Theoretical study of scalar meson $a_0(1710)$ in the $\eta_c \to \bar{K}^0 K^+ \pi^-$ reaction

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Background

- *BABAR* $\eta_c \to \eta\, \pi^+\pi^-$

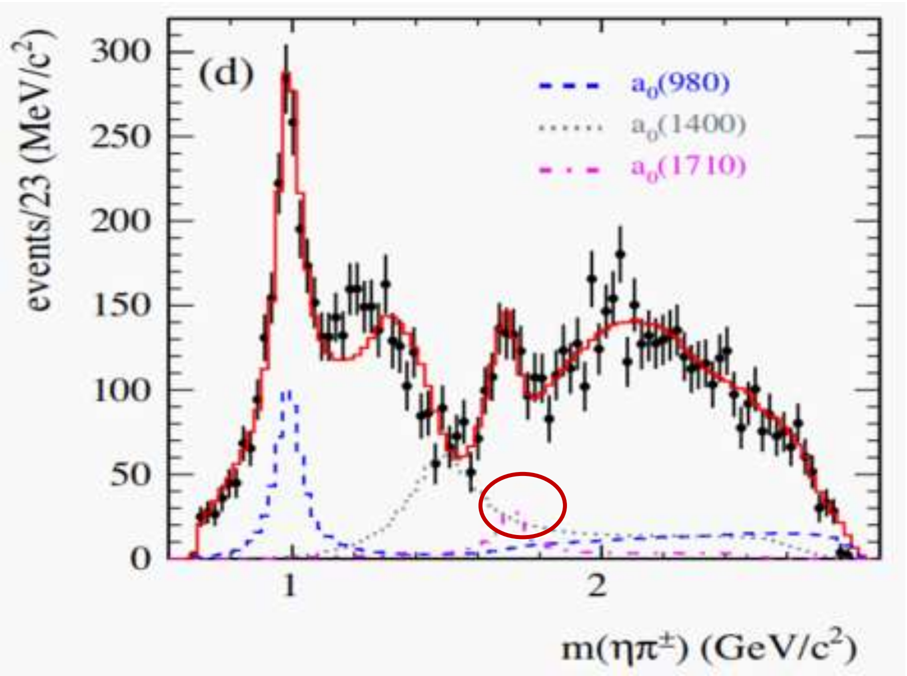
  M= (1704 ± 5 ± 2) MeV

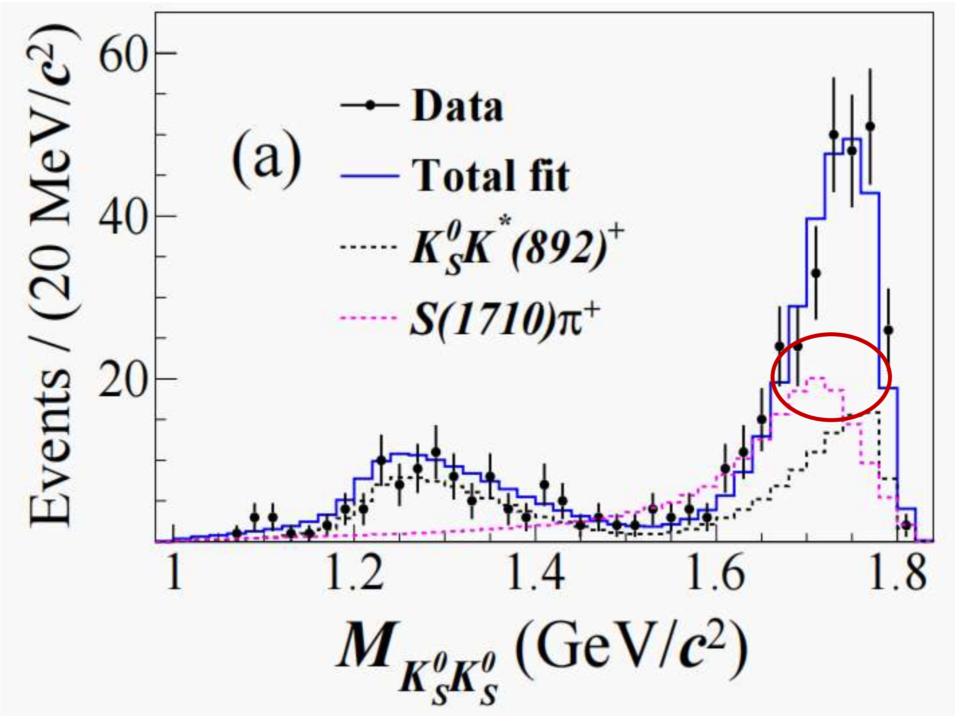
  Γ=(110 ± 15 ± 11) MeV

- BESIII $D_S^+ \to K_S^0\, K_S^0\, \pi^+$

  M= (1723 ± 11 ± 2) MeV

  Γ=(140 ± 14 ± 4) MeV





[1] BABAR Collaboration, PHYSICAL REVIEW D 104, 072002 (2021)

[2] BESIII Collaboration, PHYSICAL REVIEW D 105, L051103 (2022)

