Equation of State of Neutron-Rich Matter from Heavy-Ion Reactions

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(1)Basics of transport models for heavy-ion reactions

(2) Probes of symmetric nuclear matter EOS

(3) Probes of nuclear symmetry energy

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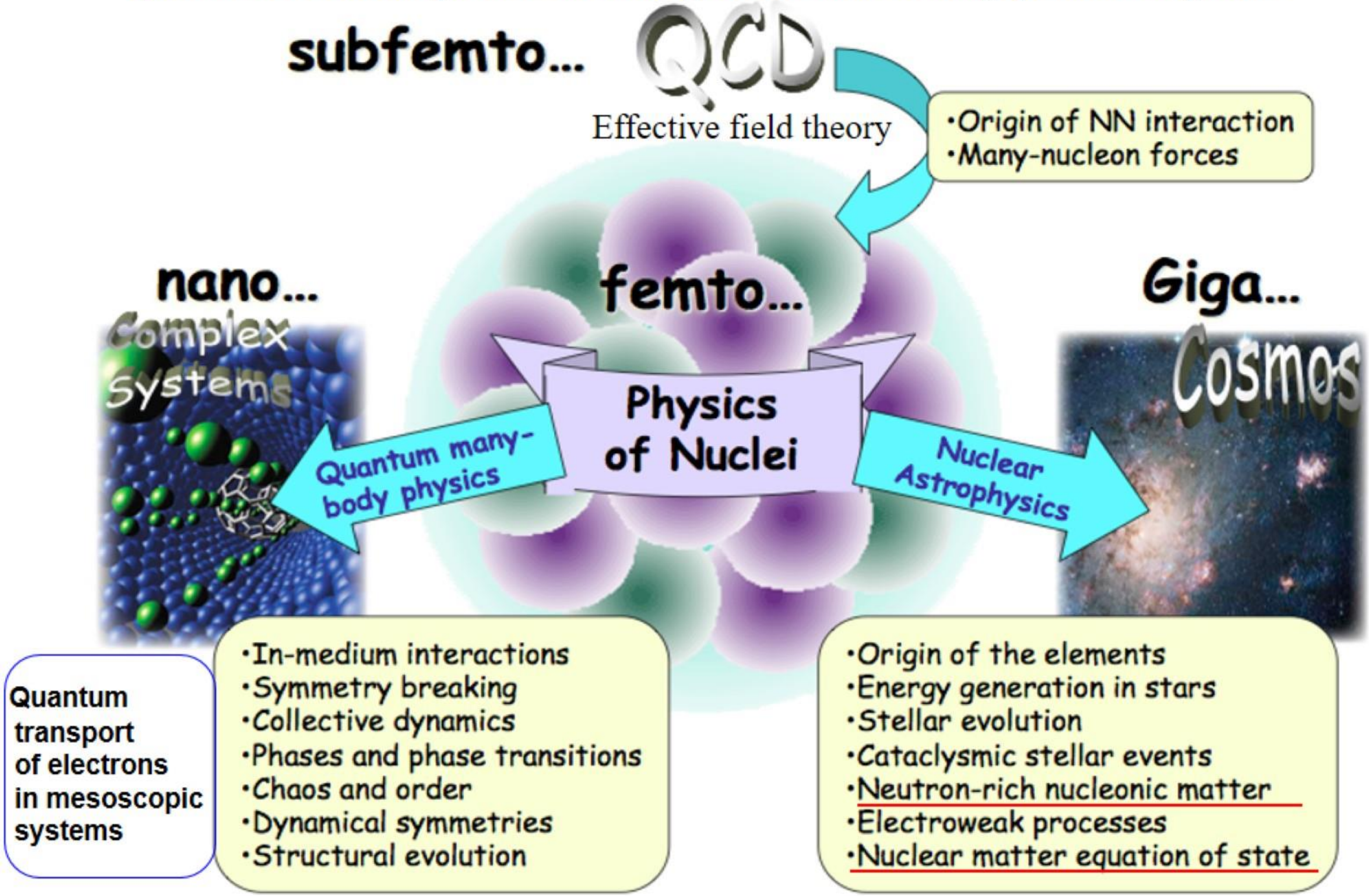
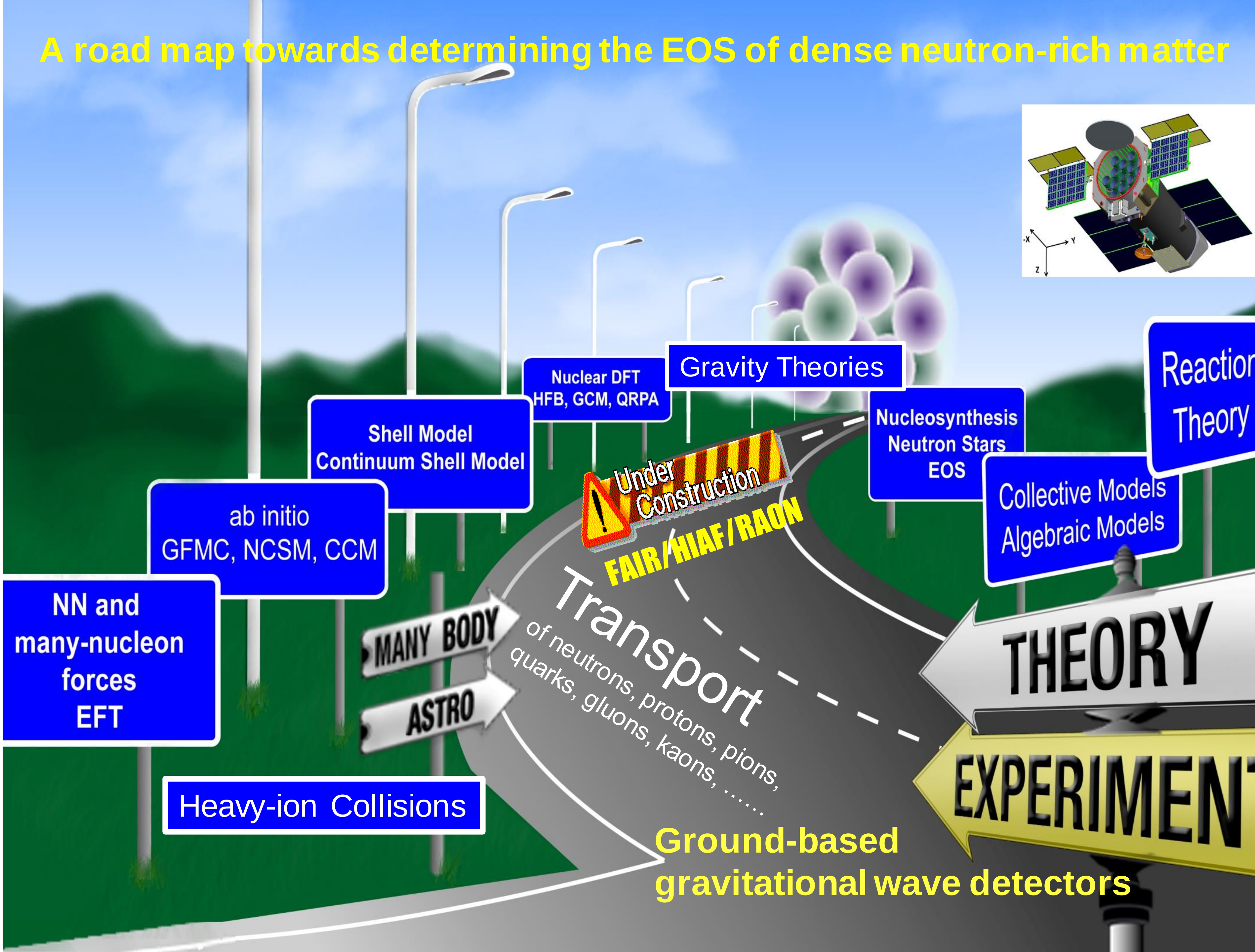
Looking at the big picture

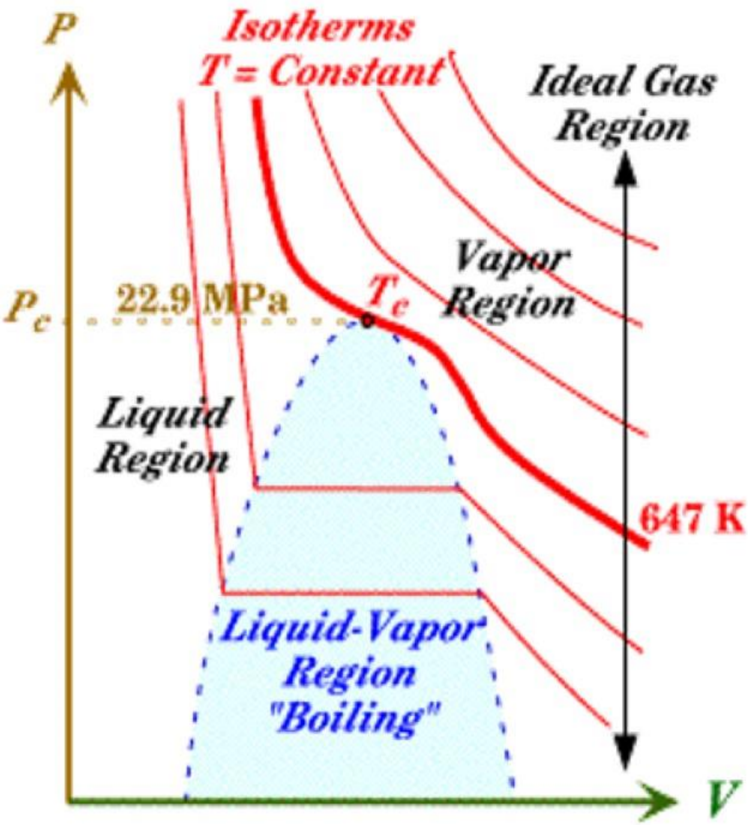
Quantum ChromoDynamics of strong interactions among quarks and gluons




A road map towards determining the EOS of dense neutron-rich matter
Nuclear DFT
HFB, GCM, QRPA
Gravity Theories
Shell Model
Continuum Shell Model
ab initio
GFMC, NCSM, CCM
NN and
many-nucleon
forces
EFT
Nucleosynthesis
Neutron Stars
EOS
Collective Models
Algebraic Models
Reaction
Theory
Under
Construction
FAIR/HIAF/RAON
MANY BODY
ASTRO
Transport
of neutrons, protons, pions,
quarks, gluons, kaons, ……
THEORY
EXPERIMENT
Heavy-ion Collisions
Ground-based
gravitational wave detectors

Equation of State: **a relationship among several state variables**

Van Der Waals EOS: $$[p + a(\frac{n}{v})^2](v - nb) = nRT$$



- The EOS depends on the interactions and properties of the particles in the matter.
- It describes how the state of the matter changes under different conditions

Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

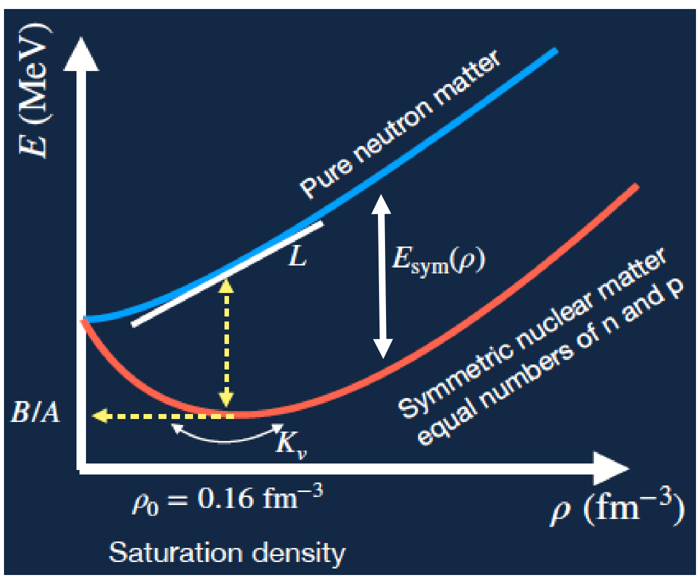
symmetry energy

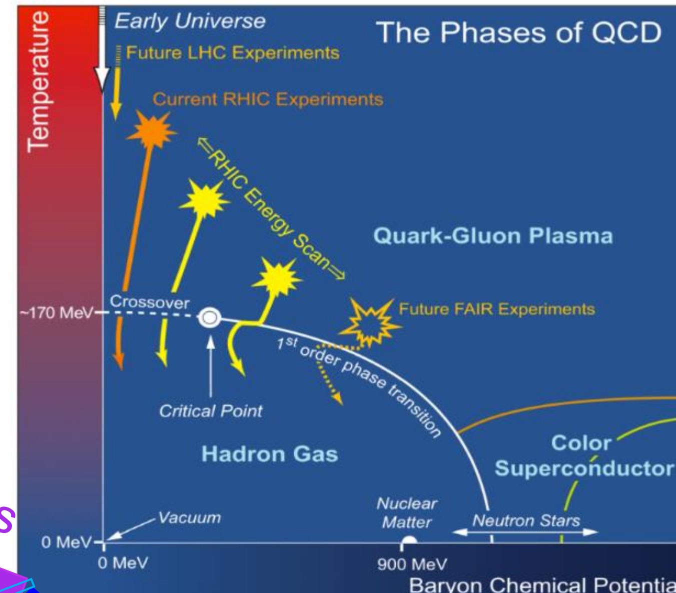
Isospin asymmetry δ

$$E(\rho_n,\rho_p)=E_0(\rho_n=\rho_p)+E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2+o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter





strangeness

Spin & magnetic field

Isospin chemical potential
$\mu_I=E_{sym}(\rho)\cdot\delta$ in n-rich matter

New opportunities

Structures and collisions of heavy-ions

Structures and mergers of neutron stars

Fundamental Microphysics Theories underlying each term in the EOS, what …, why …., where …how

Experimental and Observational Macrophysics underlying each observable and phenomenon, what …, why …., where …how

Empirical parameterizations especially useful for meta-modeling of EOS

Transport model simulations of heavy-ion collisions, energy density functionals for nuclear structures, Bayesian inferences of EOS, properties of neutron stars, waveforms of gravitational waves, ….

