The hydrodynamics description of anisotropic flow and flow fluctuations in $\sqrt{s_{\rm NN}}$=5.02 TeV Pb-Pb collisions at the LHC

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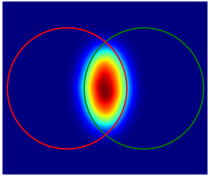
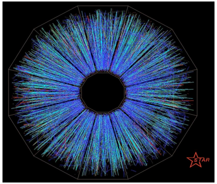
Anisotropic flow





- Flow measurement
  - Event plane method
  - scalar product method
  - two-particle correlation

$$E\frac{\mathrm{d}^3 N}{\mathrm{d}p^3} \propto 1 + 2\sum_{\mathrm{n}=1}^{\infty} v_{\mathrm{n}} \cos\left[\mathrm{n}\left(\varphi - \Psi_{\mathrm{n}}\right)\right]$$

$$v_n = \langle \cos\left[\mathrm{n}\left(\varphi - \Psi_{\mathrm{n}}\right)\right]\rangle$$

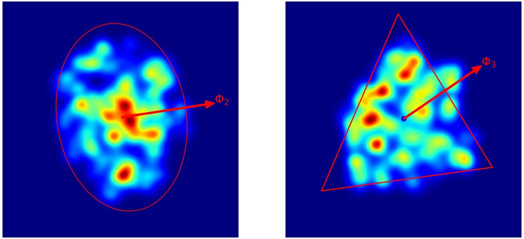
$$\vec{v}_n = v_n e^{in\Psi_n} = \left\{e^{in\varphi}\right\}$$

$$v_n\{2\}\left(p_T\right) = \frac{\left\langle\left\langle e^{in(\psi_1 - \phi_2}\right\rangle\right\rangle}{\sqrt{\langle\langle e^{in(\phi_1 - \phi_2)}\rangle\rangle}} = \frac{\langle \vec{v}_n\left(p_T\right)\vec{v}_n^*\rangle}{\sqrt{\langle \vec{v}_n \vec{v}_n^*\rangle}}$$

  - multi-particle cumulant
  - ...

Flow vector fluctuations

- The fluctuation of geometry in the initial states $\rightarrow$ anisotropic flow fluctuations of soft hadrons in the momentum space.



- For small fluctuations of v_n, one of simplest approximations is to use a Gaussian distribution.

$$\frac{dP}{d^2\vec{v}_n} = \frac{1}{2\pi\sigma_n^2} e^{-\frac{(\vec{v}_n - \bar{\vec{v}}_n)^2}{2\sigma_n^2}}$$

$$\frac{dP}{dv_n} = v_n \int d\Phi_n \frac{dP}{d^2\vec{v}_n} = \frac{v_n}{\sigma_n^2} I_0\left(\frac{v_n \bar{v}_n}{\sigma_n^2}\right) e^{-\frac{v_n^2 + \bar{v}_n^2}{2\sigma_n^2}}$$

Flow vector fluctuation

mid-central collision in large system

$$\langle v_n \rangle \approx \sqrt{\bar{v}_n^2 + \sigma_n^2}$$
$$\left\langle v_n^2 \right\rangle = \bar{v}_n^2 + 2\sigma_n^2$$
$$\left\langle v_n^4 \right\rangle = \bar{v}_n^4 + 8\bar{v}_n^2\sigma_n^2 + 8\sigma_n^4$$

$$(v_n\{2\})^2 = \left\langle v_n^2 \right\rangle = \bar{v}_n^2 + 2\sigma_n^2$$
$$(v_n\{4\})^4 = 2\left\langle v_n^2 \right\rangle^2 - \left\langle v_n^4 \right\rangle = \bar{v}_n^4$$

- By using two- and four-particle cumulants, one can subtract the mean flow and flow fluctuation.

