Chiral Effects in Heavy-Ion Collisions

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Note: There are many related works on this topic so that I cannot quote them all in this lecture.
So I will keep only a few references which are essential for the discussions.
I am sorry for not mentioning all relevant works.

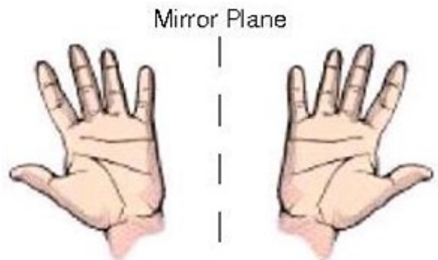
Content

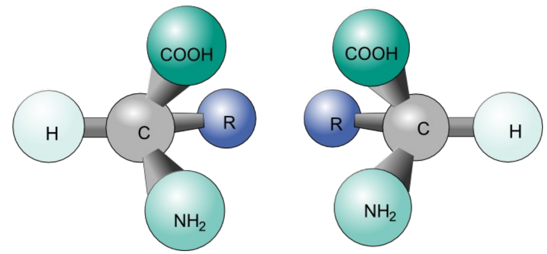
- **Chirality and chiral anomaly**
- **Anomalous chiral transport phenomena**
- **Anomalous chiral transports in heavy ion collisions**

Chirality and chiral anomaly

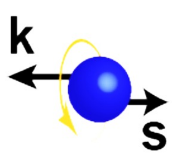
Chirality

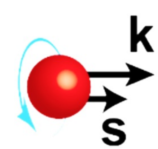
- A common concept





- For massless fermions





$|L\rangle$ $\qquad$ $|R\rangle$

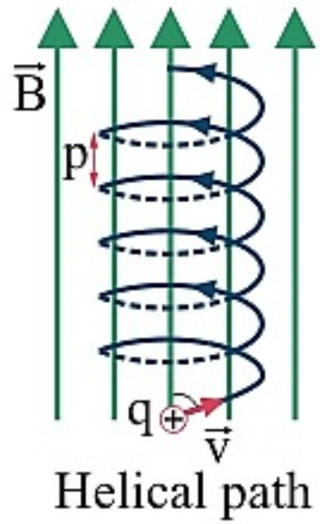
$$J_L^\mu = \bar{\psi}_L \gamma^\mu \psi_L \qquad J_R^\mu = \bar{\psi}_R \gamma^\mu \psi_R$$

- Classically

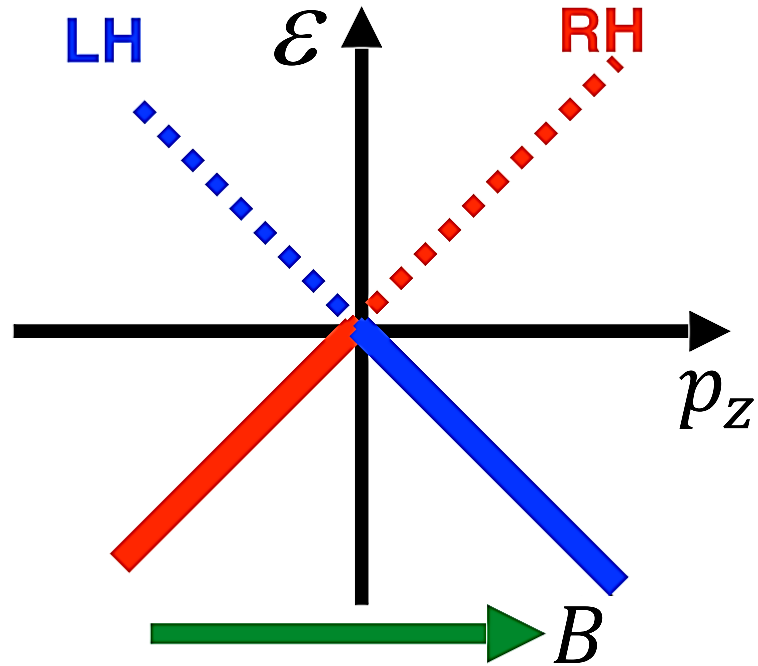
$$\partial_\mu J_V^\mu = 0 = \partial_\mu J_A^\mu \qquad \text{with} \qquad J_{V/A}^\mu = J_R^\mu \pm J_L^\mu$$

Chiral anomaly

- **Lowest Landau level of massless fermion in B**



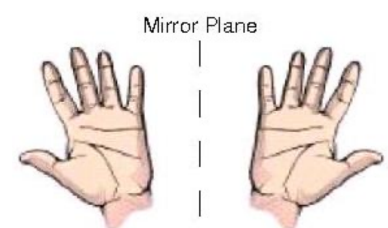
$$n = 0$$



$$E_n^2 = p_z^2 + 2n\boldsymbol{e}B$$

- **Two conserved currents with left- and right-chirality**

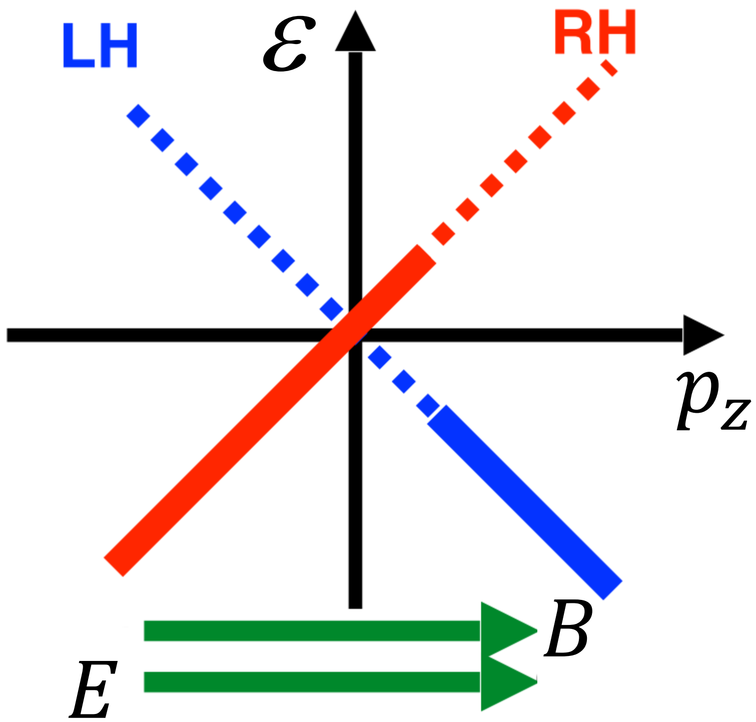
$J_R^\mu = \bar{\psi}_R \gamma^\mu \psi_R$ **and** $J_L^\mu = \bar{\psi}_L \gamma^\mu \psi_L$



Homework: Derive the Landau levels

Chiral anomaly

- Lowest Landau level of massless fermion



- One conserved current

$$J_V^\mu = J_R^\mu + J_L^\mu = \bar{\psi}\gamma^\mu\psi$$

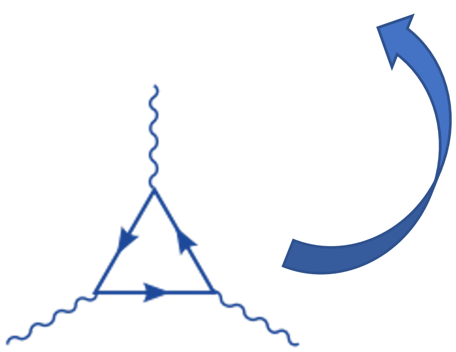
$$J_A^\mu = J_R^\mu - J_L^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

is no longer conserved:

- $N_{R/L} = V \frac{p_F^{R/L}}{2\pi}\frac{eB}{2\pi}$
- $\frac{d}{dt}N_A = \frac{d}{dt}(N_R - N_L)$

$$= V\frac{\dot{p}_F^R - \dot{p}_F^L}{2\pi}\frac{eB}{2\pi} = V\frac{eE}{\pi}\frac{eB}{2\pi}$$

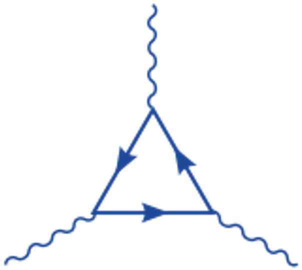
$$\Rightarrow \partial_\mu J_A^\mu = \frac{e^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}$$



Adler 1969, Bell and Jackiw 1969

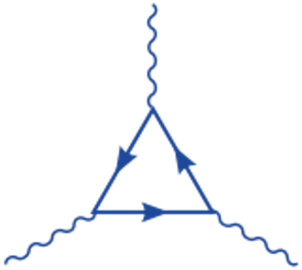
Chiral anomaly

- Triangle diagram coupled to EM field



$$\partial_\mu J_A^\mu = \frac{e^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B} = \frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

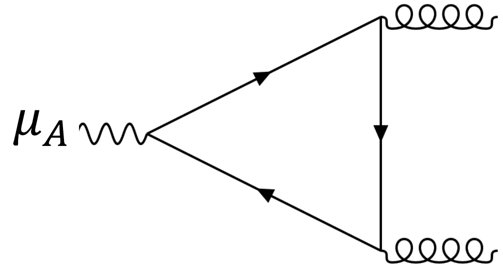
- Triangle diagram coupled to gluon field



$$\partial_\mu J_A^\mu = \frac{g^2}{16\pi^2}\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]$$

- Integrate over space and time from 0 to t

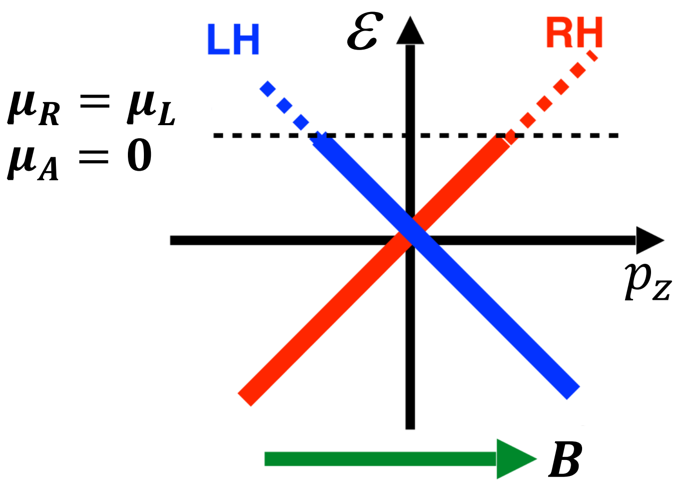
$$N_A(t) - N_A(0) = \frac{g^2}{16\pi^2}\int_0^t dt' \int d^3\mathbf{x} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

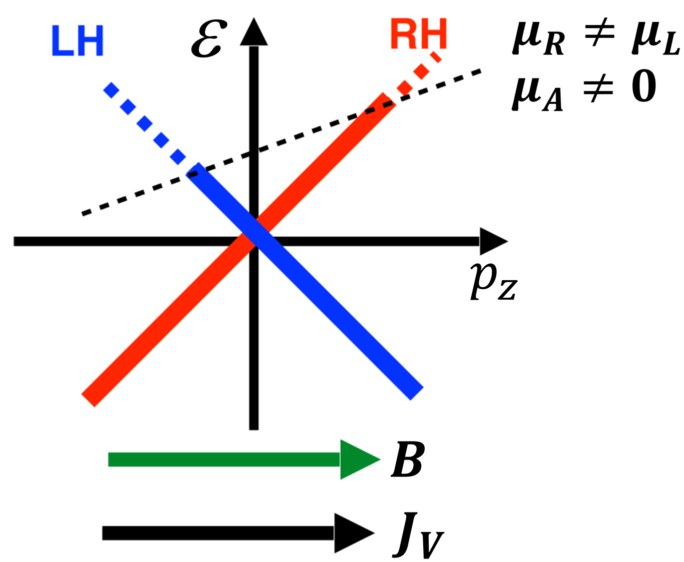


Anomalous chiral transport phenomena

Chiral magnetic effect (CME)

- Remove the E field





$$J_R = e n_R$$

$$J_L = -e n_L$$

$$n_{R/L} \equiv \frac{\mathrm{d}^3 N_{R/L}}{\mathrm{dxdydz}} = \frac{eB}{2\pi}\frac{p_\mathrm{F}^{R/L}}{2\pi}$$

$$J_V = J_R + J_L = \frac{e^2 B}{4\pi^2}\left(p_F^R - p_F^L\right)$$

$$= \frac{e^2 B}{2\pi^2}\mu_A$$

CME current

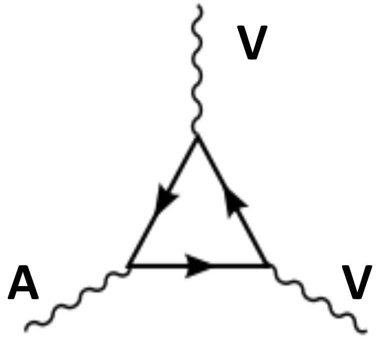
Kharzeev et al 2004-2008, Vilenkin 1980, ……

Chiral magnetic effect (CME)

- CME: vector current induced by B in matter with μ_A

$$J_V = \frac{e^2 \mu_A}{2\pi^2} B$$

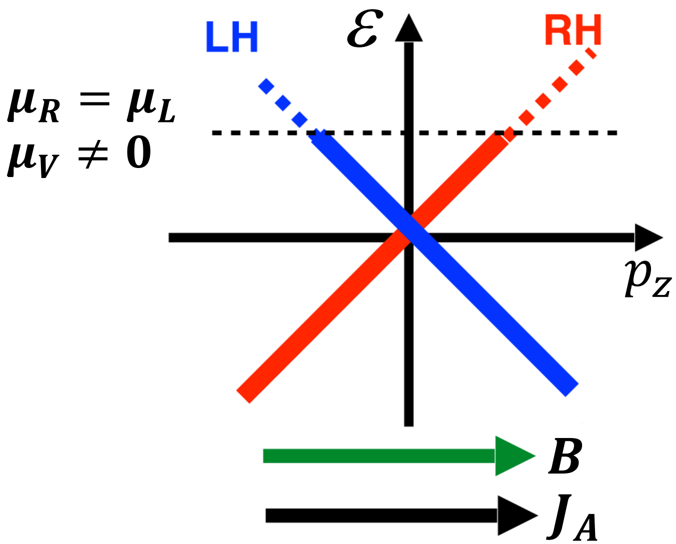
- Macroscopic quantum phenomenon
- P- and CP-odd transport
- Time-reversal even, no dissipation
- Fixed by anomaly coefficient, universal

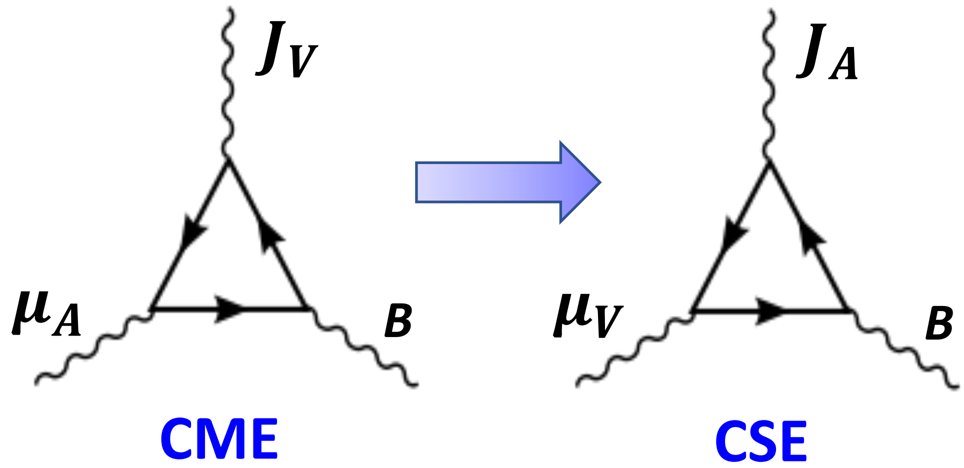


To realize CME, we need:
environmental parity violation (μ_A) and
external magnetic field (B)

Chiral separation effect (CSE)

- A dual effect to the CME: axial current induced by B in matter with μ_V





$$J_R = e n_R$$

$$J_L = -e n_L$$

$$n_{R/L} \equiv \frac{\mathrm{d}^3 N_{R/L}}{\mathrm{dxdydz}} = \frac{eB}{2\pi} \frac{p_{\mathrm{F}}^{R/L}}{2\pi}$$

$$J_A = J_R - J_L = \frac{e^2 B}{4\pi^2}\left(p_F^R + p_F^L\right)$$

$$= \frac{e^2 B}{2\pi^2} \mu_V$$

CSE current

Son and Zhitnitsky 2004 ……

Chiral vortical effect (CVE)

- **Charged particle in magnetic field and in rotation**

In magnetic field, Lorentz force:

$$\boldsymbol{F} = e(\dot{\boldsymbol{x}}\times\boldsymbol{B})$$

In rotating frame, Coriolis force:

$$\boldsymbol{F} = 2\varepsilon(\dot{\boldsymbol{x}}\times\boldsymbol{\omega}) + \boldsymbol{O}(\boldsymbol{\omega}^2)$$

