Equation of State of Neutron-Rich Matter from Heavy-Ion Reactions

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(1) Basics of transport models for heavy-ion reactions

(2) Probes of symmetric nuclear matter EOS

(3) Probes of nuclear symmetry energy

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Office of Science
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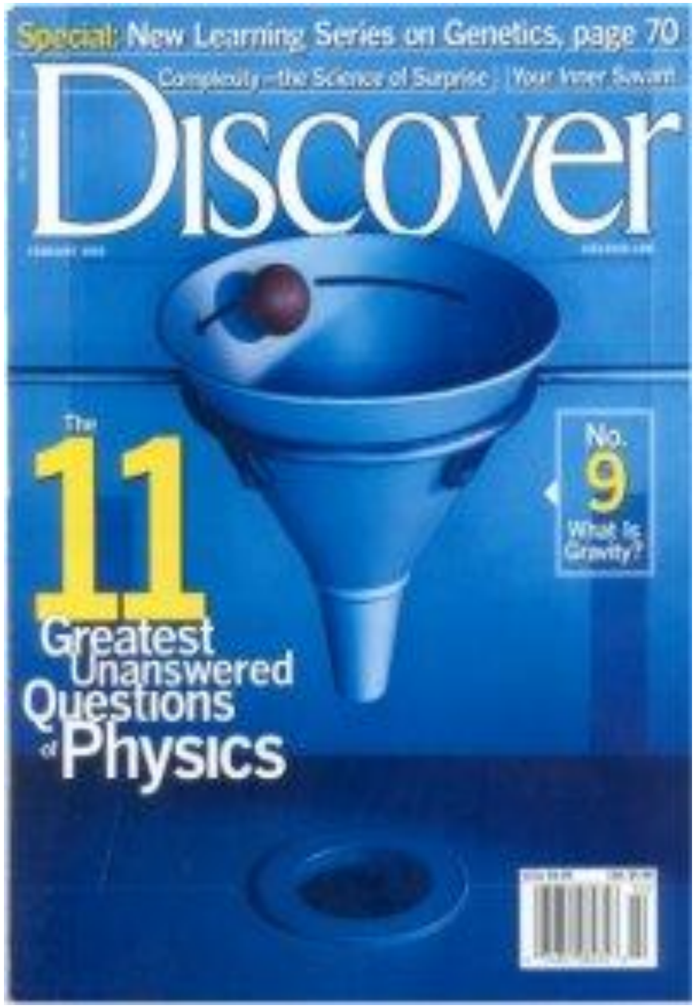
Connecting Quarks with the Cosmos: Eleven Science Questions for the New Century US National Research Council (2003)

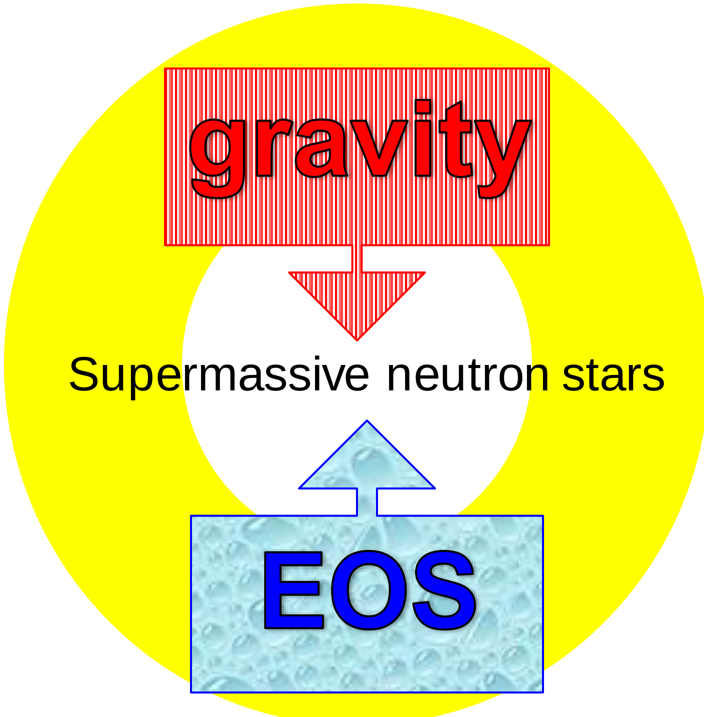
- What is the dark matter?
- What is the nature of the dark energy?
- How did the universe begin?
- What is gravity?
- Are there additional spacetime dimensions?
- What are the masses of the neutrinos, and how have they shaped the evolution of the universe?
- How do cosmic accelerators work and what are they accelerating?
- Are protons unstable?
- Are there new states of matter at exceedingly high density and temperature?
- How were the elements from iron to uranium made?
- Is a new theory of matter and light needed at the highest energies?



(1) Compact stars are natural laboratories to investigate some of these questions
(2) Nuclear EOS, especially the high-density symmetry energy is important in some cases

Gravity-EOS degeneracy in supermassive neutron stars

Strong-field gravity: Einstein's GR or Modified Gravity?



GR or [Modified Gravity]?

? **Variation of Total Action**

$$S=S_{gravity}+S_{matter}+\text{couplings}$$

Adjusting Nuclear EOS
or adding [Dark Matter]+[Dark Energy]+xyz?

Contents and stiffness of the EOS of super-dense matter

At high-densities, cold neutron-rich nucleonic matter, the most uncertain part of the EOS is the nuclear symmetry energy besides possible phase transitions

Independent information about nuclear EOS from terrestrial experiments is critical

Fundamental Microphysics Theories underlying each term in the EOS, what …, why …., where …how

Experimental and Observational Macrophysics underlying each observable and phenomenon, what …, why …., where …how

Empirical parameterizations

Transport model simulations of heavy-ion collisions, energy density functionals for nuclear structures, Bayesian inferences of EOS, properties of neutron stars, waveforms of gravitational waves, ….

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2$$

Assuming no hadron-quark phase transition

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{24}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4,$$

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho}{\rho_0}-1\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho}{\rho_0}-1\right)^2 + \frac{J_{\rm sym}}{162}\left(\frac{\rho}{\rho_0}-1\right)^3 + \mathcal{O}\left[\left(\frac{\rho}{\rho_0}-1\right)^4\right]$$

Near the saturation density ρ_0, they are Taylor expansions, appropriate for structure studies.
Just parameterizations when applied to heavy-ion collisions and the core of neutron stars

"Current" status of the restricted EOS parameter space:

Low density: $K_0 = 240 \pm 20$, $E_{\rm sym}(\rho_0) = 31.7 \pm 3.2$ and $L = 58.7 \pm 28.1$ MeV
High density: $-400 \leq K_{\rm sym} \leq 100$, $-200 \leq J_{\rm sym} \leq 800$, and $-800 \leq J_0 \leq 400$ MeV

Probes of the EOS of symmetric nuclear matter

- **Collective vibrations of nuclei probing the incompressibility K** around ρ_0
- **Particle production especially strange particles (e.g., Kaons) from heavy-ion collisions**
- **Collective flow of various particles & clusters probing K around ρ_0 and high-density EOS with the caveat that they are also sensitive to the poorly known viscosity (in-medium particle-particle scattering cross sections)**

MANY interesting issues!

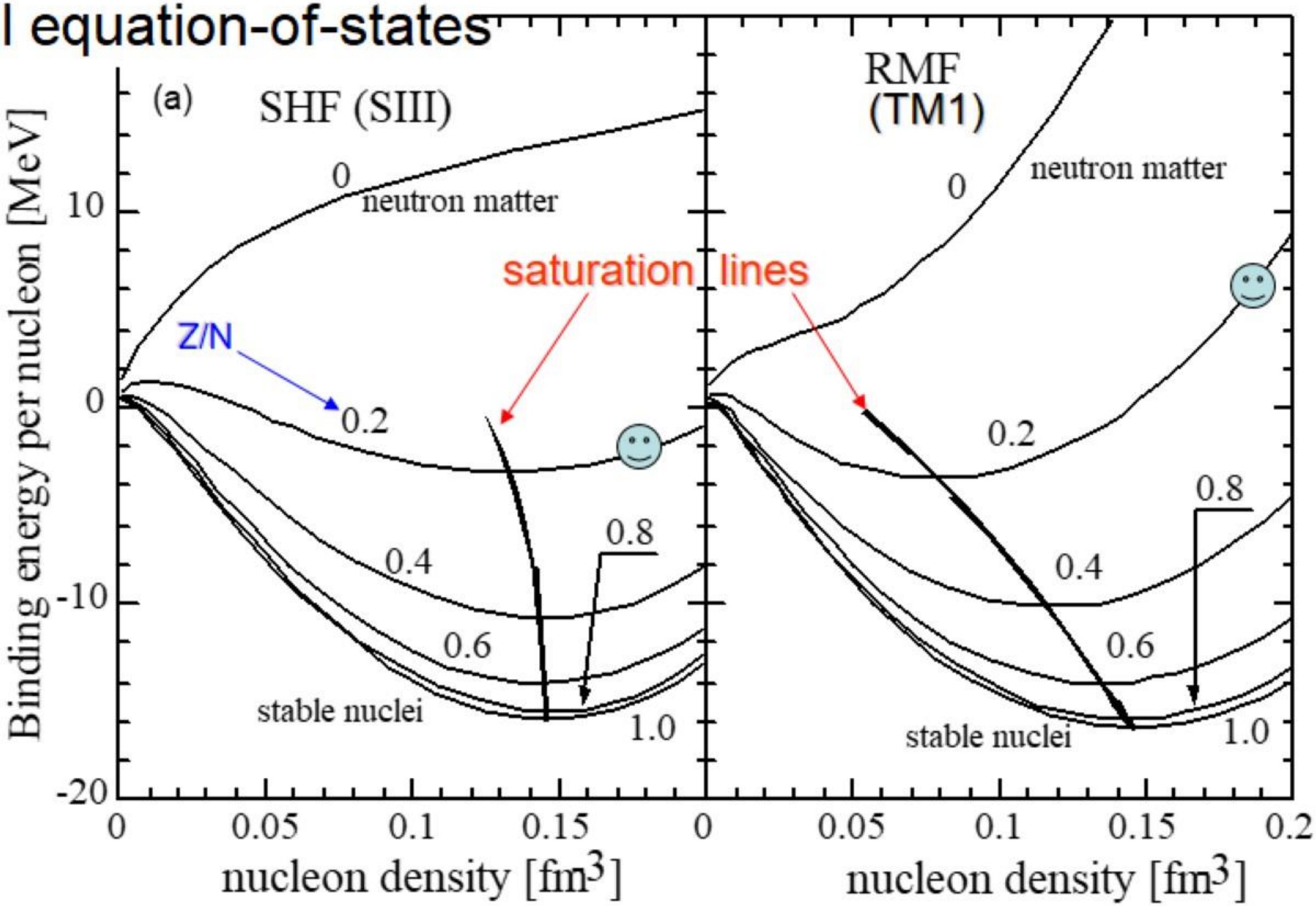
The K from different approaches may be different

Equation of State of Neutron-Rich Matter:

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2 + o(\delta^4)$$

Isospin asymmetry $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Two typical equation-of-states



K. Oyamatsu, I. Tanihata, Y. Sugahara, K. Sumiyoshi and H. Toki, NPA 634 (1998) 3.

Along the saturation line of infinite isospin-asymmetric matter

$$\rho_{\rm sat}(\delta) = \left[1 - \frac{3L}{K_0}\delta^2 + \left(\frac{3K_{\rm sym}L}{K_0^2} - \frac{3L_{\rm sym,4}}{K_0} - \frac{3J_0L^2}{2K_0^3}\right)\delta^4 + O(\delta^6)\right]\rho_0. \quad (22)$$

$$K_{\rm sat}(\delta) \equiv 9\rho_{\rm sat}^2 \frac{\partial^2 E(\rho,\delta)}{\partial\rho^2}\bigg|_{\rho=\rho_{\rm sat}} \approx K_0 + K_{\rm sat,2}\delta^2 + O(\delta^4)$$

$$K_{\rm sat,2} = K_{\rm sym} - 6L - J_0L/K_0.$$

L.W. Chen, B.J. Cai, C,M. Ko, B.A. Li,
C. Shen and J. Xu, PRC 80, 014322 (2009)

Derivations:

Step-1: Taylor expand the EOS in isospin asymmetry δ $e(\rho,\delta) = e_0(\rho) + e_2(\rho)\delta^2 + e_4(\rho)\delta^4 + \cdots + e_{2n}(\rho)\delta^{2n}.$

Step-2: Taylor expand all coefficient in density deviation z $z = (\rho - \rho_0)/\rho_0$

$$\begin{aligned} e_0(\rho) &= e_0(\rho_0) + a_{01}z + a_{02}z^2 + \cdots + a_{0n}z^n \\ e_2(\rho) &= e_2(\rho_0) + a_{11}z + a_{12}z^2 + \cdots + a_{1n}z^n \\ e_4(\rho) &= e_4(\rho_0) + a_{21}z + a_{22}z^2 + \cdots + a_{2n}z^n \\ &\vdots \\ e_{2n}(\rho) &= e_{2n}(\rho_0) + a_{n1}z + a_{n2}z^2 + \cdots + a_{nn}z^n. \end{aligned}$$

Step-3: Apply the saturation condition at a given δ

$$\frac{\partial e}{\partial z}\Big|_{z=z_{\rm sat}} = 0 \qquad \begin{aligned} &a_{01} + 2a_{02}z + 3a_{03}z^2 + \cdots + na_{0n}z^{n-1} + \\ &+(a_{11} + 2a_{12}z + 3a_{13}z^2 + \cdots + na_{1n}z^{n-1})\delta^2 + \\ &+(a_{21} + 2a_{22}z + 3a_{23}z^2 + \cdots + na_{2n}z^{n-1})\delta^4 + \\ &+\cdots+ \\ &+(a_{n1} + 2a_{n2}z + 3a_{n3}z^2 + \cdots + na_{nn}z^{n-1})\delta^{2n} = 0. \end{aligned}$$

Step-4: try the polynomial solution with $z_{\rm sat} = A_2\delta^2 + A_4\delta^4 + \cdots + A_{2k}\delta^{2k},$

Step-5: set all coefficients of δ^{2n} to zero

$$z_{\rm sat} = -\frac{3L}{K_0}\delta^2 + \left(\frac{3K_{\rm sym}L}{K_0^2} - \frac{3L_{\rm sym,4}}{K_0} - \frac{3J_0L^2}{2K_0^3}\right)\delta^4$$

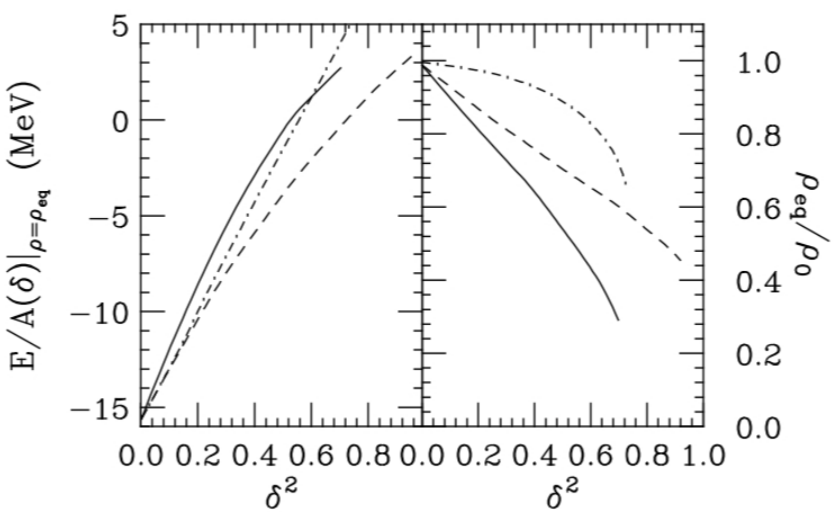
$$\begin{aligned} K_{\rm sat} = \; & K_0 + \left(K_{\rm sym} - 6L - \frac{J_0}{K_0}L\right)\delta^2 \\ & + \left(K_{\rm sym,4} - 6L_{\rm sym,4} - \frac{J_0L_{\rm sym,4}}{K_0} + \frac{9L^2}{K_0} - \frac{J_{\rm sym}L}{K_0} \right. \\ & \left. + \frac{I_0L^2}{2K_0^2} + \frac{J_0K_{\rm sym}L}{K_0^2} + \frac{3J_0L^2}{K_0^2} - \frac{J_0^2L^2}{2K_0^3}\right)\delta^4 \end{aligned}$$

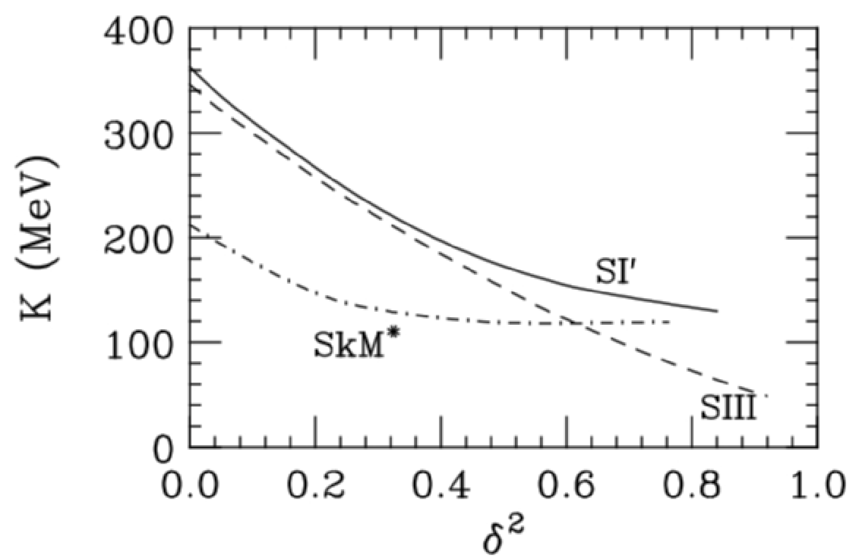
Examples: binding energy, saturation density and incompressibility

alone the saturation line of neutron-rich matter

- K. Kolehmainen et al., NPA 439, 535 (1985).
- J. Treiner et al., Ann. Phys. (N.Y.), 170, 406 (1986).
- D. Bandyopadhyay et al., NPA511, 1 (1990).

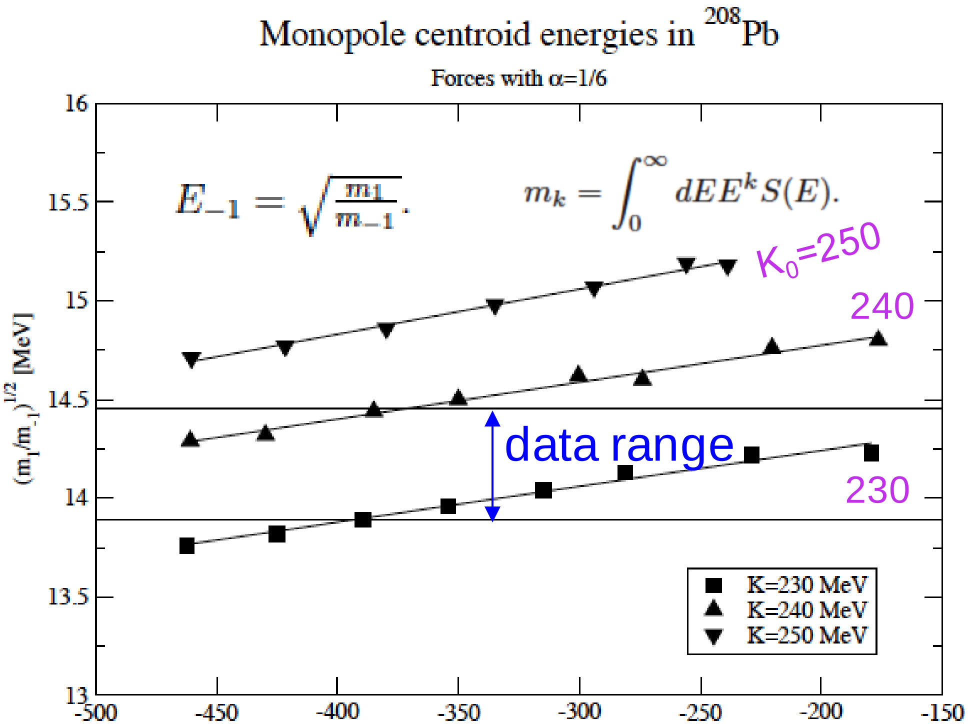
Predictions using Skyrme Hartree-Fock





The main source of the remaining uncertainty of K_0 (K_∞) from giant resonances: **Its correlation with the poorly known isospin dependent term K_τ in the incompressibility K_A of finite nuclei**

J. Colo, N. Van Giai, J. Meyer, K. Bennaceur, and P. Bonche, PRC 70, 024307 (2004).



the K_τ term $K_{\rm sym} - 6L - J_0 L/K_0$.