$$v_n^2\{2\} = \langle v_n \rangle^2 + \sigma_{v_n}^2$$
$$v_n^2\{4\} \approx \langle v_n \rangle^2 - \sigma_{v_n}^2$$

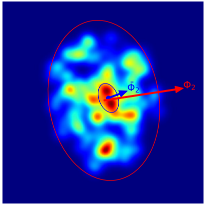
$$\Longrightarrow$$

$$\langle v_n \rangle \approx \sqrt{\frac{v_n^2\{2\} + v_n^2\{4\}}{2}}$$
$$\sigma_{v_n} \approx \sqrt{\frac{v_n^2\{2\} - v_n^2\{4\}}{2}}$$
$$F(v_n) = \frac{\sigma_{v_n}}{\langle v_n \rangle}$$

Fluctuations(separated $\mathrm{p_T}$ decorrelation)

- Traditionally, using factorization ratio r_n to describe the angle and magnitude fluctuation.

$$r_n = \frac{V_{n\Delta}\left(p_{\mathrm{T}}^a, p_{\mathrm{T}}^t\right)}{\sqrt{V_{n\Delta}\left(p_{\mathrm{T}}^a, p_{\mathrm{T}}^a\right) V_{n\Delta}\left(p_{\mathrm{T}}^t, p_{\mathrm{T}}^t\right)}} = \frac{\left\langle v_n\left(p_{\mathrm{T}}^a\right) v_n\left(p_{\mathrm{T}}^t\right) \cos n\left(\Psi_n^a - \Psi_n^t\right)\right\rangle}{\sqrt{\left\langle v_n\left(p_{\mathrm{T}}^a\right)^2\right\rangle \left\langle v_n\left(p_{\mathrm{T}}^t\right)^2\right\rangle}}$$



- However, r_n contains contributions from both the flow angle and magnitude.

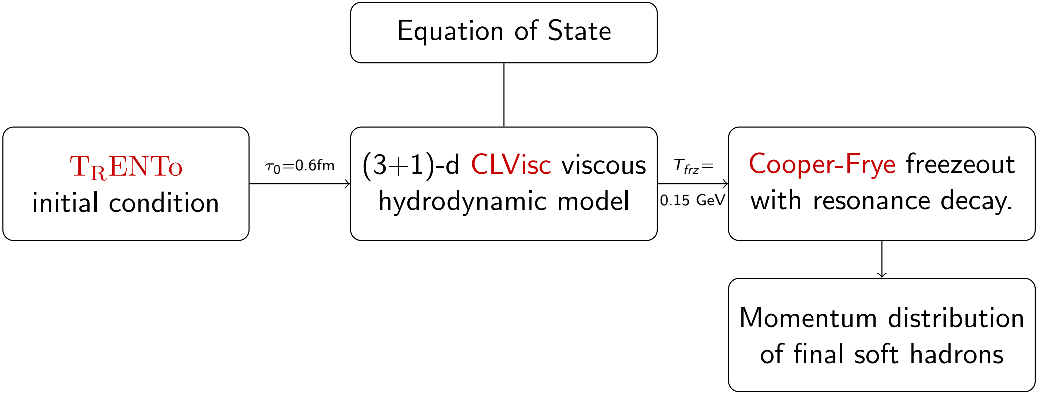
- Constructing four-particle correlation observables can help to study separated flow angle and flow magnitude fluctuations. [ALICE, 2206.04574.]