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2$$

Assuming no hadron-quark phase transition

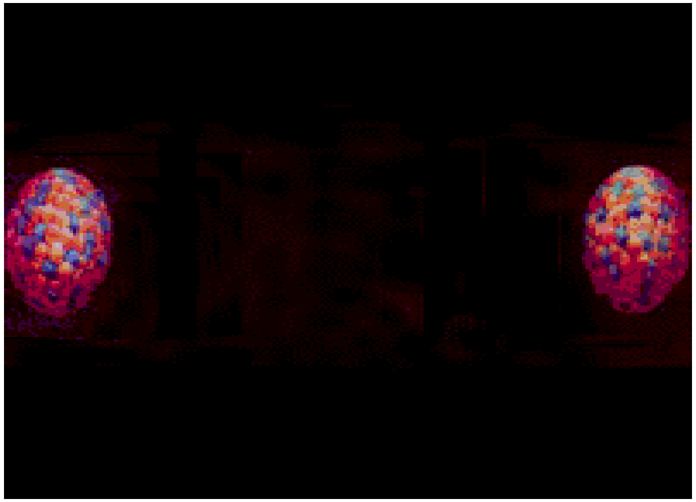
$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{24}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4,$$

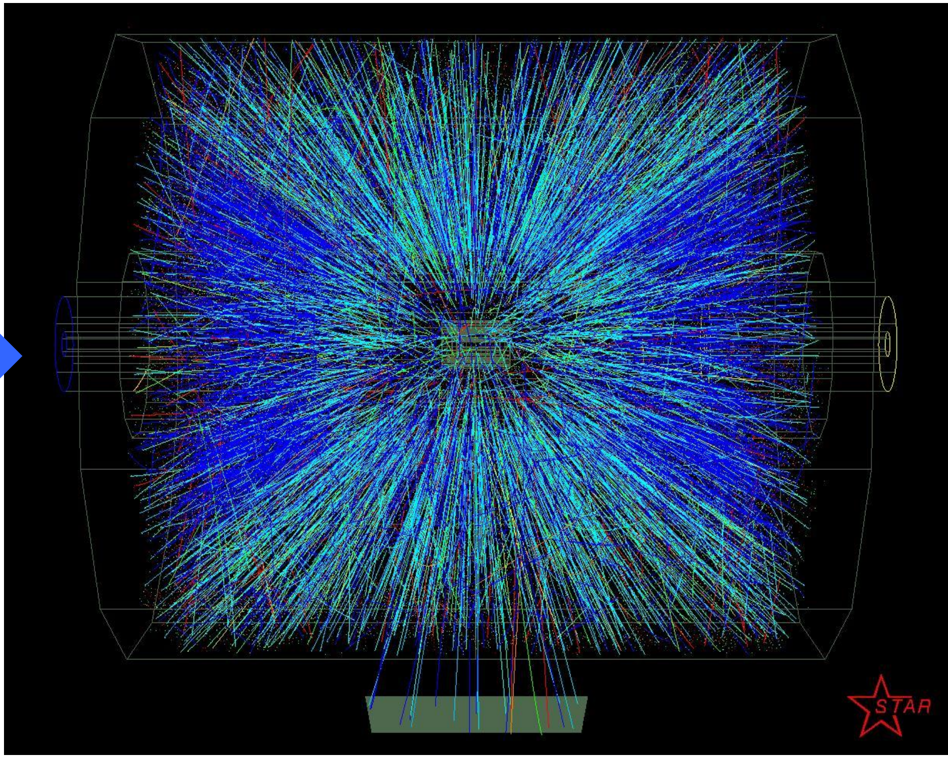
$$\boxed{E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho}{\rho_0}-1\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho}{\rho_0}-1\right)^2 + \frac{J_{\rm sym}}{162}\left(\frac{\rho}{\rho_0}-1\right)^3 + \mathcal{O}\left[\left(\frac{\rho}{\rho_0}-1\right)^4\right]}$$

Near the saturation density $\boldsymbol{\rho}_0$, they are Taylor expansions, appropriate for structure studies. Just parameterizations when applied to heavy-ion collisions and the core of neutron stars

Simulation as the Third Branch of Science

(Experiment/observation, theory and simulation)





Formation of hot and dense matter
in energetic nucleus-nucleus collisions

Basics from standard statistical mechanics textbooks

Particle number conservation → Boltzmann transport Eq. for 1-body density distribution function f(r,p,t) in phase space

Without collisions: $$f(\mathbf{r}+\mathbf{v}\,\delta t, \mathbf{p}+\mathbf{F}\,\delta t, t+\delta t) = f(\mathbf{r},\mathbf{p},t)$$

r, p
t
Newton
motion
r+δr, p+δp
t+δt

With collisions: $$\boxed{f(\mathbf{r}+\mathbf{v}\,\delta t, \mathbf{p}+\mathbf{F}\,\delta t, t+\delta t)} = f(\mathbf{r},\mathbf{p},t) + \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}} \delta t$$

Example 1:
Kerson Huang, 2nd edition,
Statistical Mechanics, 1987

Expand to first order in δt, cancel f(r,p,t) on both sides

Boltzmann transport equation: $$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{r}} + \mathbf{F}\cdot\nabla_{\mathbf{p}}\right) f(\mathbf{r},\mathbf{p},t) = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

Example 2:
L. Kadanoff and G. Baym, 2nd edition
Quantum Statistical Mechanics, 1989

Neglecting correlations, $f_2 = f(p_1)f(p_2)$

Truncating the BBGKY hierarchy for N-body correlation functions or density matrices

Hadronic transport equations for the reaction dynamics of nucleus-nucleus collisions:

Baryons: $$\frac{\partial f_b}{\partial t} + \frac{\vec{p}}{E_b}\cdot\vec{\nabla}_r f_b - \vec{\nabla}_r U_b \cdot \vec{\nabla}_p f_b = I_{bb}^{b} + I_{bm}^{b}$$

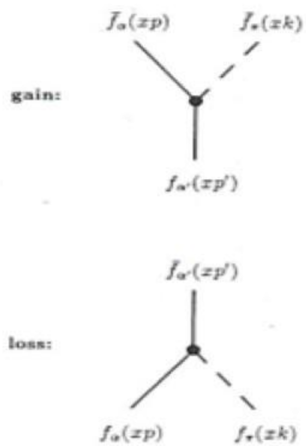
U_b is the mean-field potential for baryons

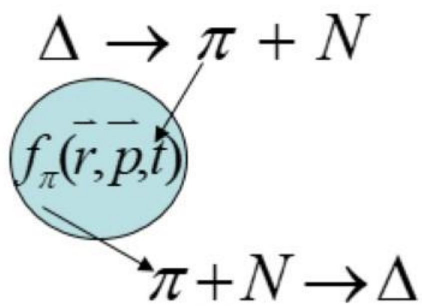
Mesons: $$\frac{\partial f_m}{\partial t} + \frac{\vec{k}}{E_m}\cdot\vec{\nabla}_r f_m = I_{mm}^{m} + I_{bm}^{m}$$

The phase space distribution functions, mean fields and collisions integrals are all isospin dependent

An example:

Collision integral $I_{b\pi}^{\pi}$: changing rate of pion phase space distribution $f_\pi(\vec{r},\vec{p},t)$ due to baryon-pion scatterings





Simulate solutions of the coupled transport equations using test-particles and Monte Carlo:

$$f(\vec{r},\vec{p},t) \cong \frac{1}{N_t}\sum_i \delta(\vec{r}-\vec{r}_i)\delta(\vec{p}-\vec{p}_i)$$

The evolution of $f(\vec{r},\vec{p},t)$ is followed on a 6D lattice

Bao-An Li, Wolfgang Bauer and George F. Bertsch, Phys. Rev. C44, (1991) 2095.

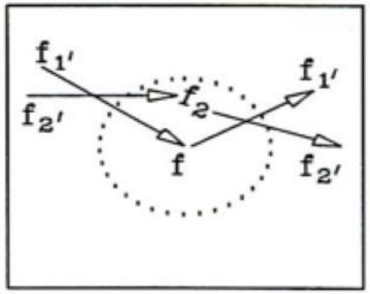
$$I_{b\pi}^{\pi}(xk) = \frac{\pi}{16}\sum_{\alpha\alpha'}\int\int \frac{M_\alpha M_{\alpha'}}{E_\alpha(p)E_{\alpha'}(p')} W_{b\pi}^{\pi}(\alpha p, \alpha' p', \pi k)\cdot \delta^{(4)}(p'-p-k)\frac{1}{(2\pi)^6}d\vec{p}d\vec{p}$$
$$[(1+f_\pi(xk))f_{\alpha'}(xp')(1-f_\alpha(xp)) \quad \text{(gain)}$$
$$-f_\pi(xk)f_\alpha(xp)(1-f_{\alpha'}(xp'))] \quad \text{(loss)}$$

S.J. Wang, B.A. Li, W. Bauer and J. Randrup, Ann. Phys. 209, 251 (1991)

Main features:

Pauli bolcking (1-f_α) for Fermions and Bose enhancement (1+f_π) for bosons are included.

A one-component model based on the Boltzmann-Uehling-Uhlenbeck (BUU) transport theory



$$
\begin{aligned}
&\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f - \vec{\nabla}_r U(\rho)\cdot\vec{\nabla}_p f \\
= &\int \frac{d^3p_{1'}\, d^3p_2\, d^3p_{2'}}{(2\pi)^9}\sigma_{12} v_{12} (2\pi)^3 \delta^3(\vec{p}+\vec{p}_2-\vec{p}_{1'}-\vec{p}_{2'}) \\
&\times\left\{f_{1'}\, f_{2'}\,(1-f)\,(1-f_2) - f\, f_2\,(1-f_{1'})\,(1-f_{2'})\right\}
\end{aligned}
$$

- $f_i(\vec{r},\vec{p},t)$: **probability density of finding particle i at position $\vec{r}$ and momentum $\vec{p}$ at time t, the density distribution** $\rho(\vec{r},t) = \int f(\vec{r},\vec{p},t)d\vec{p}$
- **left: convection terms in** $\vec{r}$ **and** $\vec{P}$ **space**
- $U(\rho)$: **mean field potential, determined by the equation of state.**
- **right: changing rate of** f **due to particle-particle scatterings with cross section** σ_{12} **at relative velocity** v_{12}
- **Pauli blocking for Fermions in the collision integral:**
  $(1-f)(1-f_2)$ **and** $(1-f_{1'})(1-f_{2'})$.

Solving the coupled, partial-differential-integral equations for a many-particle system using *testparticle approach*

- **Discretizing the continuous distribution function $f(\vec{r}, \vec{p}, t)$ using testparticles**

$$f(\vec{r}, \vec{p}, t) \cong \frac{1}{N_t} \sum_i \delta(\vec{r} - \vec{r}_i)\delta(\vec{p} - \vec{p}_i)$$

where $N_t \sim 10^3$ is the number of testparticles per nucleon.

- **Equations of motion of test particles**

$$\frac{d\vec{p}_i}{dt} = -\vec{\nabla}_r U(\rho) + \sum_j \vec{D}^i(i,j)$$
$$\frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{E_i}$$

- **Simulator equations**

$$\vec{p}_i(t + \delta t) = \vec{p}_i(t) - \delta t \vec{\nabla}_r U(\rho) + \sum_j \delta\vec{p}_i(i,j)$$
$$\vec{r}_i(t + \frac{1}{2}\delta t) = \vec{r}_i(t - \frac{1}{2}\delta t) + \delta t \frac{\vec{p}_i}{E_i}$$

where $i, j = 1, \cdots N_t \times (A_{projectile} + A_{target})$

- **Monte Carlo sampling the collision terms $\sum_j \delta\vec{p}_i(i,j)$ using experimental nucleon-nucleon elastic scattering cross sections and their anglular distributions**

- **$\rho(I_x, I_y, I_z)$ and $\vec{\nabla}_r U(\rho)$ are evaluated on a $L_x \times L_y \times L_z = 20 \times 20 \times 25$ lattice of 1 fm^3 cubes.**

$$\nabla_r f = \frac{1}{\tilde{N}} \sum_i \delta'(r - r_i)\,\delta(p - p_i)$$

$$\nabla_p f = \frac{1}{\tilde{N}} \sum_i \delta(r - r_i)\,\delta'(p - p_i)$$

$$\frac{\partial f}{\partial t} = \frac{1}{\tilde{N}} \sum_i \left(\delta'(r - r_i)\,\delta(p - p_i)\,\frac{\partial r_i}{\partial t} + \delta(r - r_i)\,\delta'(p - p$$

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} + \mathbf{F} \cdot \nabla_{\mathbf{p}}\right) f(\mathbf{r}, \mathbf{p}, t) = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

Or just use the Hamiltonian Eq. of motion:

$$\dot{\boldsymbol{p}} = -\boldsymbol{\nabla}_r U$$

and

$$\dot{\boldsymbol{r}} = \frac{\boldsymbol{p}}{\sqrt{p^2 + m^2}} + \boldsymbol{\nabla}_p U\,.$$

The simplest Skyrme potential still widely used

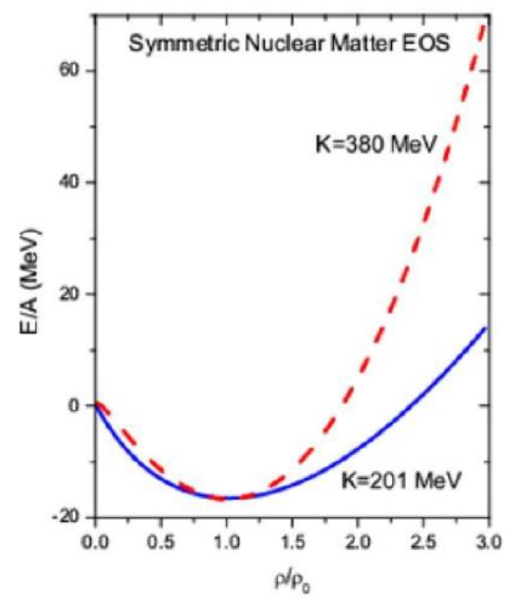
Much more complicated, isossin-momentum dependent potentials are available

$$V_q(\rho,\delta) = a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma \ + V^q_{\rm asy}(\rho,\delta) + V^q_{\rm Coulomb}.$$

a,b and σ are fixed by (1) $E_{bin}(\rho_0)$=-16 MeV, (2) $P(\rho_0)$=0 and (3) a specified incompressibility K

$$\begin{aligned} a &= -29.81 - 46.90\frac{K+44.73}{K-166.32}\ (\mathrm{MeV}), \\ b &= 23.45\frac{K+255.78}{K-166.32}\ (\mathrm{MeV}), \\ \sigma &= \frac{K+44.73}{211.05}. \end{aligned}$$

EOS of symmetric matter



K, J_0 and Z_0 are degenerate, i.e., once K is fixed, the other 2 are fixed, leading to confusions ….

$$P(\rho) = \rho^2\frac{dE_0(\rho)}{d\rho} = \frac{\rho^2}{\rho-\rho_0}[K(\frac{\rho-\rho_0}{3\rho_0})^2 + \frac{J_0}{2}(\frac{\rho-\rho_0}{3\rho_0})^3 + \frac{Z_0}{6}(\frac{\rho-\rho_0}{3\rho_0})^4].$$