Along the saturation line of finite nuclei:

$$K_A \approx K_\infty(1 + cA^{-1/3}) + K_\tau \delta^2 + K_{\rm Cou} Z^2 A^{-4/3}$$

J.P. Blaziot, Phys. Rep. 64, 171 (1980)

$$\rho_{\rm eq} \approx \rho_0 + \frac{3\rho_0}{K_0}\left(1 - LI^2 + 2a_S A^{-1/3} - a_C \frac{Z^2}{A^{4/3}}\right)$$

X. Roca-Maza and N. Paar, PPNP 101, 96 (2018)

I=δ

Isoscalar Excitation Modes of Nuclear Resonance

Lecture by a pioneer at NUSYS23

Prof. Mohsen Harakeh

K_0=240±20 MeV

$$E_{ISGMR} \approx \sqrt{\frac{K_A}{m < r^2 >}}$$

$$E_{ISGDR} \approx \sqrt{\frac{3}{7}\frac{K_A + (27/25)\varepsilon_F}{m < r^2 >}}$$

$$K_A = K_{vol} + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}}$$

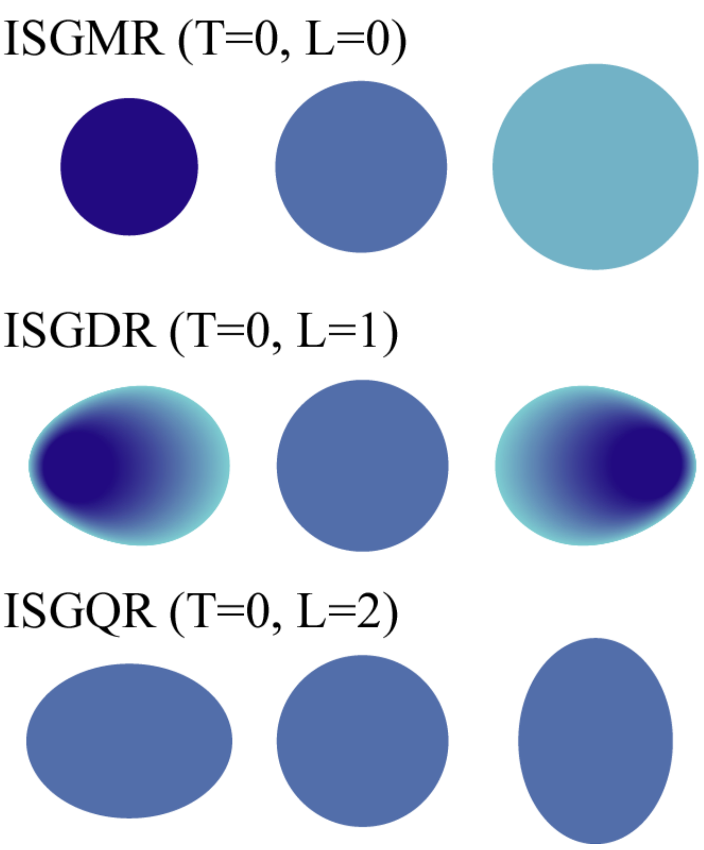
Isospin dependence of incompressibility

$$K_\tau = K_{\rm sym} - 6L(\rho_0) - \frac{J_0 L(\rho_0)}{K_0}$$

$$K_\tau = -550 \pm 100 \text{ MeV}$$

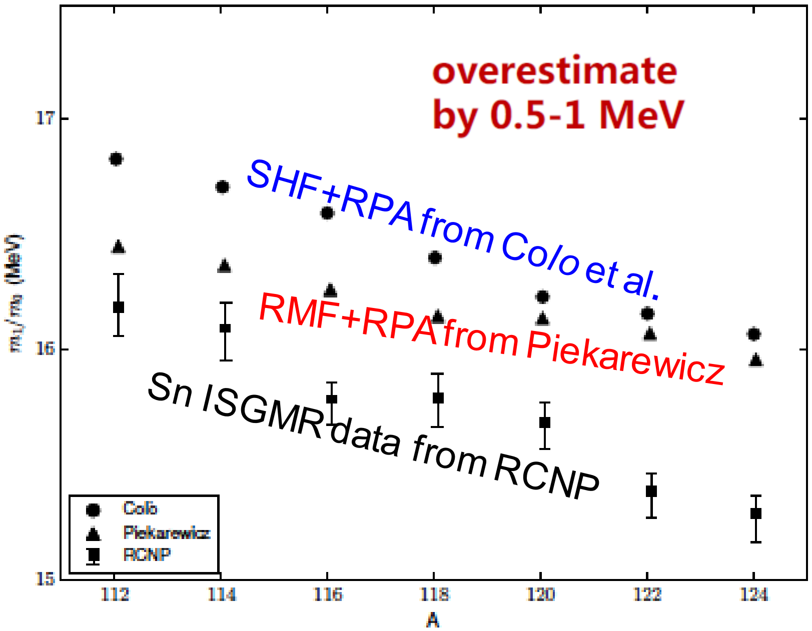
Nothing conclusive about K_{sym}

Isoscalar Giant Resonances:



G. Colò, U. Garg, H. Sagawa, Eur. Phys. J. A 50, 26 (2014)

The soft-tin ``puzzle”: both relativistic and non-relativistic models that well describe the ISGMR of ^{208}Pb can NOT reproduce the data of Sn isotopes



Two relatively recent reviews:

U. Garg and G. Colo, PPNP 101, 55 (2018)

X. Roca-Maza and N. Paar, PPNP 101, 96 (2018)

Fiducial value of K_0 since 1980: 220-260 MeV

J. P. Blaizot, Phys. Rep. 64, 171 (1980).

My possibly biased and offending (I am very sorry!) observations:

The community “consensus”: Using microscopic models to first describe well all data of finite nuclei before using them to calculate the incompressibility K_0 (K_∞) of infinite matter

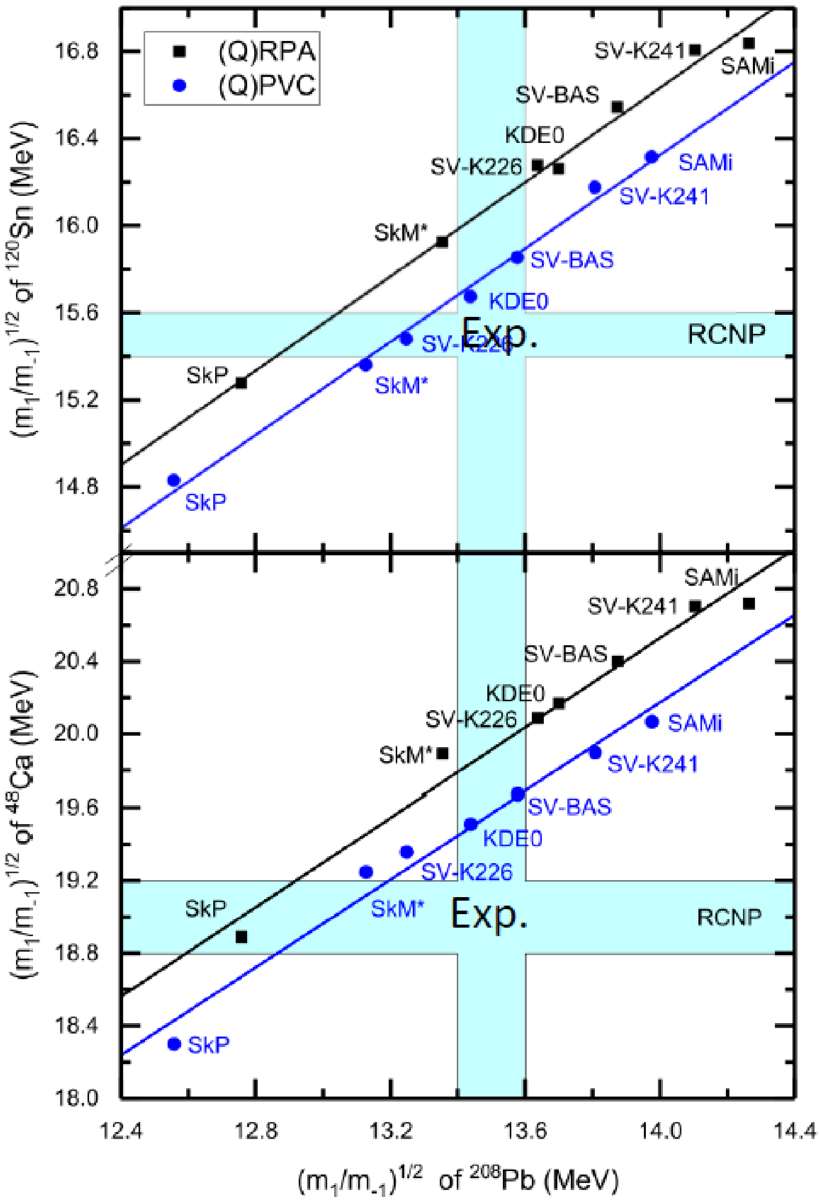
The reality: such models do NOT exist, none of the available models can describe all data. Moreover, not all data for the same nuclei from different labs are consistent

The status quo: the K_0 (K_∞) range of 220-260 MeV has been the same for about 40 years

The challenge: the 30-40 MeV error bar for K_0 (K_∞) is too big for many purposes

The need: “Forget” about the “consensus” to break the status quo

The puzzle is solved! Yifei Niu



$$E_{\mathrm{ISGMR}} = a'\sqrt{K_\infty} + b'$$

Linear correlation of GMR energies between different nuclei

- QRPA => QPVC

Simultaneous description of Sn (or Ca) and Pb is much improved!

- Best descriptions:

SV-K226 $K_\infty = 226$ MeV
KDE0 $K_\infty = 229$ MeV
consistent with $K_\infty = 240 \pm 20$ MeV

Z.Z.Li, Y.F.Niu, G.Colo, arXiv:2211.01264 (02 Nov, 2022)

Differential analysis of incompressibility in neutron-rich nuclei

Bao-An Li and Wen-Jie Xie, Phys. Rev. C 104, 034610 (2021)

$$K_A \approx K_\infty(1 + cA^{-1/3}) + K_\tau \delta^2 + K_{\rm Cou} Z^2 A^{-4/3}$$

Using K_A of any two nuclei (Z_1,A_1) and (Z_2,A_2)

$$K_\tau = \left[\frac{K_{A_1}}{S_1} - \frac{K_{A_2}}{S_2} - K_{\rm Cou}\left(\frac{Z_1^2 A_1^{-4/3}}{S_1} - \frac{Z_2^2 A_2^{-4/3}}{S_2}\right)\right] \Big/ \left(\frac{\delta_1^2}{S_1} - \frac{\delta_2^2}{S_2}\right),$$

$$K_\infty = \left[\frac{K_{A_1}}{\delta_1^2} - \frac{K_{A_2}}{\delta_2^2} - K_{\rm Cou}\left(\frac{Z_1^2 A_1^{-4/3}}{\delta_1^2} - \frac{Z_2^2 A_2^{-4/3}}{\delta_2^2}\right)\right] \Big/ \left(\frac{S_1}{\delta_1^2} - \frac{S_2}{\delta_2^2}\right),$$

$$S_i = 1 + cA_i^{-1/3}$$

$$c \approx -1.2 \pm 0.12 \quad K_{\rm Cou} \approx -5.2 \pm 0.7 \text{ MeV}$$

S. K. Patra, M. Centelles, X. Viñas, M. Del Estal, Phys. Rev. C **65**, 044304 (2002).
H. Sagawa et al., Phys. Rev. C **76**, 034327 (2007).

Errors: smallest with one n-rich and another n-poor

$$\sigma_{K_\tau} \approx \sqrt{\sigma^2_{K_{A_1}} + \sigma^2_{K_{A_2}}} \Big/ \left|\delta_1^2 - \delta_2^2\right|,$$

$$\sigma_{K_\infty} \approx \sqrt{(\delta_2^2 \cdot \sigma_{K_{A_1}})^2 + (\delta_1^2 \cdot \sigma_{K_{A_2}})^2} \Big/ \left|\delta_1^2 - \delta_2^2\right|$$

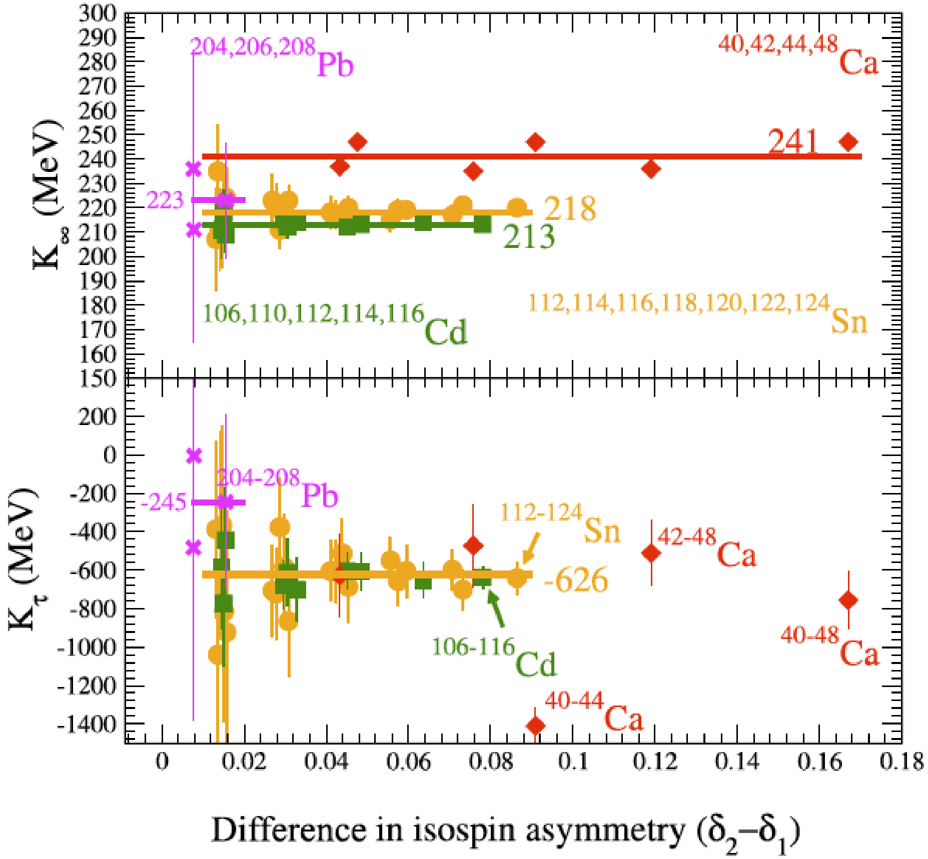
J. P. Blaizot, Phys. Rep. **64**, 171 (1980).

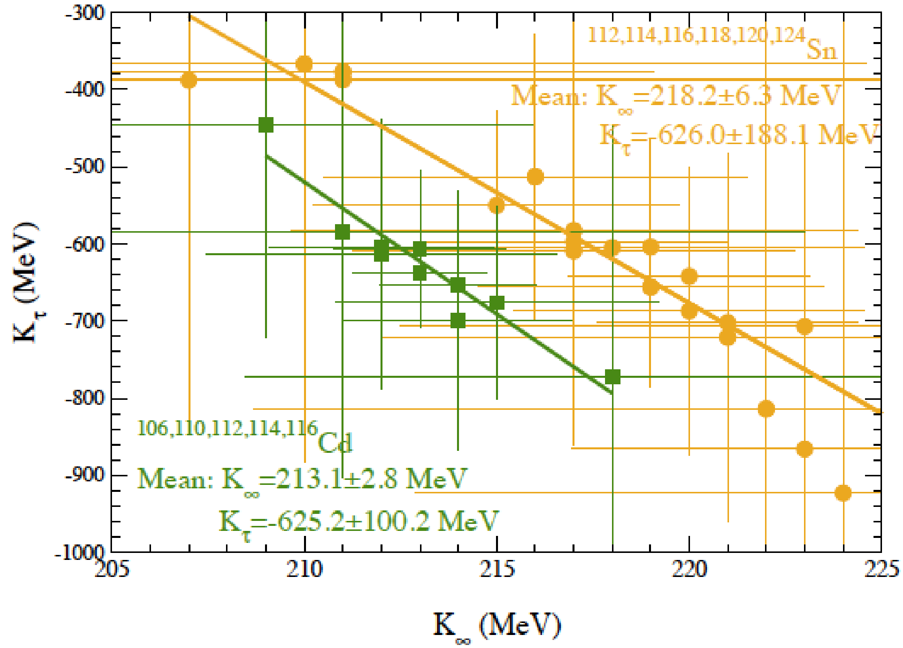
$$K_A = \left(\frac{E_{\rm ISGMR}}{\hbar c}\right)^2 Mc^2 < r^2 >$$

U. Garg and G. Colo, PPNP 101, 55 (2018)

Nucleus	K_A (MeV)	Reference
^{40}Ca	144.46 ± 0.33	[17]
^{42}Ca	139.00 ± 1.09	
^{44}Ca	137.36 ± 0.66	
^{48}Ca	131.90 ± 4.13	
^{106}Cd	127.84 ± 0.86	[14]
^{110}Cd	124.59 ± 0.86	
^{112}Cd	123.59 ± 0.77	
^{114}Cd	120.95 ± 1.24	
^{116}Cd	118.96 ± 0.86	
^{112}Sn	131.86 ± 1.53	[12, 13]
^{114}Sn	129.45 ± 1.64	
^{116}Sn	127.11 ± 1.53	All from
^{118}Sn	126.39 ± 1.54	RCNP
^{120}Sn	125.45 ± 1.63	Garg et al
^{122}Sn	121.33 ± 1.54	
^{124}Sn	120.17 ± 1.62	
^{204}Pb	136.93 ± 1.99	[15]
^{206}Pb	137.44 ± 1.99	
^{208}Pb	136.44 ± 1.99	

Bao-An Li and Wen-Jie Xie,
Phys. Rev. C 104, 034610 (2021)