Larmor theorem: $e\boldsymbol{B}\sim 2\varepsilon\boldsymbol{\omega}$

- **"Lowest Landau level" (omit centrifugal force $\boldsymbol{O}(\boldsymbol{\omega}^2)$)**

$$\left.\begin{aligned} J_R &= en_R \\ J_L &= -en_L \\ n_{R/L} &= \frac{p_F^{R/L}\omega}{2\pi}\frac{p_F^{R/L}}{2\pi} \end{aligned}\right\}$$

$$J_V = \frac{e\omega}{4\pi^2}\left((p_F^R)^2 - (p_F^L)^2\right) = \frac{e\omega}{\pi^2}\mu_V\mu_A$$

$$J_A = \frac{e\omega}{4\pi^2}\left((p_F^R)^2 + (p_F^L)^2\right) = \frac{e\omega}{2\pi^2}(\mu_V^2 + \mu_A^2)$$

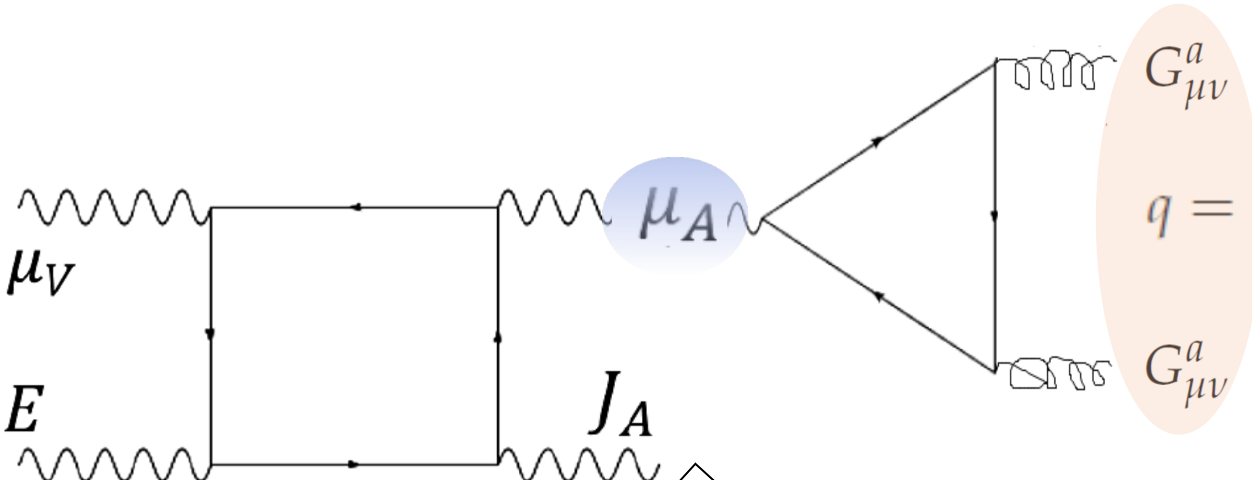
CVE currents

More rigorous calculation shows a $(T^2/6)e\boldsymbol{\omega}$ term in $\boldsymbol{J}_A$ related to gravitational anomaly or global anomaly. **(Landsteiner etal 2011, Glorioso etal 2017)**

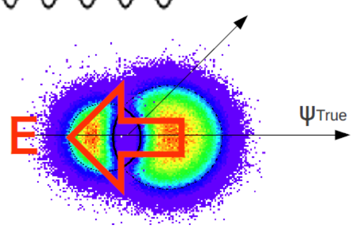
Erdmenger etal 2008, Banerjee etal 2008, Son and Surowka 2009 ......

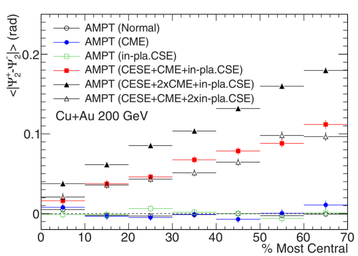
Chiral electric separation effect

- **Electric field induced anomalous transport**



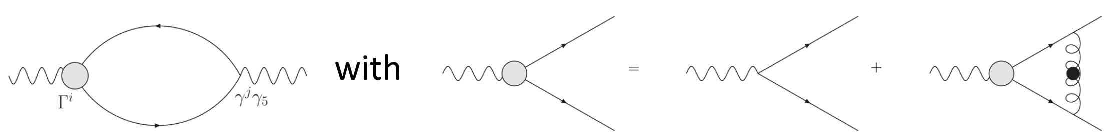
$$q = \frac{g^2}{32\pi^2}\int d^4x G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$





Chiral electric separation effect (CESE)

$$\mathbf{J}_A \approx 14.5163\,\mathrm{Tr}_f(Q_eQ_A)\frac{\mu_V\mu_A}{T^2}\frac{e^2T}{g^4\ln(1/g)}\mathbf{E}$$



XGH and Liao 2013, Jiang, XGH, Liao 2015

Ma and XGH 2015

Collective modes: chiral magnetic waves

- Consider CME and CSE in constant magnetic field

$$\boldsymbol{J}_R = \frac{e^2}{4\pi^2}\mu_R \boldsymbol{B} \qquad \boldsymbol{J}_L = -\frac{e^2}{4\pi^2}\mu_L \boldsymbol{B}$$

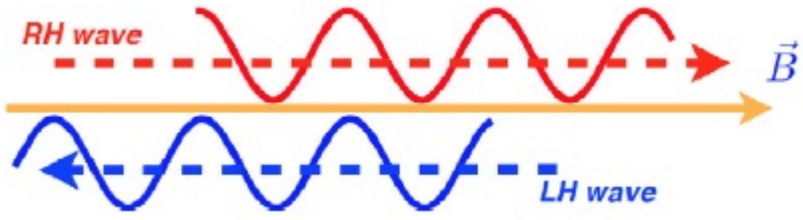
- Consider the continuity equations

$$\partial_t J^0_{R/L} + \boldsymbol{\nabla}\cdot \boldsymbol{J}_{R/L} = 0$$

- Consider small fluctuations of RH/LH densities

$$\partial_t \delta J^0_R + \frac{e^2}{4\pi^2\chi_R}\boldsymbol{B}\cdot\boldsymbol{\nabla}\delta J^0_R = 0 \qquad \partial_t \delta J^0_L - \frac{e^2}{4\pi^2\chi_L}\boldsymbol{B}\cdot\boldsymbol{\nabla}\delta J^0_L = 0$$

- These are two wave equations describing two gapless collective modes (Chiral magnetic wave) with velocities



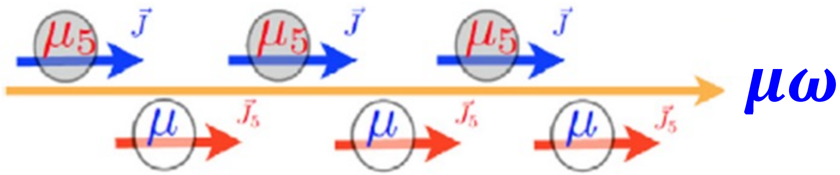
$$v_L = \frac{e^2}{4\pi^2\chi_L} \qquad v_R = \frac{e^2}{4\pi^2\chi_R}$$

Kharzeev-Yee 2011

Collective modes: chiral vortical wave

The vortical analogue of chiral magnetic wave

$$J_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}\,\boldsymbol{\omega}, \qquad J_V = \frac{1}{\pi^2}\mu_V\mu_A\boldsymbol{\omega},$$



- To reveal its dispersion we use continuity eq.

$$\partial_t n_{L,R} + \nabla \cdot \vec{J}_{L,R} = 0$$

- Substitute CVE currents. Obtain Burgers wave equation which is linearized to normal wave equation**(Homework)**

$$\partial_t n_{L,R} = \pm \frac{\omega\alpha^2}{\pi^2}\partial_x(n_{L,R}^2) \Longrightarrow \pm \boxed{\frac{2\omega\alpha^2}{\pi^2}} n_0 \partial_x(n_{L,R})$$

$\alpha = \frac{\partial\mu}{\partial n}$, ~ inverse baryon susceptiblity

CVW velocity

arXiv: 1504.03201

Collective modes: chiral electric wave

- The complete electromagnetic response of a chiral matter:

$$\begin{aligned} j_V^\mu &= \sigma E^\mu + \frac{e}{2\pi^2}\mu_A B^\mu, \\ j_A^\mu &= \sigma_5 E^\mu + \frac{e}{2\pi^2}\mu_V B^\mu. \end{aligned}$$

- Coupled evolution of vector, axial currents and E^μ, B^μ leads to several collective modes **(XGH and Liao, PRL110(2013)232302)**:
  - If $\mathbf{B} = B\hat{z}$ and $\mathbf{E} = 0$: two Chiral magnetic waves along $\mathbf{B}$

$$\omega = \pm\sqrt{(v_\chi k_z)^2 - (e\sigma_0/2)^2} - i(e\sigma_0/2)$$

  - If $\mathbf{B} = 0$ and $\mathbf{E} = E\hat{z}$ + A-background: two Chiral electric waves

$$\omega = \pm\sqrt{(v_e k_z)^2 - (e\sigma_0/2)^2} - i(e\sigma_0/2)$$

  - If $\mathbf{B} = 0$ and $\mathbf{E} = E\hat{z}$ + V-background: one Vector density wave and one Axial density wave along E-field

$$\begin{aligned} \omega_V &= v_v k_z - ie\sigma_0, \\ \omega_A &= v_a k_z \end{aligned}$$

Table of anomalous chiral transports

	$\boldsymbol{E}$	$\boldsymbol{B}$	$\boldsymbol{\omega}$
J_V	σ Ohm's law	$\frac{e^2}{2\pi^2}\mu_A$ Chiral magnetic effect	$\frac{e}{\pi^2}\mu_V\mu_A$ Vector chiral vortical effect
J_A	$\propto \frac{\mu_V\mu_A}{T^2}\sigma$ Chiral electric separation effect	$\frac{e^2}{2\pi^2}\mu_V$ Chiral separation effect	$e(\frac{T^2}{6}+\frac{\mu_V^2+\mu_A^2}{2\pi^2})$ Axial chiral vortical effect
Wave mode	$\varepsilon = \alpha_A n_A\sqrt{2\sigma_2\chi_e\alpha_V\alpha_A}\mathbf{k}\cdot\boldsymbol{E}$ Chiral electric wave	$\varepsilon = \sigma_A\sqrt{\alpha_V\alpha_A}\mathbf{k}\cdot\boldsymbol{B}$ Chiral magnetic wave	$\varepsilon = \frac{\mu_V}{2\pi^2\chi_\mu}\mathbf{k}\cdot\boldsymbol{\omega}$ Chiral vortical wave

Reviews: Kharzeev, arXiv:1312.3348;

XGH, arXiv:1509.04073;

Kharzeev-Liao-Voloshin-Wang, arXiv:1511.04050 ;

Liu-XGH, arXiv:2003.12482;

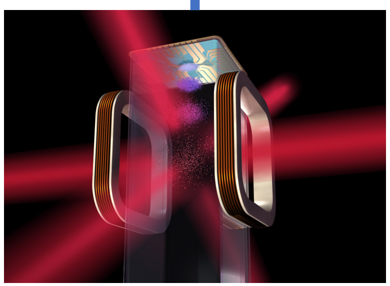
… …

Anomalous chiral transports

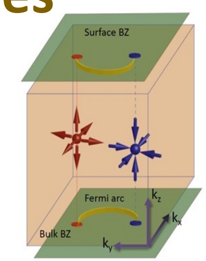
- Universal phenomena that **may** happen across a very broad hierarchy of scales.

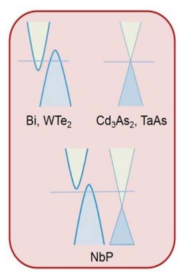
$10^{-10}K$ | $1-10^{2}K$ | $10^{10}K$ | $10^{12}K$