Symmetry energy $E_{sym}(\rho) = E_{sym}(\rho_0) u^{\gamma}, \quad u = \rho / \rho_0$

The simplest Symmetry energy/potential still widely used

Symmetry Potential

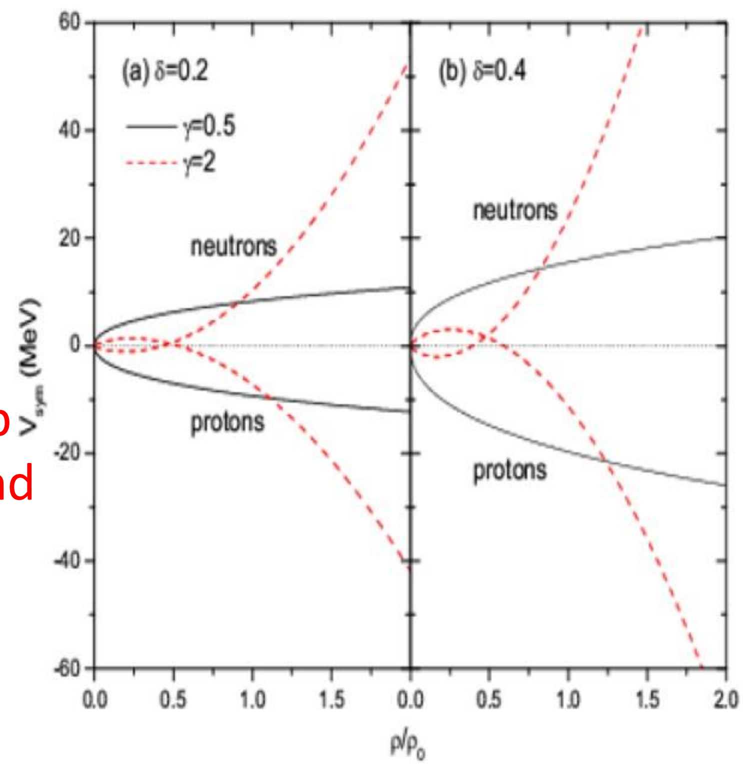
$$V_{sym}(\rho, \delta) = \frac{\partial(\rho E_{sym}^{pot})}{\partial \rho_{n/p}}$$

$$= \pm 2[E_{sym}(\rho_0) u^{\gamma} - 12.7 u^{2/3}]\delta$$

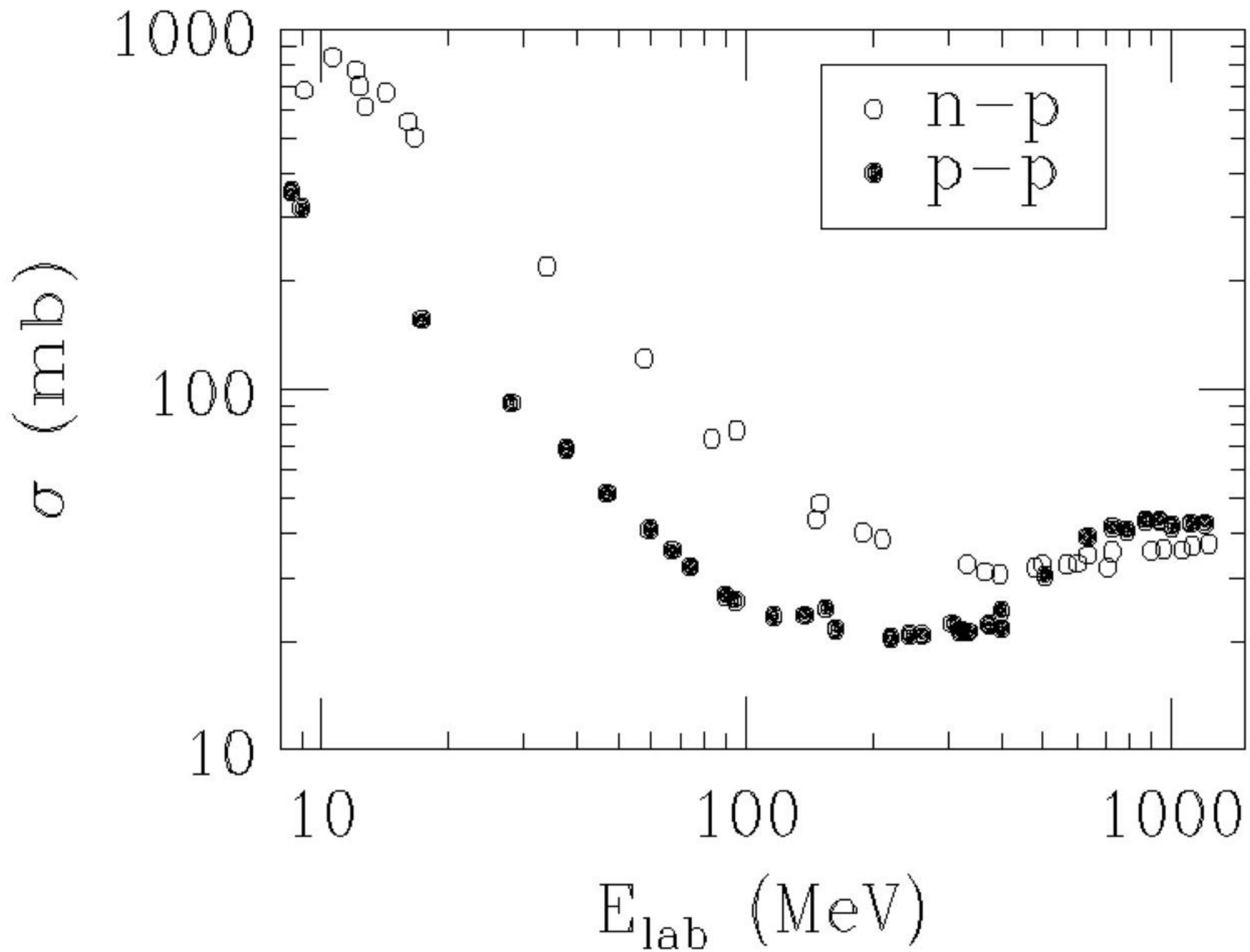
$$+ [E_{sym}(\rho_0)(\gamma - 1) u^{\gamma} + 4.2 u^{2/3}]\delta^2$$

Main feature: neutrons and protons have OPPOSITE symmetry potential (balancing the positive Coulomb potential for protons, just as the Coulomb energy and symmetry energy in the mass formula are balancing each other in determining the beta-stability line

Symmetry Potential



Nucleon-nucleon cross sections in free-space



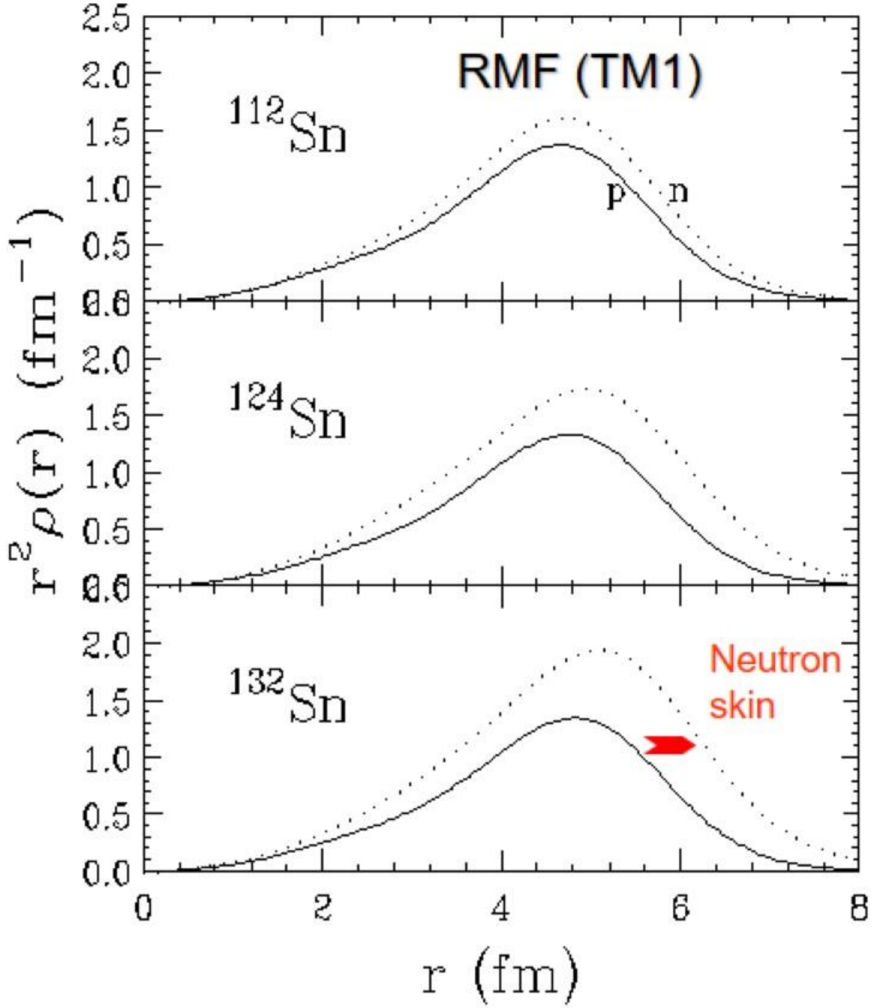
Attempts you may not approve, suggestions are more than welcome

Initialization procedures:

(1) r-space: distribute n and p according to predictions by structure models**, e.g., RMF

(2) p-space: using local Thomas-Fermi

(3) check stability



** Trade-off: drawbacks/advantages

- The initial state may not be the ground state corresponding to the effective interactions used in the subsequent reactions.

- None of the existing transport models can generate the initial nucleon density profiles BETTER than dedicated structure models for radioactive nuclei, and there is no data to constrain the neutron density profiles.

- If one insists on using the same effective interactions in generating the initial state and carrying out the subsequent reactions, it is then hard to tell whether differences in final observables obtained from using different interactions, e.g., symmetry energies, are from the different initial states or from the reaction dynamics.

A compromise has to be made before better approaches are found

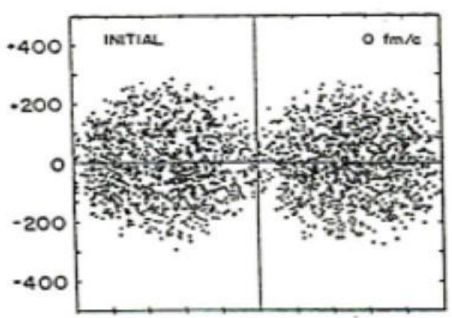
Initialization in phase space $(\vec{r}, \vec{p})$

- Initial momentum due to Fermi motion

$$
\begin{aligned}
P &= P_F (X_1)^{1/3} \\
cos\theta &= 1 - 2X_2 \\
\phi &= 2\pi X_3
\end{aligned}
$$

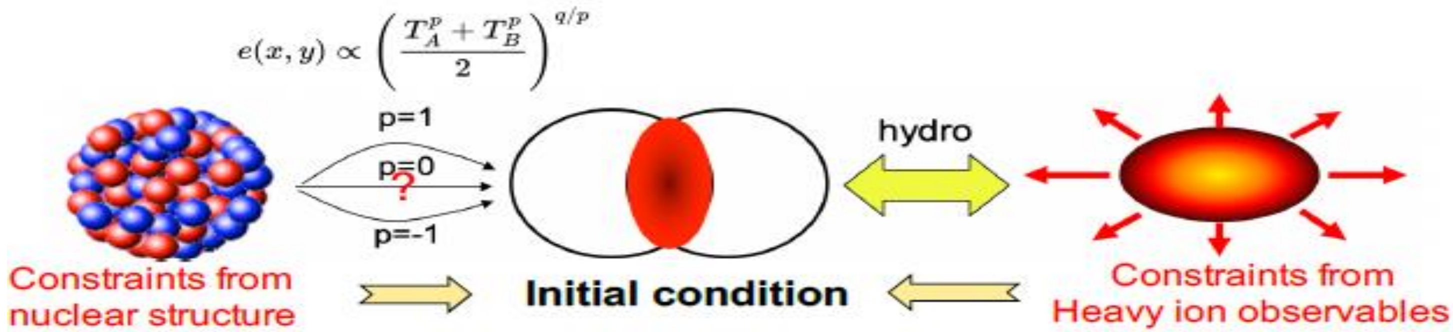
where $X_i = Ran(Iseed)$, i=1,2,3 and $P_F = 270$ MeV/c.

$$
\begin{aligned}
p_x &= P \cdot sin\theta \cdot cos\phi \\
p_y &= P \cdot sin\theta \cdot sin\phi \\
p_z &= P \cdot cos\theta
\end{aligned}
$$



- Initial coordinates
  Replace P_F with $R_A = 1.2A^{1/3}$, and P with R to generate (x,y,z) coordinates.

MANY interesting issues related to nuclear structure in the initial state!



Imaging the initial condition of heavy-ion collisions and nuclear structure across the nuclide chart, arXiv:2209.11042v1, B. Bally et al.

Examples: deformation, neutron-skin in coordinate, proton-skin in momentum, short-range correlation, bubbles in nuclei…

Separating the Impact of Nuclear Skin and Nuclear Deformation in High-Energy Isobar Collisions
Jiangyong Jia, G. Giacalone, and Chunjian Zhang PRL 131, 022301 (2023)

Evidence of Hexadecapole Deformation in Uranium-238 at the Relativistic Heavy Ion Collider,
W. Ryssens, G. Giacalone, B. Schenke and Chun Shen, PRL 130, 212302 (2023)

Evidence of quadrupole and octupole deformations in 96Zr+96Zr and 96Ru+96Ru collisions at ultra-relativistic energies, Chunjian Zhang, Jiangyong Jia, PRL 128, 022301 (2022)

Impact of Nuclear Deformation on Relativistic Heavy-Ion Collisions,
G. Giacalone, Jiangyong Jia and Chunjian Zhang, PRL 127, 242301 (2021)

Modeling Particle - Particle Collisions

Will P_1 and P_2 collide during the time interval δt ?

MANY interesting issues!