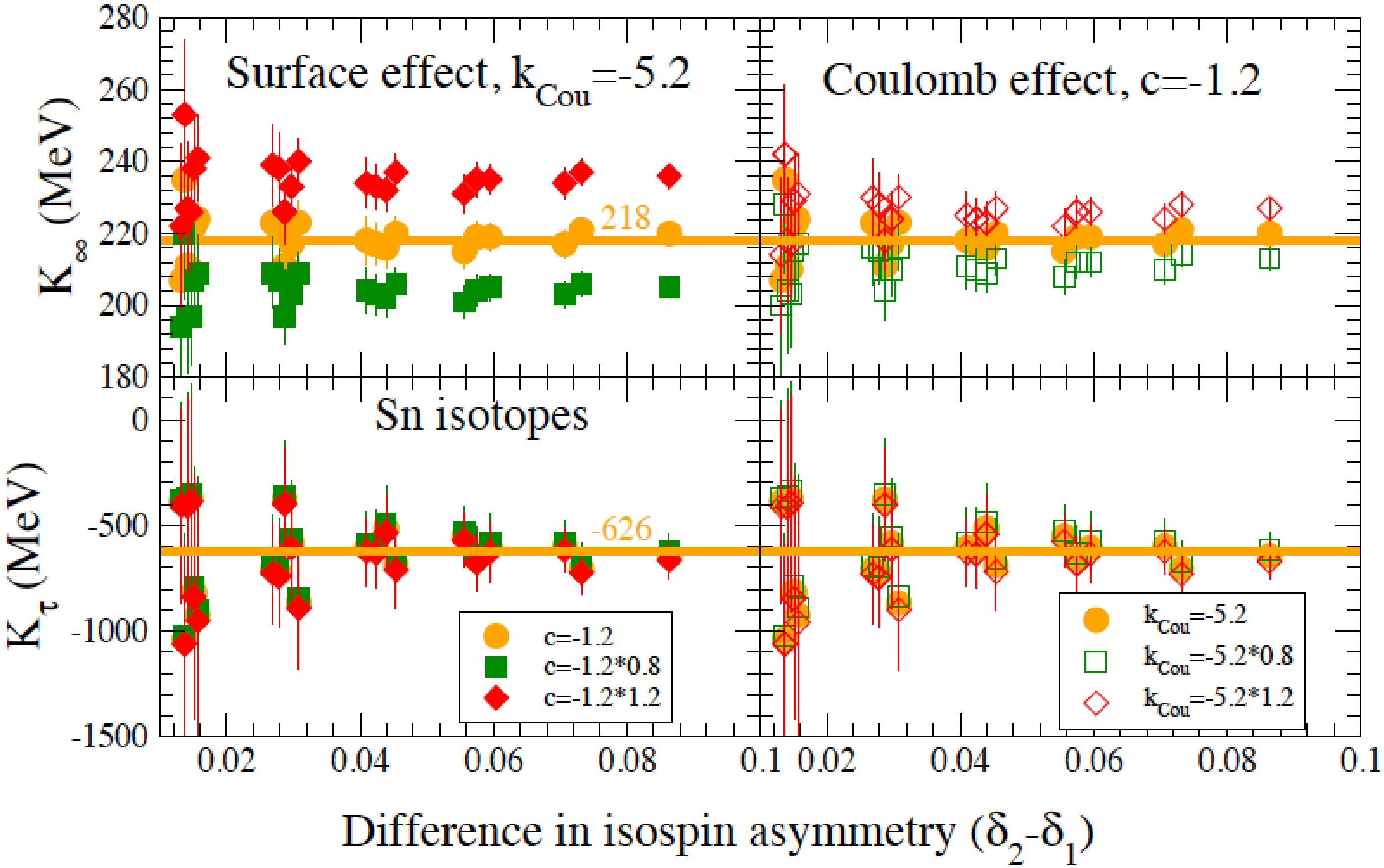


(1) Pb has only 3 points and all with large errors, needs more data for Pb isotopes
(2) Pb appears harder because its K_τ is much higher while its K_∞ is consistent with Sn
(3) Pairs with the largest difference in isospin asymmetry is the best (smallest error)
(4) Ca isotopes are probably too light for the liquid drop formula to work properly
(5) Sn looks normal than Pb compared to other nuclei

Soft Sn or Hard Pb puzzle?

J. Piekarewicz and M. Centelles, PRC 79, 054311 (2009)
E. Khan, PRC 80, 011307 and 057302 (2009)

Surface and Coulomb effects



(1) Varying the surface (c) and/or Coulomb (k_{cou}) by 40% has little effect on K_τ

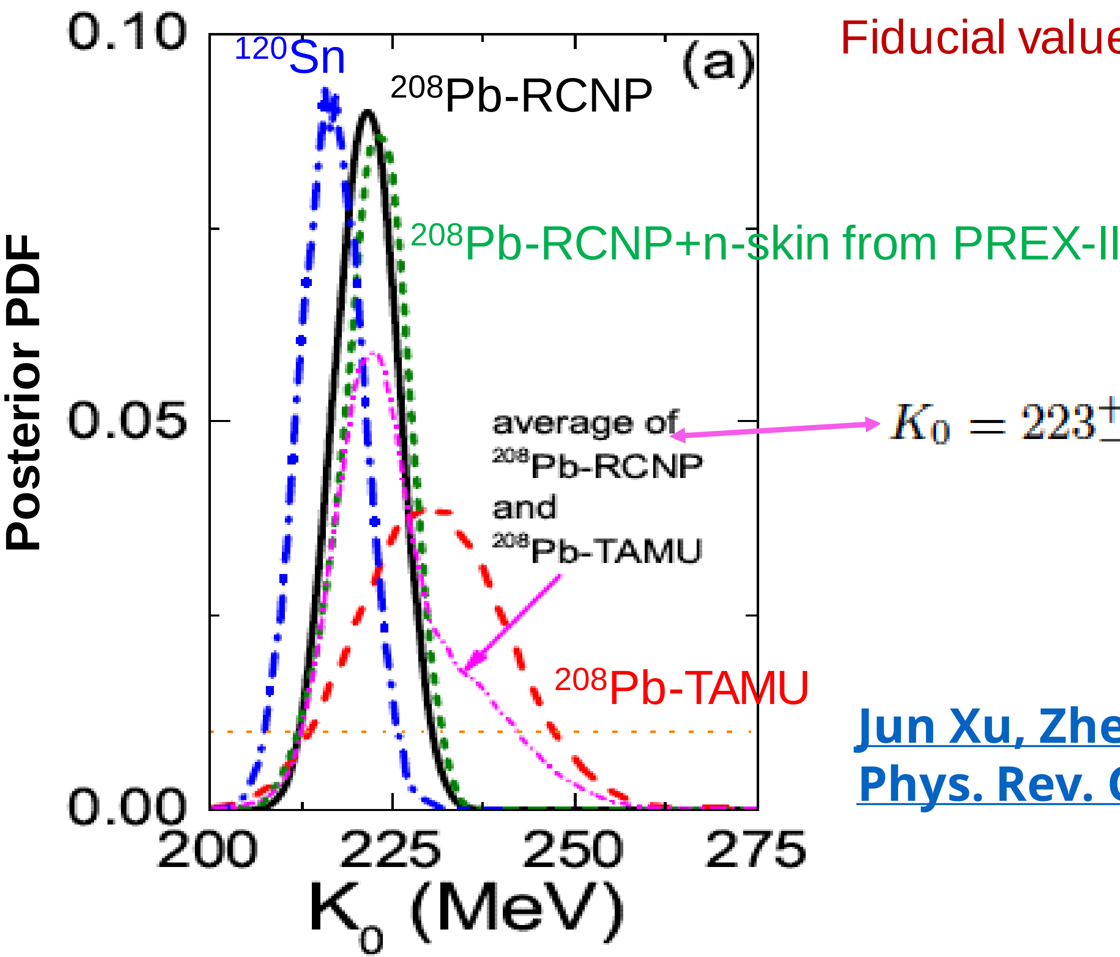
(2) Varying the surface (c) parameter by 40% leads to 210 < K_∞ < 240 MeV

Bao-An Li and Wen-Jie Xie, Phys. Rev. C 104, 034610 (2021)

Bayesian inference of K_0 from combined data of centroid energy and electrical Polarizability of IVGDR, n-skin, and centroid energy of ISGMR using SHF+RPA

G. Colò, L. Cao, N. Van Gia, and L. Capelli, Comput. Phys.Commun. 184, 142 (2013).

	E_{-1} (MeV)	α_D (fm^3)	Δr_{np} (fm)	E_{ISGMR} (MeV)	E_b (MeV)	R_c (fm)
^{208}Pb-TAMU	13.46 ± 0.10	19.6 ± 0.6	0.170 ± 0.023	14.17 ± 0.28	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{208}Pb-RCNP	13.46 ± 0.10	19.6 ± 0.6	0.170 ± 0.023	13.9 ± 0.1	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{208}Pb-RCNP-PREXII	13.46 ± 0.10	19.6 ± 0.6	0.283 ± 0.071	13.9 ± 0.1	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{120}Sn	15.38 ± 0.10	8.59 ± 0.37	0.150 ± 0.017	15.7 ± 0.1	$-8.504548 \pm 3\%$	$4.6543 \pm 3\%$

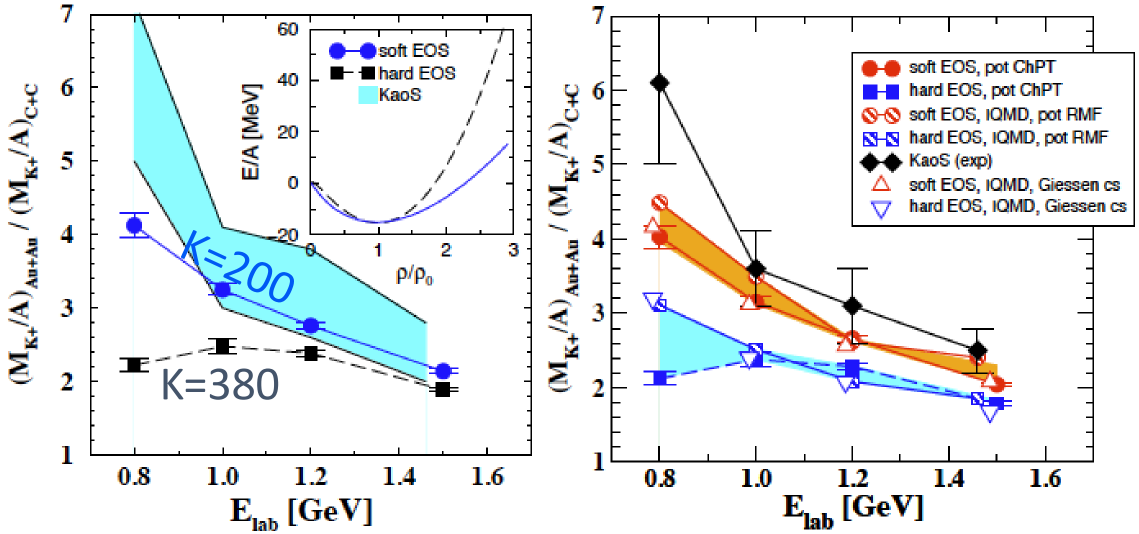


Fiducial value of K_0 since 1980: 220-260 MeV

$$K_0 = 223^{+7}_{-8} \text{ MeV at } 68\% \text{ confidence level}$$

Jun Xu, Zheng Zhang and Bao-An Li, Phys. Rev. C104, 054324 (2021)

Constraints on K from Kaon production in heavy-ion collisions



C. Fuchs et al., PRL 86 (2001) 1974

C. Hartnack et al., PRL 96 (2006) 012302

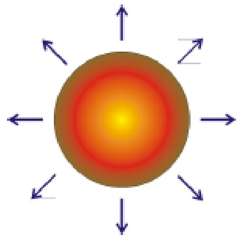
Indication: the incompressibility K should be less than 200 MeV

Azimuthal Anisotropy

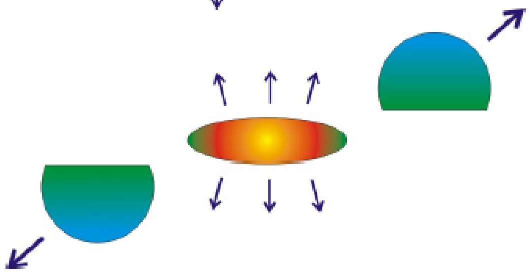
$$E\frac{dN^3}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1 + 2v_1\cos(\phi - \psi_R) + 2v_2\cos 2(\phi - \psi_R) + \ldots\right)$$

isotropic directed elliptic

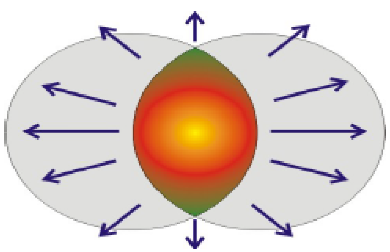
Radial flow
- Isotropic expansion of participant zone
- Measurable via slope parameter of spectra



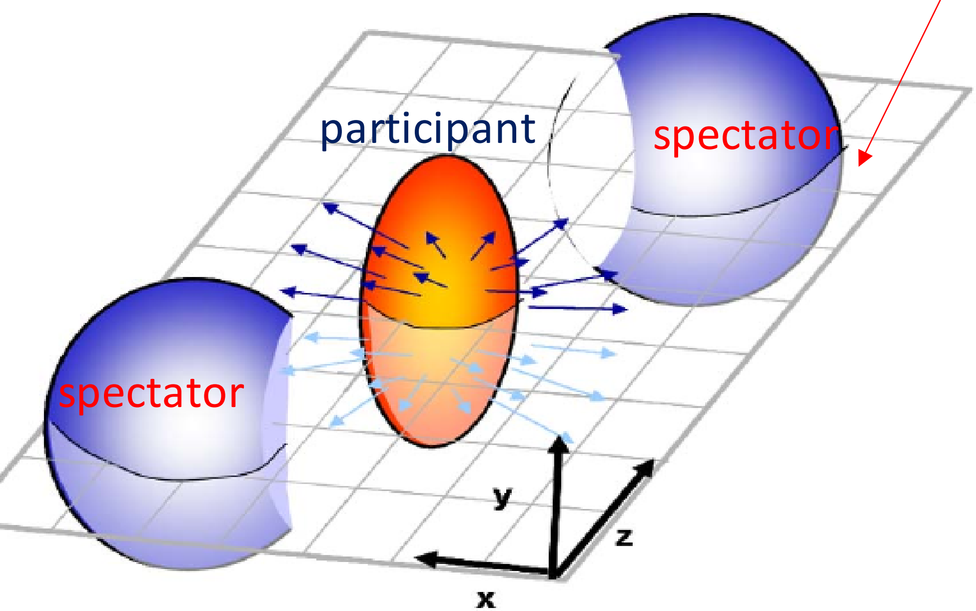
Directed flow (v_1)
- Spectators deflected from dense reaction zone
- Sensitive to pressure
- Strong sensitivity to EoS

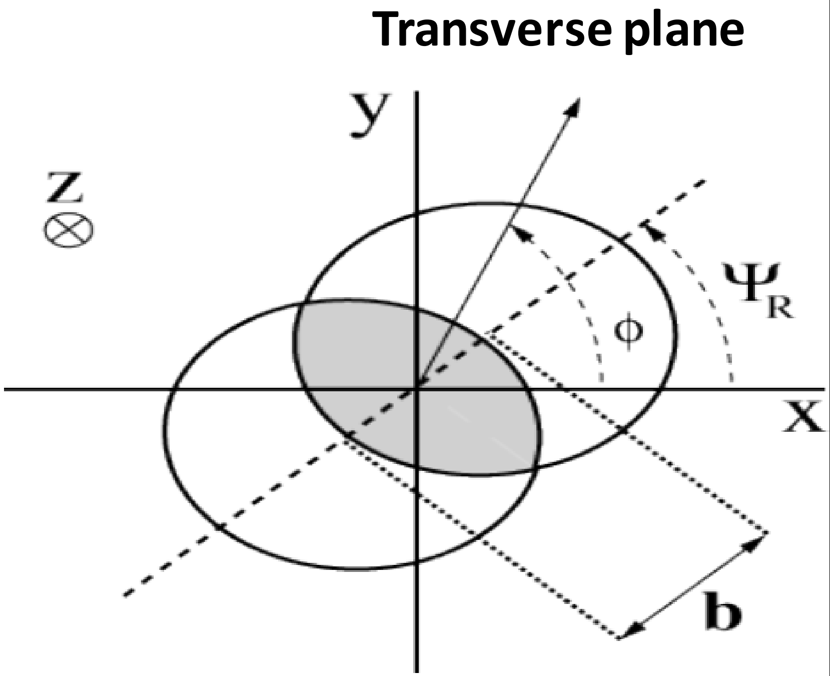


Elliptic flow (v_2)
- Asymmetry out- vs. in-plane emission
- Emission mostly during early phase
- Sensitive to EoS



Reaction Plane





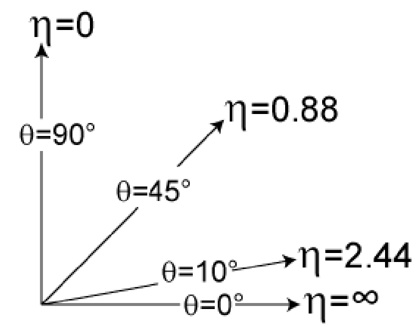
- Reaction plane provides natural coordinate system in non central collisions
- Experimentally the reaction plane Ψ_r is <u>unknown</u>
- Experimental estimater of the reaction plane is event plane
- Anisotropic flow ≡ azimuthal correlation with the reaction plane

Momentum **P=(p_x,p_y,p_z)**=(p_T,y)

