Exploring critical piont by light nuclei production in relativistic heavy-ion collision

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Outline

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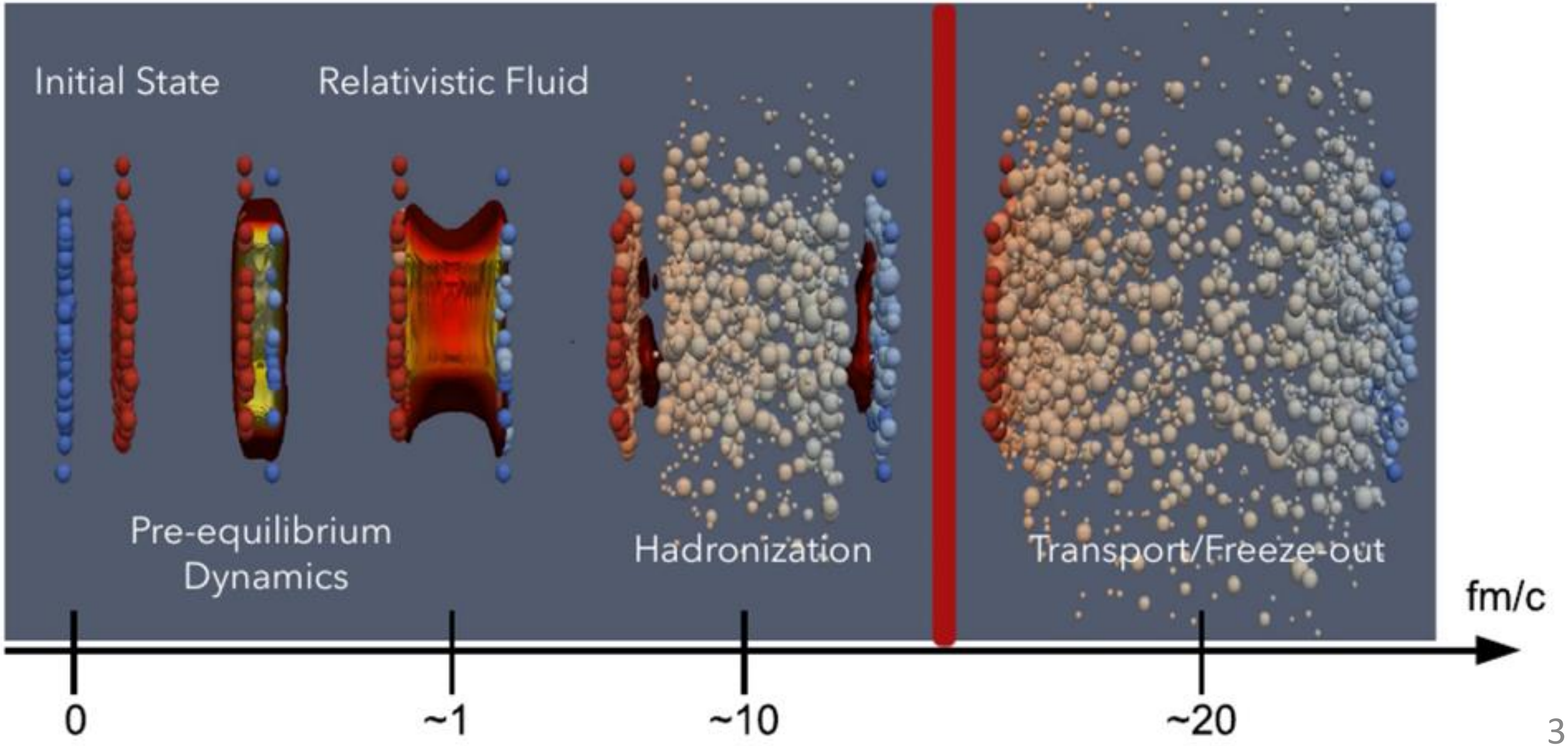
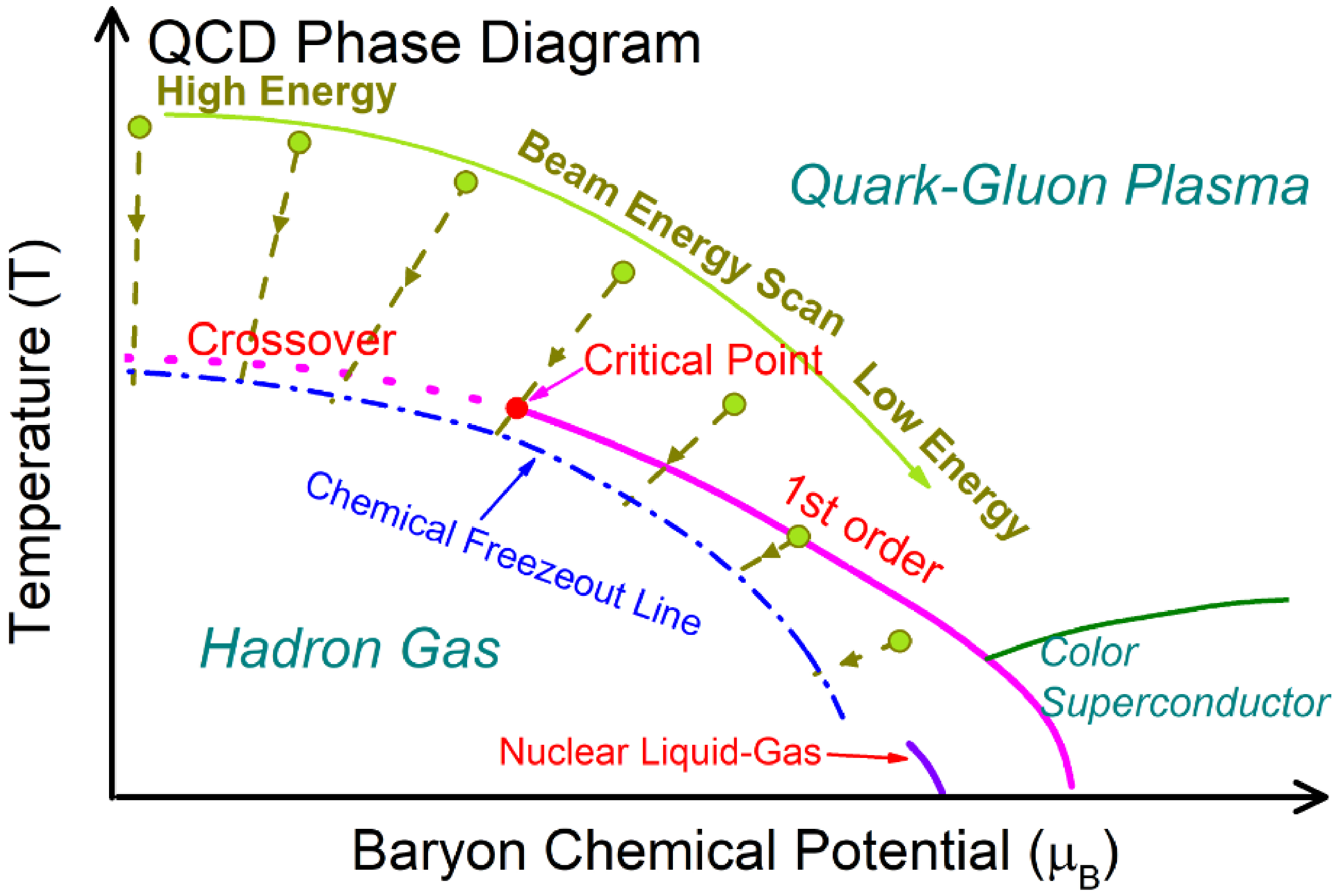
- **The calculation of $C_3(x_1, x_2, x_3)$**
- **The yield ratio $N_t N_p / N_d^2$ of the light nuclei**
- **Summary**