$$A_n^f = \frac{\langle\langle \cos n[\phi_1^a + \phi_2^a - \phi_3 - \phi_4]\rangle\rangle}{\langle\langle \cos n[\phi_1^a + \phi_2 - \phi_3^a - \phi_4]\rangle\rangle} = \frac{\langle v_n^2(\mathrm{p_T^a}) v_n^2 \cos 2n[\Psi_n(\mathrm{p_T^a}) - \Psi_n]\rangle}{\langle v_n^2(\mathrm{p_T^a}) v_n^2\rangle} \approx \langle \cos 2n[\Psi_n(\mathrm{p_T^a}) - \Psi_n]\rangle_{\mathrm{w}}$$

$$M_n^f = \frac{\langle\langle \cos n[\phi_1^a + \phi_2 - \phi_3^a - \phi_4]\rangle\rangle}{(\langle\langle \cos n[\phi_1^a - \phi_3^a]\rangle\rangle\langle\langle \cos n[\phi_2 - \phi_4]\rangle\rangle)} \Bigg/ \frac{\langle\langle \cos n[\phi_1 + \phi_2 - \phi_3 - \phi_4]\rangle\rangle}{\langle\langle \cos n[\phi_1 - \phi_2]\rangle\rangle^2} = \frac{\langle v_n^2(\mathrm{p_T^a}) v_n^2\rangle / (\langle v_n^2(\mathrm{p_T^a})\rangle\langle v_n^2\rangle)}{\langle v_n^4\rangle / \langle v_n^2\rangle^2}$$

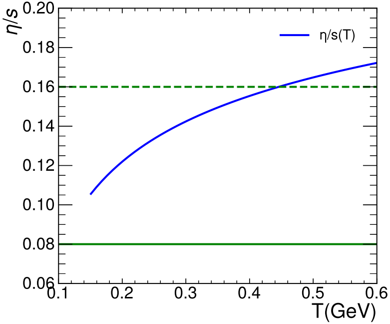
Model setup

Simulate the dynamical evolution of the QGP fireball



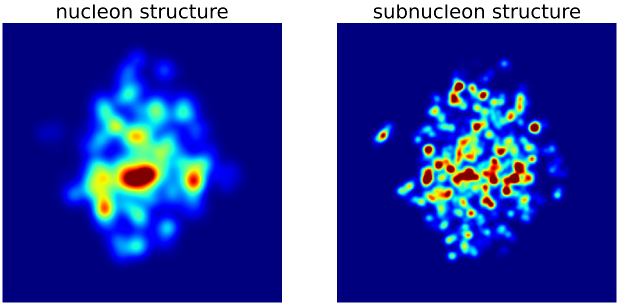
TRENTo: Phys. Rev. C 92 , 011901 (2015)
CLVisc1.0: Phys. Rev. C 97, 064918 (2018)
CLVisc2.0: Phys. Rev. C 105, 034909 (2022)

Viscosity and substructure



$\eta/s(\mathrm{T})$ from Phys. Rev.C 104, 054904 (2021)

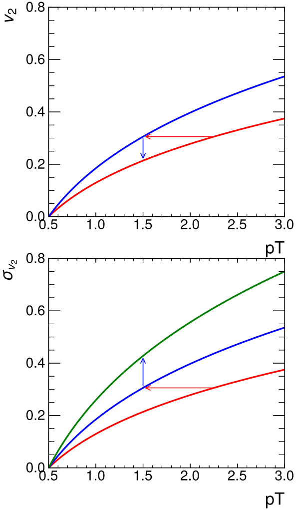
- $\eta/s \downarrow$, $\langle v_2 \rangle \uparrow$.

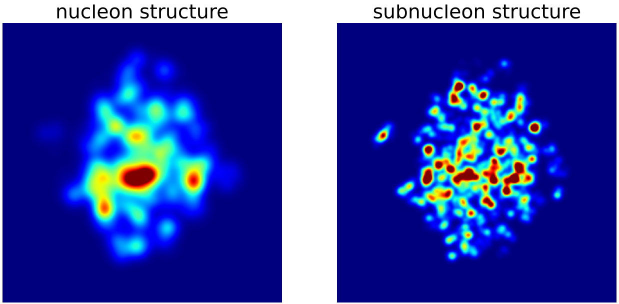


Substructure of nucleon will

- suppress blue shift.
- enhance σ_{v_2} and suppress anisotropy.
- become important in peripheral collisions and small system.

