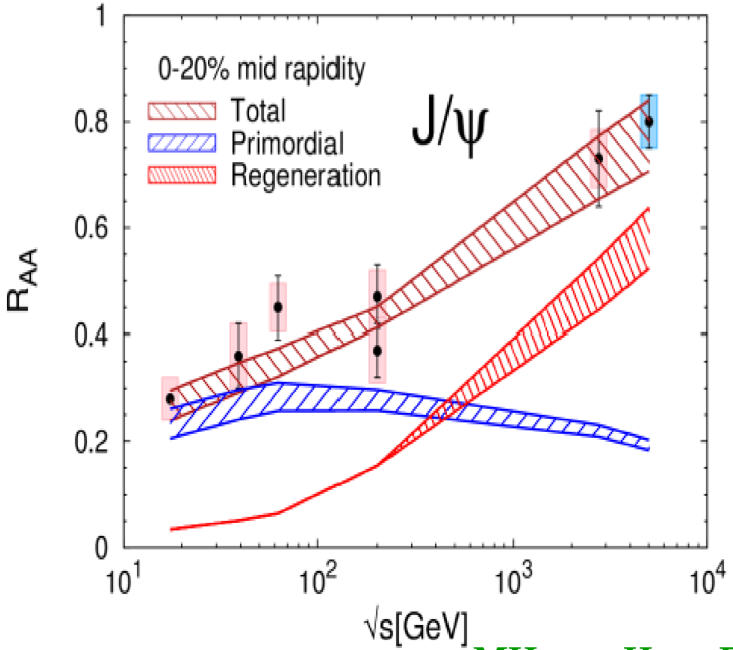


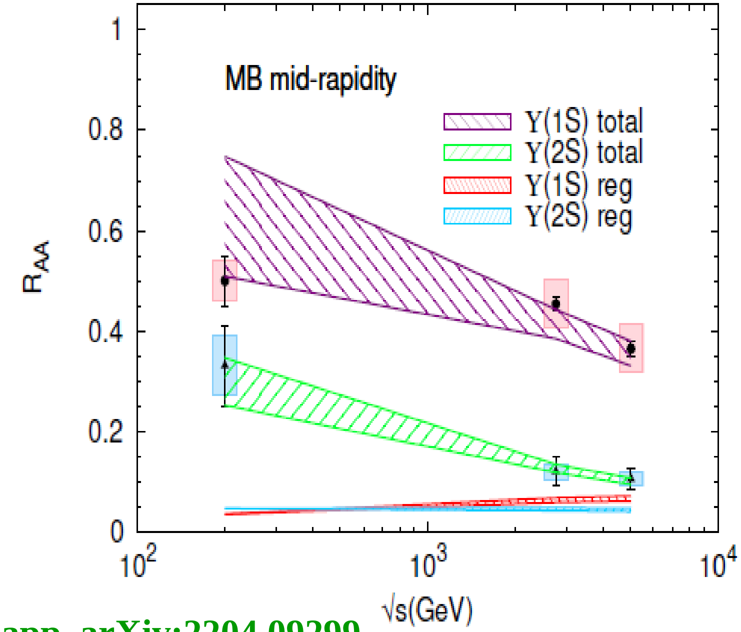
- SHM charmonia: prediction of ψ(2S)/J/ψ~0.05, a factor 3 smaller than pp integrated R_{AA} sensitive to input charm cross section





Quarkonium excitation function: R_{AA} vs sqrt(s)