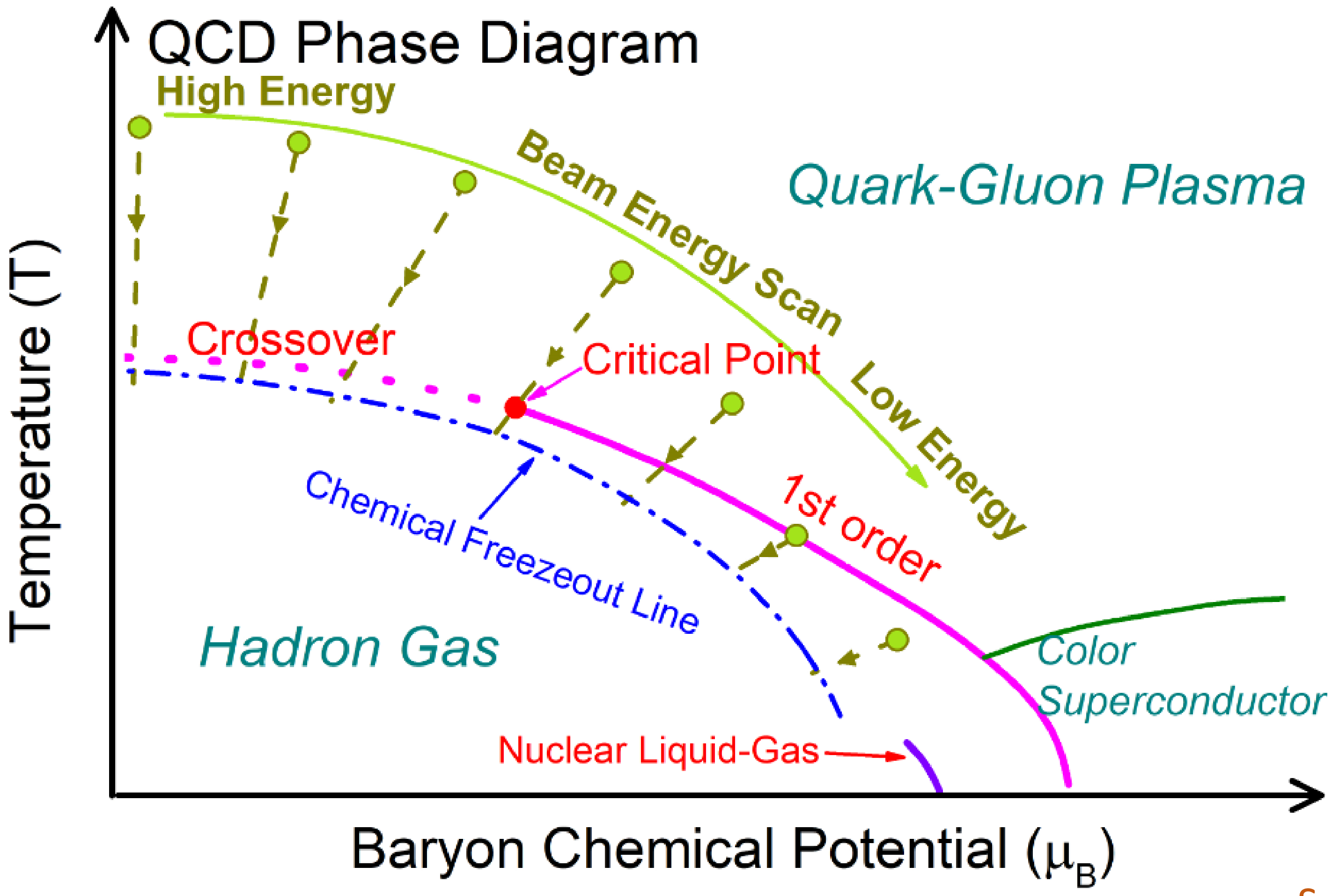
Introduction of QGP



Overview of QCD Phase Diagram



Overview of QCD Phase Diagram



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

First-order phase transition

Second-order phase transition

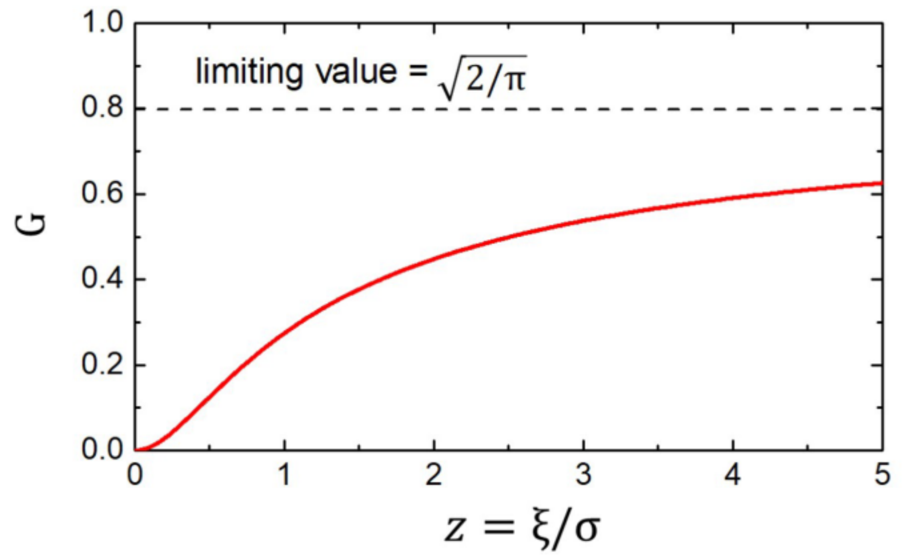


FIG. 1: The dependence of the function $G(\xi/\sigma)$ on the correlation length ξ with σ being the width parameter in the deuteron or triton Wigner function.

Sun, K. J. , Li, F. , & Ko, C. M. . (2021). Effects of qcd critical point on light nuclei production. Physics Letters B, 816(30), 136258.

Overview of QCD Phase Diagram

$$N_d \approx N_d^{(0)} + \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} \times \int d^3\boldsymbol{x}_1\, d^3x_2 C_2(x_1, x_2) \frac{e^{-\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{2\sigma_d^2}}}{(2\pi\sigma_d^2)^{\frac{3}{2}}}$$

$$N_t \approx N_t^{(0)} + \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^{3} \int d^3 x_1 d^3x_2 d^3x_3 \rho_{nnp}(x_1, x_2, x_3) \times \frac{e^{-\frac{(x_1-x_2)^2}{2\sigma_t^2}-\frac{(x_1+x_2-2x_3)^2}{6\sigma_t^2}}}{3^{3/2}(\pi\sigma_t^2)^3}$$

Three-nucleon joint ensity distribution function:

$$\rho_{nnp}(x_1, x_2, x_3) \approx \rho_n(x_1)\rho_n(x_2)\rho_p(x_3) + C_2(x_1, x_2)\rho_p(x_3) + C_2(x_2, x_3)\rho_n(x_1) + C_2(x_3, x_1)\rho_n(x_2) + C_3(x_1, x_2, x_3)$$

Using the nucleon coalescence model, which can naturally take into account the correlations in the nucleon density distribution

The number of the light nuclei

The number of deuteron

$$N_d \approx N_d^{(0)} \left(1 + \frac{\alpha^2}{N_p\langle\rho_n\rangle} G_{c2}\right)$$

$$N_d^{(0)} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} N_p\langle\rho_n\rangle$$

$$G_{c2} \approx (-\frac{1}{2}\alpha)^{-2} \int d^3\boldsymbol{x}_1\, d^3x_2 C_2(x_1, x_2) \frac{e^{-\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{2\sigma_d^2}}}{(2\pi\sigma_d^2)^{\frac{3}{2}}}$$

