Giant Resonances

Fundamental Collective Modes of Nuclear Excitation

Electric Modes

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In the following:

IS = Iso-Scalar

IV = Iso-Vector

S = Spin

G = Giant

M = Monopole

D = Dipole

Q = Quadrupole

O = Octupole

e.g., ISGMR = Isoscalar giant monopole resonance

ISGDR = Isoscalar giant dipole resonance

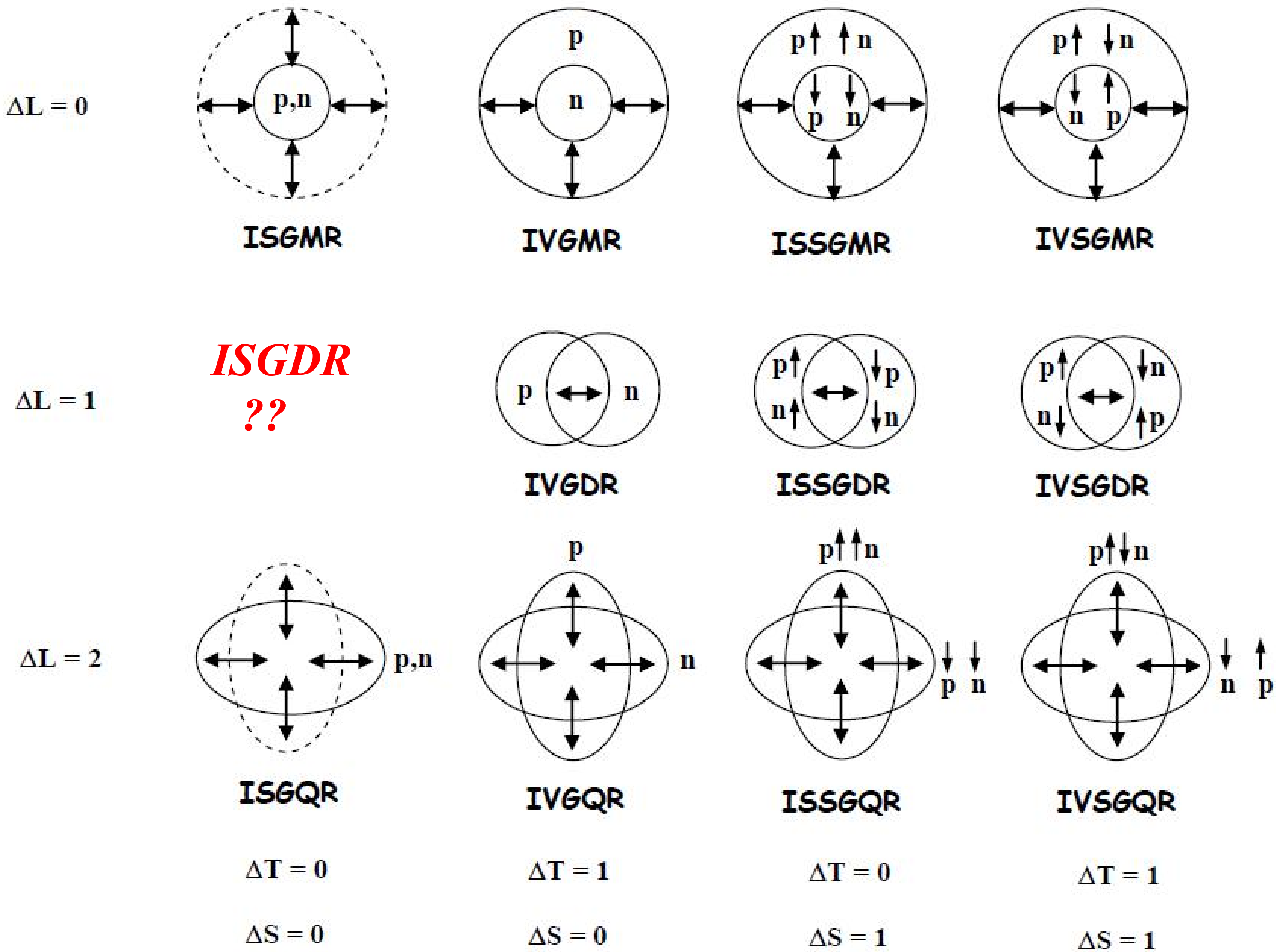
IVGDR = Isovector giant dipole resonance

IVSGMR= Isovector spin giant monopole resonance

IVSGDR = Isovector spin giant dipole resonance

Giant Resonances in hydrodynamic models

Coherent vibrations of nucleonic fluids (p & n; ↑ & ↓) in a nucleus



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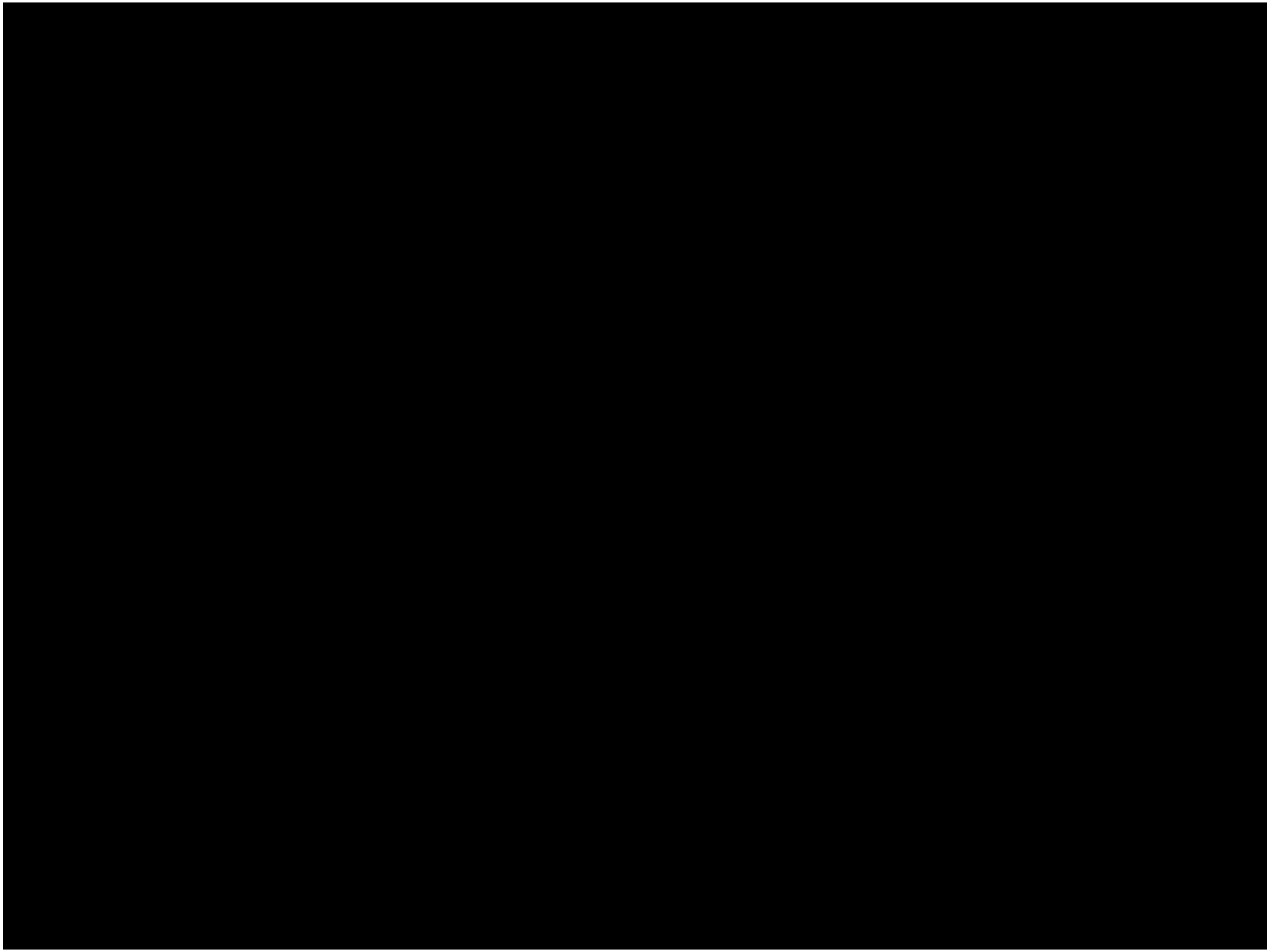
Vibrations of a liquid drop in weightlessness



Operators and Microscopic Structure

Microscopic picture: GRs are coherent (1p-1h) excitations induced by single-particle operators.

- **Excitation energy depends on**

  ***i*) multipole *L* (*Lħω*, since radial operator $\propto r^L$; except for ISGMR and ISGDR, *2ħω* & *3ħω*, respectively),**

  ***ii*) strength of effective interaction and**

  ***iii*) collectivity.**
- **Exhaust appreciable % of EWSR**
- **Acquire a width due to coupling to continuum and to underlying 2p-2h configurations.**

Microscopic structure of ISGMR & ISGDR

Transition operators: Long-wave length limit
Expand Bessel function

$$O^{L=0} = \cancel{\sum_i r_i^0 Y_0^0} + \frac{1}{2}\sum_i r_i^2 Y_0^0 + \ \dots$$

Constant   Overtone

***2ħω* excitation**

$$O^{L=1} = \cancel{\sum_i r_i^1 Y_0^1} + \frac{1}{2}\sum_i r_i^3 Y_0^1 + \ \dots$$

Spurious c.o.m. motion   Overtone

***3ħω* excitation (overtone of c.o.m. motion)**

Nucleus → **Many-body system with a finite size**

Vibrations → **Multipole expansion with r, Y_{lm}, τ, σ**

	$\Delta S=0,\ \Delta T=0$	$\Delta S=0,\ \Delta T=1$	$\Delta S=0,\ \Delta T=1$	$\Delta S=1,\ \Delta T=1$	$\Delta S=1,\ \Delta T=1$
$L=0$: Monopole	ISGMR $r^2 Y_0$	IAS τY_0	IVGMR $\tau r^2 Y_0$	GTR $\tau\sigma Y_0$	IVSGMR $\tau\sigma r^2 Y_0$
$L=1$: Dipole	ISGDR $(r^3 - 5/3\langle r^2\rangle r) Y_1$		IVGDR $\tau r Y_1$		IVSGDR $\tau\sigma r Y_1$
$L=2$: Quadrupole	ISGQR $r^2 Y_2$		IVGQR $\tau r^2 Y_2$		IVSGQR $\tau\sigma r^2 Y_2$
$L=3$: Octupole	LEOR, HEOR $r^3 Y_3$				

Dropped $\Delta S=1$, $\Delta T=0$ operators because excitations are very weak

IVGDR

$\tau\, r Y_1$

$\Delta N = 1$ **E1 (IVGDR)**

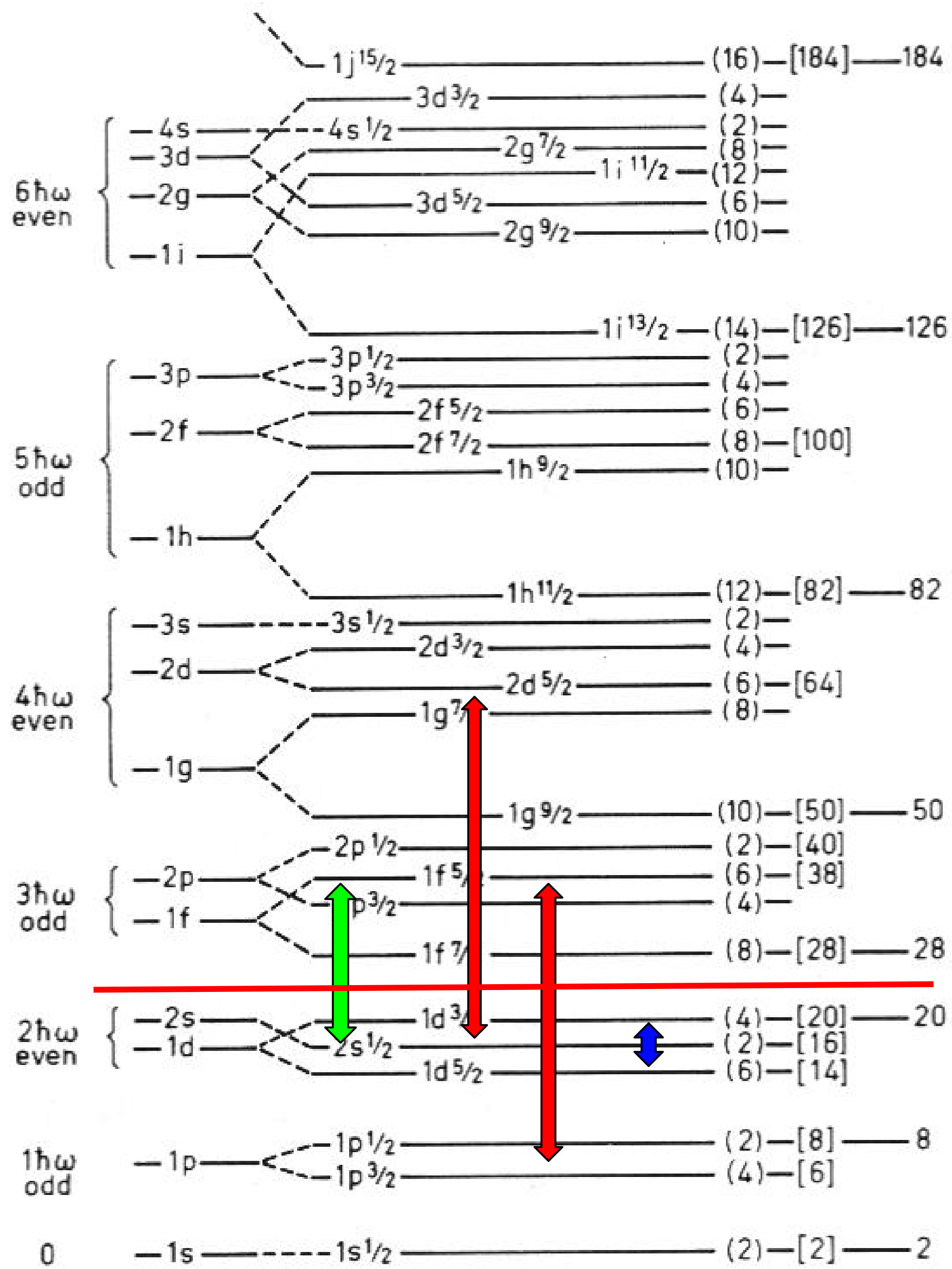
$\Delta N = 2$ **E2 (ISGQR)**

&

$\Delta N = 0$ **E0 (ISGMR)**

ISGMR $r^2 Y_0$

ISGQR $r^2 Y_2$



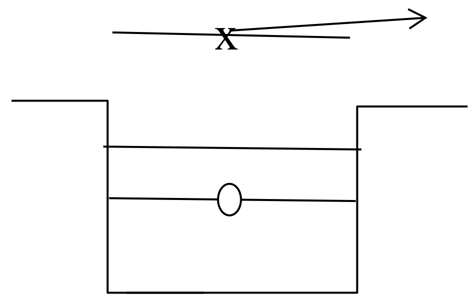
Decay of giant resonances

- **Width of resonance**

  $\Gamma, \Gamma^{\uparrow}, \ \Gamma^{\downarrow} \ (\Gamma^{\downarrow\uparrow}, \Gamma^{\downarrow\downarrow})$

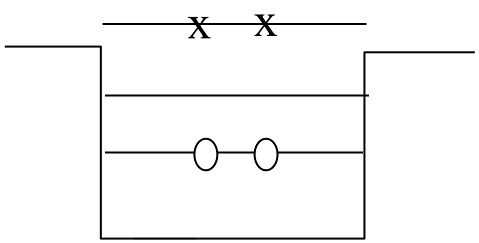
  - $\Gamma^{\uparrow}$: **direct or escape width**
  - $\Gamma^{\downarrow}$: **spreading width**

    $\Gamma^{\downarrow\uparrow}$: **pre-equilibrium,** $\Gamma^{\downarrow\downarrow}$: **compound**



$\Gamma^{\uparrow}$

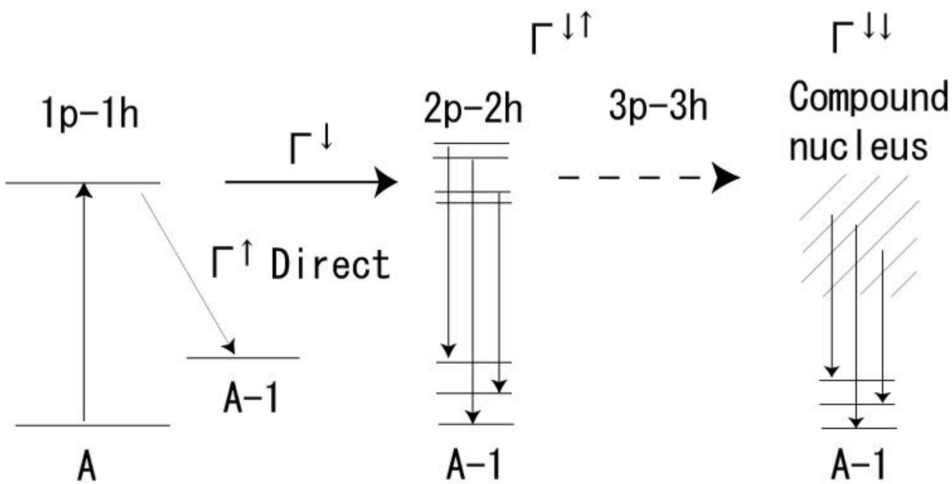
$\Gamma^{\downarrow}$



- **Decay measurements**

  ⇒ Direct reflection of damping processes

  Allows detailed comparison with theoretical calculations



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Energy-Weighted Sum Rules

$$P_{0\mu} = \frac{1}{2}\sum_i r_i^2$$

$$\sum_n (E_n - E_0) B(E0, 0 \to n) = S_0 = \frac{\hbar^2}{2m} A < r^2 >$$

$$P_{1\mu} = \frac{1}{2}\sum_i r_i^3 Y_{1\mu}(\hat{r}_i)$$

$$\sum_n (E_n - E_0) B(E1, 0 \to n) = S_1 = \frac{\hbar^2}{8\pi m}\frac{3}{4} A[11 < r^4 > - \frac{25}{3} < r^2 >^2 - 10\varepsilon < r^2 >]$$

$$\varepsilon = (\frac{4}{E_2} + \frac{5}{E_0})\frac{\hbar^2}{3mA}$$

$$Q_{\lambda\mu} = \sum_i r_i^\lambda Y_{\lambda\mu}(\hat{r}_i)$$

$$\sum_n (E_n - E_0) B(E\lambda, 0 \to n) = S_\lambda = \frac{\hbar^2}{8\pi m}\lambda(2\lambda+1)^2 A < r^{2\lambda-2} >$$

$$\int_0^\infty \sigma(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2 \hbar}{mc}\frac{NZ}{A} = 60\frac{NZ}{A}\ MeV\,mb$$

This is the TRK sum rule for a nucleus.

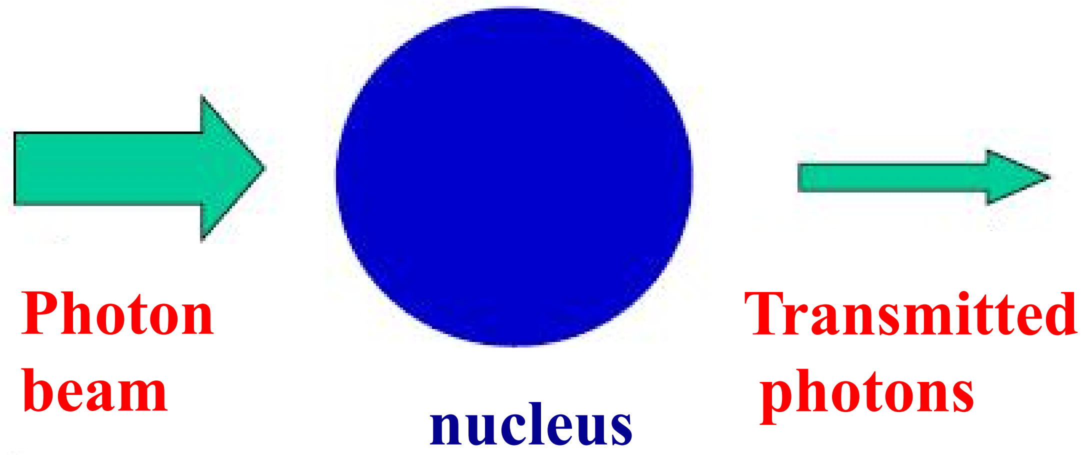
$$\sum_n (E_n - E_0)B(E1,0 \to n) = \frac{9\hbar^2}{8\pi m}\frac{NZ}{A}e^2$$

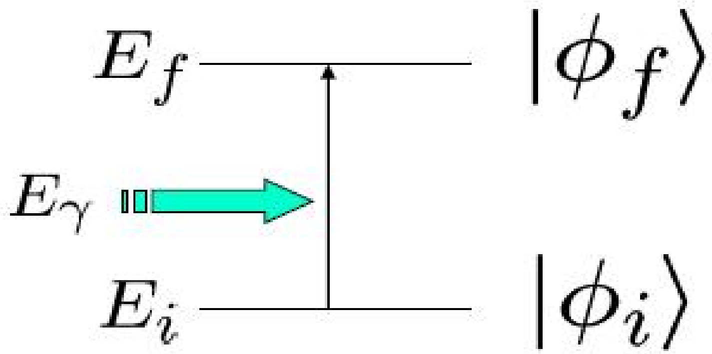
IVGDR

Consider isovector electric dipole excitations.