Viscosity and substructure





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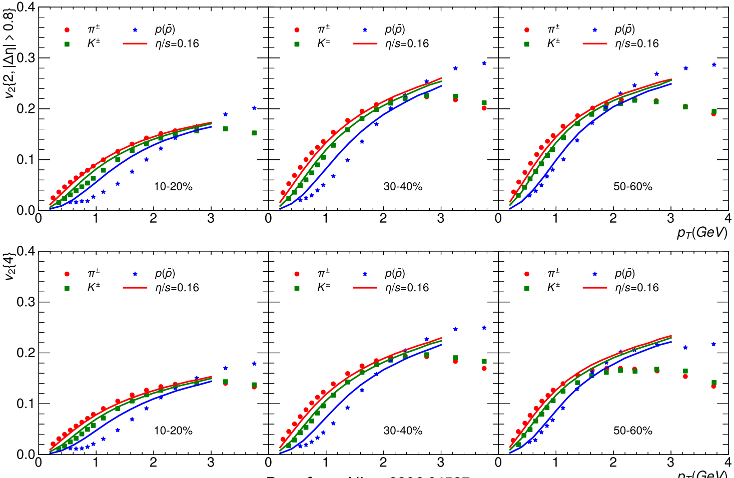
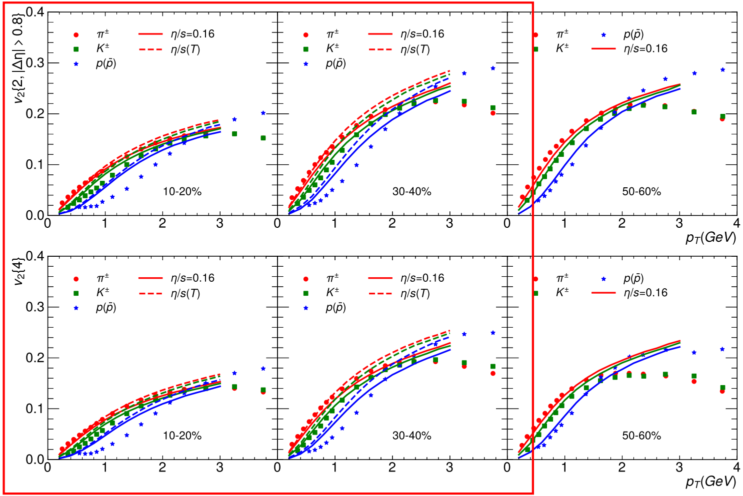
Results(cumulant)



Data from Alice, 2206.04587

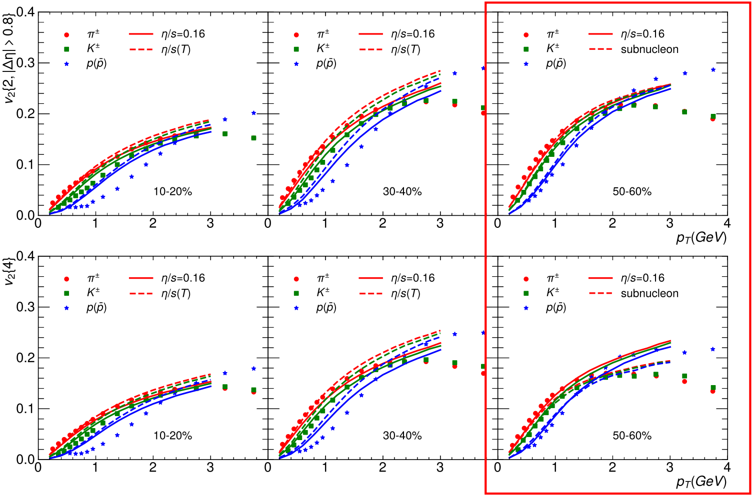
- The simulation with η/s=0.16 can reproduce experimental flow well except in peripheral collisions and proton case.