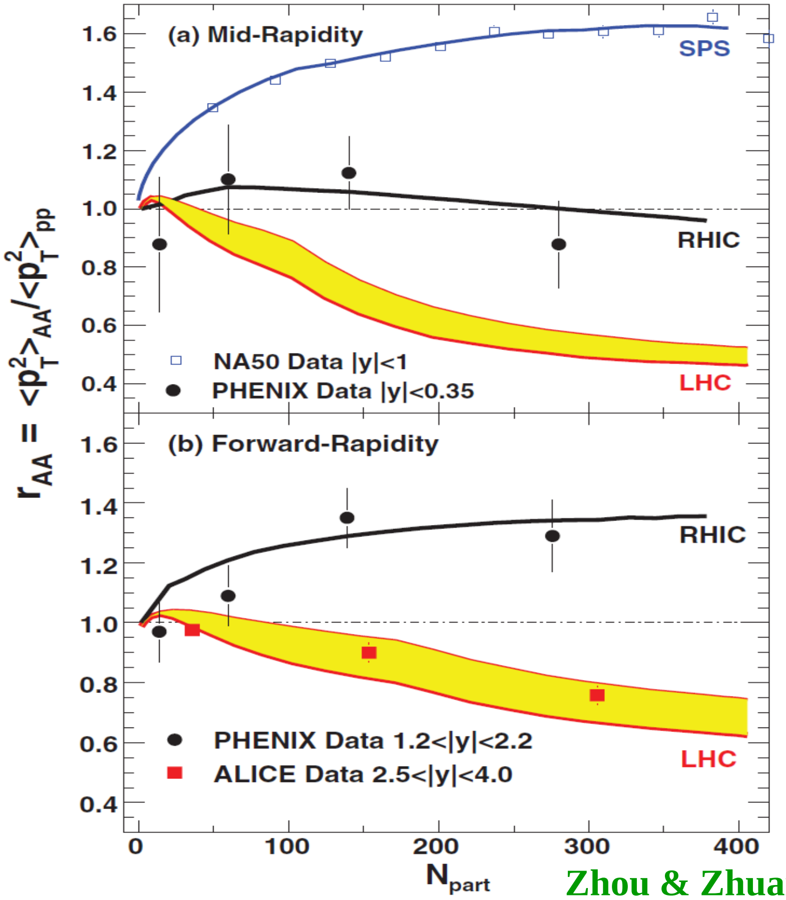


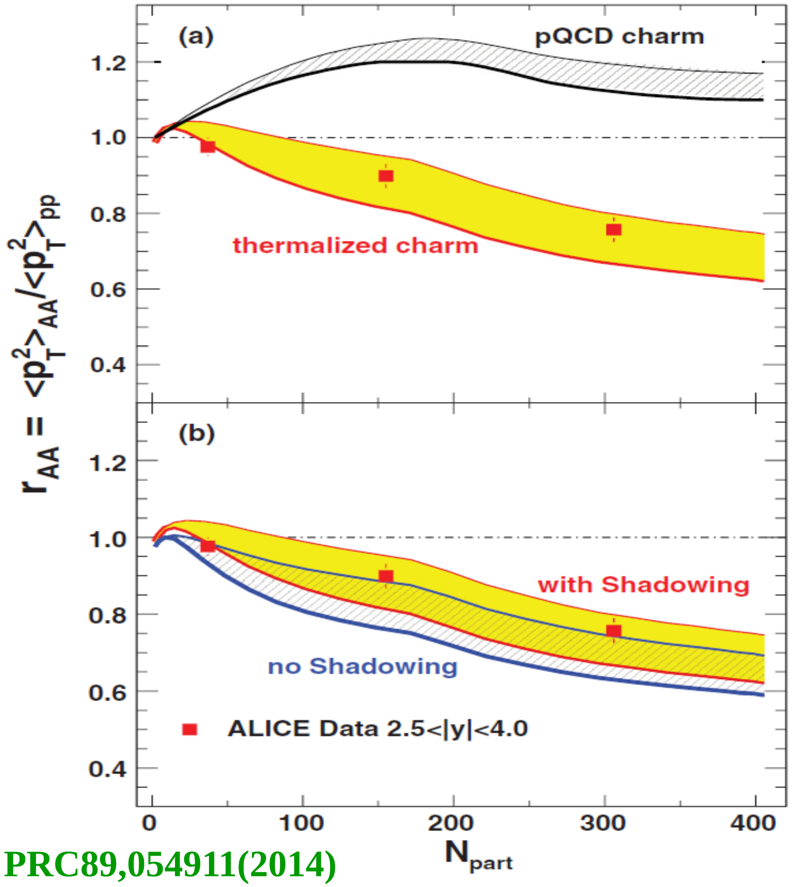


MH, van Hees, Rapp, arXiv:2204.09299

- J/ψ: at SPS, T<=230 MeV (from dilepton data), J/ψ suppression due to hot dissociation of excited states (χ_c & ψ(2S)) + nuclear absorption (CNM) [direct J/ψ not affected]
- J/ψ: at RHIC & LHC: higher T & hot suppression stronger, but regeneration from abundant near-thermalized c-cbar becomes efficient ➔ R_{AA} becomes larger toward LHC energies
- Υ(2S) similar vacuum binding energy as J/ψ, but very different R_{AA} , because b-bbar small number and less thermalization ➔ nearly no regeneration ➔ Υ(nS) sequential suppression and stronger suppression toward LHC energies (higher T)