How does a nucleus respond to an external perturbation, e.g., real photons?

$\Rightarrow$ Photo-absorption cross section
γ-rays from bremsstrahlung or positron capture in flight





The state is strongly excited when
$E_f - E_i = E_\gamma$

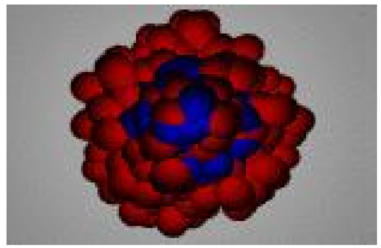
Nuclear Collective response
Giant Resonances

Isovector Electric Giant Resonances

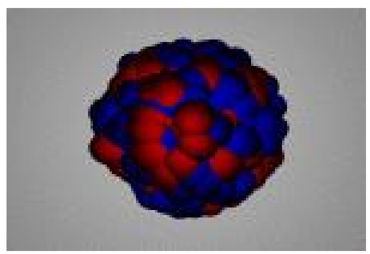
Isovector

Photo-neutron cross sections

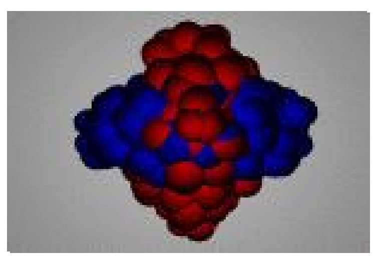
Monopole
(IVGMR)



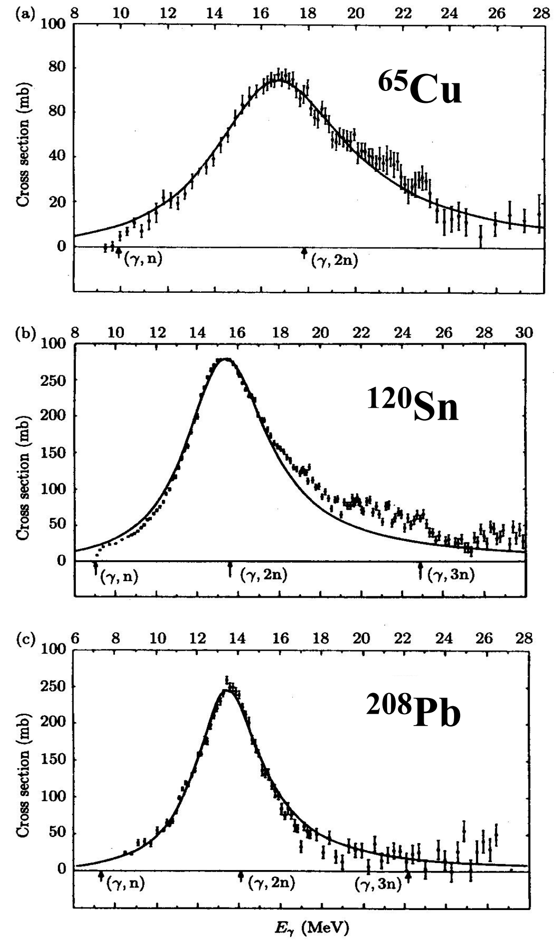
Dipole
(IVGDR)



Quadrupole
(IVGQR)



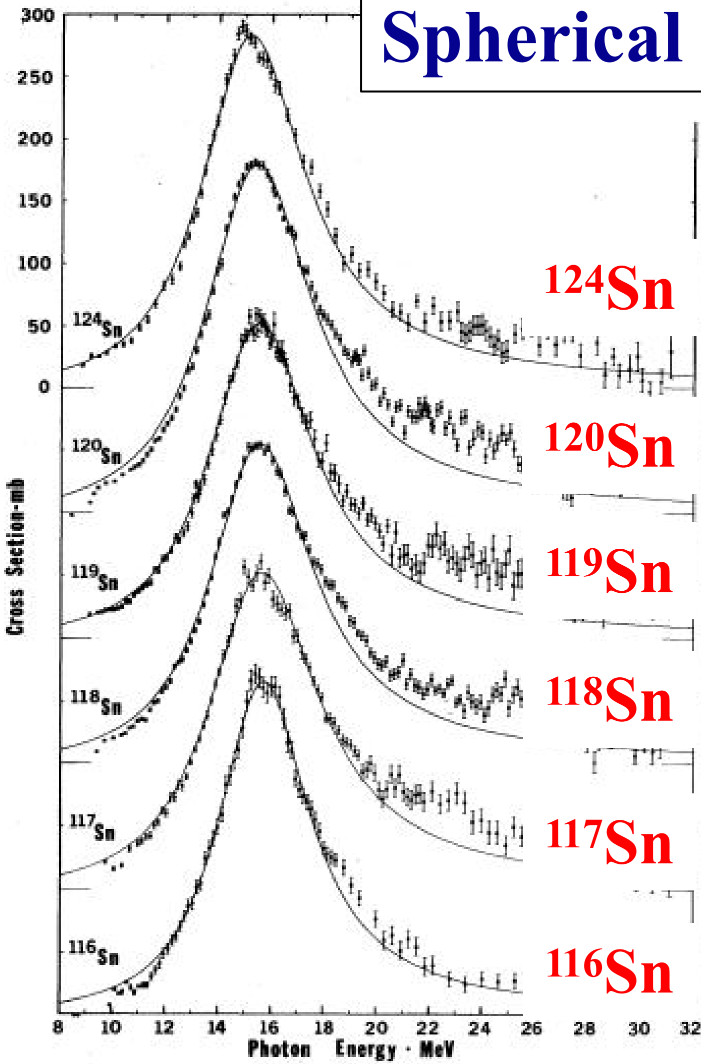
Berman and Fultz, Rev. Mod. Phys. 47 (1975) 713



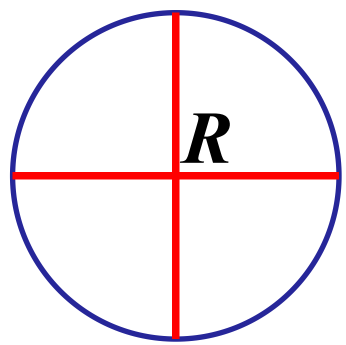
B. L. Berman and S. C. Fultz, Rev. Mod. Phys. 47 (1975) 713

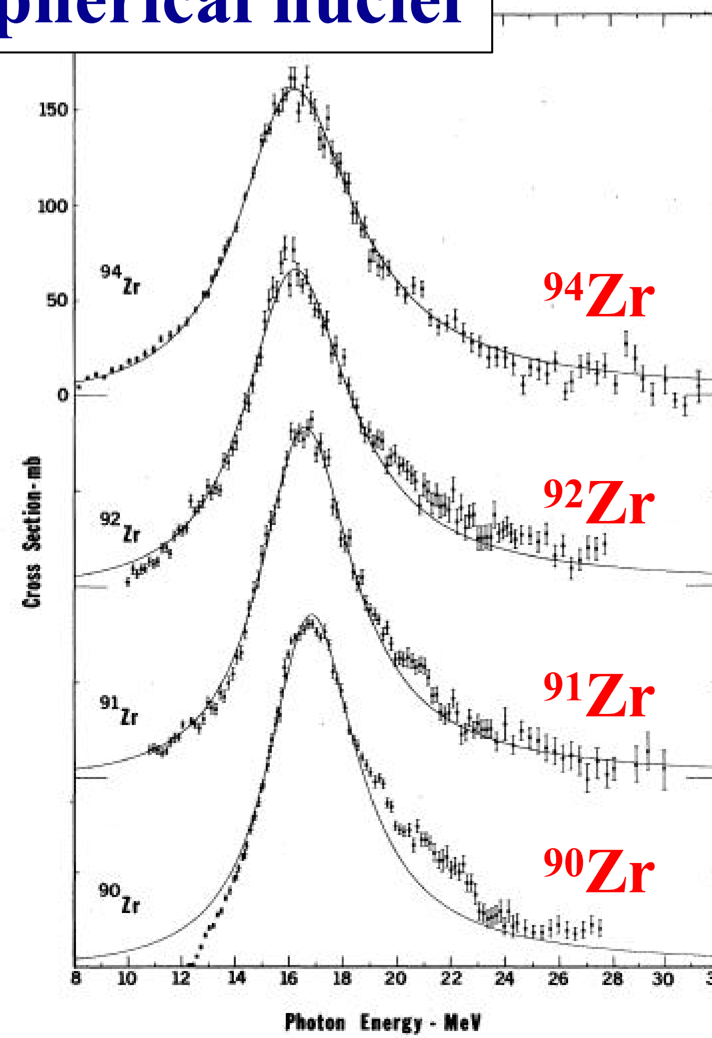
Isovector Giant Dipole Resonances: Photo-neutron cross section

Spherical and nearly spherical nuclei

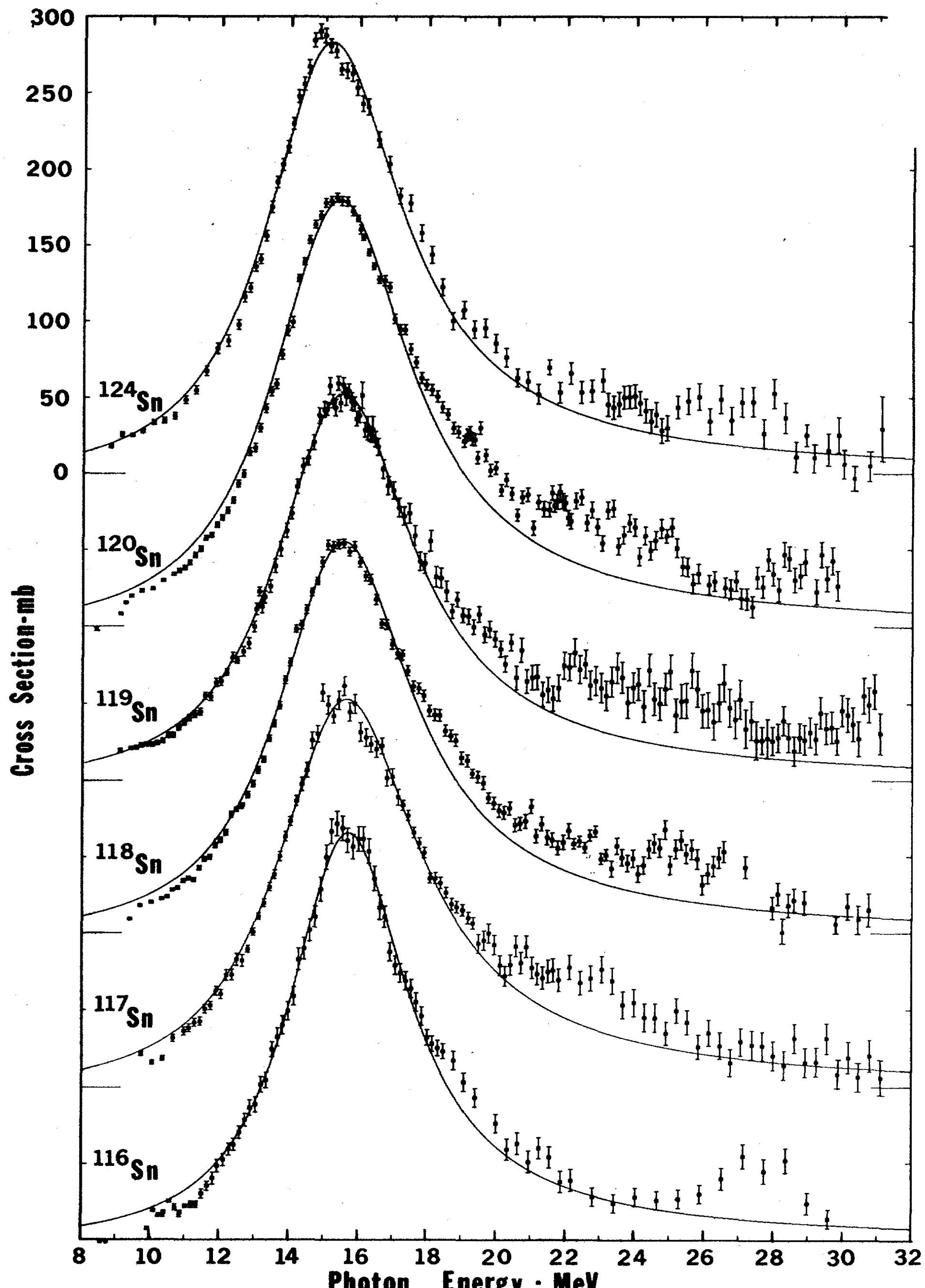


$$\omega \propto \frac{1}{R} \propto A^{-\frac{1}{3}}$$





B. L. Berman and S. C. Fultz, Rev. Mod. Phys. 47 (1975) 713



Measurement of the giant dipole resonance with mono-energetic photons

B.L. Berman and S.C. Fultz
Rev. Mod. Phys. 47 (1975) 713

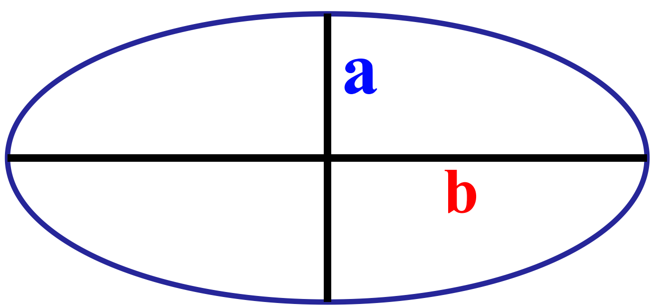
Nucleus	Centroid (MeV)	Width (MeV)
^{116}Sn	15.68	4.19
^{117}Sn	15.66	5.02
^{118}Sn	15.59	4.77
^{119}Sn	15.53	4.81
^{120}Sn	15.40	4.89
^{124}Sn	15.19	4.81

Photo-neutron cross section in deformed nuclei:

Deformed Nucleus

$$R(\theta,\phi)=R_0(1+\beta_2 Y_{20}(\theta,\phi))$$

β_2 (^{150}Nd) = 0.285(3)



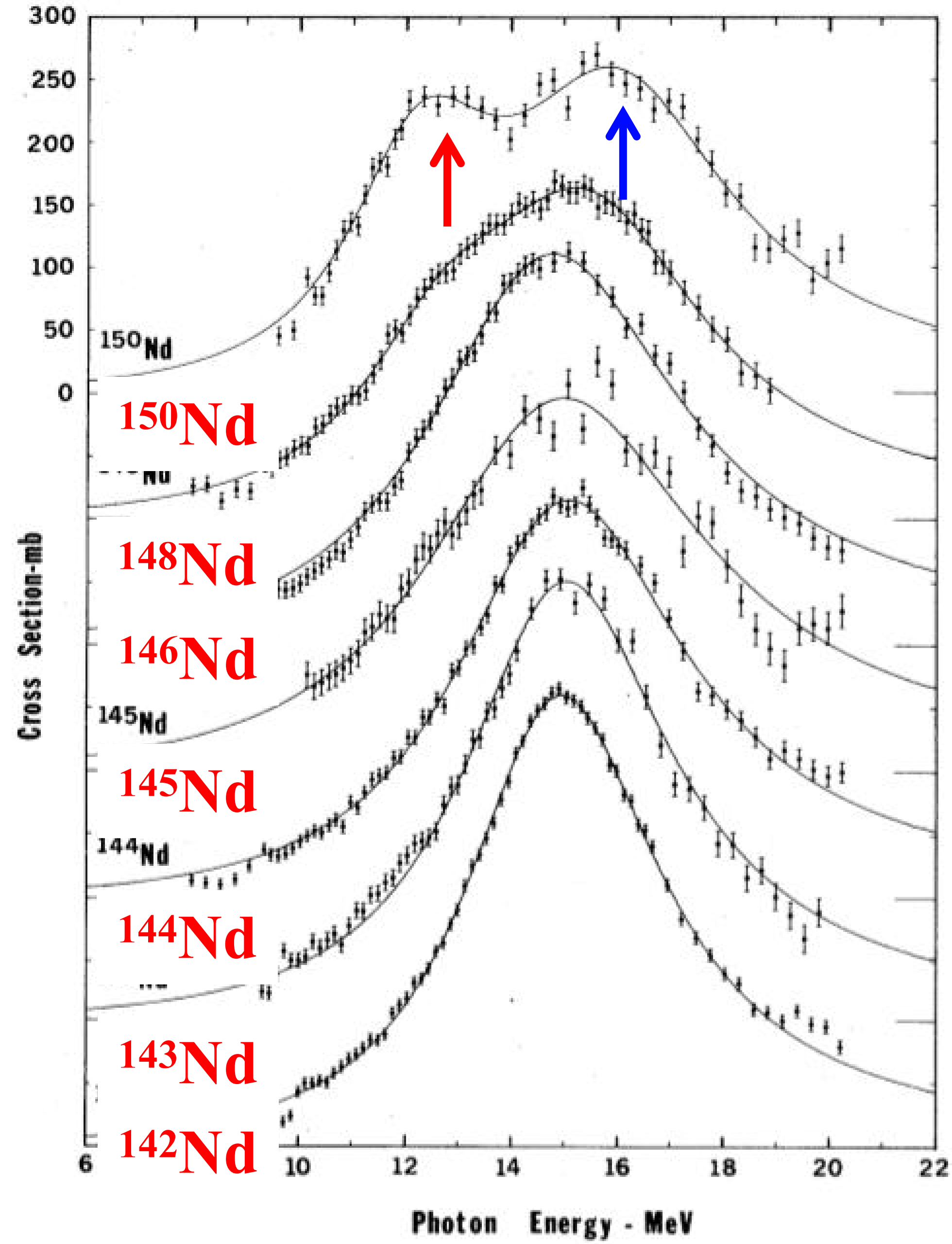
Excitation energies:

$E_2/E_1 = 0.911\eta + 0.089$

Where $\eta = b/a$

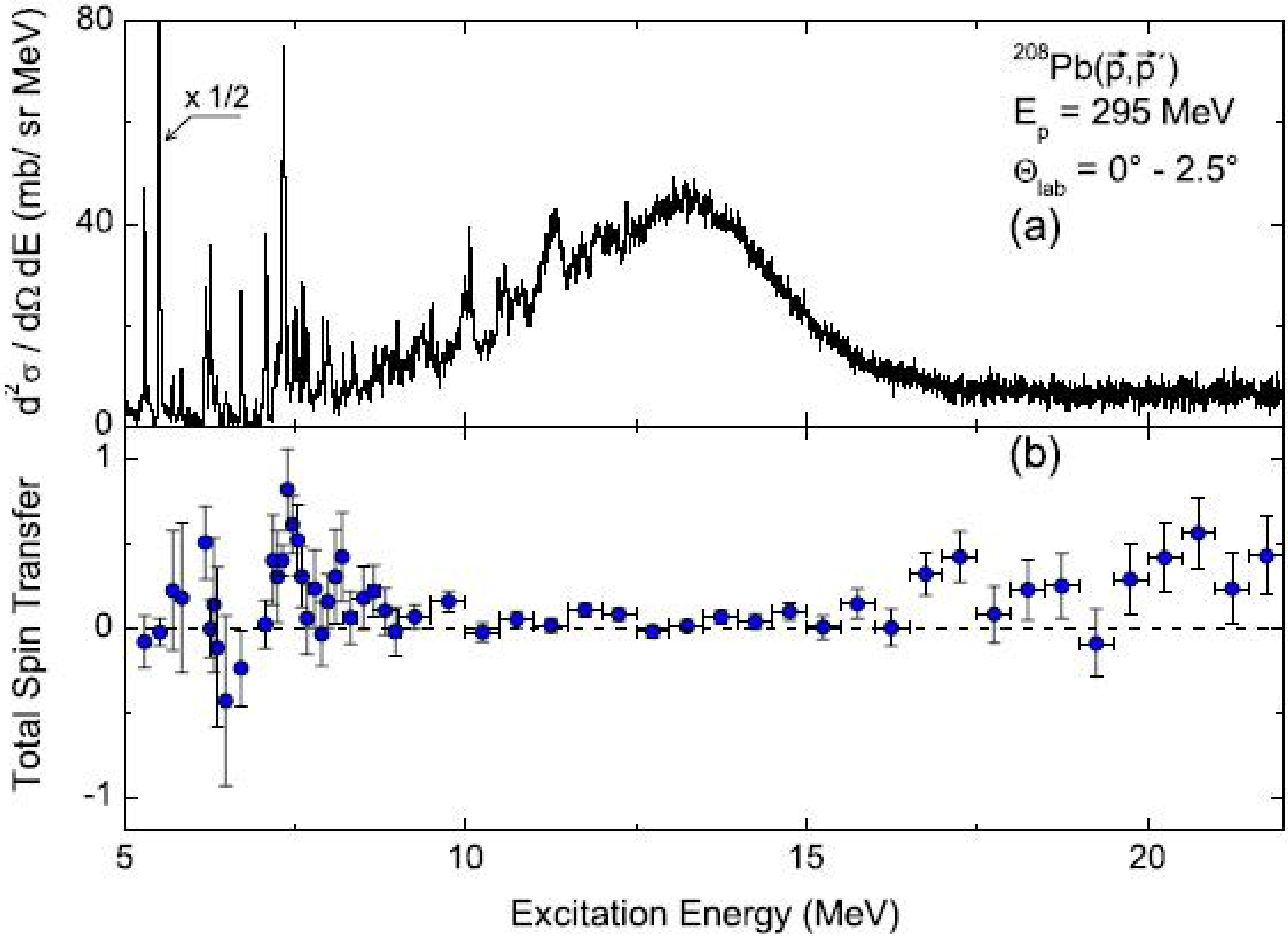
$S_1/S_2 = 1/2$

B. L. Berman and S. C. Fultz,
Rev. Mod. Phys. 47, 713 (1975)



Experiments at RCNP, Osaka University

- **(*p*,*p′*) reaction at 295 MeV**
  - **High-resolution spectrometer “Grand Raiden”**



A. Tamii *et al.*, Phys. Rev. Lett. 107 (2011) 062502

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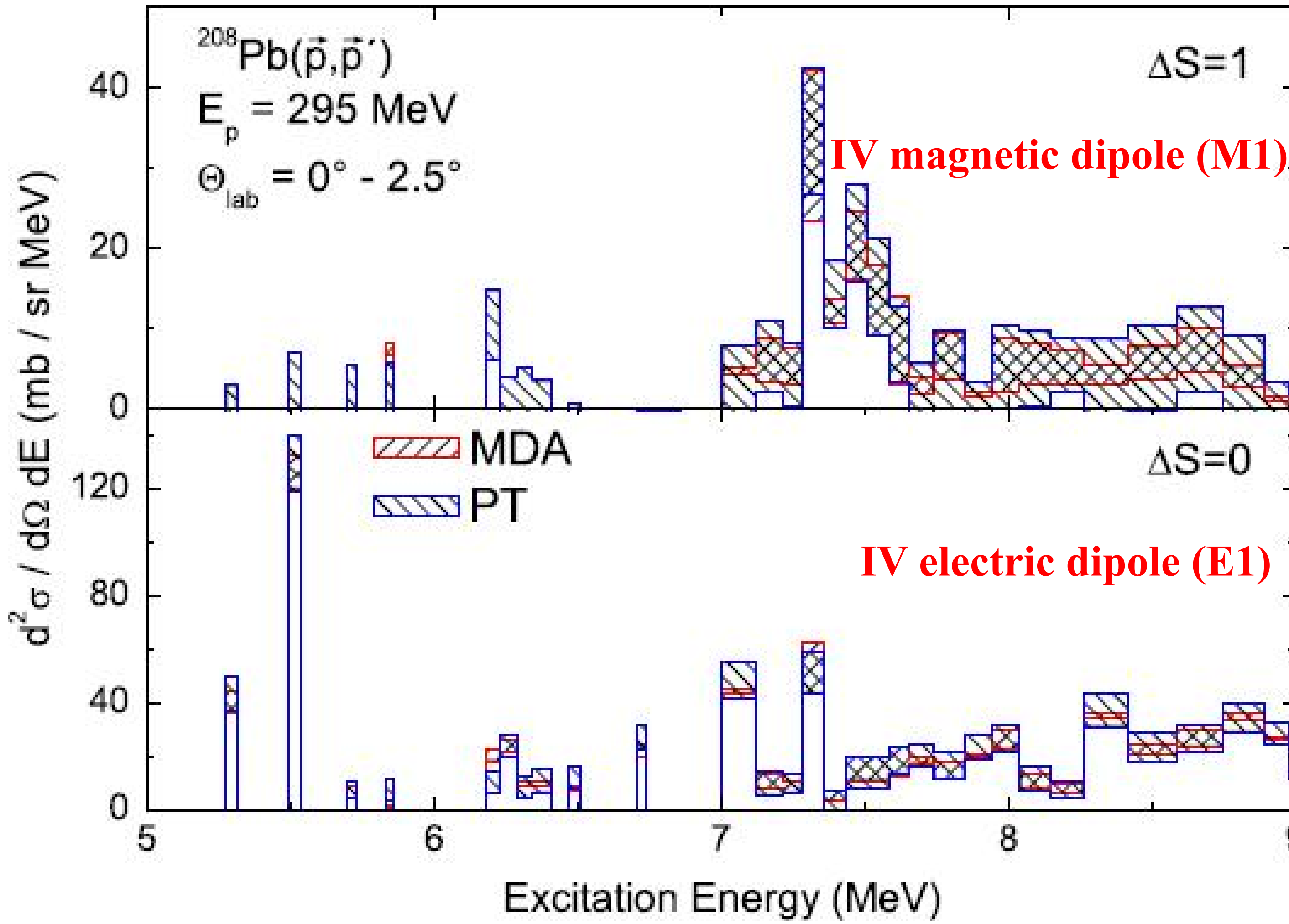
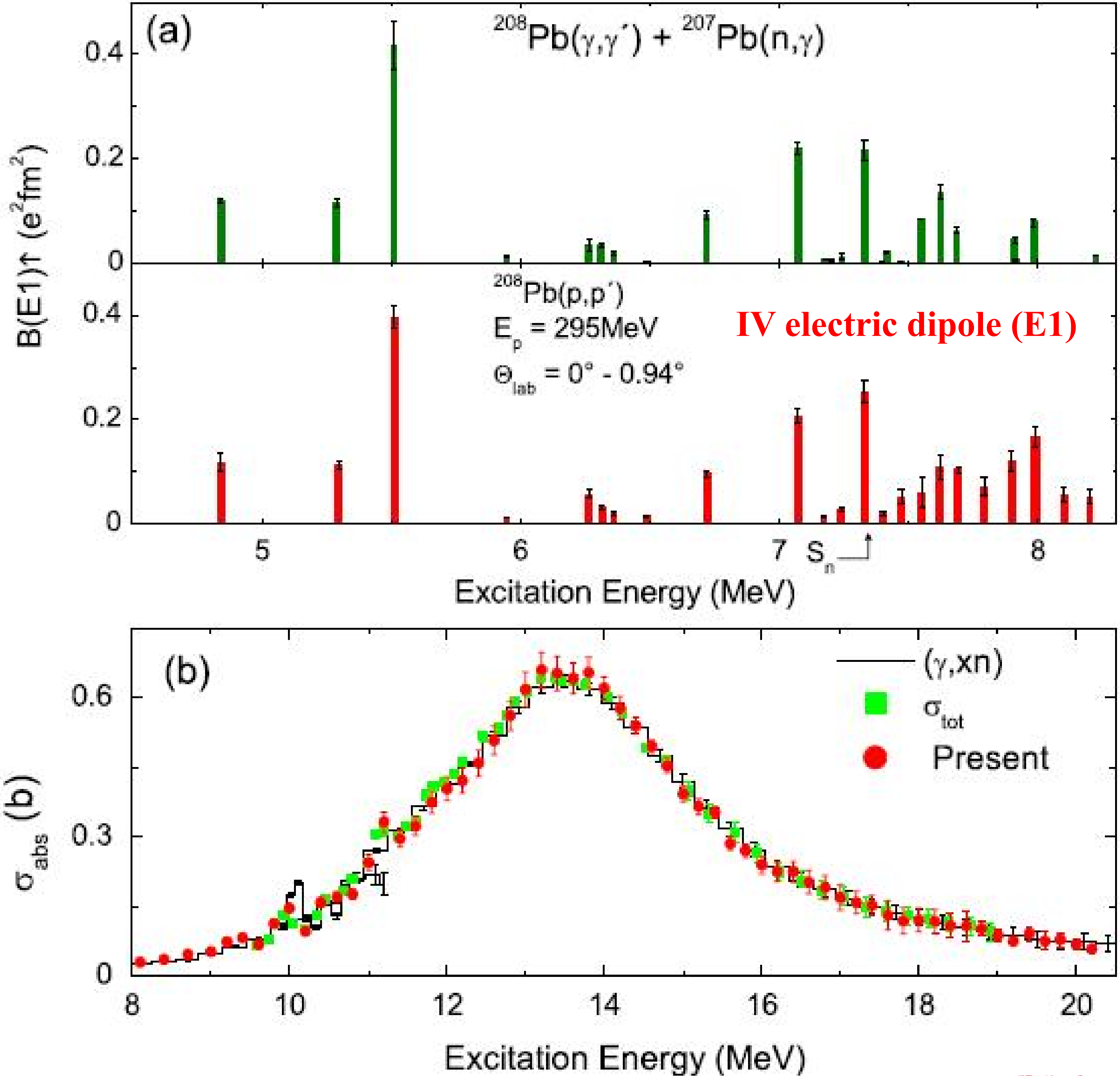


MDA = Multipole-Decomposition Analysis

PT = Polarisation Transfer

A. Tamii *et al.*, Phys. Rev. Lett. 107 (2011) 062502



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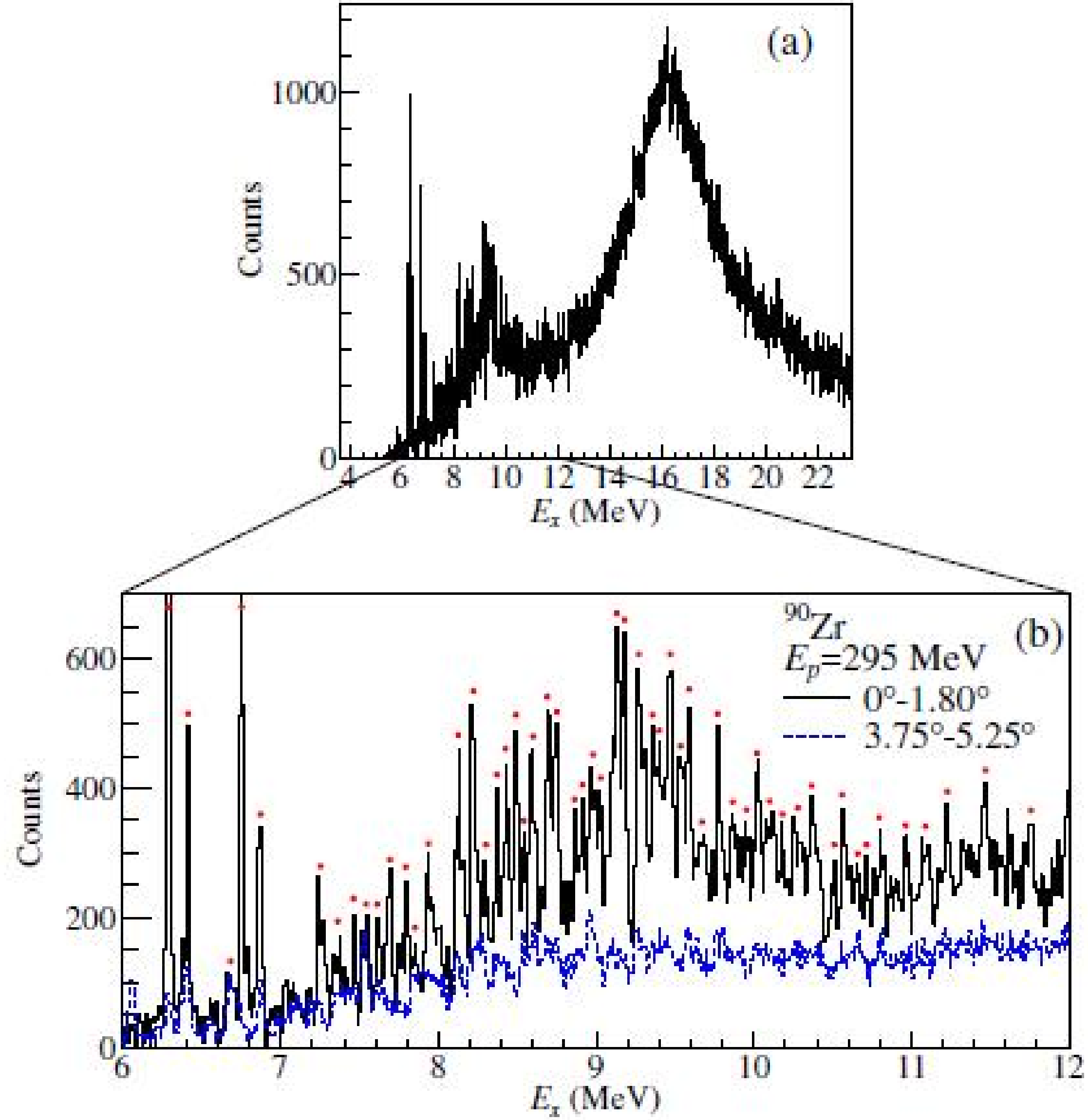
0°-1.8° inelastic proton scattering spectrum shows in addition to IVGDR low-lying E1 and M1 structures.

Peaks with (*) have been selected for multipole-decomposition analysis.

3.75°-5.25° inelastic proton scattering spectrum is almost structure-less.



C. Iwamoto *et al.*, Phys. Rev. Lett. 108 (2012) 262501

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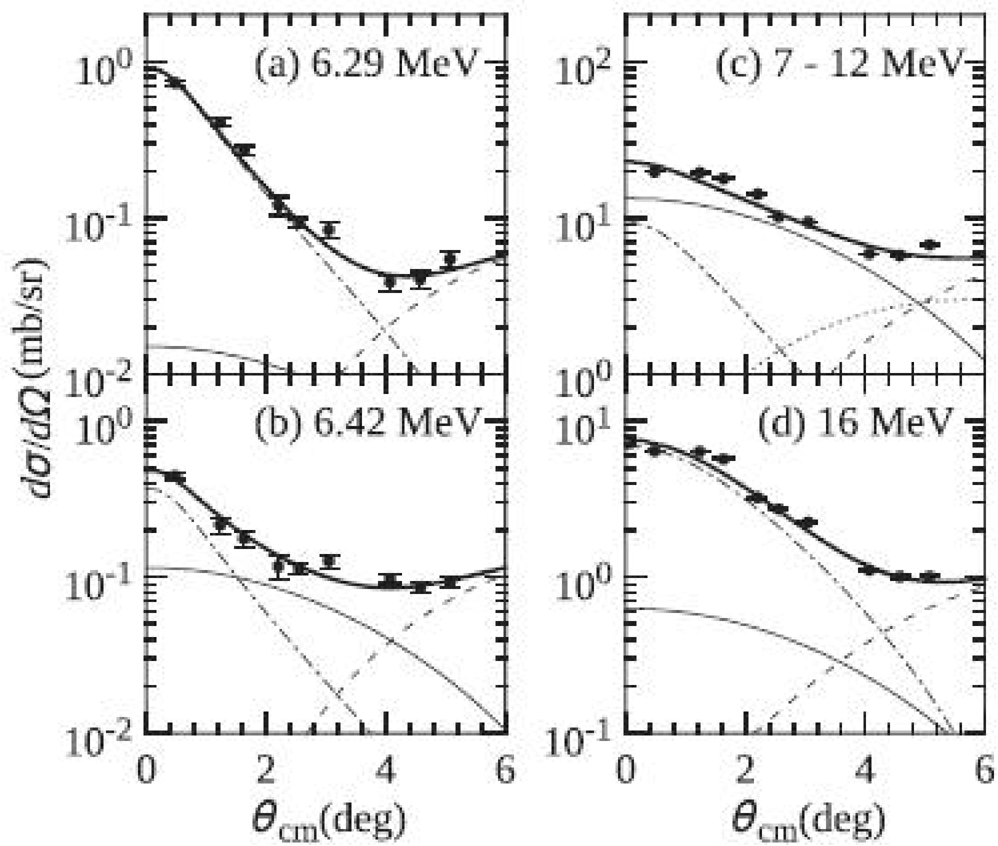
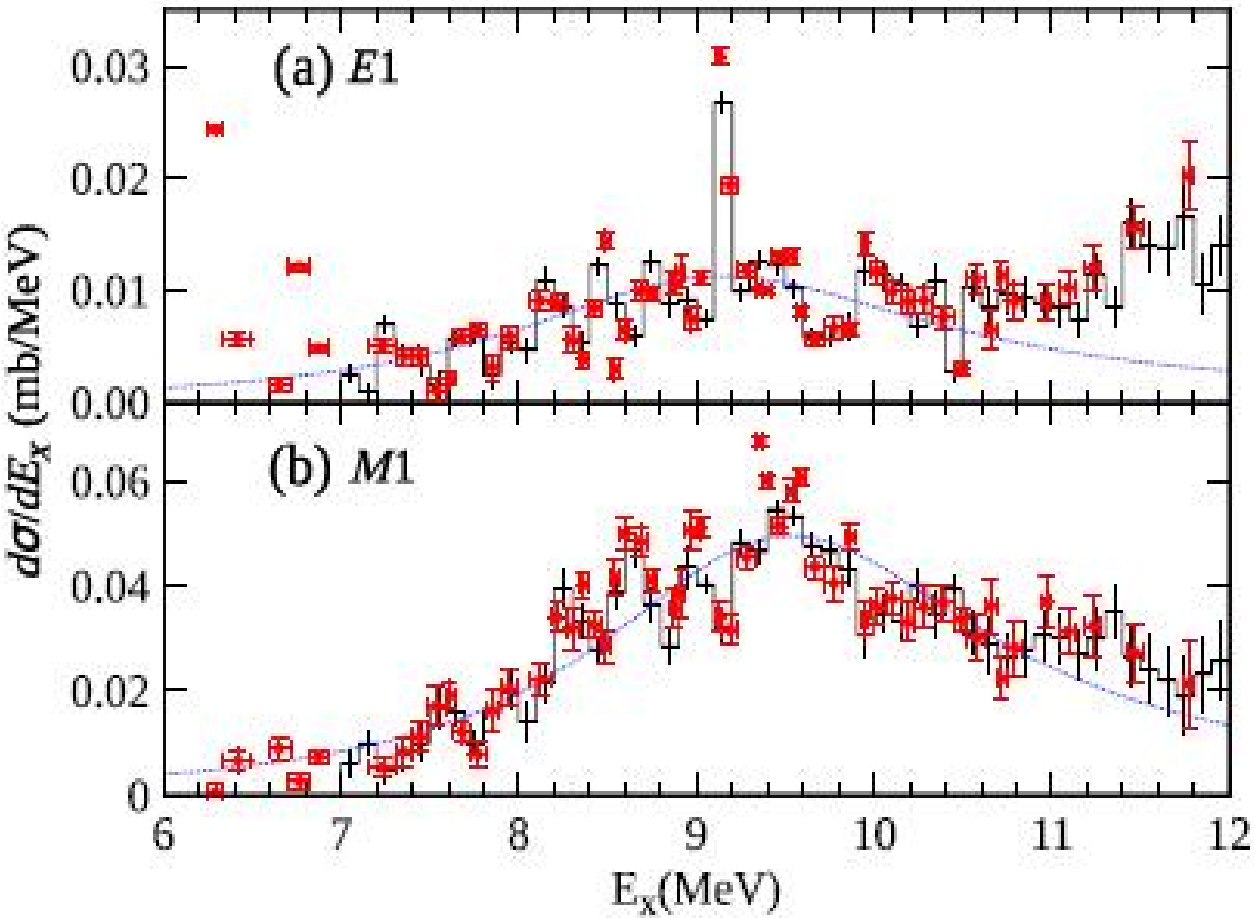
E1 (dash-dotted line)

M1 (solid line)

E2 (dashed line)



C. Iwamoto *et al.*, Phys. Rev. Lett. 108 (2012) 262501



Histograms MDA of 100 keV bins and red circles with errors are of selected peaks.