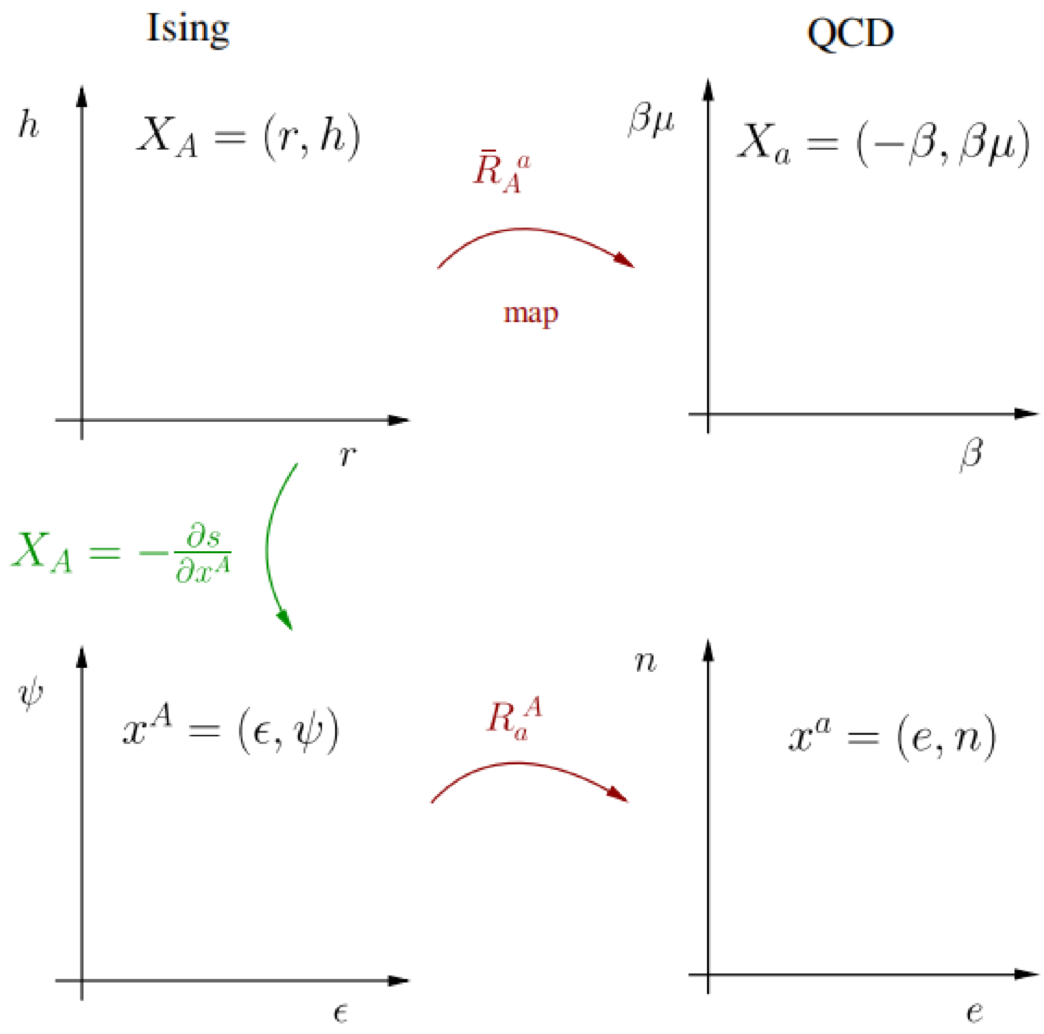
The number of triton

$$N_t = N_t^{(0)} \left[1 + \frac{1}{N_p\langle\rho_n\rangle^2} \left(12\sigma_t^{-2}\rho(-\frac{1}{2}\alpha)^2 G_{c2} + (-\frac{1}{2}\alpha)^3 G_{c3}\right)\right]$$

$$N_t^{(0)} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2$$

$$G_{C3} \approx \left(-\frac{1}{2}\alpha\right)^{-3} \int d^3 x_1 d^3 x_2 d^3 x_3 C_3(x_1, x_2, x_3) \times \frac{1}{3^{3/2}(\pi\sigma_t^2)^3} e^{-\frac{(x_1-x_2)^2}{2\sigma_t^2} - \frac{(x_1+x_2-2x_3)^2}{6\sigma_t^2}}$$

The calculation of $C_2(x_1,x_2)$

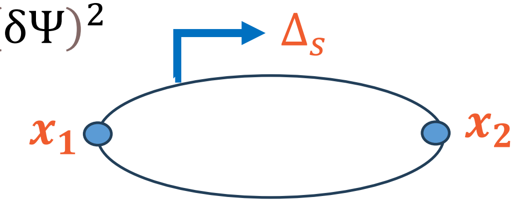


1. Mapping QCD to Ising model

$$\begin{pmatrix}\delta e\\ \delta n\end{pmatrix} \approx \begin{pmatrix}0 & 1\\ -\frac{1}{2} & 0\end{pmatrix}\begin{pmatrix}\delta\varepsilon\\ \delta\Psi\end{pmatrix} \qquad \begin{pmatrix}t\\ \bar{\mu}\end{pmatrix} \approx \begin{pmatrix}0 & 1\\ -2 & 0\end{pmatrix}\begin{pmatrix}r\\ h\end{pmatrix}$$

$\varepsilon = \delta G_{Ising}\,[t,\Psi]/\delta\, t_{Ising}$

$\delta n \propto \delta\varepsilon \propto (\delta\Psi)^2$



$$c_2^n(x_1,x_2) = \left(-\frac{1}{2}\alpha\right)^2 \int \frac{d\omega_1 d\omega_2 d^3\vec{k_1} d^3\vec{k_2}}{(2\pi)^8} e^{-i(\vec{k_1}+\vec{k_2})(\vec{x_1}-\vec{x_2})} \Delta_s\left(\omega_1,\vec{k_1}\right)\Delta_s\left(\omega_2,\vec{k_2}\right)$$

Martinez, M. , Schaefer, T. , & Skokov, V. . (2019). Critical behavior of the bulk viscosity in qcd.