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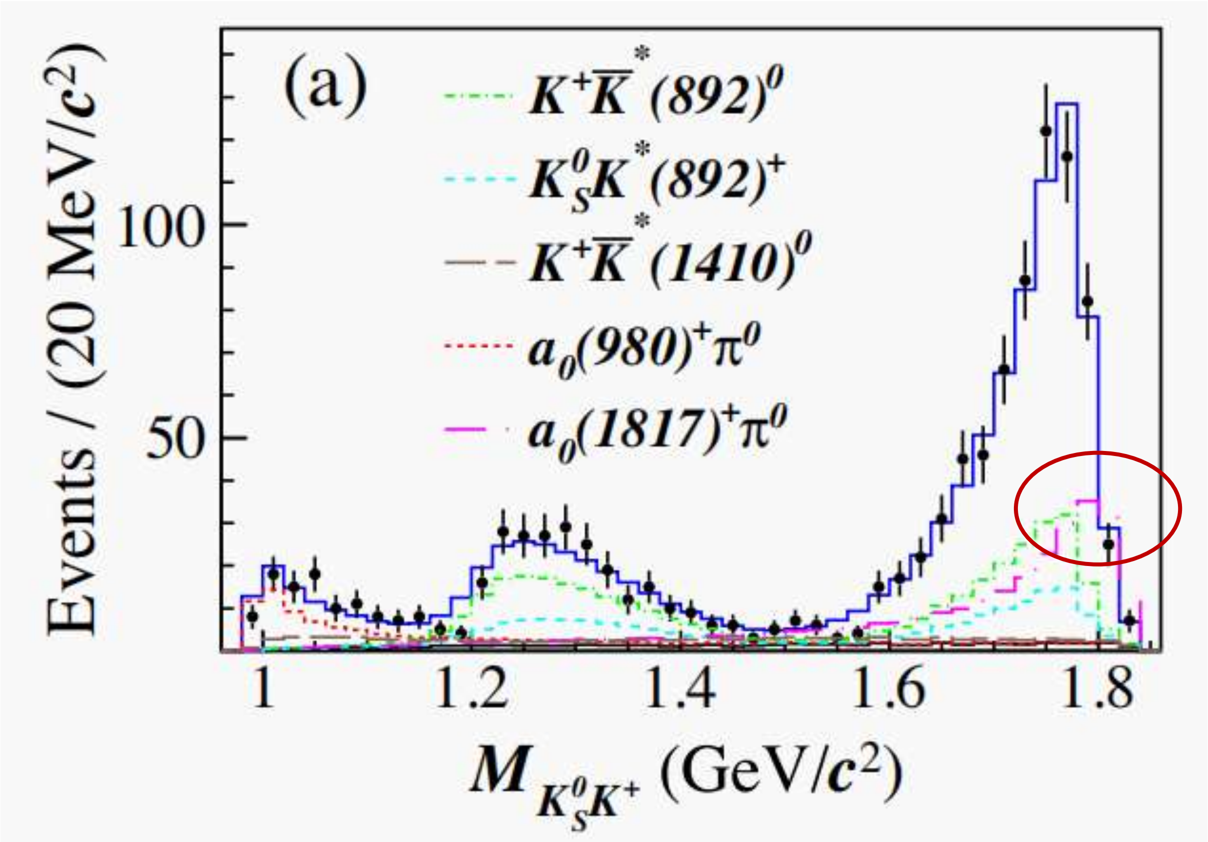
- **BESIII** $D_S^+ \to K_S^0 K^+ \pi^0$

$M=$ **(1817 ± 8 ± 20) MeV** Γ**=(197 ± 22 ± 15) MeV**



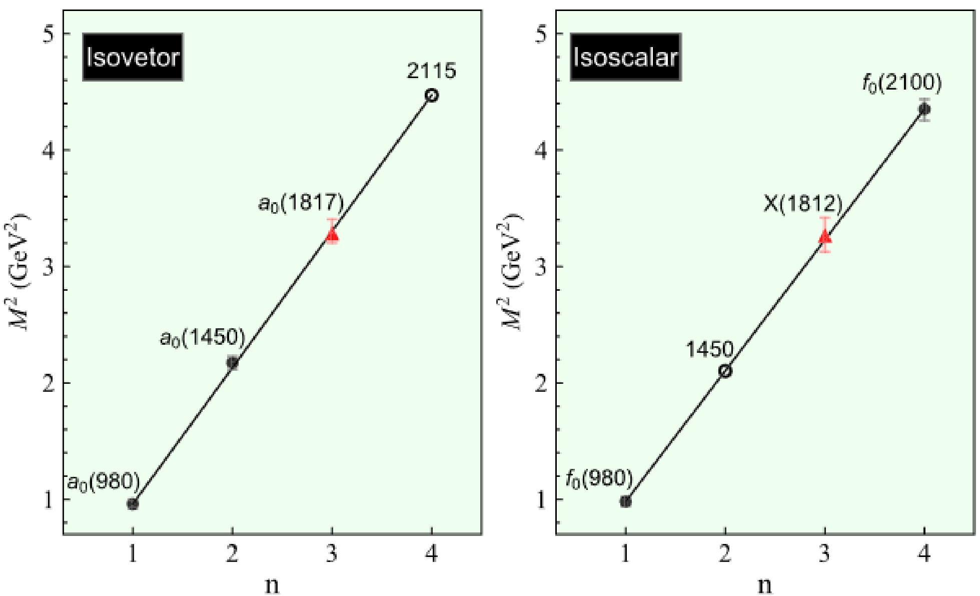
[3] BESIII Collaboration, PHYSICAL REVIEW LETTERS 129, 182001 (2022)

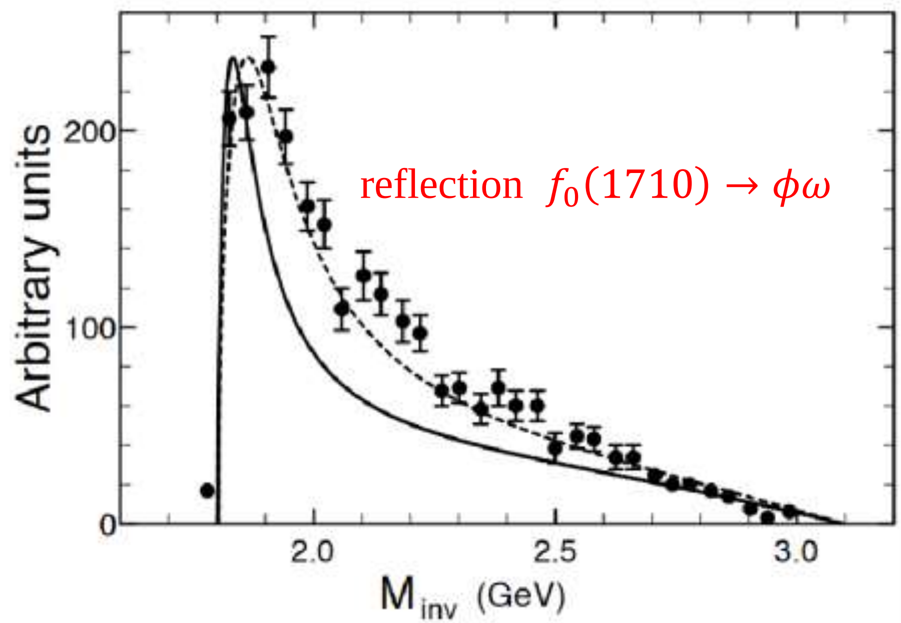
Background

- Geng Dai Oset ZhuXin $D_S^+ \to K_S^0 K_S^0 \pi^+$
- ZhuXin Xie $D_S^+ \to K_S^0 K^+ \pi^+$

$\Longrightarrow$ $K^* \overline{K}^*$ molecular state

- Liu Xiang $a_0(1817)$ X(1812) $\Longrightarrow$ 3^3P_0





[4] Geng Dai Oset , EPJC(2022)116010
[5] ZhuXin, PHYSICAL REVIEW D 105, 116010 (2022)
[6] ZhuXin, PHYSICAL REVIEW D 107, 034001 (2022)
[7] LiuXiang, PHYSICAL REVIEW D 105, 114014 (2022)
[8] A. Martinze, PLB719, 388 (2013)