C. Iwamoto *et al.*, Phys. Rev. Lett. 108 (2012) 262501

Compression Modes
ISGMR & ISGDR

The Collective Response of the Nucleus: Giant Resonances

Compression modes
ISGMR & ISGDR

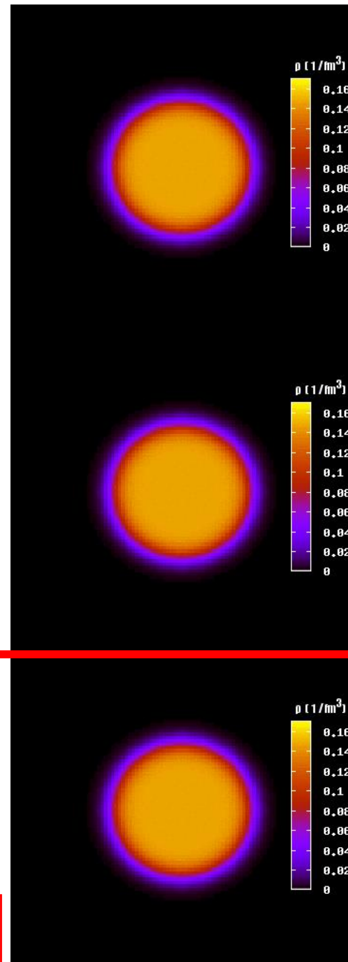
Isoscalar (In phase)
$\Delta T = 0$

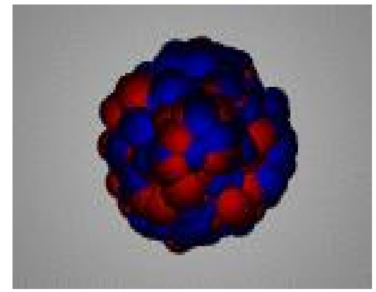
Isovector (Out of phase)
$\Delta T = 1$

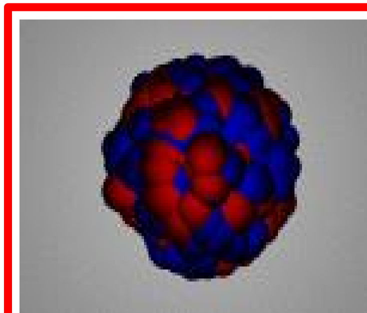
Monopole
$\Delta L = 0$
(ISGMR)
Breathing mode

Dipole
$\Delta L = 1$
(ISGDR)
Squeezing mode

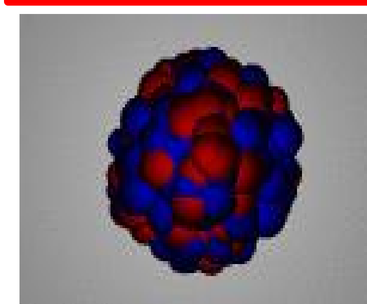
Quadrupole
$\Delta L = 2$
(ISGQR)







IVGDR



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In fluid mechanics, compressibility is a measure of the relative volume change of a fluid as a response to a pressure change.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial P}$$

where P is pressure, V is volume.

Incompressibility or bulk modulus (K) is a measure of a substance's resistance to uniform compression and can be formally defined:

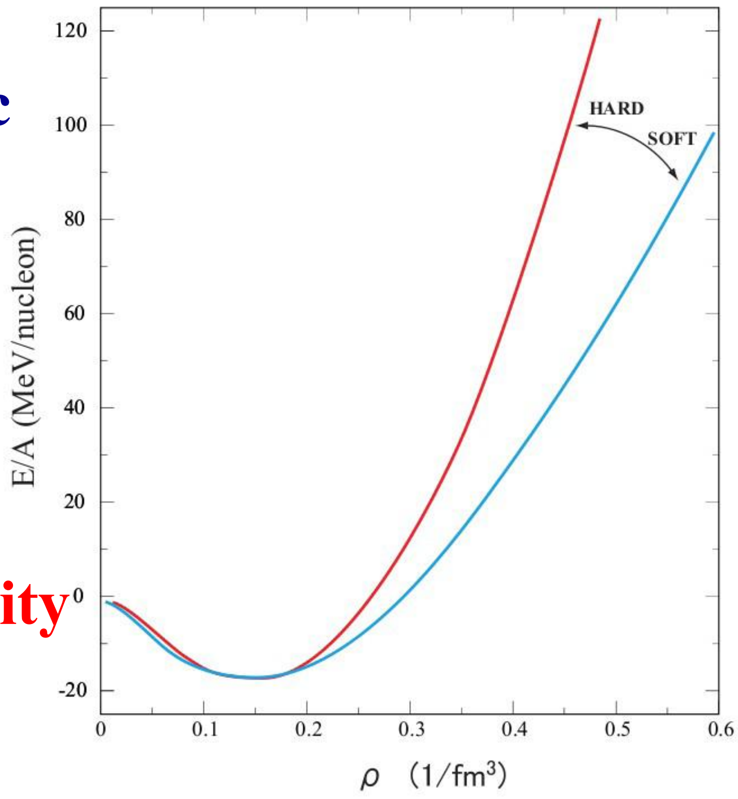
$$K = -V\frac{\partial P}{\partial V}$$

For the equation of state of symmetric nuclear matter at saturation nuclear density:

$$\left[\frac{d(E/A)}{d\rho}\right]_{\rho=\rho_0} = 0$$

and one can derive the incompressibility of nuclear matter:

$$K_{nm} = \left[9\rho^2 \frac{d^2(E/A)}{d\rho^2}\right]_{\rho=\rho_0}$$



E/A: **binding energy per nucleon**

ρ : **nuclear density**

ρ₀ : **nuclear density at saturation**

J.P. Blaizot, Phys. Rep. 64 (1980) 171

Equation of state (EOS) of nuclear matter

More complex than for infinite neutral liquids
Neutrons and protons with different interactions
Coulomb interaction of protons

1. **Governs the collapse and explosion of giant stars (supernovae)**

2. **Governs formation of neutron stars (mass, radius, crust)**

3. **Governs collisions of heavy ions.**

4. **Important ingredient in the study of nuclear properties.**

Isoscalar Excitation Modes of Nuclei

Hydrodynamic models/Giant Resonances

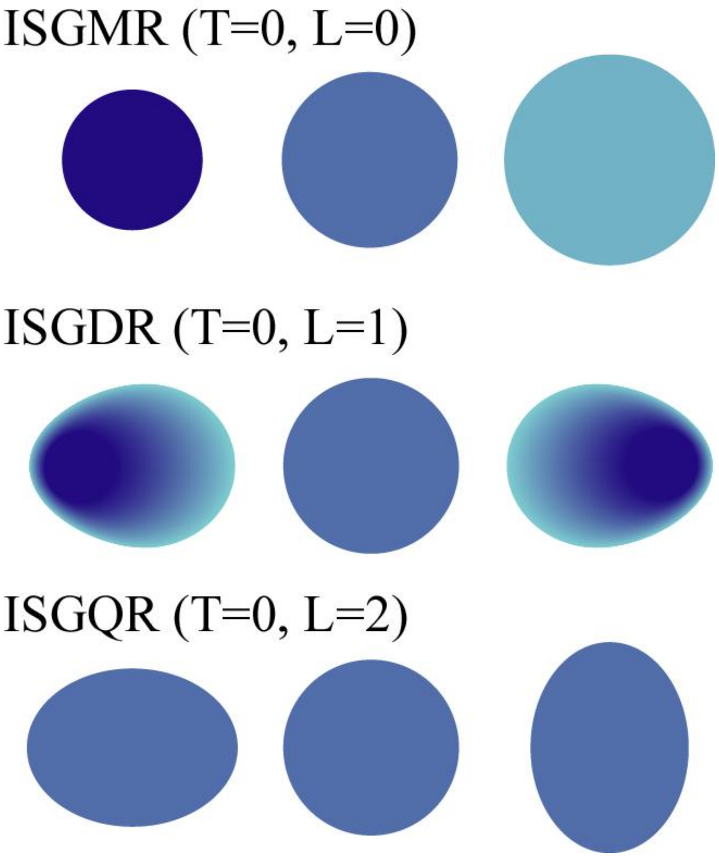
Coherent vibrations of nucleonic fluids in a nucleus.

Compression modes: ISGMR, ISGDR

In Constrained and Scaling Models:

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

$$E_{ISGDR} = \hbar\sqrt{\frac{7}{3}\frac{K_A + \frac{27}{25}\varepsilon_F}{m\langle r^2\rangle}}$$



ε_F is the Fermi energy and the nucleus incompressibility:

$$\rightarrow K_A = \left[r^2(d^2(E/A)/dr^2)\right]_{r=R_0}$$

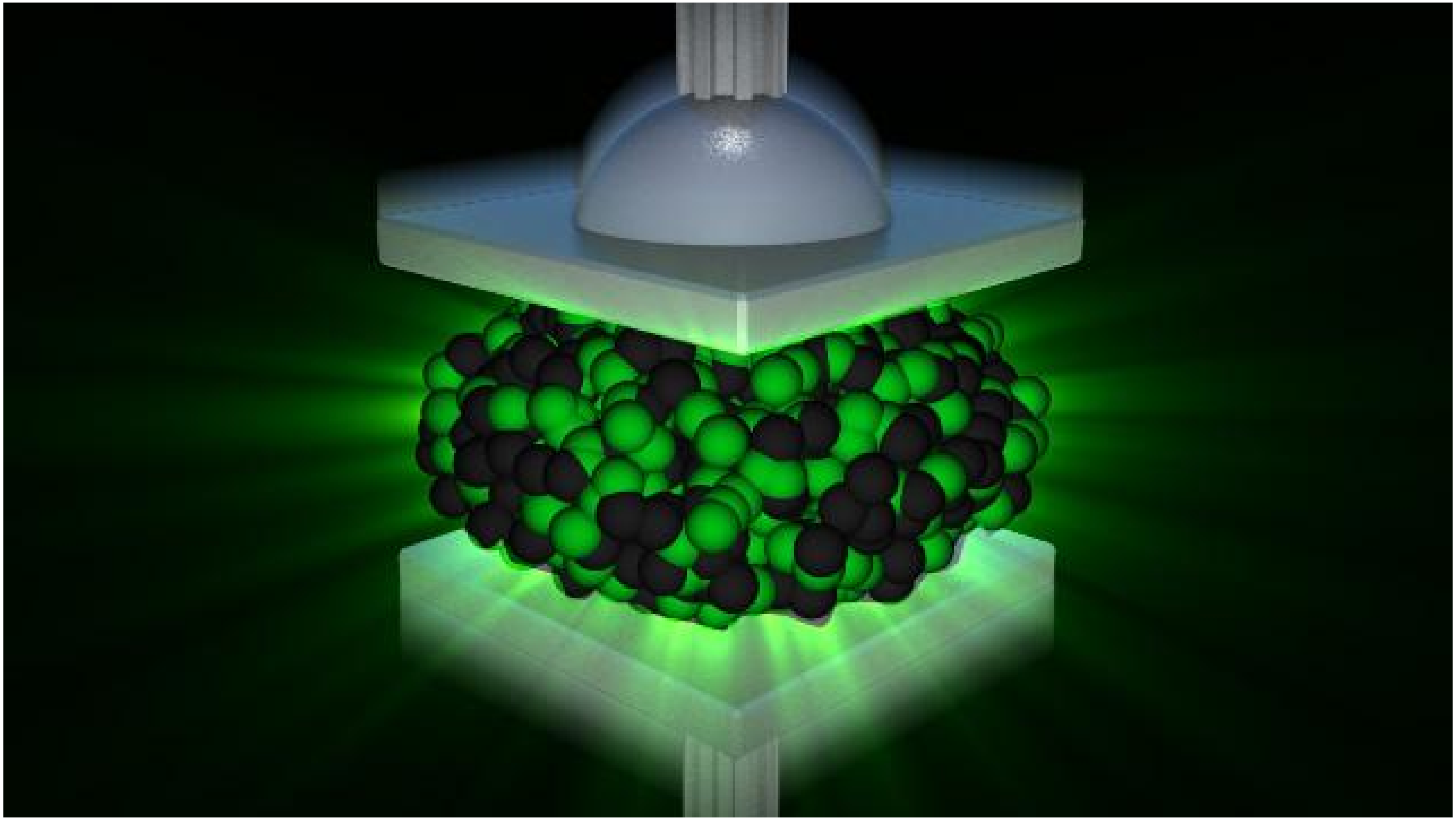
J.P. Blaizot, Phys. Rep. 64 (1980) 171

Giant resonances

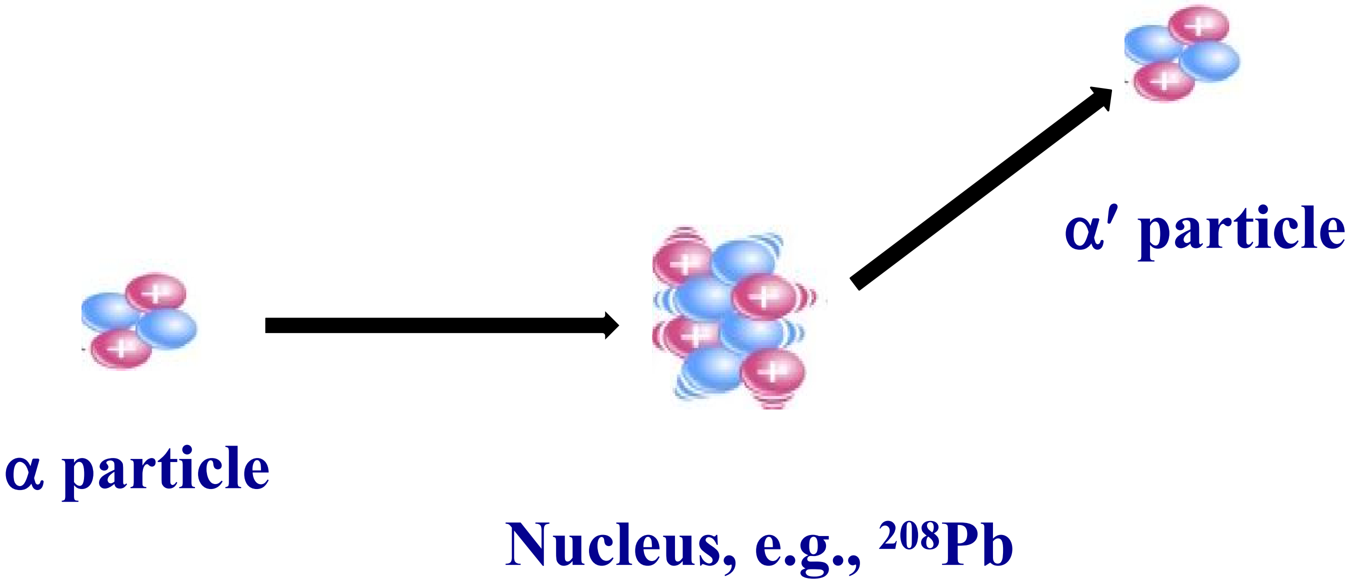
- **Macroscopic properties: E_x, Γ, %EWSR**
- **Isoscalar giant resonances; compression modes**

  ISGMR, ISGDR $\Rightarrow$ Incompressibility, symmetry energy

$$K_A = K_{vol} + K_{surf} A^{-1/3} + K_{sym}((N-Z)/A)^2 + K_{Coul} Z^2 A^{-4/3}$$

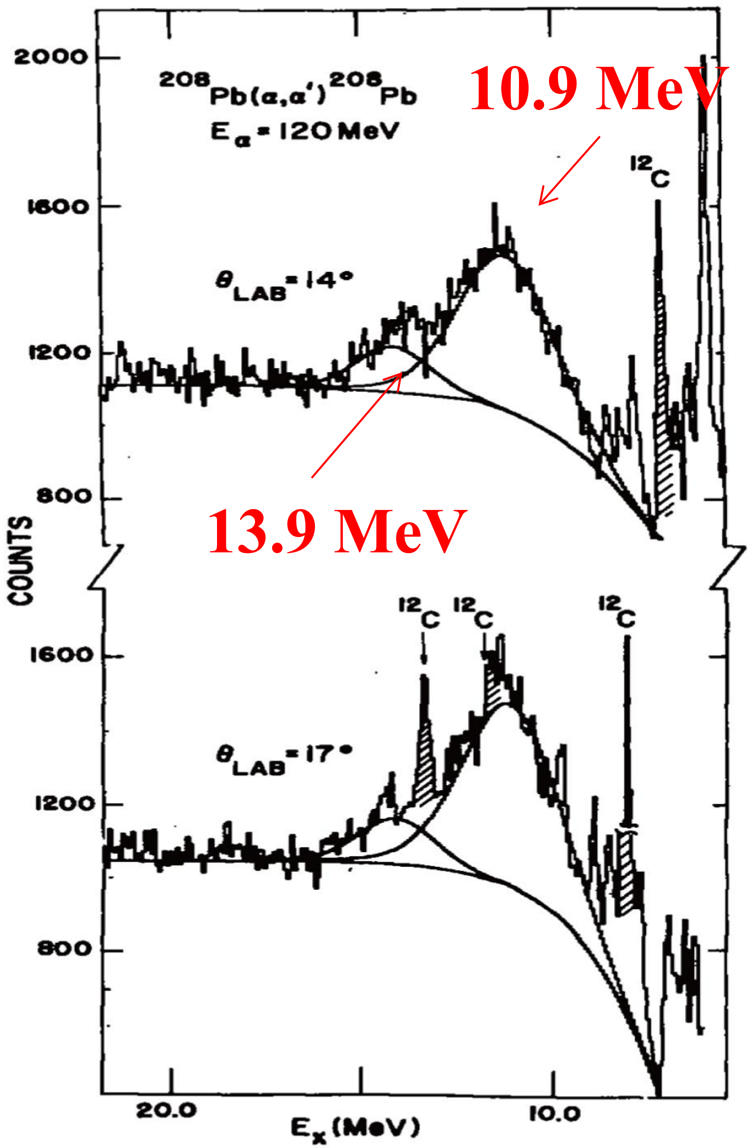


Thomas Flechel



Inelastic α scattering

ISGQR, ISGMR

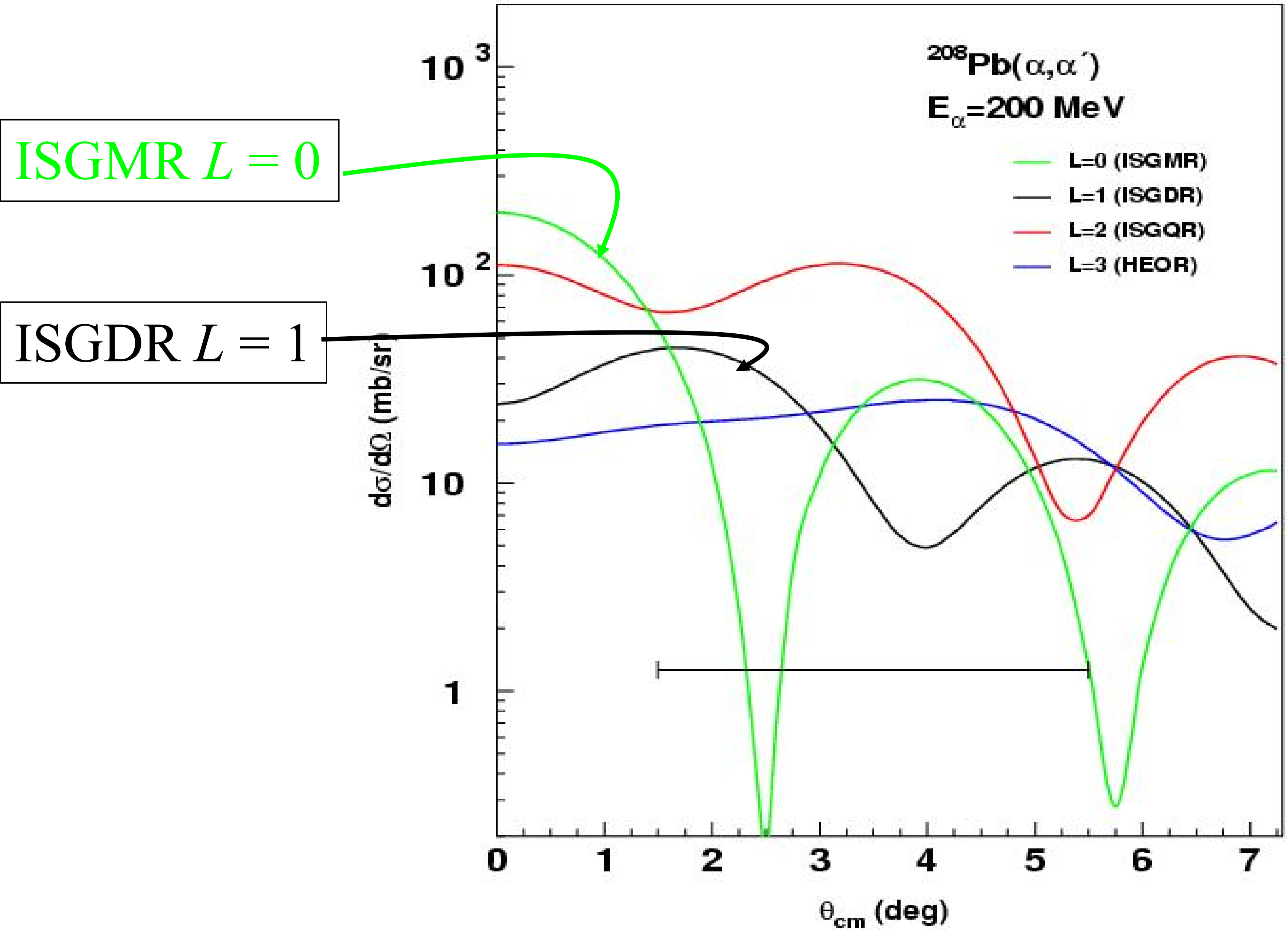


⇐ ^{208}Pb(α,α′) at E_{α}=120 MeV

Measurement made with solid-state detectors.