Viscosity effect



- Temperature-dependent shear viscosity $\eta/\mathrm{s}(\mathrm{T})$ enhances $v_2\{4\}$ and $v_2\{2\}$ at high transverse momentum and in peripheral collisions as a result of the lower shear viscosity at lower temperature in $\eta/\mathrm{s}(\mathrm{T})$.

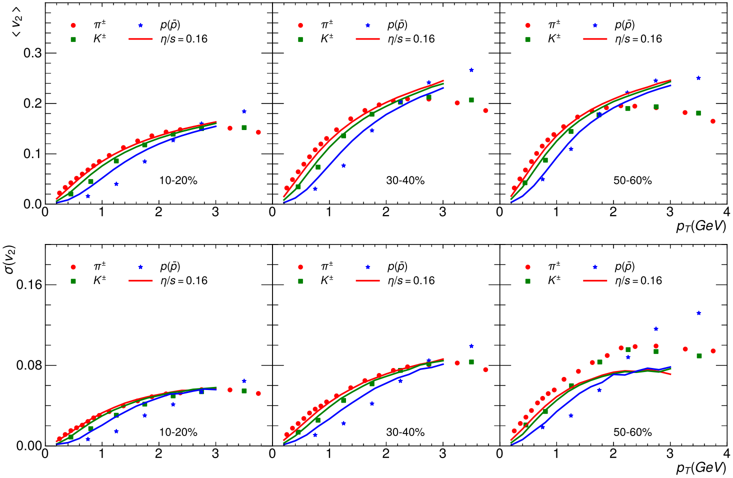
Substructure effect



- The subnucleon structure increases the $v_2\{2, |\Delta\eta| > 0.8\}$and suppresses the $v_2\{4\}$ due to the effect of fluctuation.

$$v_n^2\{2\} = \langle v_n \rangle^2 + \sigma_{v_n}^2, \, v_n^2\{4\} \approx \langle v_n \rangle^2 - \sigma_{v_n}^2$$

Results(flow fluctuation)



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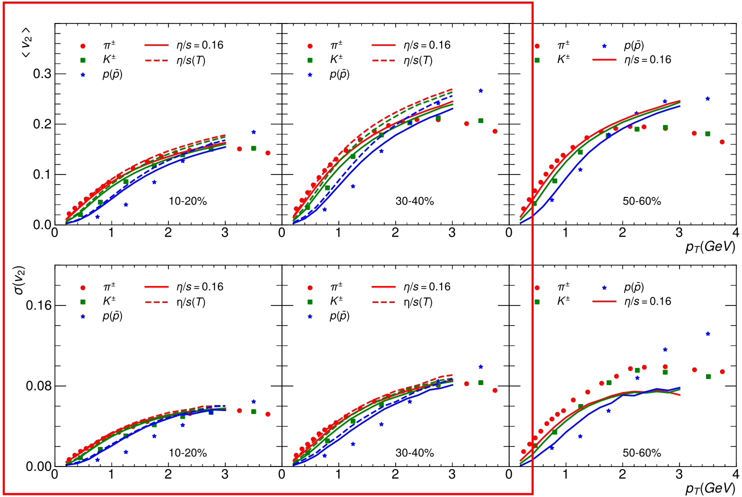
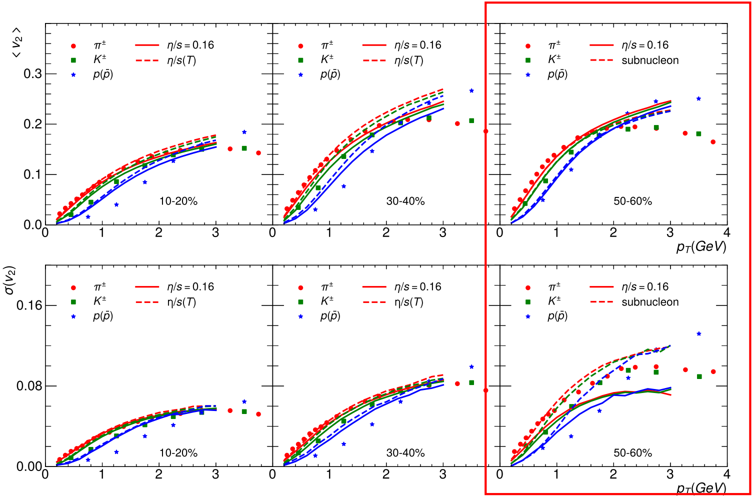