Motivation

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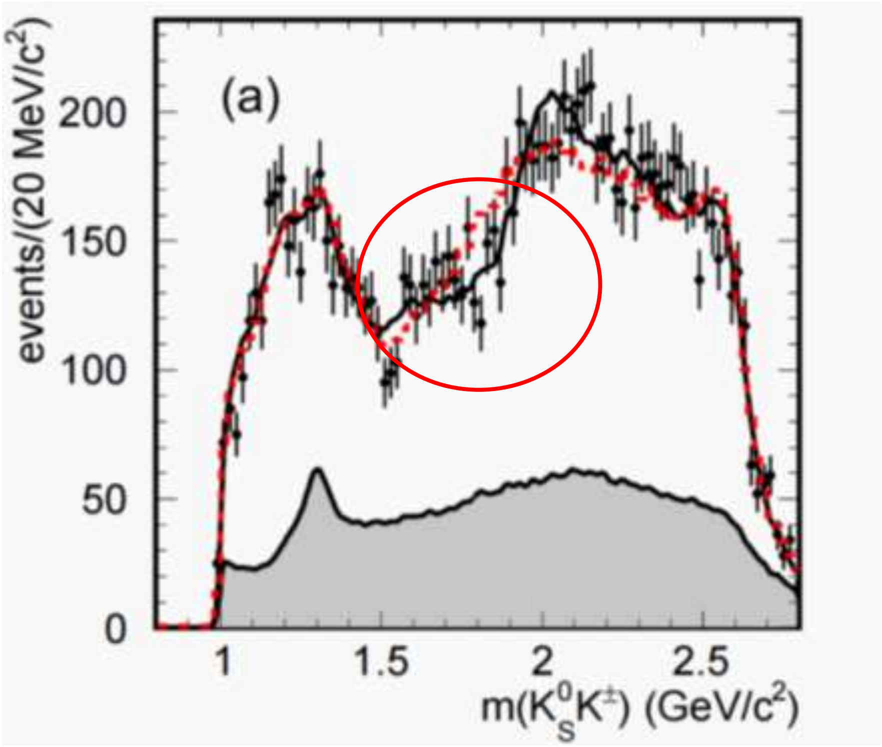
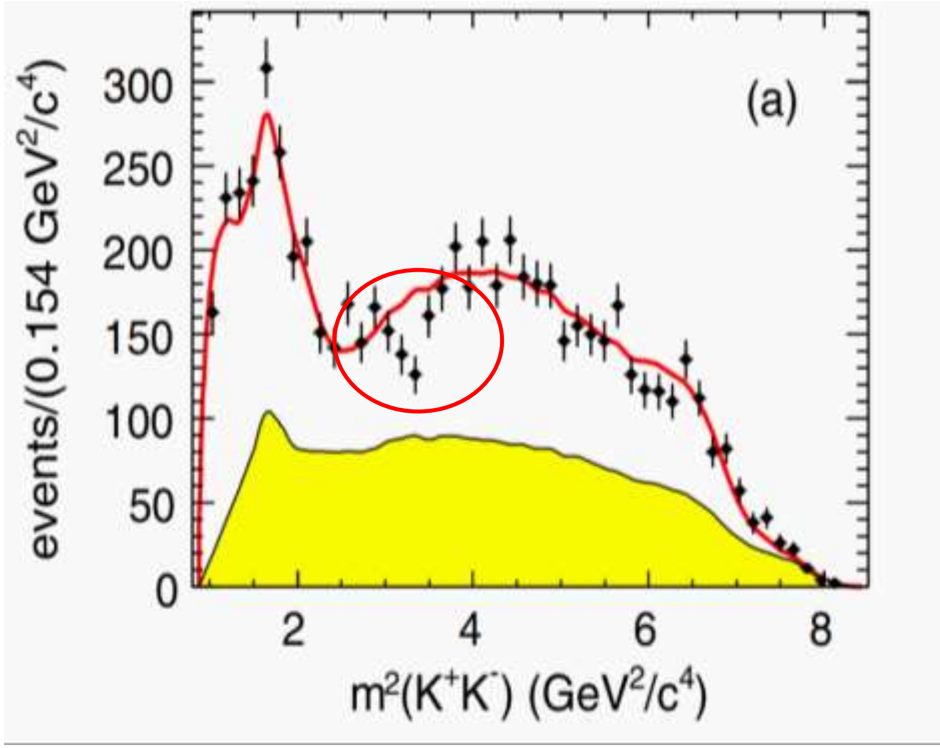
BABAR

$\gamma\gamma \rightarrow \eta_c \rightarrow K_S^0 K^{\pm}\pi^{\mp}$



BABAR

$\eta_c \rightarrow K^+K^-\eta/K^+K^-\pi^0$



[7] BABAR Collaboration, PHYSICAL REVIEW D 93(2016)012005

[8] BABAR Collaboration, PHYSICAL REVIEW D 89, 112004 (2014)

Mechanism of the $\eta_c \to \bar{K}^0 K^+ \pi^-$

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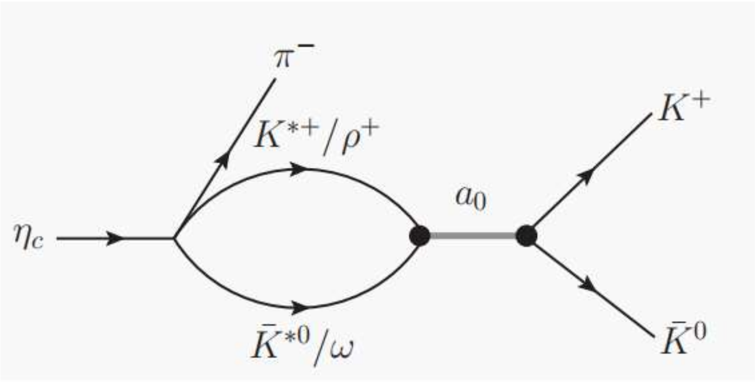
- $\eta_c \to \bar{K}^0 K^+ \pi^-$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}$$

$$\begin{aligned} &< V\,V\,P > \\ &= < V\,V >_{12} P_{21} \\ &= \pi^- \textstyle\sum_i V_{1i} V_{i2} \\ &= \pi^- \left[\rho^+ \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right) + \left(-\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right) \rho^+ + \bar{K}^{*0} K^{*+} \right] \\ &= \pi^- \left(\sqrt{2}\rho^+ \omega + \bar{K}^{*0} K^{*+}\right) \end{aligned}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & \bar{K}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