Large instrumental background and nuclear continuum!

M. N. Harakeh *et al.*, Phys. Rev. Lett. 38 (1977) 676



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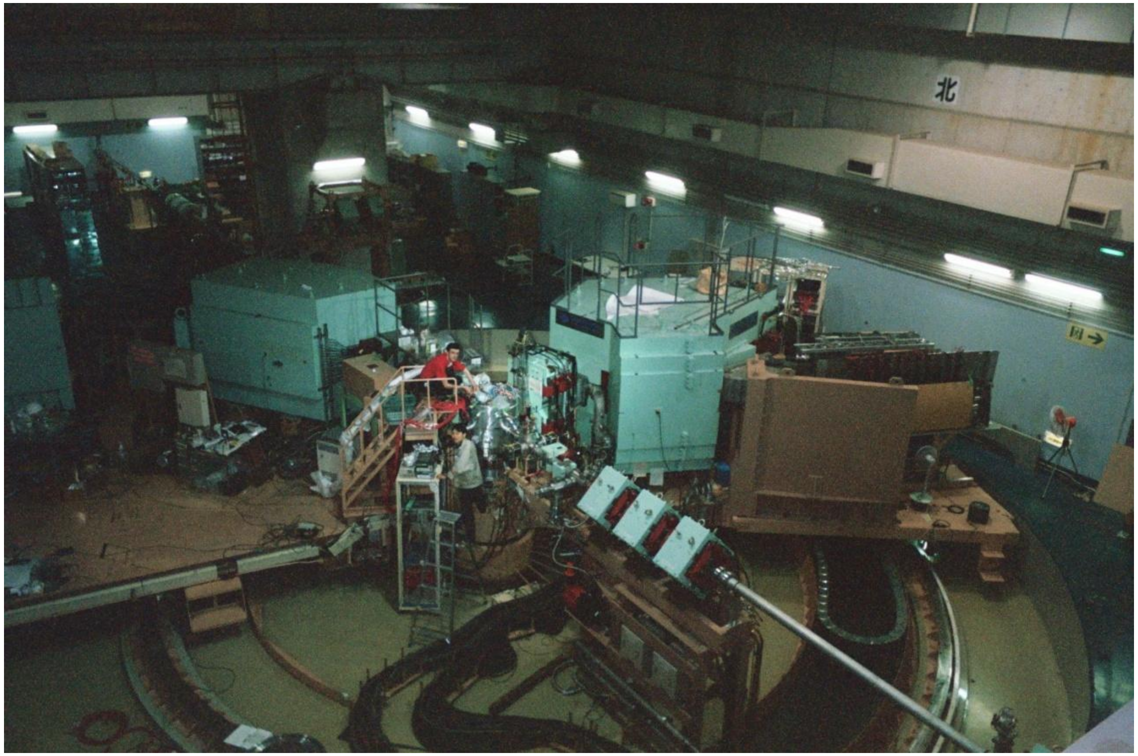
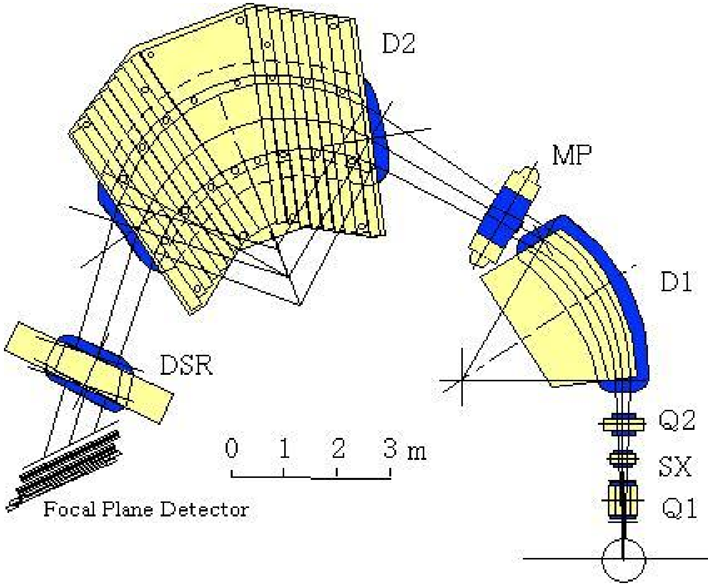
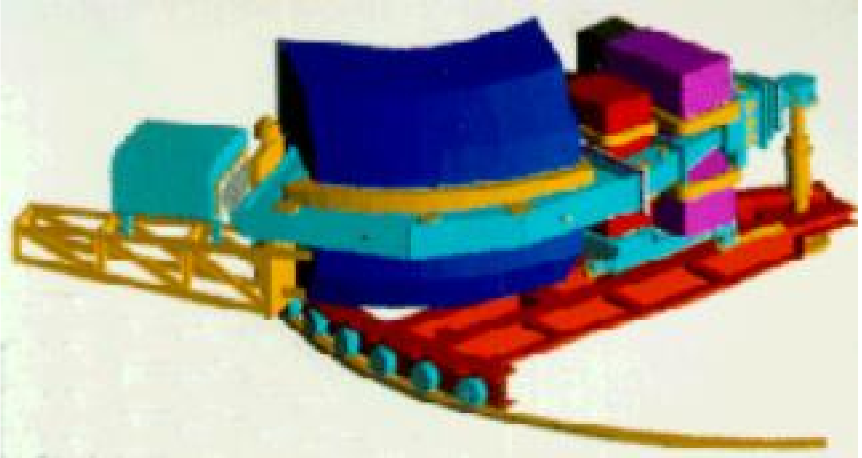