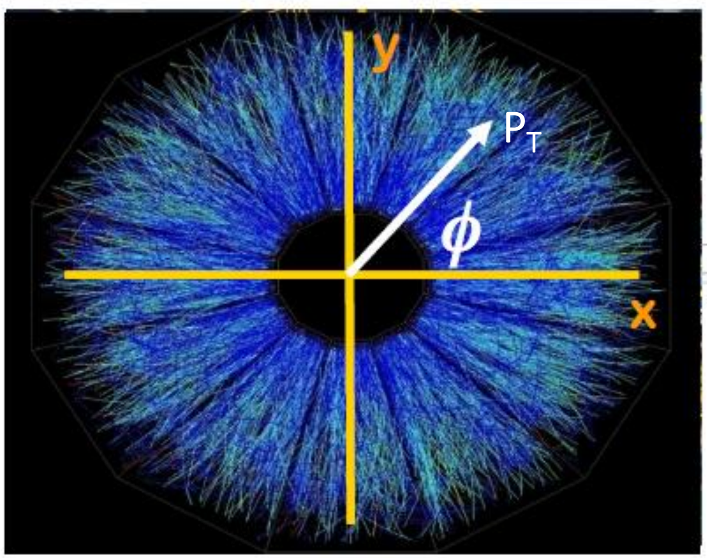
pseudo-rapidity η

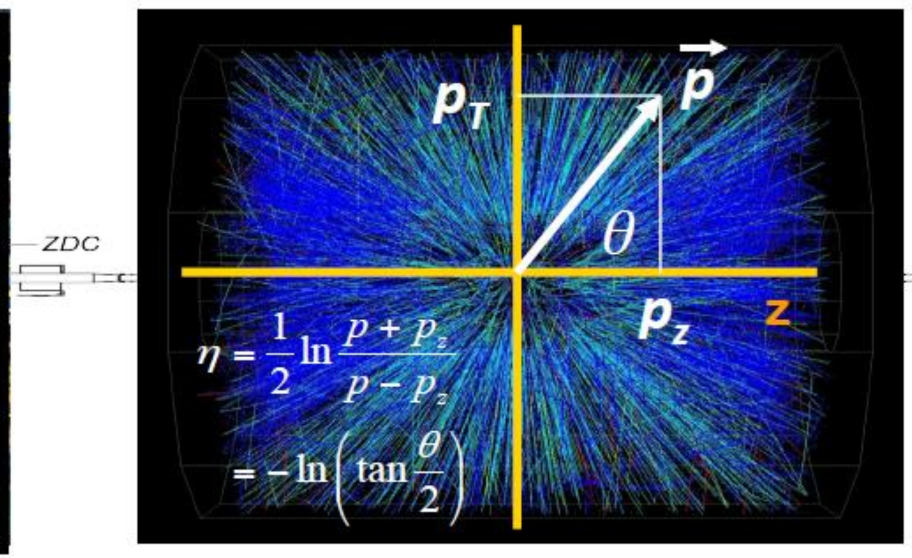
Rapidity y

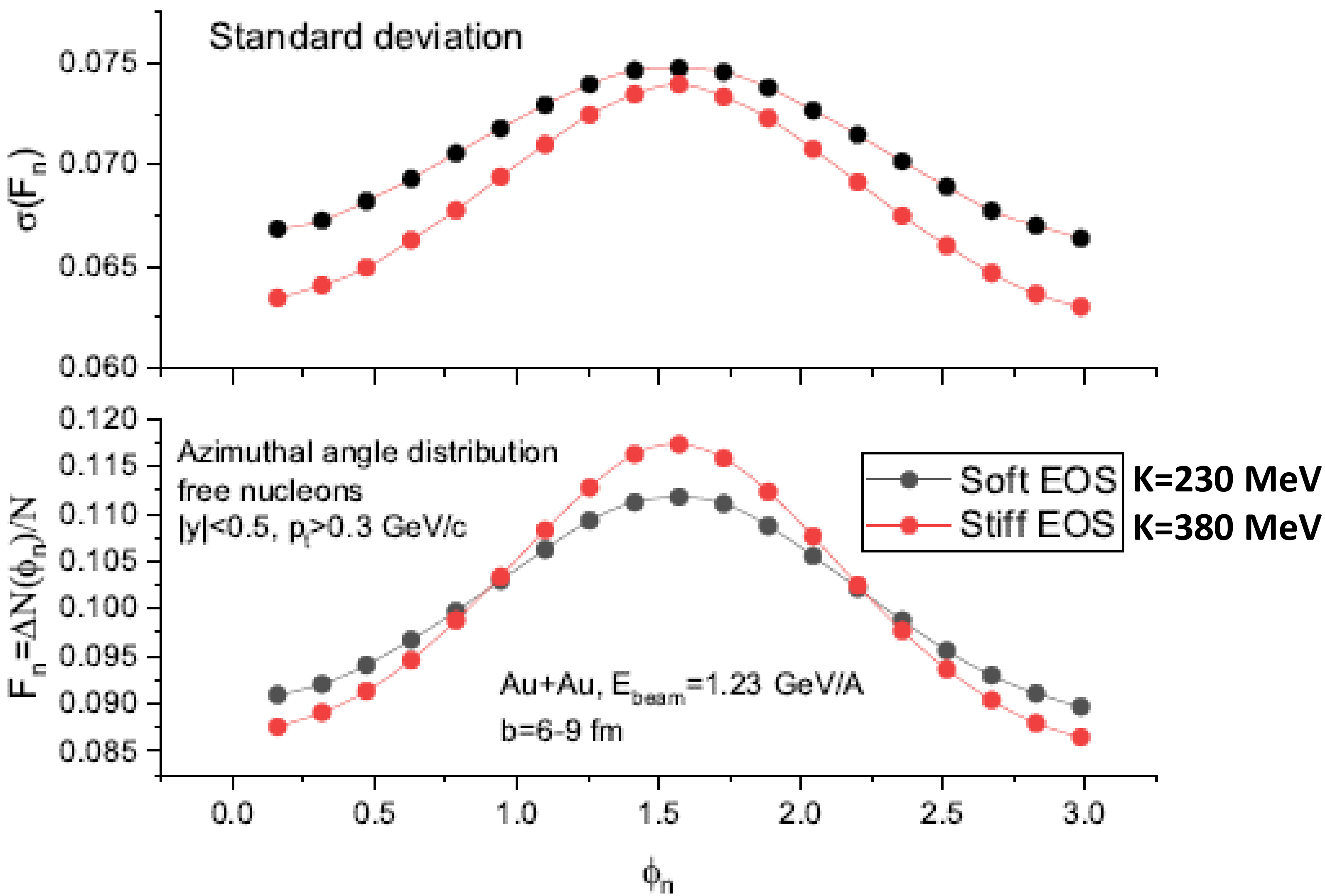
$$y = \frac{1}{2}\ln\left(\frac{E+p_z c}{E-p_z c}\right) \xrightarrow{\text{Highly relativistic}} y \simeq \boxed{-\ln\tan\frac{\theta}{2}}.$$



Transverse plane







IBUU transport model simulations

Bao-An Li and Jake Richter, Nucl. Phys. A 1034 (2023) 122640

LBL-18600
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

TRANSVERSE MOMENTUM ANALYSIS OF COLLECTIVE MOTION IN RELATIVISTIC NUCLEAR COLLISIONS

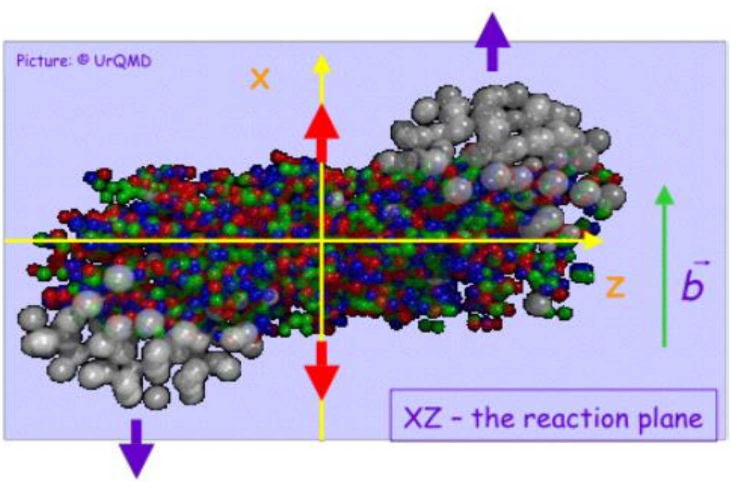
P. Danielewicz and G. Odyniec

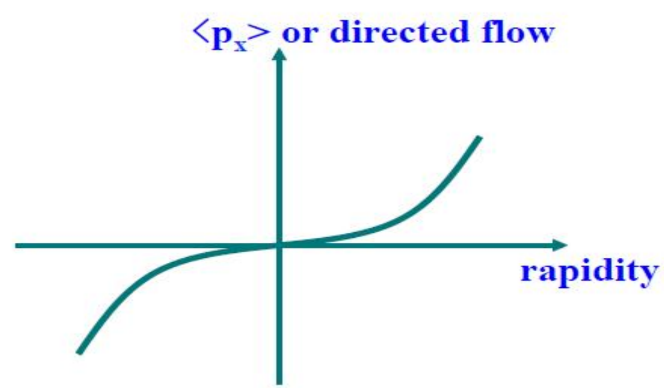
Phys. Lett. B 157 (1985) 146

Directed flow (v_1)

$$v_1(y,p_t) = < cos(\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}}{p_{it}}$$

Directed flow is quantified by the first harmonic (v_1)



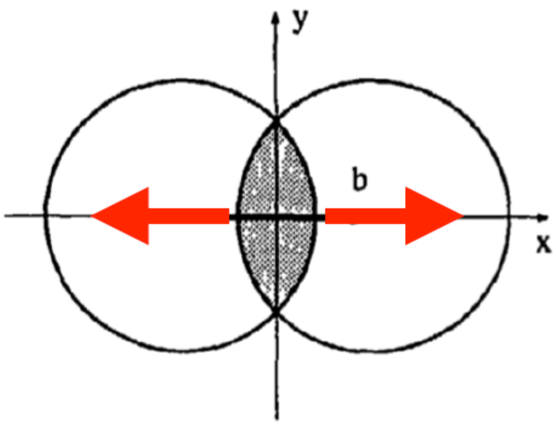


Azimuthal anisotropy in non-central collisions

Jean-Yves Ollitrault, IPhT Saclay, France

JYO, Phys. Rev. D 46, 229 (1992)

In hydrodynamics, fluid acceleration is proportional to pressure gradient: Larger acceleration along smaller dimension x. Azimuthal anizotropy is generated.



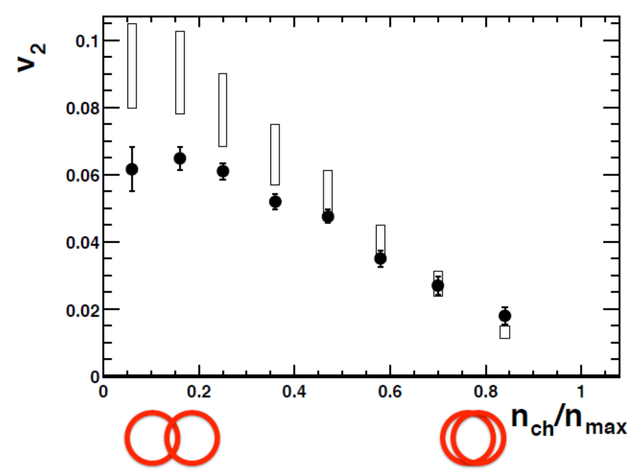
Transverse plane

Euler Eq. For flow velocity U in ideal fluid:
DU/Dt=-gradient of Pressure/density
+ external force

$$v_2(y,p_t) =< cos(2\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}^2 - p_{iy}^2}{p_{it}^2}$$

Elliptic flow at RHIC

STAR nucl-ex/0009011

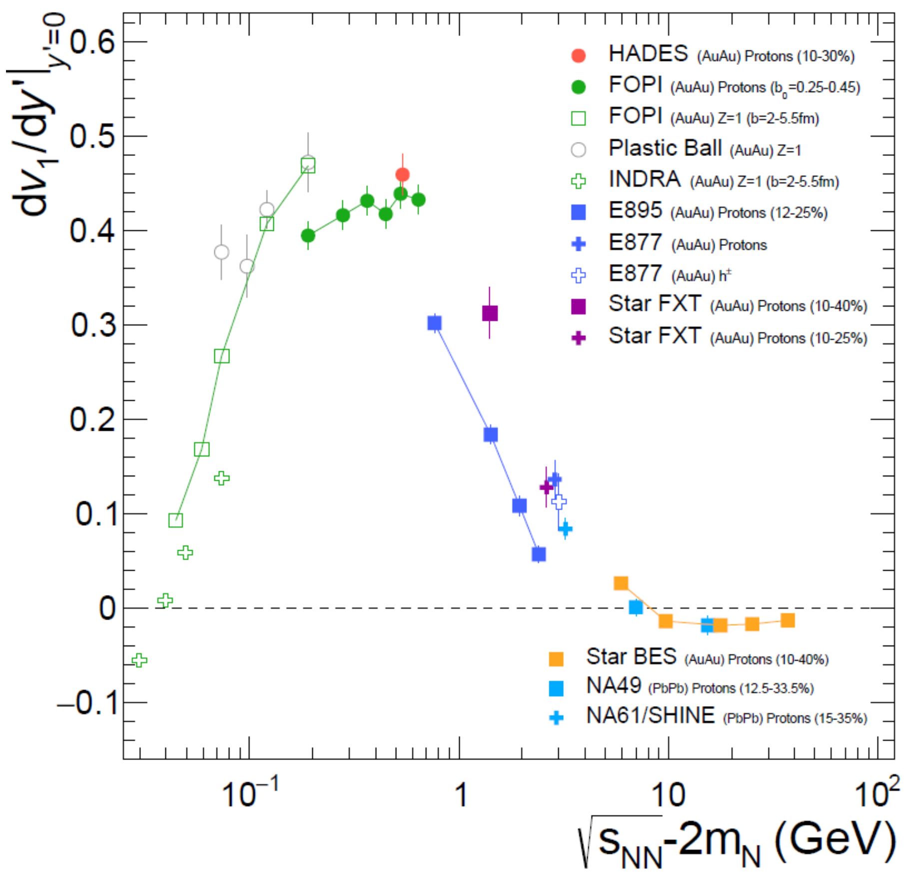


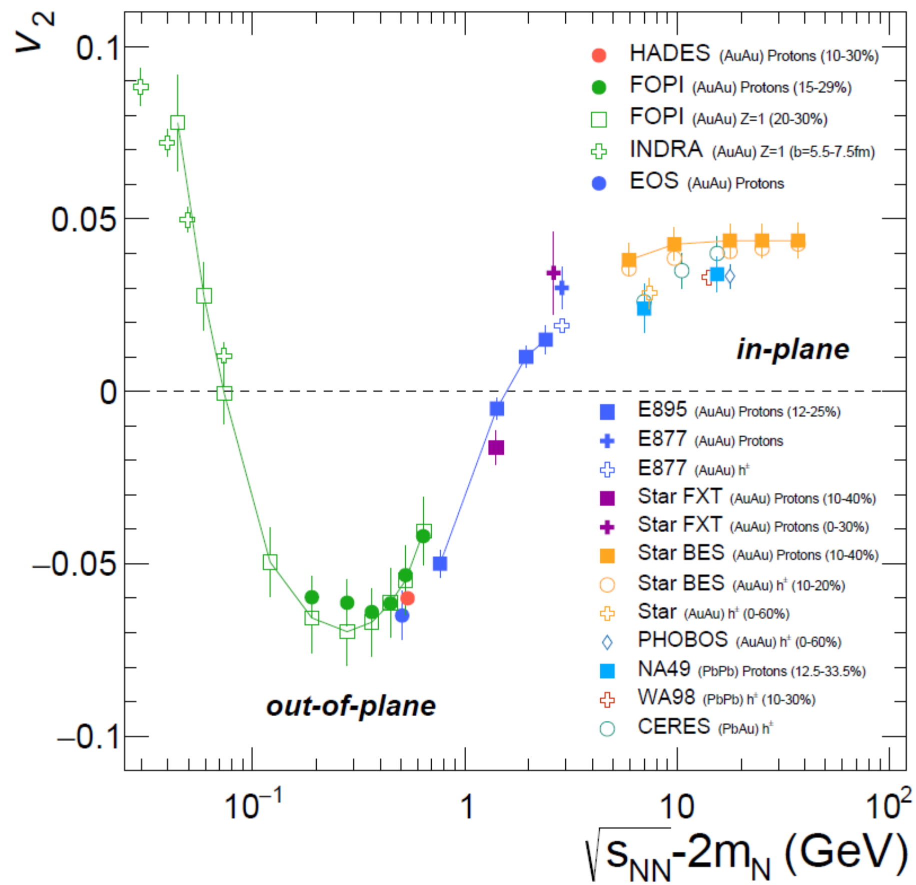
The observation of a large v_2, compatible with hydrodynamic predictions, came as a surprise, and soon established hydrodynamics as the only way of modeling the expansion

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n(\phi - \Psi_{\mathrm{PP}_n})\right]$$

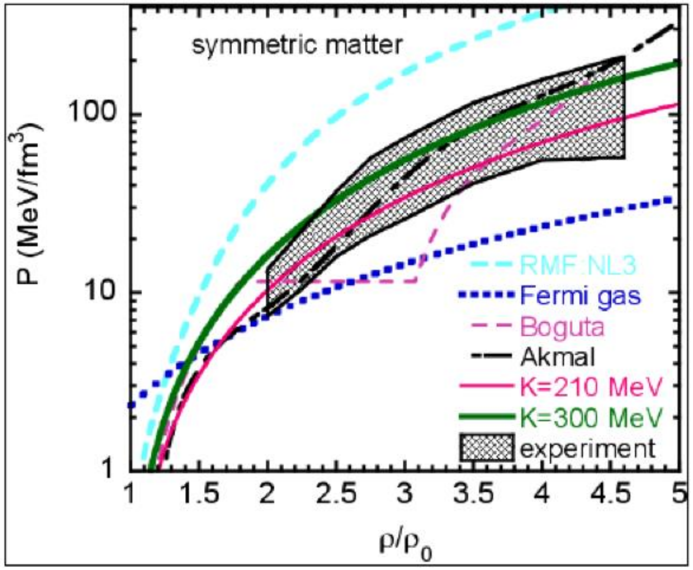
$$v_1(y,p_t) = < cos(\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}}{p_{it}} \qquad v_2(y,p_t) = < cos(2\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}^2 - p_{iy}^2}{p_{it}^2}$$

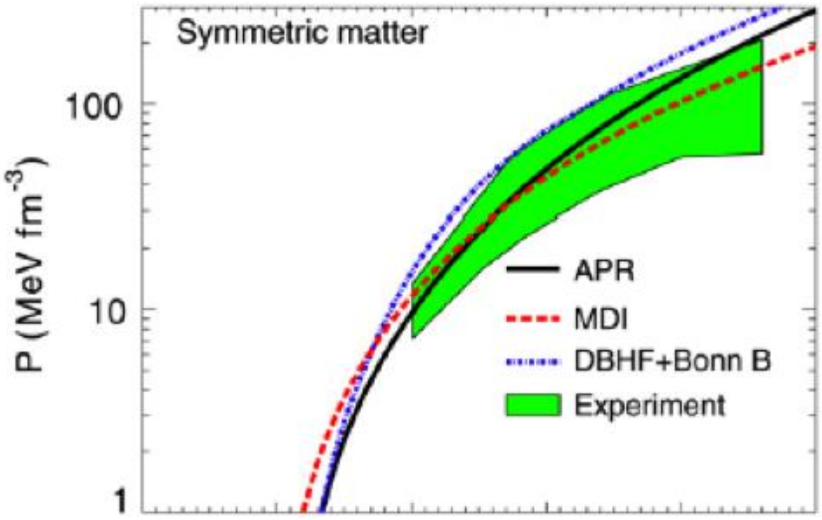
v_1 and v_2 are anti-correlated





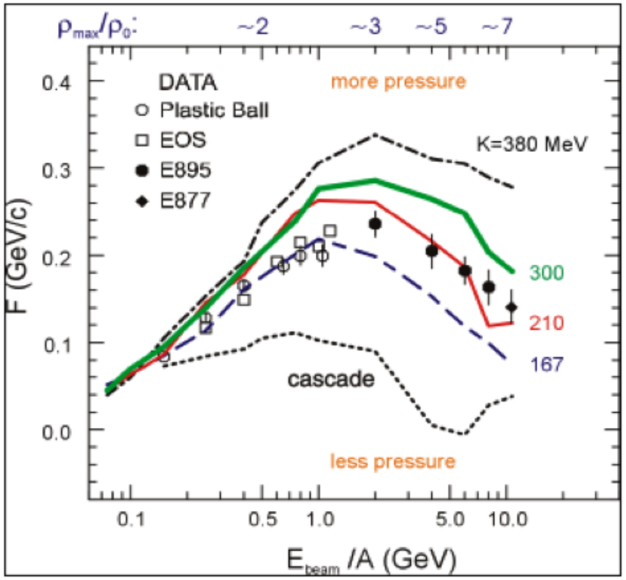
Danielewicz P. I., Lacey R and Lynch W G 2002 *Science* 298 1592

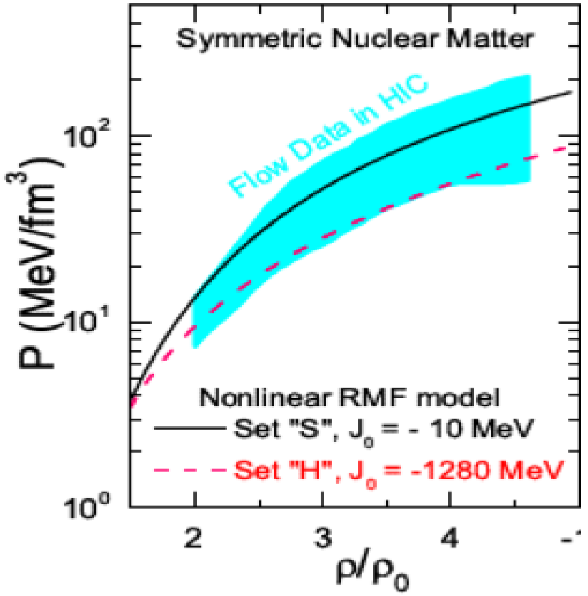




P. G Krastev, Bao-An Li, A. Worley, PLB 668, 1 (2008)

Flow favors K=167-210 MeV





BJ Cai, LW Chen, NST 28 (12), 1-9 (2017)