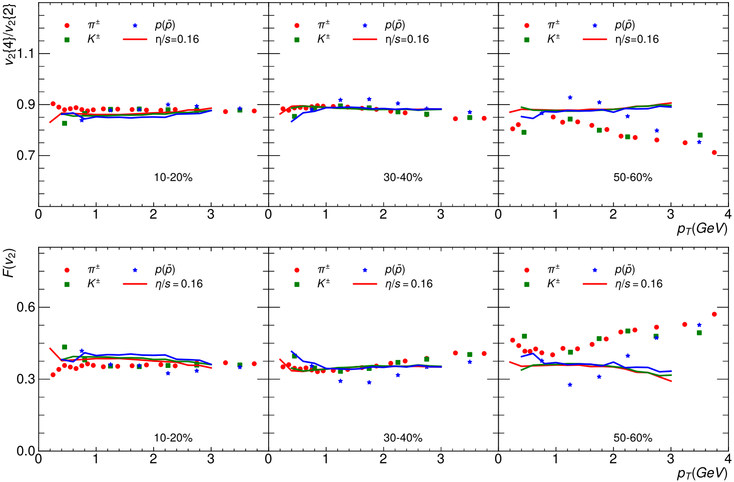
- Specific shear viscosity has little effect on fluctuation width and a comparable effect on mean value of v_2, especially in peripheral collisions.

Substructure effect



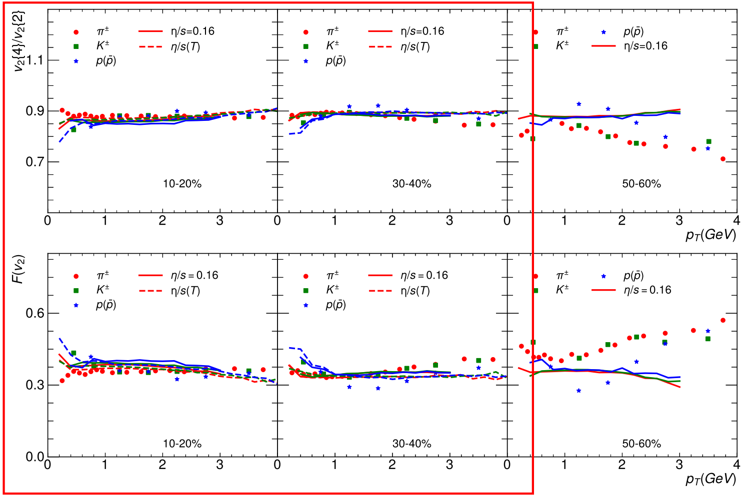
- The substructure of the nucleon enhances the fluctuation of various species of particles, but does not have an effect on the mean value of the elliptic flow.