G.F. Bertsch and S. Das Gupta
Phys. Rep. 160, 189 (1988)

- **Before the collision:** Calculate the $E_{cms}(p_1, p_2)$, look up a nuclear database to get the corresponding collision cross section σ_{12}.
- **Collision criteria:**
  1. If $r_{12} \leq r_g (\equiv \sqrt{(}\sigma_{12}/\pi$, is the geometrical collision radius).
  2. If p_1 and p_2 will pass their closet approach r_c in δt.
- **During the collision:** Determine the reaction channel
  If $X < \sigma_{elastic}/(\sigma_{elastic} + \sigma_{inelastic}) \Longrightarrow$ elastic collision
  else inelastic collision $\Longrightarrow$ further branching.
- **After the collision:**
  1. Find the final momenta p'_1 and p'_2
  2. Generate the outgoing angles using $f(cos\theta)$
     Example: to generate θ, $X = \frac{\int_0^\theta f(cos\theta)dcos\theta}{\int_0^\pi f(cos\theta)dcos\theta}$
  3. Generate masses for various resonances $(\Delta, N^*, \rho, \omega, \cdots)$ using F(mass) and rejection method.
  4. Assign new positions for p'_1, p'_2 and newly created particles.
  5. Check quantum statistics (Pauli blocking) of final states

Inelastic baryon-baryon collisions included in ART:

B.A. Li and C.M. Ko,
PRC 52 (1995) 2037

$$NN \leftrightarrow N\Delta, \quad (1)$$
$$NN \leftrightarrow NN^*(1440), \quad (2)$$
$$NN \leftrightarrow NN^*(1535), \quad (3)$$
$$NN \leftrightarrow \Delta\Delta, \quad (4)$$
$$NN \leftrightarrow \Delta N^*(1440), \quad (5)$$
$$NN \rightarrow NN\rho, \quad (6)$$
$$NN \rightarrow NN\omega, \quad (7)$$
$$NN \rightarrow \Delta\Delta\pi, \quad (8)$$
$$NN \rightarrow \Delta\Delta\rho, \quad (9)$$
$$NN \rightarrow \Delta\Delta\omega, \quad (10)$$
$$N\Delta \leftrightarrow NN^*(1440), \quad (11)$$
$$N\Delta \leftrightarrow NN^*(1535), \quad (12)$$
$$\Delta\Delta \leftrightarrow NN^*(1440), \quad (13)$$
$$\Delta\Delta \leftrightarrow NN^*(1535), \quad (14)$$
$$\Delta N^*(1440) \leftrightarrow NN^*(1535), \quad (15)$$

and those producing kaons:

$$NN \rightarrow N\Lambda(\Sigma)K, \quad (16)$$
$$NN \rightarrow \Delta\Lambda(\Sigma)K, \quad (17)$$
$$NR \rightarrow N\Lambda(\Sigma)K, \quad (18)$$
$$NR \rightarrow \Delta\Lambda(\Sigma)K, \quad (19)$$
$$RR \rightarrow N\Lambda(\Sigma)K, \quad (20)$$
$$RR \rightarrow \Delta\Lambda(\Sigma)K, \quad (21)$$

where R is Δ, $N^*(1440)$ or $N^*(1535)$.

Meson-baryon collisions:

1. elastic:
   $\pi N \leftrightarrow \Delta(N^*)$
   $\eta N \leftrightarrow N^*(1535)$
   $KN \longrightarrow KN$

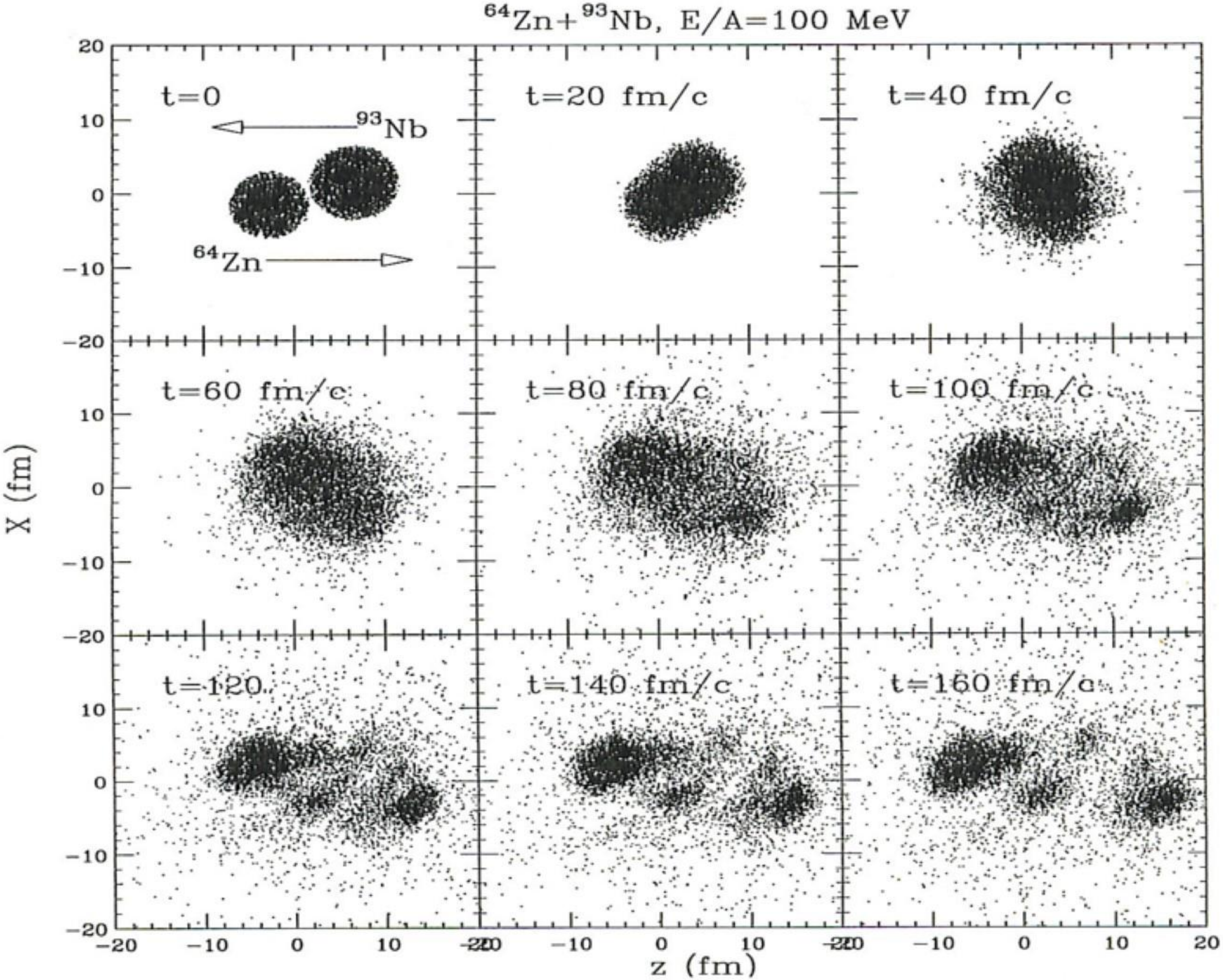
2. Inelastic:
   $\pi \mathbf{N} \longrightarrow \boldsymbol{\Delta}\pi, +[N\rho, \Delta\rho, N]$
   $\pi N \longrightarrow \Lambda(\Sigma)K$

$\pi - \pi$ collisions:

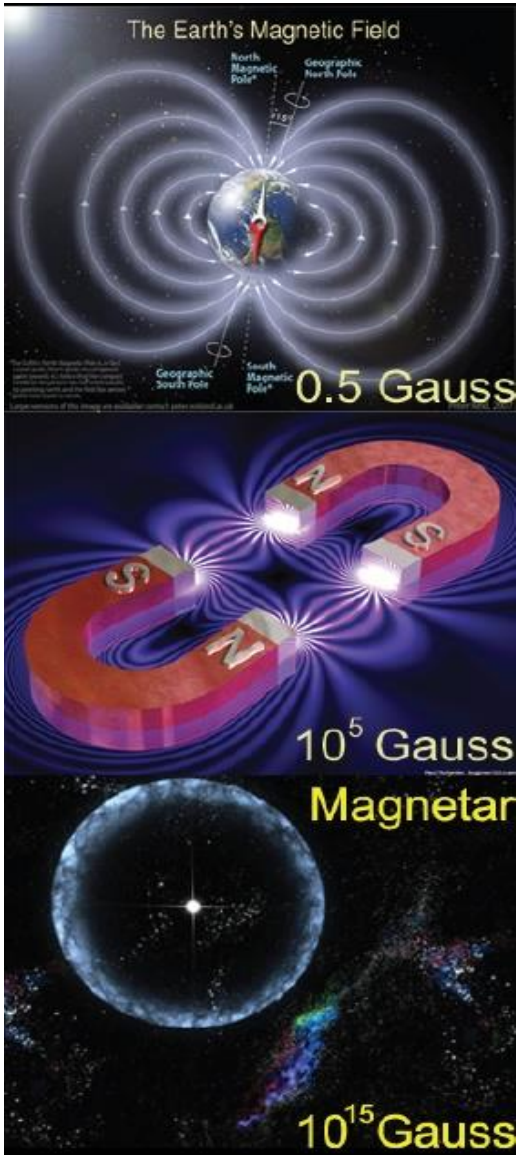
1. elastic: $\pi\pi \longrightarrow \pi\pi$

2. inelastic $\pi\pi \longrightarrow K^+K^-$

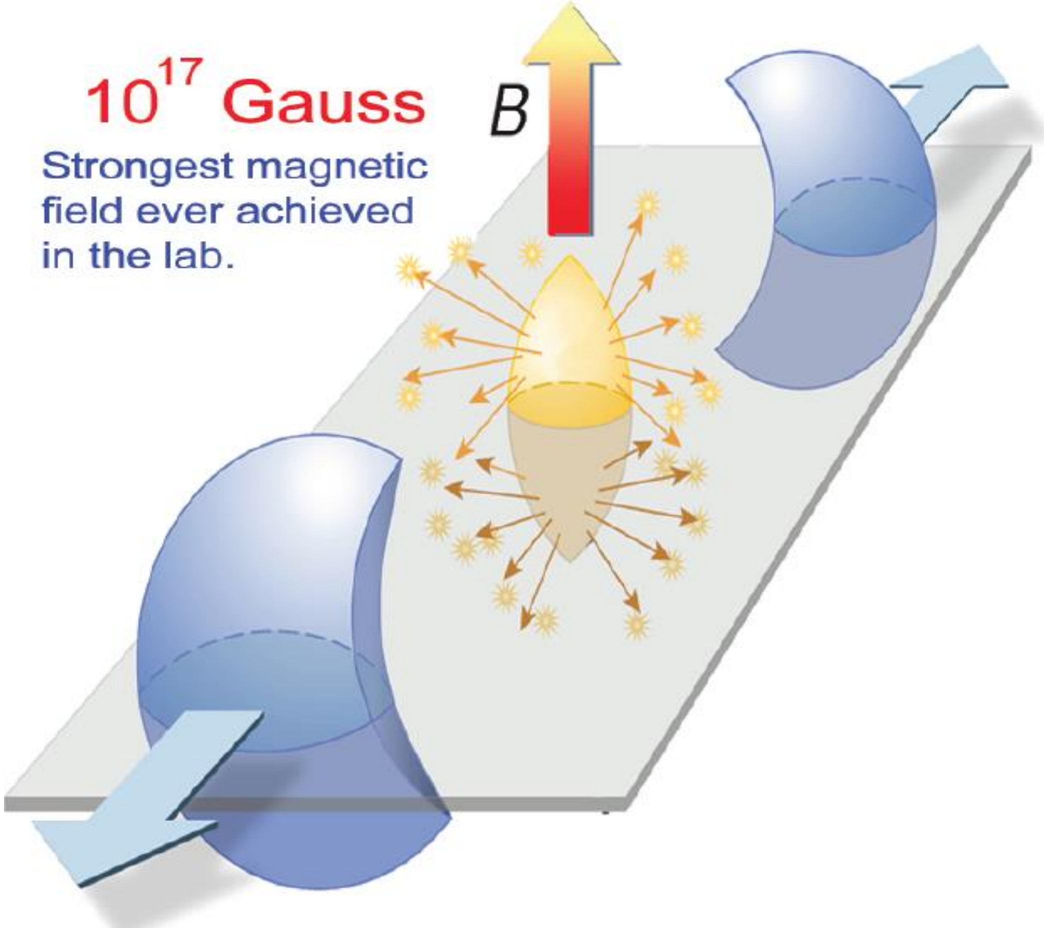
Motion of Test Particles in the Reaction Plane



Super-strong magnetic field formation in heavy-ion collisions



In off-central Au+Au collisions at RHIC,
D. Kharzeev, L. McLerran and H.Warringa, NPA803:227 (2008)



In sub-Coulomb barrier U+U collisions, the magnetic field B is on the order of 10^{14} G
J. Rafelski and B. Muller, PRL 36, 517 (1976)

Terrestrial laboratory to study properties of nuclear matter under extreme magnetic field

Model

The Boltzmann-Uhling-Uhlenbeck transport equation including electomagnetic fields:

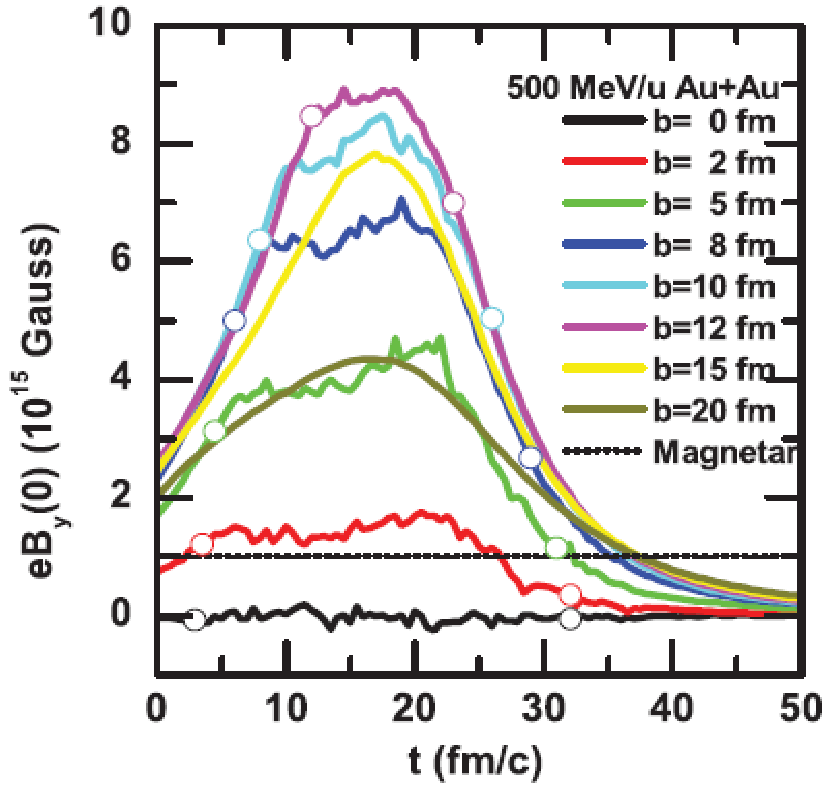
$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{P}}{E}\nabla_r - (\nabla_r \boldsymbol{U} - q\boldsymbol{v} \times \boldsymbol{B} - q\boldsymbol{E})\nabla_p\right] f(\boldsymbol{r},\boldsymbol{p},t) = I(\boldsymbol{r},\boldsymbol{p},t). \quad (1)$$