Temperature

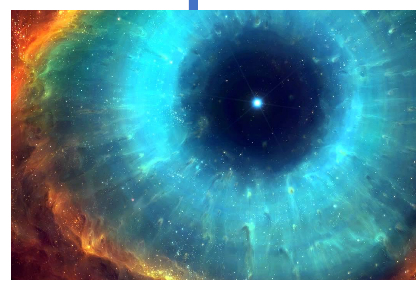


Cold atomic gases

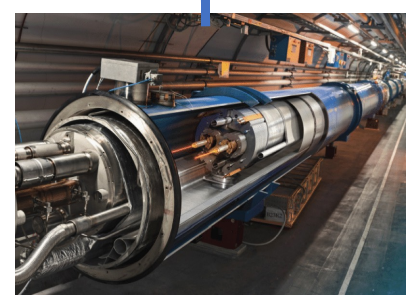




Weyl/Dirac semimetals



Supernovae



Heavy-ion collisions

Electromagnetic force

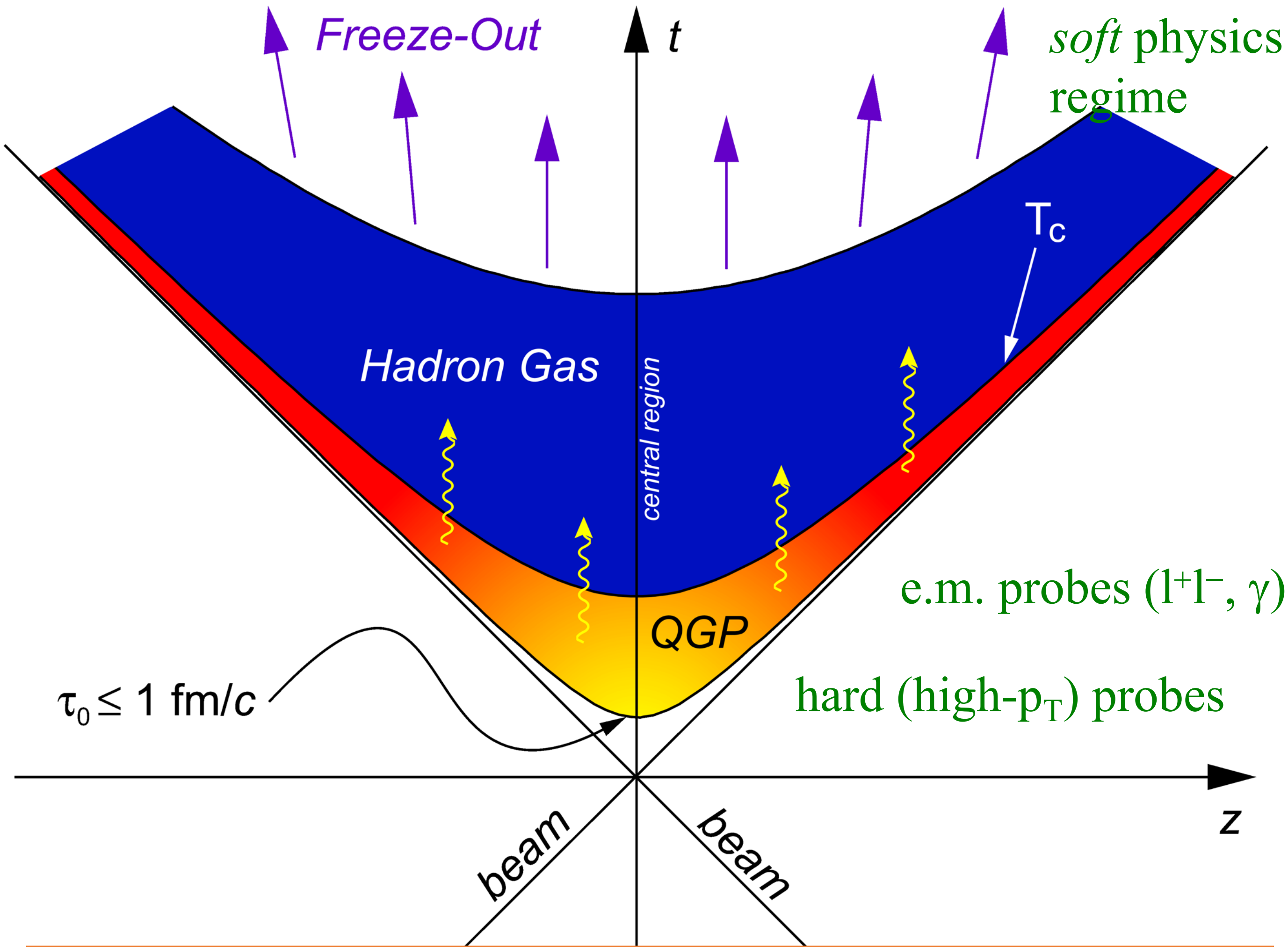
Electroweak force

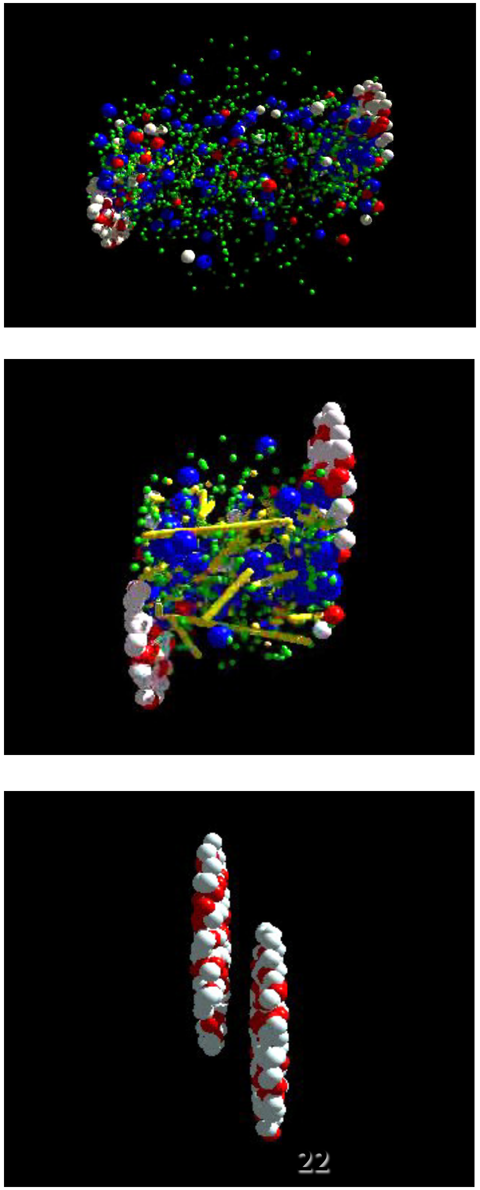
Strong force

Anomalous chiral transports in heavy ion collisions

Question: **Electromagnetic fields and vorticity in heavy-ion collisions?**

Time Scales of a Relativistic Heavy Ion Collisions





Chemical freezeout ($T_{ch} \leq T_c$) : inelastic scattering stops

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering stops

Highest artificial temperature

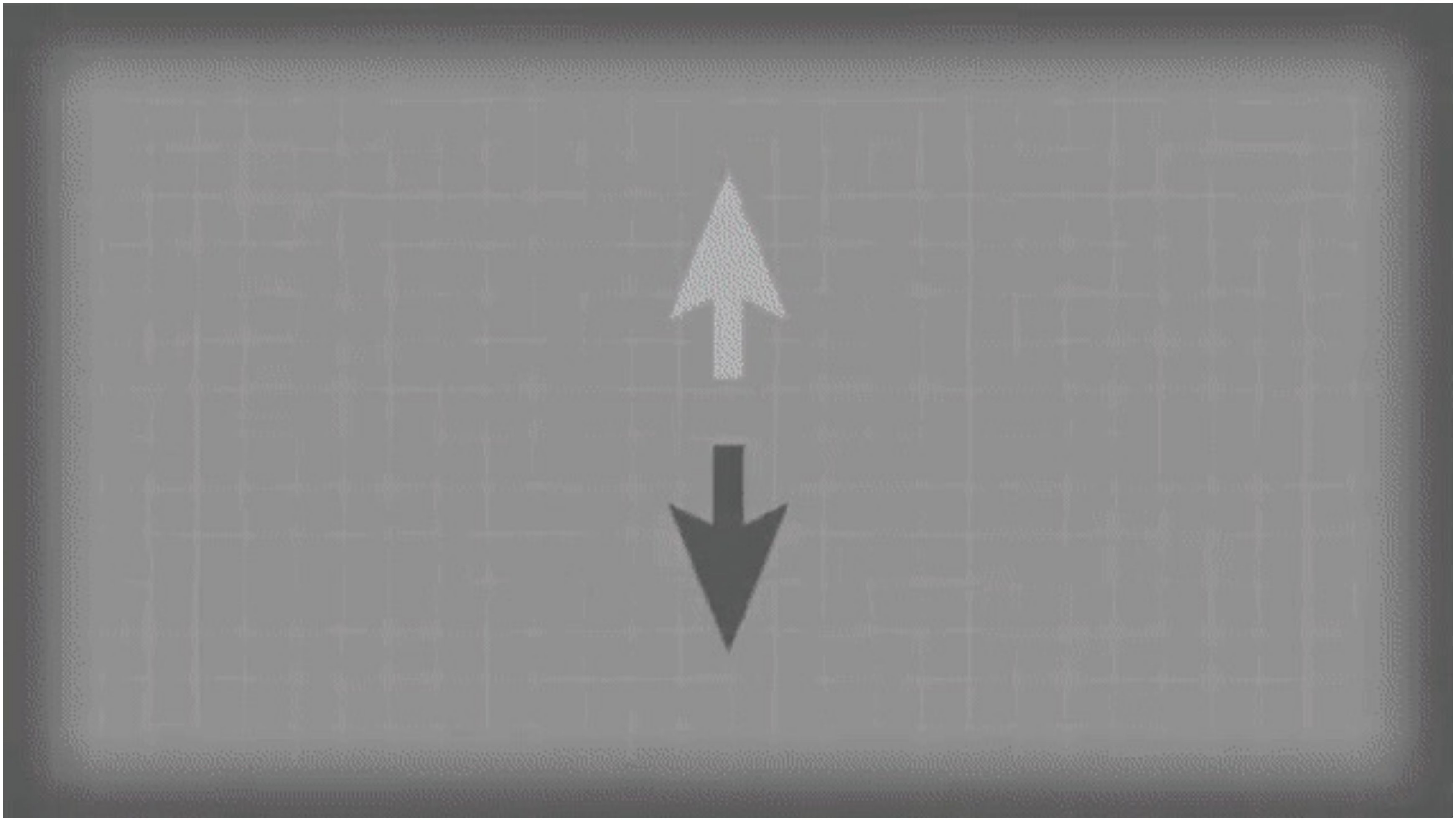
https://www.guinnessworldrecords.com/world-records/highest-man-made-temperature

Who	What
CERN, LARGE HADRON COLLIDER	5X10^12 DEGREE(S) KELVIN

Where	When
SWITZERLAND ()	13 AUGUST 2012

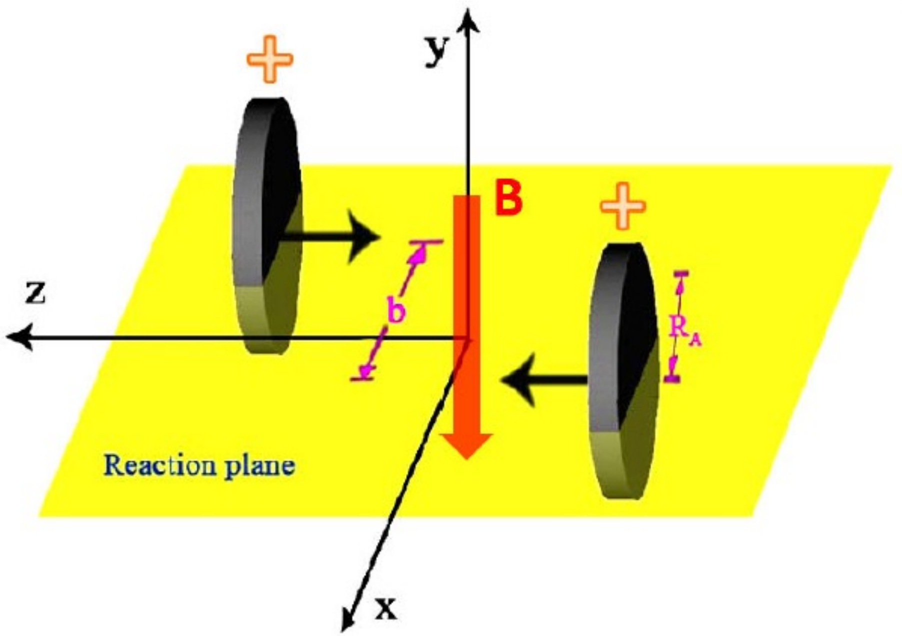
On 13 August 2012 scientists at CERN's Large Hadron Collider, Geneva, Switzerland, announced that they had achieved temperatures of over 5 trillion K and perhaps as high as 5.5 trillion K. The team had been using the ALICE experiment to smash together lead ions at 99% of the speed of light to create a quark gluon plasma – an exotic state of matter believed to have filled the universe just after the Big Bang.

The strongest magnet!



Cartoon from BNL

The strongest magnet!

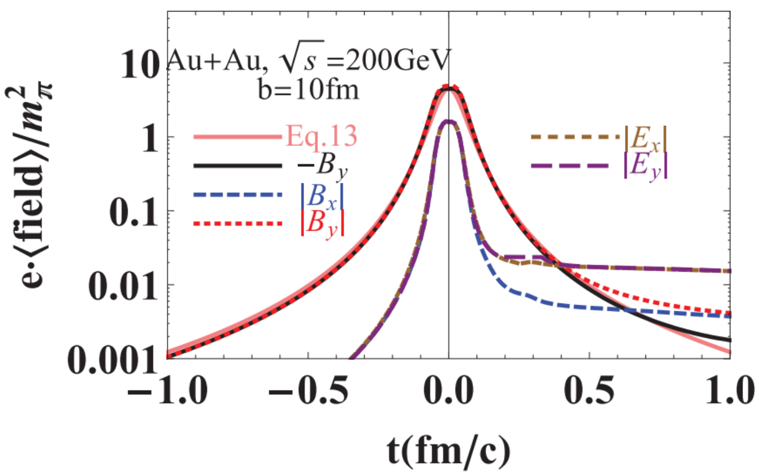


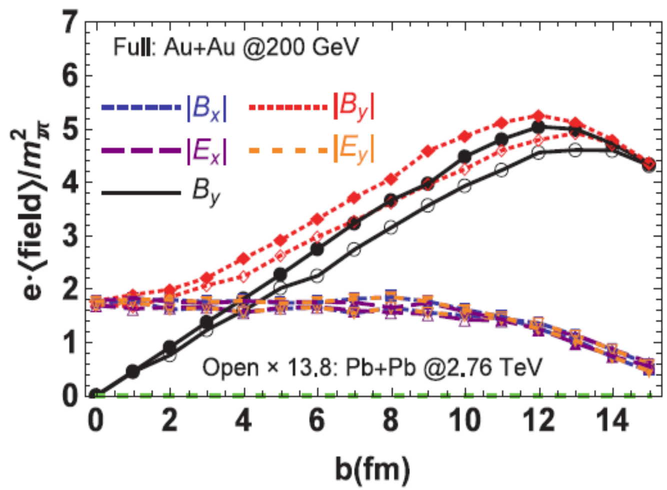
- RHIC Au+Au collision, $Z = 79$, $\sqrt{s} = 200$ GeV ($\Rightarrow v_z \simeq 0.99995c$), impact parameter $b = 5$ fm
- The B field at the colliding time, $t = 0$. Biot-Savart law

$$eB_y \sim 2 \times \gamma \frac{e^2}{4\pi} Z v_z (2/b)^2 \approx 40 m_\pi^2 \sim 10^{19}\text{Gauss}$$

$$1\ \text{MeV}^2 = e \cdot 1.6904 \times 10^{14}\ \text{Gauss}$$

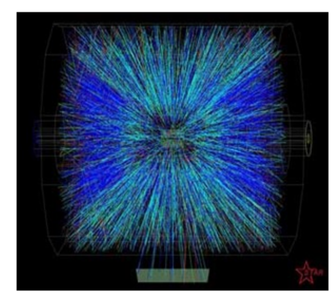
The strongest magnet!

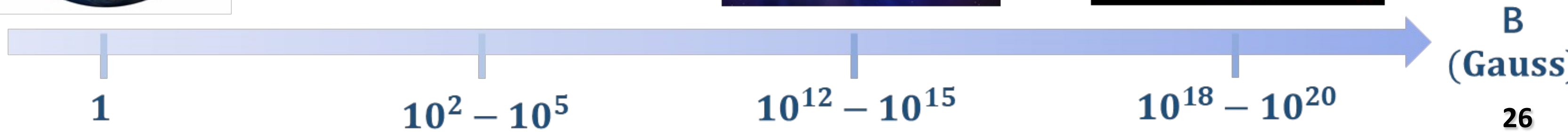




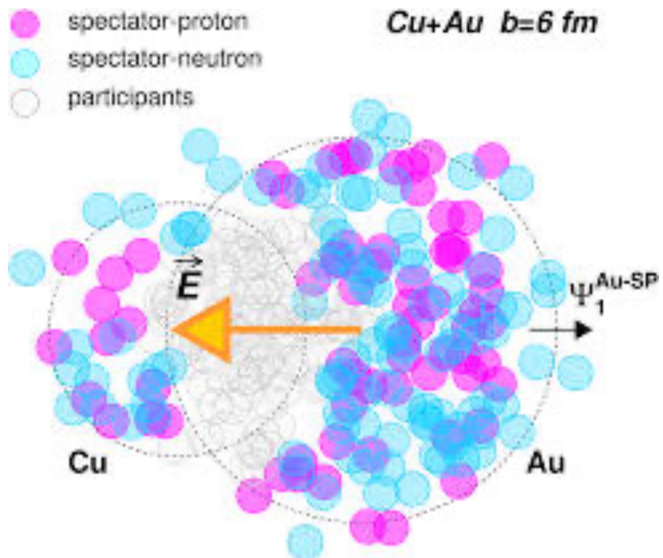
[arXiv:1201.5108](https://arxiv.org/abs/1201.5108) [arXiv:1509.04073](https://arxiv.org/abs/1509.04073)

Strongest B fields we have known in current universe:
$eB \sim 10^{18}$ G (RHIC)- 10^{20} G (LHC)

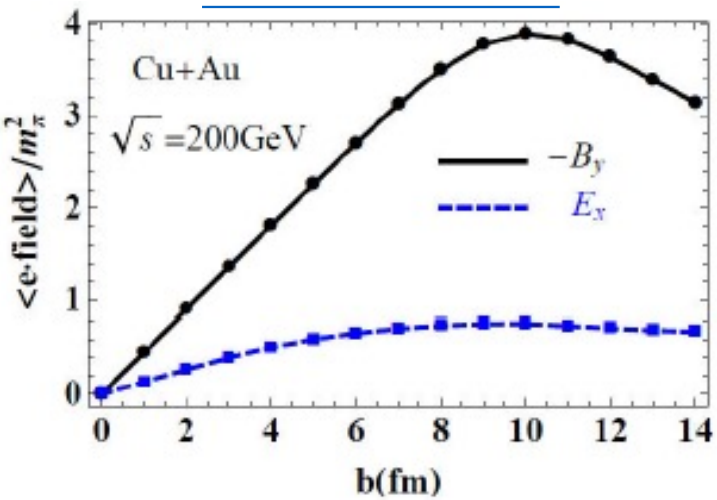
Earth	Magnet	Neutron star	Heavy-ion collisions
			



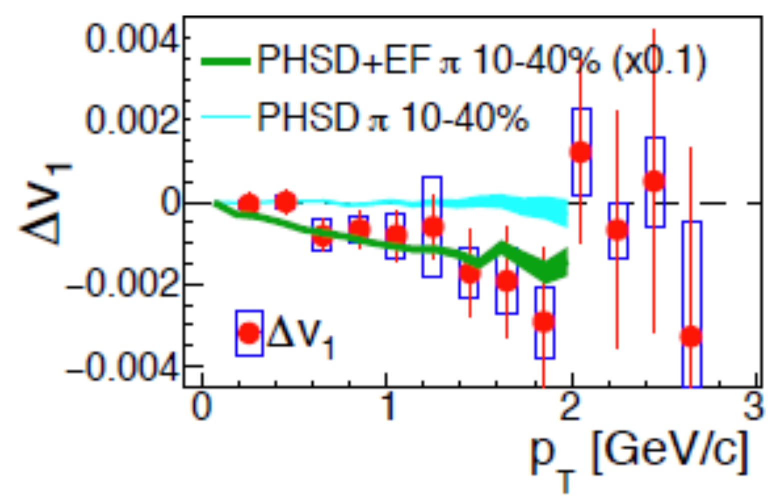
The strongest electric field!



arXiv:1411.2733



Charge dependence of v1



How to compute the EM fields?

Consider a moving point charge (for example, a proton):

In the rest frame (with a prime):

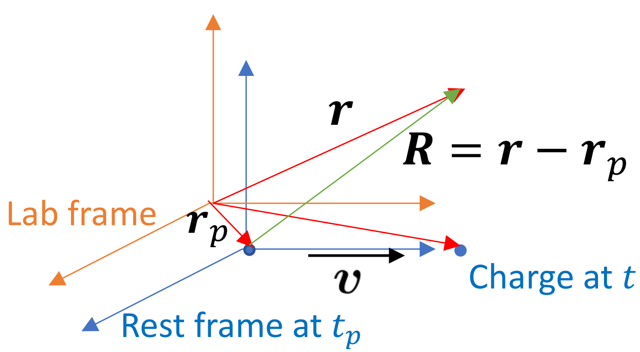
$$e\boldsymbol{E}' = \frac{e^2}{4\pi}\frac{\boldsymbol{r}'}{r'^3},$$
$$e\boldsymbol{B}' = 0.$$

In the Lab frame:

$$E_z = E'_z, \;\; B_z = B'_z,$$

$$\boldsymbol{E}_\perp = \gamma(\boldsymbol{E}' - \boldsymbol{v}\times\boldsymbol{B}')_\perp, \boldsymbol{B}_\perp = \gamma(\boldsymbol{B}' + \boldsymbol{v}\times\boldsymbol{E}')_\perp$$

Coordinate relation:



$$z' = \gamma[Z - v(t - t_p)], x' = X, y' = Y$$

$$t - t_p = R$$

Retardation effect

How to compute the EM fields?