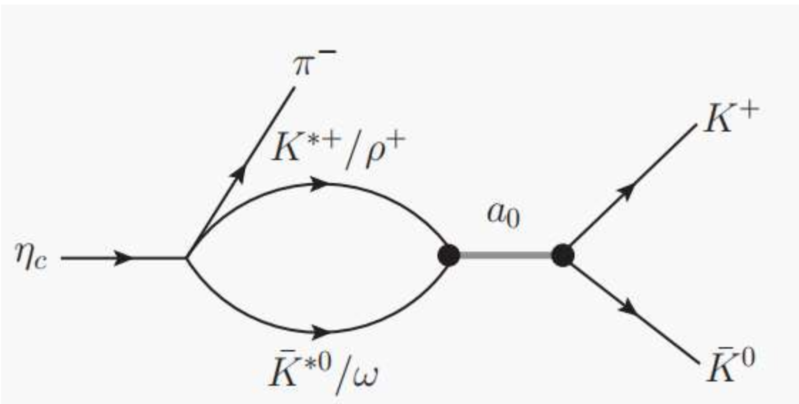


V P are the SU(3) vector and pseudoscalar matrices

[9] Eulogio Oset, PHYS. REV. D 100, 114011 (2019)

Mechanism of the $\eta_c \to \bar{K}^0 K^+ \pi^-$

➢ $\eta_c \to \bar{K}^0 K^+ \pi^-$



$$t^a = V_P \sum_i h_i \, G_i \, t_{i \to \bar{K}^0 K^+}$$

$$h_1 = \sqrt{2} \quad h_2 = 1$$

1 , 2 for $\rho^+\omega$ and $K^{*+}\ \bar{K}^{*0}$

$$t_{i \to \bar{K}^0 K^+} = \frac{g_{K^*\bar{K}^*/\rho\omega}\, g_{K\bar{K}}}{M_{\bar{K}^0 K^+} - m_{a_0}{}^2 + i M_{a_0} \Gamma_{a_0}}$$

G_i is the loop function taking into account the vector width,

$$G_{\omega\rho}(M_{\bar{K}^0 K^+}) = \int_{m_{1-}^2}^{m_{1+}^2} \int_{m_{2-}^2}^{m_{2+}^2} d\tilde{m}_1{}^2 d\tilde{m}_2{}^2 \times \omega(\tilde{m}_1{}^2) \times \omega(\tilde{m}_1{}^2) \tilde{G}(M_{\bar{K}^0 K^+}, \tilde{m}_1{}^2, \tilde{m}_2{}^2)$$

$$\omega(\tilde{m}_i{}^2) = \frac{1}{N} \mathrm{Im}\left(\frac{1}{\tilde{m}_i{}^2 - m_i^2 + i\Gamma(\tilde{m}_i{}^2)\tilde{m}_i{}^2}\right)$$

$$N = \int_{m_{i-}^2}^{m_{i+}^2} d\tilde{m}_i{}^2 \, \mathrm{Im}\left(\frac{1}{\tilde{m}_i{}^2 - m_i^2 + i\Gamma(\tilde{m}_i{}^2)\tilde{m}_i{}^2}\right)$$

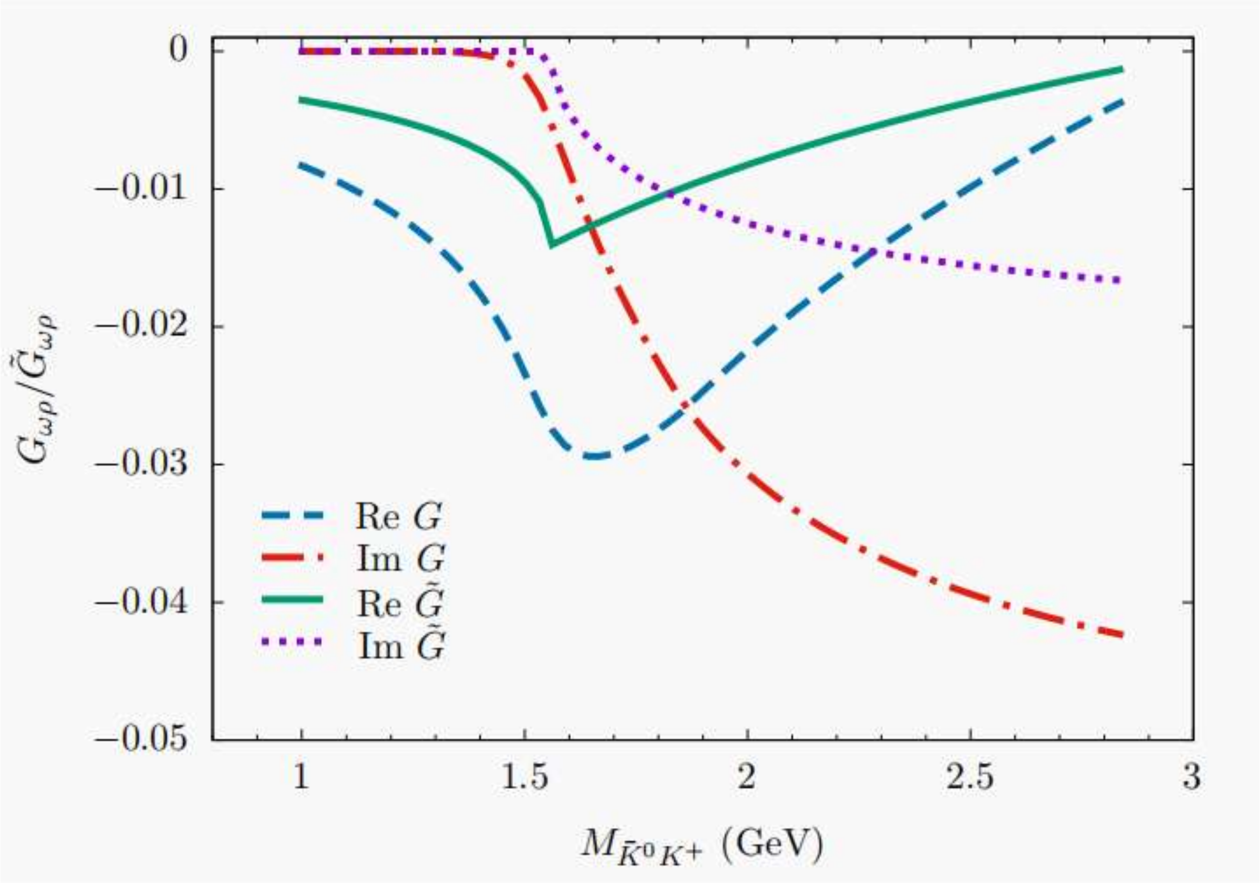
For $\omega\rho^+$: $m_2 = \rho^+$ $m_{p_1} = \pi^0$ $m_{p_2} = \pi^+$