Hadronic transport equations:

Mean-field potential for baryons

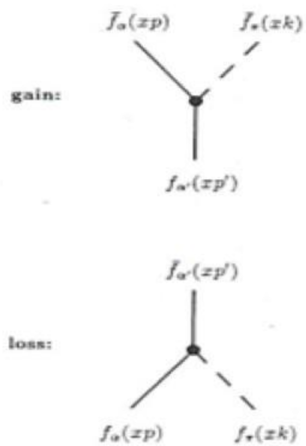
Baryons: $$\frac{\partial f_b}{\partial t} + \frac{\vec{p}}{E_b} \cdot \vec{\nabla}_r f_b - \vec{\nabla}_r U \cdot \vec{\nabla}_p f_b = I_{bb}^b + I_{bm}^b$$

Mesons: $$\frac{\partial f_m}{\partial t} + \frac{\vec{k}}{E_m} \cdot \vec{\nabla}_r f_m = I_{mm}^m + I_{bm}^m$$

Assuming no mean-field for mesons

An example:

Collision integral $I_{b\pi}^{\pi}$: changing rate of pion phase space distribution $f_\pi(\vec{r}, \vec{p}, t)$ due to baryon-pion scatterings



$\Delta \to \pi + N$

$f_\pi(\vec{r}, \vec{p}, t)$

$\pi + N \to \Delta$

Simulate solutions of the coupled transport equations using test-particles and Monte Carlo:

$$f(\vec{r}, \vec{p}, t) \cong \frac{1}{N_t} \sum_i \delta(\vec{r} - \vec{r}_i)\delta(\vec{p} - \vec{p}_i)$$

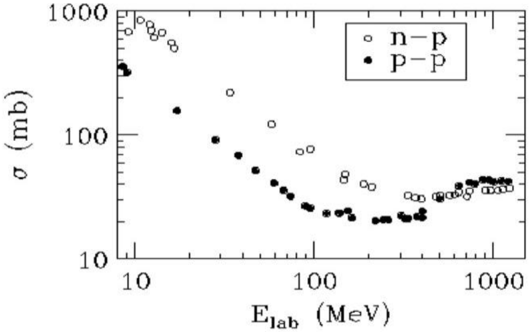
The evolution of $f(\vec{r}, \vec{p}, t)$ is followed on a 6D lattice

$$I_{b\pi}^{\pi}(xk) = \frac{\pi}{16} \sum_{\alpha\alpha'} \int\int \frac{M_\alpha M_{\alpha'}}{E_\alpha(p) E_{\alpha'}(p')} W_{b\pi}^{\pi}(\alpha p, \alpha' p', \pi k) \cdot \delta^{(4)}(p' - p - k) \frac{1}{(2\pi)^6} d\vec{p} d\vec{p}$$
$$[(1 + f_\pi(xk)) f_{\alpha'}(xp')(1 - f_\alpha(xp)) \quad \text{(gain)}$$
$$- f_\pi(xk) f_\alpha(xp)(1 - f_{\alpha'}(xp'))] \quad \text{(loss)}$$

Main features:

Pauli bolcking (1-f_α) for Fermions and Bose enhancement (1+f_π) for bosons are included.

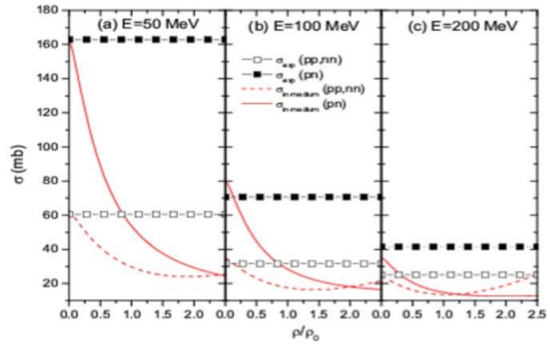
NN cross section in free-space



$$\sigma_{np}^{free} = -70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta \text{ (mb)},$$
$$\sigma_{pp}^{free} = 13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{4} \text{ (mb)},$$

Where β=v/c is the speed of the projectile nucleon

In-medium NN cross sections



$$\sigma_{np}^{medium} = \left[31.5 + 0.092 abs(20.2 - E_{lab}^{0.53})^{2.9}\right] \cdot \frac{1.0 + 0.0034 E_{lab}^{1.51}\rho^2}{1.0 + 21.55\rho^{1.34}} \text{ (mb)}$$
$$\sigma_{pp}^{medium} = \left[23.5 + 0.0256(18.2 - E_{lab}^{0.5})^4\right] \cdot \frac{1.0 + 0.1667 E_{lab}^{1.05}\rho^3}{1.0 + 9.704\rho^{1.2}} \text{ (mb)}.$$

G.Q. Li and R. Machleidt,
Phys. Rev. C48, 11702 and C49, 566 (1994).

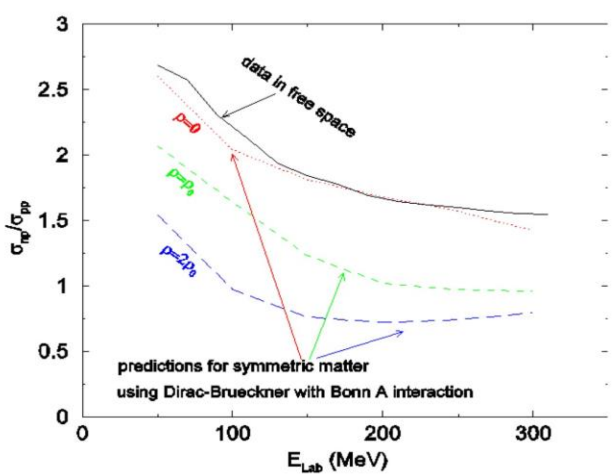
Opposing conclusions with other models:

1. Q. Li et al., PRC 62, 014606 (2000)
2. G. Giansiracusa et al., PRC 53, R1478 (1996)
3. H.-J. Schulze et al., PRC 55, 3006 (1997)
4. M. Kohno et al., PRC 57, 3495 (1998)

Probing the isospin-dependence of in-medium NN cross sections

How does the σ_{np}/σ_{pp} ratio change in neutron-rich medium?

Bao-An Li, Pawel Danielewicz and Bill Lynch, Phys. Rev. C71, 054603 (2005)



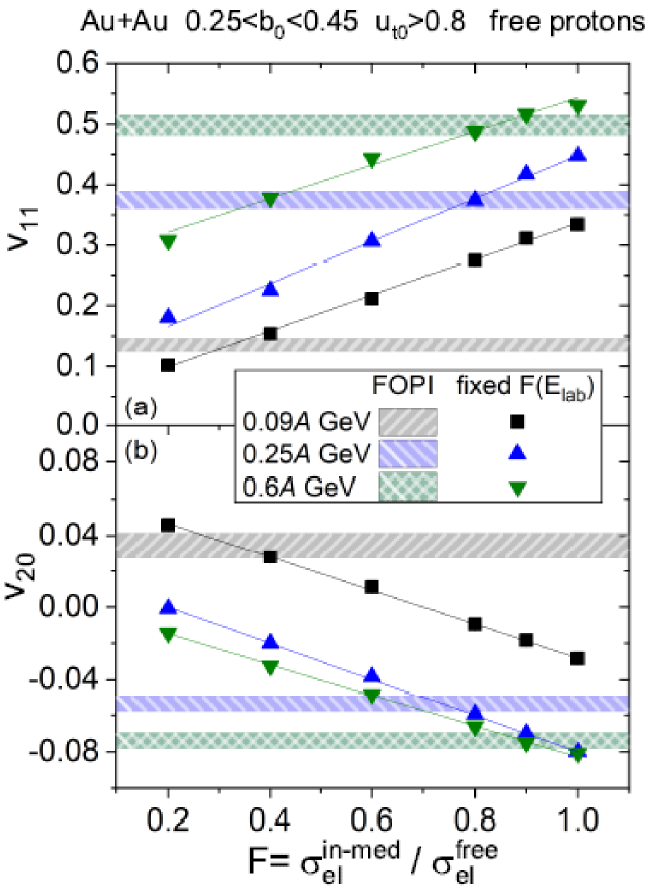
Fermi's Golden Rule for scattering:

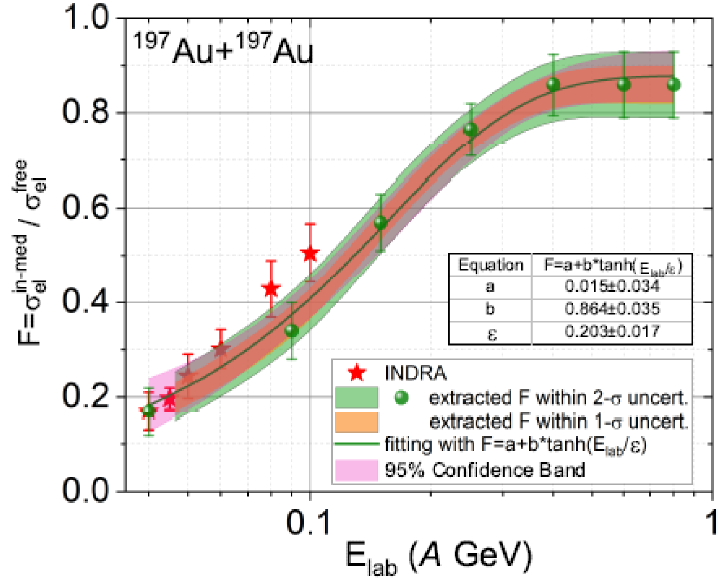
Xsection~ M^2*Density of final state*Pauli Blocking/Incoming current

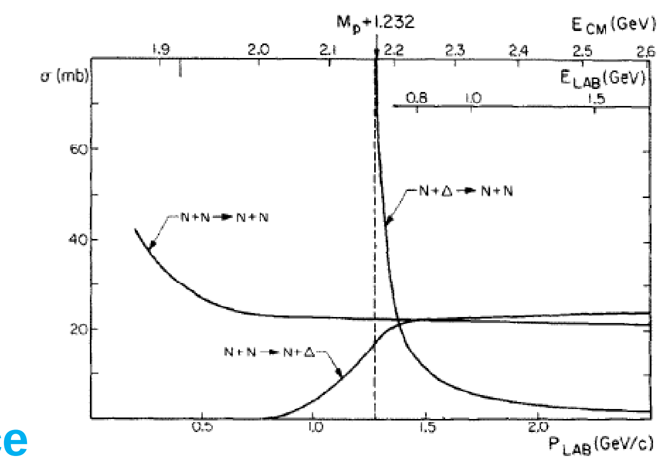
Accessing the in-medium effects on nucleon-nucleon elastic cross section with collective flows and nuclear stopping

Pengcheng Li,[1,2] Yongjia Wang,[2] Qingfeng Li,[2,3,*] and Hongfei Zhang[4]

Using UrQMD, Phys. Lett. B 828 (2022) 137019







Only elastic xsections are modified by F
Inelastic xsections are kept the same as in free-space

PION COLLECTIVITY IN RELATIVISTIC HEAVY-ION COLLISIONS

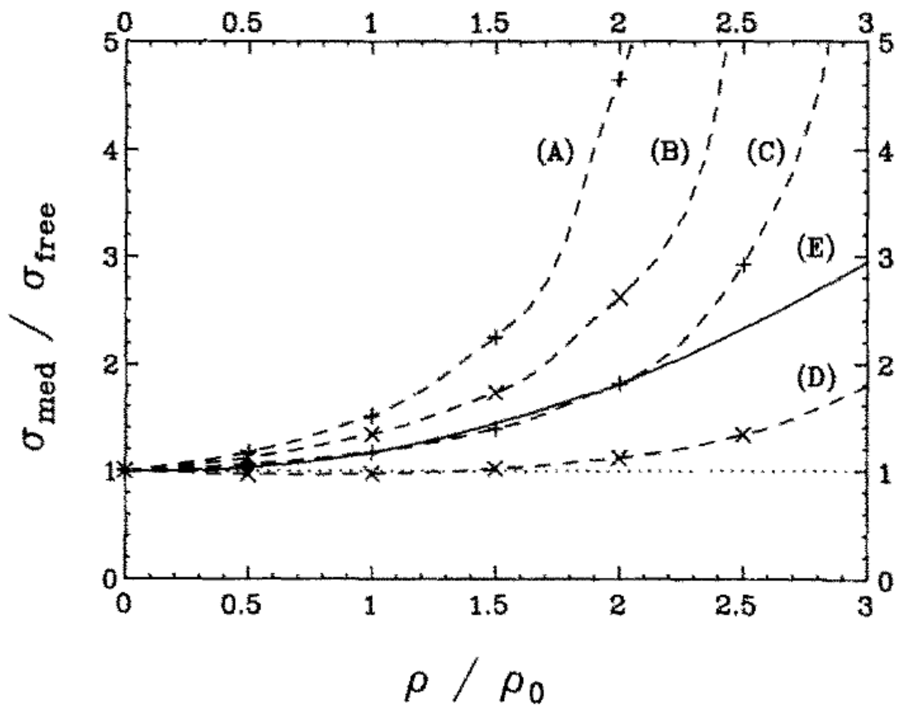
George F. BERTSCH[1], Gerald E. BROWN[2], Volker KOCH[2*] and Bao-An LI[1]

Nuclear Physics **A490** (1988) 745–755

$NN \to N\Delta$ cross section :
$$\frac{d\sigma}{d\Omega} = \frac{1}{|v_1 - v_2|} \frac{m^2}{E_{i1}E_{i2}} \frac{1}{(2\pi)^3} \int dm^{*2} \frac{\frac{1}{2}\Gamma}{(m^* - m_\Delta)^2 + \frac{1}{4}\Gamma^2} \times \int d\Omega_3 \frac{mp_3}{\sqrt{s}} |\bar{M}|^2_{NN\to N\Delta} ,$$

Pion dispersion/mean-field affects:
(1) Delta width through phase space
(2) Pion propagator, thus NN interaction

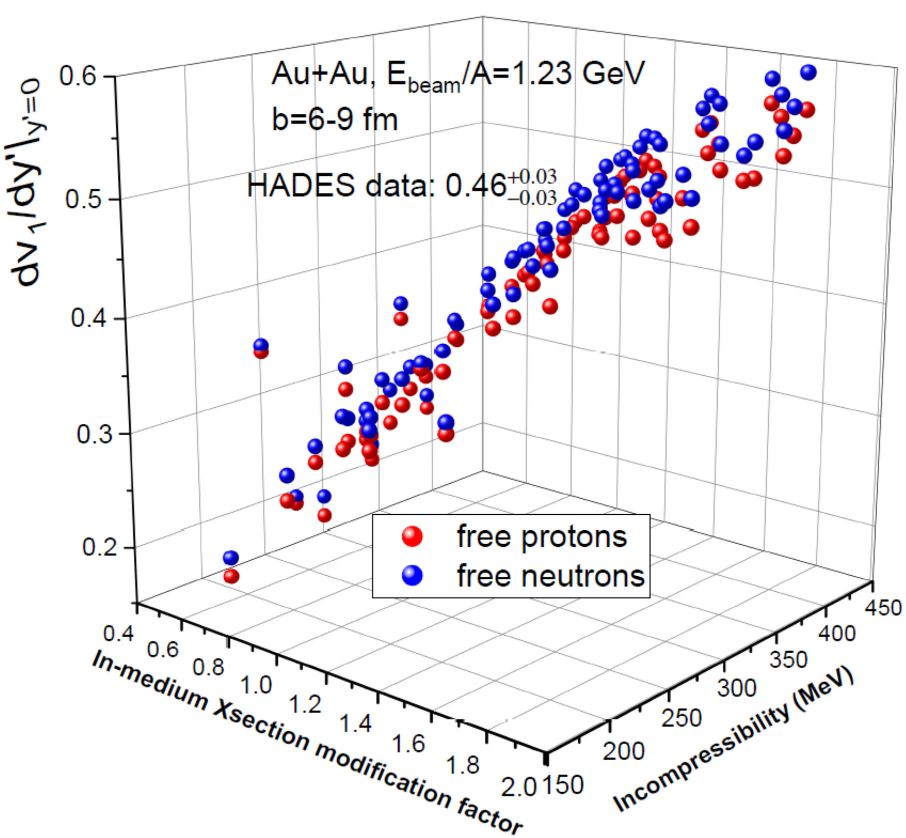
set A	$\Lambda_\pi = 780$ MeV $g'_{N\Delta} = \frac{1}{3}$	$\Lambda_\rho = 1800$ MeV $f^\rho_{NN} = 6.24$ $f^\rho_{N\Delta} = 10.54$ $m_\rho = 770$ MeV
set B	$\Lambda_\pi = 565$ MeV $g'_{N\Delta} = \frac{1}{3}$	no rho exchange
set C	$\Lambda_\pi = 560$ MeV $g'_{N\Delta} = 0.4$	no rho exchange
set D	$\Lambda_\pi = 545$ MeV $g'_{N\Delta} = 0.5$	no rho exchange

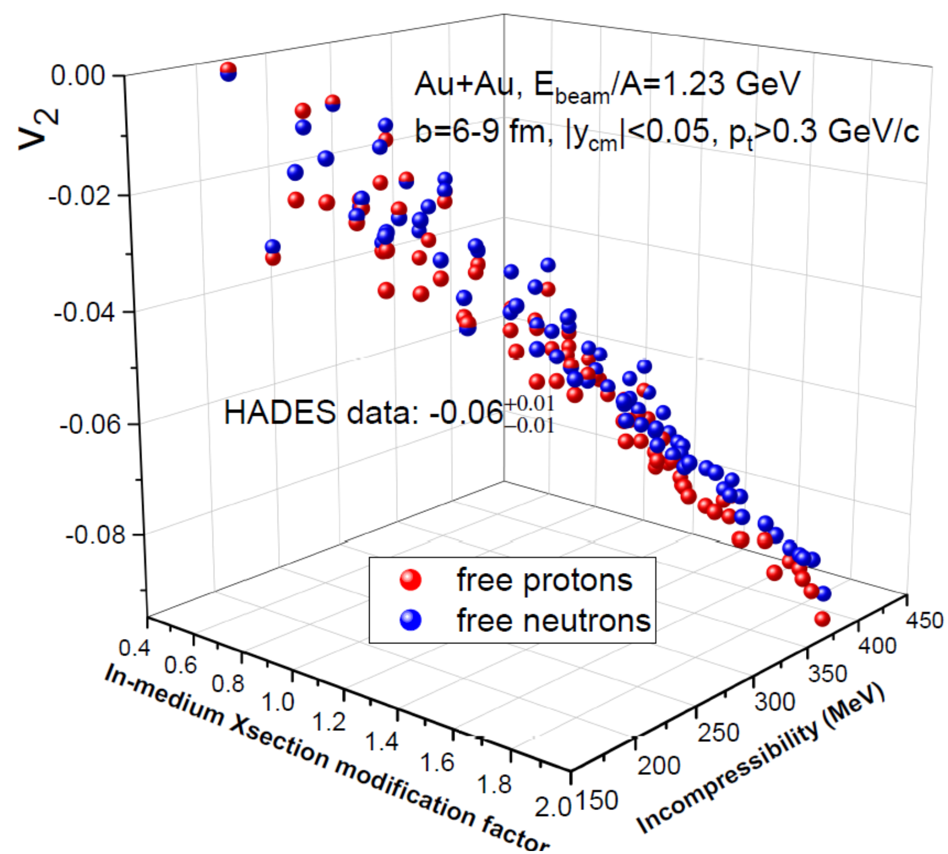


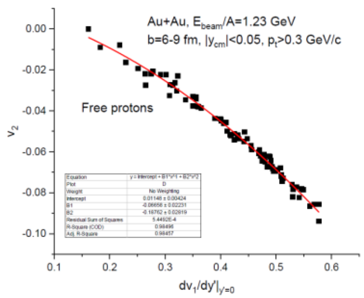
Bayesian inference of in-medium baryon-baryon scattering cross sections from HADES proton flow data