Combine the equations above (Homework):

$$e\boldsymbol{E}(t,\boldsymbol{r}) = \frac{e^2}{4\pi}\frac{(\boldsymbol{R}-R\boldsymbol{v})(1-v^2)}{(R-\boldsymbol{R}\cdot\boldsymbol{v})^3},$$

$$e\boldsymbol{B}(t,\boldsymbol{r}) = e\boldsymbol{v}\times\boldsymbol{E}(t,\boldsymbol{r}) = \frac{e^2}{4\pi}\frac{(\boldsymbol{v}\times\boldsymbol{R})(1-v^2)}{(R-\boldsymbol{R}\cdot\boldsymbol{v})^3}$$

Extended to N particles (e.g., for Au, N=79):

$$e\boldsymbol{E}(t,\boldsymbol{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\boldsymbol{R}_n - R_n\boldsymbol{v}_n}{(R_n-\boldsymbol{R}_n\cdot\boldsymbol{v}_n)^3}(1-v_n^2),$$

$$e\boldsymbol{B}(t,\boldsymbol{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\boldsymbol{v}_n\times\boldsymbol{R}_n}{(R_n-\boldsymbol{R}_n\cdot\boldsymbol{v}_n)^3}(1-v_n^2),$$

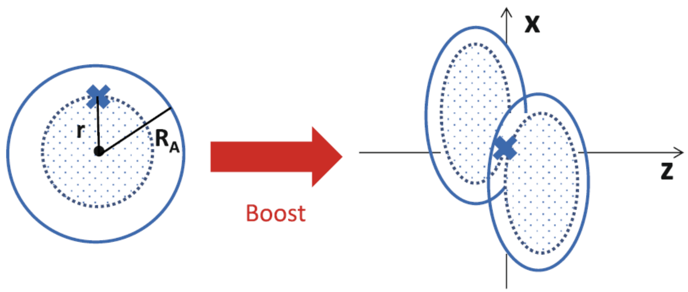
Z_n, v_n = charge and velocity of the nth particle

This is the Lienard-Wiechert formula

How to compute the EM fields?

From HIJING or AMPT or UrQMD or other event generators, applying these formulas give the EM fields shown before.

As an exercise, we consider continuous charge distribution as an approximation.



$$Z_{\mathrm{eff}}^{\pm}(t, \boldsymbol{x}) = 4\pi \int_0^{r^\pm} \mathrm{d}r'\, r'^2 \rho(r') \qquad r^\pm = \sqrt{(x \pm b/2)^2 + y^2 + \gamma^2 (z - v^\pm t)^2}$$

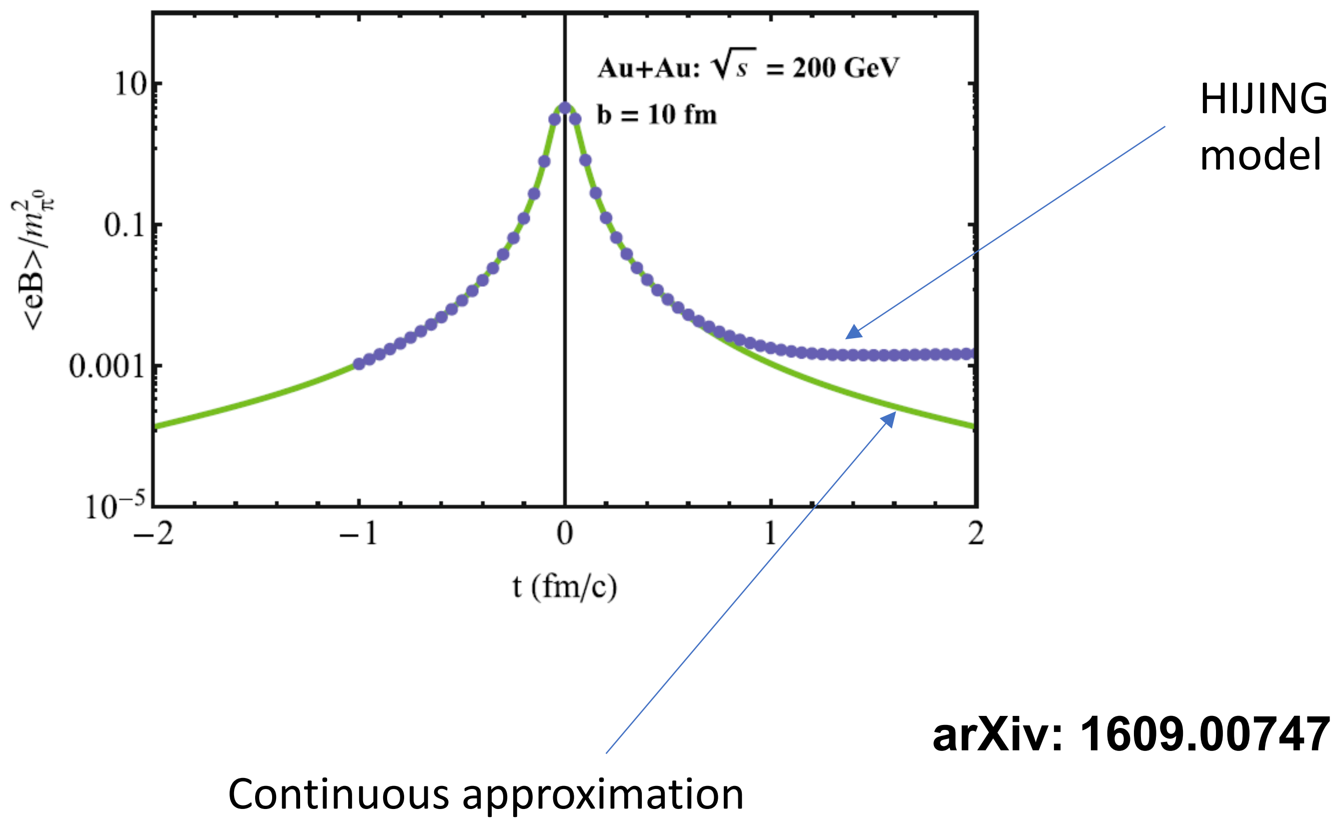
$$\rho(r) = N_Z / \{1 + \exp[(r - R_A)/a]\} \text{ with } N_Z = Z\{4\pi \textstyle\int_0^{R_A} \mathrm{d}r' r'^2 \rho(r')\}^{-1}$$

$Z = 79$, $R_A = 6.38$ fm, and $a = 0.535$ fm for Au, and
$Z = 82$, $R_A = 6.62$ fm, and $a = 0.546$ fm for Pb.

How to compute the EM fields?

Homework: using this continuous approximation, calculate the magnetic field at the center of the collision region in Au + Au collisions at 200 GeV.

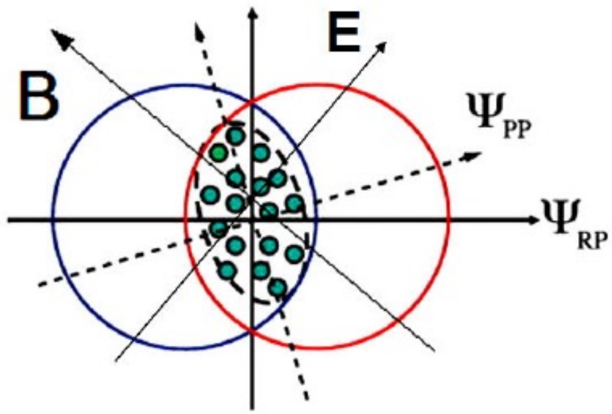
Answer:

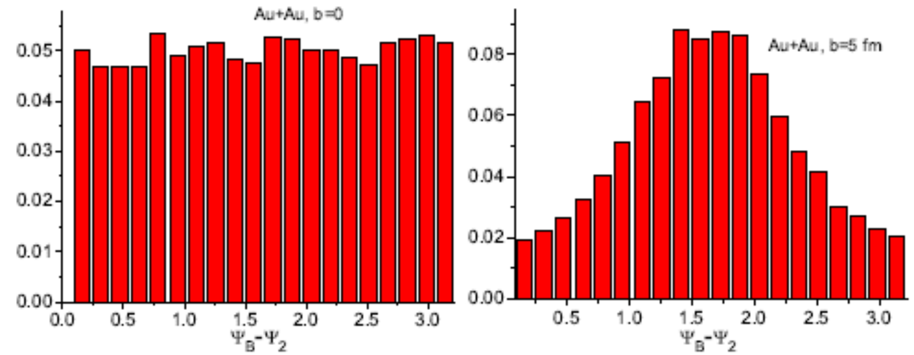


arXiv: 1609.00747

Some properties of EM field in HICs

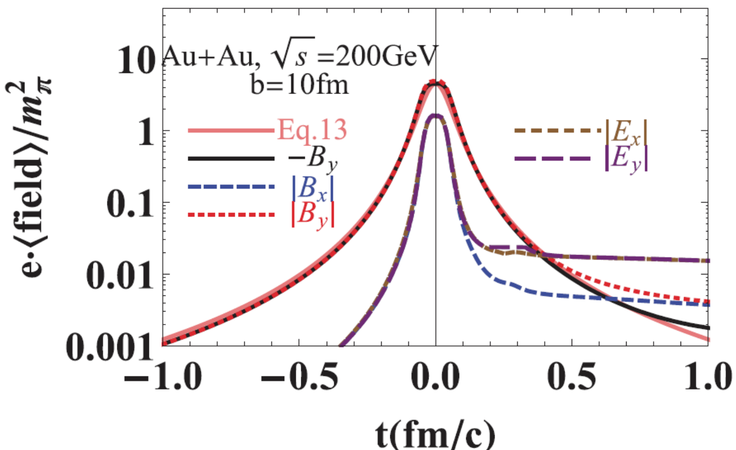
Azimuthal fluctuation





arXiv: 1209.6594

Time evolution of the B field (vacuum)



arXiv:1509.04073

Well fitted by

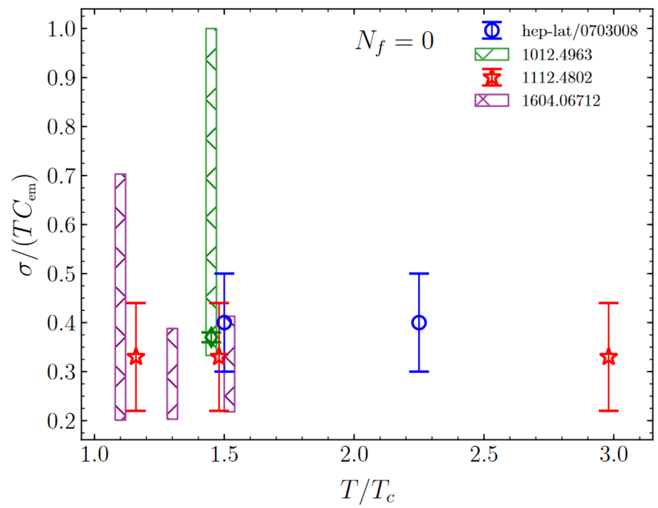
$$\langle eB_y(t)\rangle \approx \frac{\langle eB_y(0)\rangle}{(1+t^2/t_B^2)^{3/2}}$$

Life time of B field

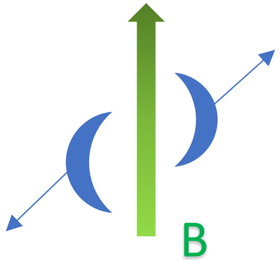
$$t_B \approx R_A/(\gamma v_z) \approx \frac{2m_N}{\sqrt{s}} R_A$$

Some properties of EM field in HICs

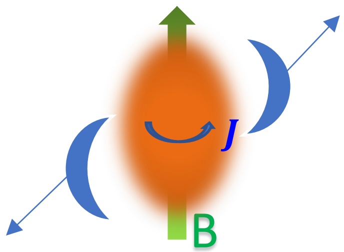
Quark gluon plasma (QGP) is a very good conductor



Time evolution of the B field (conducting medium)



In vacuum:
moving charges



In conductor:
Faraday effect

Some properties of EM field in HICs

Time evolution of the B field (conducting medium): Solving the coupled Boltzmann and Maxwell equations

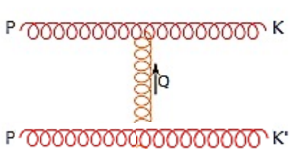
$$
\begin{cases}
[p^\mu \partial_\mu + eQ_a p_\mu F^{\mu\nu} \partial_{p^\nu}] f_a(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f_a] \\
\partial_\mu F^{\mu\nu} = j^\nu \\
j^\mu = e \sum_F Q_F s_F \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_p} p^\mu \left(f_q^F - f_{\bar{q}}^F \right)
\end{cases}
\qquad a = q, \bar{q}, g
$$

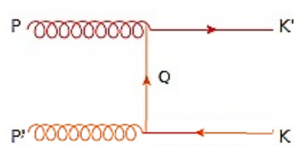
arXiv: 2104.00831

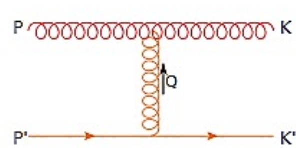
Initial condition for EM field: moving colliding nuclei in vacuum
Initial condition for q and g: CGC inspired distribution

$$
\begin{aligned}
\mathcal{C}[f_{\mathbf{p}}^a] = & \frac{1}{2E_p \nu_a} \sum_{b,c,d} \frac{1}{s_{cd}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} \frac{d^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}} \frac{d^3\mathbf{k}'}{(2\pi)^3 2E_{\mathbf{k}'}} \\
& \times (2\pi)^4 \delta^{(4)}(P + P' - K - K') |\mathcal{M}_{cd}^{ab}|^2 \\
& \times \left[f_{\mathbf{k}}^c f_{\mathbf{k}'}^d (1 + \epsilon_a f_{\mathbf{p}}^a)(1 + \epsilon_b f_{\mathbf{p}'}^b) - f_{\mathbf{p}}^a f_{\mathbf{p}'}^b (1 + \epsilon_c f_{\mathbf{k}}^c)(1 + \epsilon_d f_{\mathbf{k}'}^d) \right]
\end{aligned}
$$

$$
|\mathcal{M}|^2 \ni gg \leftrightarrow q\bar{q},\ gq \leftrightarrow gq,\ g\bar{q} \leftrightarrow g\bar{q},\ gg \leftrightarrow gg
$$



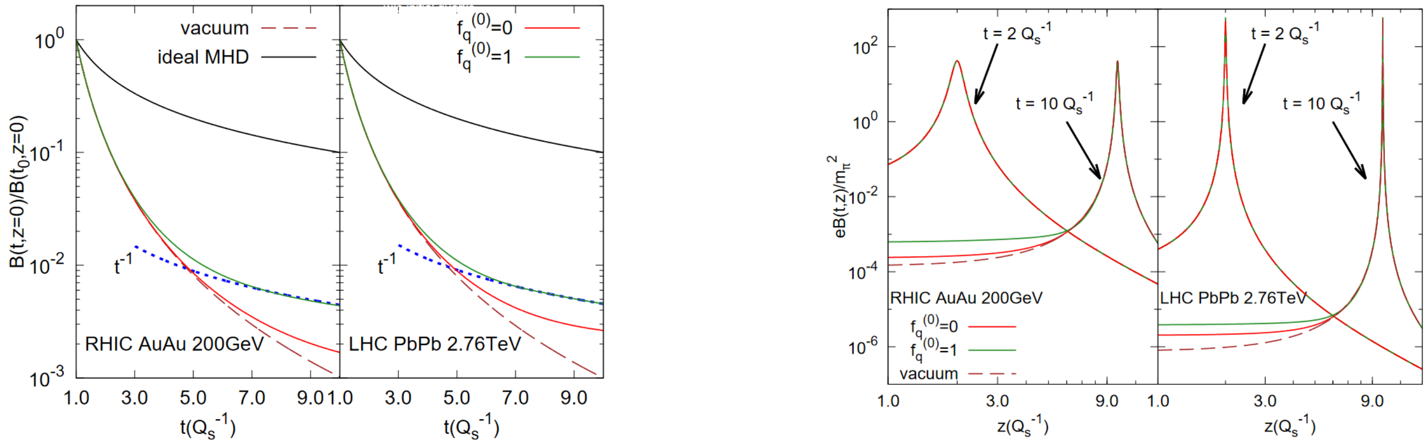




Plus s, u channels

Some properties of EM field in HICs

Time evolution of the B field (conducting medium): Solving the coupled Boltzmann and Maxwell equations

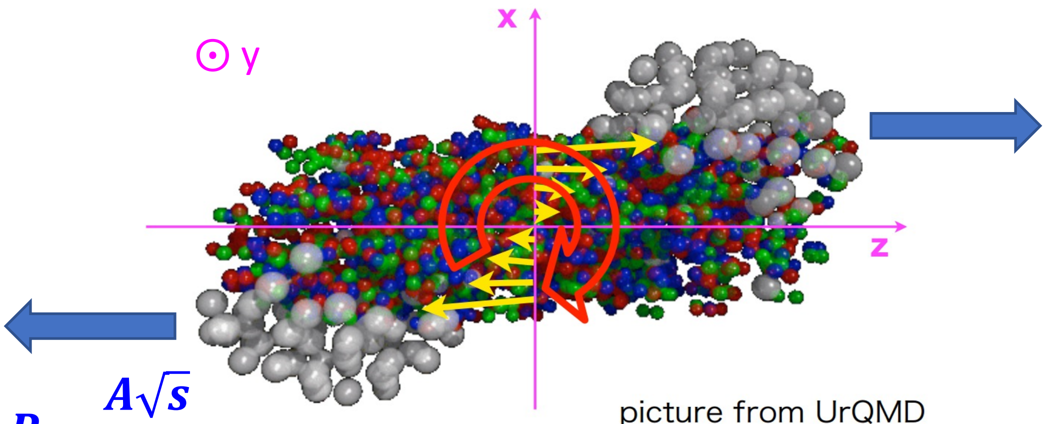


arXiv: 2104.00831

After the kinetic evolution, the system is thermalized, and after that the time evolution can be obtained by solving relativistic magnetohydrodynamics.