J/ψ r_{AA} to characterize regeneration

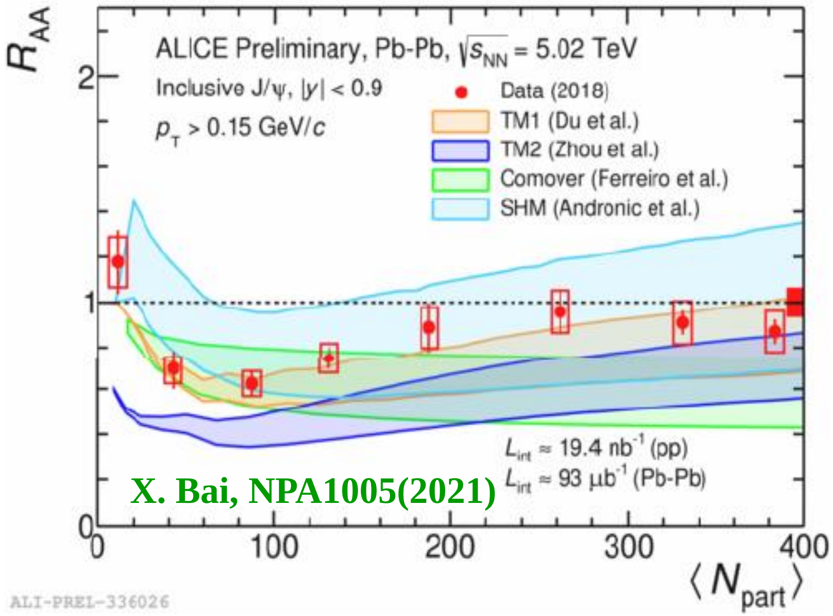




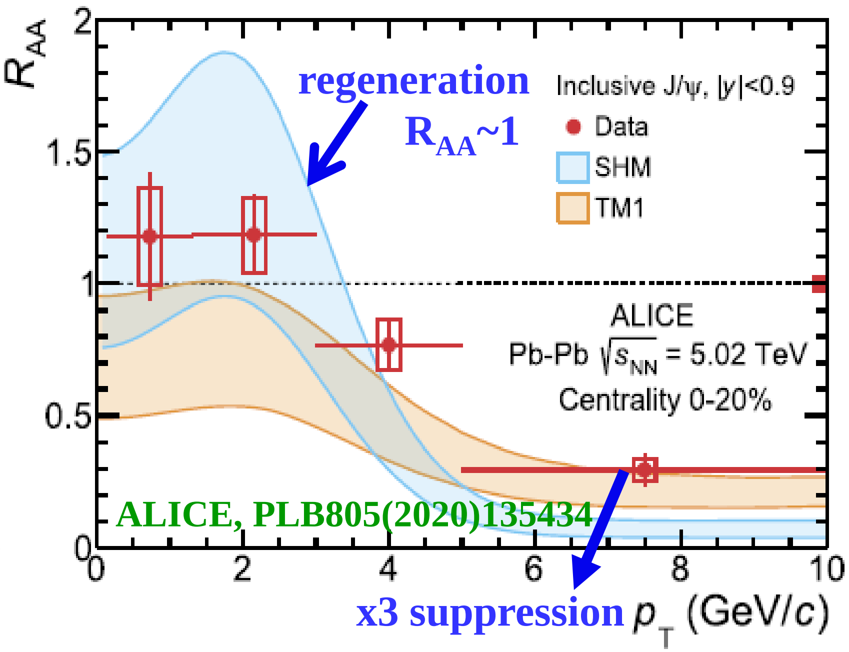
Zhou & Zhuang, PRC89,054911(2014)

- J/ψ r_{AA} =1.5 (SPS) → 1 (RHIC) → 0.5 (LHC): transition from primordial production at SPS to regeneration production from a nearly thermal source at LHC
- Stronger thermalization (lower mean p_T) of c-cbar enhances regeneration of J/ψ

J/ψ: suppression vs regeneration



X. Bai, NPA1005(2021)



regeneration R_{AA}~1

x3 suppression

ALICE, PLB805(2020)135434

- semi-classical transport: regeneration component p_T-spectrum modeled with a blast wave → falling off two fast

Du et al. ’15, Zhou et al. ’14, Ferreiro et al. ’14

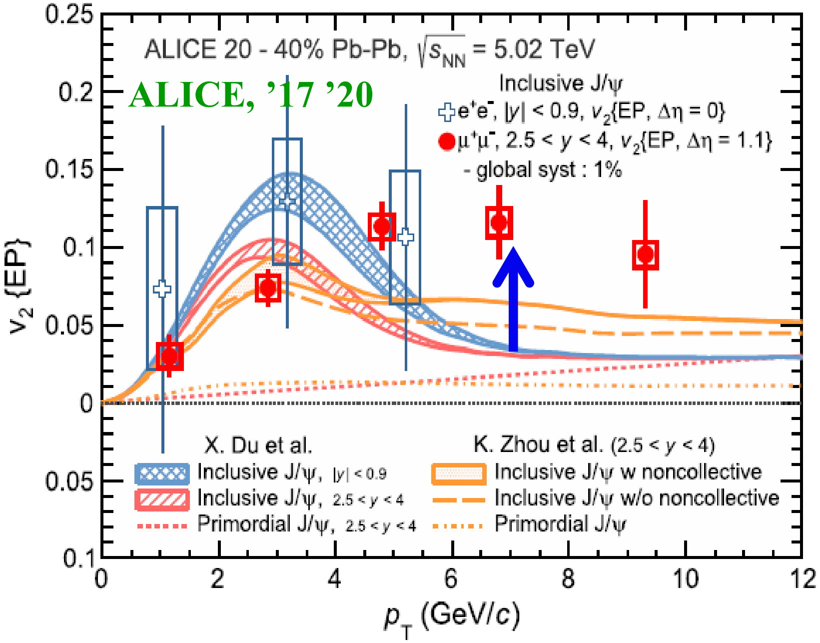
$$\frac{dN_\Psi^{\rm reg}}{p_t\,dp_t} \propto m_t \int_0^R r\,dr\,K_1\left(\frac{m_t\cosh y_t}{T}\right) I_0\left(\frac{p_t\sinh y_t}{T}\right)$$