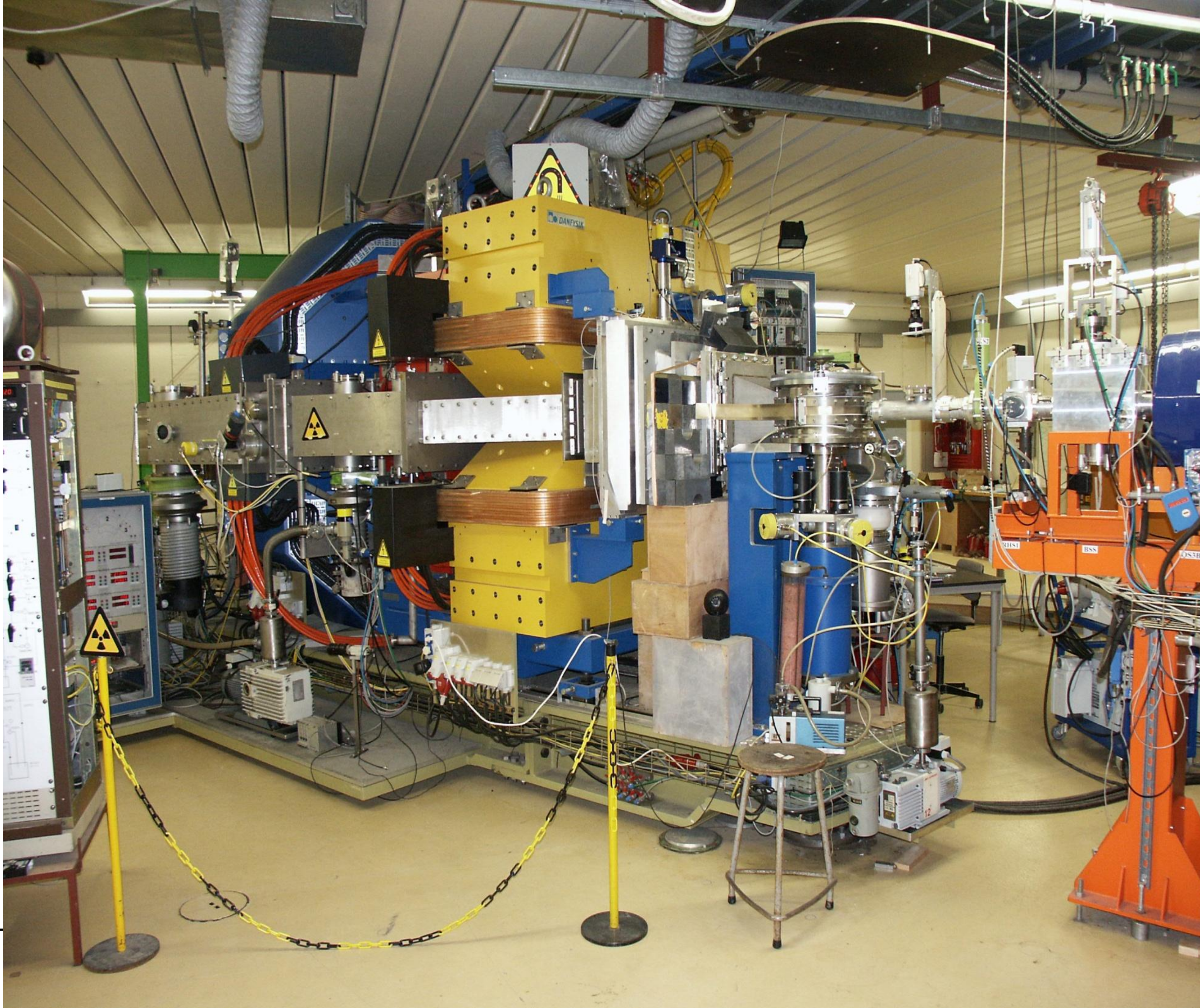


Grand Raiden@ RCNP

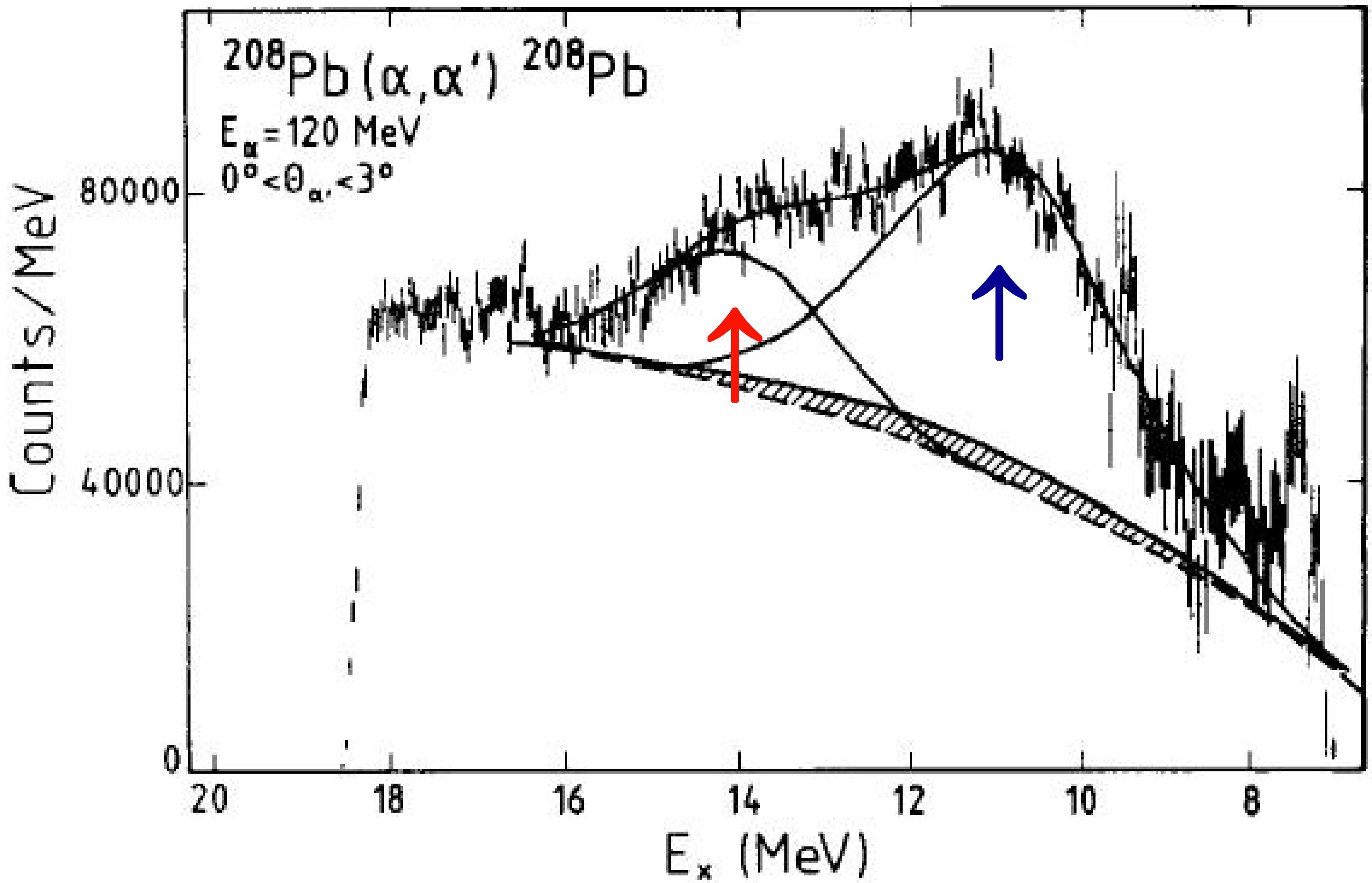


BBS@KVI

(p,p') at E_p~ 300 (α,α') at E_α~ 400 & 200 MeV at RCNP & KVI, respectively


DANFYSIK
RHS1
BS8
QS3B

ISGQR at 10.9 MeV
ISGMR at 13.9 MeV



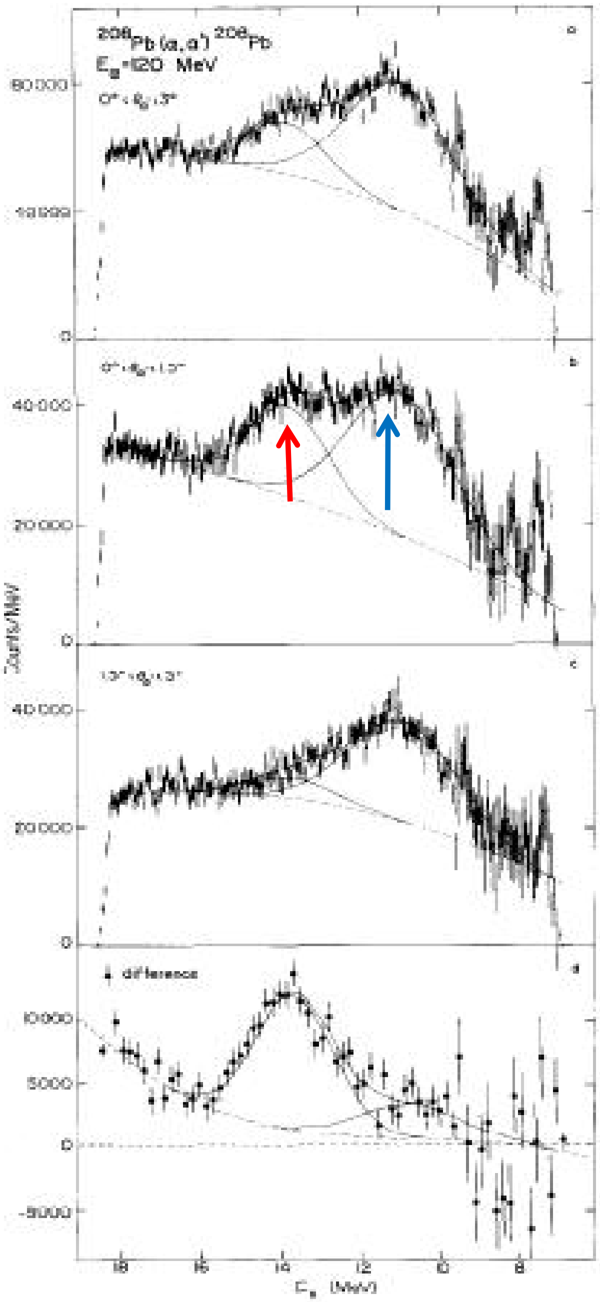
Difference of spectra

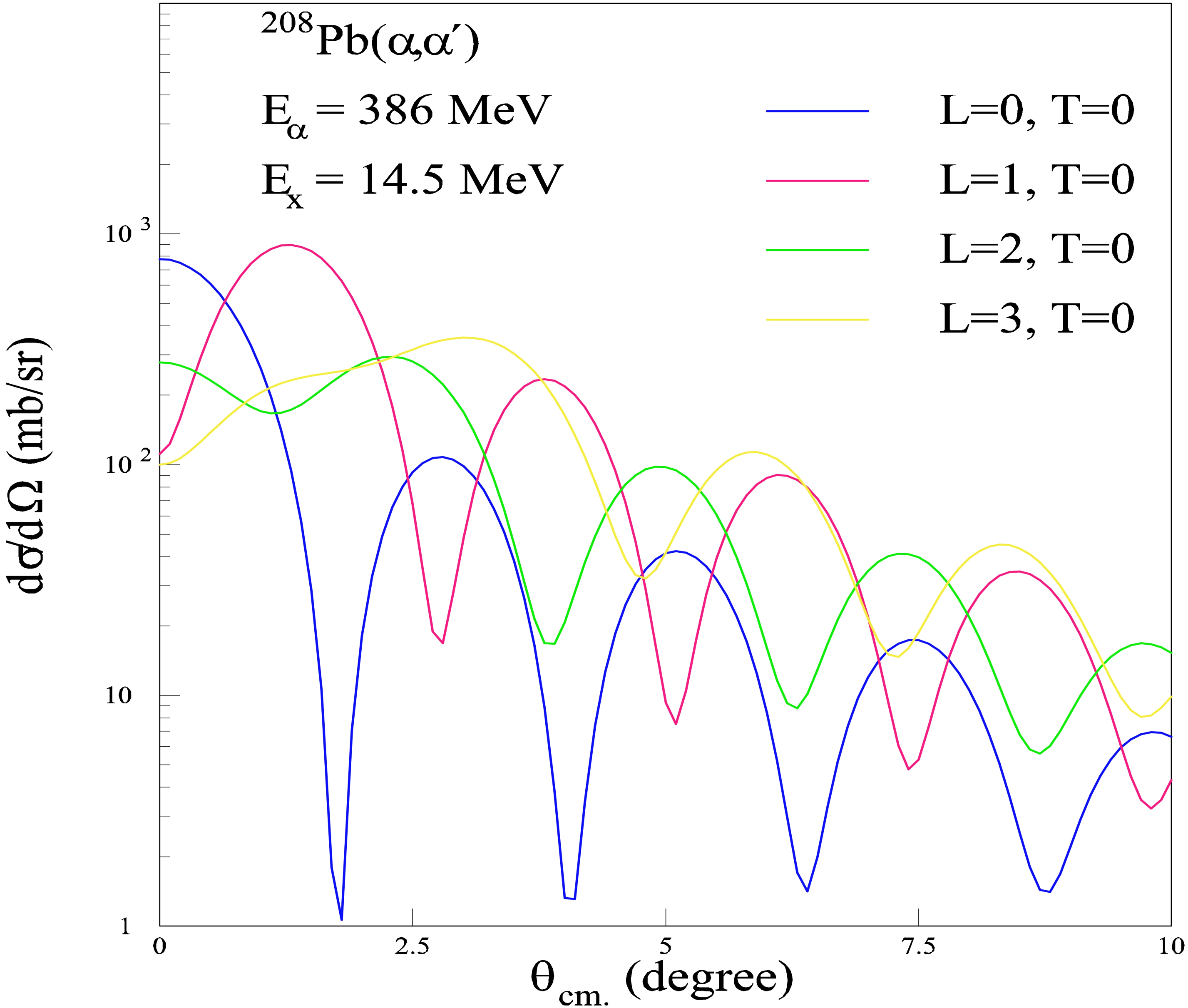
$0° < \theta_{\alpha'} < 3°$

$0° < \theta_{\alpha'} < 1.5°$

$1.5° < \theta_{\alpha'} < 3°$

Difference





ISGMR, ISGDR

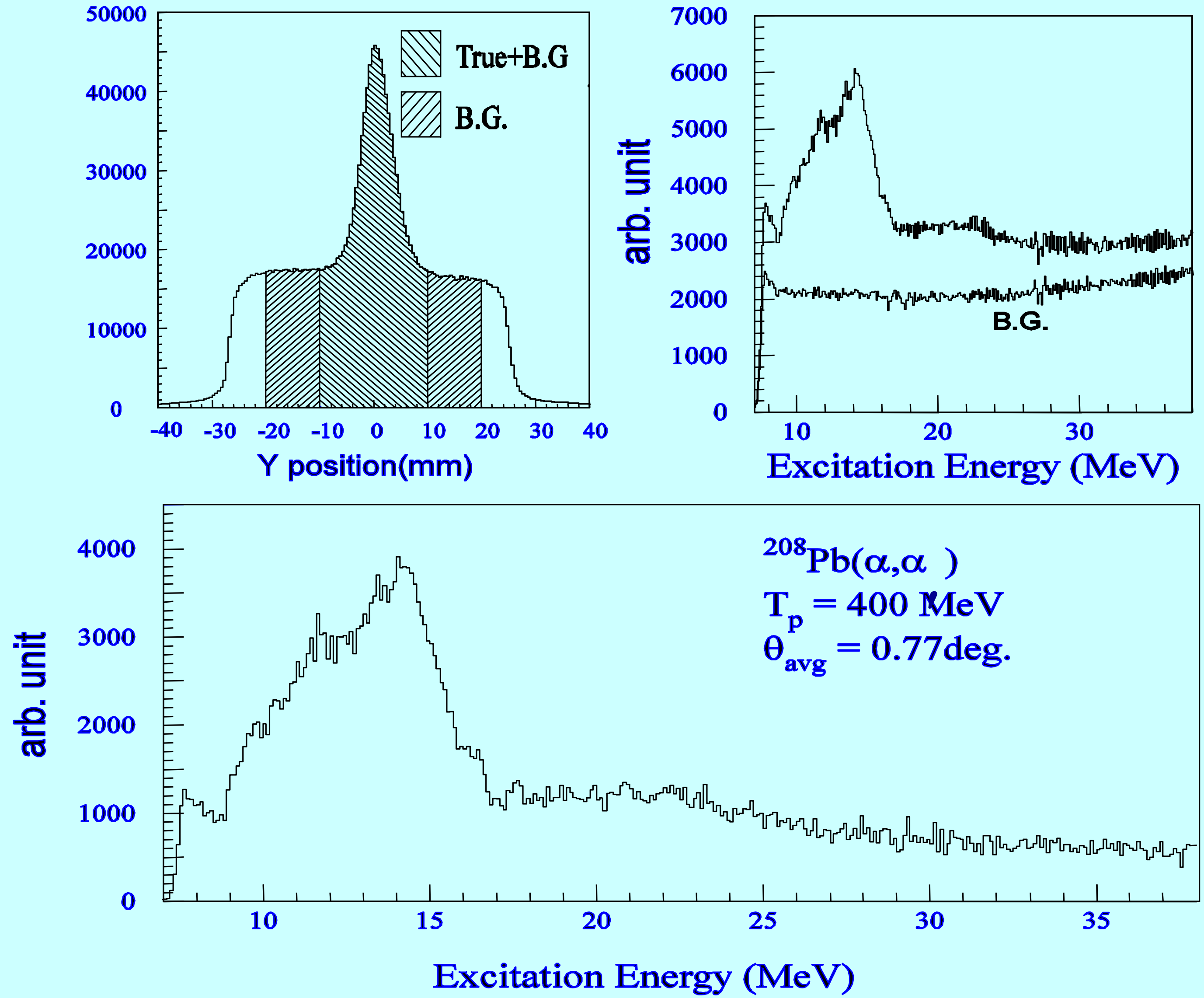
ISGQR, HEOR

100 % EWSR

At E_x= 14.5 MeV

Experiments at RCNP, Osaka University

- (α,α') reaction at 386 MeV
  - High-resolution spectrometer "Grand Raiden"



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Multipole decomposition analysis (MDA)

$$\left(\frac{d^2\sigma}{d\Omega dE}(\vartheta_{c.m.},E)\right)^{\exp.} = \sum_L a_L(E)\left(\frac{d^2\sigma}{d\Omega dE}(\vartheta_{c.m.},E)\right)_L^{calc.}$$

$$\left(\frac{d^2\sigma}{d\Omega dE}(\vartheta_{c.m.},E)\right)^{\exp.} : \text{Experimental cross section}$$

$$\left(\frac{d^2\sigma}{d\Omega dE}(\vartheta_{c.m.},E)\right)_L^{calc.} : \text{DWBA cross section (unit cross section)}$$

$a_L(E)$: EWSR fraction

a. **ISGR (L<7)+ IVGDR (calculated through Coulomb excitation)**

b. **DWBA formalism; single folding ⇒ transition potential**

$$\delta U_L(r,E) = \int d\vec{r}'\,\delta\rho_L(\vec{r}',E)\left[V(|\vec{r}-\vec{r}'|,\rho_0(r')) + \rho_0(r')\frac{\partial V(|\vec{r}-\vec{r}'|,\rho(r'))}{\partial\rho_0(r')}\right]$$

$$U(r) = \int d\vec{r}'\,V(|\vec{r}-\vec{r}'|,\rho_0(r'))\,\rho_0(r')$$

Transition density

- **ISGMR Satchler, Nucl. Phys. A472 (1987) 215**

$$\delta\rho_0(r,E) = -\alpha_0[3 + r\frac{d}{dr}]\rho_0(r)$$

$$\alpha_0^2 = \frac{2\pi\hbar^2}{mA < r^2 > E}$$

- **ISGDR Harakeh & Dieperink, Phys. Rev. C23 (1981) 2329**

$$\delta\rho_1(r,E) = -\frac{\beta_1}{R\sqrt{3}}[3r^2\frac{d}{dr} + 10r - \frac{5}{3} < r^2 > \frac{d}{dr} + \varepsilon(r\frac{d^2}{dr^2} + 4\frac{d}{dr})]\rho_0(r)$$

$$\beta_1^2 = \frac{6\pi\hbar^2}{mAE}\frac{R^2}{(11 < r^4 > -(25/3) < r^2 >^2 - 10\varepsilon < r^2 >)}$$

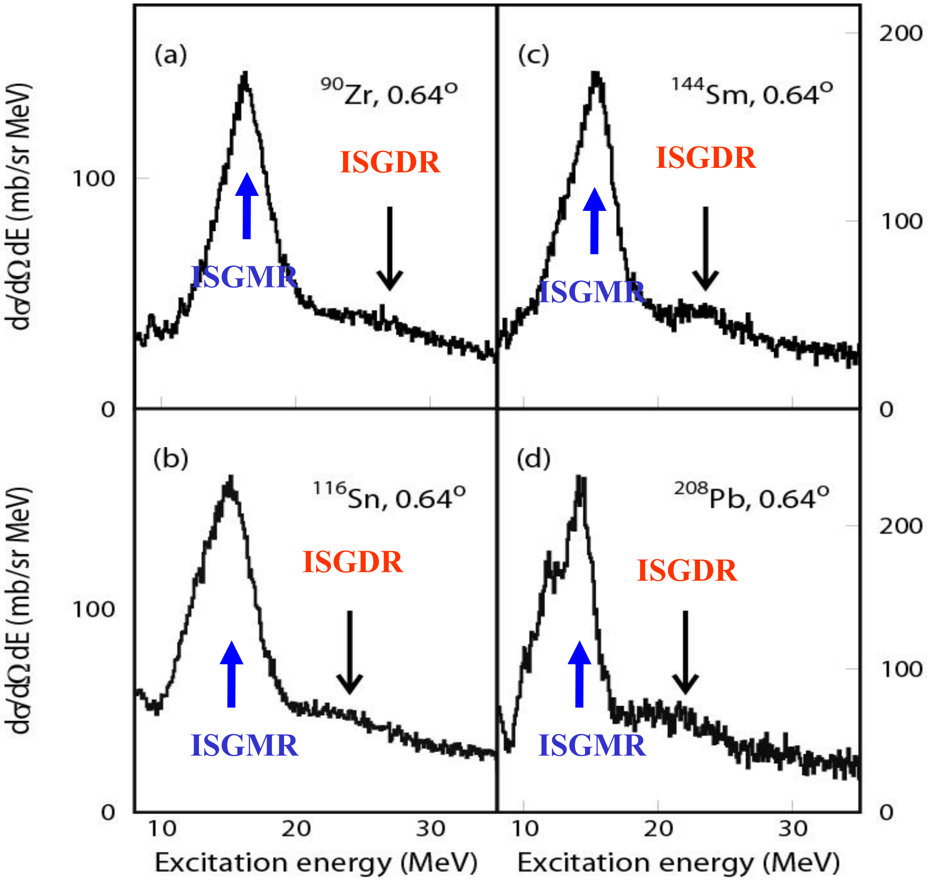
- **Other modes Bohr-Mottelson (BM) model**

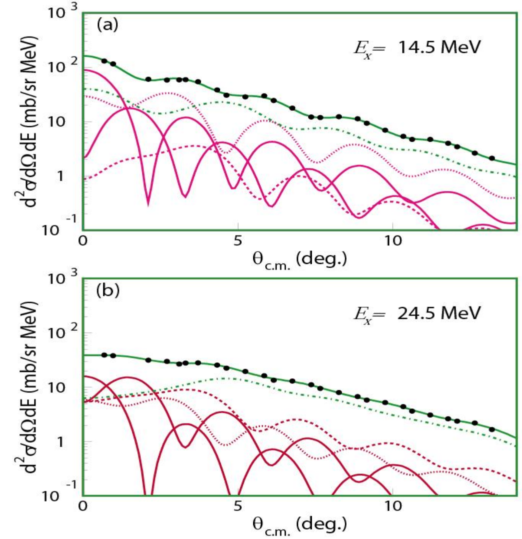
$$\delta\rho_L(r,E) = -\delta_L\frac{d}{dr}\rho_0(r)$$

$$\delta_L^2 = (\beta_L c)^2 = \frac{L(2L+1)^2}{(L+2)^2}\frac{2\pi\hbar^2}{mAE}\frac{< r^{2L-2} >}{< r^{L-1} >^2}$$

Uchida *et al.*,
Phys. Lett. B557 (2003) 12
Phys. Rev. C69 (2004) 051301

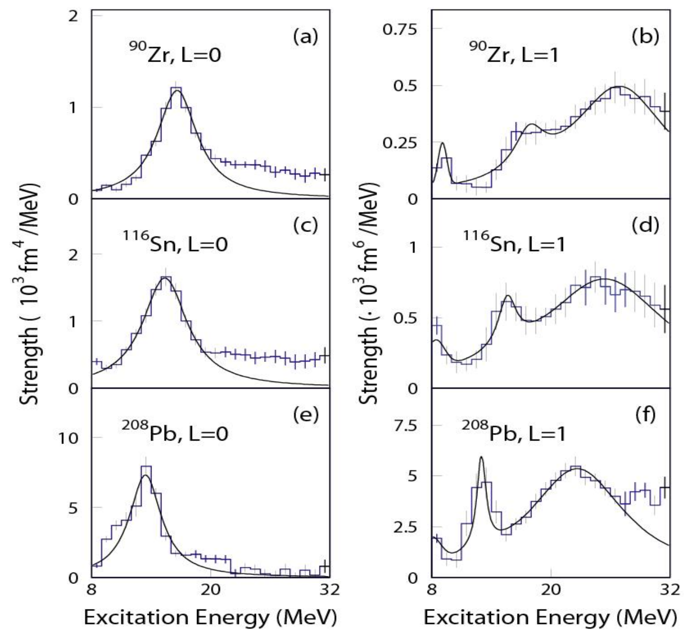
(α,α′) spectra at 386 MeV

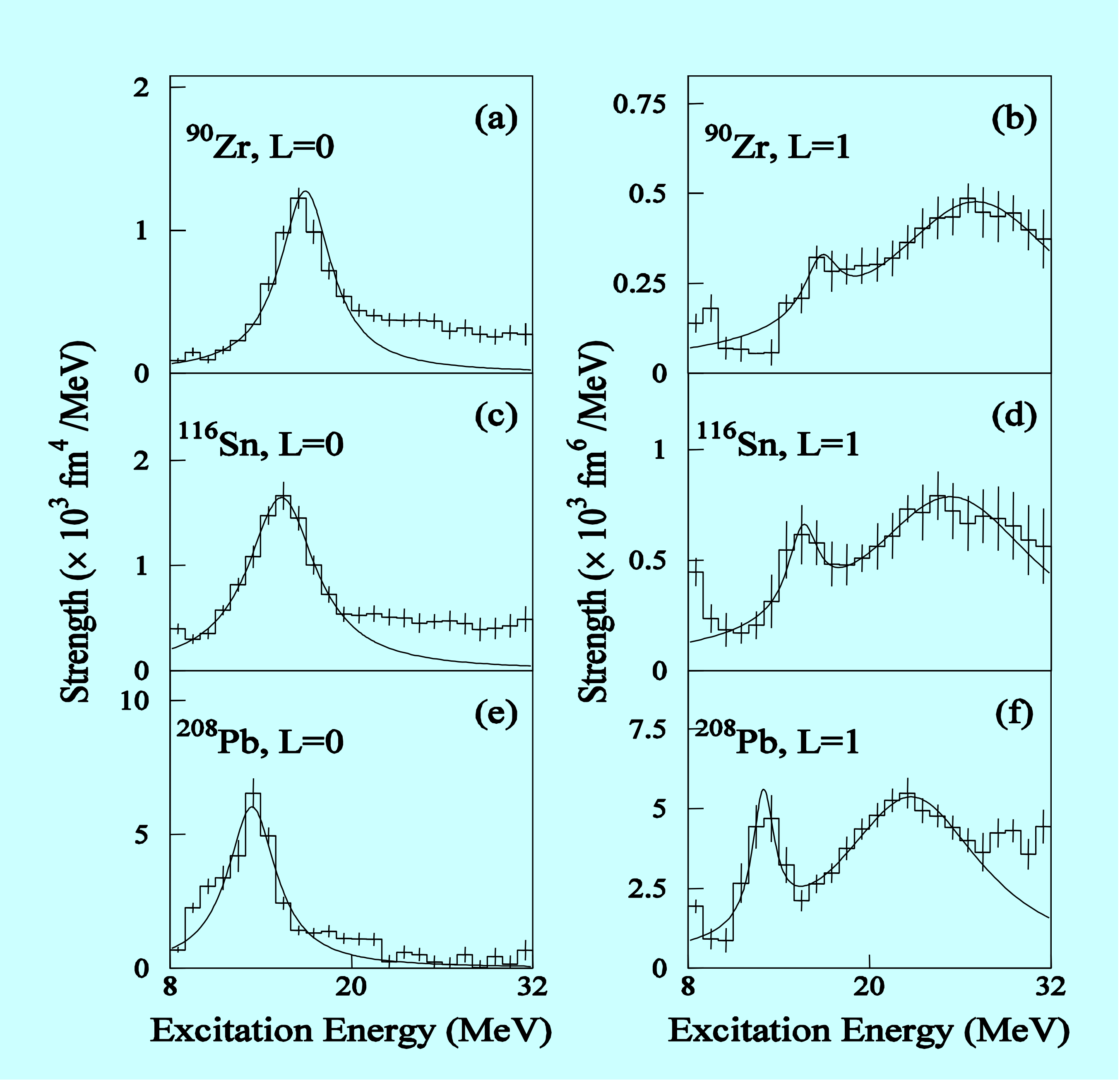




^{116}Sn

MDA results for *L=0* and *L=1*




(a)
^{90}Zr, L=0
(b)
^{90}Zr, L=1
(c)
^{116}Sn, L=0
(d)
^{116}Sn, L=1
(e)
^{208}Pb, L=0
(f)
^{208}Pb, L=1
Strength ($\times 10^3$ fm^4 /MeV)
Strength ($\times 10^3$ fm^6 /MeV)
Excitation Energy (MeV)
Excitation Energy (MeV)
8
20
32

In HF+RPA calculations,

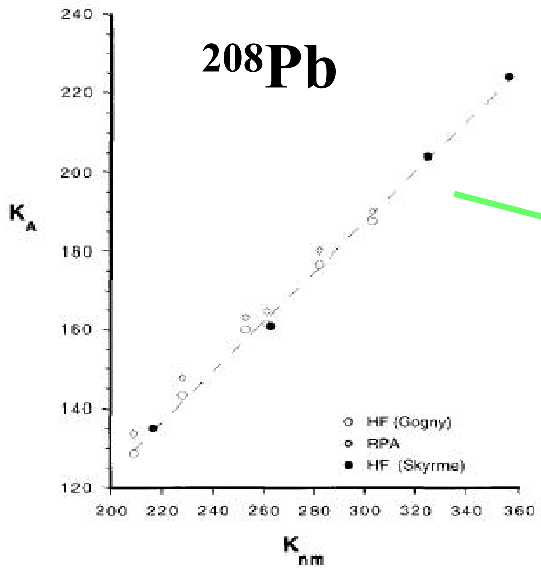
$$K_{nm} = \left[9\rho^2 \frac{d^2(E/A)}{d\rho^2} \right]_{\rho=\rho_0}$$