But the full time evolution is still unknown.

Angular momentum in HICs



$$P_z \sim \frac{A\sqrt{s}}{2}$$

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

Global angular momentum

(RHIC Au+Au 200 GeV, b=10 fm)

Fluid vorticity in HICs

Global angular momentum

Local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$$

(Angular velocity of fluid cell)

Let us estimate the vorticity:

$$\boldsymbol{J} \sim \int d^3x I(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x})$$

$I(\boldsymbol{x}) \sim [\boldsymbol{x}^2 - (\boldsymbol{x}\cdot\hat{\boldsymbol{\omega}})^2]\varepsilon(\boldsymbol{x})$ is the moment of inertia density

$$\varepsilon = \left[2(N_c^2 - 1) + \frac{7}{4}2N_cN_f\right]\frac{\pi^2}{30}T^4$$

Fluid vorticity in HICs

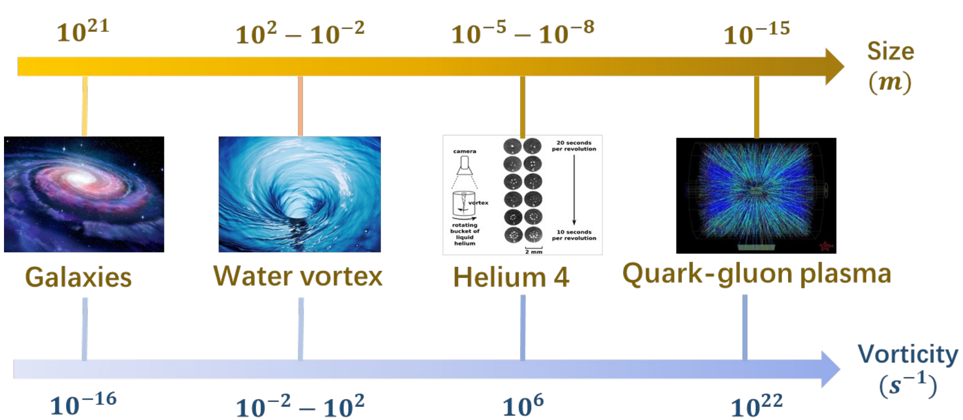
Let us estimate the vorticity:

$$J \sim \int d^3x I(x)\omega(x)$$

$I(x) \sim [x^2 - (x\cdot\hat{\omega})^2]\varepsilon(x)$ is the moment of inertia density

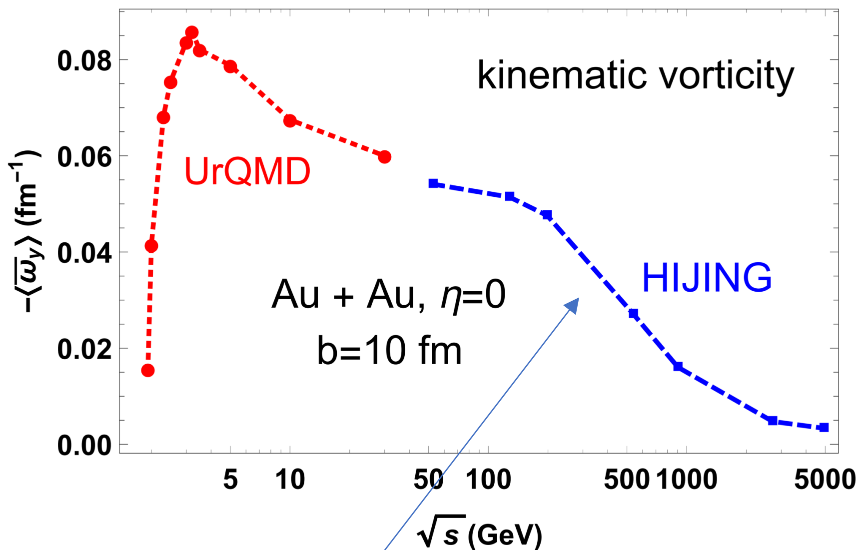
$$\varepsilon = \left[2(N_c^2-1) + \frac{7}{4}2N_cN_f\right]\frac{\pi^2}{30}T^4$$

Consider Au+Au@200GeV, $T = 300$MeV, system size 10 fm (Homework): $\omega \sim 10^{22}s^{-1}$



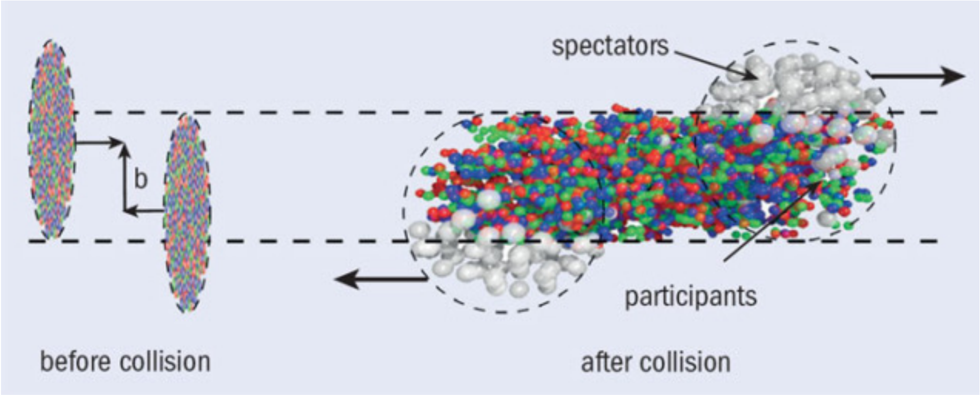
Fluid vorticity in HICs

More rigorous computation:

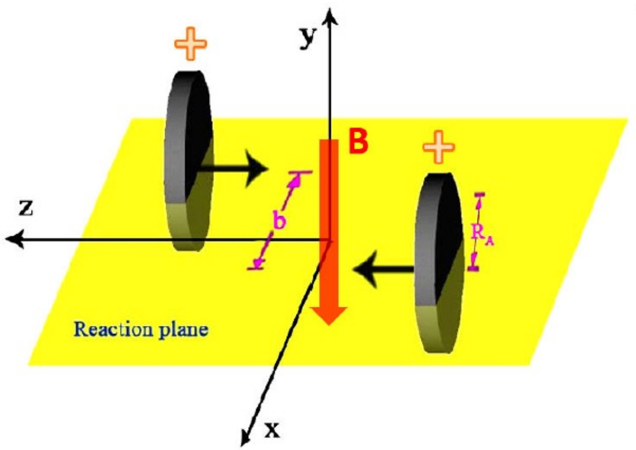


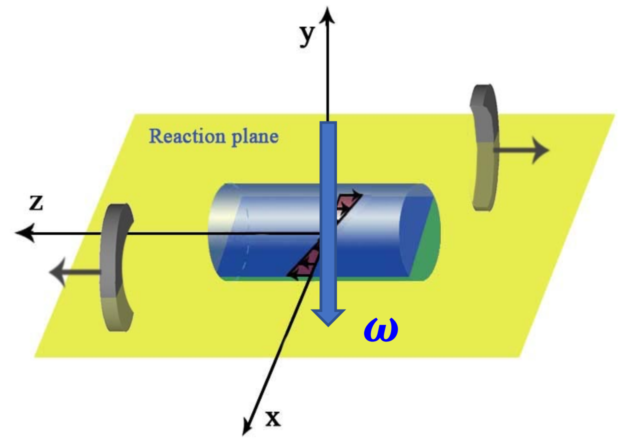
arXiv:1603.06117,
arXiv:2001.01371,
arXiv:2002.07549

Why decreasing with increasing energy?



- **Heavy-ion collisions can generate**





Strongest EM fields

Largest local vorticity

Anomalous chiral transports
But where μ_A comes from?

QED vacuum

In 3+1 dimension, QED Hamiltonian

$$H_{\text{photon}} = \frac{1}{2}\int d^3\mathbf{x}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$$

QED vacuum

$$\boldsymbol{E} = \boldsymbol{B} = \mathbf{0}$$

Thus QED vacuum is described by pure gauge $A_\mu = \partial_\mu f$

Consider the Chern-Simons term (magnetic helicity) which counts the winding number of magnetic lines (and thus is topological)

$$h = \int d^3\mathbf{x}\epsilon_{ijk}A_i\partial_j A_k$$

It is zero in (3+1)-D QED vacuum. So QED vacuum is topologically trivial.

*Note: in (1+1)-D, QED vacuum is topologically nontrivial.

$$N_{\mathrm{CS}} = -\frac{i}{2\pi}\int d\theta U^{-1}\partial_\theta U\,,\ U \in U(1)$$

QCD vacuum

In 3+1 dimension, QCD Hamiltonian

$$H_{\text{gluon}} = \frac{1}{2}\sum_a \int d^3\mathbf{x}(\boldsymbol{E}_a^2 + \boldsymbol{B}_a^2)$$

QCD vacuum

$$\boldsymbol{E}_a = \boldsymbol{B}_a = \mathbf{0}$$

Thus QCD vacuum is described by pure gauge $\mathcal{A}_i(\boldsymbol{x}) = \mathrm{i}g^{-1}U^{-1}(\boldsymbol{x})\partial_i U(\boldsymbol{x})$

$$U(x) \in SU(3)$$

Consider the Chern-Simons term (the coupling constant is absorbed into gauge field)

$$\begin{aligned} h &= \int d^3\mathbf{x}\epsilon_{ijk}\mathrm{Tr}\left[\mathcal{A}_i\partial_j\mathcal{A}_k - \frac{2}{3}i\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\right] \\ &= \frac{1}{3}\int d^3\mathbf{x}\epsilon_{ijk}\mathrm{Tr}\left[U^{-1}\partial_i UU^{-1}\partial_j UU^{-1}\partial_k U\right] \\ &= 8\pi^2 N_{\mathrm{CS}} \end{aligned}$$

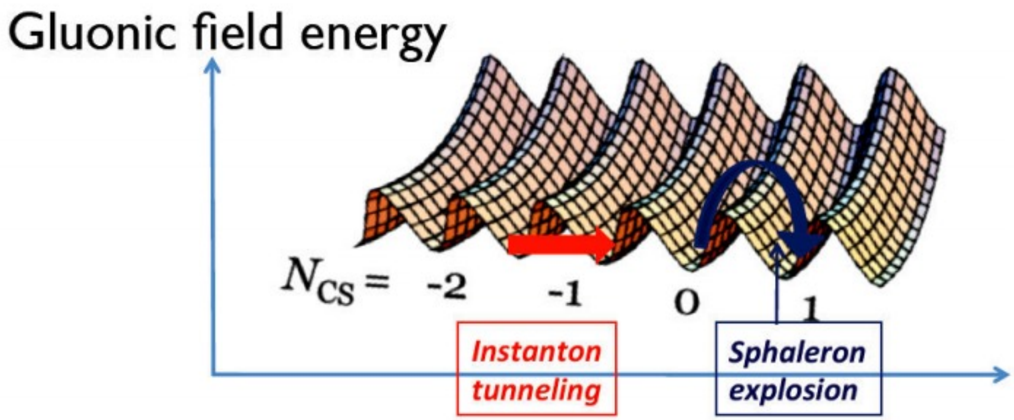
$$\boldsymbol{N_{CS} = \frac{1}{24\pi^2}\int d^3x\, \varepsilon^{ijk}\mathrm{tr}\left[\left(U^{-1}\partial_i U\right)\left(U^{-1}\partial_j U\right)\left(U^{-1}\partial_k U\right)\right], U \in SU(3)}$$

It is non-zero! So in (3+1)-D QCD vacuum is topologically non-trivial!

*Homework: Show that N_CS is an integer and counting the winding number from S^3 to S^3

QCD vacuum

QCD topologically nontrivial vacuum: The theta vacuum

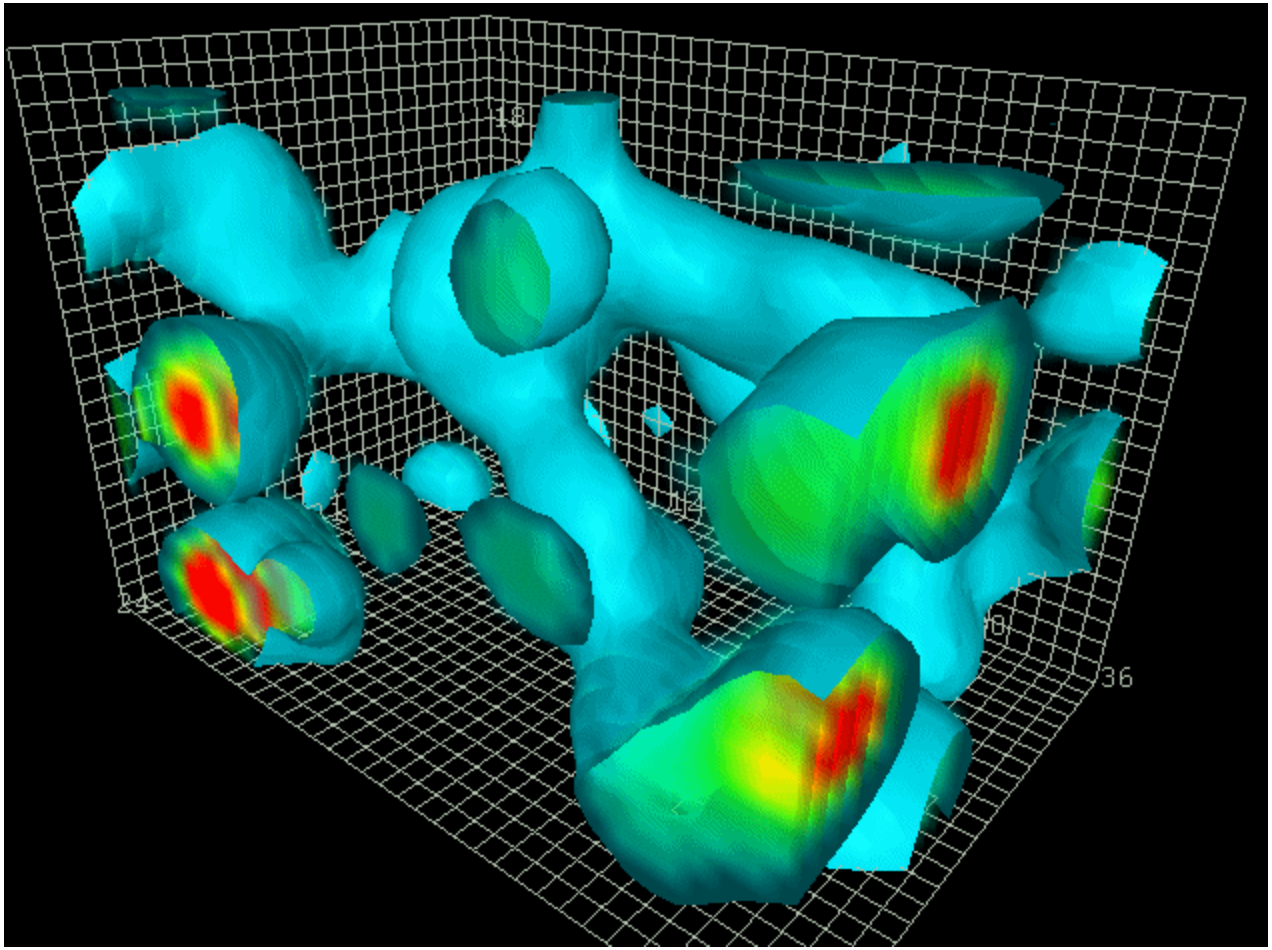


$$N_{CS} = \frac{1}{24\pi^2} \int d^3x\, \varepsilon^{ijk} \mathrm{tr}\left[\left(U^{-1}\partial_i U\right)\left(U^{-1}\partial_j U\right)\left(U^{-1}\partial_k U\right)\right], U \in SU(3)$$