The calculation of $C_2(x_1, x_2)$

2. $\boldsymbol{C_2(X_1, X_2)}$ ***can be transferred into***

$$c_2^n(x_1,x_2) = (-\frac{1}{2}\alpha)^2 \int \frac{d\omega_1 d\omega_2 d^3\overrightarrow{k_1} d^3\overrightarrow{k_2}}{(2\pi)^8} e^{-i(\overrightarrow{k_1}+\overrightarrow{k_2})(\overrightarrow{x_1}-\overrightarrow{x_2})} \Delta_s\left(\omega_1, \overrightarrow{k_1}\right) \Delta_s\left(\omega_2, \overrightarrow{k_2}\right)$$

where

$$\Delta_s(\omega, k) = 2\chi_k T \frac{\Gamma_k}{\omega^2 + \Gamma_k^2} \qquad \Gamma_k = \frac{T\xi^{-3}}{6\pi\eta_0} K(k\xi)\chi_k = \frac{\chi_0}{1 + (k\xi)^{2\eta}} \qquad \xi = \xi_0 R^{-\nu} g_\xi^{1/2}(\theta)$$

$$K(x) = \frac{3}{4}\{[1 + x^2 + (x^3 + x^{-1})arctan(x)]\} \qquad g_\xi(\theta) \simeq (1 - 5\theta^2/18)$$

3. ***Transforming*** $\boldsymbol{(R, \theta)}$ ***into*** $\boldsymbol{(\bar{\mu}, t)}$ ***coordinate system***

$$\begin{pmatrix} t \\ \bar{\mu} \end{pmatrix} \approx \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} r \\ h \end{pmatrix} \qquad \begin{matrix} r = R(1 - \theta^2) \\ \xi = \xi_0 R^{-\nu} g_\xi^{1/2}(\theta) \end{matrix} \qquad \begin{matrix} h = h_0 R^{\beta\delta} \tilde{h}(\theta) \\ \tilde{h}(\theta) = \theta + h_1\theta^3 + h_2\theta^5 \end{matrix}$$

The calculation of $C_2(x_1, x_2)$

The $C_2(x_1, x_2)$ contribution to the number of the light nuclei

$$G_{c2} \approx \left(-\frac{1}{2}\alpha\right)^{-2} \int d^3\boldsymbol{x}_1\, d^3x_2\, C_2(x_1, x_2)\, \frac{e^{-\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{2\sigma_d^2}}}{(2\pi\sigma_d^2)^{\frac{3}{2}}}$$

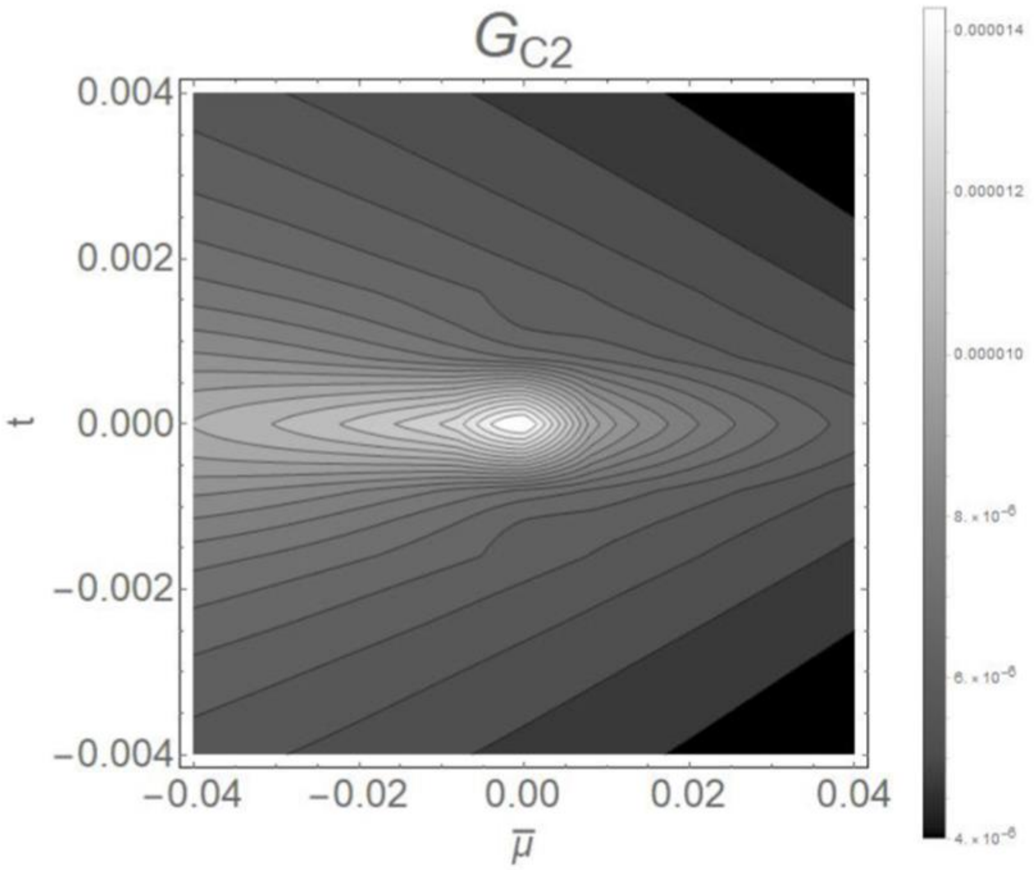
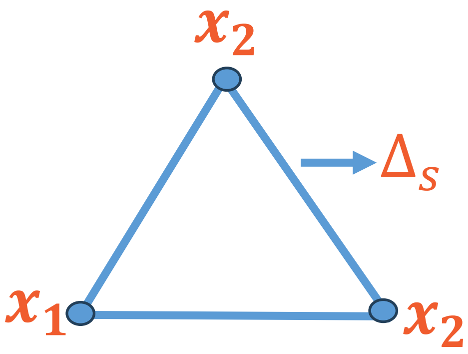


FIG. 1. (Color online)Contour plot of the temperature and chemical potential dependent the G_{C2} near the CEP.