Nuclear matter

E/A: **binding energy per nucleon** ***K_A***: **incompressibility**

ρ : **nuclear density**

ρ_0 : **nuclear density at saturation**



K_A is obtained from excitation energy of ISGMR & ISGDR

$$K_A = 0.64 K_{nm} - 3.5$$

J.P. Blaizot, Nucl. Phys. A591 (1995) 435

From GMR data on ^{208}Pb and ^{90}Zr,

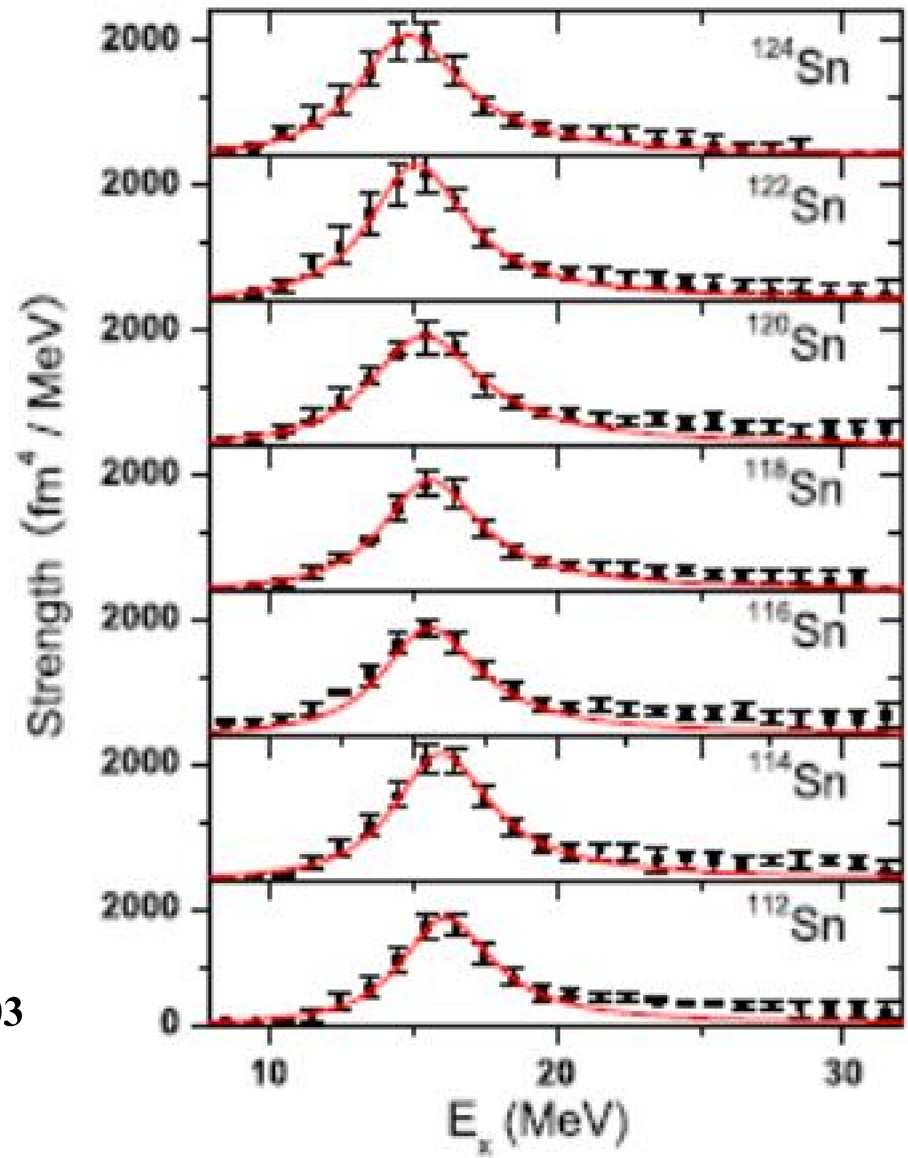
QRPA calculation =>

$K_\infty = 240 \pm 10\ (\pm 20)$ MeV

[See, *e.g.*, G. Colò *et al.*, Phys. Rev. C 70 (2004) 024307]

This number is consistent
with both ISGMR and ISGDR Data
and
with non-relativistic and relativistic calculations

Isoscalar GMR strength distribution in Sn-isotopes obtained by Multipole Decomposition Analysis of singles spectra obtained in ^{A}Sn(α,α′) measurements at incident energy 400 MeV and angles from 0º to 9º



T. Li et al., Phys. Rev. Lett. 99 (2007) 162503
T. Li et al., Phys. Rev. C81 (2010) 034309

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$$K_A = K_{vol} + K_{surf} A^{-1/3} + K_{sym}((N-Z)/A)^2 + K_{coul} Z^2 A^{-4/3}$$

$$K_A \sim K_{vol}(1 + cA^{-1/3}) + K_\tau((N - Z)/A)^2 + K_{Coul} Z^2 A^{-4/3}$$

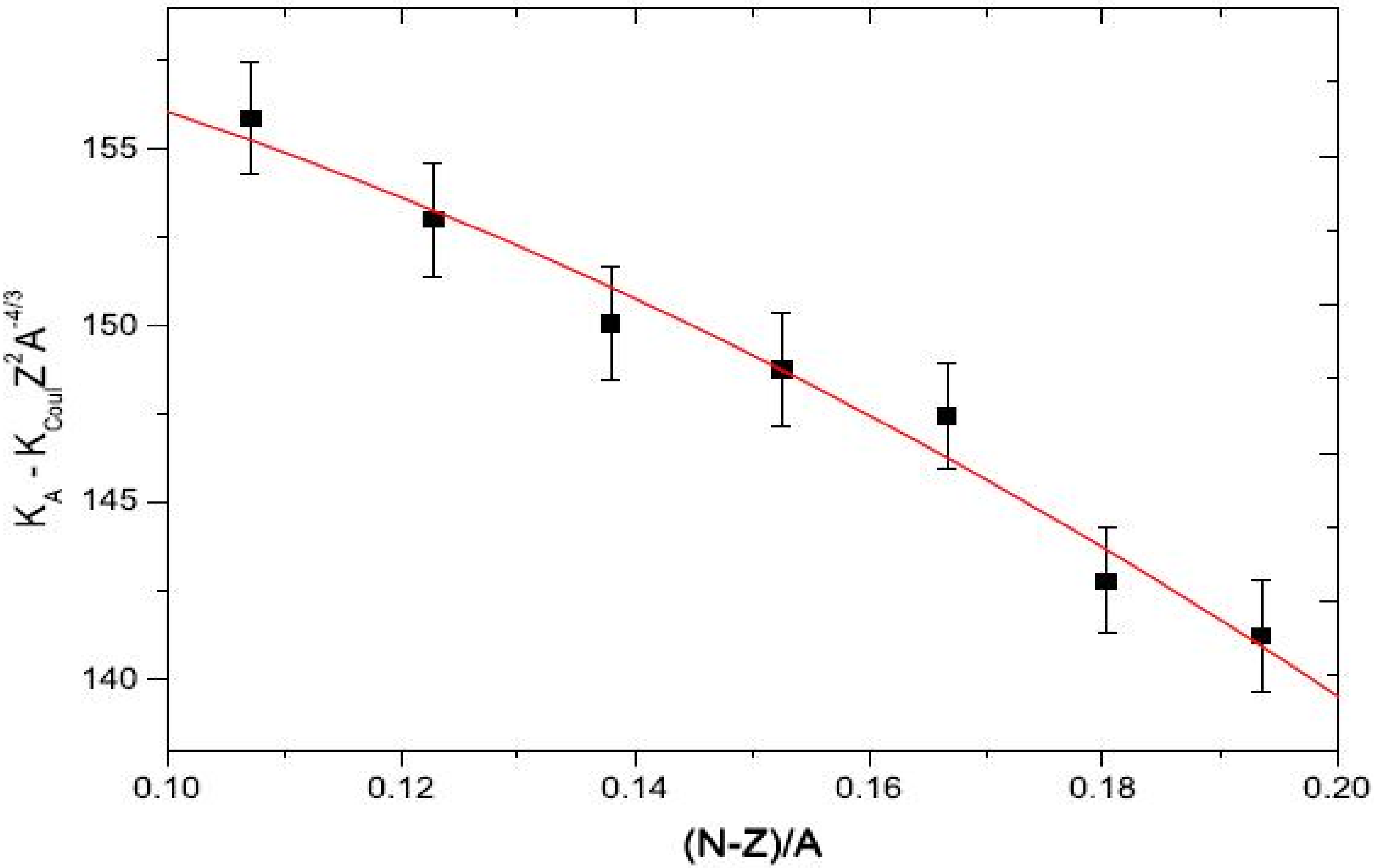
$$K_A - K_{Coul} Z^2 A^{-4/3} \sim K_{vol}(1 + cA^{-1/3}) + K_\tau((N - Z)/A)^2$$

$$\sim \text{Constant} + K_\tau((N - Z)/A)^2$$

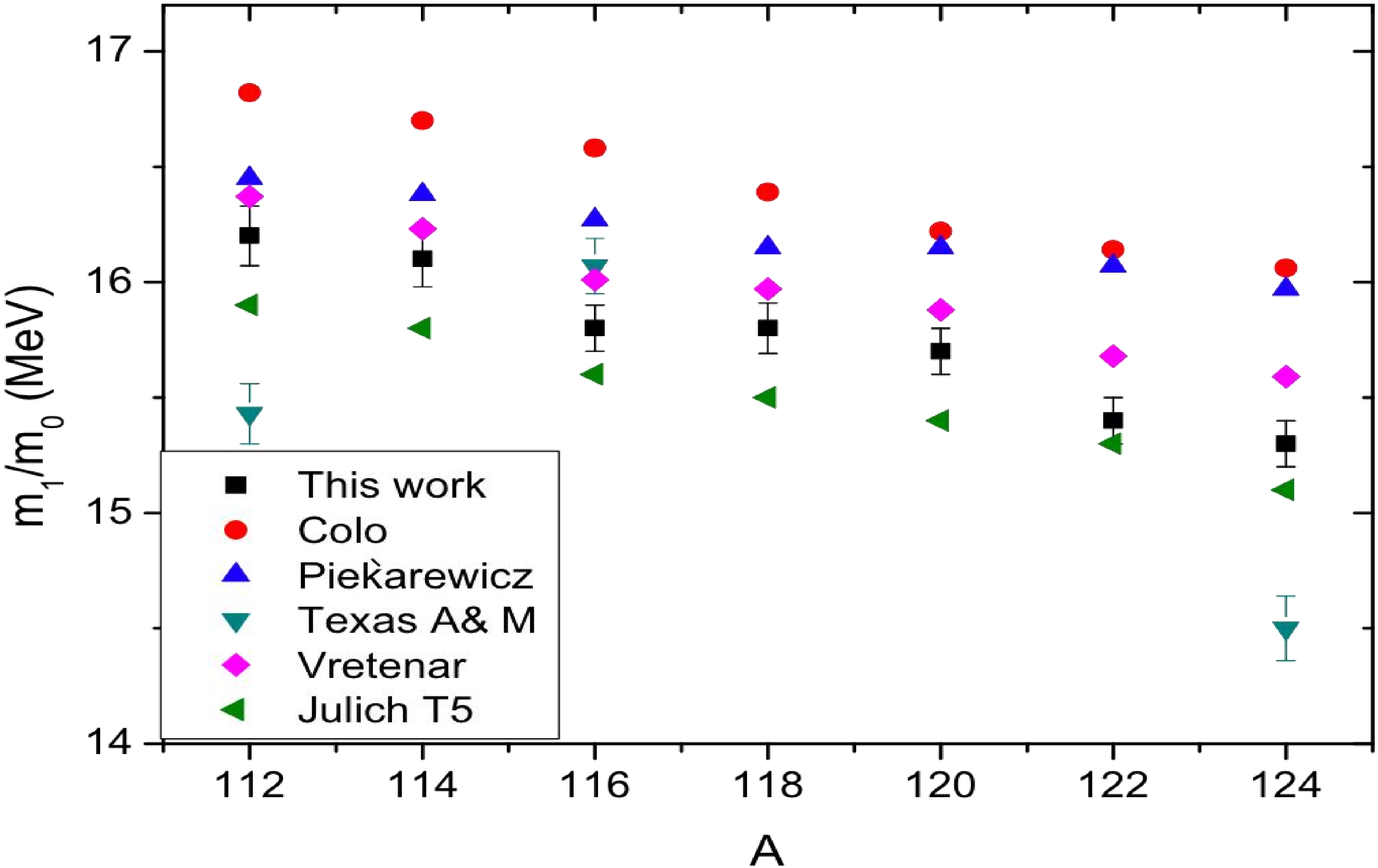
We use K_{Coul} = - 5.2 MeV (from Sagawa)

$(N - Z)/A$

^{112}Sn – ^{124}Sn: 0.107 – 0.194



Sn isotopes ⇒ $K_\tau = -550 \pm 100$ MeV



17
16
15
14
m_1/m_0 (MeV)
112
114
116
118
120
122
124
A
This work
Colo
Pieka̋rewicz
Texas A& M
Vretenar
Julich T5

Colò *et al.*: Non-relativistic RPA (without pairing) reproduces ISGMR in ^{208}Pb and ^{90}Zr.

Piekarewicz: Relativistic RPA (FSUGold model) reproduces g.s. observables and ISGMR in ^{208}Pb, ^{144}Sm and ^{90}Zr [K_∞ = 230 MeV]

Vretenar: Relativistic mean field (DD-ME2: density-dependent mean-field effective interaction).

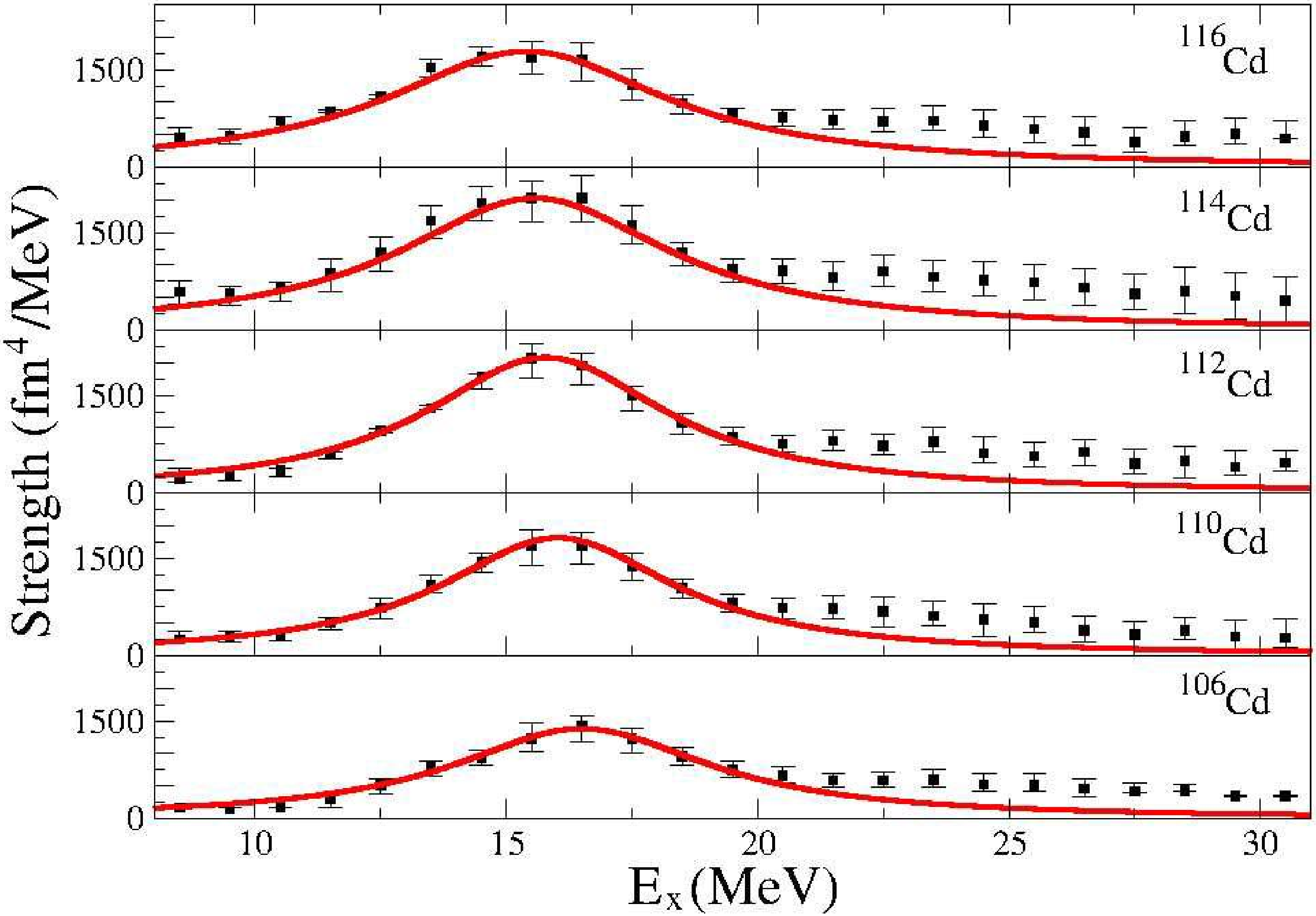
[K_∞ = 240 MeV]. Possibly agreement is fortuitous since strength distributions are not much different from those by Colò *et al*. and Piekarewicz.

Tselyaev *et al*.: Quasi-particle time-blocking approximation (QTBA) (T5 Skyrme interaction)

[K_∞ = 202 MeV?!]

Softness of Sn-nuclei is still unresolved

Monopole strength Distribution



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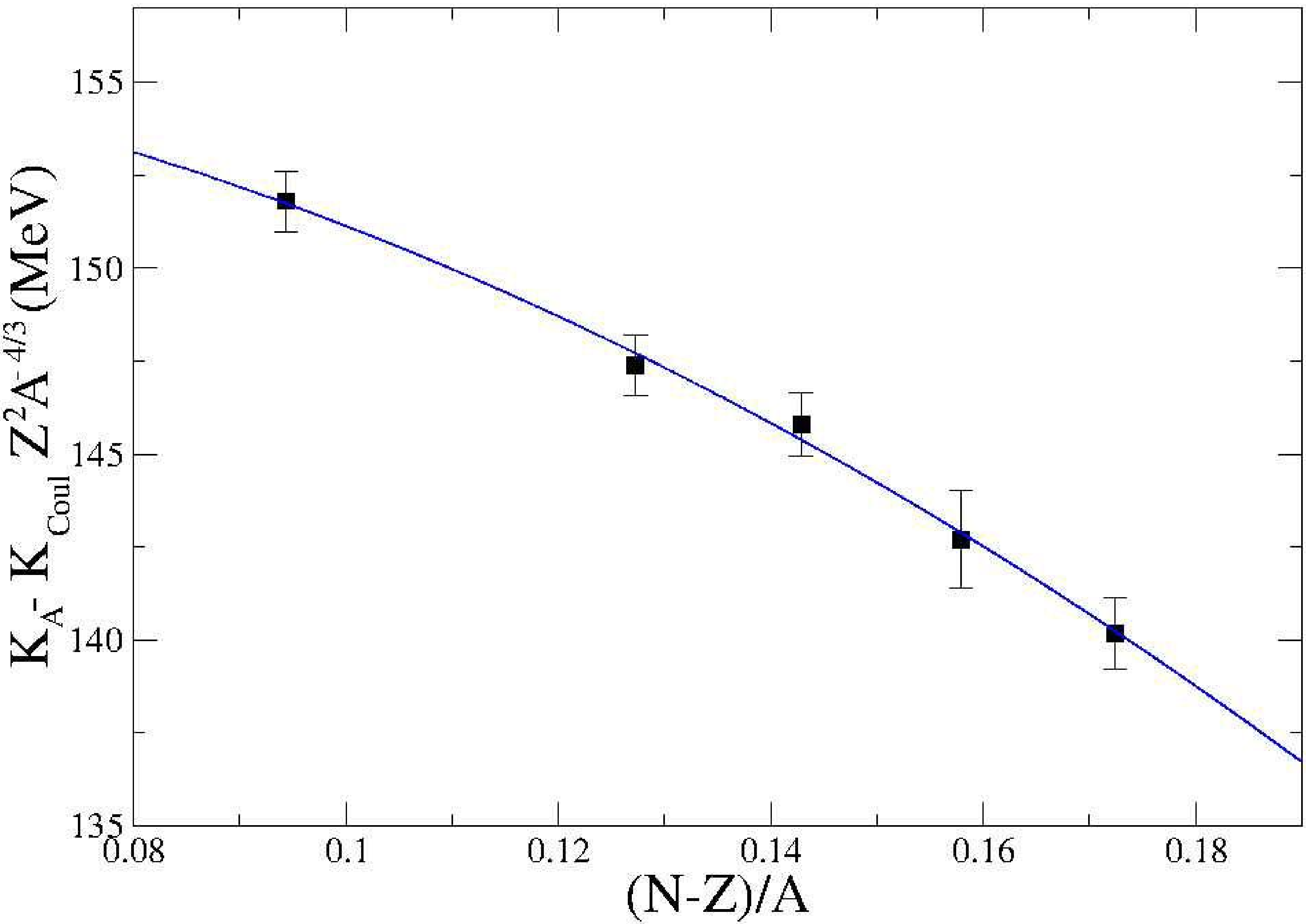
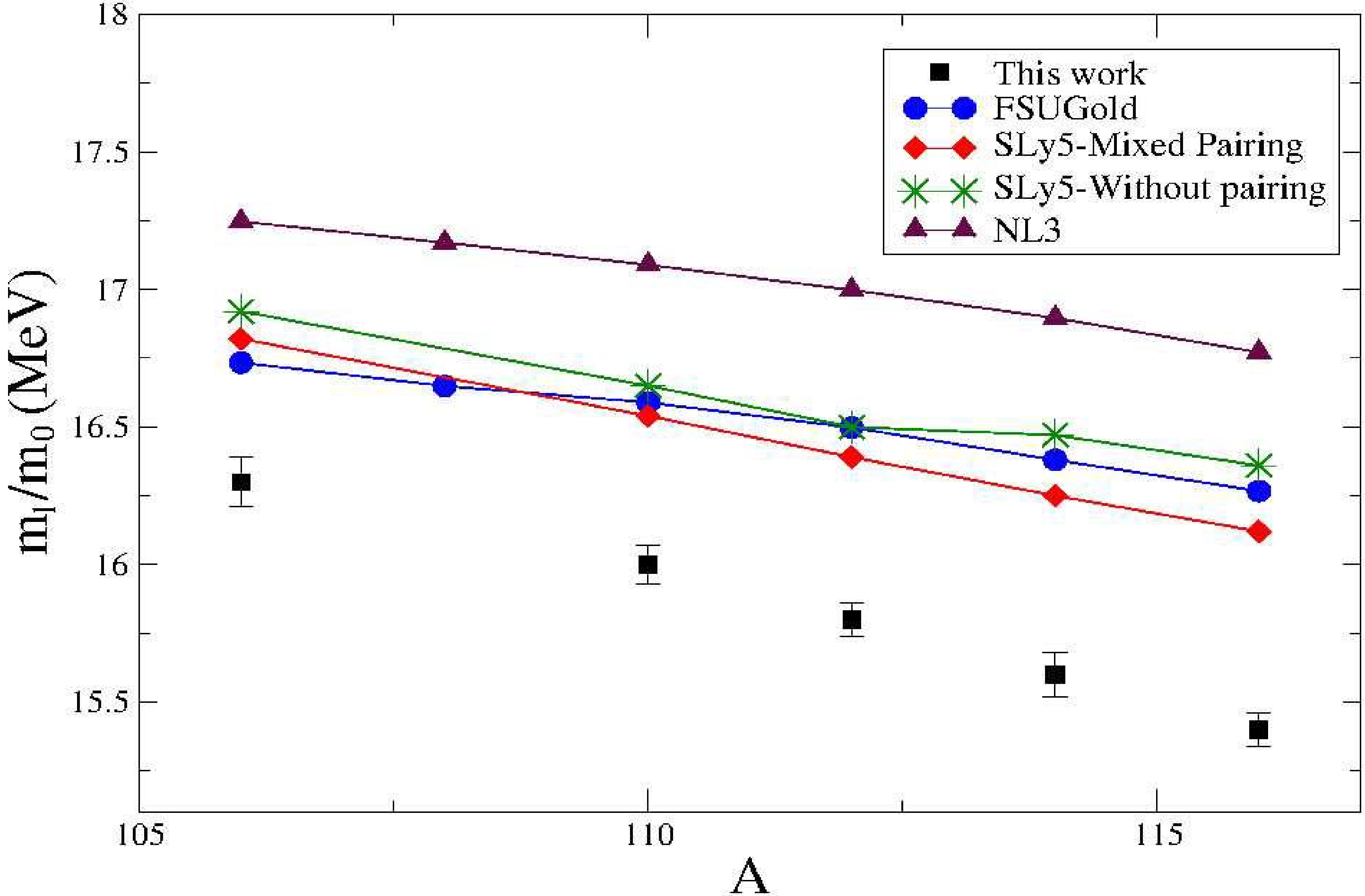