Electric and magnetic field strength with Liénard-Wiechert potentials and the retardation effect are calculated according to

$$e\boldsymbol{E}(\boldsymbol{r},t) = \frac{e^2}{4\pi\epsilon_0} \sum_n Z_n \frac{c^2 - v_n^2}{(cR_n - \boldsymbol{R}_n \cdot \boldsymbol{v}_n)^3}(c\boldsymbol{R}_n - R_n\boldsymbol{v}_n); \quad (2)$$

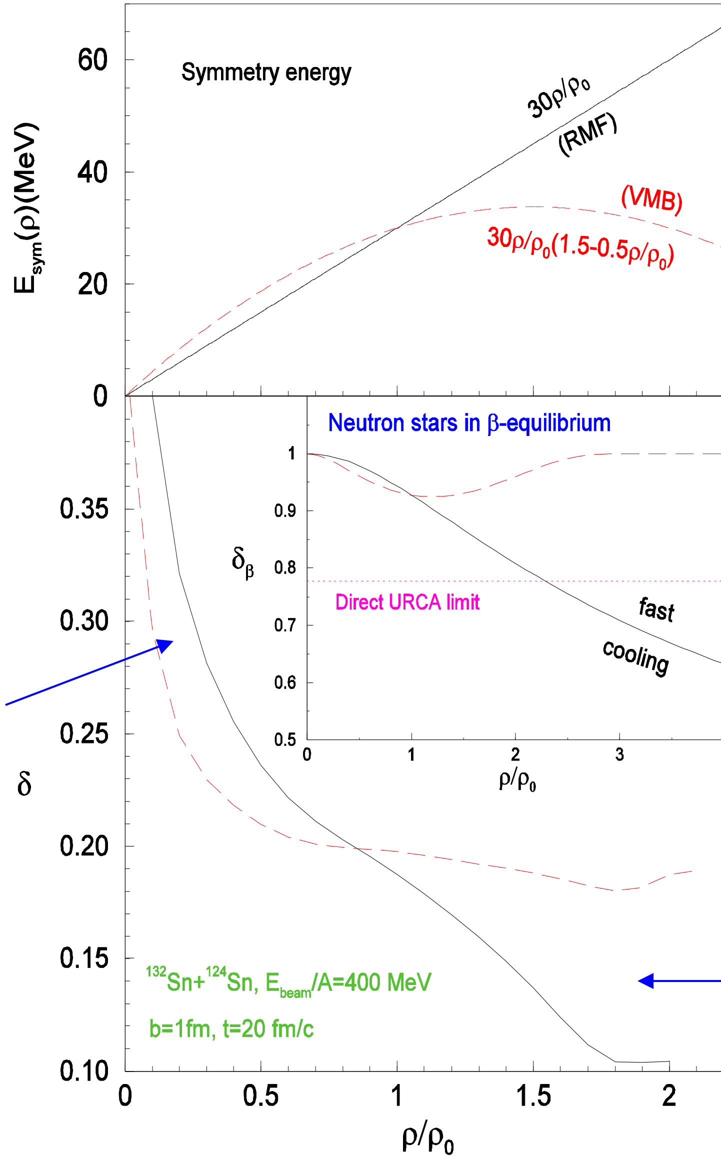
$$e\boldsymbol{B}(\boldsymbol{r},t) = \frac{e^2}{4\pi\epsilon_0 c} \sum_n Z_n \frac{c^2 - v_n^2}{(cR_n - \boldsymbol{R}_n \cdot \boldsymbol{v}_n)^3}\boldsymbol{v}_n \times \boldsymbol{R}_n. \quad (3)$$

$\boldsymbol{R}_n = \boldsymbol{r} - \boldsymbol{r}'_n$, $\boldsymbol{r}'_n$ is the position vector of particle, $\boldsymbol{v}_n$ is particle velocity. $\boldsymbol{v}_n$ and $\boldsymbol{R}_n$ are taken at the retarded time $t_{rn} = t - |\boldsymbol{r} - \boldsymbol{r}'_n(t_{rn})|$.



Li Ou and Bao-An Li, Physical Review C84, 064605 (2011)

Effects of symmetry energy on isospin fractionation



$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2$$

high density region is more neutron-rich with soft symmetry energy

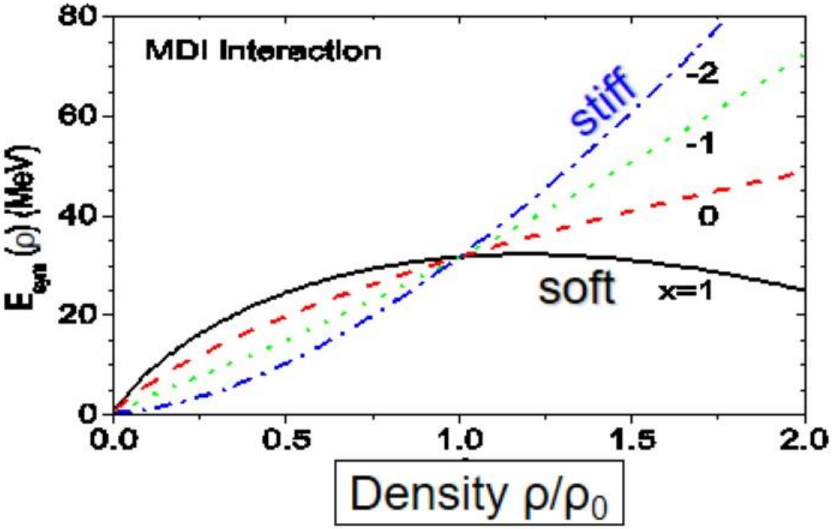
n/p spectrum ratio of pre-equilibrium emission probing neutron-proton effective mass splitting

The density dependence of isospin asymmetry in neutron stars and heavy-ion reactions are similar

π^-/π^+ ratio at freeze-out and neutron-proton differential flow probing high-density E_{sym}

Bao-An Li, Phys. Rev. Lett. 88 (2002) 192701

Symmetry energy and single nucleon potential used in the IBUU04 transport model



The x parameter is introduced to mimic various predictions by different microscopic Nuclear many-body theories using different Effective interactions

Single nucleon potential within the HF approach using a modified Gogny force:

$$U(\rho,\delta,\bar{p},\tau,x) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma (1 - x\delta^2) - 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}$$

$$+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3p'\frac{f_\tau(r,p')}{1+(p-p')^2/\Lambda^2} + \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3p'\frac{f_{\tau'}(r,p')}{1+(p-p')^2/\Lambda^2}$$

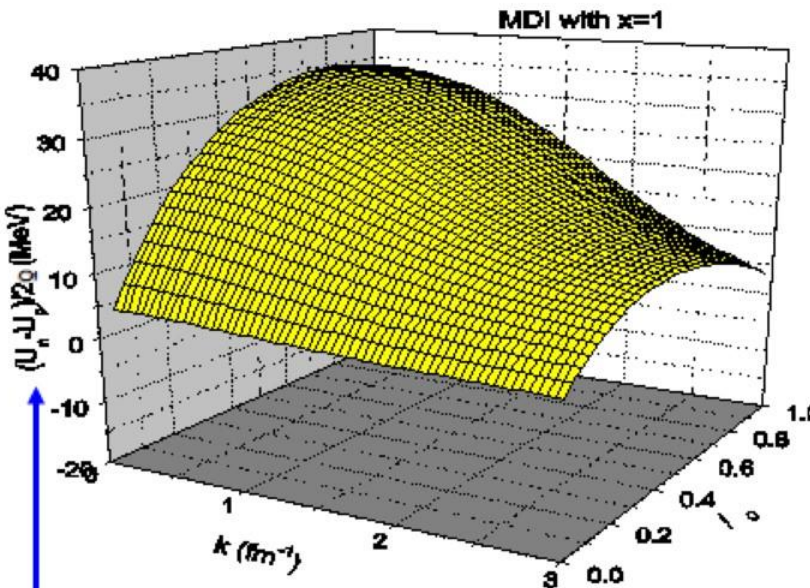
$$\tau,\tau' = \pm\frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma+1}, A_u(x) = -96 - \frac{2Bx}{\sigma+1}, K_0 = 211MeV$$

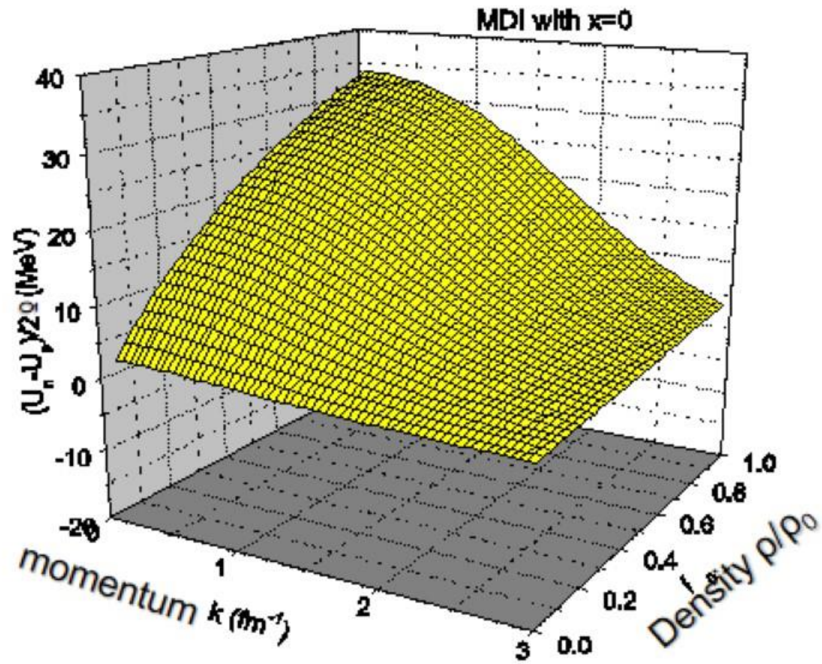
The momentum dependence of the nucleon potential is a result of the non-locality of nuclear effective interactions and the Pauli exclusion principle

C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC **69**, 034614; NPA **735**, 563 (2004).

Momentum and density dependence of the symmetry potential $U_{n/p} \approx U_{isoscalar} \pm U_{Lane} \cdot \delta$





Lane potential extracted from n/p-nucleus scatterings and (p,n) charge exchange reactions provides only a constraint at ρ_0:

$$U_{Lane} \equiv (U_n - U_p)/2\delta \approx V_1 - \varepsilon_R \cdot E_{kin},$$

$$V_1 \approx 28 \pm 6 MeV, \varepsilon_R \approx 0.1 - 0.2$$

for $E_{kin} < 100$ MeV

Although the uncertainities are large, NOT a single experiment was analyzed assuming the symmetry potential would INCREASE with energy because then the errors will be even larger

P.E. Hodgson, The Nucleon Optical Model, World Scientific, 1994

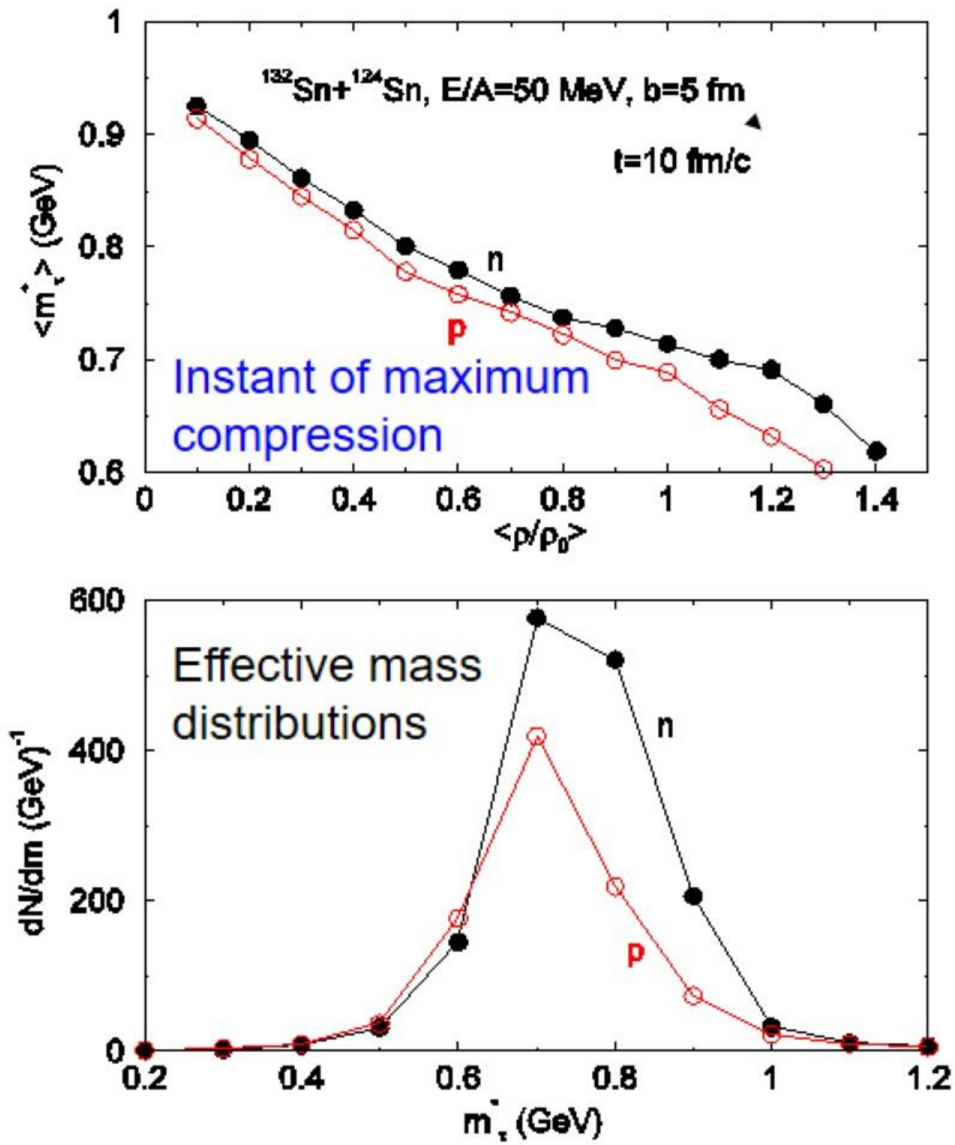
G.W. Hoffmann et al., PRL, 29, 227 (1972).