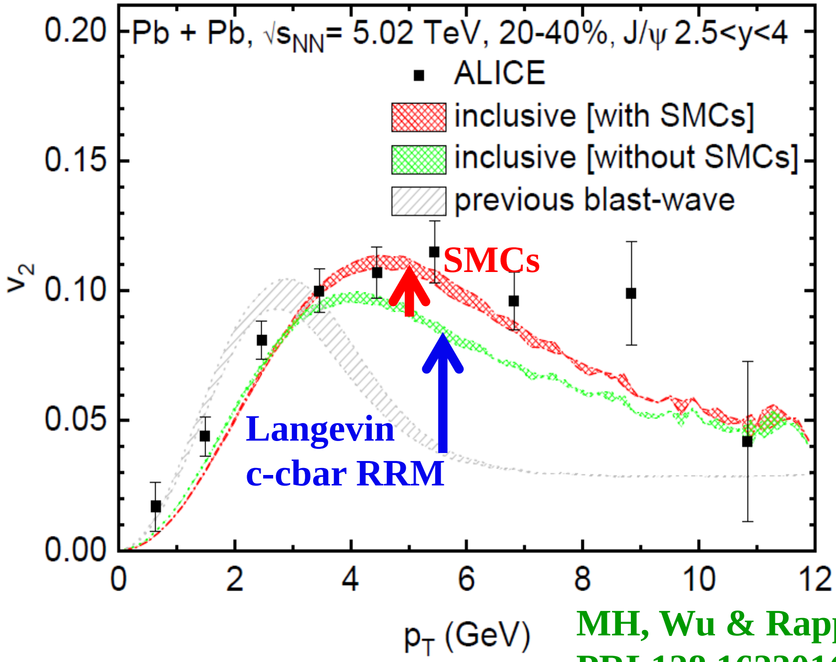
- SHMc: hydrodynamic blastwave spectrum + pp corona

Andronic et al. ’19

$$\frac{\mathrm{d}N(h^j_{hc})}{\mathrm{d}y} = g_c^2\,V\,n_j^{\rm th} \quad \propto g_c^{\,2} \leftarrow d\sigma^{ccbar}/dy$$

J/ψ "v_2 puzzle"





MH, Wu & Rapp, PRL128,162301(2022)