Bao-An Li and Wen-Jie Xie, 2303.10474 [nucl-th], Nucl. Phys. A (2023) in press

Using the simplest Skyrme force



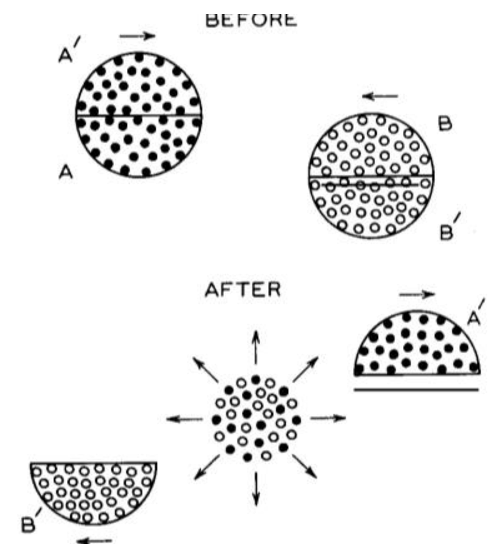




$$v_2(y,p_t) =< cos(2\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}^2 - p_{iy}^2}{p_{it}^2}$$

$$v_1(y,p_t) =< cos(\phi) > (y,p_t) = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ix}}{p_{it}}$$

v_1 and v_2 are anti-correlated



Reaching the stationary state in the Markov Chain Monte Carlo (MCMC) process

Bayes' theorem:

Prior distribution

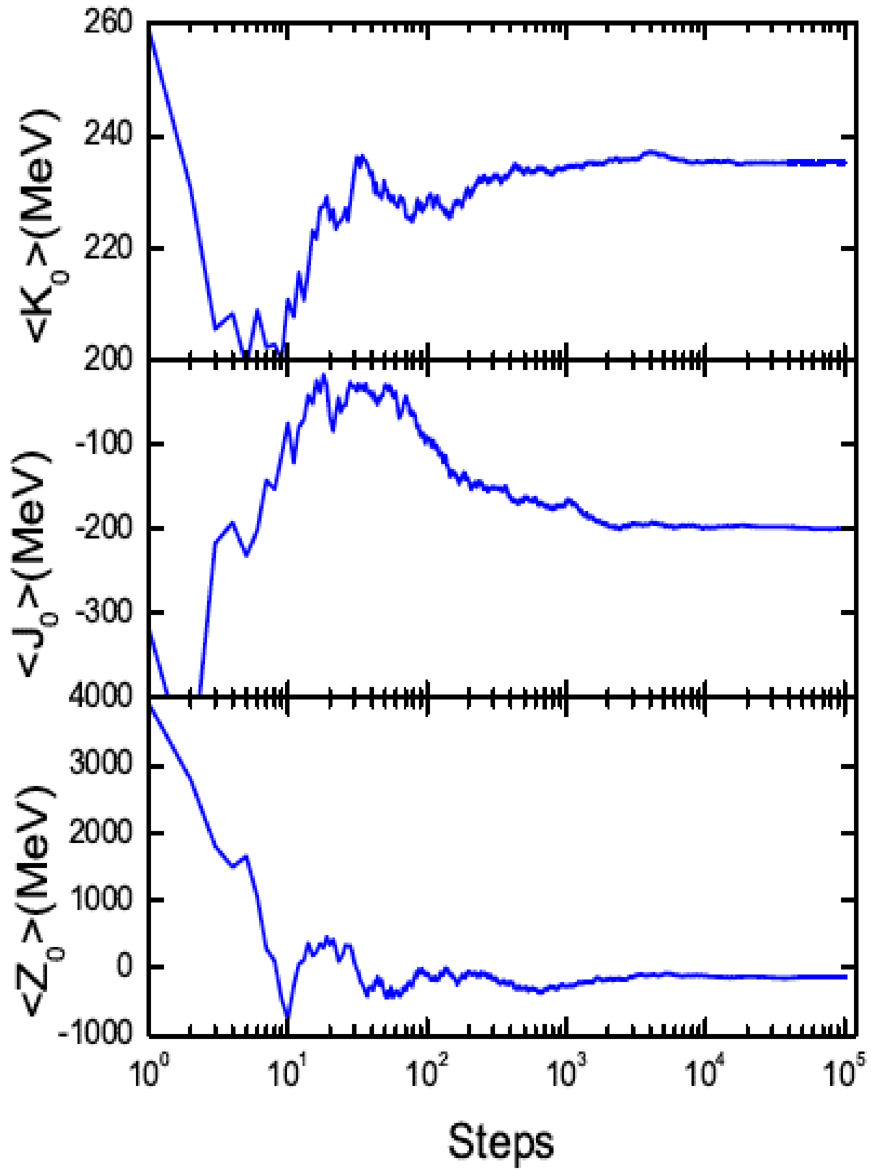
$$P(\mathcal{M}(K_0, J_0, Z_0)|D) = CP(D|\mathcal{M}(K_0, J_0, Z_0))P(\mathcal{M}(K_0, J_0, Z_0)),$$

Likelihod function

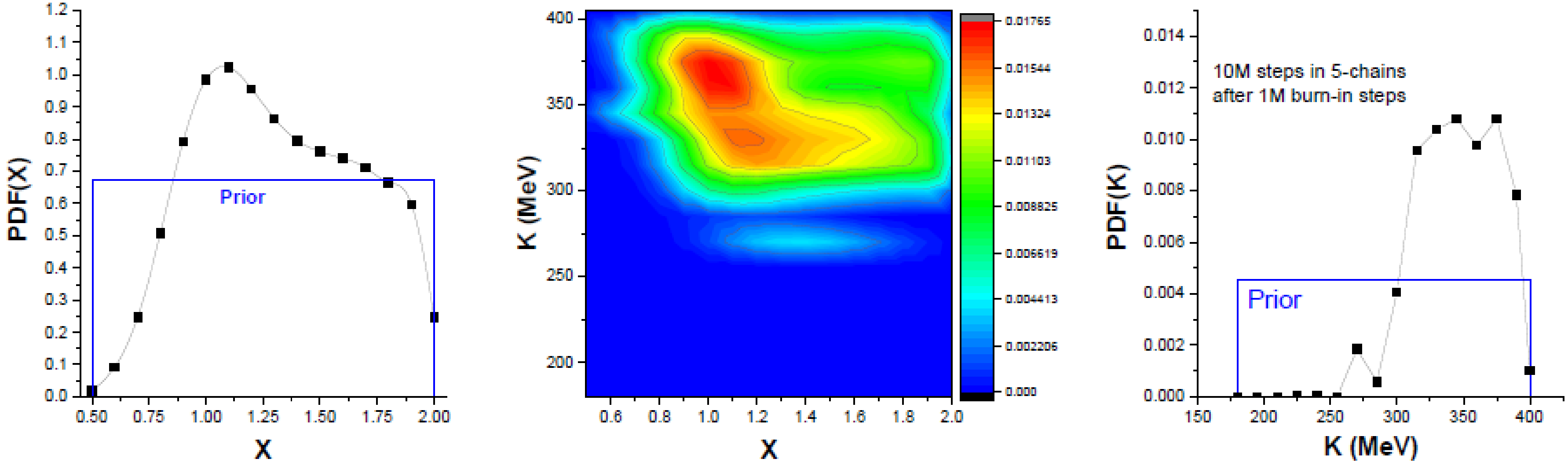
Calculations data

$$P[D|\mathcal{M}(K_0, J_0, Z_0)] = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{D,j}} \exp\left[-\frac{(P_{\mathrm{th},j} - P_{D,j})^2}{2\sigma_{D,j}^2}\right]$$

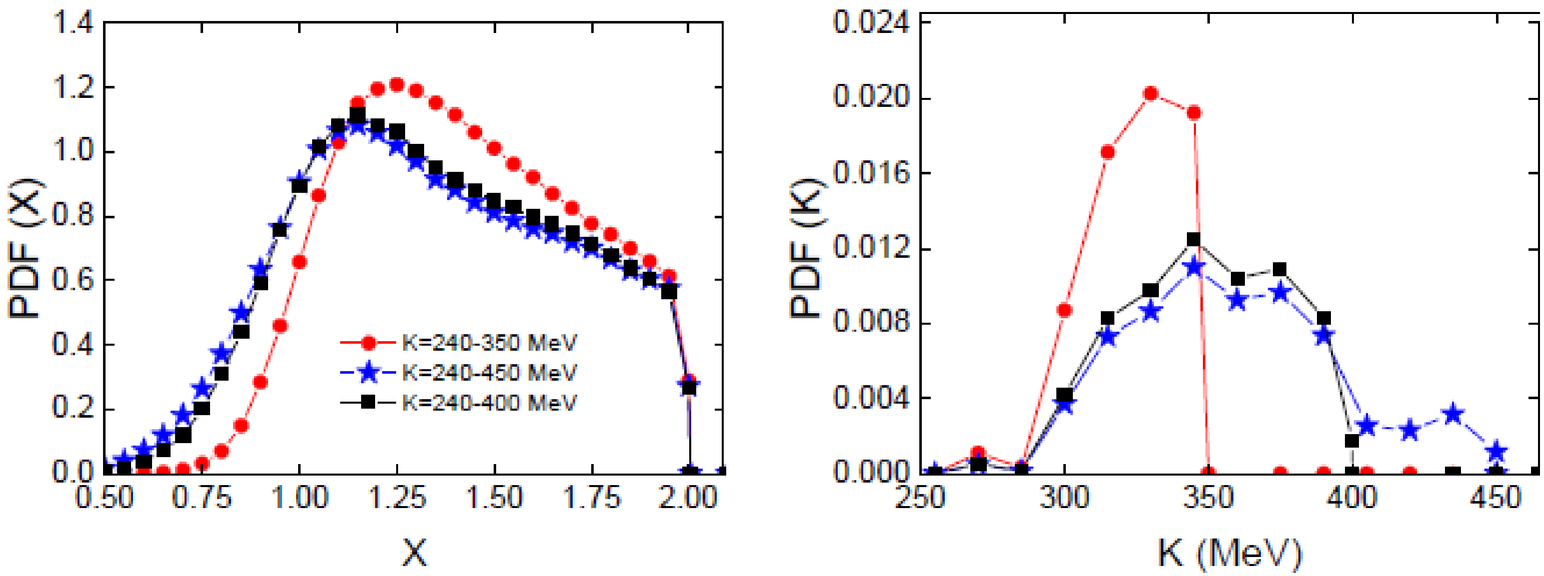
Metropolis-Hastings algorithm
in deciding each step



Indication of enhanced in-medium NN cross sections



Effects of prior ranges



HADES data prefers an ENHANCED in-medium Section and a stiff K much higher than what GMR data indicates

Prior range of K (MeV)	Mean of X	Mean of K	MPV of X	MPV of K
240-350	1.38,	325.38,	$1.25^{+0.40}_{-0.15}$,	330^{+15}_{-15}
240-400	1.32,	349.98,	$1.20^{+0.45}_{-0.15}$,	375^{+15}_{-45}
240-450	1.31,	358.97,	$1.15^{+0.50}_{-0.20}$,	375^{+15}_{-45}

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{24}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4.$$

Boltzmann-Uehling-Uhlenbeck equation for one componenet system

$$P(\rho) = \rho^2 \frac{dE_0(\rho)}{d\rho} = \frac{\rho^2}{\rho-\rho_0}[K(\frac{\rho-\rho_0}{3\rho_0})^2 + \frac{J_0}{2}(\frac{\rho-\rho_0}{3\rho_0})^3 + \frac{Z_0}{6}(\frac{\rho-\rho_0}{3\rho_0})^4].$$

$$\frac{\partial f(\vec{r},\vec{p},t)}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f - \vec{\nabla}_r V \cdot \vec{\nabla}_p f = I_c(f, \sigma_{NN})$$

$$V = V_0 + V_{sym}$$

Isospin-independent potential

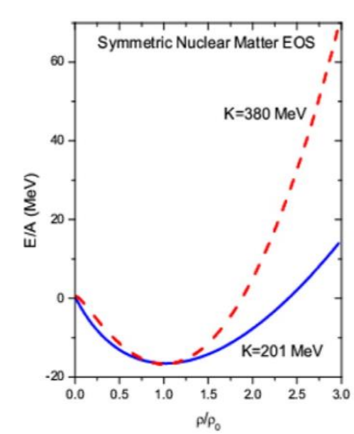
$$V_0 = a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma$$

$$a = -29.81 - 46.90\frac{K+44.73}{K-166.32}\ (\text{MeV}),$$

$$b = 23.45\frac{K+255.78}{K-166.32}\ (\text{MeV}),$$

$$\sigma = \frac{K+44.73}{211.05}.$$

EOS of symmetric matter

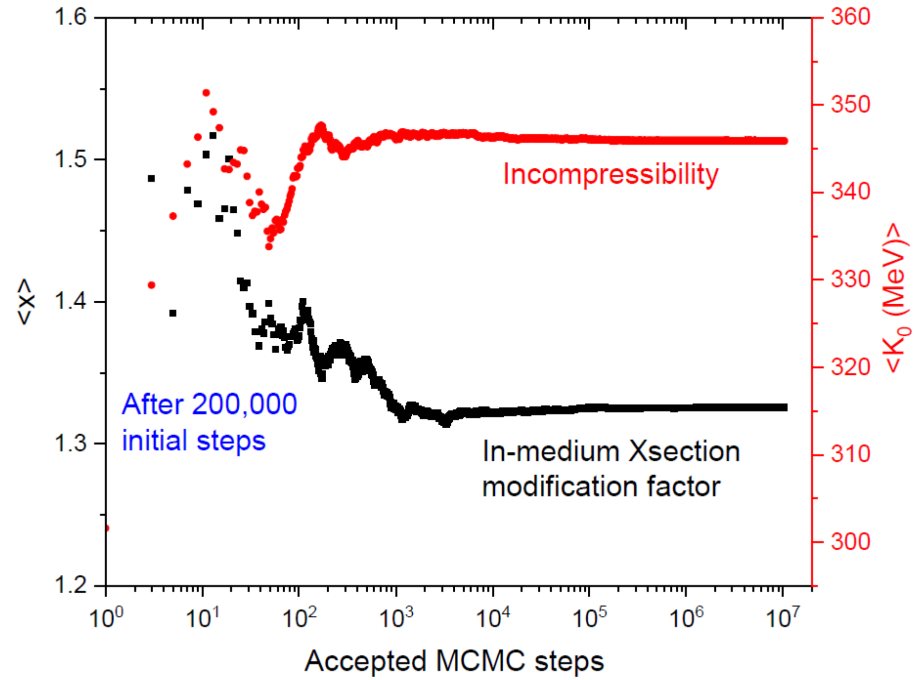


Probes of the EOS of symmetric nuclear matter

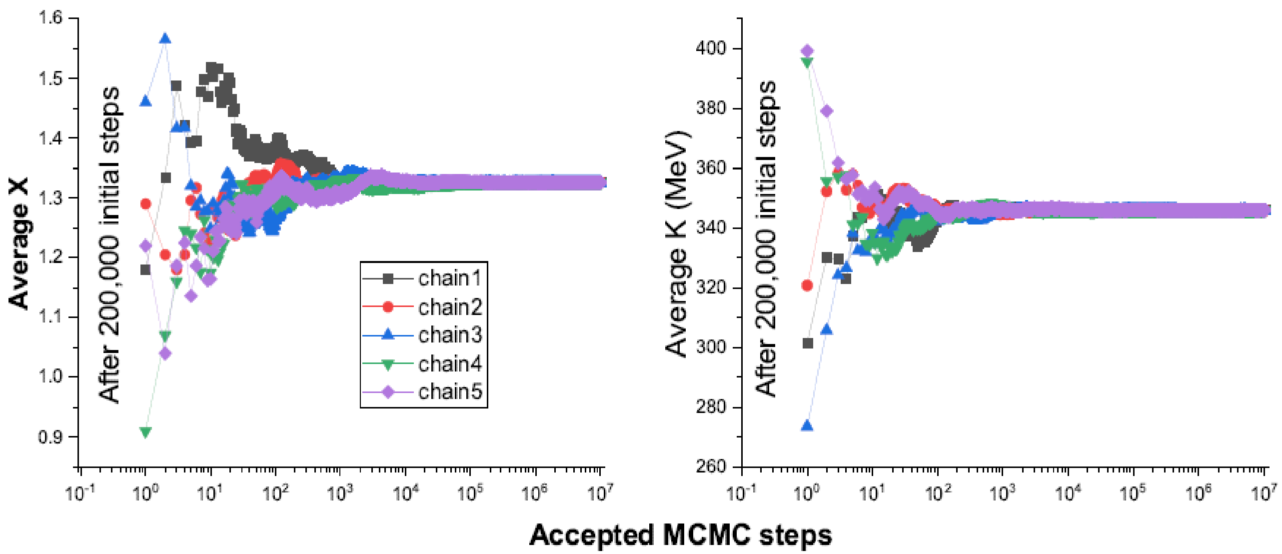
- **Collective vibrations of nuclei probing the incompressibility K** around ρ_0
- **Particle production especially strange particles (e.g., Kaons) from heavy-ion collisions**
- **Collective flow of various particles & clusters probing K around ρ_0 and high-density EOS with the caveat that they are also sensitive to the poorly known viscosity (in-medium particle-particle scattering cross sections)**