Transition between 2 vacua is topological (e.g., Instanton, sphaleron)

$$\mathrm{Q} = \frac{1}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a = N_{CS}(\mathbf{t} = \infty) - N_{CS}(\mathbf{t} = -\infty)$$

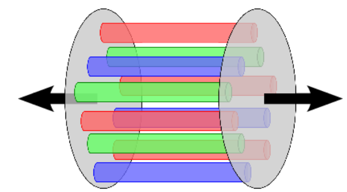
Topological fluctuation of QCD vacuum

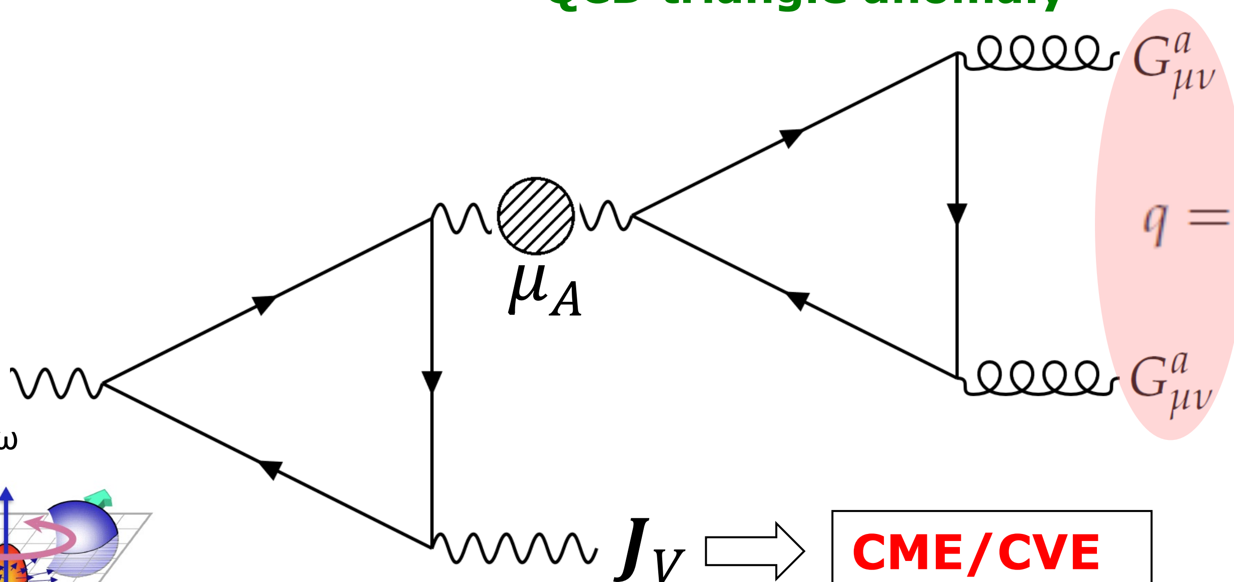


Derek Leinweber

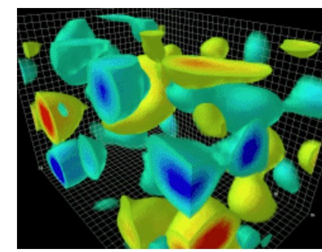
Anomalous transports in HICs

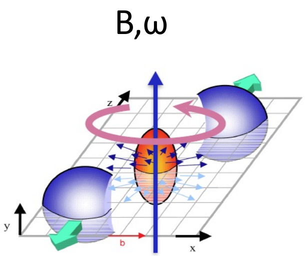
QCD triangle anomaly





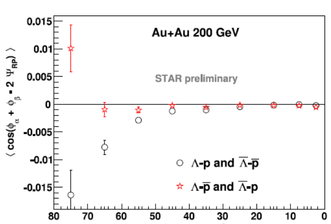
$$q = \frac{g^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

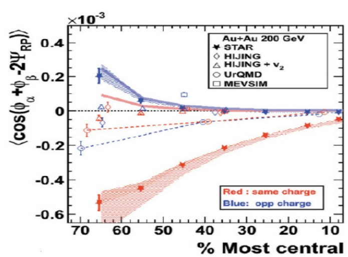


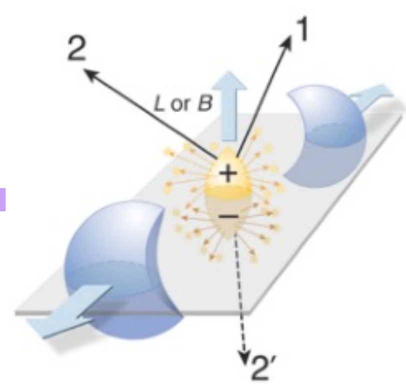


CME/CVE

QED triangle anomaly







Initial state topological fluctuations

How to detect CME, CMW, etc?

Experimental test of CME

Event-by-event charge separation wrt. reaction plane

We investigate the charge dependent two-particle correlations with respect to the reaction plane:

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2a_{\pm} sin(\phi^{\pm} - \Psi_{RP})$$

Direct measurement of "a" would yield zero value. So we need "three point-correlator"— observable "γ"!

$$\gamma = \langle cos(\phi_\alpha + \phi_\beta - 2\psi_{RP})\rangle$$

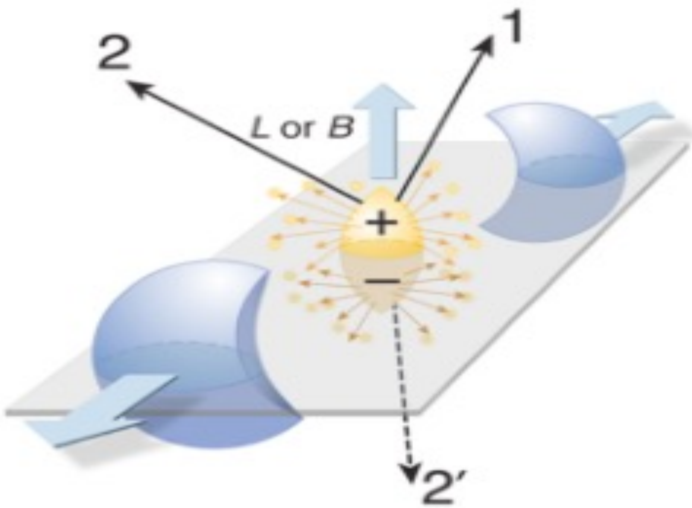
$$= [\boxed{\langle v_{1,\alpha} v_{1,\beta}\rangle} + \boxed{B_{in}}] - [\boxed{\langle a_\alpha a_\beta\rangle} + \boxed{B_{out}}]$$

Directed flow: expected to be same for "same sign" and "opposite sign"

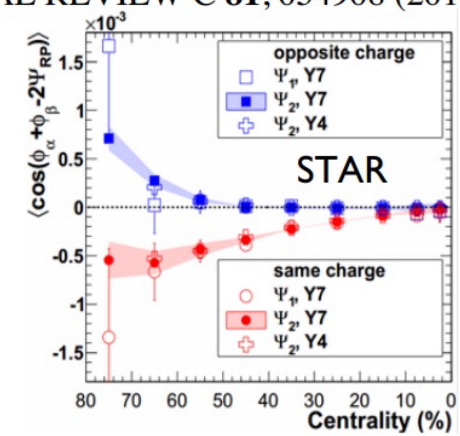
Background effects: could cancel out, but flow-related background may still exist.

P-even quantity: still sensitive to separation effect, i.e., different for "same sign" and "opposite sign"

S. Voloshin, Physical Review C. 70 (2004) 057901

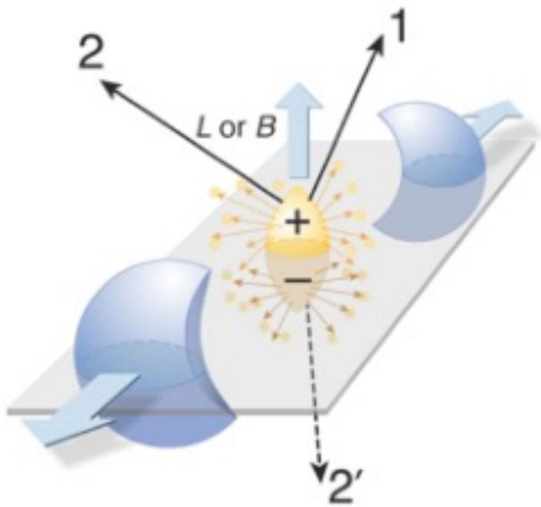


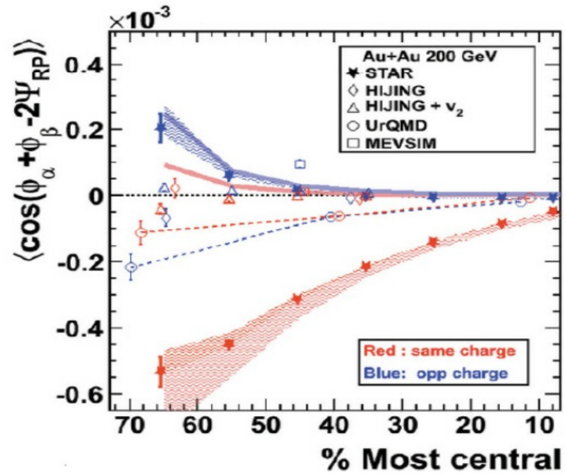
PHYSICAL REVIEW C **81**, 054908 (2010)



Experimental test of CME

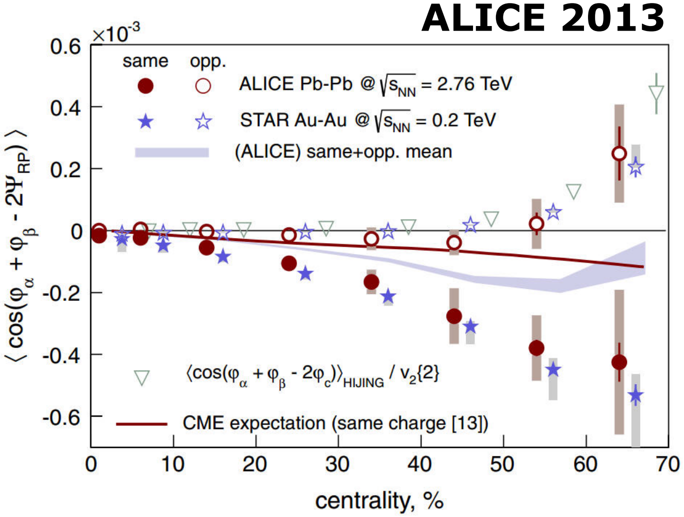
Event-by-event charge separation wrt. reaction plane



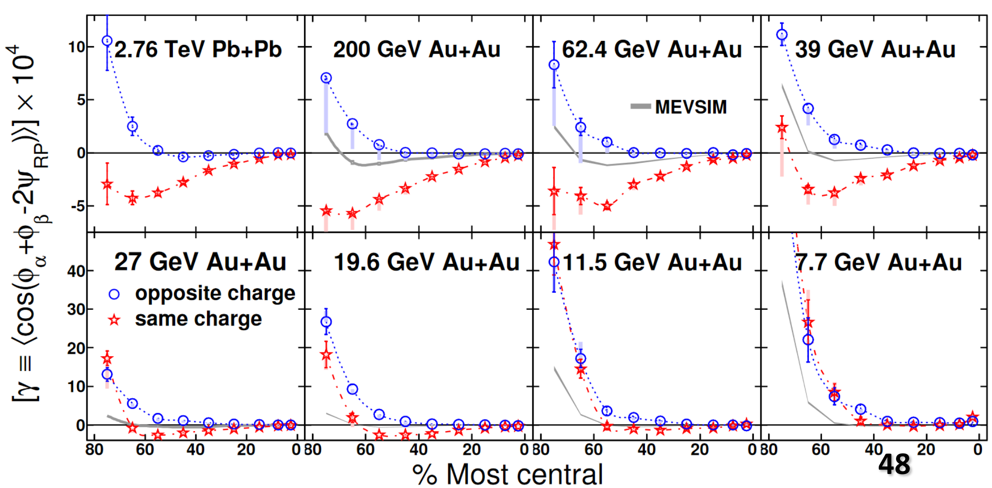


STAR 2010

ALICE 2013



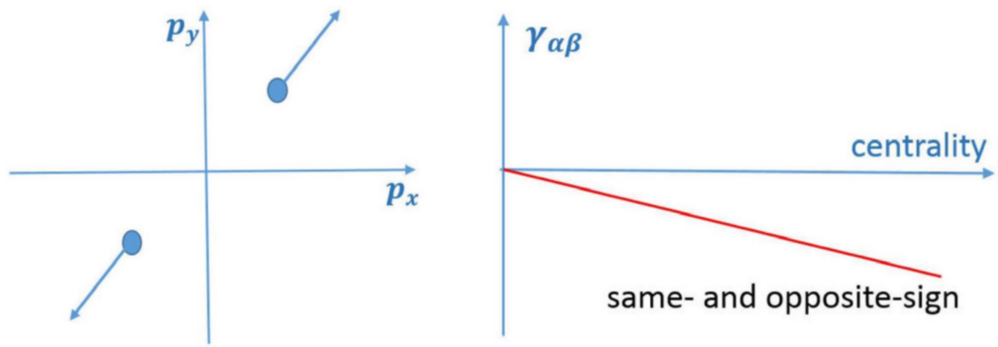
STAR 2014



Back-ground contributions to CME

Back-ground contributions to gamma correlator

Transverse momentum conservation (Pratt 2010; Liao, Bzdak,Koch 2011):



- **Charge blind**
- $\gamma \propto -v_2/N$
- **Can be subtracted in** $\Delta\gamma = \gamma_{OS} - \gamma_{SS}$

$$\gamma_{SS}^{TMC} = \gamma_{OS}^{TMC} = \left\langle \frac{\sum_{i\neq j}\cos(\phi_i+\phi_j-2\Psi_{RP})}{\sum_{i\neq j}} \right\rangle$$

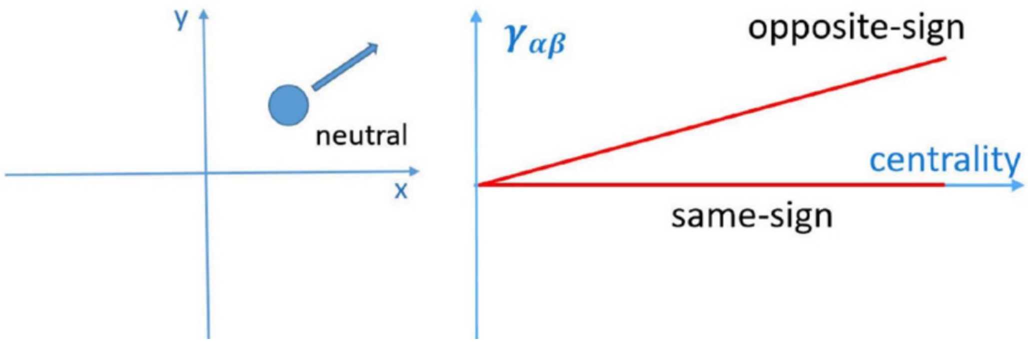
$$= \left\langle \frac{[\sum_i \cos(\phi_i-\Psi_{RP})]^2 - [\sum_i \sin(\phi_i-\Psi_{RP})]^2 - \sum_i \cos(2\phi_i - 2\Psi_{RP})}{\sum_{i\neq j}} \right\rangle$$

$$= -\frac{1}{N-1}\langle\cos(2\phi_i-2\Psi_{RP})\rangle \approx -\frac{v_2}{N},$$

Back-ground contributions to CME

Back-ground contributions to gamma correlator

Local charge conservation (Pratt, Schlichting 2011) or neutral resonance decay (Wang 2010) :



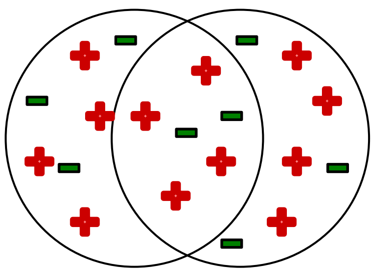
$$\gamma_{OS} \propto v_2/N, \gamma_{SS} \sim 0$$

$$\begin{aligned}
\gamma^{\rm LCC}_{+-} &= \left\langle \frac{\sum_{i,q_i=+}\sum_{j,q_j=-}\cos(\phi_i+\phi_j-2\Psi_{\rm RP})}{N_+N_-} \right\rangle \\
&\approx \left\langle \frac{\sum_c \sum_{i\in c,q_i=+}\sum_{j\in c,q_j=-}\cos(\phi_i+\phi_j-2\Psi_{\rm RP})}{N_+N_-} \right\rangle \\
&\approx \left\langle \frac{M^2\sum_c \cos(2\phi_c-2\Psi_{\rm RP})}{4N_+N_-} \right\rangle \\
&\approx \frac{M}{N} v_2,
\end{aligned}$$

Main challenge: how to separate the background elliptic flow effects?

