QCD相变的非微扰连续场论方法研究

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Outline

Ⅰ. Introduction

Ⅱ. A Continuum npQCD Approach

Ⅲ. Criterion of the PT

Ⅳ. Some of the Results

Ⅴ. Summary & Remarks

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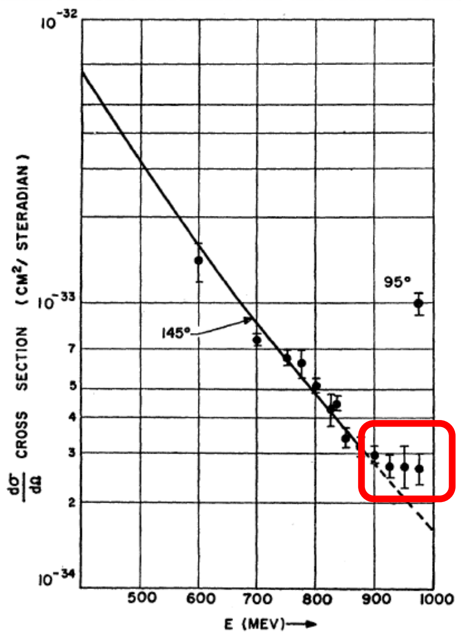
I. Introduction

1. Fundamental Ingredients of Observable Matter

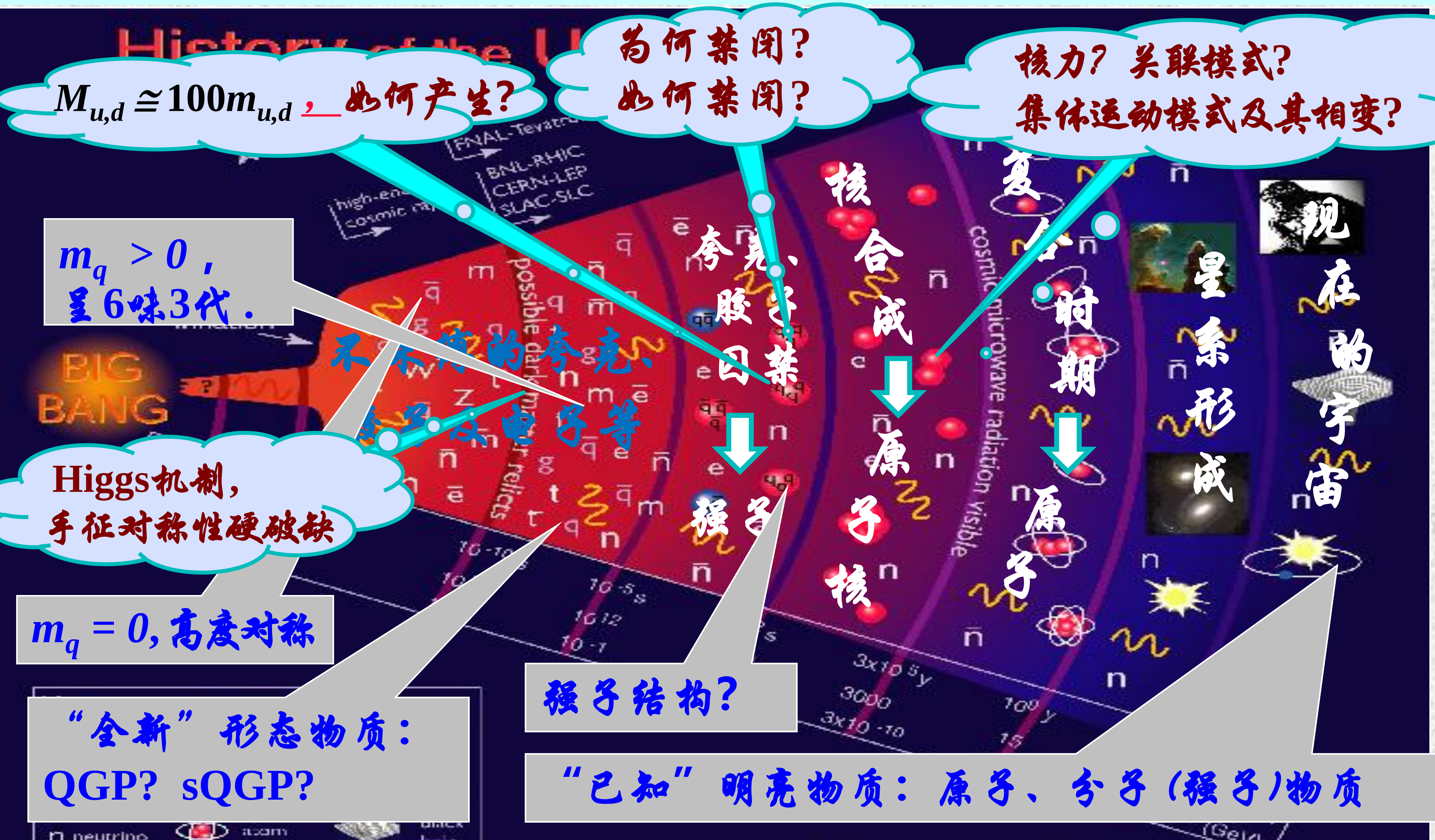
- Observable matter is composed of atoms, more deeply, nuclei, hadrons. On mass, they are strong interaction matter.
- e-p scattering experiments (PR 98, 217(1955); PRL 5, 263(1960); PR 124, 1623(1961); etc) reveal that proton is **not point particle** but **composite one**.



➔Hadrons: composite particles.
Compositing particle: partons;
➔ quarks and gluons.

➔ **Strong interaction matter in early universe is quark-gluon matter, even QGP.**

2. Overview of the Strong Interaction Matter Evolution & the nucleon mass crisis



3. General view of the mass generation

(1) Generally it is believed that mass is generated from the chiral symmetry breaking,
Specifically the usual Higgs mechanism is not enough to generate the observable mass.

♠ Concept of Symmetry

- Symmetry: invariance under a operation/transformation
  Noether Theorem: a continuous symmetry
  corresponds to a conservation charge.
  Wigner Theorem: symmetric transformation affects on
  physical quantity,
  the observed value does not change; …

- Theory of Symmetry: Group & Algebra theory;
  Continuous one: Lie Group & Lie Algebra.

♠ Largest Symmetry of a Quantum Particle

♣ Quantum state: Wave function $\psi(q)$;
$|\psi(q)|^2$ describes the probability
distribution of the particle at the state.

♣ Symmetry: $|\psi(q)|^2$ maintains its distribution
under the operation, i.e., $\left|\hat{O}\psi(q)\right|^2 = |\psi(q)|^2$,
➔ the operation holds unitary symmetry.

♣ Realization of the Symmetry

Mathematically:

u(n) algebra ➔ matrices: $\{E_{pp'}|p, p' = 1, 2, \cdots, n\}$,
with $[E_{pp'},\ E_{qq'}] = \delta_{p'q}E_{pq'} - \delta_{pq'}E_{qp'}$.

Physically: $E_{pp'} = a_p^{\dagger}a_{p'}$, or $b_q^{\dagger}b_{q'}$,

➔ u(n) can be realized with fermions or bosons.

♠ Contraction of u(n) Algebra & Dynamical Symmetry Breaking

♣ U(n) ⊃ O(n)

Mathematically: o(n) algebra → matrices:

$$\{I_{pp'} = E_{pp'} - E_{p'p} | p, p' = 1, 2, \cdots, n\},$$

with $\left[I_{pp'},\ I_{qq'}\right] = \delta_{pq'} I_{qp'} + \delta_{p'q} I_{pq'} - \delta_{pq} I_{p'q'} - \delta_{p'q'} I_{pq}$.

Physically: $I_{pq} = b_p^{\dagger} b_q - b_q^{\dagger} b_p$.

➔ o(n) can be realized with bosons, and there exists contraction u(n) ⊃ o(n).

♣ Similarly, sp(n) (n=even) can be realized with fermions, and the elements are

$$\Xi_{pq} = a_p^{\dagger} a_q + a_q^{\dagger} a_p \quad .$$

There exists then contraction u(n) ⊃ sp(n).

♣ Symmetry Breaking

There exists algebra contraction (group chain):

$\mathrm{u}(n) \supset \mathrm{o}(n)$,   for boson system;

$\mathrm{u}(n) \supset \mathrm{sp}(n)$,   for fermion system.

With the irreducible representation of the $\mathrm{u}(n)$, $\mathrm{o}(n)$ ($\mathrm{sp}(n)$) being labelled as Λ, λ, the basis of the states are: $\{|\Lambda\, \lambda \cdots \rangle\}$.

Degeneracy is reduced by the λ, the symmetry is broken.

If the Lagrangian (Hamiltonian) involves terms of the Casimir operator(s) of the subgroup(s), it is the dynamical symmetry breaking (DSB).

If not, it is the spontaneous breaking (SSB).

♣ Ex.: Classification of light-flavor hadrons

♣ Color Structure

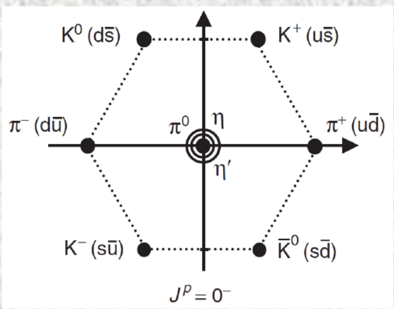
Hadron is colorless, corresponds to IRREP. [0].

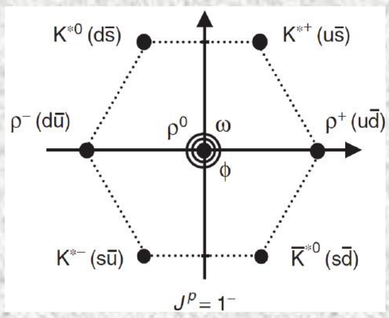
♣ Flavor Structure

Approximately $SU_F(3)$ symmetry,

- mesons, $(\bar{q}q)$,

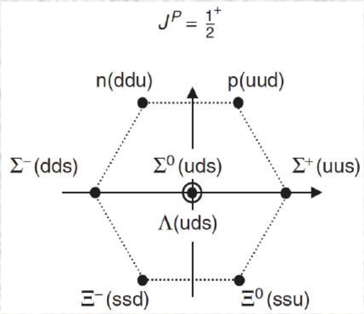
  $\bar{3} \times 3 = 8 + 1$;

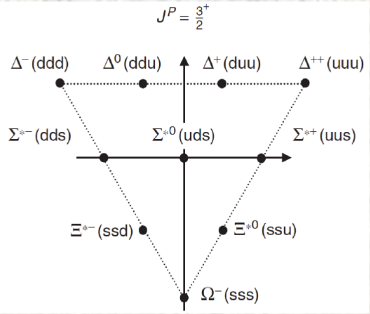




- baryons, (qqq),

  $3 \times 3 \times 3 = 10 + 8 + 8 + 1$;





♣ Mass Spectrum

(2) Chiral Symmetry & Its Breaking

♠ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \sum_{f=1}^{N_f} \bar{q}_f(x)(i\not{D} - m_f^0)q_f(x) - \frac{1}{4}G^a_{\mu\nu}(x)G_a^{\mu\nu}(x),$$

where $G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + gf^{abc}A^b_\mu(x)A^c_\nu(x)$

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \quad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu},$$

Chiral rotation operator $\mathcal{P}^{L,R} = \frac{1}{2}(1 \pm \gamma_5),$

where

$$\gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -I_{2\times 2} \\ -I_{2\times 2} & 0 \end{pmatrix}, \quad \{\gamma_5, \gamma_\mu\} = 0 \ \ (\mu = 1, 2, 3, 4).$$

one has then

$$q^{L,R} = \mathcal{P}^{L,R}q = \frac{1}{2}(1 \pm \gamma_5)q\,, \qquad \overline{q^{L,R}} = (\gamma_4 q^{L,R})^\dagger = \bar{q}\frac{1}{2}(1 \mp \gamma_5)\,.$$

and in turn,

$$\bar{q}_f(x) \not{D} q_f(x) = \bar{q}^L_f(x) \not{D} q^L_f(x) + \bar{q}^R_f(x) \not{D} q^R_f(x) \,.$$

i.e., the kinetic energy part is invariant under the chiral transformation $e^{i\gamma_5\Theta}$.

The gauge field part is definitely invariant,

→ the system holds chiral symmetry if $m_0^f \equiv 0$.

However, $m^0_f \bar{q}_f(x) q_f(x) = m^0_f \left(\bar{q}^R_f(x) q^L_f(x) + \bar{q}^L_f(x) q^R_f(x) \right) ,$

is not invariant!

→ The chiral symmetry is broken, if $m_0^f \neq 0$.

→ Chiral symmetry breaking induces a mass for a quark.