G.R. Satchler, Isospin Dependence of Optical Model Potentials, 1968

Nucleon effective k-masses during heavy-ion reactions

B.A. Li and L.W. Chen, Phys. Rev. C72, 064611 (2005).

$$\frac{m^*_\tau}{m_\tau} = [1 + \frac{m_\tau}{p}\frac{\partial U}{\partial p}]^{-1}_{p^\tau_F}$$



Probing the symmetry energy at supra-saturation densities

- π^-/π^+, n/p ratio of squeezed-out nucleons
- Neutron-proton differential flow
- Neutrino flux of supernova explosions
- Strength and frequency of gravitational waves

Clarifying effects of the completely unknown isovector potential for Delta(1232) resonances

$$NN \rightarrow N\Delta$$
$$\Delta \rightarrow N+\pi$$

QCD Aspects of Resonances:

- hadronic (*soft scale*) *molecular-type* components $|N_s\rangle$
- QCD (*hard scale*) confined components $|N_h\rangle$

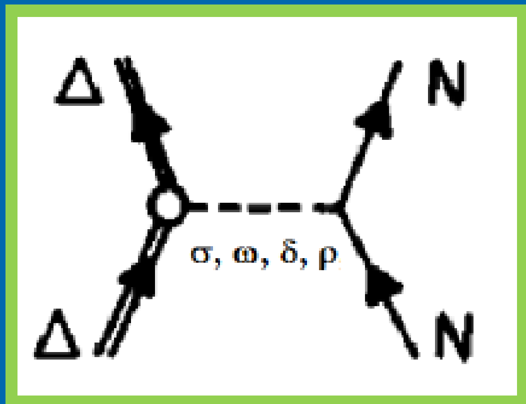
$$\left|N^*\right\rangle = \left|N_s^*\right\rangle + \left|N_h^*\right\rangle = x_1\left|mB\right\rangle + x_2\left|qqq\right\rangle + x_3\left|qqq\right\rangle \otimes \left|q\bar{q}\right\rangle + \ldots$$

$$|N_s^*\rangle = |MB\rangle \qquad\qquad |N_h^*\rangle = |qqq\rangle + |\mathrm{m.c.}\rangle$$

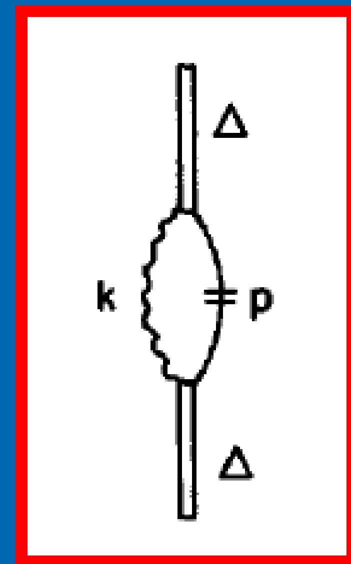
Strong Medium Dependence   **Weak Medium Dependence**

Taken from Horst Lenske, 2014

Delta Self-Energy in Nuclear Matter



+



Direct Self-energy → Hartree-Potential

$$U_\Delta^{(H)} = U_0 + U_1 \tau_\Delta \cdot \tau_N$$

$$U_\Delta^{(H)} \sim U_0 + U_1 t_z^{(\Delta)} \cdot \frac{N-Z}{A}$$

Polarization Self-Energy → dispersive (optical) potential

$$\Sigma_{\mathrm{pol}}^{(\Delta)} \sim \Sigma_0 + \Sigma_1 t_z^{(\Delta)} \frac{N-Z}{A}$$

$$\Sigma_\alpha = V_\alpha - iW_\alpha$$

...see e.g.:

E. Oset, L.L. Salcedo, NPA 468 (1987) 631; G.E. Brown, W. Weise, Phys. Rept. 22 (1975) 279

$U_0(\Delta)$ is 0-30 MeV deeper than $U_0(N)$ at ρ_0 from e+A, π+A and γ+A scattering, but nothing is known about the $U_1(\Delta)$

In the pi-N molecule model, assuming pions have no mean-field,the Delta isovector potential is linked to the nucleon isovector potential

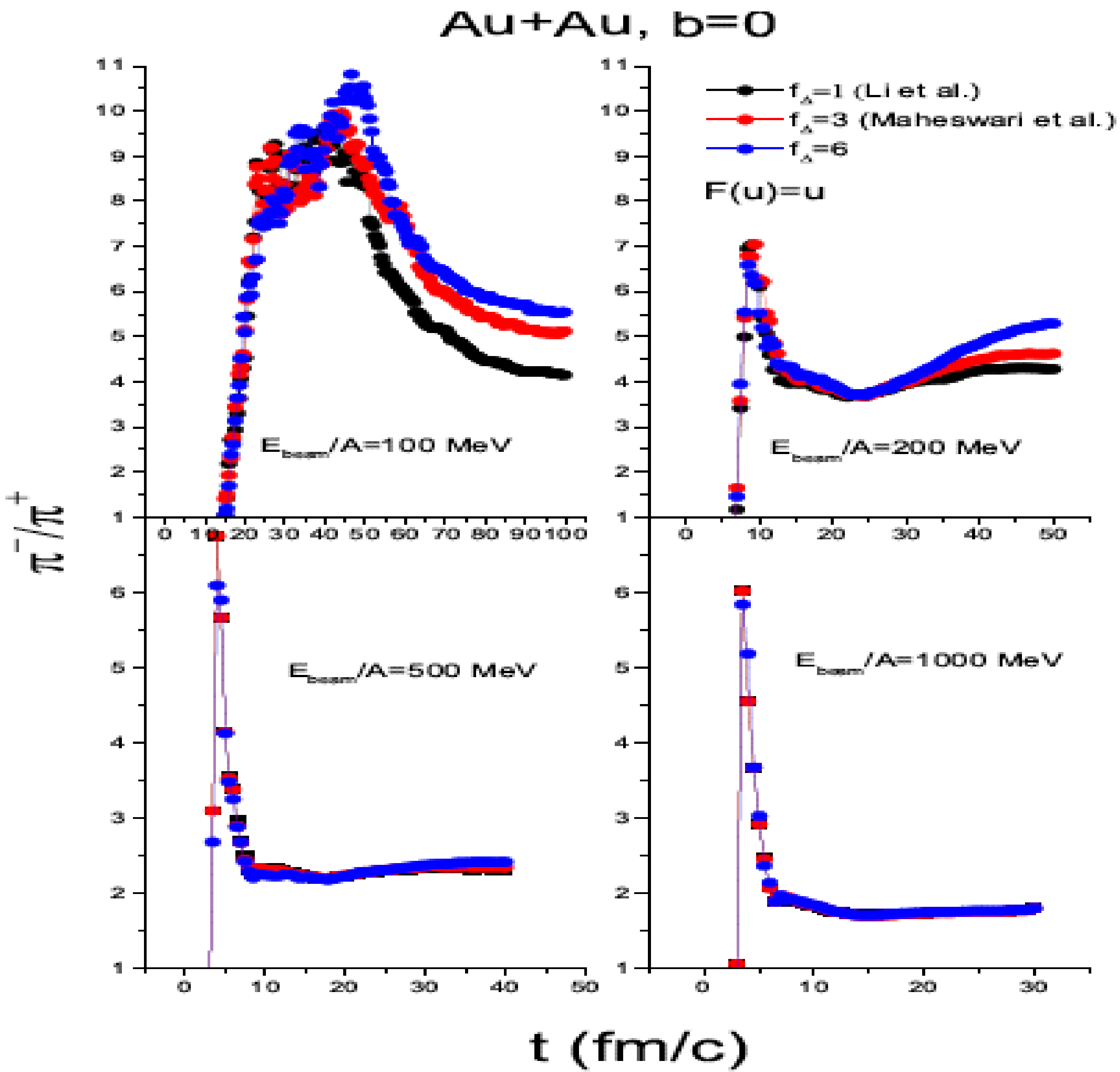
Bao-An Li, PRL88, 192701 (2002) and NPA 365 (2002)

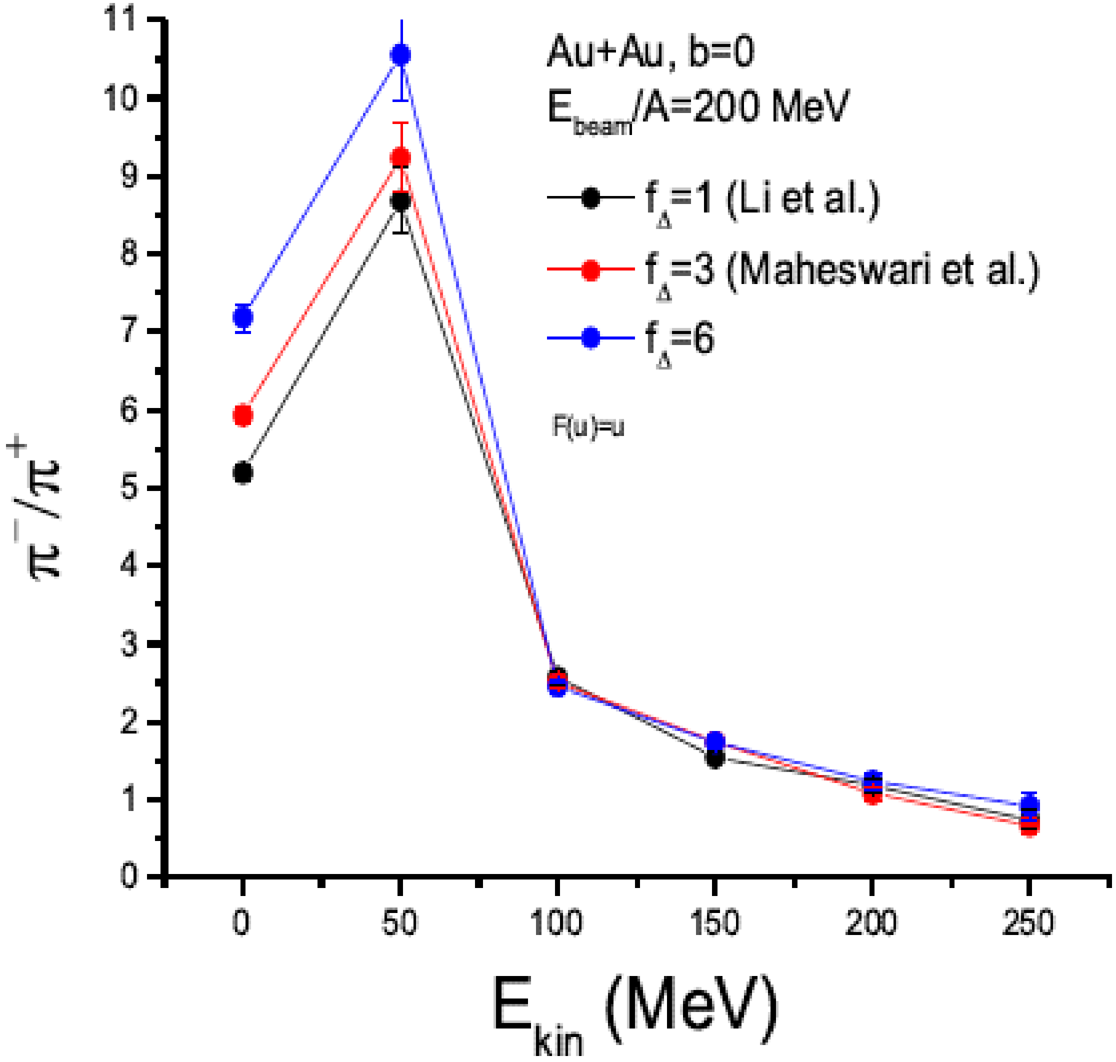
$$
\begin{aligned}
v_{asy}(\Delta^-) &= v_{asy}(n), \\
v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n).
\end{aligned}
$$

To study effects of the completely unknown Delta isovector potential, multiply the above with a Delta-probing-factor:

$$f_\Delta = 1,\ 3 \text{ and } 6.$$

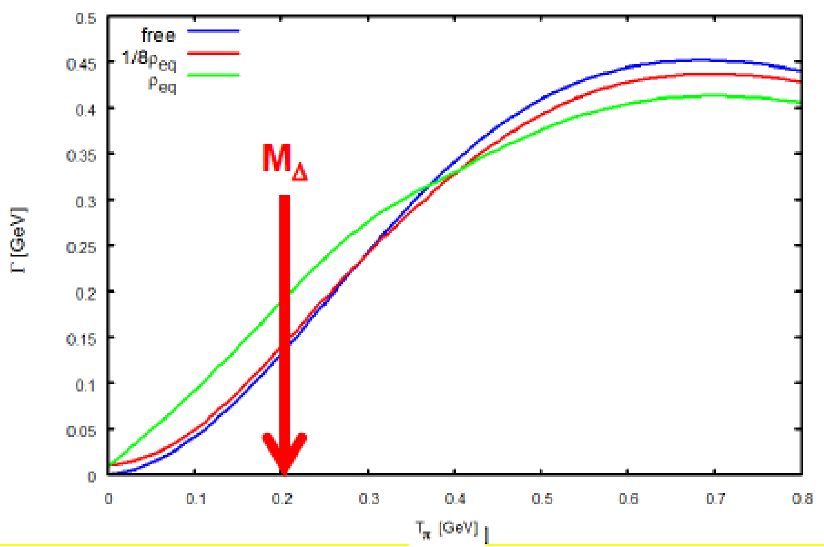
[Symmetry potential of Δ(1232) resonance and its effects on the $\pi-/\pi+$ ratio in heavy-ion collisions near the pion-production threshold](), B.A. Li, *PRC* 92 (2015), 034603