The calculation of $C_3(x_1, x_2, x_3)$



$$C_3^{n}(x_1, x_2, x_3) = (-\frac{1}{2}\alpha)^3 \int \frac{d\omega_1 d\omega_2 d\omega_3 d^3\overrightarrow{k_1} d^3\overrightarrow{k_2} d^3\overrightarrow{k_3}}{(2\pi)^{12}} e^{-i(\overrightarrow{k_1}+\overrightarrow{k_2})(\overrightarrow{x_1}+\overrightarrow{x_2})} \Delta_s\left(\omega_1, \overrightarrow{k_1}\right) \Delta_s\left(\omega_2, \overrightarrow{k_2}\right) \Delta_s\left(\omega_3, \overrightarrow{k_3}\right)$$

The $C_3(x_1, x_2, x_3)$ contribution to the number of the light nuclei

$$G_{C3} \approx \left(-\frac{1}{2}\alpha\right)^{-3} \int d^3x_1 d^3x_2 d^3x_3 C_3(x_1, x_2, x_3) \times \frac{1}{3^{3/2}(\pi\sigma_t^2)^3} e^{-\frac{(x_1-x_2)^2}{2\sigma_t^2} - \frac{(x_1+x_2-2x_3)^2}{6\sigma_t^2}}$$

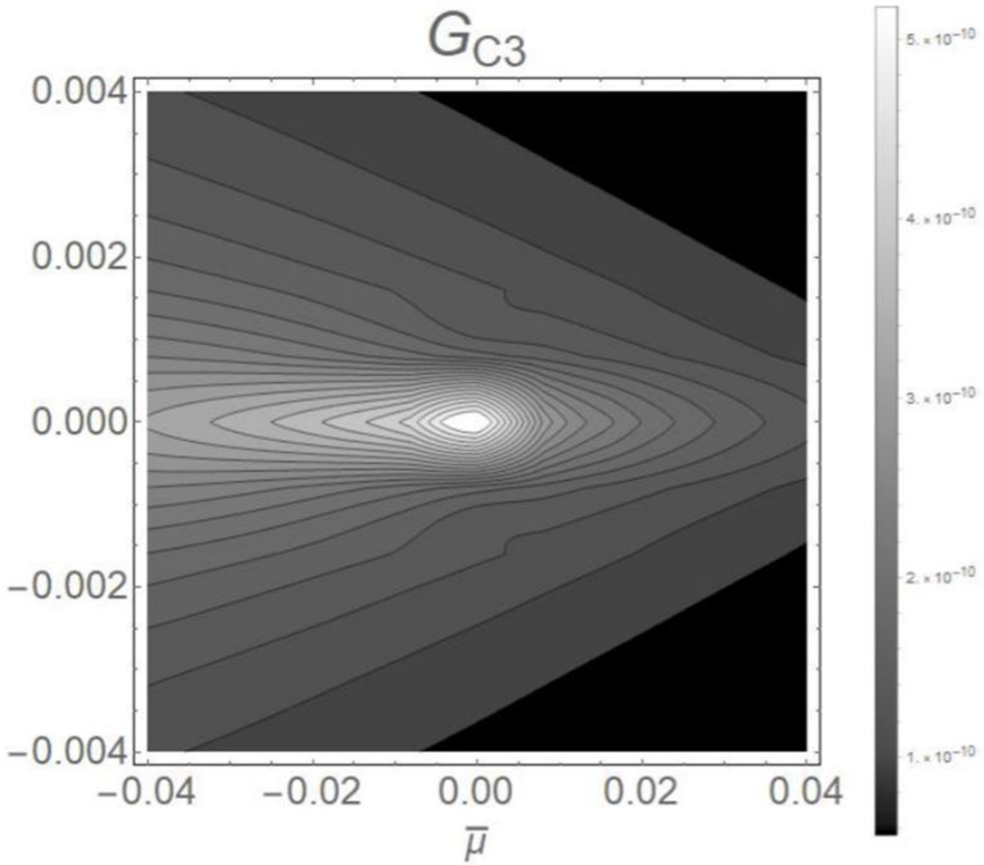


FIG. 2. (Color online)Contour plot of the temperature and chemical potential dependent the G_{C3} near the CEP.