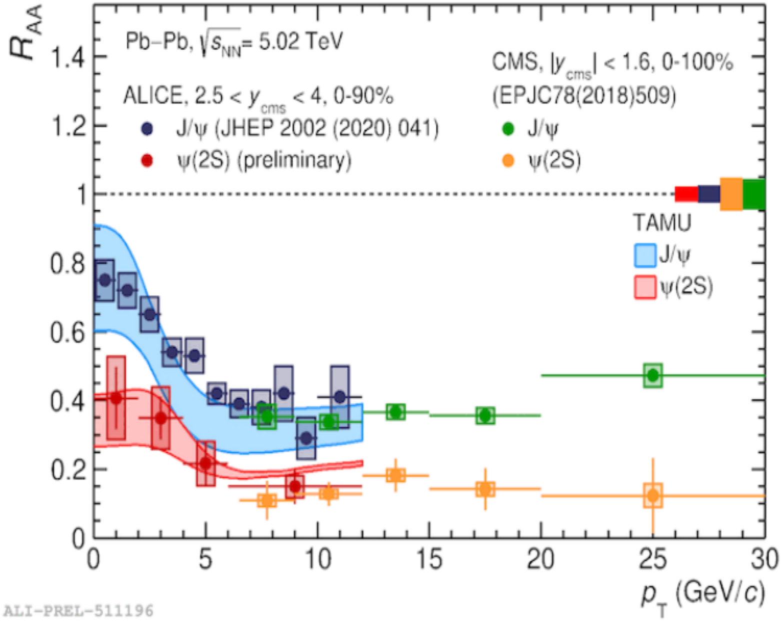
- regeneration via RRM

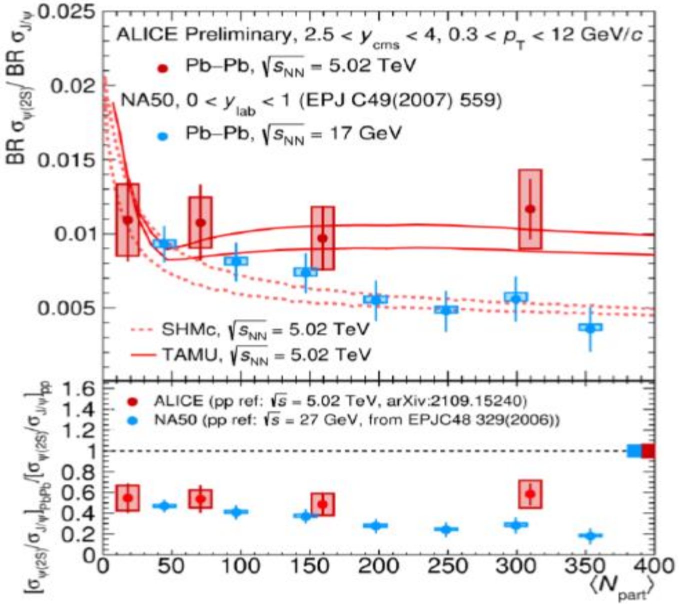
$$f_\Psi(\vec{x},\vec{p}) = C_\Psi \frac{E_\Psi(\vec{p})}{m_\Psi \Gamma_\Psi} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} \underbrace{f_c(\vec{x},\vec{p}_1) f_{\bar{c}}(\vec{x},\vec{p}_2)}_{\text{transported } c \,\&\, \bar{c} \text{ quark spectra constrained by D-meson observables}} \times \sigma_\Psi(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2) \delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

- off-equilibrium $c/\bar{c}$ spectra + space-momentum correlations (SMCs)
  ➔ regeneration up to p_T~8 GeV ➔ v_2 enhanced

quantitative connections open- ↔ hidden-charm transport

Latest SQM22: J/ψ vs ψ(2S)

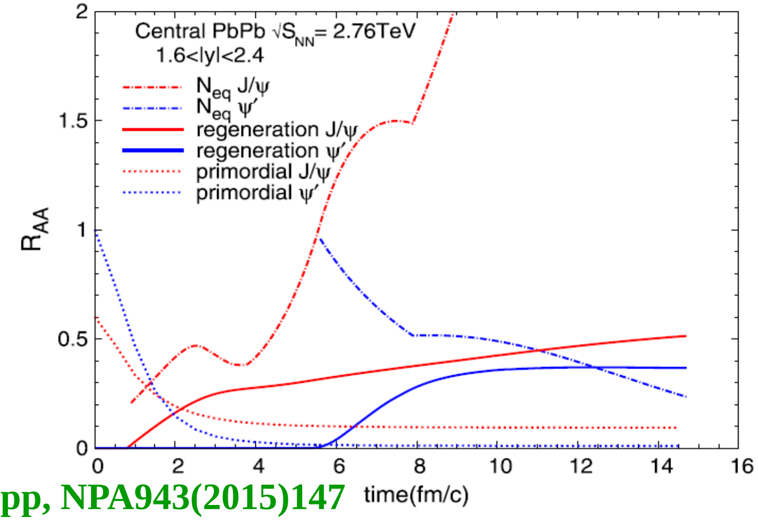




- Sequential regeneration: $T_d(J/\psi) > T_d(\psi(2S))$ ➔ regeneration of ψ(2S) is much later (at T~150-160 MeV)

- $R_{AA}(J/\psi) > R_{AA}(\psi(2S))$: production of both of them at low p_T are dominated by regeneration

- ALICE data of ψ(2S)/J/ψ favors transport calculation; SHM is disfavored



Du&Rapp, NPA943(2015)147

Open quantum system approach to bottomonia

- Regeneration of bottomonia is insignificant ➔ more direct window on suppression mechanisms
  semi-classical transport: Γ(T) vs E_B(T) ➔ a potential much stronger than F is needed

Du,MH&Rapp, PRC96, 054901 (2017)
Strickland et al., PRD 97, 016017 (2018)

- semi-classicsl transport: well-defined eigenstates (bound states) during in-medium evolution
  eigenstates being dissociated, but no quantum transition between them

- Evolution of a single b-bbar pair wave-packet: time-dependent Schrodinger equation with complex in-medium Q-Qbar potential $V(r) = V_R(r) + iV_I(r)$

$$V_R(r) = \Re[V(r)] = \begin{cases} V_{\rm KMS}(r) & \text{if } V_{\rm KMS}(r) \le V_{\rm vac}(r_{\rm SB}) \\ V_{\rm vac}(r_{\rm SB}) & \text{if } V_{\rm KMS}(r) > V_{\rm vac}(r_{\rm SB}) \end{cases}$$

$$V_{\rm KMS}(r) = U = -\frac{a}{r}(1+m_D r)e^{-m_D r} + \frac{2\sigma}{m_D}\left[1-e^{-m_D r}\right] - \sigma r e^{-m_D r}$$

Strickland et al., JHEP03(2021)235

$$V_I(r) = \Im[V(r)] = -C_F \alpha_s T \phi(m_D r)$$

Purely radial potential ➔ angular momentum is conserved $u(r,\theta,\phi,t) = r\psi(r,\theta,\phi,t)$

$$u(r,\theta,\phi,t) = \sum_{\ell m} u_{\ell m}(r,t) Y_{\ell m}(\theta,\phi) \qquad \hat{H}_\ell = \frac{\hat{p}^2}{2m} + V_{{\rm eff},\ell}(r,t)$$

