Phase Transition - 00 (Gen. Intro.)

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Reference:

- <Scaling and Renormalization in Statistical Physics> by John Cardy
- <相变与临界现象>郝柏林
- <Statistical Mechanics> by Pathria

Target:

- A general picture of phase transition (symmetry, order parameter, ...)
- Hamiltonian → Phase diagram (Under M.F. approx.)
- 1st order phase transition (spinodal instabitlity, metastable, ...)
- Critical phenomena (scaling, univisality, RG→Critical exponents, slowing...)
- Observables

General Introduction on Phase Transition

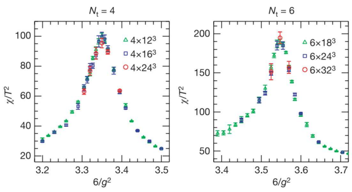
Symmetry & Order-Parameter

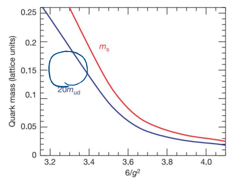
- List:

Symmetry	Order-Parameter	Phase-Transition	Breaking terms
$Z_2: x \to -x$	$\langle S_z \rangle$	Para-ferromagnetic (Ising model)	$h_z S_z$
(Approx.)	$\rho_{liq.} - \rho_{gas}$	Liquid-Gas	$\mu\rho$
	$\langle \bar{q} q \rangle$	chiral	$m\langle \bar{q} q \rangle$
$Z_3: z \to e^{2i\pi/3} z$	Polyakov-loop	Confine-deconfinement	
O(2): $z \to e^{i\theta} z$	$\langle S_x + iS_y \rangle$	Superfluid (XY-model)	
O(3): $\vec{r} \to O(3)\vec{r}$	$\langle \vec{S} \rangle$	Para-ferromagnetic (Heisenberg model)	$\vec{h} \cdot \vec{S}$
O(∞): $\vec{r} \to O(\infty)\vec{r}$		Polymer chain	

- Phase transition: Spontaneous symmetry breaking ⇒ order paramter from 0 to finite
  Dining table senario (Z_2, 1-D):
  Initially, you have a pair of chopsticks on your right hand side, and another pair on your left hand side. ⇒ Left-right symmetry in system.
  After someone pick-up the pair on his/her left hand side, everyone follows. ⇒ left-right symmetry breaks down.
  https://www.zhihu.com/question/403076246/answer/1308003563

- Classification of phase transition ($t \equiv T/T_c - 1$) :
  - Cross-over:
    with t & h, order paramter transforms continuously and smoothly
  - 1st order:
    with t, order parameter jumps from 0 to finite
    With h, jumps from one value to the other
  - Critical point:
    with t, order parameter transforms continuously but not smoothly from 0 to finite
    With h, transforms from $-\langle O \rangle$ to $\langle O \rangle$ with an infinite slope at $h = 0 \Rightarrow \chi \equiv \partial\langle O \rangle / \partial h|_{h=0} = \infty$
  - QCD phase transition at zero chemical potential:

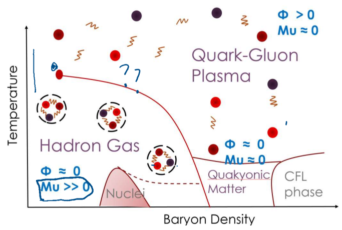


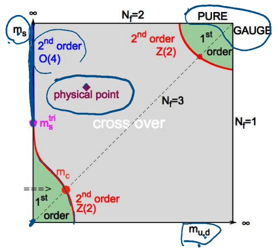


- Correlation length: Length scale that the FLUCTUATIONs are correlated
  In a normal phase: Close to the microscopic scale ($\xi \sim a$)
  Closed to critical point: much larger than the microscopic scale ($\xi \gg a$)
  Anomalous dimension: near CEP, ξ is the ONLY large scale. If only the IR physics is concerned, one can write down any quantity F as $F = F(\xi) = \xi^?$, with '?' obtained via dimension analysis.
  e.g., $[\chi] = -2 \Rightarrow \chi \propto \xi^2$ (χ diverges at CEP). But in fact, $\chi \propto \xi^{2-\eta}$
  Origin of η: $\chi(\xi) \to \chi(\xi, a/\xi) \propto \xi^2 (a/\xi)^\eta$

- Critical exponents:
  - Heat capacity: $C \sim A_\pm |t|^{-\alpha}$ ($\alpha_+ = \alpha_-, h = 0$)
  - Order parameter:
    - $\langle O \rangle \propto (-t)^\beta$ ($h = 0$)
    - $\langle O \rangle \propto |h|^\delta$ ($t = 0$)
  - Susceptibility: $\chi \propto |t|^\gamma$ ($h = 0$)
  - Correlation length: $\xi \propto |t|^{-\nu}$(h=0)
  - Correlation EXACTLY @ CEP: $G(r) \sim r^{-(d-2-\eta)}$
  - Relaxation: $\tau \propto \xi^z$ (critical slowing down)

Schematic QCD Phase Diagrams





General Introduction on Phase Transition

Lee-Yang Theory:

- Analytical continuation on parameters q (temperature or coupling constant e.g.,) in partition functions Z(q) to a complex number
- Find q^* satisfies $Z(q^*) = 0$
- Phase transition occurs if q* is real & positive.
- Example (1-d Ising model):
  $-T \ln Z(j) = -NT(j + \ln(1 + e^{-2j}))$

General Introduction on Phase Transition

A mean field view

Free energy: $F = F(t, h, O) = F_0 - btO^2 + u(t)O^4 \cdots$