Calculation results

The behavior of G_{C2} and G_{C3} near the CEP

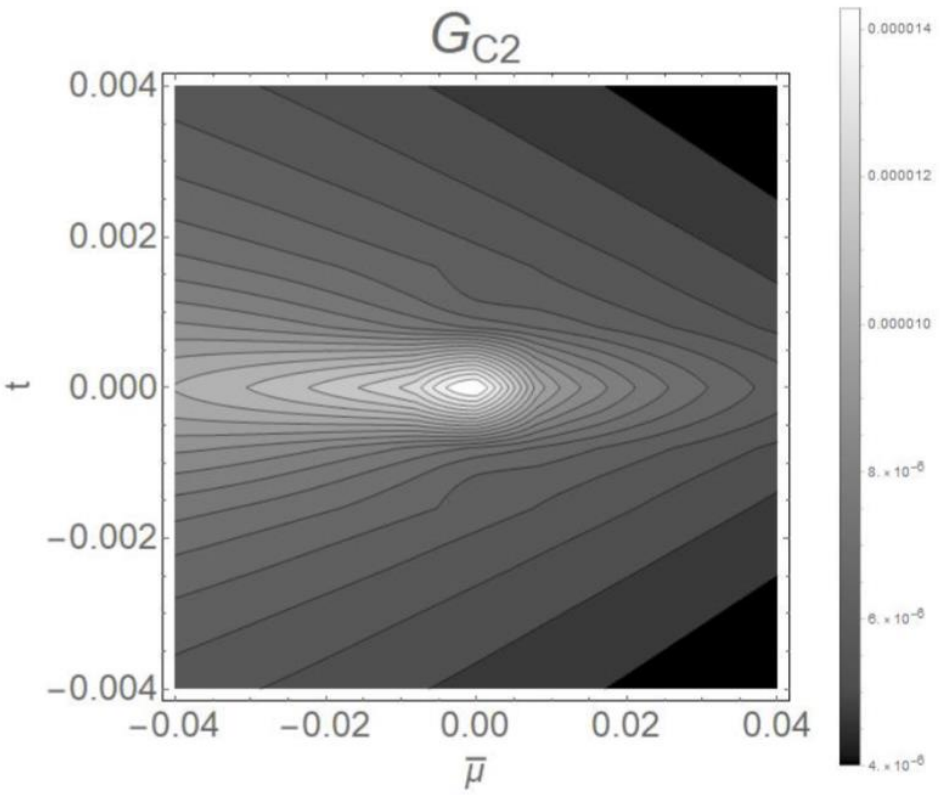


FIG. 1. (Color online)Contour plot of the temperature and chemical potential dependent the G_{C2} near the CEP.

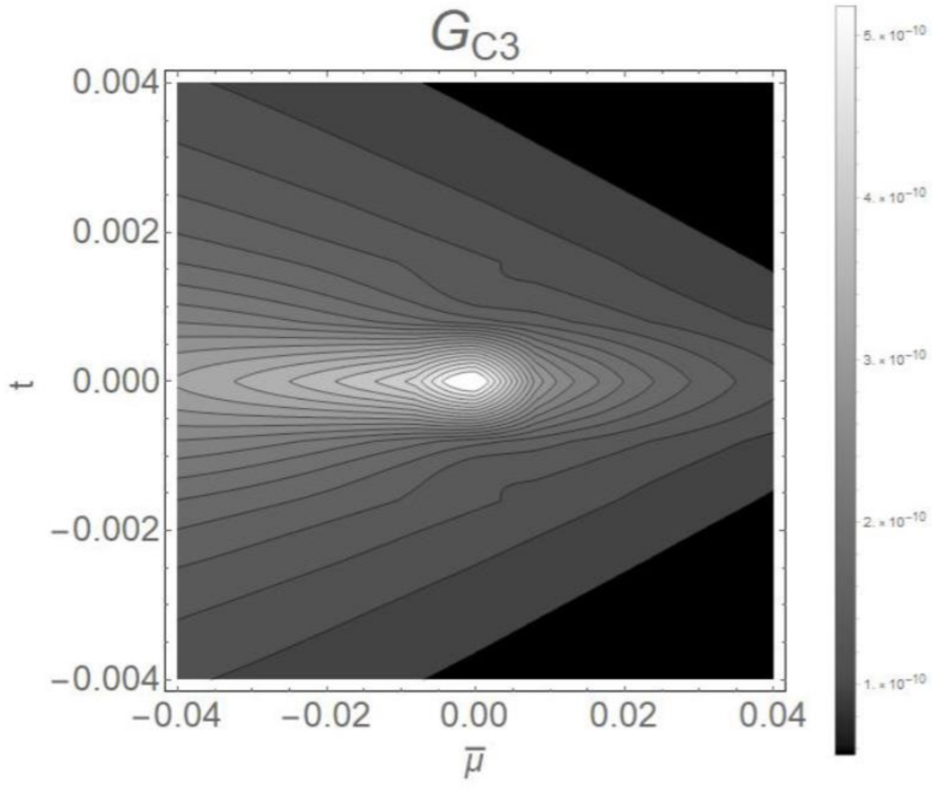


FIG. 2. (Color online)Contour plot of the temperature and chemical potential dependent the G_{C3} near the CEP.