- Independent time evolution of different l states: $u_\ell(r,t+\Delta t) = \exp(-i\hat{H}_\ell \Delta t) u_\ell(r,t)$

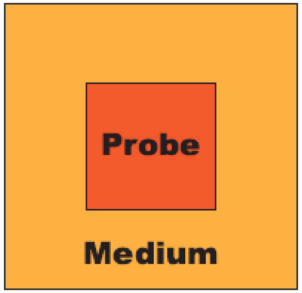
  with initial wave-function $u_\ell(r,\tau=0) \propto r^{\ell+1}\exp(-r^2/\Delta^2)$ ➔ finally projecting onto vacuum eigenstates

- The total norm of a single b-bbar pair is conserved, but are being redistributed into different bound eigenstates and unbound (dissociated) state due to ImV [if there's only ReV, expansion coefficient $c_n(t)=c_n(0)\exp(-iE_n t)$, probabilities not changing]

OQS: reduced density matrix

- b-bbar as an open quantum system interacting with the medium described by a reduced density matrix:

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\text{tot}} = H_{\text{probe}} \otimes I_{\text{medium}} + I_{\text{probe}} \otimes H_{\text{medium}} + H_{\text{int}}$$

- Total density matrix

$$\rho_{\text{tot}} = \sum_j p_j |\psi_j\rangle\langle\psi_j| \longrightarrow \frac{d}{dt}\rho_{\text{tot}} = -i[H_{\text{tot}}, \rho_{\text{tot}}]$$

- Reduced density matrix Brambilla et al. 2018, 2019, 2021; Yao 2021; Akamatsu, 2021

$$\rho_{\text{probe}} = \text{Tr}_{\text{medium}}[\rho_{\text{tot}}] \longrightarrow$$ **Evolution equation?**

OQS: pNRQCD + Lindblad equation

- Non-relativistic bottomonium + medium scale hierarchy: marginally satisfied only by Coulombic Υ(1S)

$$M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E.$$

Brambilla et al. 2017, 2018, 2019, 2021

environment correlation time τ_E. ~1/πT,

system/bottomonium intrinsic time τ_S ~1/E, ➔

system relaxation time 1/Γ: $\tau_R \sim \frac{1}{\Sigma_s} \sim \frac{1}{a_0^2(\pi T)^3}$

$\tau_R \gg \tau_E$: Markovian approximation, Insensitive to prior evolution

$\tau_S \gg \tau_E$. bottomonium quantum Brownian motion

- Non-relativistic bottomonium: pNRQCD, singlet & octet as d.o.f. + dipole coupling to medium gluons

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + \mathrm{Tr}\left\{S^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)S + O^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_o\right)O\right\}$$

$$+V_A \mathrm{Tr}\left\{O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, S + S^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O\right\} \rightarrow$$

$O^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{S}$

$$+\frac{V_B}{2}\mathrm{Tr}\left\{O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O + O^\dagger O\, \mathbf{r}\cdot g\mathbf{E}\right\} \rightarrow$$

$O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, \mathrm{O}\}$

Singlet and octet potentials

$$V_s(r) = -C_F\frac{\alpha_s}{r}$$

$$V_o(r) = \frac{\alpha_s}{2N_c r}$$

- Lindblad equation for the reduced density matrix (diagonal in color singlet & octet space)

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i\left(C_i\rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

OQS: pNRQCD + Lindblad equation

- Lindblad equation for the reduced density matrix (diagonal in color singlet & octet space)

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Transport coefficient: heavy quark mom. diffusion coefficient & quarkonium mass shift ➔ lattice QCD

$$\kappa = \frac{g^2}{18} \mathrm{Re} \int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0}) \rangle$$

$$\gamma = \frac{g^2}{18} \mathrm{Im} \int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0}) \rangle$$

- Reorganized into

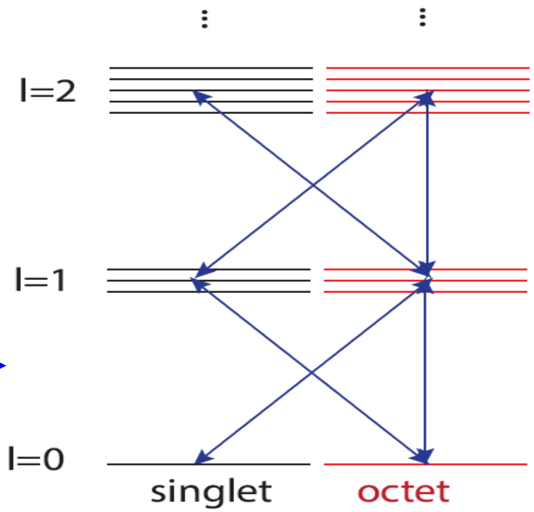
$$\frac{d\rho_{\mathrm{probe}}}{dt} = -iH_{\mathrm{eff}}\rho_{\mathrm{probe}} + i\rho_{\mathrm{probe}}H_{\mathrm{eff}}^{\dagger} + \sum_n C_n\, \rho_{\mathrm{probe}}\, C_n^{\dagger}$$

with non-Hermitian effective Hamiltonian with total width Γ~ ImV
➔ no mixing between different color & L

$$H_{\mathrm{eff}} = H_{\mathrm{probe}} - \frac{i}{2}\Gamma \qquad \Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