♠ Ex.: the CS & CSB of two flavor system

♣ Chiral symmetry

- **Lagrangian**

$$\mathcal{L}_{\rm QCD} = \bar{u}(x)(i\not{D} - m_u^0)u(x) + \bar{d}(x)(i\not{D} - m_d^0)d(x) - \frac{1}{4}G^a_{\mu\nu}(x)G_a^{\mu\nu}(x)\,.$$

- **Chiral symmetry**

Isospin operators: $t_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad t_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$

$\mathcal{L}$ of the system in chiral limit $(m_u^0 = m_d^0 = 0)$ is invariant under Flavor transformation

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} u' \\ d' \end{pmatrix} = e^{i(t_k\Theta^{V,k} + \gamma_5 t_k \Theta^{A,k})}\begin{pmatrix} u \\ d \end{pmatrix}\,.$$

Decomposing the isospin operator as $\vec{t} = \vec{t}^L + \vec{t}^R\,,$

the $\vec{t}^L = \frac{1}{2}(1+\gamma_5)\vec{t}, \quad \vec{t}^R = \frac{1}{2}(1-\gamma_5)\vec{t}$ form two su(2) algebras,

$\mathcal{L}$ maintains invariant, → $SU_L(2) \otimes SU_R(2)$ Sym.

Defining $\vec{x} = \vec{t}^L - \vec{t}^R = \gamma_5 \vec{t}\,,$

and noticing $[t_i,\ t_j] = i\varepsilon_{ijk} t_k\,, \quad [t_i,\ x_j] = i\varepsilon_{ijk} x_k\,, \quad [x_i,\ x_j] = i\varepsilon_{ijk} t_k\,,$,

➔ the $SU_L(2) \otimes SU_R(2)$ symmetry can be written in other form.

Taking $\vec{V}^\mu = i\bar{q}\gamma^\mu \vec{t} q\,, \qquad \vec{A}^\mu = i\bar{q}\gamma^\mu \gamma_5 \vec{t} q\,,$

one has $\partial_\mu \vec{V}^\mu = \partial_\mu \vec{A}^\mu = 0\,.$

Defining (charges) $\hat{T} = \int d^3x V^4\,, \qquad \hat{X} = \int d^2 A^4$,

they behave $[T_i,\ T_j] = i\varepsilon_{ijk} T_k\,, \quad [T_i,\ X_j] = i\varepsilon_{ijk} X_k\,, \quad [X_i,\ X_j] = i\varepsilon_{ijk} T_k\,,$

the quark field transforms as

$$[\vec{T}\,,\ \ q] = -\vec{t} q\,, \qquad\qquad [\vec{X}\,,\ \ q] = -\vec{x} q\,.$$

➔ The chiral symmetry can be rewritten as

$$SU_L(2) \otimes SU_R(2) \cong SU_V(2) \otimes SU_{AV}(2)\,.$$

• Chiral Symmetry Breaking

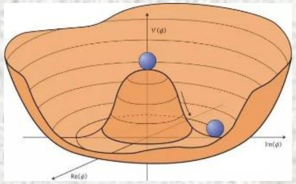
Above mentioned symmetry means that there exist chiral partner states with opposite parities.

In fact, $m_\pi \approx 138$ MeV, $\neq$ $m_\sigma \approx (400\sim550)$ MeV;

$m_\rho \approx 775$ MeV, $\neq$ $m_{a1} \approx 1230$ MeV .

➔ The chiral symmetry must be broken.

Since there is always $SU_V(2)$, the breaking should be $SU_L(2) \otimes SU_R(2) \supset SU_V(2) \otimes U_A(1)$.



➔ in case $m_q^0 = 0$, Hadrons & Goldstone bosons;

in case $m_q^0 \neq 0$, Goldstone b. $\Rightarrow$ pseudo ones;

e.g., pion, GOR relation: $m_\pi^2 f_\pi^2 = -\left(m_u^0 + m_d^0\right)\langle \bar{q} q \rangle$.

♣ Classification of the CSB

- **Explicit Breaking (ECSB)**

The current mass induced CSB.

- **Spontaneous/Dynamical Breaking (DCSB)**

If the Lagrangian maintains the symmetry but that of the vacuum is broken, the Br is SCSB.

e.g., $\mathcal{L} = \bar{\psi} i\gamma \cdot p\psi + \lambda(\bar{\psi}\psi\bar{\psi}\psi)\,,$

Self-energy ➔ $M = \lambda Tr(S) = \lambda \int_0^{\Lambda^2} \frac{M}{p^2+M^2} dp^2 = \lambda M \Big[\Lambda^2 - M^2 \ln\big(1+\frac{\Lambda^2}{M^2}\big)\Big],$

i.e., $M(\Lambda^2 - \frac{1}{\lambda}) = M^3 \ln\Big(1+\frac{\Lambda^2}{M^2}\Big)\,.$

The Eq. has always solution $M \equiv 0$;

As $\lambda > \Lambda^2$, the equation has non-zero solution(s).

➔ As the interaction is strong enough, the CS is broken simultaneously, in turn SCSB/DCSB.

4. 可见物质及其质量起源归结为QCD相变

- 上述演化各阶段物质的性质各具特色，相差甚远；任何两阶段间的演化也各自千差万别。

♣ 如何具体描述是物理学人致力探索的瑰宝！

♠ 不均匀的状态中，可以由力学手段分离的组分相同、物理和化学性质相同的均匀部分的状态称为物质的相；外部条件不变情况下，由一相到另一相的演化称为相变。

♠ 强相互作用物质

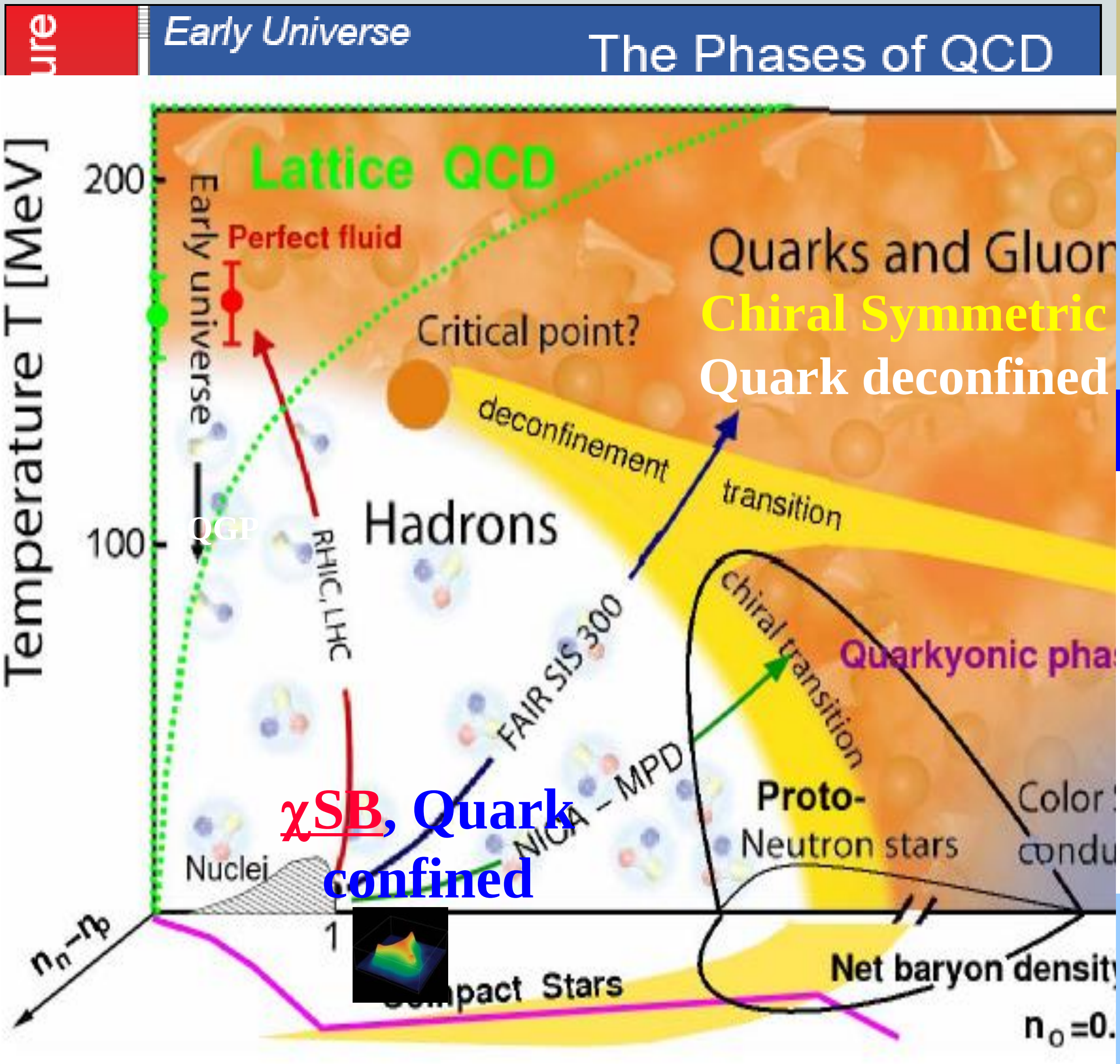
原子核形成的物质、强子形成的物质、夸克及胶子形成的物质、以及强子与夸克和胶子共存的系统都称为强相互作用物质。

➔ 上述演化过程可表述为强相互作用物质的相变！

➔ 相即确定对称性的态，相变即对称性破缺 or 恢复的过程！

➔ 早期宇宙强相互作用物质的演化即强作用系统的(手征)对称性动力学破缺和色禁闭过程！亦即QCD相变过程。

→基本问题归结为QCD相变，研究正如火如荼



Schematic QCD phase diagram for nuclear matter. The solid lines show the phase boundaries for the indicated phases. The solid circle depicts the critical

The Frontiers of Nuclear Science

A LONG RANGE PLAN

December 2007

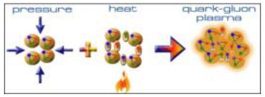
涉及相变：

禁闭（强子化）—退禁闭

手征对称性破缺 – 恢复

影响QCD相变的因素：

介质效应：温度，



密度（化学势）

有限尺度

内禀因素：流质量，

跑动耦合强度，

色味结构，•••

研究方法：

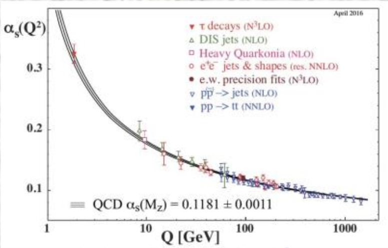
实验：RHIC、Ast-Obs.

理论：离散场论、连续场论

计算：实现理论、模拟

♠ Requirement for the theoretical approach to describe the DCSB

- Phenomena related
  the DCSB & its Restoration ,
  the Confinement & Deconfinement ;
  Generally denoted as QCD phase transition.
- Energy scale: 10^2 MeV ,
  typically nonperturbative !



- The approaches should be the ones involving simultaneously the charters
  of the DCSB & its Restoration ,
  the Confinement & Deconfinement ;

i.e., the NP QCD !

Ⅱ. Dyson-Schwinger Equations — A Continuum npQCD Approach

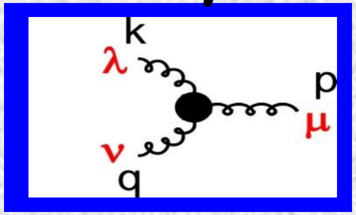
1. Outline of the DS Equations of QCD

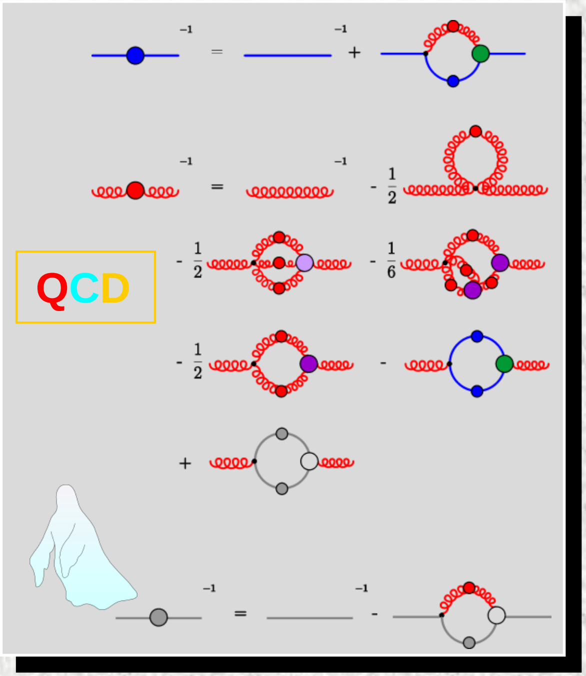
DSEs: of the most probable stationary field configurations.