MANY interesting issues remain to be resolved

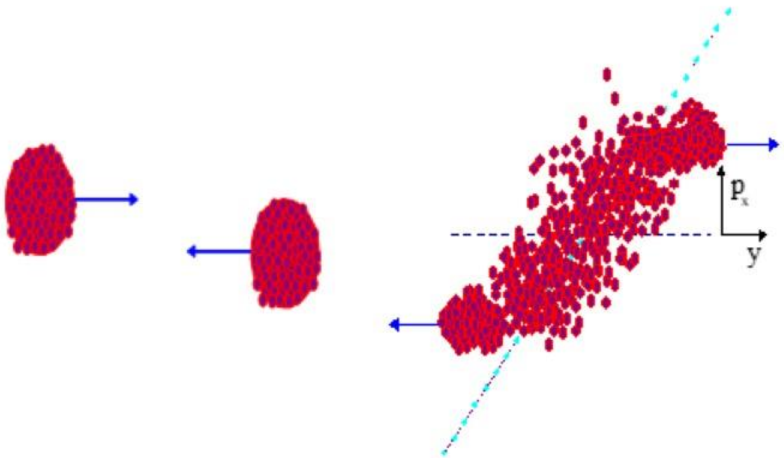
A single MCMC chain

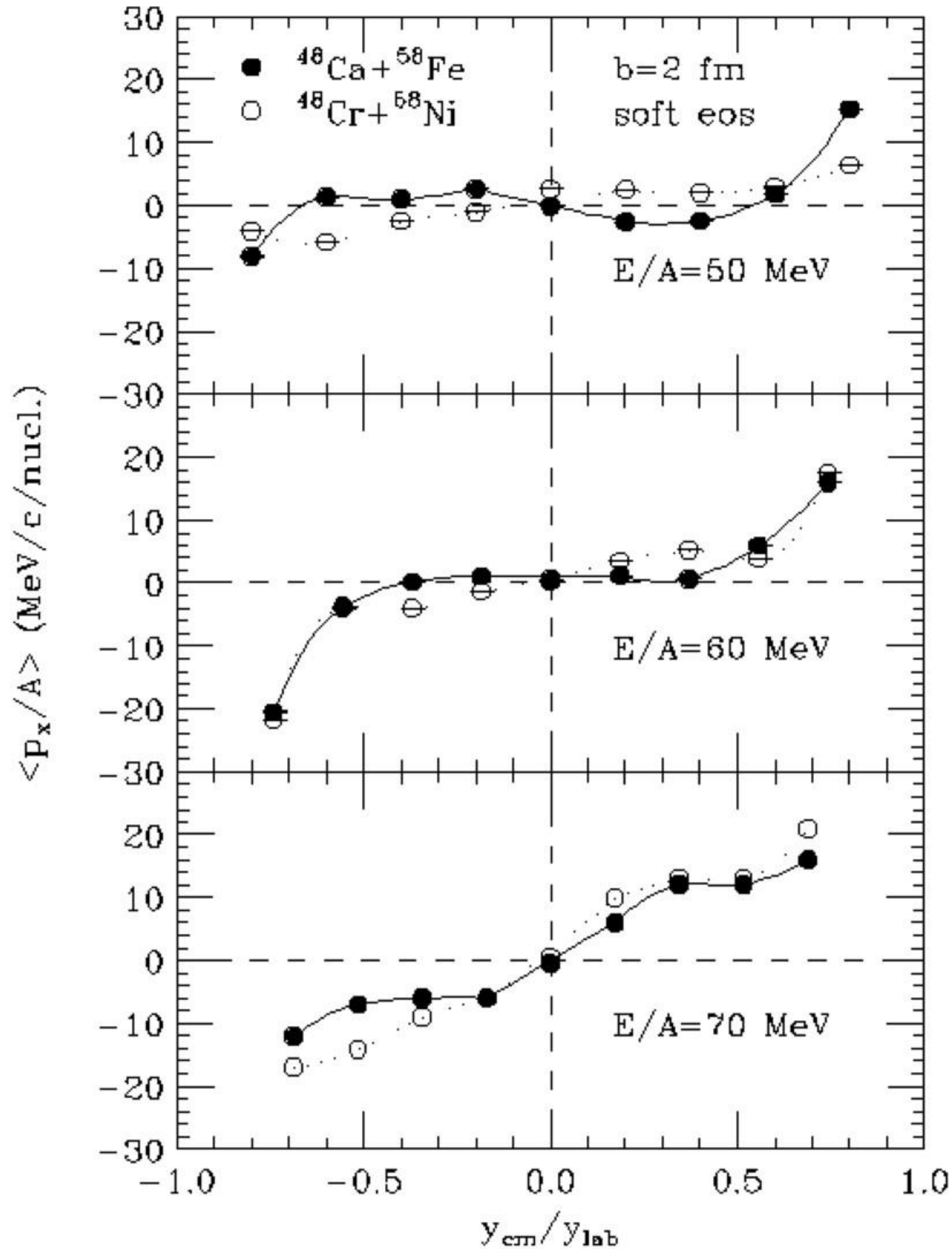


Multi MCMC chains



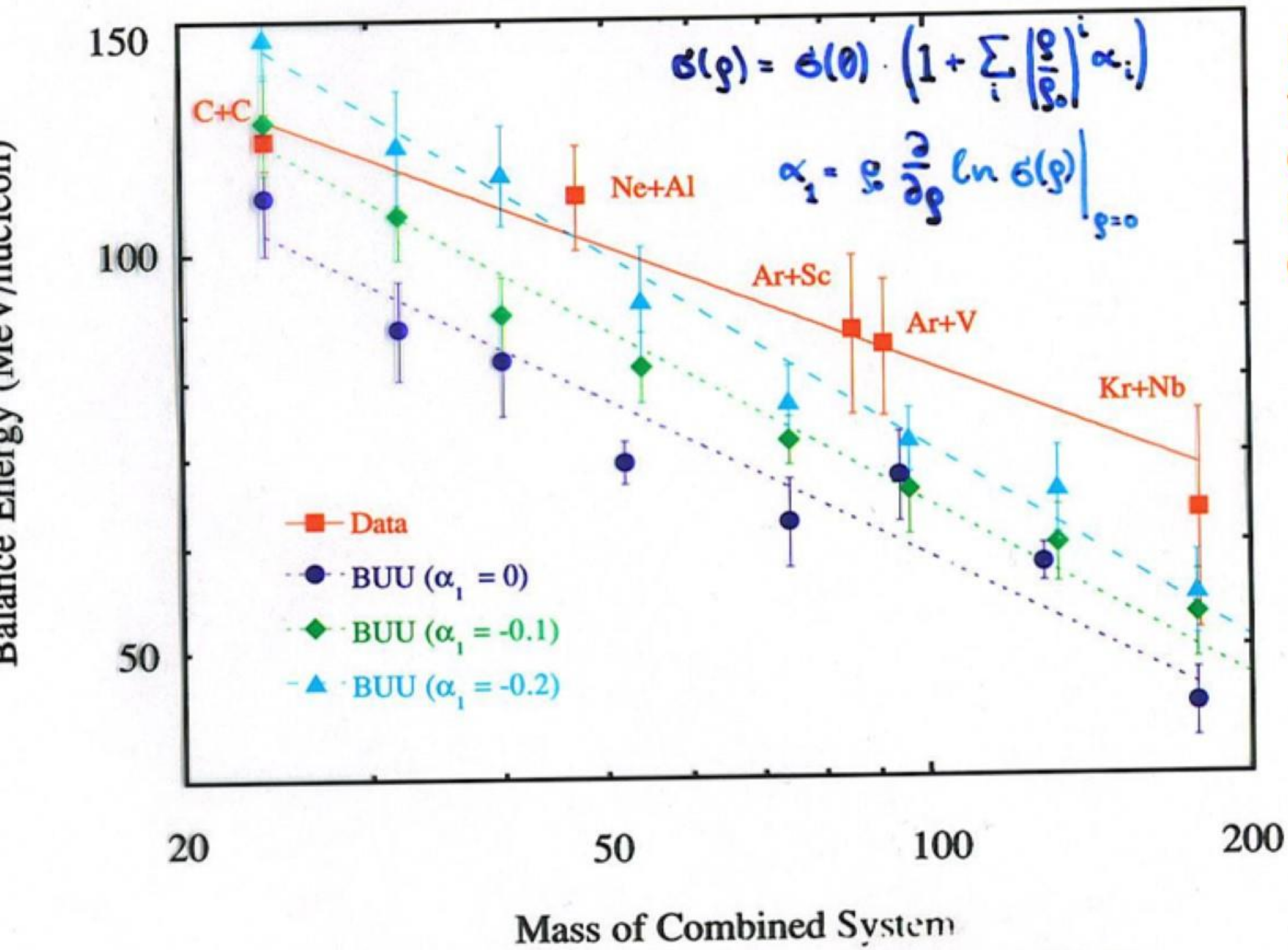
Transverse momentum analysis of collective flow near the balance energy





$$\sigma_{\text{medium}} = \sigma_{\text{free space}} \bullet \left(1 + \alpha \frac{\rho}{\rho_0}\right)$$

Mass Dependence of the Disappearance of Flow



No information about the isosoin dependence of the in-medium NN cross section is obtained

Data: G. Westfall et al., PRL 71, 1986 (1993)

Calculations:, D. Klakow, G. Welke, W.B., PRC 48, 1982 (1993)

Bayesian inference of high-density SNM EOS parameters from heavy-ion reaction data

	K_0	J_0	Z_0
Av	235	−200	−146
σ	30	200	1728
Min	145	−800	−5330
Max (3σ)	325	400	5038

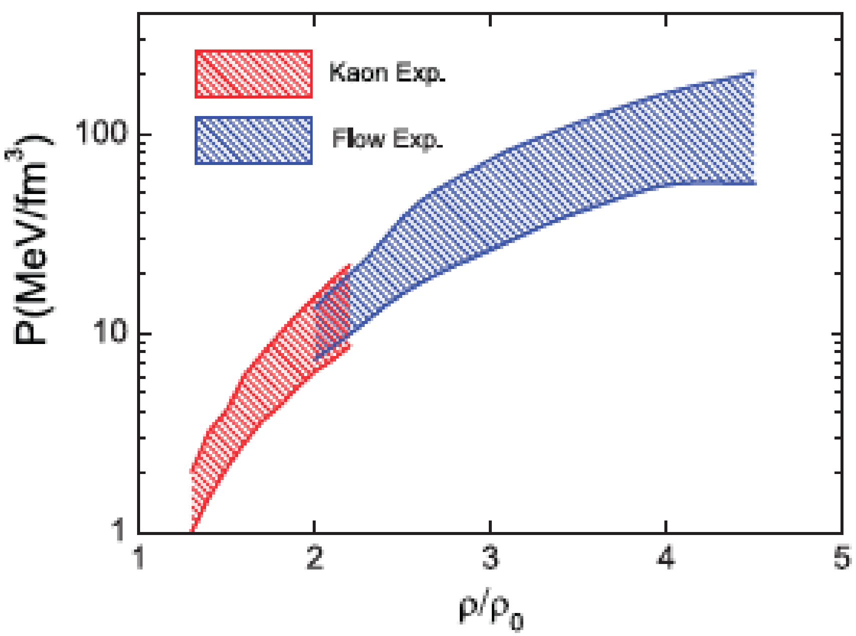
Prior ranges of SNM EOS parameters based on theories and data available

Margueron J, Hoffmann C R and Gulminelli F 2018 PRC 97, 025805 and 025806

Antic S, Chatterjee D, Carreau T and Gulminelli F 2019 J. Phys. G: Nucl. Part Phys. 46 065109

The pressure in symmetric nuclear matter

$$P(\rho) = \rho^2 \frac{\mathrm{d}E_0(\rho)}{\mathrm{d}\rho} = \frac{\rho^2}{\rho - \rho_0}\left[K_0\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4\right].$$



Constraints on the EOS of symmetric nuclear matter from heavy-ion collisions

Danielewicz P. I., Lacey R and Lynch W G 2002 *Science* 298 1592

Fuchs C 2006 *Prog. Part. Nucl. Phys.* 56 1

Lynch W G *et al* 2009 *Prog. Part. Nucl. Phys.* 62 427

Bayesian inference of the EOS parameters from the empirical pressure as a function of density "data"

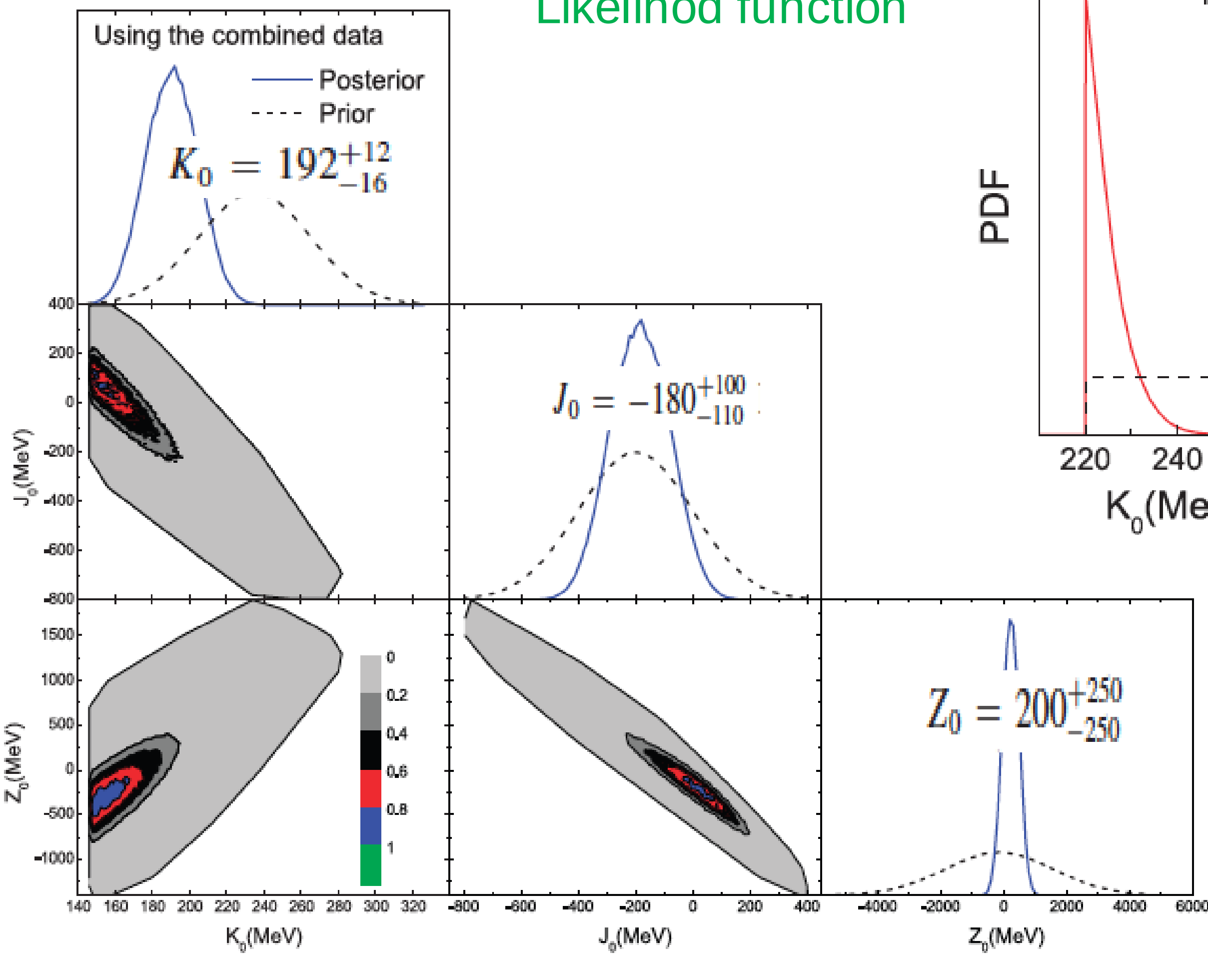
Wen-Jie Xie and Bao-An Li
JPG 48, 025110 (2021)

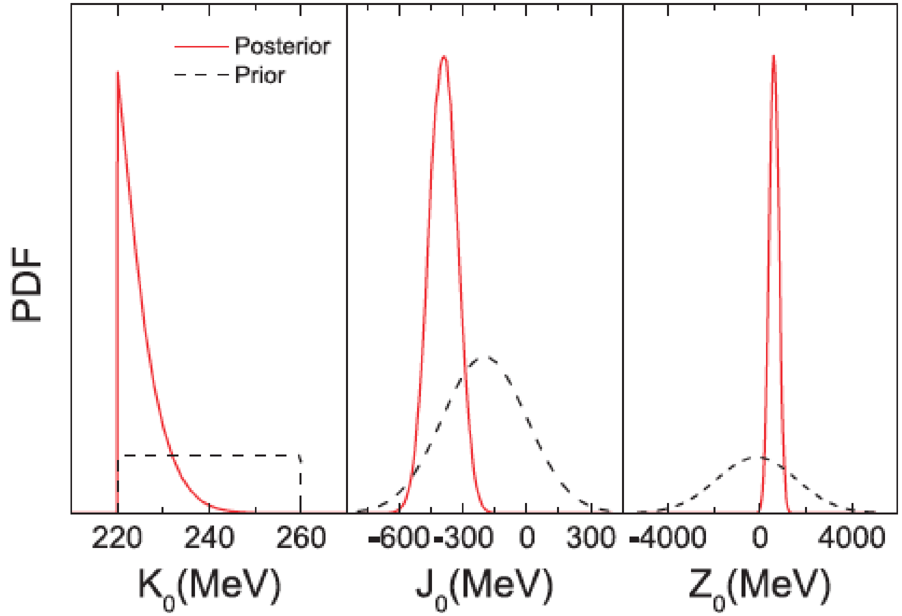
Posterior PDF: $P(\mathcal{M}(K_0, J_0, Z_0)|D) = CP(D|\mathcal{M}(K_0, J_0, Z_0))P(\mathcal{M}(K_0, J_0, Z_0)),$

Prior PDF

$$P[D|\mathcal{M}(K_0, J_0, Z_0)] = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{D,j}} \exp\left[-\frac{(P_{\text{th},j} - P_{D,j})^2}{2\sigma_{D,j}^2}\right]$$

Likelihod function





Kaon production and elliptical flow data from GSI favors a K_0 smaller than the fiducial value from ISGMR data: 240±20 MeV

A. Le Fe`vre et al., NPA945, 112 (2016).

Y.J. Wang et al., PLB778, 207 (2018)

Transverse flow

IBUU Calculations

Bao-An Li and A. Sustich

PRL 82, 5004 (1999)

Differential transverse flow

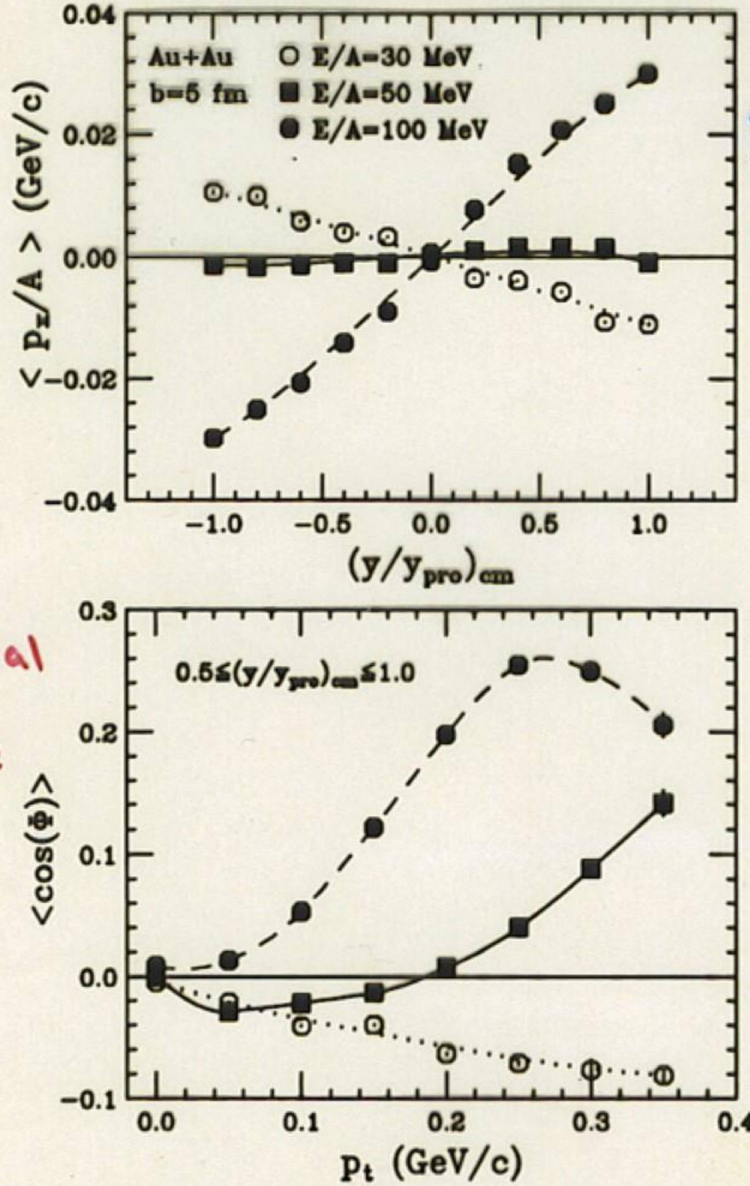


FIG. 1. Total (upper) and differential (lower) transverse flow analysis for the reaction of Au+Au

Isospin Dependence of Collective Flow in Heavy-Ion Collisions at Intermediate Energies

Bao-An Li,[1] Zhongzhou Ren,[2,3] C. M. Ko,[1] and Sherry J. Yennello[4]
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[2]Ganil, BP5027, F14021 Caen Cedex, France
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(Received 11 January 1996)