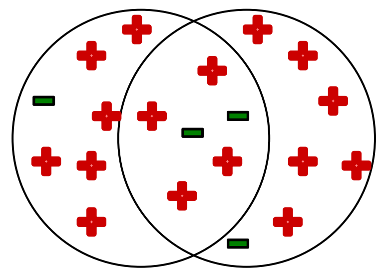
Experimental methods

Fix the flow, but vary the magnetic field: isobar collisions



$$^{96}_{40}\mathrm{Zr} + {}^{96}_{40}\mathrm{Zr}$$

Vs

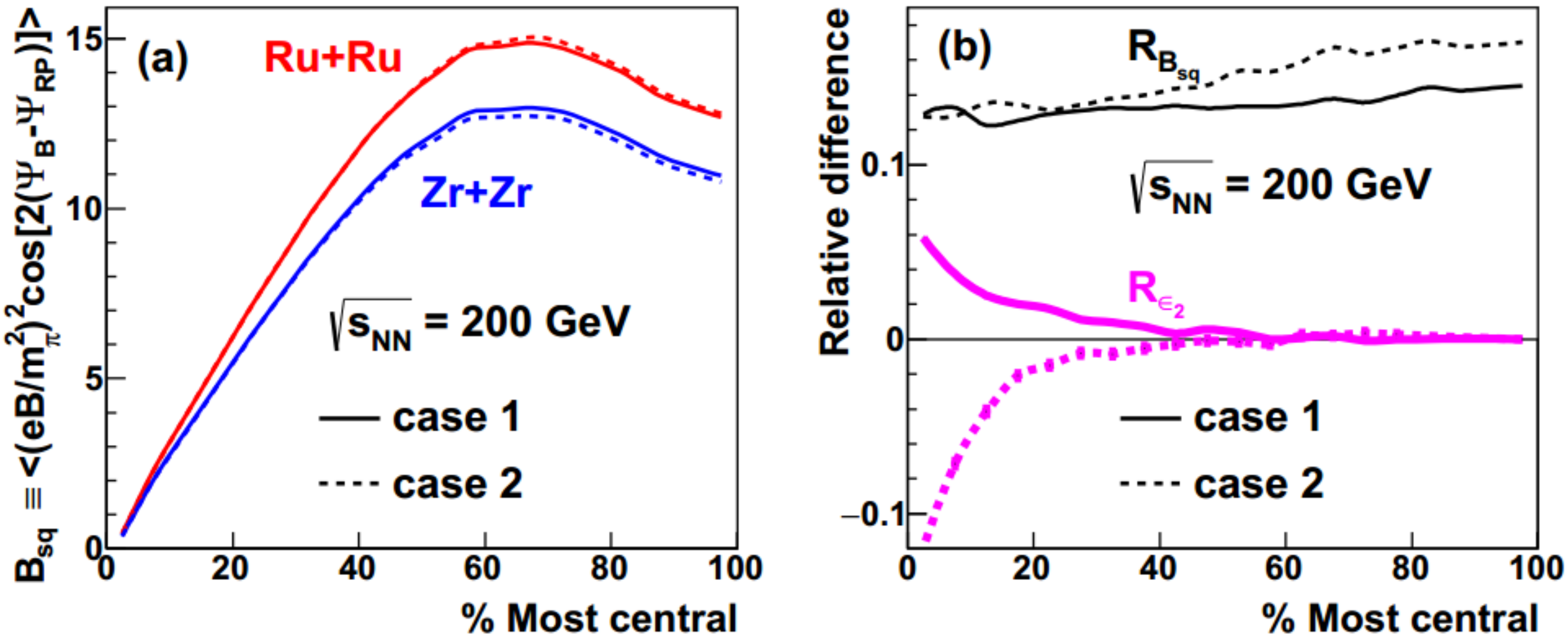


$$^{96}_{44}\mathrm{Ru} + {}^{96}_{44}\mathrm{Ru}$$

At same energy, same centrality, they would have equal elliptic flow but 10% difference in magnetic field.

Isobar collisions

Deng, XGH, Ma, and Wang, 2016



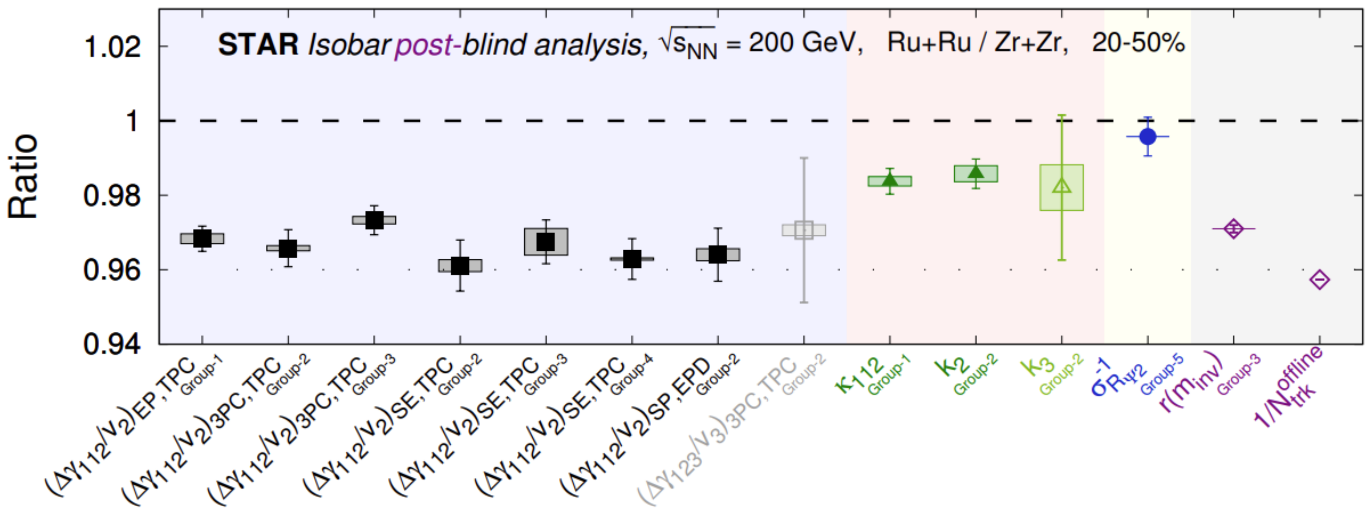
Centrality 20-60%: sizable difference in B ($R_{B_{sq}} \sim 10-20\%$) but small difference in eccentricity ($R_{\varepsilon_2} < 2\%$)

First run: 2018 @ RHIC has done!
Results published in 2021.

Isobar collisions

- **Experimental result**

First run: 2018 @ RHIC
3.1B events for each type of collision



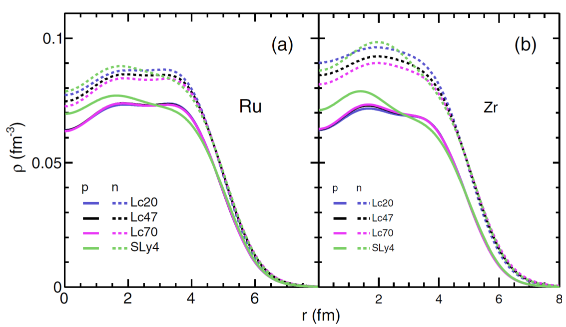
If bg=90%, theory prediction is here

(STAR 2021)

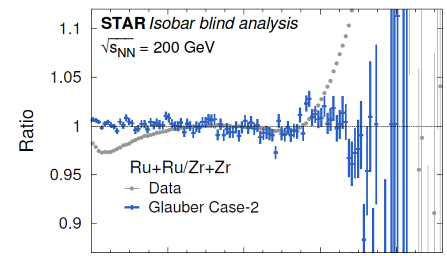
How to understand the data?

Isobar collisions

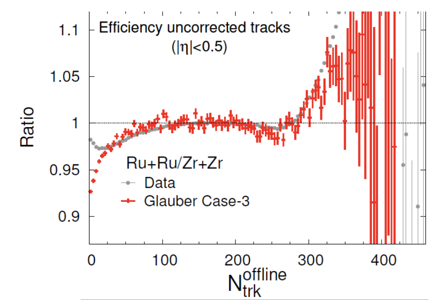
- **A failed assumption. Nuclear structure may play an essential role!**



(Li-Xu-Zhou-Wang-Zhao-Chen-Wang, PRL2020)

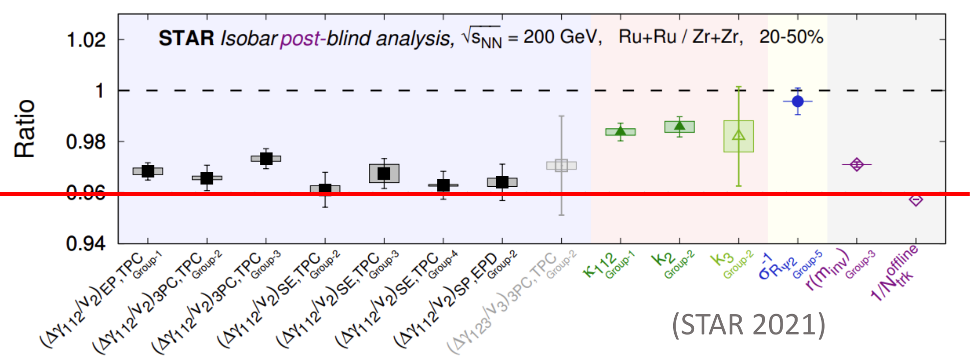


Our parameters:
W/o neutron skin



NDF:W/ neutron skin

- **Experimental result**



(STAR 2021)

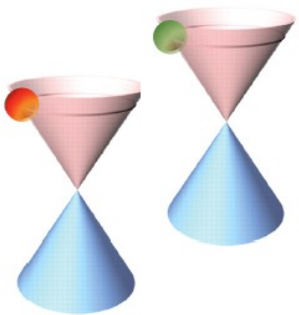
The baseline should be here rather than 1.

1) Room for CME appears.
2) Need calculation with neutron skin.
3) Use heavy ion collisions to study nuclear structure.

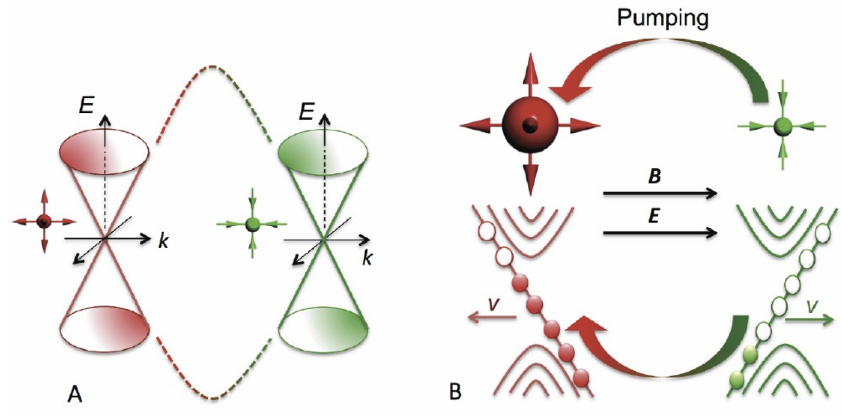
CME on desktop

- **Chiral fermions in 3D semimetals**

Weyl semimetal
(non-degenerated bands)

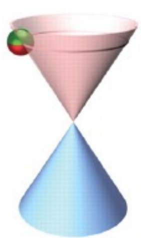


TaAs
NbAs
NbP
TaP

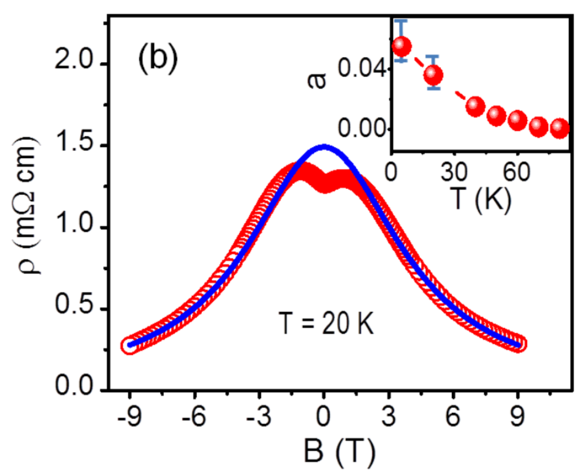


$$\partial_\mu J_5^\mu = C_A \vec{E} \cdot \vec{B}$$

Dirac semimetal
(doubly degenerated bands)



$ZrTe_5$
Na_3Bi,
Cd_3As_2



$$J^i_{\mathrm{CME}} = \sigma^{ik}_{\mathrm{CME}}\ E^k$$

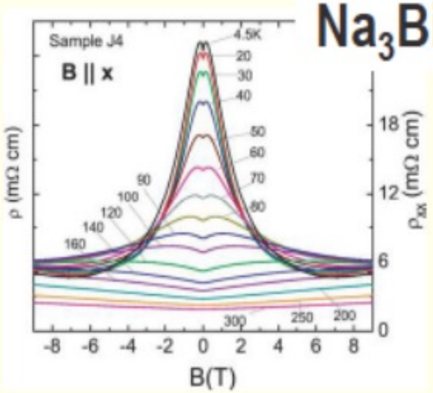
$$\sigma^{zz}_{\mathrm{CME}} = \frac{e^2}{\pi\hbar}\ \frac{3}{8}\frac{e^2}{\hbar c}\ \frac{v^3}{\pi^3}\ \frac{\tau_V}{T^2 + \frac{\mu^2}{\pi^2}}\ B^2$$

Li et al 2015, … …

CME on desktop

Dirac semimetals:

Na_3Bi



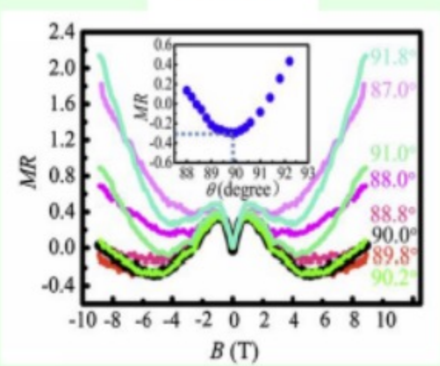
$ZrTe_5$ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.) arXiv:**1412.6543**; doi:10.1038/NPHYS3648

Na_3Bi - J. Xiong, N. P. Ong et al (Princeton Univ.) arxiv:**1503.08179**; Science 350:413,2015

Cd_3As_2- C. Li et al (Peking Univ. China) arxiv:**1504.07398**; Nature Commun. 6, 10137 (2015).

Weyl semimetals

TaAs



TaAs - X. Huang et al (IOP, China) arxiv:**1503.01304**; Phys. Rev. X 5, 031023

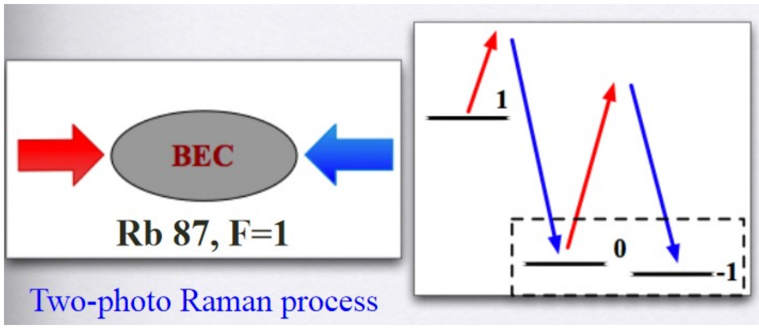
NbAs - X. Yang et al (Zhejiang Univ. China) arxiv:**1506.02283**

NbP - Z. Wang et al (Zhejiang Univ. China) arxiv:**1504.07398**

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden) arxiv:**1506.06577**

CME on desktop

- Spin-orbit coupling (SOC) in cold atomic gases

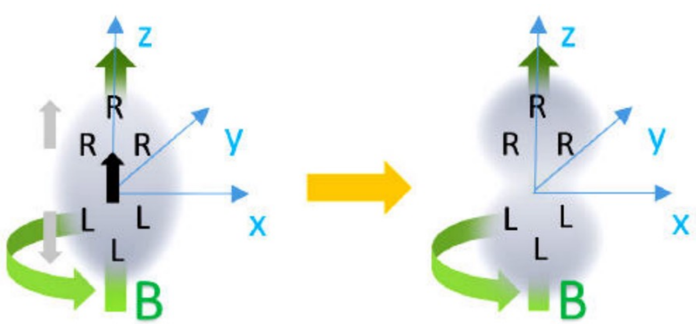


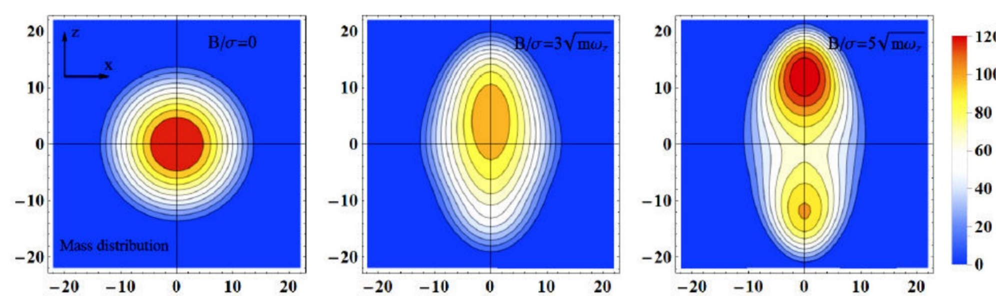
The Weyl SOC may also be realized (Spielman 2012; Andersen etal 2013; … …)