Results(relative fluctuation)



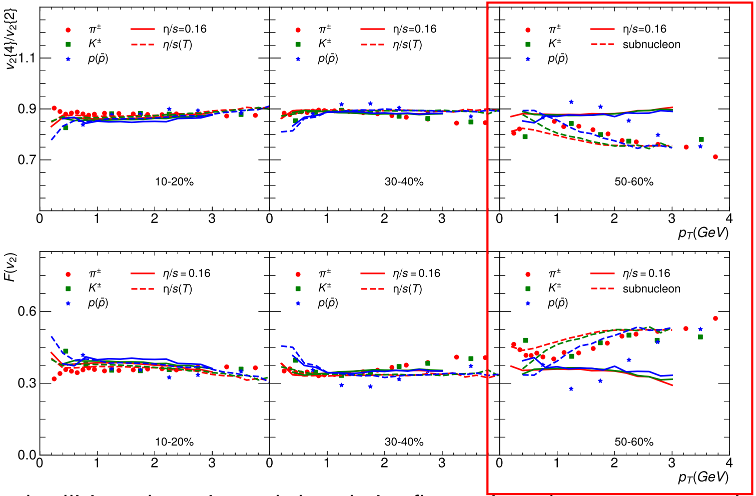
- The ratios and the relative fluctuations have no dependence on particle species and transverse momentum which is consistent with the experimental results in central collisions.

Viscosity effect



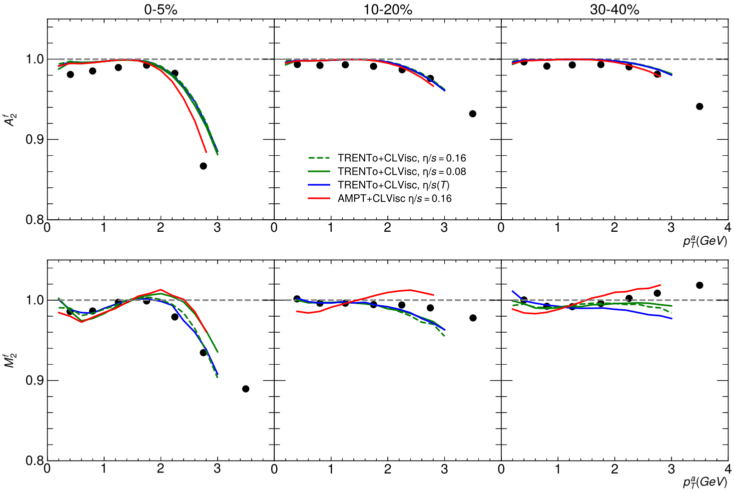
- Due to the linear relation between $v_2\{n\}$ and $\epsilon_2\{n\}$, ratios of cumulants are expected to be independent of the hydrodynamic response which can be properly reproduced in our simulation.

Substructure effect



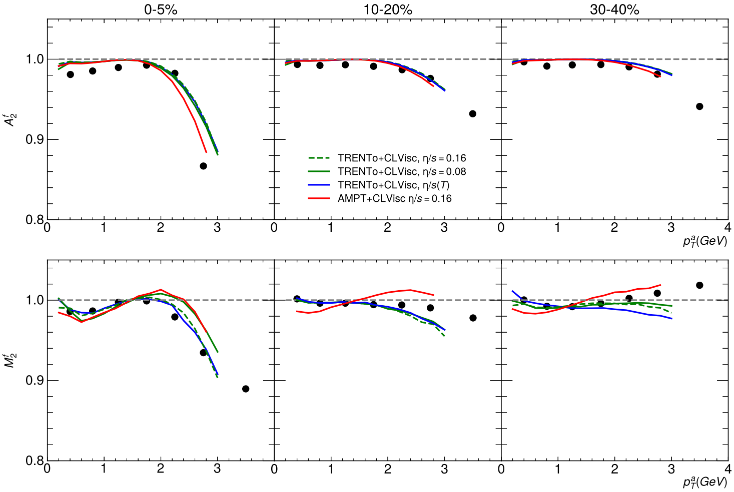
- In peripheral collisions the ratios and the relative fluctuations show non-monotonic dependence on transverse momentum and particle type grouping in $1 < \mathrm{p_T} < 3\mathrm{GeV}$, which can be qualitatively reproduced if the subnucleon structure is considered.