QCD: SU(3) gauge field theory

$$\mathcal{L}_S = \bar{\psi}_k (i\gamma^{\mu} D_{\mu} - m_k)\psi_k - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} \, ,$$

Slavnov-Taylor Identity





axial gauges

$$k_{\lambda}\, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; \Pi^{\mu\nu}(p) \;-\; \Pi^{\mu\nu}(q)$$

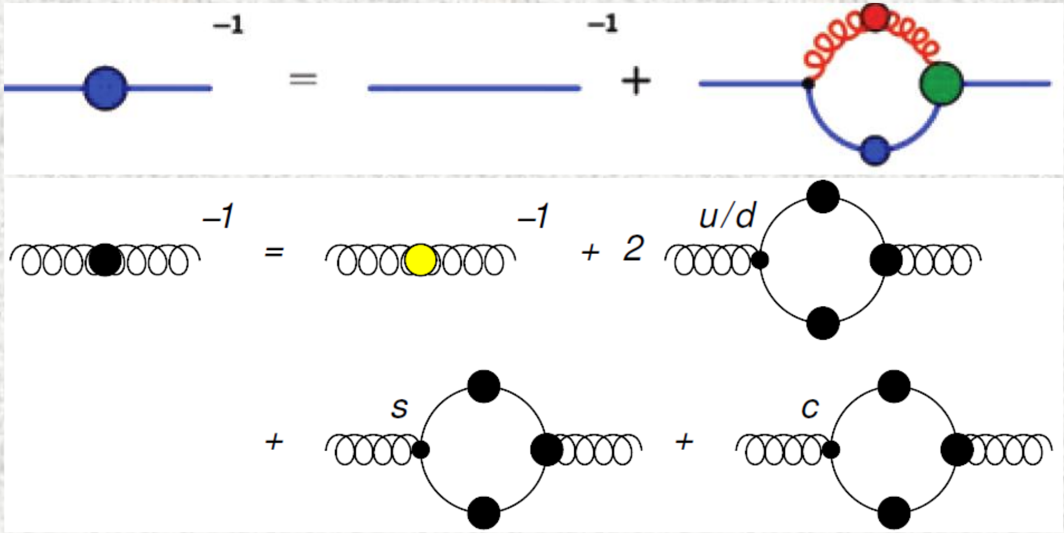
covariant gauges

$$k_{\lambda}\, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; H(k^2)\; \left[G_{\mu,\sigma}(q,-k)\, \Pi^{T}_{\sigma,\nu}(p) \;-\; G_{\nu\sigma}(p,-k)\, \Pi^{T}_{\sigma\mu}(q) \right]$$

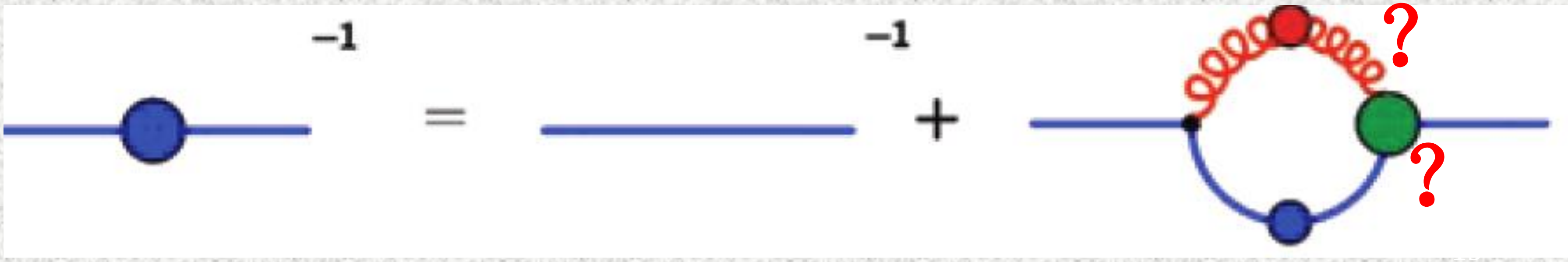
C. D. Roberts, et al, PPNP 33(1994), 477; R. Alkofer, et. al, Phys. Rep. 353(2001), 281;
A. Bashir, ···, LYX, Roberts, et al., CTP 58 (2012), 79; C.S. Fischer, PPNP 105(2019), 1;

2. Algorithm of Solving the DSEs of QCD

- **Solving the coupled** quark, ghost and gluon **equations** (parts of the diagrams) :



- **Solving the truncated quark equation** with the symmetries being preserved.



3. Expression of the quark gap equation

♠ Truncation: Preserving Symm. ➔ Quark Eq.

$$S^{-1}(p) = Z_2(-i\not{p} + Z_m m) + Z_1 g^2 \int \frac{d^4q}{(2\pi)^4} [t^a \gamma_\mu S(q) \Gamma^b_\nu(p,q) D^{ab}_{\mu\nu}(p-q)]$$

♠ Decomposition of the Lorentz Structure

- Quark Eq. in Vacuum :

$$S^{-1}(p) = i\not{p} A(p^2, \Lambda^2) + B(p^2, \Lambda^2)$$

with

$$\begin{cases} A(x) = 1 + \dfrac{1}{6\pi^3} \displaystyle\int dy \frac{yA(y)}{yA^2(y) + B^2(y)} \Theta_A(x,y) \\ B(x) = \dfrac{1}{2\pi^3} \displaystyle\int dy \frac{yB(y)}{yA^2(y) + B^2(y)} \Theta_B(x,y) \end{cases}$$

where $\Theta_A(x,y)$ & $\Theta_B(x,y)$ are functions of the vertex model & the gluon model.

• Quark Eq. in Medium

Matsubara Formalism

Temperature T : ➔ Matsubara Frequency

$$\omega_n = (2n+1)\pi T$$

Density ρ : ➔ Chemical Potential μ

$$S^{-1}(p) \Longrightarrow S^{-1}(p, \omega_n, \mu)$$

Decomposition of the Lorentz Structure

$$S^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2), \qquad \Longrightarrow$$

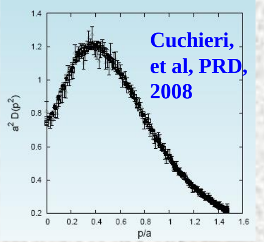
$$S^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\tilde{p}) + \cdots$$

♠ Models of the eff. gluon propagator

$$g^2 D_{\rho\sigma}(k) = 4\pi \frac{\mathcal{G}(k^2)}{k^2}\left(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)$$

- **Commonly Used: Maris-Tandy Model** (PRC 56, 3369)

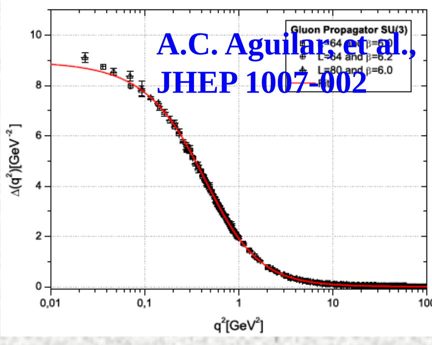
$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \frac{1 - \exp(-t/[4m_{\mathcal{F}}^2])}{t} \quad (3)$$



- **Recently Proposed: Infrared Constant Model**

(Qin, Chang, Liu, Roberts, Wilson, Phys. Rev. C 84, 042202(R), (2011).)

Taking $t/\omega^2 = k^2/\omega^2 = 1$ in the coefficient of the above expression



- **Derivation and analysis in PRD 87, 085039 (2013) (Zwanziger) show that the one in 4-D should be infrared constant.**

♠ Models of quark-gluon interaction vertex

$$\Gamma^a_\mu(q,p) = t^a \Gamma_\mu(q,p)$$

- **Bare Ansatz**

$$\Gamma_\mu(q,p) = \gamma_\mu$$ **(Rainbow Approx.)**

- **Ball-Chiu (BC) Ansatz**

$$\Gamma^{BC}_\mu(p,q) = \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} - i[B(p^2)-B(q^2)]\}$$

Satisfying W-T Identity, L-C. restricted

- **Curtis-Pennington (CP) Ansatz**

$$\Gamma^{CP}_\mu(p,q) = \Gamma^{BC}_\mu(p,q) + \frac{1}{2}(A(p^2)-A(q^2))\frac{\gamma_\mu(p^2-q^2)-(k+p)_\mu\gamma\cdot(p+q)}{d(p,q)},$$

$$d(p,q) = \frac{(p^2-q^2)^2+[M^2(p^2)+M^2(q^2)]^2}{p^2+q^2}。$$

Satisfying Prod. Ren.

- **CLRQ** (BC+ACM, Chang, etc, PRL 106,072001('11); Qin, etc, PLB 722,384('13); C. Tang, F. Gao, & YXL, Phys. Rev. D 100, 056001 (2019))

$$\Gamma^{\rm acm}_\mu(p_f,p_i) = \Gamma^{{\rm acm}_4}_\mu(p_f,p_i) + \Gamma^{{\rm acm}_5}_\mu(p_f,p_i),$$

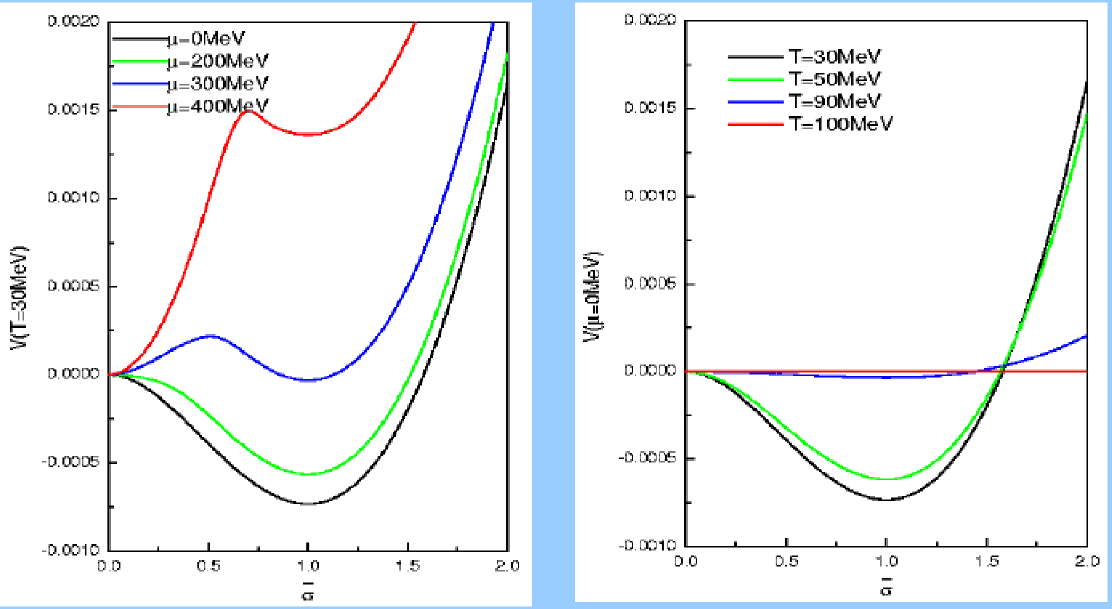
Ⅲ Criterion to Describe the QCD Phase Transition

1. Conventional Criterion

Order Parameter: chiral cond. $\langle\bar{q}q\rangle$!

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C\,\frac{-\langle\bar{q}q\rangle}{p^2\left[\ln p/\Lambda_{QCD}\right]^d}$$

Procedure: Analyzing the ThDyn Potential



Signature of PT: $\frac{\partial^2\Omega}{\partial T^2}$, $\frac{\partial^2\Omega}{\partial \mu^2}$, etc., change sign .

Question:

In complete nonperturbation, one can not have the thermodynamic potential.

The conventional criterion fails.

One needs then new criterion!