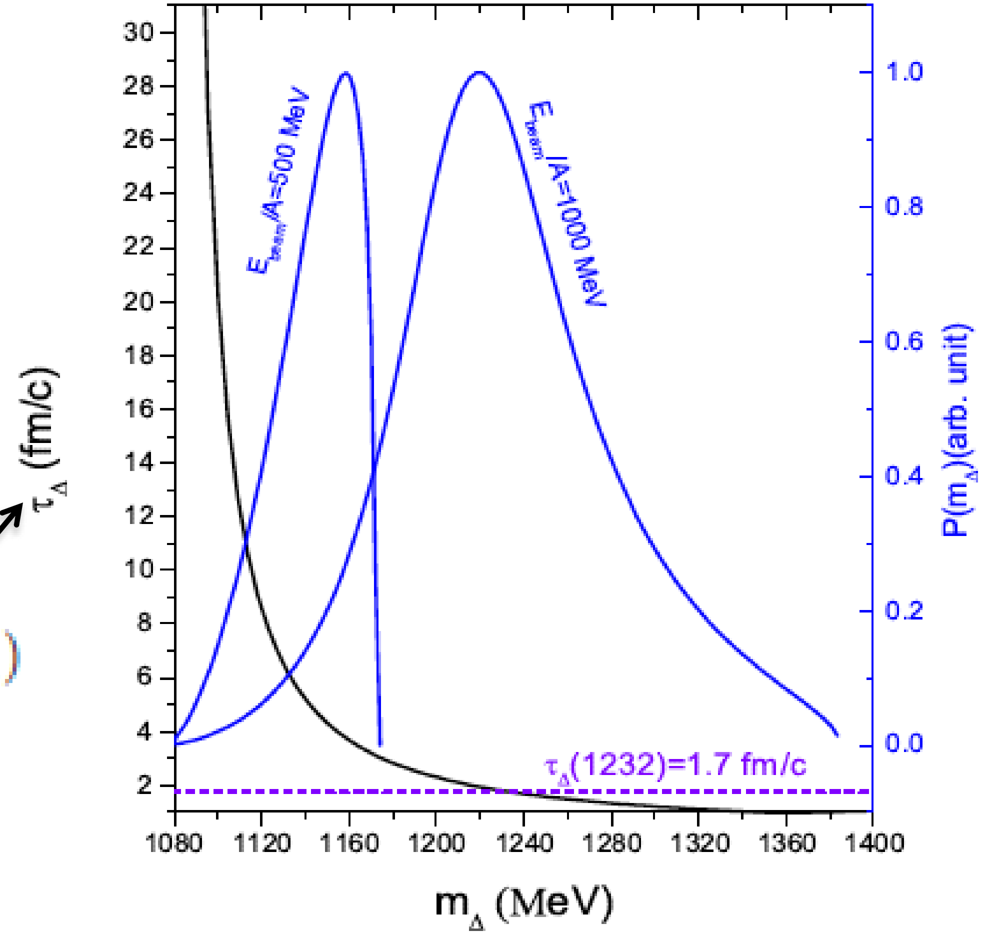


Delta isovector potential has NO effect on the high-energy spectrum!

In-medium Delta width, H. Lenske et al.



Delta lifetime and mass distribution in Au+Au collisions at b=0



$$\tau_\Delta = \hbar/\Gamma(m_\Delta)$$

WHY?

High-mass Delta produced in energetic collisions dies too quickly to feel any mean-field effect!

Recent Review:

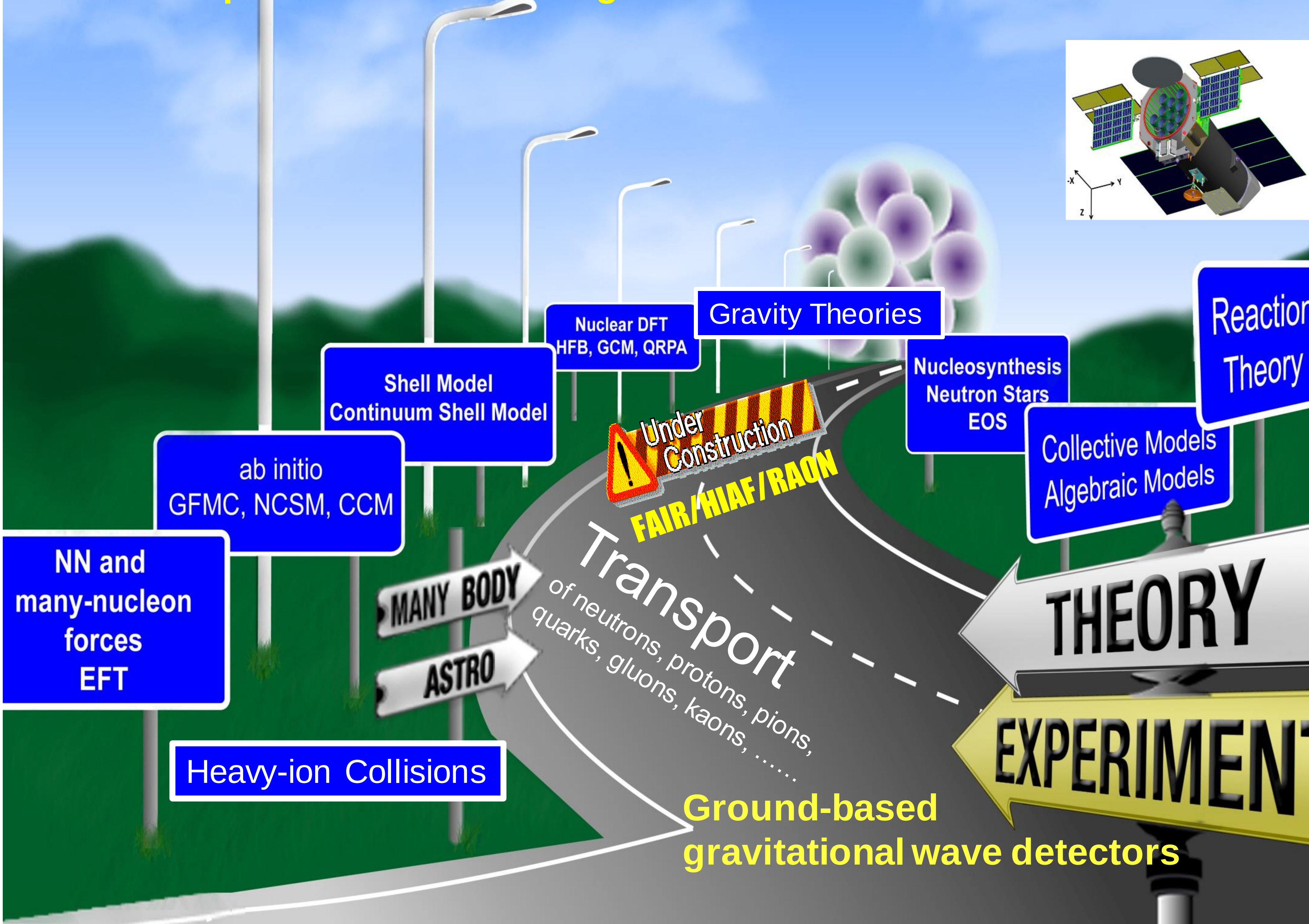
Transport model comparison studies of intermediate-energy heavy-ion collisions
TMEP Collaboration, Hermann Wolter et al. *Prog. Part. Nucl. Phys.* 125 (2022) 103962

White Papers for 2023 US Nuclear Physics Long Range Plan:

Long Range Plan: Dense matter theory for heavy-ion collisions and neutron stars
Alessandro Lovato et al., 2211.02224 [nucl-th]

Dense Nuclear Matter Equation of State from Heavy-Ion Collisions
Agnieszka Sorensen et al., 2301.13253 [nucl-th]

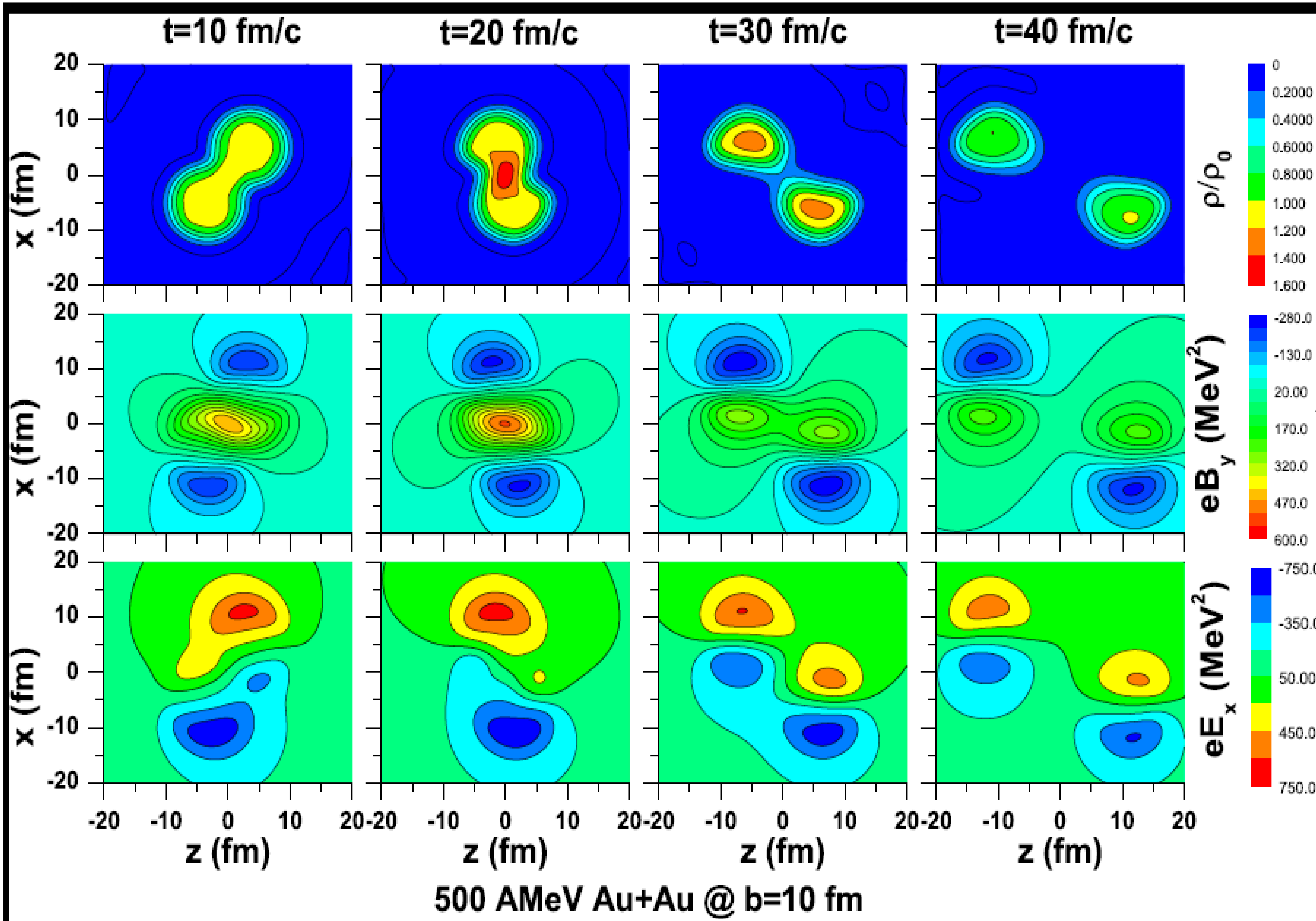
A road map towards determining the EOS of dense neutron-rich matter



Formation of neutron-star matter in terrestrial nuclear laboratories

Li Ou and Bao-An Li, Physical Review C84, 064605 (2011)

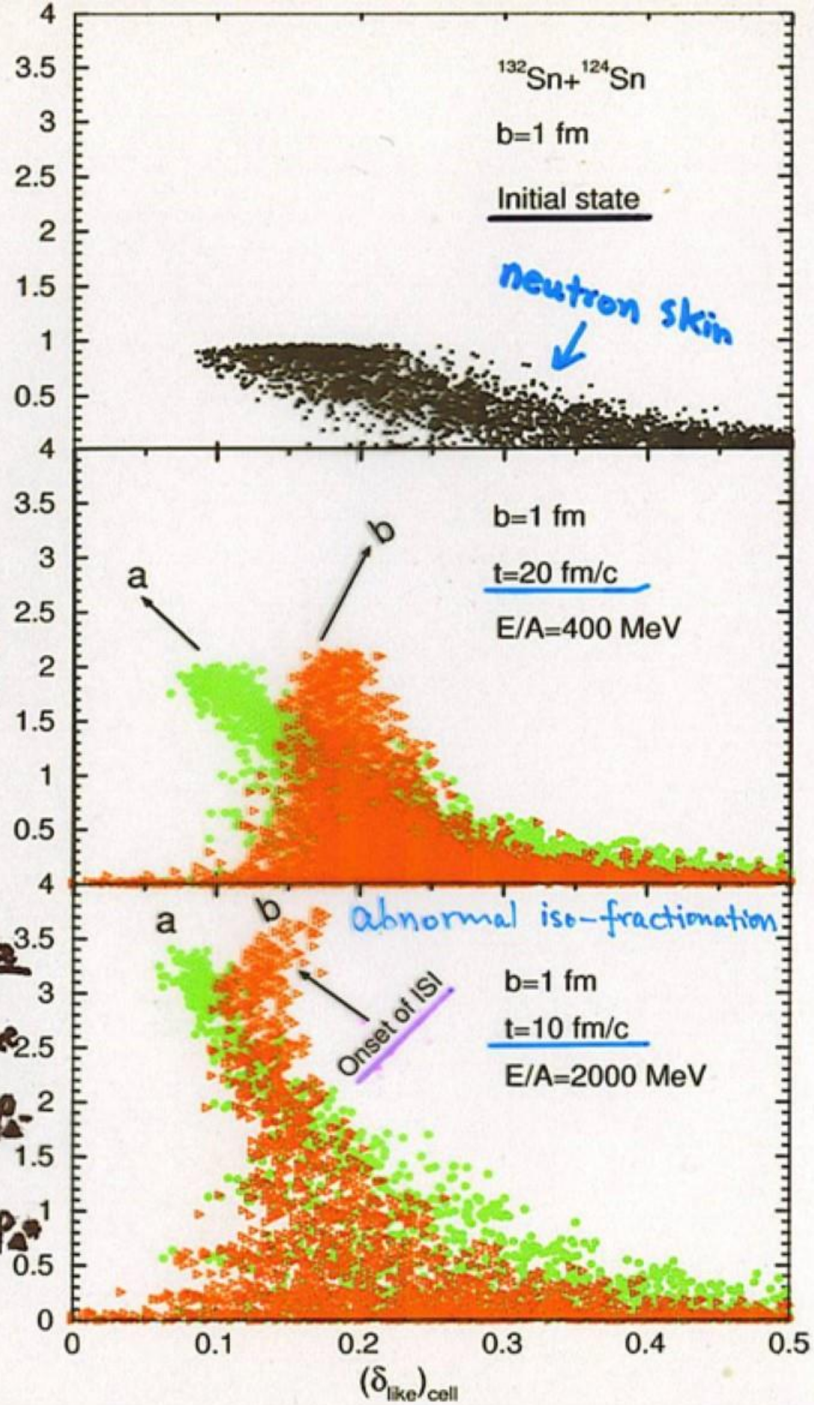
Highly-magnetized high-density matter is formed in heavy-ion reactions



$*1.44\ 10^{13}$ G

Calculations are done on a density matrix with cells of 1 fm^3 in size.