Angle fluctuation



- The simulation can roughly produce the small deviation of angle fluctuations in low $\mathrm{p_T}$ and mid-central collisions as well as the large deviation in high $\mathrm{p_T}$ and central collisions.

Magnitude fluctuation



- The flow magnitude fluctuations are sensitive to the initial condition and slightly dependent on the shear viscosity except the central collision.

Summary and outlook

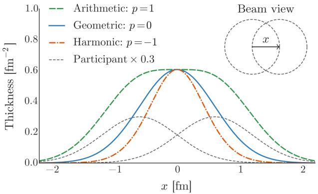
- Summary
  - Flow fluctuation is sensitive to the initial state model, especially the granularity of the initial state fluctuations.
  - The transverse momentum decorrelations of the flow angle and magnitude are sensitive to initial model and insensitive to transport properties of the QGP.
- Outlook
  - Non-gaussian distribution can be considered to extend the study to central collisions and small system.
  - Hadron cascade and coalescense model can be added to extend the study to medium p_T and more particle species.
  - Flow fluctuation can be used to constrain initial model using bayesian analysis.

Thanks for your attention!

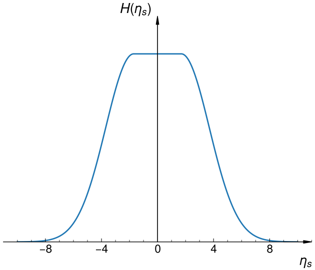
backup

Model setup $T_R$ento

$$dS/\,dy\big|_{\tau=\tau_0} \propto T_R\left(p;\, T_A,\, T_B\right) \equiv \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$



$$T_R = \begin{cases} \max\left(T_A,\, T_B\right), & p \to +\infty \\ \left(T_A + T_B\right)/2, & p = +1 \text{ (arithmetic)} \\ \sqrt{T_A T_B}, & p = 0 \text{ (geometric)} \\ 2\,T_A T_B/\left(T_A + T_B\right), & p = -1 \text{ (harmonic)} \\ \min\left(T_A,\, T_B\right), & p \to -\infty \end{cases}$$



$$H\left(\eta_s\right) = \exp\left[-\frac{\left(|\eta_s| - \eta_w\right)^2}{2\sigma_\eta^2}\theta\left(|\eta_s| - \eta_w\right)\right]$$

backup

Model setup hydro

CLVisc hydrodynamic evolution

- $T^{\mu\nu} = eU^{\mu}U^{\nu} - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$, $[e(1), P(1), U^{\mu}(3), \Pi(1), \pi^{\mu\nu}(5)]$

$$\partial_{\mu} T^{\mu\nu} = 0 \qquad [4]$$

$$U^{\mu}\partial_{\mu}\Pi = \frac{1}{\tau_{\Pi}}(\Pi + \xi\theta) - \frac{4}{3}\Pi\theta \qquad [1]$$

$$\Delta^{\mu\nu}_{\alpha\beta} u^{\lambda}\partial_{\lambda}\pi^{\alpha\beta} = -\frac{1}{\tau_{\pi}}\left(\pi^{\mu\nu} - \eta\sigma^{\mu\nu}\right) - \frac{4}{3}\pi^{\mu\nu}\theta \qquad [5]$$

$$e = e(P) \qquad [1]$$

Cooper-Frye freeze-out: $\dfrac{dN_i}{dYp_T dp_T d\phi} = \dfrac{g_i}{(2\pi)^3}\displaystyle\int p^{\mu} d\Sigma_{\mu} f_{\rm eq}(1 + \delta f)$

$$f_{\rm eq} = \frac{1}{\exp\left[\left(p \cdot u - \mu_i\right)/T_{\rm frz}\right] \pm 1}, \quad \delta f = (1 \mp f_{\rm eq})\frac{p_{\mu}p_{\nu}\pi^{\mu\nu}}{2T^2_{\rm frz}(\varepsilon + P)}$$