& 6 collapse operators ➔ mixing between different L by color-dipole singlet-octet transition

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \qquad C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



Open quantum system approach to Υ states

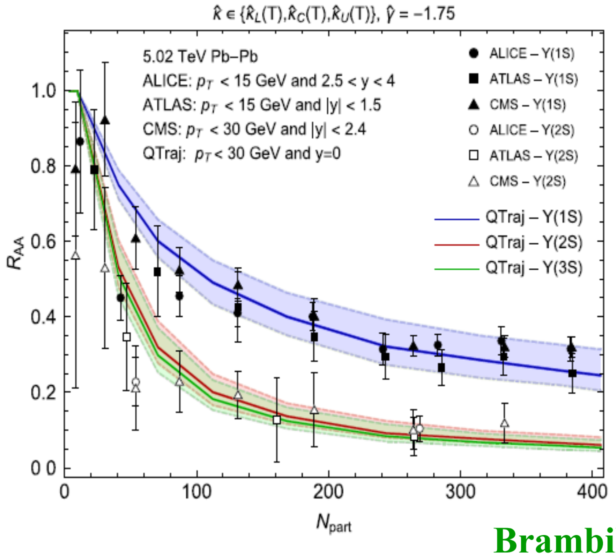
- OQS + pNRQCD ➔ Lindblad equation

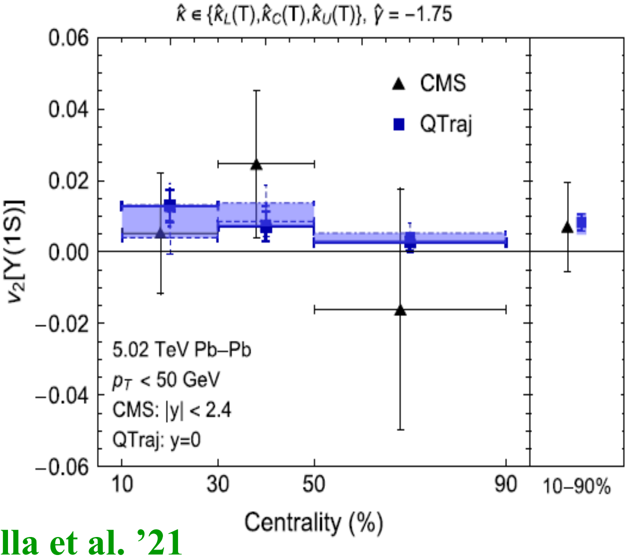
Brambilla et al. ’17-21, Yao et al., ’21, Blaizot ’18
Akamatsu ’21, Rothkopf ’20, Gossiaux et al. ’21

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho(t)\} \right)$$

$$M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E.$$

- quantum transition between different states included, lacking in semi-classical
- regeneration currently limited to diagonal $b\bar{b}$
- Coulomb potential + transport coefficient κ encoded in C_n





Brambilla et al. ’21

- Υ(1S) in-medium width $\Gamma_{\Upsilon(1S)} = 3a_0^2\kappa \sim$ 50 MeV at T~250 MeV
- values & results comparable to semi-classical approach **Strickland et al. ’15**