2. New Criterion: Chiral Susceptibility

♠ **Def.: Response of the order parameter to control variables**

$$\frac{\partial M}{\partial T},\ \frac{\partial M}{\partial \mu};\quad \frac{\partial\langle\bar{q}q\rangle}{\partial T},\ \frac{\partial\langle\bar{q}q\rangle}{\partial \mu};\quad \frac{\partial B}{\partial T},\ \frac{\partial B}{\partial \mu};\quad \frac{\partial B}{\partial m_0};$$

♠ **Simple Demonst. Equiv. of NewC to ConvC**

(刘玉鑫，《热学》，北京大学出版社，2016年第1版)

TD Potential: $\Omega(T,\eta) = \Omega_0(T) + \frac{1}{2}\alpha\eta^2 + \frac{1}{4}\beta(\eta^2)^2 + \frac{1}{6}\gamma(\eta^2)^3 + \cdots$

Stability Condition: $\frac{\partial\Omega}{\partial\eta} = \alpha\eta + \beta\eta^3 + \gamma\eta^5 = 0$

$$\frac{\partial^2\Omega}{\partial\eta^2} = \alpha + 3\beta\eta^2 + 5\gamma\eta^4 > 0,\ St.;\ < 0,\ Unst..$$

Derivative of ext. cond. against control. var.:

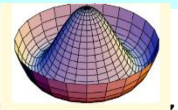
$$[\alpha + 3\beta\eta^2 + 5\gamma\eta^4]\left(\frac{\partial\eta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta\left(\frac{\partial\alpha}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial\beta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial\gamma}{\partial\varsigma}\right)_{\varsigma=\zeta_c} = 0$$

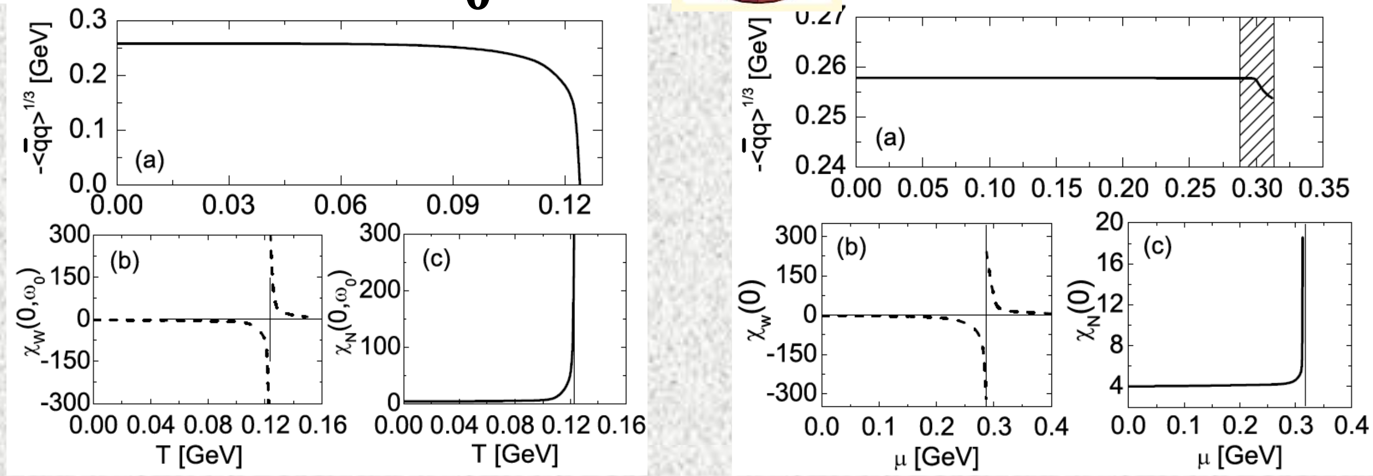
we have: $\chi = \left(\frac{\partial\eta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} = -\frac{\eta\left(\frac{\partial\alpha}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial\beta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial\gamma}{\partial\varsigma}\right)_{\varsigma=\zeta_c}}{\left(\frac{\partial^2\Omega}{\partial\eta^2}\right)_{\frac{\partial\Omega}{\partial\eta}=0}}$

At field theory level, see Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016).

♠ Demonstration of the New Criterion

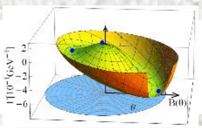
In chiral limit ($m_0 = 0$)



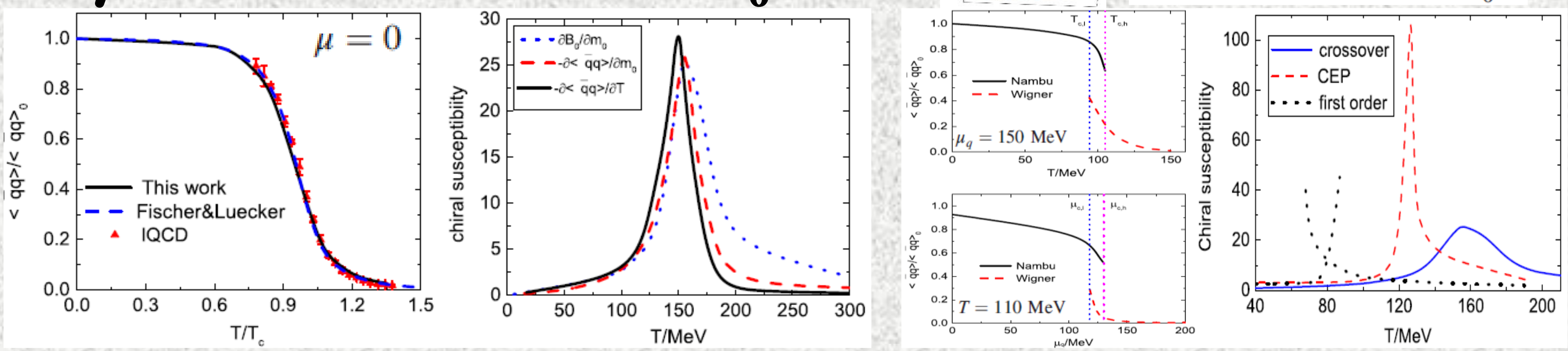


S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

Beyond chiral limit ($m_0 \neq 0$)



$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{m_0} - m_0 \frac{\partial \langle \bar{q}q \rangle_{m_0}}{\partial m_0}$$



Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

♠ Characteristic of the New Criterion

As 2nd order PT (Crossover) occurs,
the χs of the two (DCS, DCSB) phases
diverge (take maximum) at same states.

As 1st order PT takes place,
χs of the two phases diverge at dif. states.

➔ the χ criterion can not only give the phase boundary, but also determine the position the CEP.

For multi-flavor system,
one should analyze the maximal eigenvalue of the susceptibility matrix (L.J. Jiang, YXL, et al., PRD 88, 016008), or the mixed susceptibility (F. Gao, YXL, PRD 94, 076009).

IV. Some of the Results via the DSE Approach

1. Dynamical chiral symmetry breaking

$$\boxed{\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle\bar{q}q\rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}}$$

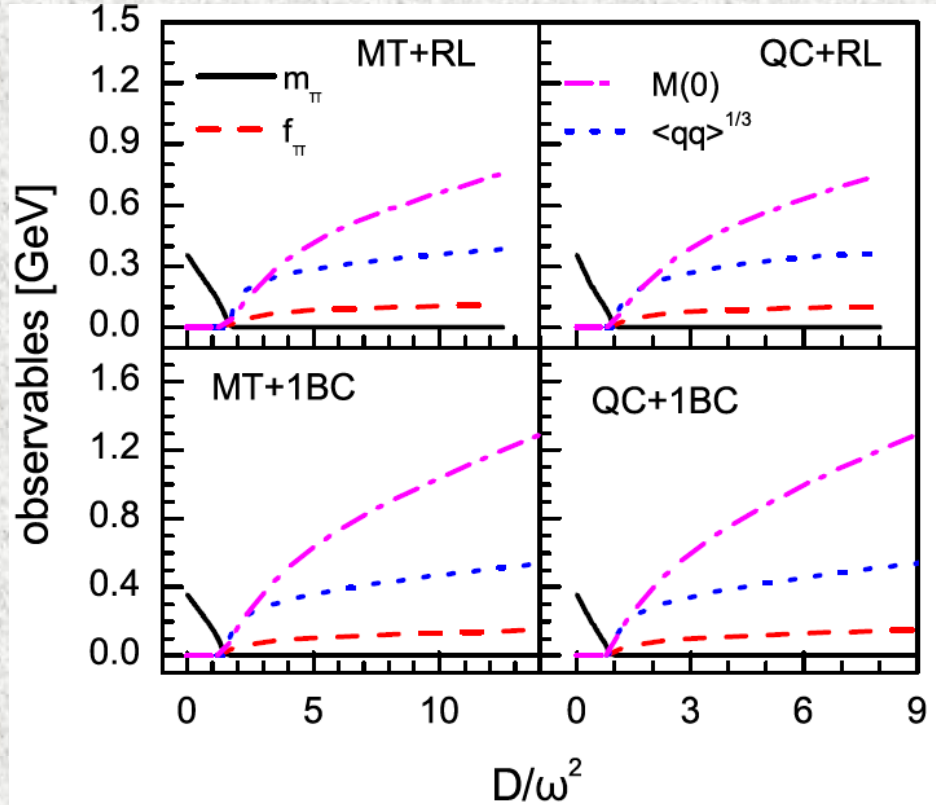
$\langle\bar{q}q\rangle \neq 0 \Longrightarrow$ **DCSB**

In DSE approach

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$

♠ In chiral limit

→ Increasing the interaction strength induces the dynamical mass generation

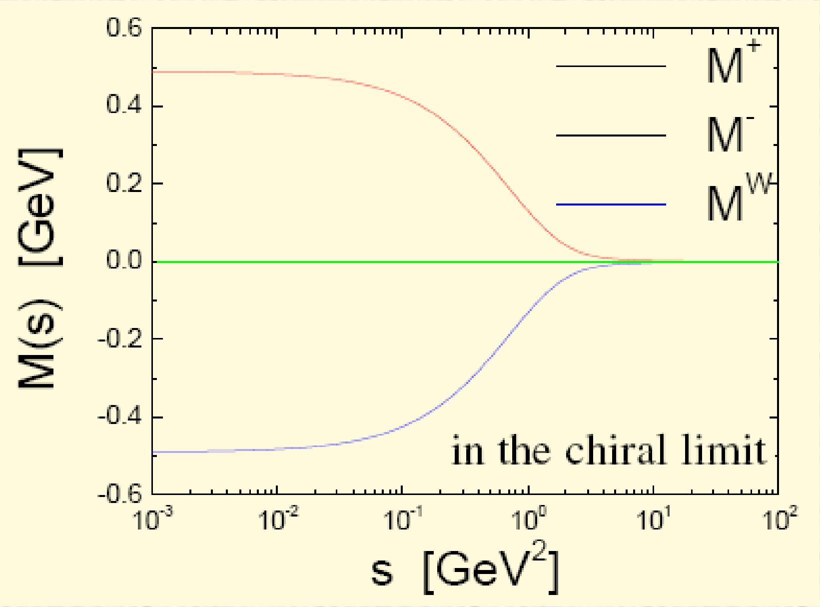


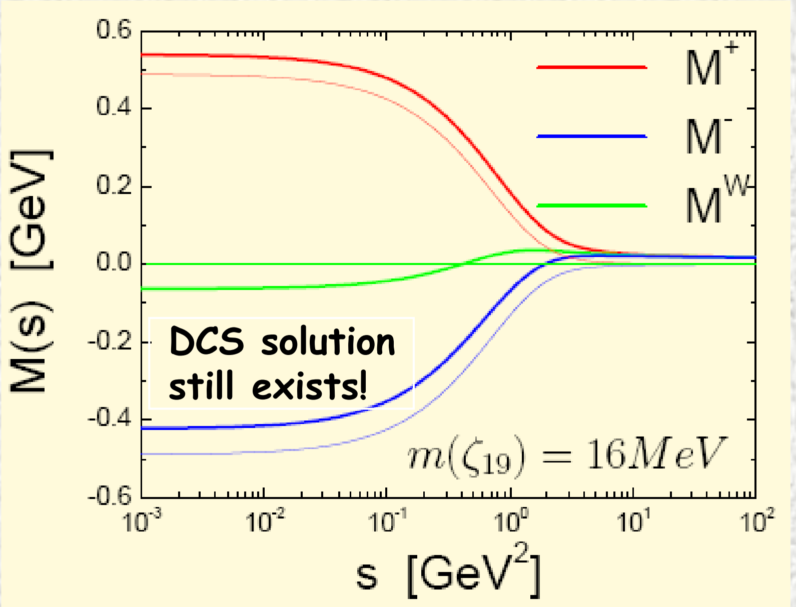
K.L. Wang, YXL, et al., PRD 86,114001('12);

♠ Beyond the chiral limit

L. Chang, Y. X. Liu, C. D. Roberts, et al, arXiv: nucl-th/0605058;
R. Williams, C.S. Fischer, M.R. Pennington, arXiv: hep-ph/0612061;
K. L. Wang, Y. X. Liu, & C. D. Roberts, Phys. Rev. D 86, 114001 (2012).