For $\bar{K}^{*0}\ K^{*+}$: $m_1 = \bar{K}^{*0}$ $m_{p_1} = \pi$ $m_{p_2} = K$

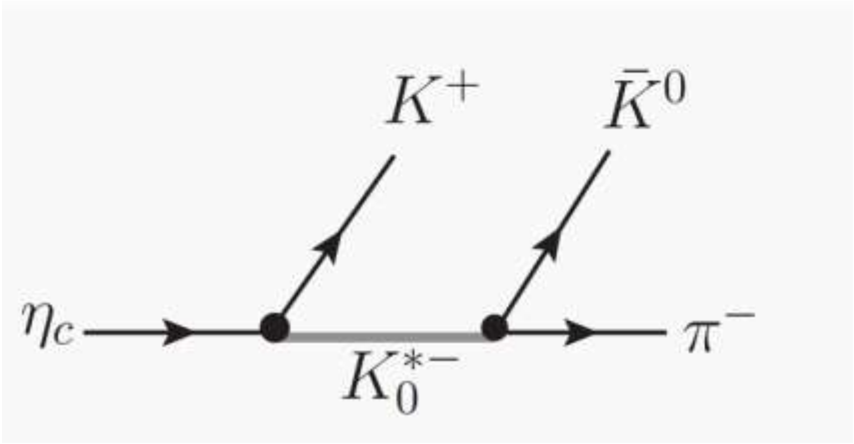
$m_2 = \bar{K}^{*+}$ $m_{p_1} = \pi$ $m_{p_2} = K$

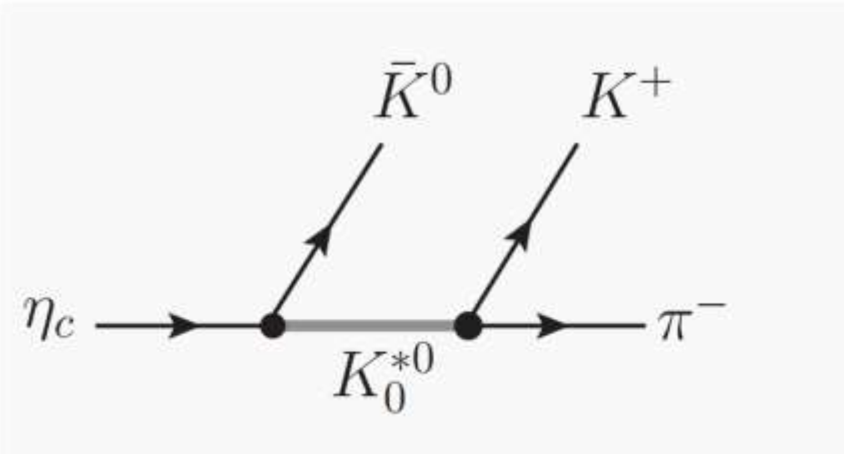
$$G_{\omega\rho}(M_{\bar{K}^0 K^+}) = \int_{m_{1-}^2}^{m_{1+}^2}\int_{m_{2-}^2}^{m_{2+}^2} d\tilde{m}_1^2 d\tilde{m}_2^2 \times \omega(\tilde{m}_1^2) \times \omega(\tilde{m}_1^2)\tilde{G}(M_{\bar{K}^0 K^+}, \tilde{m}_1^2, \tilde{m}_2^2)$$

For $\omega\rho^+$



- $\eta_c \to \bar{K}^0 K^+\pi^-$

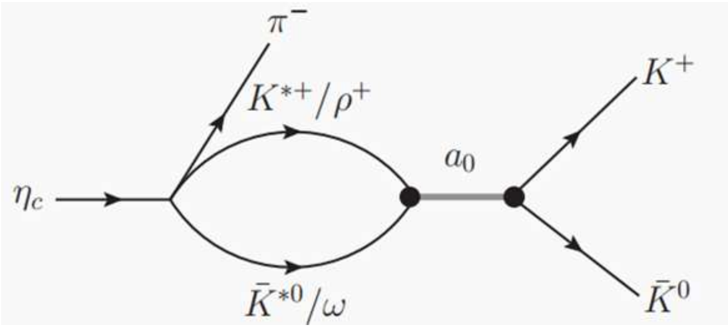




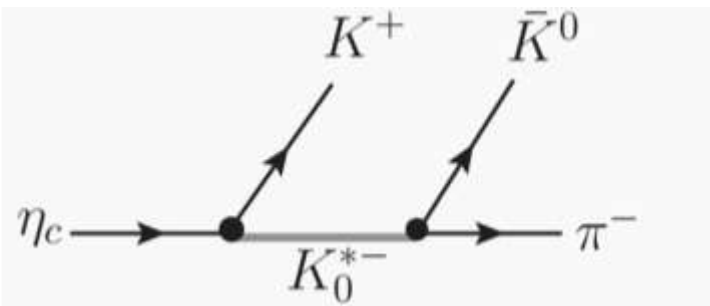
$$t^b = \frac{g_{\eta_c K^+ {K_0}^{*-}}\, g_{{K_0}^{*-} \bar{K}^0 \pi^-}}{{m_{\bar{K}^0\pi^-}}^2 - {m_{{K_0}^*}}^2 + iM_{{K_0}^*}\Gamma_{{K_0}^*}} \qquad t^c = \frac{g_{\eta_c \bar{K}^0 {K_0}^*}\, g_{{K_0}^{*0} K^+ \pi^-}}{{m_{K^+\pi^-}}^2 - {m_{{K_0}^*}}^2 + iM_{{K_0}^*}\Gamma_{{K_0}^*}}$$

$m_{{K_0}^*} = 1425\ \text{MeV} \quad \Gamma_{{K_0}^*} = 270\ \text{MeV}$

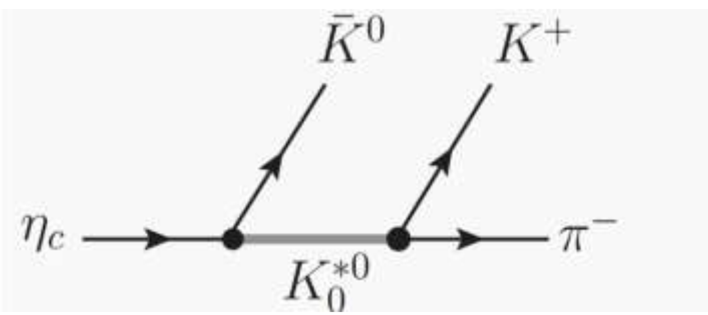
Mechanism of the $\eta_c \to \bar{K}^0\, K^+\pi^-$



$$t^a = V_P \sum_i h_i\, G_i\, t_{i\to \bar{K}^0 K^+}$$



$$t^b = \frac{g_{\eta_c K^+ K_0{}^{*-}}\, g_{K_0{}^{*-} K^0 \pi^-}}{m_{K^0\pi^-}{}^2 - m_{K_0{}^*}{}^2 + iM_{K_0{}^*}\Gamma_{K_0{}^*}}$$