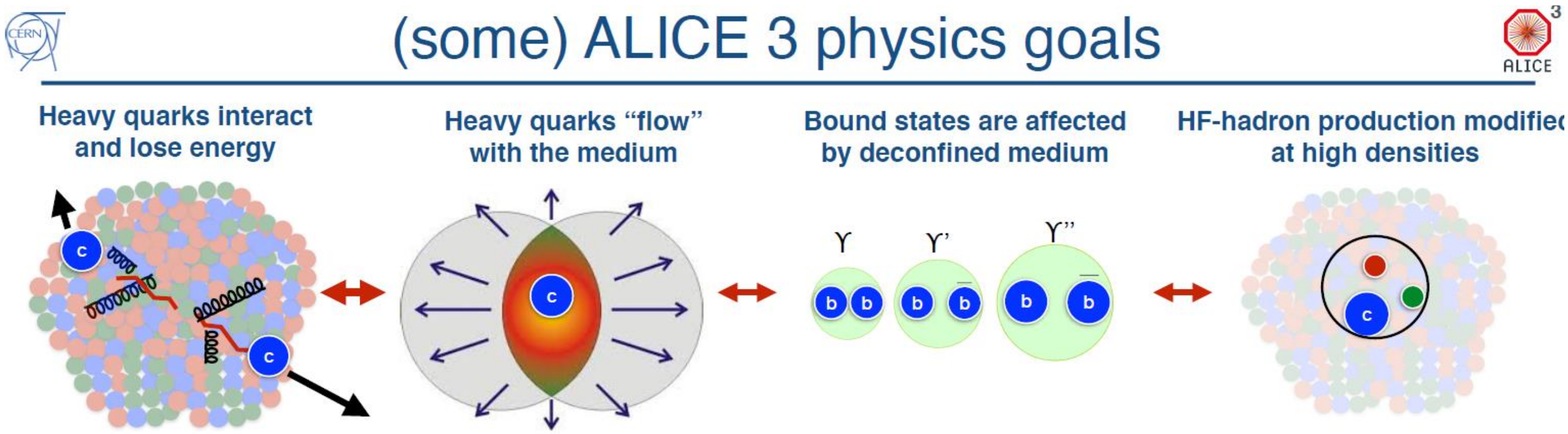
Summary & outlook

- HFs: excellent probes of sQGP structure, transport properties, in-medium force & hadronization
  - a small open HF diffusion coefficient $\mathcal{D}_s$
  - recombination/color neutralization important
  - quarkonia melting by large reaction rates

  ⟷ strong V(r,T)

  ➔ connection between open- & hidden-HF, e.g. via J/ψ regeneration

- HFs: outlook into Run3 & ALICE3 **see ALICE Collab.: Letter of intent for ALICE 3**



(some) ALICE 3 physics goals

Heavy quarks interact and lose energy

Heavy quarks “flow” with the medium

Bound states are affected by deconfined medium

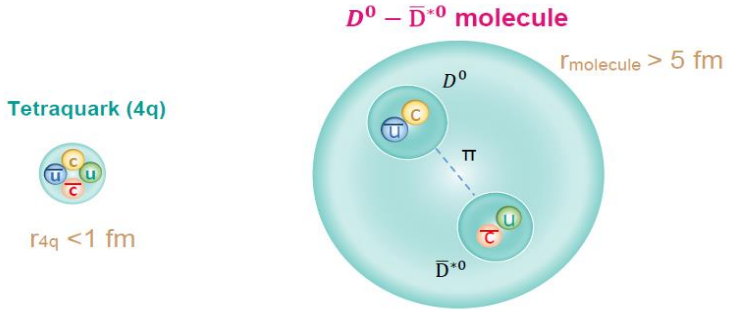
HF-hadron production modified at high densities

Back-up

The following are back-up pages

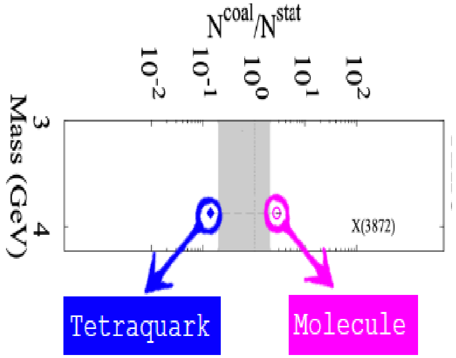
X(3872) production in HIC

❖ inner structure: compact tetraquark vs loosely bound molecule

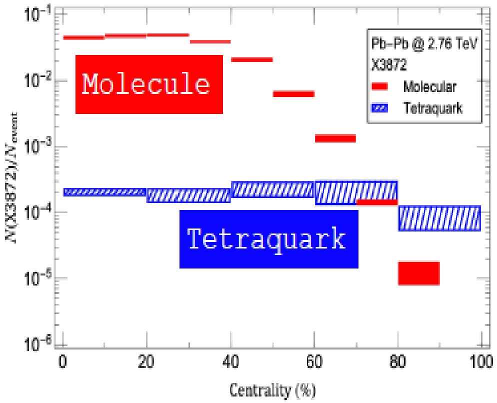


coalescence model

Cho et al. '11

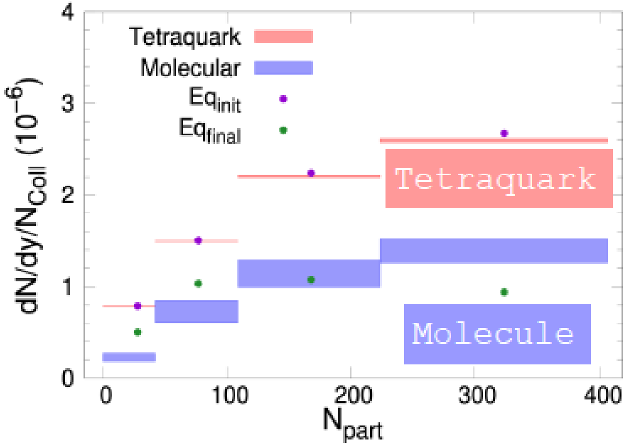


coalescence within AMPT **zhang et al. '21**



transport model

Wu et al. '21



- $N_{molecule} > N_{tetraquark}$ by x10 or 100, yet no account of hadron phase reactions πX <-->DD*
  ➔to be better constrained
- $N_{tetraquark} > N_{molecule}$ by x2, molecule regenerated in late hadronic phase, tetraquark chem. freezeout at T_c