Solutions of the DSE with MT model and QC model for the effective gluon propagator, bare & 1BC model for the quark-gluon interaction vertex :

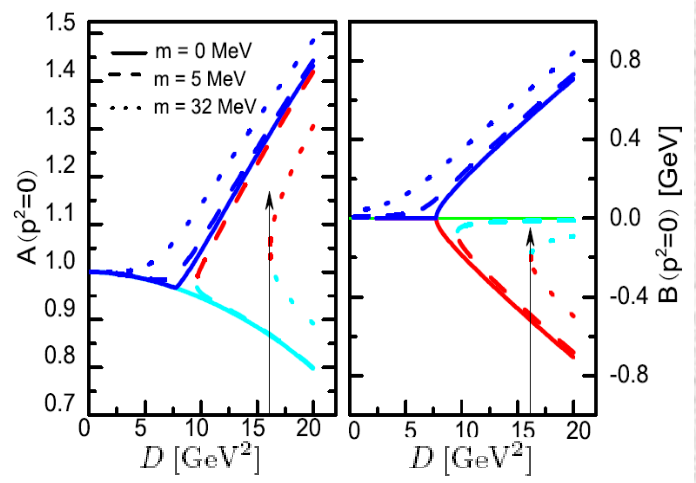


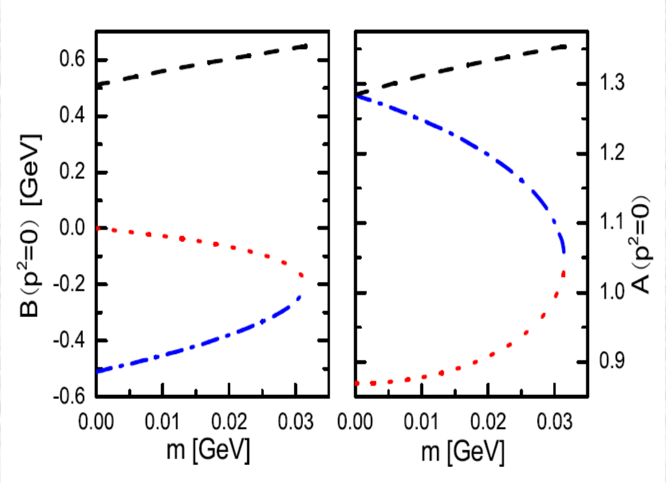


with $D = 16$ **GeV^2**, $\omega = 0.4$ **GeV**

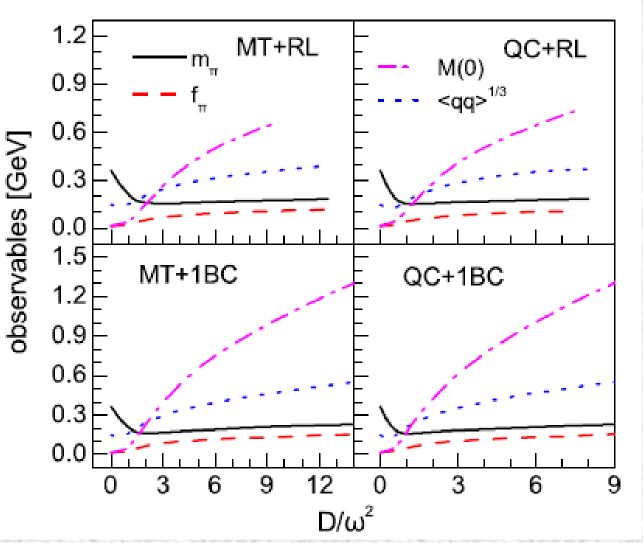
♠ Beyond the chiral limit (continued)

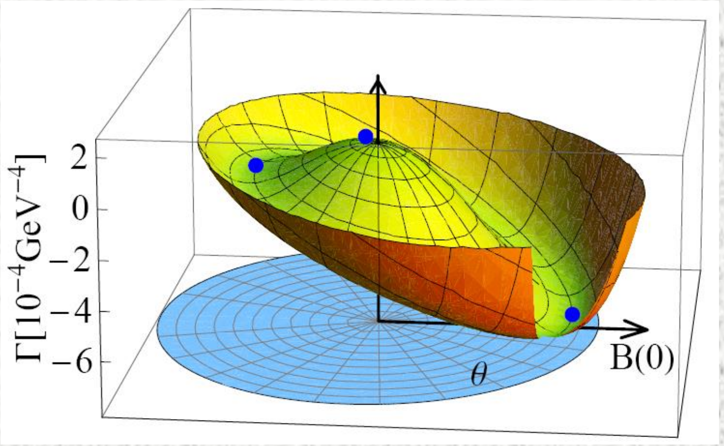
Solutions continued





Conditions: Interaction strength large enough, m not very large.



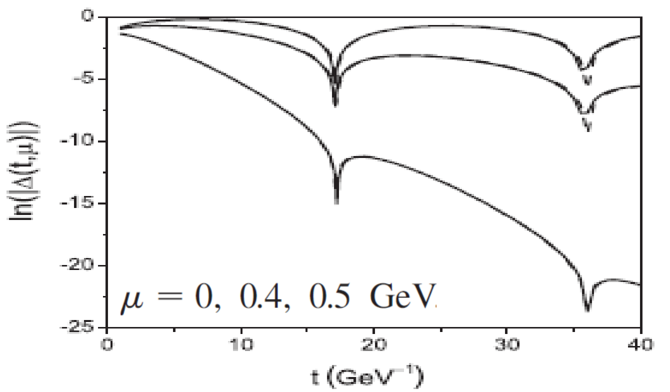


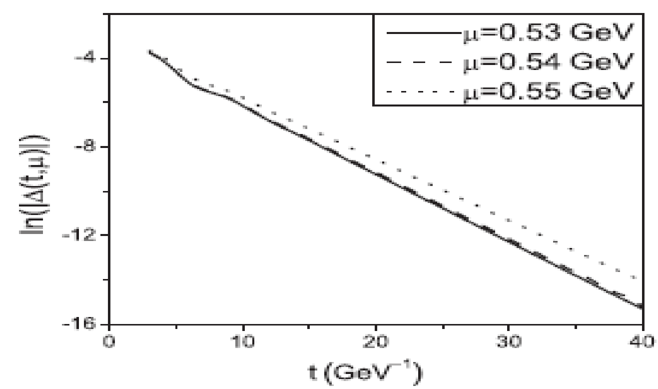
➔ DCSB still exists beyond chiral limit,

but the 2nd order PT shifts to crossover.

2. Confinement

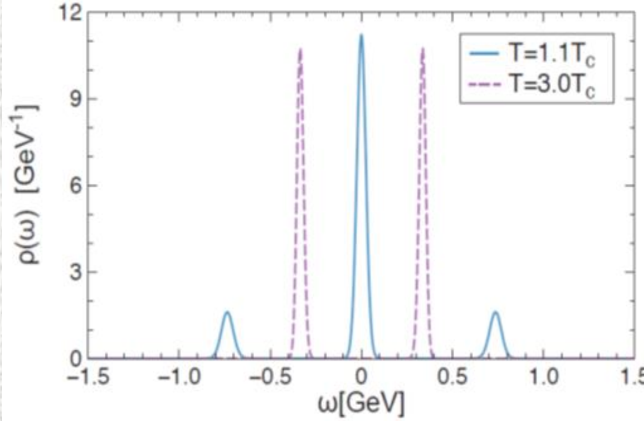
Positivity of the spectral density function is violated at low T → quarks are confined.

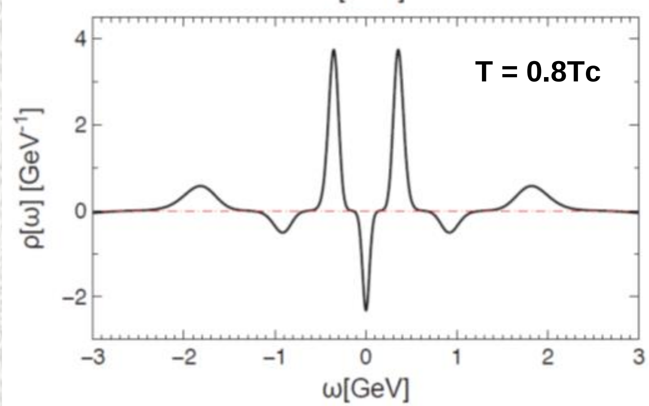




$$\Delta(\tau,\mu) = \int \frac{d^4 p}{(2\pi)^4} e^{i\vec{p}\cdot\vec{x}+ip_4\tau}\delta(\vec{p})\sigma_B(p;\mu).$$

H. Chen, YXL, et al., Phys. Rev. D 78, 116015 (2008)





S.X. Qin, D. Rischke, Phys. Rev. D 88, 056007 (2013)

$$S^R(\omega,\vec{p}) = S(i\omega_n,\vec{p})|_{i\omega_n\to\omega+i\epsilon}$$

$$S(i\omega_n,\vec{p}) = \int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\frac{\rho(\omega',\vec{p})}{i\omega_n-\omega'}$$

$$\rho(\omega,\vec{p}) = -i\vec{\gamma}\cdot\vec{p}\,\rho_v(\omega,\vec{p}^2) + \gamma_4\omega\,\rho_e(\omega,\vec{p}^2) + \rho_s(\omega,\vec{p}^2)$$

In MEM

$$P[\rho|M(\alpha)] = \frac{1}{Z_S}e^{\alpha S[\rho,m]},$$

$$S[\rho,m] = \int_{-\infty}^{+\infty} d\omega\left[\rho(\omega)-m(\omega)-\rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$$

$$m(\omega) = m_0\theta(\Lambda^2-\omega^2).$$

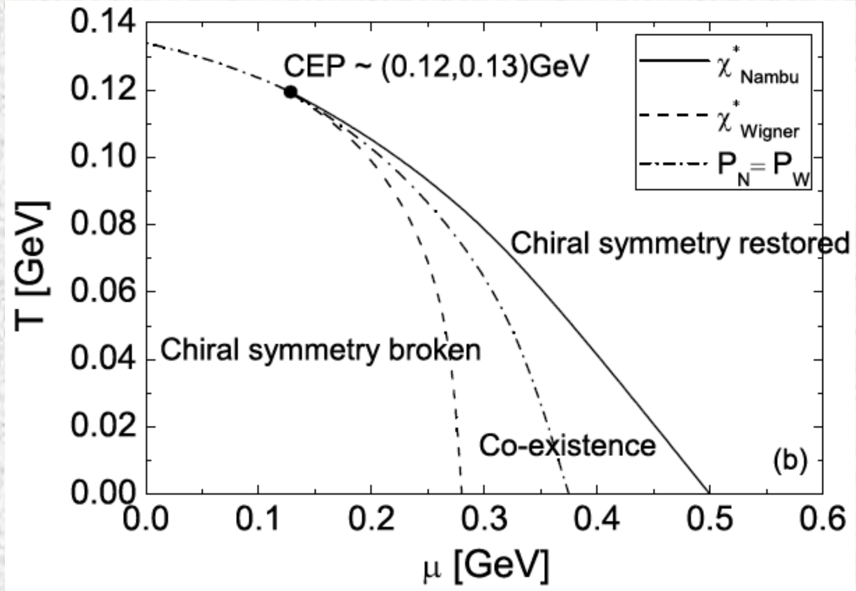
3. QCD Phase Diagrams and the position of the CEP

(1) Result without the feedback

- **In chiral limit**

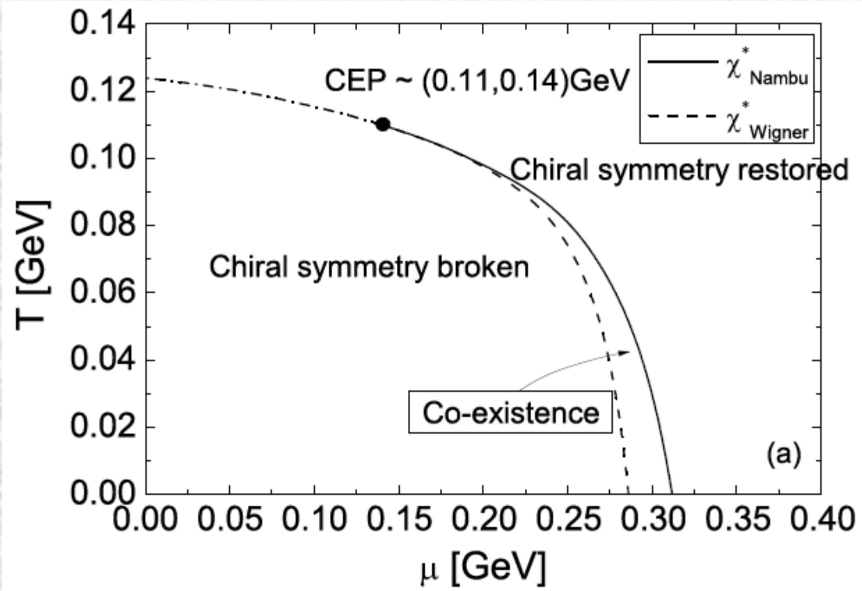
With bare vertex

(ETP is available, the PB is shown as the dot-dashed line)



With Ball-Chiu vertex

(ETP is not available, but the coexistence region is obtained)