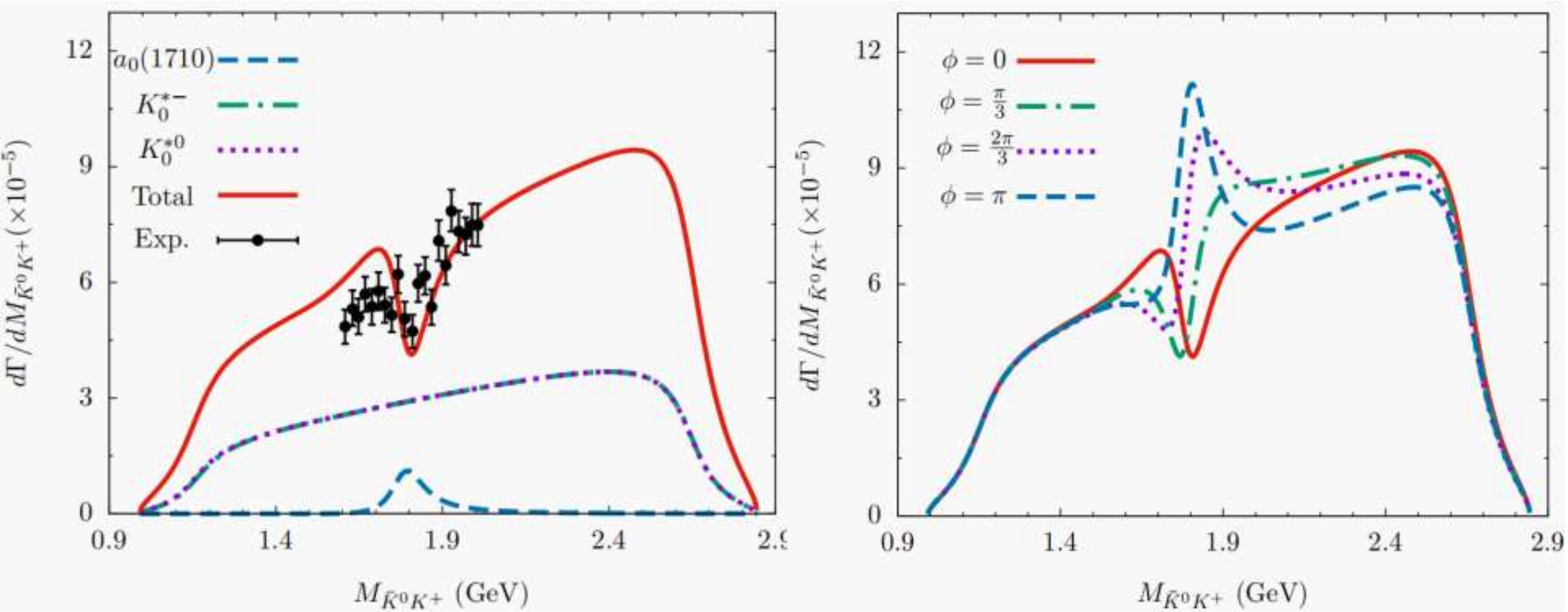
$$t^c = \frac{g_{\eta_c K^0 K_0{}^*}\, g_{K_0{}^{*0} K^+ \pi^-}}{m_{K^+\pi^-}{}^2 - m_{K_0{}^*}{}^2 + iM_{K_0{}^*}\Gamma_{K_0{}^*}}$$

$$\frac{d^2\Gamma}{dM_{K^+\bar{K}^0}\, dM_{\bar{K}^0\pi^-}} = \frac{1}{128\pi^3}\frac{M_{\bar{K}^0K^+} M_{\bar{K}^0\pi^-}}{M_{\eta_c}^2}\left|t^a + t^b + t^c\right|^2$$

Results

- $\bar{K}^0 K^+$

$$\frac{d^2\Gamma}{dM_{K^+\bar{K}^0}dM_{\bar{K}^0\pi^-}} = \frac{1}{128\pi^3}\frac{M_{\bar{K}^0K^+}M_{\bar{K}^0\pi^-}}{M_{\eta_c}^2}\left|t^a e^{i\phi} + t^b + t^c\right|^2$$