The yield ratio $N_t N_p / N_d^2$ of the light nuclei

The yield ratio $\frac{N_t N_p}{N_d^2}$ *will show a maximum behavior near CEP when* $0 < \boldsymbol{\alpha} < 0.97$, *and a minimum behavior near CEP when* $\boldsymbol{\alpha} > 1.16$

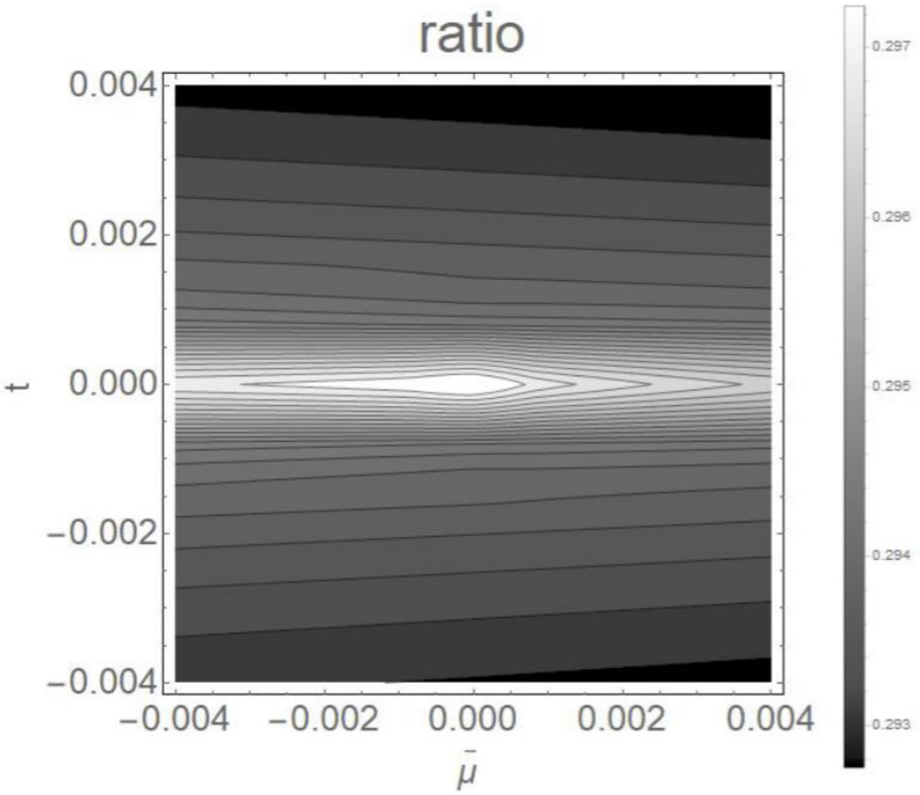


FIG. 3. (Color online)Contour plot of the temperature and chemical potential dependent the yield ratio $N_t N_p / N_d^2$ when $\alpha = 0.5$ near the CEP.

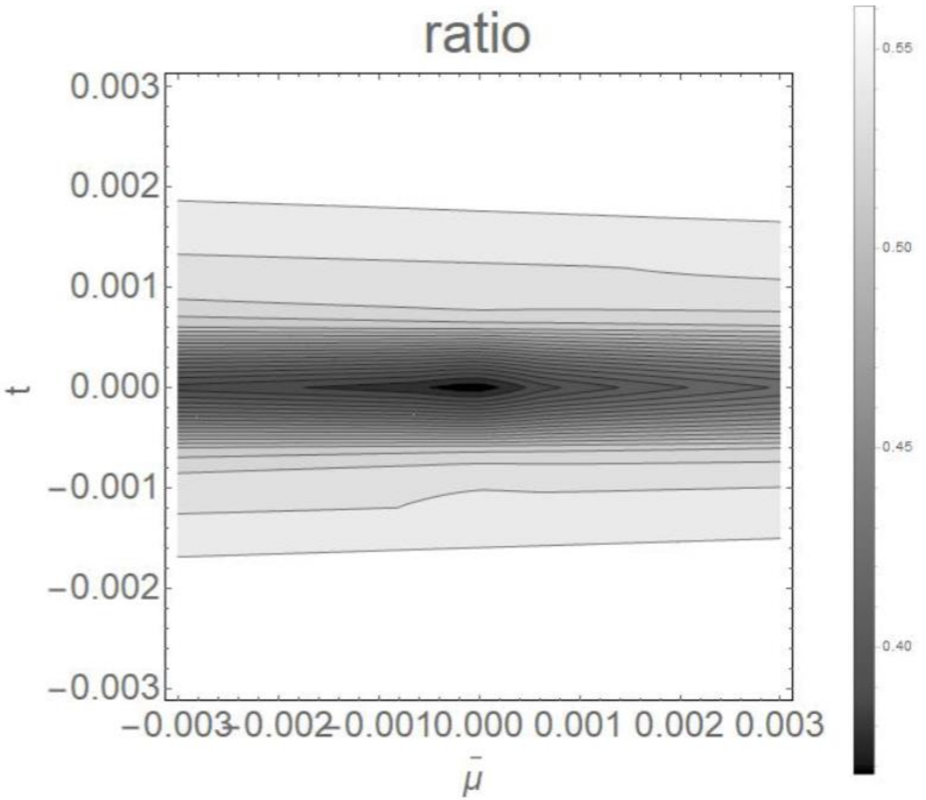


FIG. 4. (Color online)Contour plot of the temperature and chemical potential dependent the yield ratio $N_t N_p / N_d^2$ when $\alpha = 2.5$ near the CEP.

Summary

①Based on a map between QCD and Ising model, we calculate $C_2(x_1, x_2)$ and $C_3(x_1, x_2, x_3)$, and investigate their contributions to the light nuclei yields in the vicinity of CEP.

②$The\ yield\ ratio$ $\mathrm{N_t N_p / N_d^2}$ $shows\ a\ maximum$ at $CEP\ when$ $0 < \alpha < 0.97$,$and\ a\ minimum$ at $CEP\ when$ $\alpha > 1.16$

Thanks