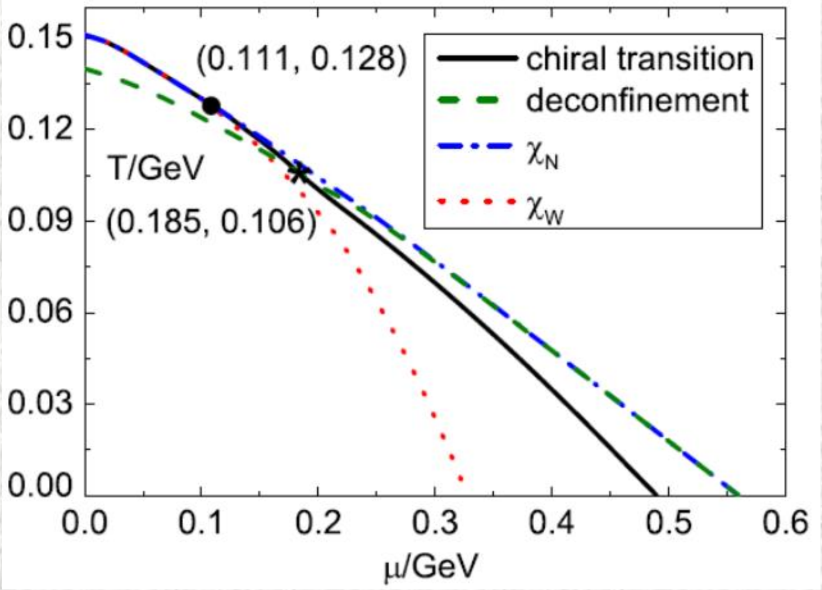
S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

(1) Result without the feedback (continued)

- **Beyond chiral limit**

With bare vertex

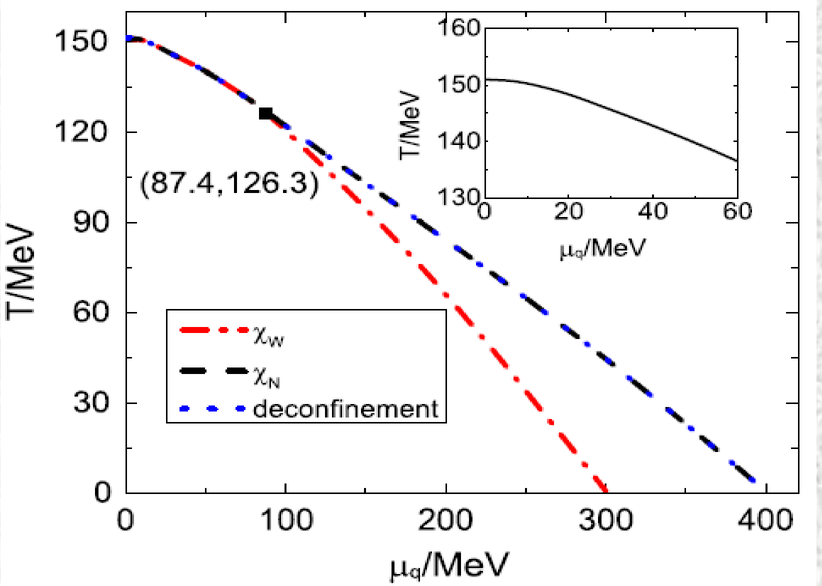
(ETP is available, the PB is shown as the dot-dashed line)



F. Gao, J. Chen, Y.X. Liu, et al., Phys. Rev. D 93, 094019 (2016).

With CLRQ vertex

(ETP is not available, but the coexistence region is obtained)



F. Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016).

(2) Result with the feedback

• Beyond chiral limit

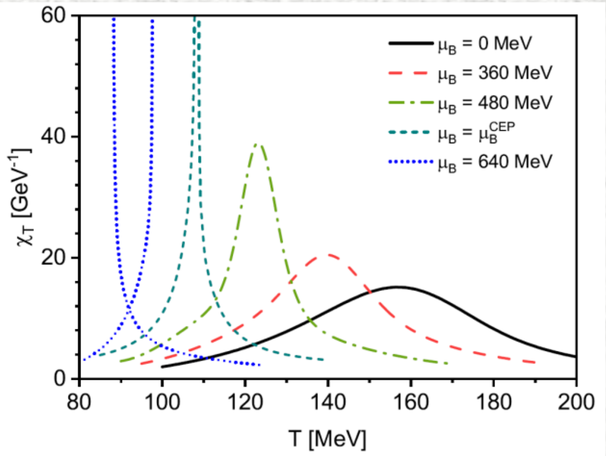
CLRQ vertex (consistent with the solutions of the coupled DSEs) represents the DCSB effect on the quark-gluon interaction

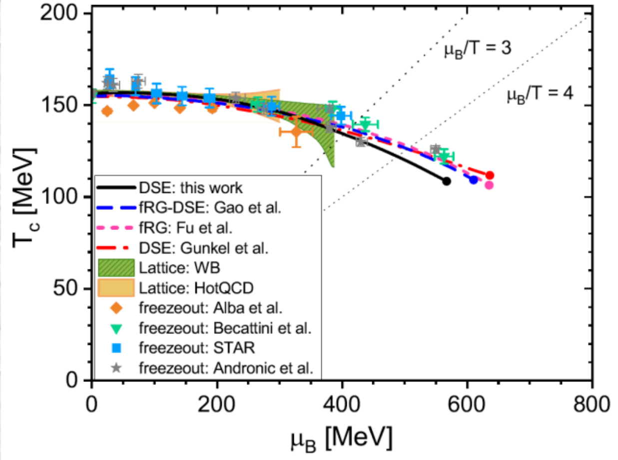
$$\Gamma_\mu(k;q,p) = \gamma_\mu F(k^2)\Sigma_A(q,p) + \Gamma_\mu^{\mathcal{T}_4}(k;q,p). \qquad \Gamma_\mu^{\mathcal{T}_4} = -i\left[\lambda_4^E\, P_{\mu\nu}^E(k) + \lambda_4^M\, P_{\mu\nu}^M(k)\right] \not{k}\gamma^\nu,$$

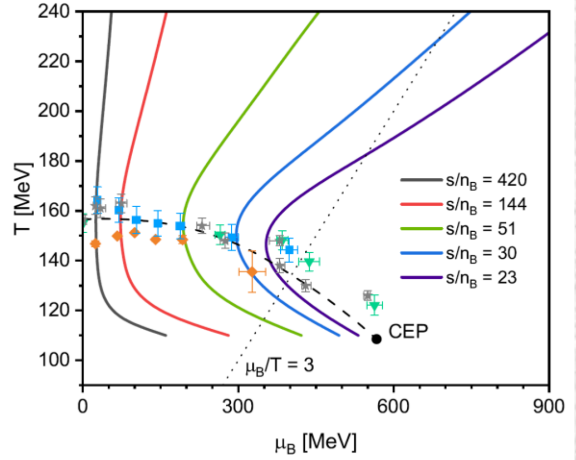
$$\lambda_4^{E,M}(k;\tilde{q},\tilde{p}) = Z_{E,M}^{-1/2}(k^2)\,\Delta_B(\tilde{q},\tilde{p}).$$

$$\Sigma_A(q,p) = \frac{A(p)+A(q)}{2} \qquad \Delta_B(p,q) = \frac{B(p)-B(q)}{p^2-q^2}.$$

Obtained Phase diagram







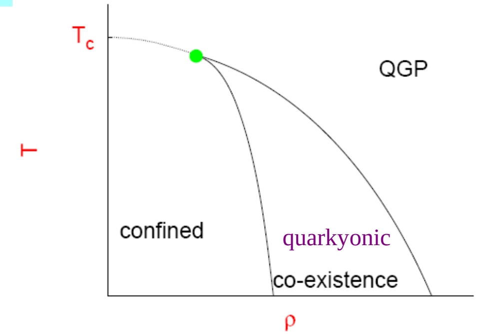
Y. Lu, F. Gao, Y.X. Liu, et al., In preparation.

4. Relation between the Chiral PT & the Confinement-Deconfinement PT

♣ **Lattice QCD Calculation**

de Forcrand, et al.,

Nucl. Phys. B Proc. Suppl. 153, 62 (2006); ···

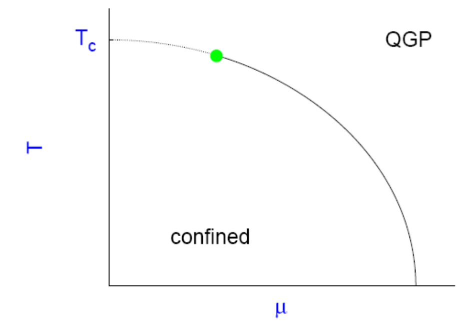


and **General (large-N_C) Analysis**

McLerran, et al., NPA 796, 83 (‘07);

NPA 808, 117 (‘08);

NPA 824, 86 (‘09), ···



claim that there exists a quarkyonic phase.

♣ **Coleman-Witten Theorem (PRL 45, 100 (‘80)):**

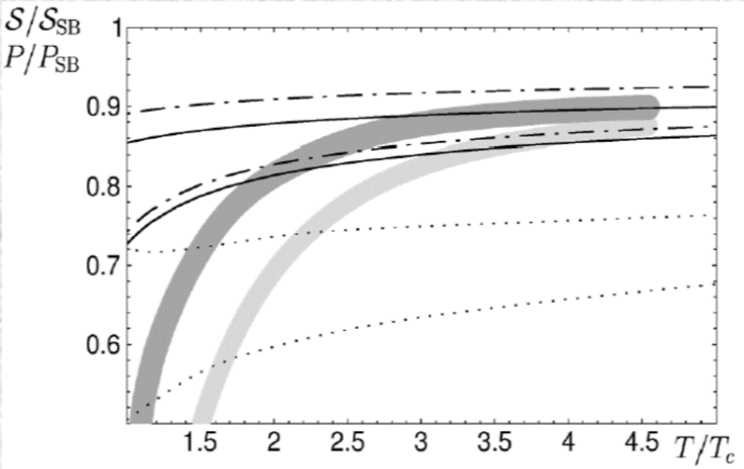
Confinement coincides with DCSB !!

♣ **Inconsistence really exists?!**

Nature of the Quarkyonic Phase ?!

♣ Hadronization Process

- For the system at **thermodynamic limit**,



J.-P. Blaizot, et al., PRL 83, 2906 (1999);

etc.

The monotonic behavior manifests that the entropy density of the quark-gluon phase is always larger than that of hadron phase.

(Nonaka, et al., Phys. Rev. C **71**, 051901R (2005) ; tec.,)

---- Entropy Puzzle.

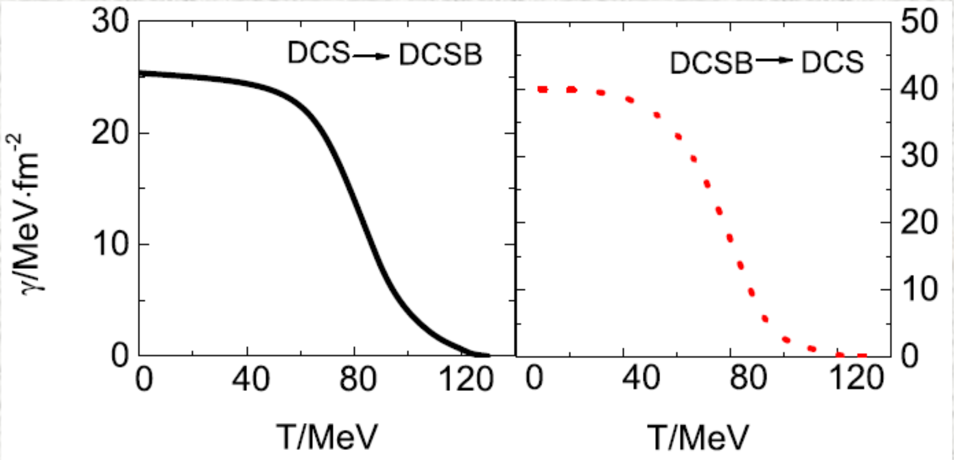
In fact, there exists finite interface, which contributes to the entropy of the system.