Results

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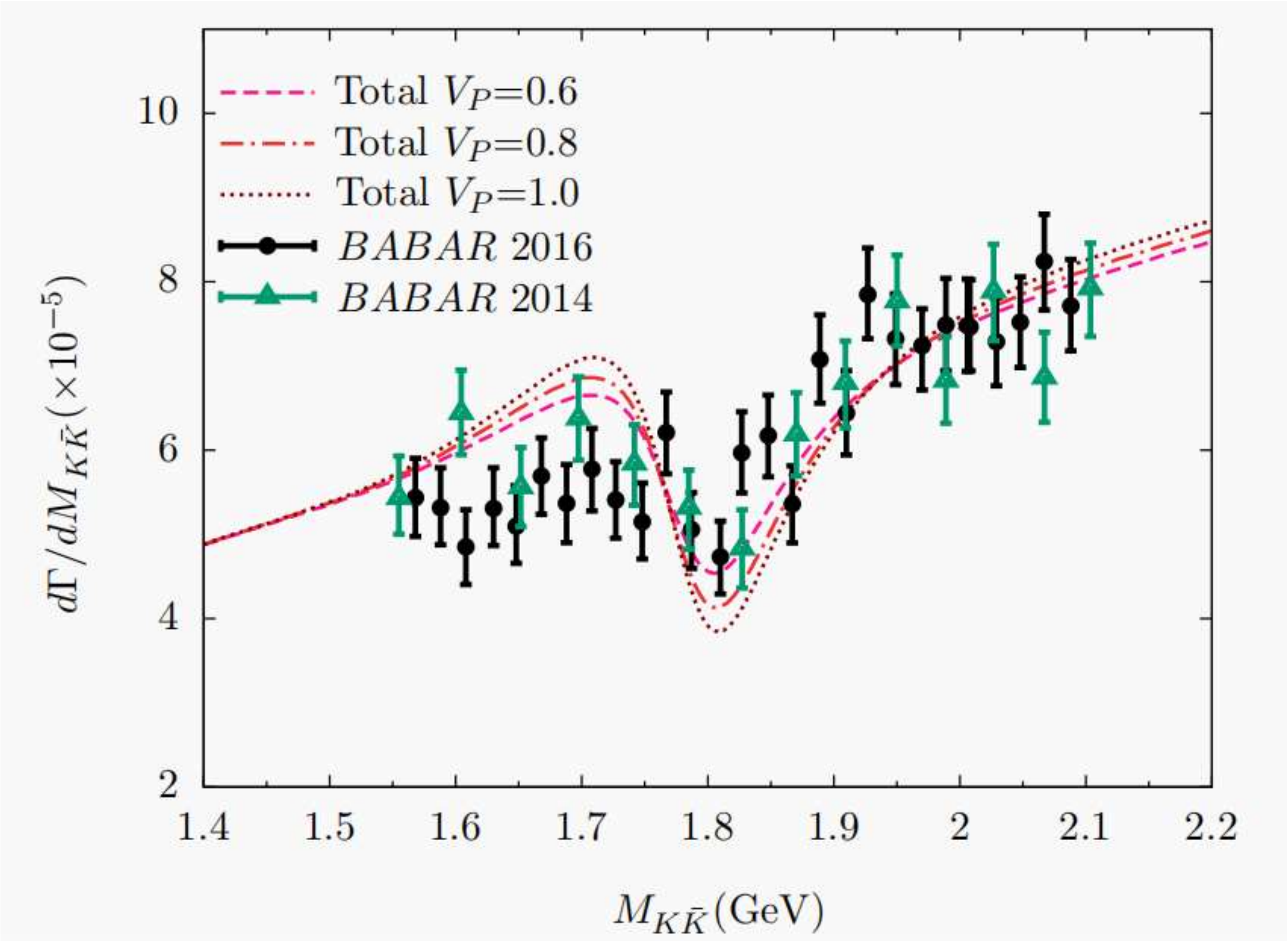
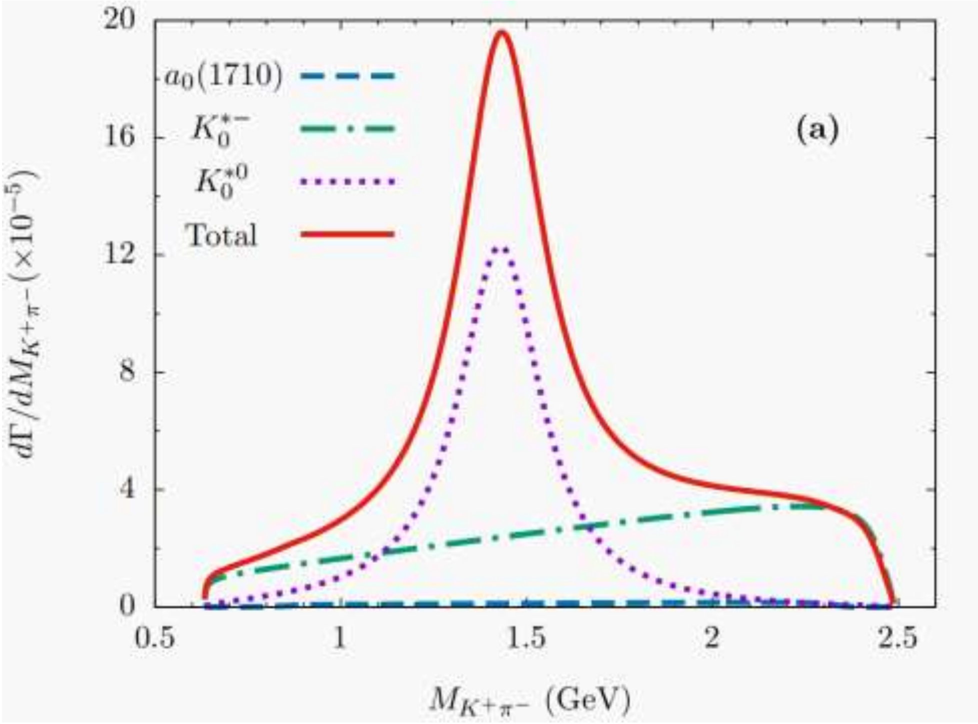
➢ *BABAR*-comparison