Spielman et al 2011: Equal Rashba-Dresselhaus SOC Bose gas; MIT 2012, Shanxi U. 2012, for Fermi gas

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda\sigma\cdot\mathbf{p}$$

- A rotating Weyl SOC atomic gas will show CME:

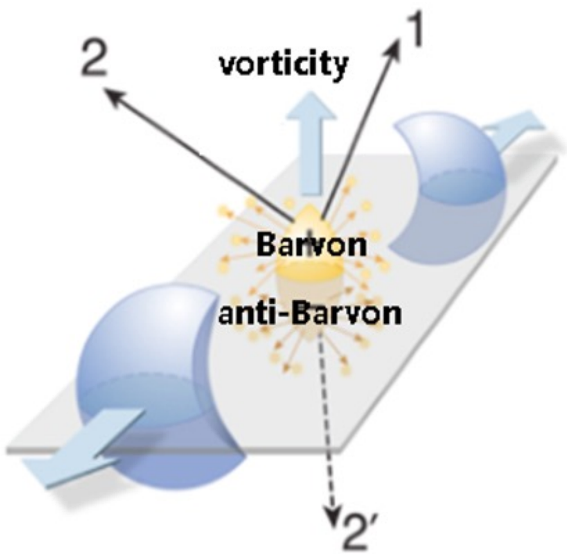




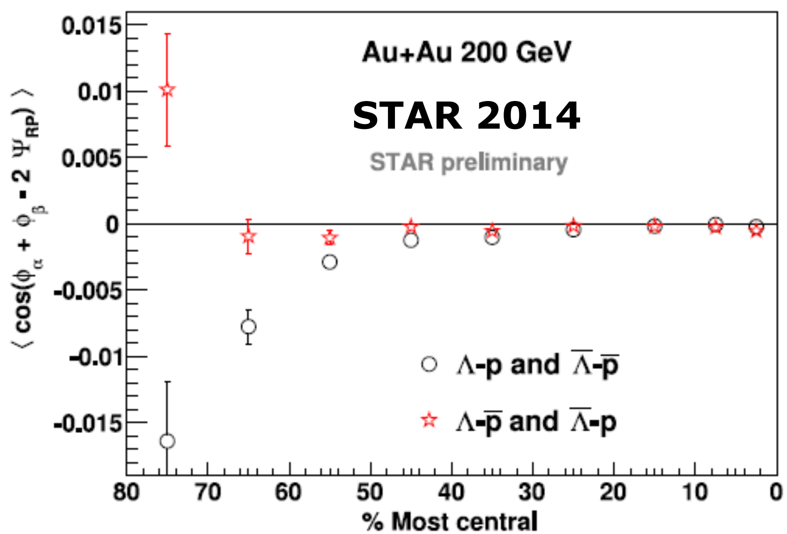
arXiv: 1506.03590

Experimental test of CVE

Event-by-event baryon separation wrt. reaction plane



The vortical gamma correlator

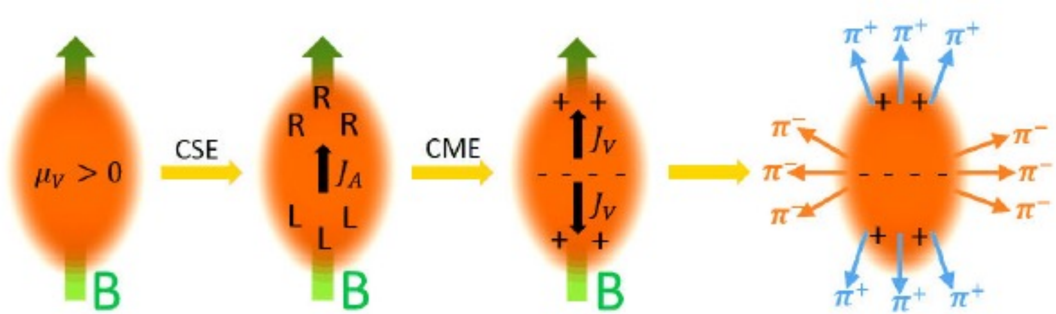


- **Positive opposite-sign correlation, negative same-sign correlation**
- **Increase with centrality = vorticity increases with centrality**

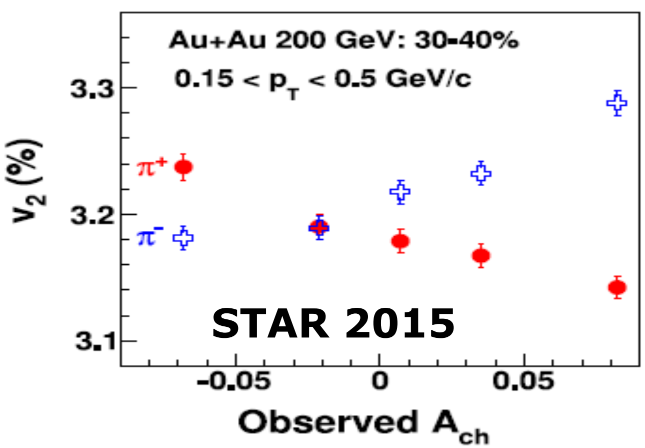
Experimental test of CMW

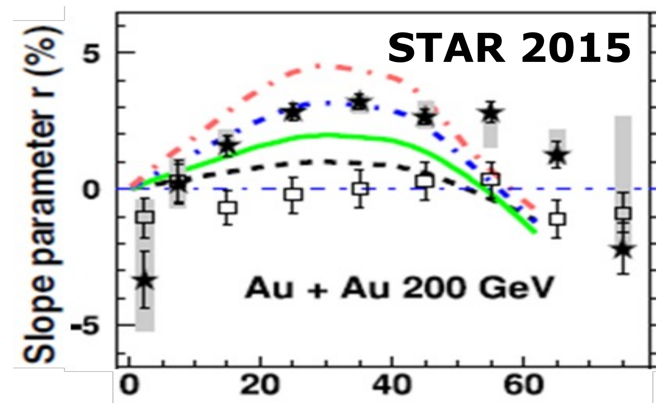
Phenomenology of CMW in heavy-ion collisions:
Elliptic flow splitting of charged pions **(Burnier, Kharzeev, Liao, Yee 2011)**

Intuitive picture



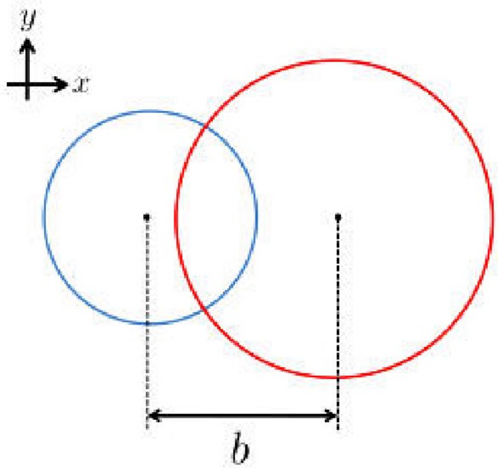
- CMW $\Rightarrow$ $v_2(\pi^-) \neq v_2(\pi^+)$: $v_2(\pi^-) - v_2(\pi^+) \approx r A_\pm$: linear approx. in net charge asymmetry $A_\pm = (N_+ - N_-)/(N_+ + N_-)$

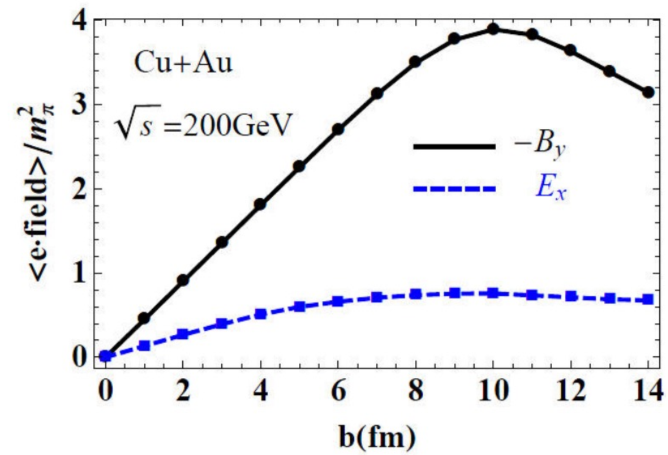




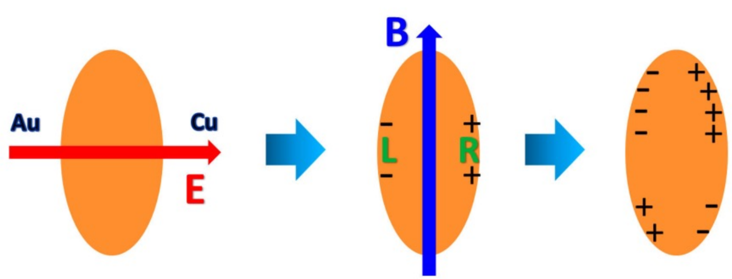
Potential experimental test of CESE (1)

- Possible implication: Recall that in-plane E-field in AuCu collisions.





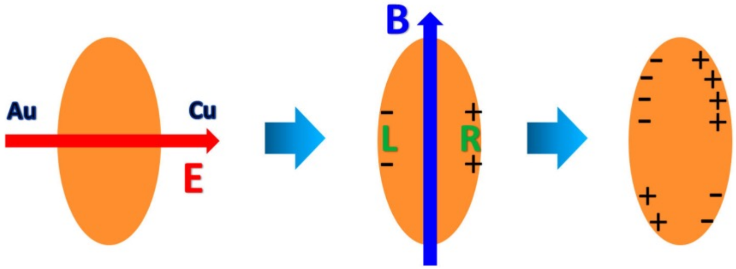
- In-plane dipole due to usual Ohm conduction + out-of-plane dipole due to CME + quadrupole due to CESE and CME in Cu + Au collisions.



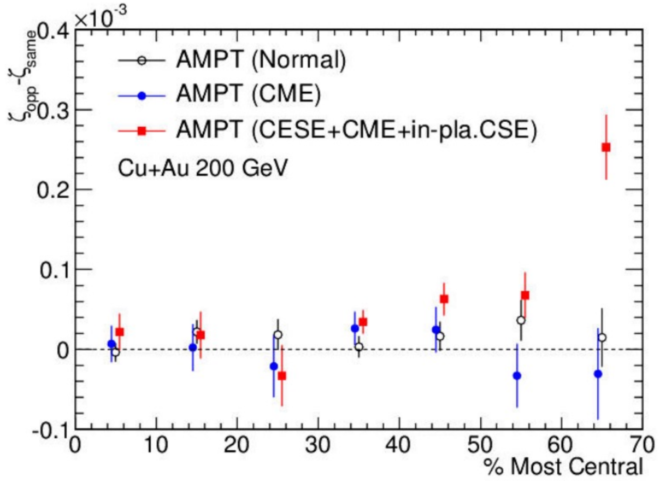
$$
\begin{aligned}
f_1(q,\phi) \quad &\propto \quad 1 + 2v_1^0\cos(\phi-\psi_1) + 2qd_E\cos(\phi-\psi_E) + 2\chi qd_B\cos(\phi-\psi_B) \\
&\quad +2v_2^0\cos[2(\phi-\psi_2)] + 2\chi qh_B\cos[2(\phi-\psi_c)] + \text{higher harmonics}
\end{aligned}
$$

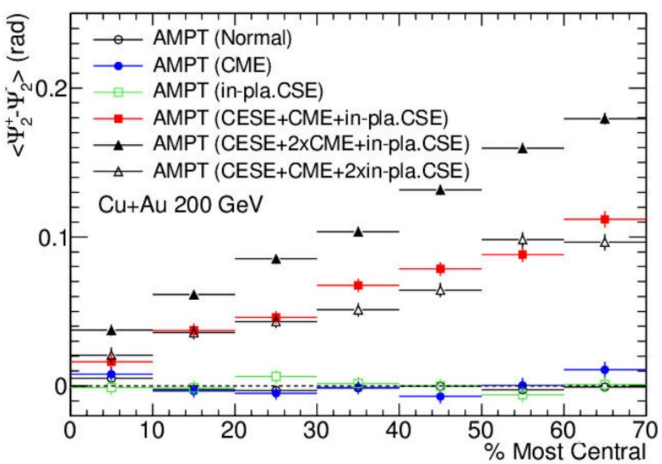
Potential experimental test of CESE (2)

- Signals for CESE in Cu + Au: $\zeta_{\alpha\beta} = \langle\cos[2(\phi_\alpha + \phi_\beta - 2\psi_{\mathrm{RP}})]\rangle$ and Ψ_2^q (the event-plane for hadrons of charge q).



- $\Delta\zeta = \zeta_{opp} - \zeta_{same}$ and $\Delta\Psi = \langle|\Psi_2^+ - \Psi_2^-|\rangle$ sensitive to CESE, survive final interaction **(Ma and XGH, PRC 91(2015)054901)**

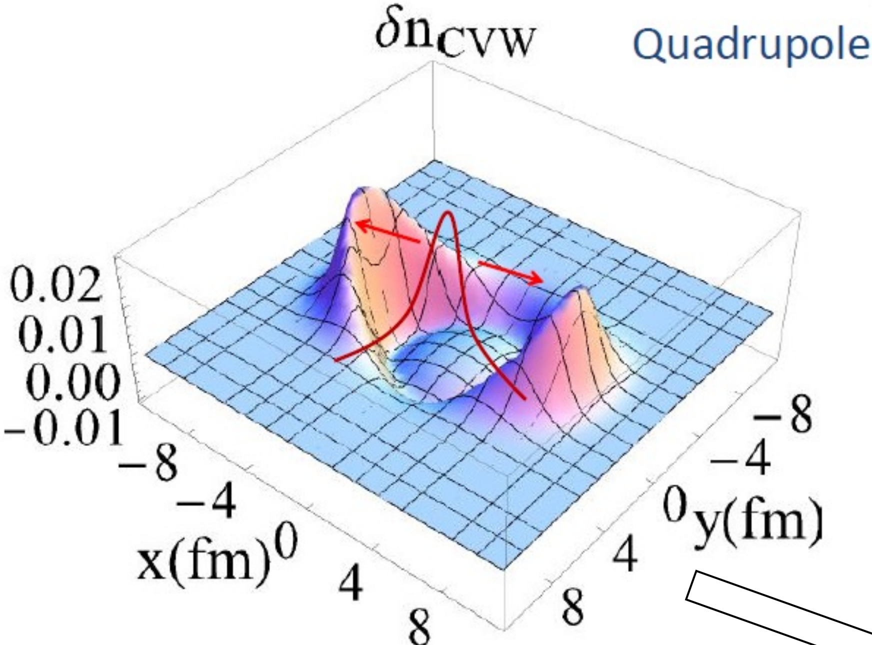




- Possible backgrounds for $\Delta\zeta = \zeta_{opp} - \zeta_{same}$: local charge conservation, chiral magnetic wave. Need more studies.

Potential experimental test of CVW

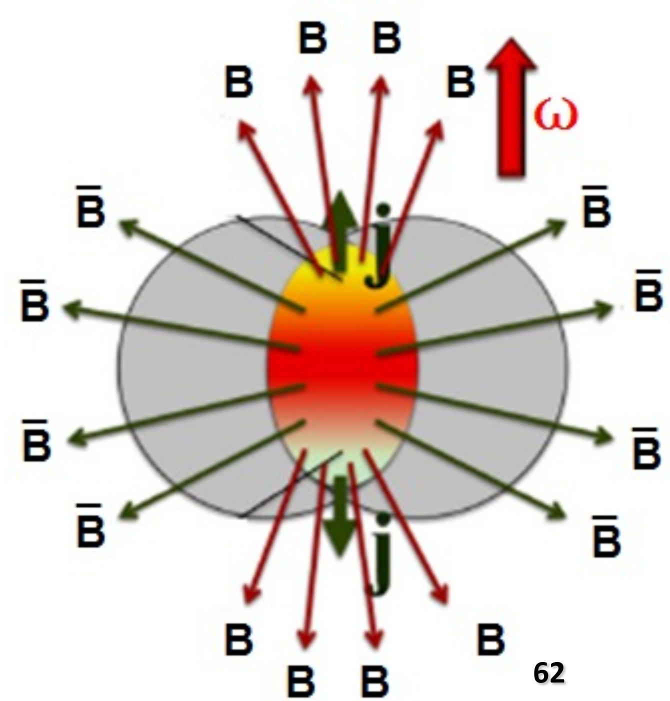
Experimental implication: baryon charge quadrupole



arXiv: 1504.03201

- More baryon charges at the tips of the fireball, more antibaryon charges at the center

- Stronger in-plane radial expansion lets antibaryons get larger elliptic flow than baryons



Thank you!

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