♣ Solving the entropy puzzle

- **Interface tension between the DCS-unconf. phase and the DCSB-confined phase**

$$\gamma(T) = \int_{n_L}^{n_H} \sqrt{\frac{C}{2}\Delta F_T(n)}\, dn \ .$$

J. Randrup, PRC 79, 054911 (2009)



Parameterized as $\gamma(T) = a + be^{(c/T + d/T^2)}$,

with parameters

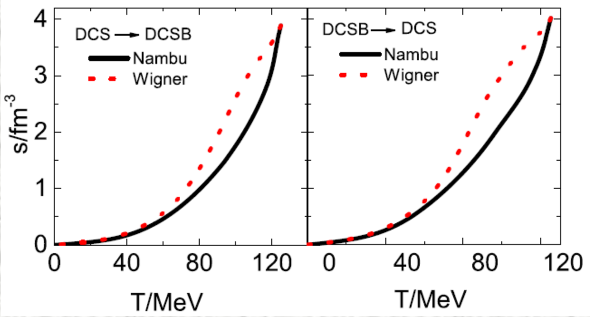
	$a/(\mathrm{MeV/fm^2})$	$b/(\mathrm{MeV/fm^2})$	$c/$ MeV	$d/\mathrm{GeV^2}$
DCS → DCSB	25.4	−1.5	736	−0.048
DCSB → DCS	40.0	−8.1	399	−0.025

Fei Gao, & Yu-xin Liu, Phys. Rev. D 94, 094030 (2016)

• Interface eff. in the hadronization Proc.

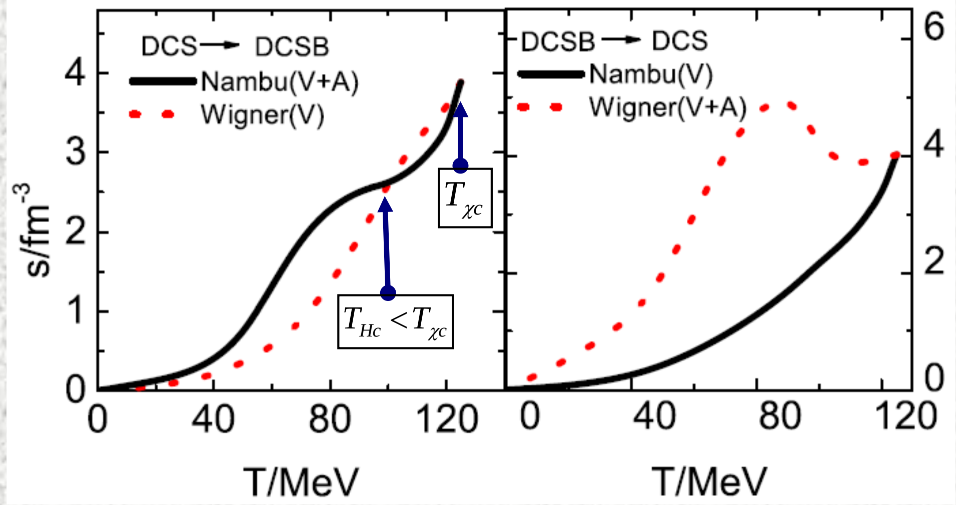
Solving the entropy puzzle

In thermodynamic limit



$$s_V = \frac{1}{T}(\epsilon + P - \mu n) = \frac{\partial P}{\partial T}.$$

With the interface entropy density $s_A = -(\frac{\partial \gamma}{\partial T})_{VA}$ being included, we have

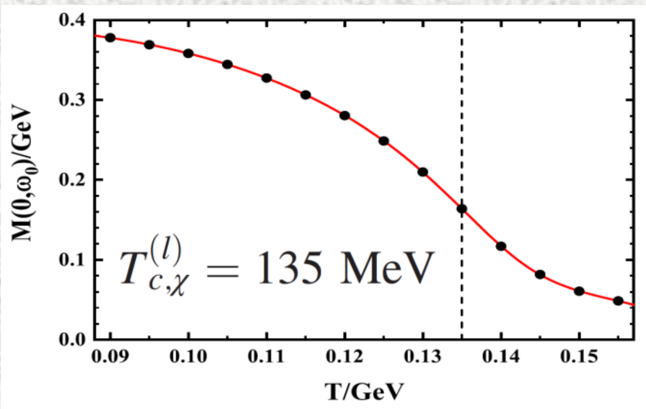


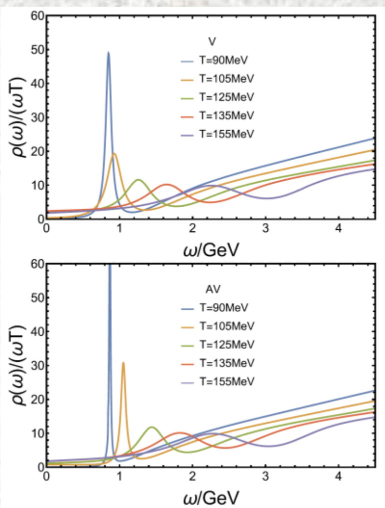
F. Gao, & Y.X. Liu, Phys. Rev. D 94, 094030 (2016);

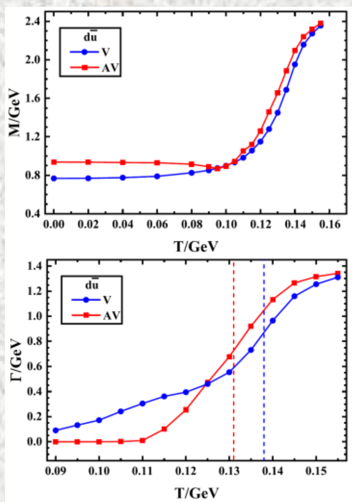
••••••

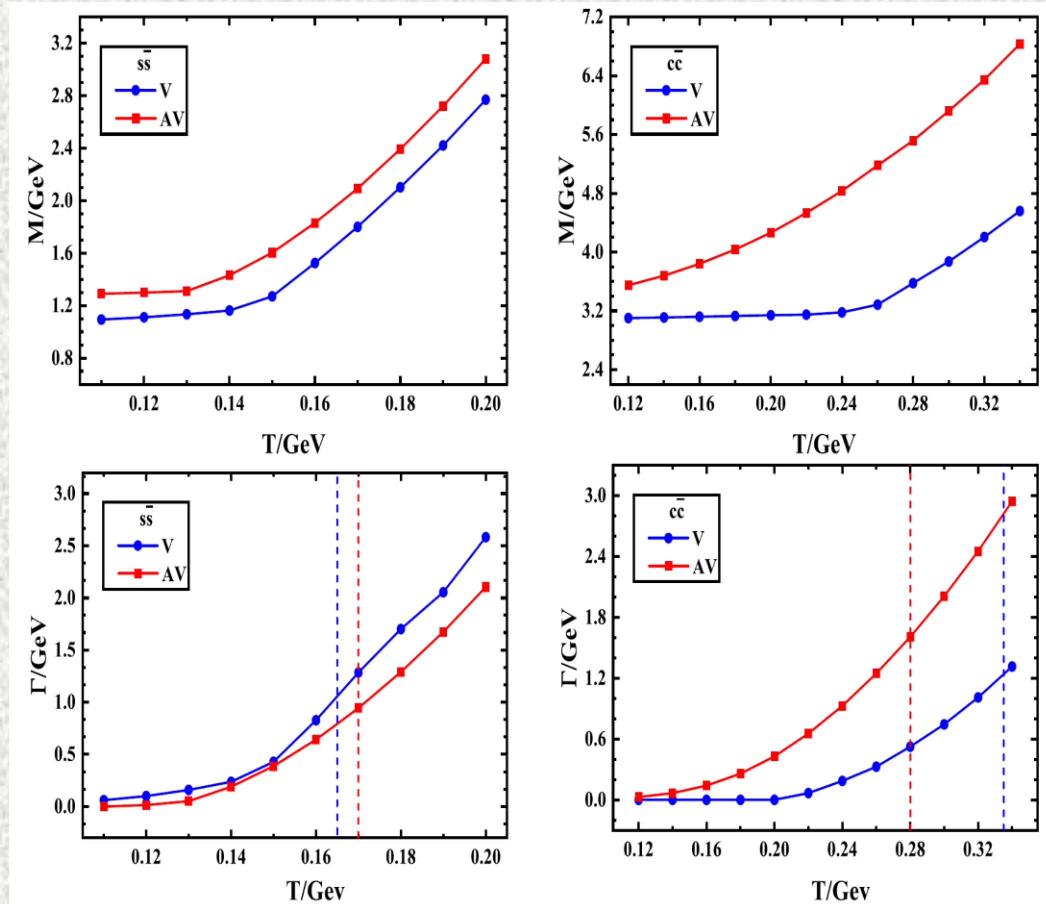
$T_{Hc} < T_{\chi c}$ means that the hadronization (confinement) temperature may really be different from the chiral pseudocritical temperature, which is just a demonstration of the Ultracold Phenomenon due to the interface.

♣ Flavor-dependence of the dissociation Temperature & the relation with the $T_{c,\chi}$







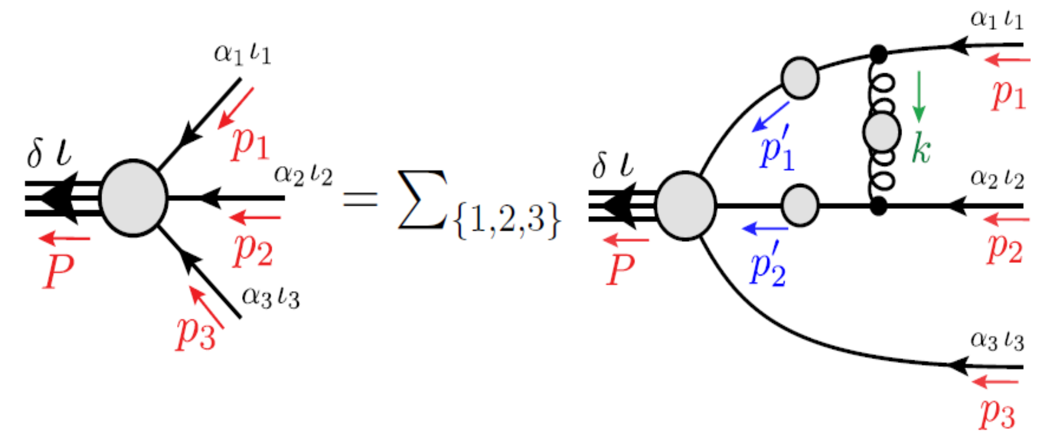


Light flavor: coincident; Heavy flavor: $T_{c,d} > T_{c,}$!

Ling-feng Chen, Si-xue Qin, and YXL, Phys. Rev. D 102, 054012 (2020)

2. Hadron Mass Spectrum

(1) Nucleon — Relativistic three-body problem, Poincare covariant Faddeev Eq.



$$\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3) = \sum_{j=1,2,3} [\mathcal{K}SS\Psi]_j,$$

$$[\mathcal{K}SS\Psi]_3 = \int_{dk} \mathcal{K}^{\alpha_1\alpha_1',\alpha_2\alpha_2'}_{\iota_1\iota_1'\iota_2\iota_2'}(p_1,p_2;p_1',p_2') S^{\alpha_1'\alpha_1''}_{\iota_1'\iota_1''}(p_1') S^{\alpha_2'\alpha_2''}_{\iota_2'\iota_2''}(p_2') \Psi^{\alpha_1''\alpha_2''\alpha_3;\delta}_{\iota_1''\iota_2''\iota_3;\iota}(p_1',p_2',p_3),$$

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'},$$

$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k), \qquad k^2T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu.$$

$$\frac{1}{Z_2^2}\tilde{\mathcal{G}}(s) = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln[\tau + (1+s/\Lambda^2_{QCD})^2]},$$

- **Numerical results in RL approximation**

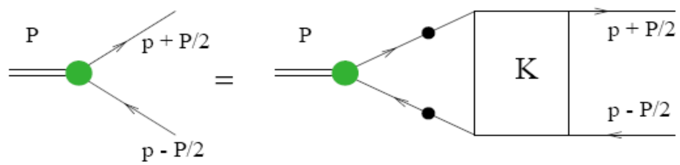
Baryon	quarks	Empirical [90]			Herein				
		$(0,\frac{1}{2}^+)_3$	$(1,\frac{1}{2}^+)_4$	$(0,\frac{1}{2}^-)_5$	$(0,\frac{1}{2}^+)_6$	$(1,\frac{1}{2}^+)_7$	$(0,\frac{1}{2}^-)_8$	$(1,\frac{1}{2}^+)^*_9$	$(0,\frac{1}{2}^-)^*_{10}$
N	uud	0.938	1.440	1.535	0.948	1.279	1.144	1.440	1.542
Λ	uds	1.116	1.600	1.670	1.114	1.474	1.316	1.582	1.581
Σ	uus	1.189	1.660	1.620	1.114	1.474	1.316	1.582	1.581
Ξ	uss	1.315			1.279	1.670	1.487	1.723	1.620
Λ_c	udc	2.286		2.595	2.184	2.543	2.401	2.650	2.666
Σ_c	uuc	2.455			2.184	2.543	2.401	2.650	2.666
Λ_b	udb	5.619		5.912	5.394	5.809	5.650	5.916	5.916
Σ_b	uub	5.811			5.394	5.809	5.650	5.916	5.916
Ξ_c	usc	2.468		2.790	2.350	2.738	2.572	2.792	2.705
Ξ_c'	usc	2.577			2.350	2.738	2.572	2.792	2.705
Ξ_{cc}	ucc	3.621			3.421	3.807	3.657	3.861	3.790
Ξ_b	usb	5.792			5.560	6.004	5.822	6.058	5.955
Ξ_b'	usb	5.945			5.560	6.004	5.822	6.058	5.955
Ξ_{cb}	ucb				6.631	7.073	6.907	7.127	7.040
Ξ_{cb}'	ucb				6.631	7.073	6.907	7.127	7.040

S.X. Qin, C.D. Roberts, S.M. Schmidt, Few-body Syst. 60, 26 (2019); S.X. Qin, et al. Phys. Rev. D 97,114017('18); etc.