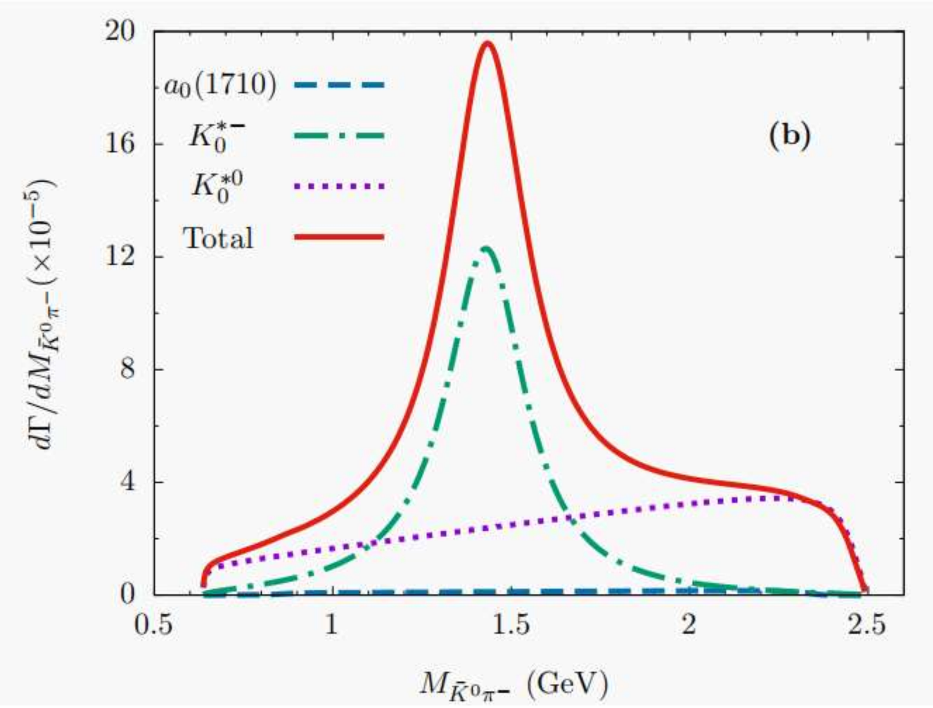


Results

➢ ***Kπ***



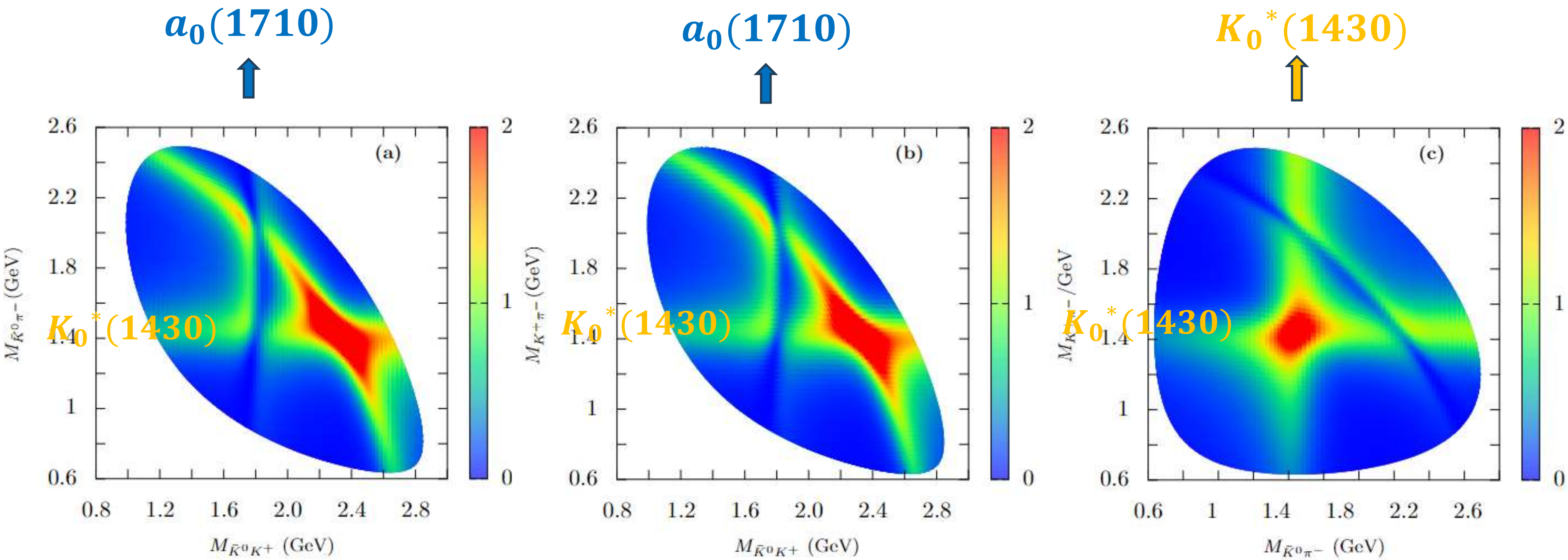
20
16
12
8
4
0
0.5
1
1.5
2
2.5
$a_0(1710)$
K_0^{*-}
K_0^{*0}
Total
(a)
$d\Gamma/dM_{K^+\pi^-}(\times 10^{-5})$
$M_{K^+\pi^-}$ (GeV)



20
16
12
8
4
0
0.5
1
1.5
2
2.5
$a_0(1710)$
K_0^{*-}
K_0^{*0}
Total
(b)
$d\Gamma/dM_{\bar{K}^0\pi^-}(\times 10^{-5})$
$M_{\bar{K}^0\pi^-}$ (GeV)

Results

➢ Dalitz plots



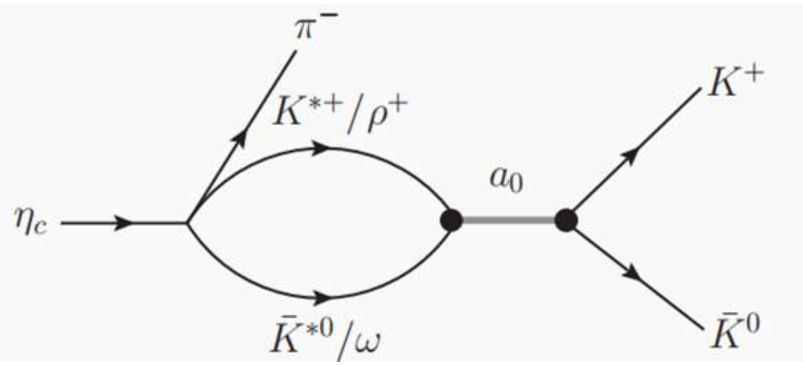
$M_{\overline{K}^0K^+}$ vs. $M_{\overline{K}^0\pi^-}$

$M_{\overline{K}^0K^+}$ vs. $M_{K^+\pi^-}$

$M_{\overline{K}^0\pi^-}$ vs. $M_{K^+\pi^-}$

Results

➢ Branching fractions estimation



① $\eta_c \to \bar{K}^{*0} K^{*+} \pi^-$

$$\mathcal{B}(\eta_c \to \bar{K}^{*0} K^{*+} \pi^-) = \frac{1}{\Gamma_{\eta_c}} \int \left(\frac{d\Gamma}{dM_{\bar{K}^{*0}K^{*+}}}\right) dM_{\bar{K}^{*0}K^{*+}}$$

$$= 5.5 \times 10^{-3}$$

② $\eta_c \to \omega \rho^+ \pi^-$

$$\mathcal{B}(\eta_c \to \omega\rho^+\pi^-) = \frac{1}{\Gamma_{\eta_c}} \int \left(\frac{d\Gamma}{dM_{\omega\rho^+}}\right) dM_{\omega\rho^+}$$

$$= 7.9 \times 10^{-3}$$

$$t^a = V_P \sum_i h_i G_i t_{i \to K^0 K^+}$$

$$V_P = 0.8 \qquad h_1 = \sqrt{2} \quad h_2 = 1$$

1 , 2 for $\omega\rho^+$ and $\bar{K}^{*0}K^{*+}$

Γ_{39}	$K^+K^-\pi^+\pi^-\pi^0$	(3.4 ±0.5) %
Γ_{40}	$K^0K^-\pi^+\pi^-\pi^+$ +c.c.	(5.7 ±1.6) %
Γ_{41}	$K^+K^-2(\pi^+\pi^-)$	(7.6 ±2.4) $\times 10^{-3}$
Γ_{42}	$2(K^+K^-)$	(1.38±0.29) $\times 10^{-3}$
Γ_{43}	$\pi^+\pi^-\pi^0$	< 5 $\times 10^{-4}$
Γ_{44}	$\pi^+\pi^-\pi^0\pi^0$	(4.8 ±1.1) %
Γ_{45}	$2(\pi^+\pi^-)$	(8.7 ±1.1) $\times 10^{-3}$
Γ_{46}	$2(\pi^+\pi^-\pi^0)$	(16.2 ±2.1) %

Summary

- **The $a_0(1710)$ mass and width are crucial to understand its internal structure.**
- **The intermediate state $K_0^*(1430)$ plays an important role in those processes.**
- **Our results are in good agreement with the *BABAR* measurements, which supports the $K^*\bar{K}^*$ molecule of $a_0(1710)$.**
- **We have estimated the branching fractions of $\eta_c \to \bar{K}^{*0}K^{*+}\pi^-$ and $\eta_c \to \omega\rho^+\pi^-$.**

Thank you very much!