Cd isotopes ⇒ $K_\tau = -555 \pm 75$ MeV



RRPA: FSUGold [K_∞ = 230 MeV] B.G. Todd-Rutel and J. Piekarewicz

NL3 [K_∞ = 271 MeV] G.A. Lalazissis, J. Konig, and P. Ring

QRPA: SLy5 [K_∞ = 230 MeV] L-G. Cao, H. Sagawa, and G. Colò

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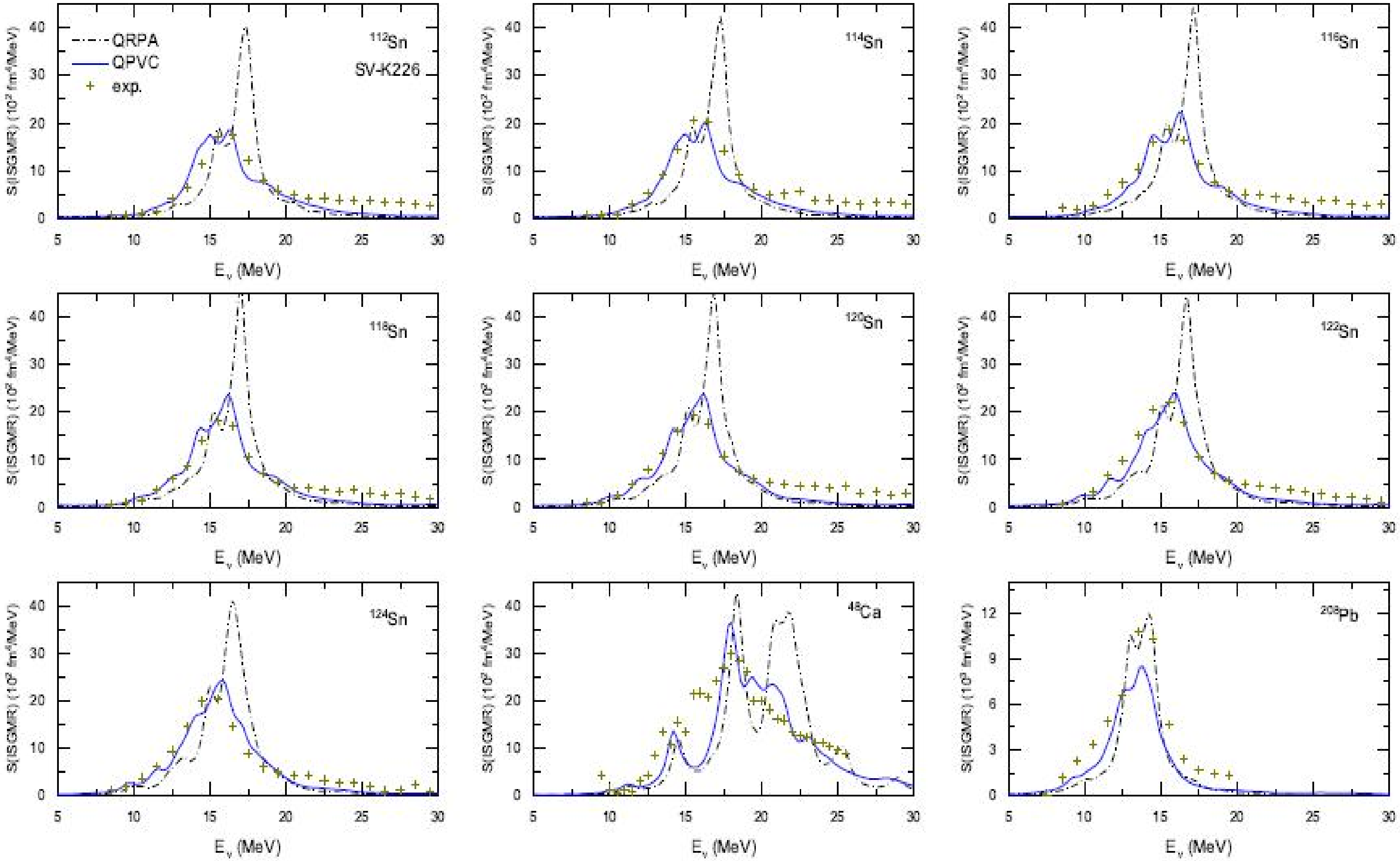
Z.Z. Li, Y.F. Niu, and G. Colò, Accepted Phys. Rev. Lett. recently

Fully self-consistent Quasiparticle Random-Phase Approximation plus Quasiparticle-Vibration Coupling model (QPVC) have been developed, based on the Skyrme-Hartree-Fock-Bogoliubov (SHFB) framework. Both QPVC effects and pairing effects are considered self-consistently.

Results suggest that (Q)PVC effects are crucial in order to reach a unified description of the ISGMR in Ca, Sn, and Pb isotopes at the same time.



ISGMR strength functions in even-even $^{112\text{-}124}$Sn, ^{48}Ca, and ^{208}Pb isotopes, calculated either by (Q)RPA using a smoothing with Lorentzian having a width of 1 MeV (dash-dotted [black] line), or (Q)RPA+(Q)PVC (solid [blue] line). The SV-K226 Skyrme force is used. The experimental data are given by green crosses

Z.Z. Li, Y.F. Niu, and G. Colò, preprint

	SkP	SkM*	SV-K	KDE0	SV-bas	SV-K	SAMi
K_∞	201	217	226	229	233	241	245
				(Q)RPA			
^{48}Ca	0.11	0.89	1.09	1.17	1.40	1.70	1.72
^{120}Sn	0.22	0.43	0.78	0.76	1.05	1.31	1.34
^{208}Pb	0.74	0.14	0.14	0.20	0.37	0.60	0.76
				(Q)RPA & (Q)PVC			
^{48}Ca	0.70	0.25	0.36	0.51	0.67	0.90	1.07
^{120}Sn	0.67	0.14	0.02	0.18	0.36	0.68	0.82
^{208}Pb	0.94	0.37	0.25	0.06	0.08	0.31	0.48

The deviation of ISGMR energies from experimental data [$|E^{theo.} - E^{exp.}|$ (MeV)] in ^{48}Ca, ^{120}Sn, and ^{208}Pb, calculated by (Q)RPA and (Q)PVC using the Skyrme parameter sets SkP, SkM*, SV-K226, KDE0, SV-bas, SV-K241, and SAMi. The experimental data are taken from [1, 2, 3].

1- D. Patel, et al., Phys. Lett. **B 726**, 178 (2013). **-- Pb**
2- T. Li, et al., Phys. Rev. Lett. **99**, 162503 (2007). **-- Sn**
3- S. D. Olorunfunmi, et al., Phys. Rev. **C 105**, 054319 (2022). **-- Ca**

	SkP	SkM*	SV-K	KDE0	SV-bas	SV-K	SAMi
K_∞	201	217	226	229	233	241	245
			(Q)RPA & (Q)PVC				
^{48}Ca	0.70	0.25	0.36	0.51	0.67	0.90	1.07
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^{208}Pb	0.94	0.37	0.25	0.06	0.08	0.31	0.48

Including (Q)PVC effects, ^{48}Ca prefers SkM* and SV-K226, ^{120}Sn prefers SkM*, SV-K226, and KDE0, while ^{208}Pb prefers SVK226, KDE0, and SV-bas.

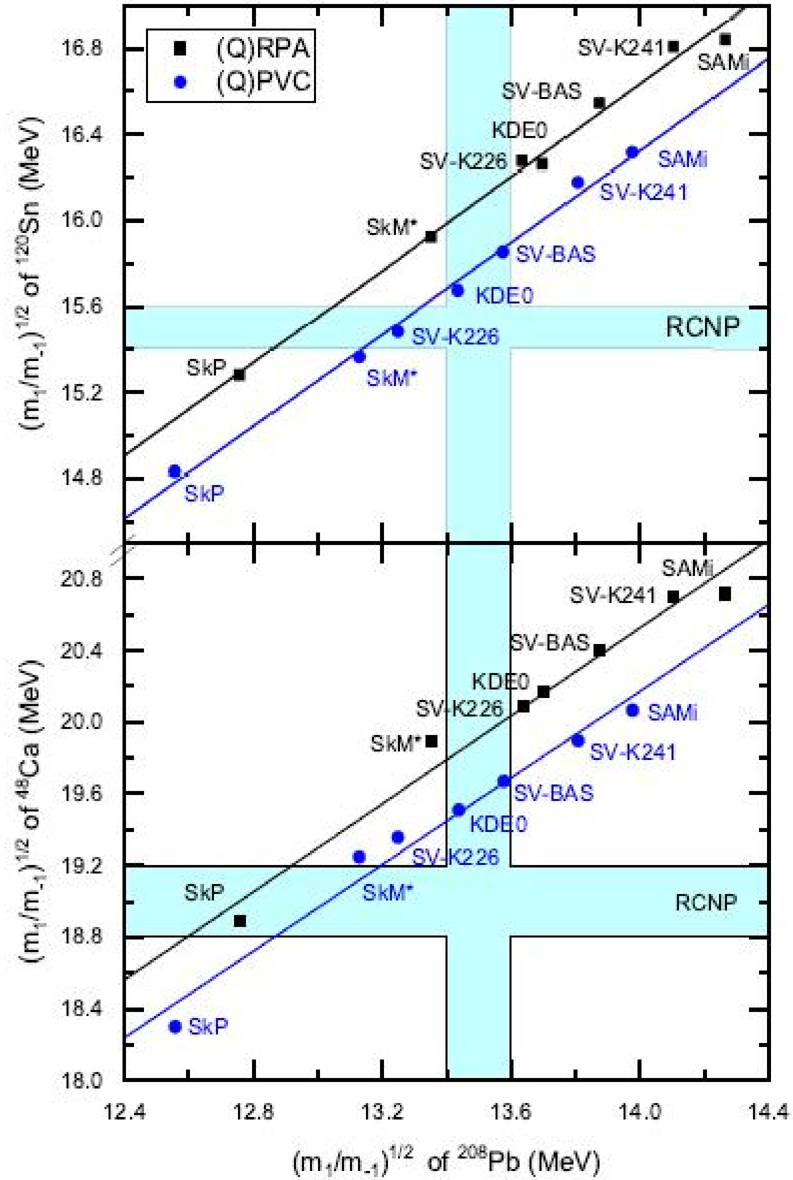
=> SV-K226 and KDE0 describe all three nuclei very well at the same time, **with K_∞ = 226 MeV and 229 MeV, respectively: this is consistent with the constraint 240 ± 20 MeV**, obtained previously from the ISGMR of ^{90}Zr and ^{208}Pb in QRPA .

Thank you for your attention

Including (Q)PVC effects, ^{48}Ca prefers SkM* and SV-K226, ^{120}Sn prefers SkM*, SV-K226, and KDE0, while ^{208}Pb prefers SVK226, KDE0, and SV-bas.

=> SV-K226 and KDE0 describe all three nuclei very well at the same time, **with K_∞ = 226 MeV and 229 MeV respectively: this is consistent with the constraint 240±20 MeV**, obtained previously from the ISGMR of ^{208}Pb in QRPA .

The ISGMR energies in ^{208}Pb vs. the ones in ^{120}Sn (upper panel), and ^{48}Ca (lower panel), calculated by (Q)RPA (black squares), and by (Q)RPA+(Q)PVC (blue circles) using 7 different Skyrme parameters. The regression lines are obtained from the (Q)RPA results and (Q)RPA+(Q)PVC results, respectively. The experimental data and their uncertainties, taken from [1, 2, 3], are displayed by means of cyan-coloured bands.



we get:

$$\mathcal{M}(E\lambda,\mu) = \sum_k e\left(\frac{1}{2} - t_{zk}\right) r_k^\lambda Y_{\lambda\mu}(\Omega_k)$$

$$\mathcal{M}(E\lambda,\mu) = \frac{1}{2} e \sum_k r_k^\lambda Y_{\lambda\mu}(\Omega_k) - e \sum_k t_{zk}\, r_k^\lambda Y_{\lambda\mu}(\Omega_k)$$

For the isoscalar *E0* and *E1*, 1st order leads to a constant and c.o.m. coordinate, respectively. Expanding to 2nd order (taking only dependence on *r*) we get:

$$\mathcal{M}(E0) = \underbrace{\frac{1}{4} e \sum_k r_k^2}_{\text{Isoscalar}} - \underbrace{\frac{1}{2} e \sum_k t_{zk} r_k^2}_{\text{Isovector}}$$

$$\mathcal{M}(E1,\mu) = \frac{1}{4} e \sum_k r_k^3 Y_{1\mu}(\Omega_k)$$

Isovector term neglected

Thomas Reiche Kuhn (TRK) sum rule is originally obtained for an atomic system assuming an electric field directed along z-axis:

$$S_e(E1) = \sum_f (E_f - E_i) \left| \langle f | \textstyle\sum_k z_k | i \rangle \right|^2$$

The total absorption cross section is in the long-wave length limit:

$$\int_0^\infty \sigma(E_\gamma) dE_\gamma = \frac{4\pi^2 e^2}{\hbar c} \sum_f (E_f - E_i) \left| \langle f | \textstyle\sum_k z_k | i \rangle \right|^2$$

For a Hermitian operator and using closure relation

($\sum_f |f><f| = 1$), we obtain:

$$\int_0^\infty \sigma(E_\gamma) dE_\gamma = \frac{4\pi^2 e^2}{\hbar c} \frac{1}{2} \left\langle i \middle| \left[\textstyle\sum_k z_k, [H, \textstyle\sum_k z_k] \right] \middle| i \right\rangle$$

Consider only kinetic term of Hamiltonian:

$$\int_0^\infty \sigma(E_\gamma)dE_\gamma = \frac{4\pi^2 e^2}{\hbar c}\frac{1}{2}\left\langle i\left\|\left[\sum_k z_k, \left[\frac{p_z^2}{2m_e}, \sum_k z_k\right]\right]\right\|i\right\rangle$$

$$\int_0^\infty \sigma(E_\gamma)dE_\gamma = \frac{4\pi^2 e^2}{\hbar c}\frac{\hbar^2 I}{2m_e}$$

I is number of electrons. For a nucleus (see later):

$$e_{eff}^2 I = Ze_{peff}^2 + Ne_{neff}^2 = \frac{NZ}{A}e^2$$

Therefore:

$$\int_0^\infty \sigma(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2 \hbar}{mc}\frac{NZ}{A} = 60\frac{NZ}{A}\ MeV\ mb$$

This is the TRK sum rule for a nucleus.

$$B\left(E\lambda, J_i \rightarrow J_f\right) = \sum_{\mu M_f} \left|\left\langle \Psi_f \middle| \mathcal{M}(E\lambda, \mu) \middle| \Psi_i \right\rangle\right|^2$$

$$B\left(E\lambda, J_i \rightarrow J_f\right) = \sum_{\mu M_f} \left\langle J_i M_i \lambda \mu \middle| J_f M_f \right\rangle^2 \left|\left\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \right\rangle\right|^2$$

$$B\left(E\lambda, J_i \rightarrow J_f\right) = \frac{2J_f + 1}{2J_i + 1} \left|\left\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \right\rangle\right|^2$$

$$S_\lambda(E\lambda) = \sum_f \left(E_f - E_i\right) |\langle f | \mathcal{M}(E\lambda, \mu) | i \rangle|^2$$

$$\Rightarrow S_\lambda(E\lambda) = \frac{1}{2} \left|\left\langle i \middle| \left[\mathcal{M}(E\lambda, \mu), [H, \mathcal{M}(E\lambda, \mu)]\right] \middle| i \right\rangle\right|$$

Introducing for $\mathcal{M}(E\lambda, \mu)$ the isoscalar *E0*, *E1* and $E\lambda$ operators, we obtain using a similar procedure as for TRK sum rule (using Hermitian property and closure relation) we obtain the isoscalar *E0*, *E1* and $E\lambda$ energy-weighted sum rules (EWSR).

Isovector E1 operator

$$\mathcal{M}(E1) = \sum_{k=1}^{A} e\left(\frac{1}{2} - t_{zk}\right)\vec{r}_k^{\,int}$$

$$\vec{r}_k = \vec{R} + \vec{r}_k^{\,int} \quad where\ \vec{R} = \sum\nolimits_k \vec{r}_k / A$$

$$\mathcal{M}(E1) = e\sum_{k=1}^{A}\left(\frac{1}{2} - t_{zk}\right)(\vec{r}_k - \vec{R})$$

$$\mathcal{M}(E1) = -e\sum_{k=1}^{A} t_{zk}\,(\vec{r}_k - \vec{R})$$

$$\mathcal{M}(E1) = e\sum_{k=1}^{A}\left(\frac{N-Z}{2A} - t_{zk}\right)\vec{r}_k$$

$\Rightarrow$ Effective charges for neutrons and protons

$$e_D = e\left(\frac{N-Z}{2A} - t_{zk}\right) = \begin{cases} \frac{N}{A}e & for\ proton \\ -\frac{Z}{A}e & for\ neutron \end{cases}$$

$$\sum_n (E_n - E_0) B(E\lambda, 0 \to n) = S_\lambda = \frac{\hbar^2}{8\pi m} \lambda (2\lambda + 1)^2 A \langle r^{2\lambda - 2} \rangle$$

For isovector *E1*, λ=1 and *A* becomes $Ze^2_{peff} + Ne^2_{neff}$, which leads to:

$$\sum_n (E_n - E_0) B(E1,0 \to n) = \frac{\hbar^2}{8\pi m} 9 \left[Z \left(\frac{N}{A} \right)^2 e^2 + N \left(\frac{Z}{A} \right)^2 e^2 \right]$$

$$\sum_n (E_n - E_0) B(E1,0 \to n) = \frac{9\hbar^2}{8\pi m} \frac{NZ}{A} e^2$$