(2) Meson — Poincare covariant BSE with DSE

Quantum field theory bound states: BSE

$$\Gamma_M(p;P) = \int_k^{\Lambda} K(p,k;P)\, S(k_+)\, \Gamma_M(k;P)\, S(k_-)$$



L. Chang,
C.D. Roberts,
PRL 103,
081601
(2009); ……

Interaction	Eq. (6)	Eq. (8)	Eq. (8)	Eq. (8)	Eq. (8)
$(D\omega)^{1/3}$	0.72	0.8	0.8	0.8	0.8
ω	0.4	0.4	0.5	0.6	0.7
$m^{\zeta}_{u,d}$	0.0037	0.0034	0.0034	0.0034	0.0034
m^{ζ}_{s}	0.084	0.082	0.082	0.082	0.082
$A(0)$	1.58	2.07	1.70	1.38	1.16
$M(0)$	0.50	0.62	0.52	0.42	0.29
m_π	0.138	0.139	0.134	0.136	0.139
f_π	0.093	0.094	0.093	0.090	0.081
$\rho_\pi^{1/2}$	0.48	0.49	0.49	0.49	0.48
m_K	0.496	0.496	0.495	0.497	0.503
f_K	0.11	0.11	0.11	0.11	0.10
$\rho_K^{1/2}$	0.54	0.55	0.55	0.55	0.55
m_ρ	0.74	0.76	0.74	0.72	0.67
f_ρ	0.15	0.14	0.15	0.14	0.12
m_ϕ	1.07	1.09	1.08	1.07	1.05
f_ϕ	0.18	0.19	0.19	0.19	0.18
m_σ	0.67	0.67	0.65	0.59	0.46
$\rho_\sigma^{1/2}$	0.52	0.53	0.53	0.51	0.48

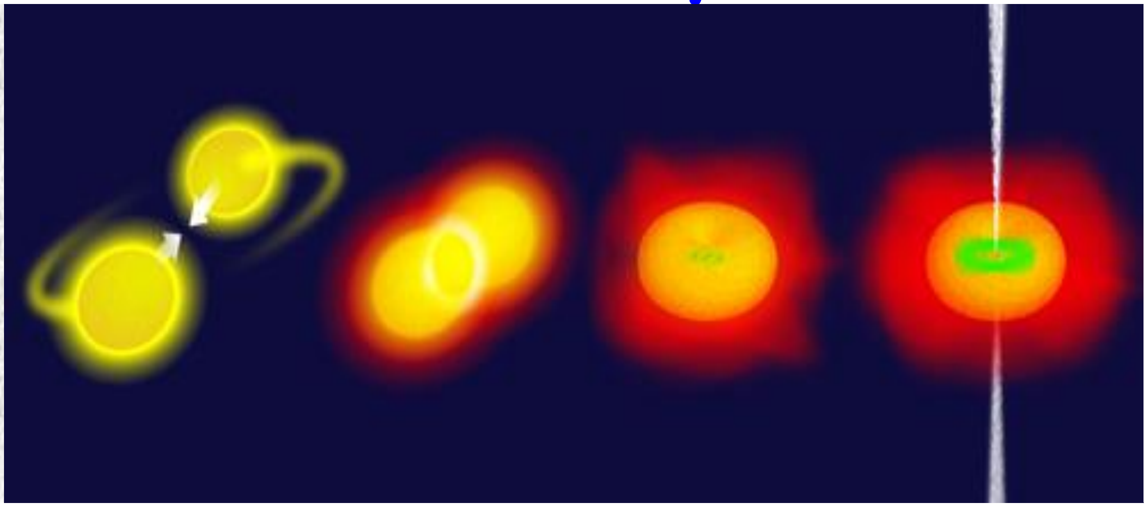
(S.X. Qin, et al., Phys. Rev. C 84, 042202(R) (2011) ,

	via CLRQ vertex	Expt.	RL-Padé	RL-direct
m_π	0.138	0.138	0.138	0.137
m_ρ	0.84 ± 0.03	0.777	0.754	0.758
m_σ	1.13 ± 0.01	$0.4 - 1.2$	0.645	0.645
m_{a_1}	1.28 ± 0.01	1.24 ± 0.04	0.938	0.927
m_{b_1}	1.24 ± 0.10	1.21 ± 0.02	0.904	0.912
$m_{a_1} - m_\rho$	0.44 ± 0.04	0.46 ± 0.04	0.18	0.17
$m_{b_1} - m_\rho$	0.40 ± 0.14	0.43 ± 0.02	0.15	0.15

(L. Chang, et al.,
Phys. Rev. C 85, 052201(R) (2012))

♠ an Excellent Astronomic Observation Signal: Gravitational Mode Oscillation Frequency

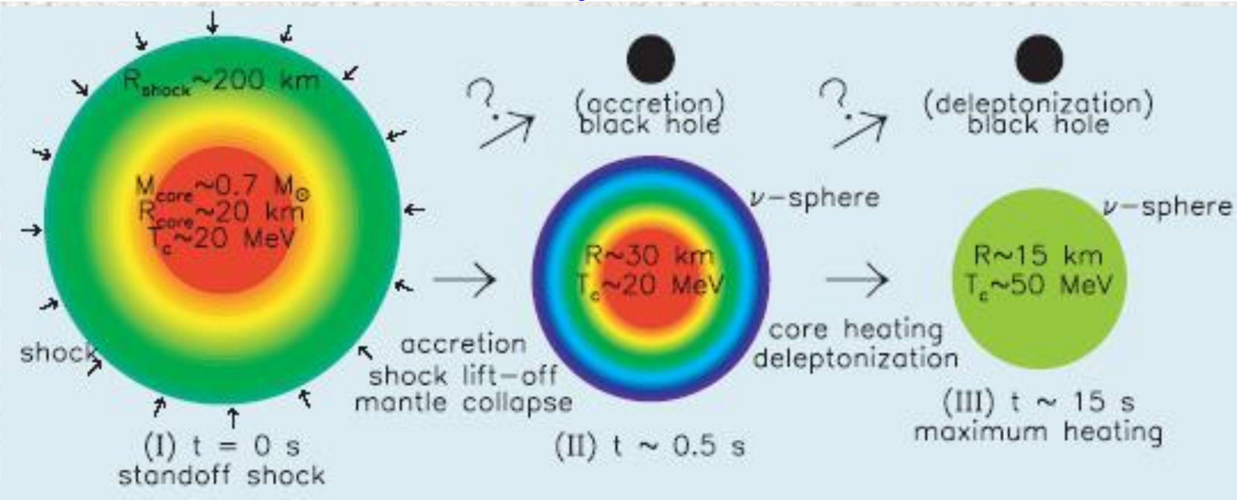
- **G-Wave in Binary Neutron Star Merger**

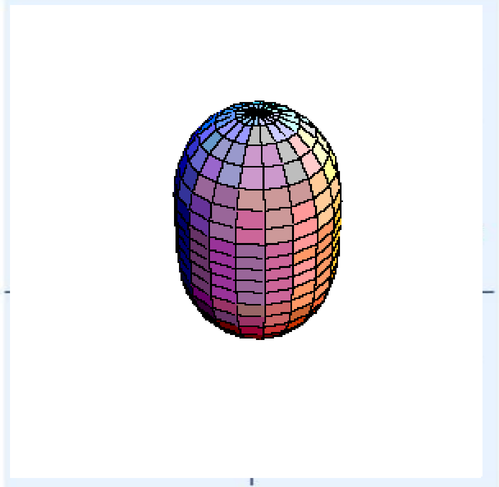


$F_{postmerger} \in (1.84, 3.73)$kHz, with width<200Hz, (PRD 86, 063001(2012))

$F_{spiral} < F_{postmerger}$

- **G-Wave in Newly Born NS/QS after the SNE**





- Comparison of G-mode Oscillation Frequencies of the two kind nb Stars

Neutron Star: RMF, Quark Star: Bag Model

→ Frequency of the G-mode oscillation

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
$n=1$	717.6	774.6	780.3	82.3	78.0	63.1
$n=2$	443.5	467.3	464.2	52.6	45.5	40.0
$n=3$	323.8	339.0	337.5	35.3	30.8	27.8

W.J. Fu, H.Q. Wei, and Y.X. Liu, arXiv: 0810.1084, Phys. Rev. Lett. 101, 181102 (2008)

• Comparison with other modes

Neutron Star: RMF, Quark Star: Bag Model

➡ **Frequencies of the f- & p-mode oscillations**

Modes	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
${}_2f$	1103	1133	1176	2980	2997	3016
${}_2p_1$	2265	2426	2494	18282	17330	16792
${}_2p_2$	3780	4054	4179	28792	27288	26438
${}_2p_3$	5319	5702	5869	38988	36950	35798

♣ G-mode oscillation in quark star has very low freq. !

W.J. Fu, Z. Bai, Y.X. Liu, arXve:1701.00418

IV. Summary & Remarks

♠ The framework of QCD is surveyed,

♠ DS equations, — A continuum npQCD approach is presented.

♠ The observable mass generation & confinement are described with the DSE approach.

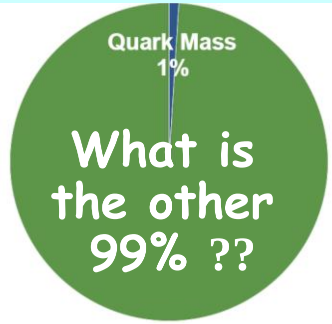
♠ Hadron mass spectra are described with the DSE approach.

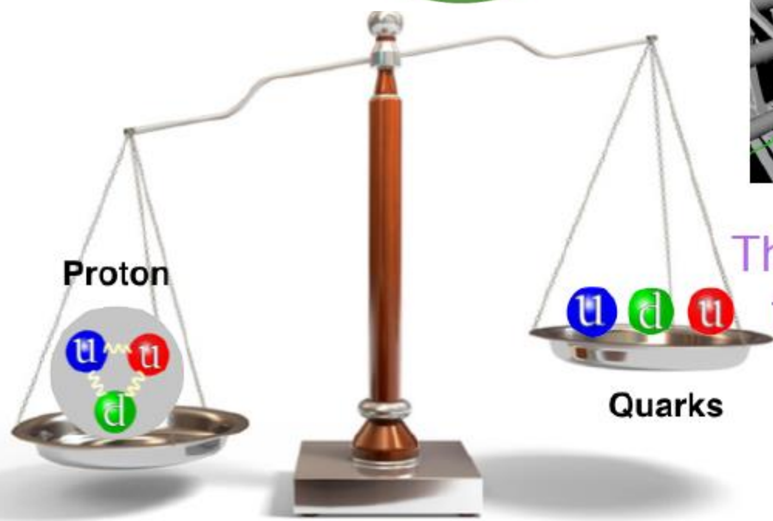
♠ Continuum QCD has contributed great to fundamental issues of physics, and is promising! ……

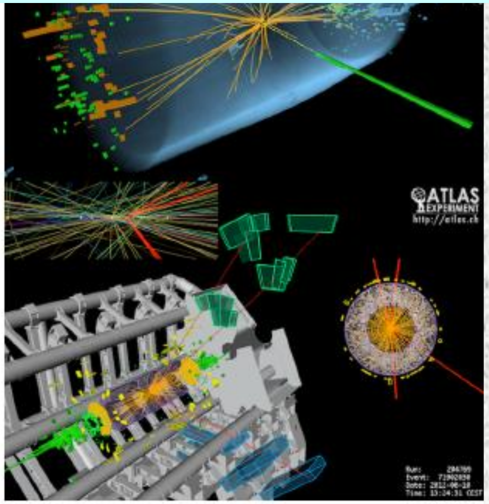
Thanks !

♠ The Nucleon Mass Crisis

How does the mass of nucleon arise ??







The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(09)$ MeV
$m_d = 4.73(12)$ MeV

http://flag.unibe.ch/2019/Quark%20masses

But the mass of the proton is
938.272046(21) MeV.
~100 times of the sum of the quark masses!

"核子质量起源"与"夸克囚禁"共同构成一个世纪大奖问题!