• each point represents one cell

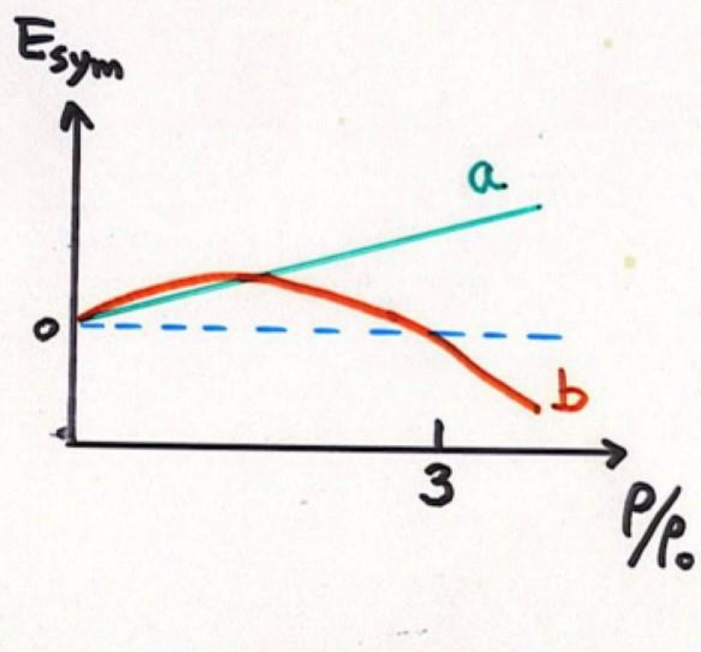


$$\delta_{Like} \equiv \frac{(\rho_n)_{Like} - (\rho_p)_{Like}}{(\rho_n)_{Like} + (\rho_p)_{Like}}$$

$$(\rho_n)_{Like} = \rho_n + \frac{2}{3}\rho_{\Delta^0} + \frac{1}{3}\rho_{\Delta^+} + \rho_{\Delta^-}$$

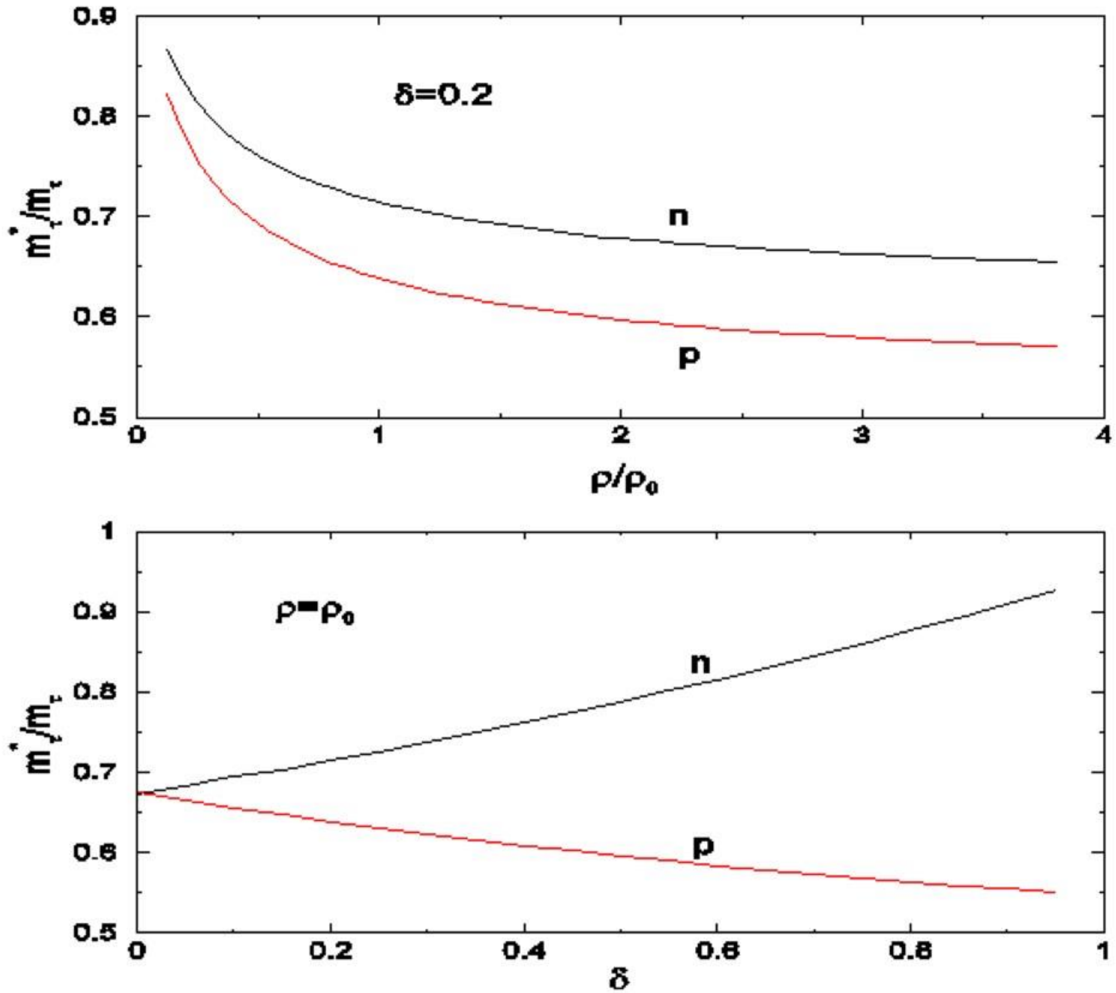
$$(\rho_p)_{Like} = \rho_p + \frac{2}{3}\rho_{\Delta^+} + \frac{1}{3}\rho_{\Delta^0} + \rho_{\Delta^{++}}$$

Density – isospin asymmetry correlation at the instants of maximum compression



Neutron-proton effective k-mass splitting
in neutron-rich matter at zero temperature

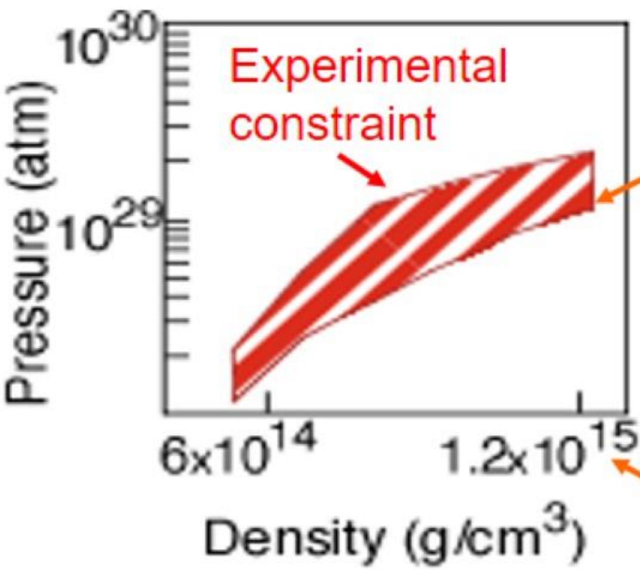
$$\frac{m_\tau^*}{m_\tau} = [1 + \frac{m_\tau}{p}\frac{\partial U}{\partial p}]^{-1}_{p_F^\tau}$$



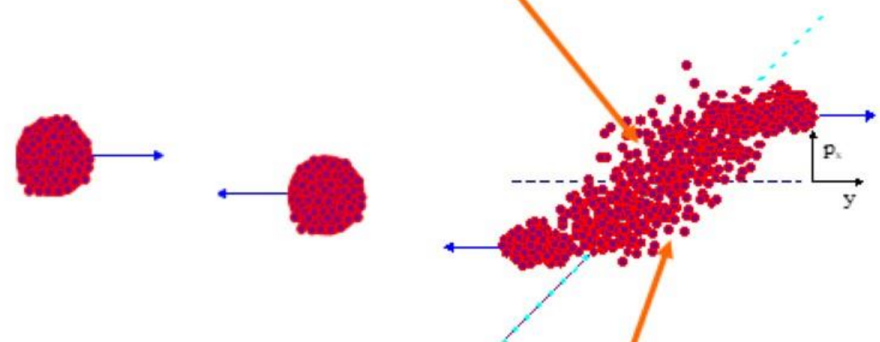
With the modified Gogny effective interaction

Equation of State of *symmetric* nuclear matter is relatively well determined
after 30 years of hard work of many people in the nuclear physics community

P. Danielewicz, R. Lacey and W.G. Lynch, *Science 298, 1592 (2002).*



The highest pressure recorded under laboratory controlled conditions in nucleus-nucleus collisions



Normal density of nuclear matter $\rho_0 \approx 2.7\times10^{14}\,\mathrm{g/cm^3}$

The density of water is 1 g/cm^3

High density nuclear matter
2 to 5ρ_0

1st Law of thermodynamics:
dU=Tds-PdV+μdN, E=U/N, ρ=N/V→ Pressure $P(\rho) = \rho^2 \left(\frac{\partial E}{\partial \rho}\right)_s$

EOS of cold matter: P(ρ) or P(ρE) for studying neutron stars

Development and applications of Transport Theory

1. Development of transport models

   Transport codes often implement extra physical assumptions and dynamical mechanisms which go beyond the equations used to motivate their designs. These algorithms often undergo evolutions with time as we make progresses in our R&D efforts and also as our needs for including new processes arise. They may involve many phenomenological parameters which are not all well experimentally constrained yet because of the lack of the relevant experimental data, and some of them are exactly what we want to infer because they can not be measured directly.

2. Applications of transport models

   Widely used in astrophyics, plasma physics, semiconductor and nanostructures, particle and nuclear physics, nuclear stockpile stewardship.

Isospin-dependence of nucleon-nucleon cross sections in neutron-rich matter

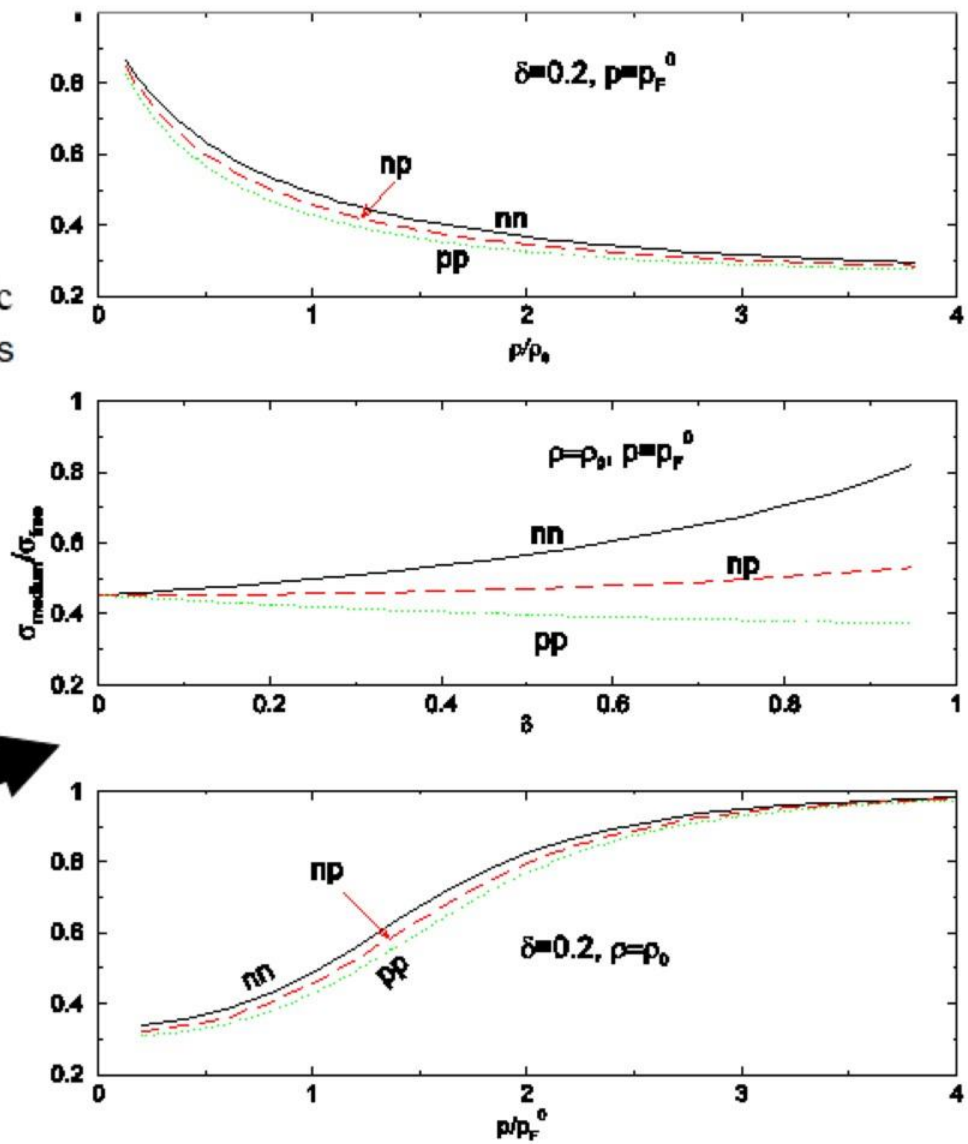
The effective mass scaling model:

$$\sigma_{medium} / \sigma_{free} \approx \left(\frac{\mu^*_{NN}}{\mu_{NN}} \right)^2$$

μ^*_{NN} is the reduced effective mass of the colliding nucleon pair NN

valid for $\rho \leq 2\rho_0$ and relative momenta ≤ 240 MeV/c according to Dirac-Brueckner-Hatree-Fock calculations F. Sammarruca and P. Krastev, nucl-th/0506081; Phys. Rev. C73, 014001 (2005) .

$\sigma_{medium} / \sigma_{free}$ in neutron-rich matter at zero temperature



Applications in symmetric nuclear matter:

J.W. Negele and K. Yazaki, PRL 47, 71 (1981)
V.R. Pandharipande and S.C. Pieper, PRC 45, 791 (1992)
M. Kohno et al., PRC 57, 3495 (1998)
D. Persram and C. Gale, PRC65, 064611 (2002).

Application in neutron-rich matter:
nn and pp xsections are splitted due to the neutron-proton effective mass slitting

Bao-An Li and Lie-Wen Chen, nucl-th/0508024, Phys. Rev. C72, 064611 (2005).