Within the framework of an isospin-dependent Boltzmann-Uehling-Uhlenbeck model using initial proton and neutron densities calculated from the nonlinear relativistic mean-field theory, we compare the strength of transverse collective flow in reactions $^{48}Ca + ^{58}Fe$ and $^{48}Cr + ^{58}Ni$, which have the same mass number but different neutron/proton ratios. The neutron-rich system ($^{48}Ca + ^{58}Fe$) is found to show significantly stronger negative deflection and consequently has a higher balance energy, especially in peripheral collisions. [S0031-9007(96)00431-0]

Isospin Dependence of Collective Transverse Flow in Nuclear Collisions

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Collective transverse flow of nuclear matter was measured as a function of the ratio of neutrons to protons (N/Z) of the interacting system for the first time. The collisions of three isotopically pure beams of $A = 58$ nuclei with two $A = 58$ targets were studied at 55 MeV/nucleon. The results for the flow variables demonstrate the sensitivity of transport models to elementary aspects of the nucleon-nucleon collisions. [S0031-9007(97)02359-4]

Isospin Dependence of the Balance Energy

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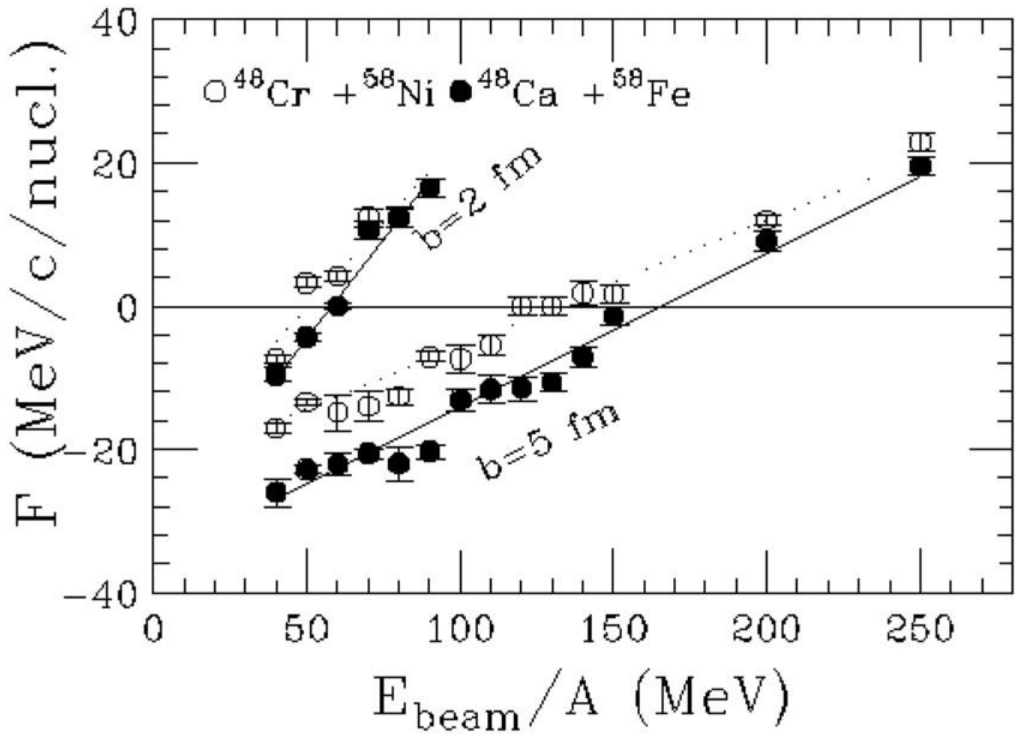
(Received 20 August 1996)

The energy at which collective transverse flow in the reaction plane disappears, the balance energy E_{bal}, is found to depend on the isospin of the system using the reactions $^{58}Fe + ^{58}Fe$ and $^{58}Ni + ^{58}Ni$. The more neutron-rich system exhibits higher balance energies for all measured impact parameters, in agreement with the predictions of a transport model which incorporates an

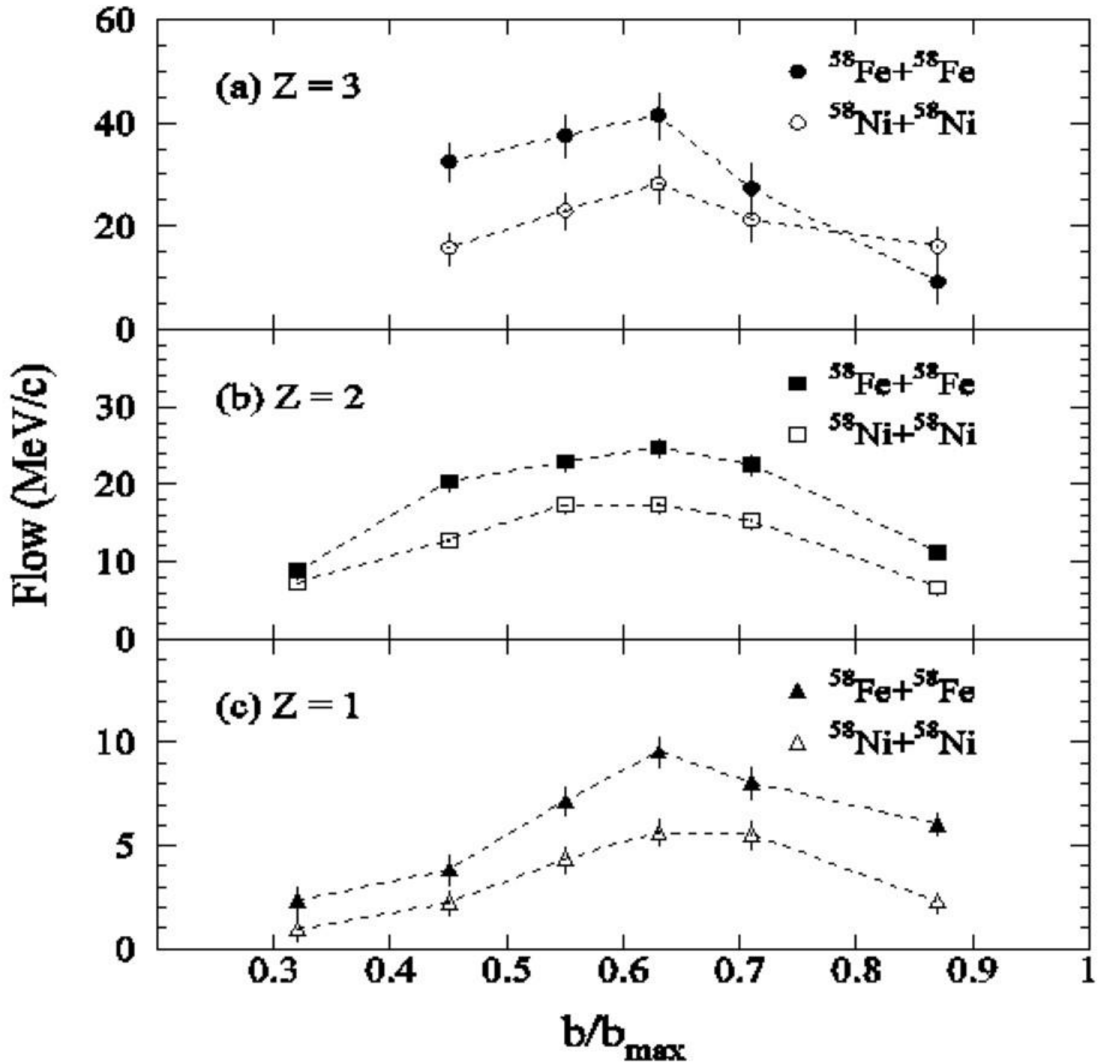
Isospin dependence of collective flow
and excitation function of the flow parameter $F=dP_x/dy$



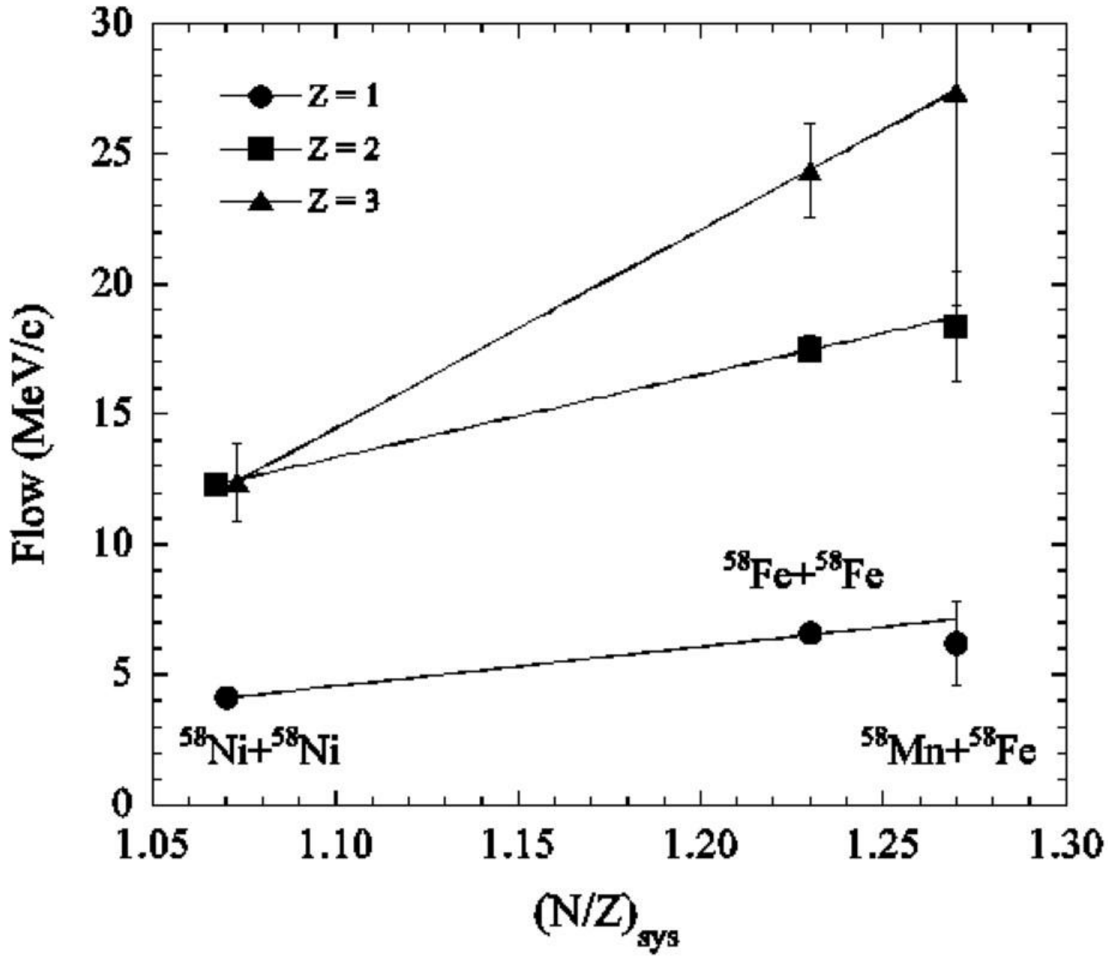
B.A. Li, Z.Z. Ren, C.M. Ko and S.J. Yennello, PRL 76, 4492 (1996).

Isospin dependence of collective flow

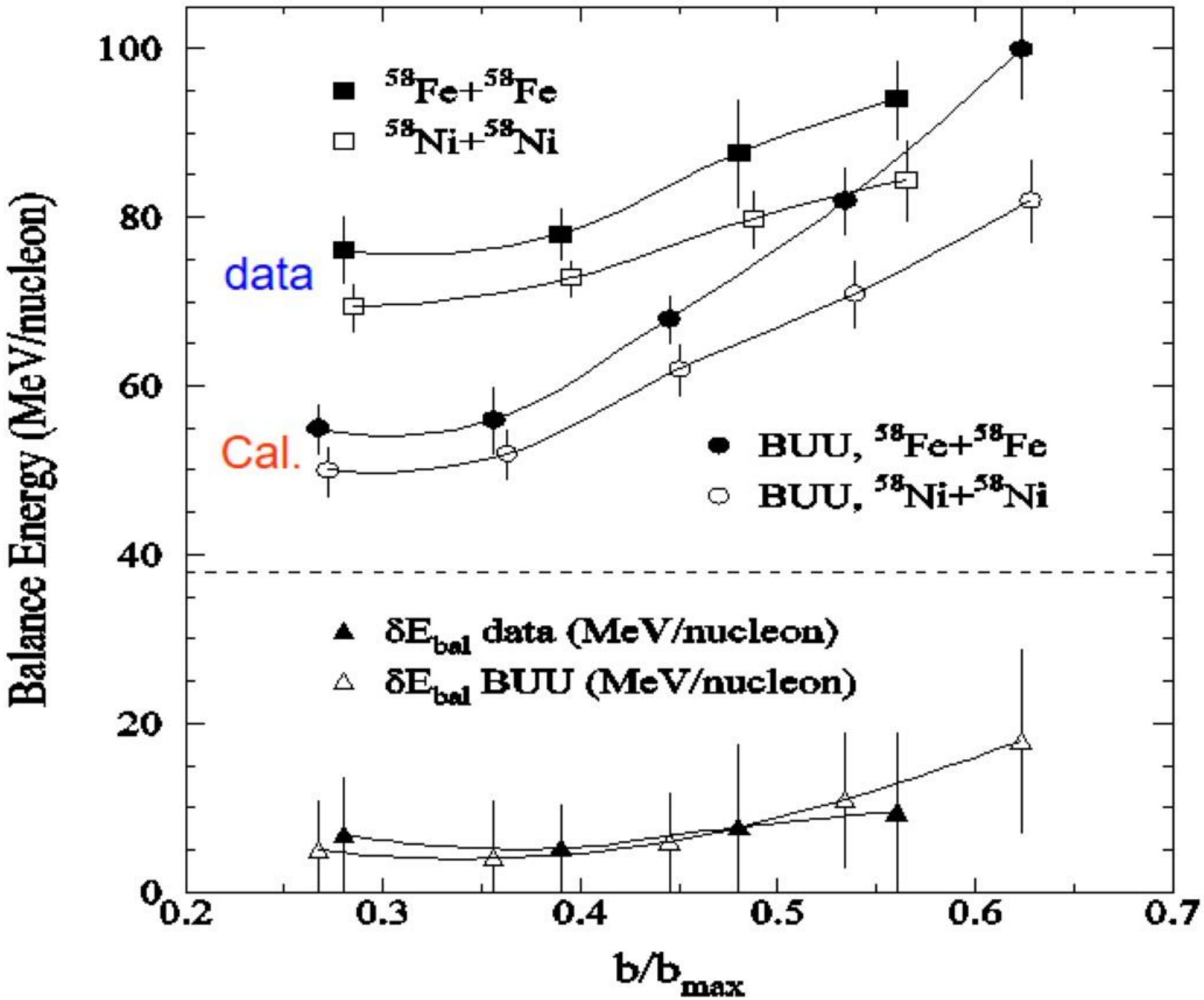
R. Pak et al., PRL 78, 1022 and 1026 (1997).



Isospin-dependence of flow
G.D. Westfall et al.



Comparing with the experimental data



Indication: reduced, possibly isospin-dependent, in-medium NN cross sections are required !

Isospin-dependence of nucleon-nucleon cross sections in neutron-rich matter

The effective mass scaling model:

$$\sigma_{medium} / \sigma_{free} \approx \left(\frac{\mu^*_{NN}}{\mu_{NN}} \right)^2$$

μ^*_{NN} is the reduced effective mass of the colliding nucleon pair NN

valid for $\rho \leq 2\rho_0$ and relative momenta ≤ 240 MeV/c according to Dirac-Brueckner-Hatree-Fock calculations
F. Sammarruca and P. Krastev, nucl-th/0506081
Phys. Rev. C (2005) in press.

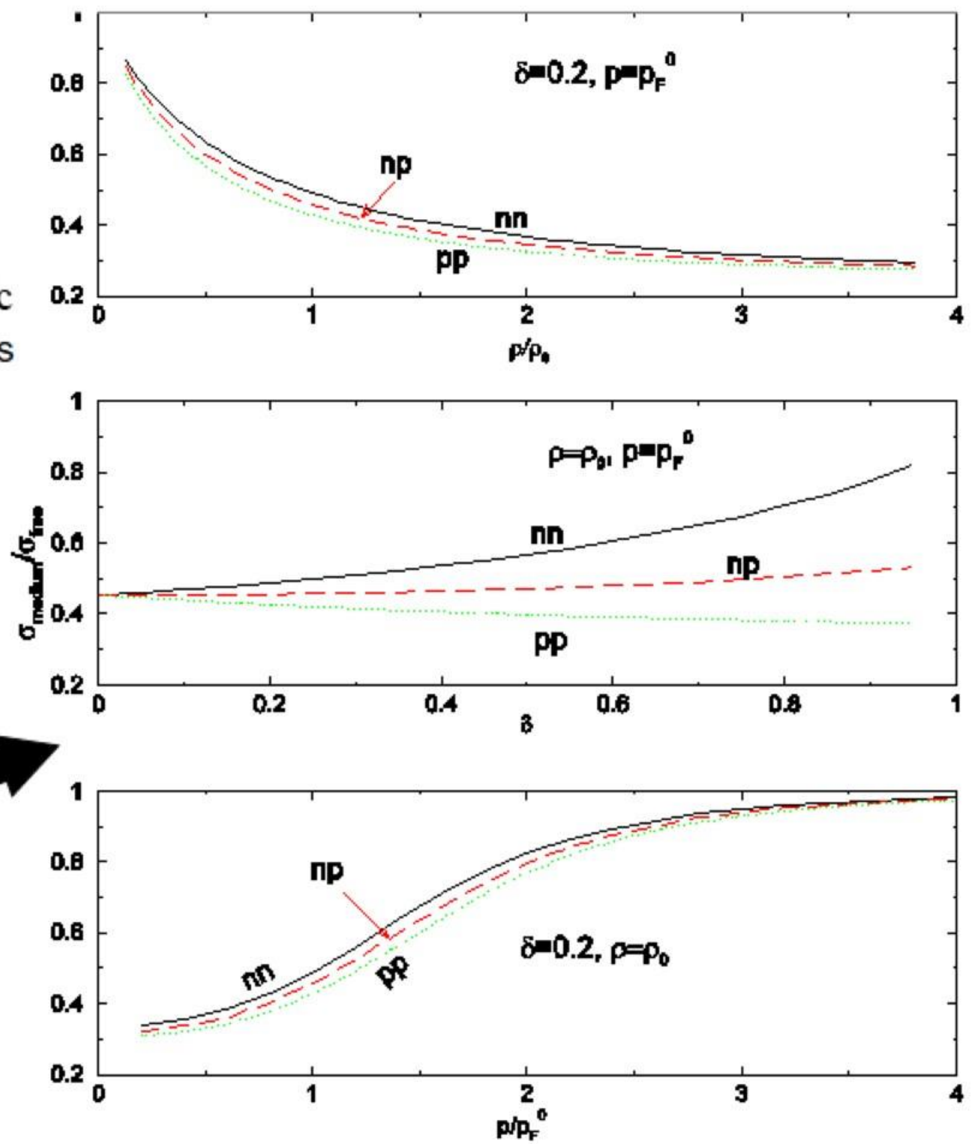
Applications in symmetric nuclear matter:

J.W. Negele and K. Yazaki, PRL 47, 71 (1981)
V.R. Pandharipande and S.C. Pieper, PRC 45, 791 (1992)
M. Kohno et al., PRC 57, 3495 (1998)
D. Persram and C. Gale, PRC65, 064611 (2002).

Application in neutron-rich matter:
nn and pp xsections are splitted due to the neutron-proton effective mass slitting

Bao-An Li and Lie-Wen Chen, nucl-th/0508024, Phys. Rev. C (2005) in press.

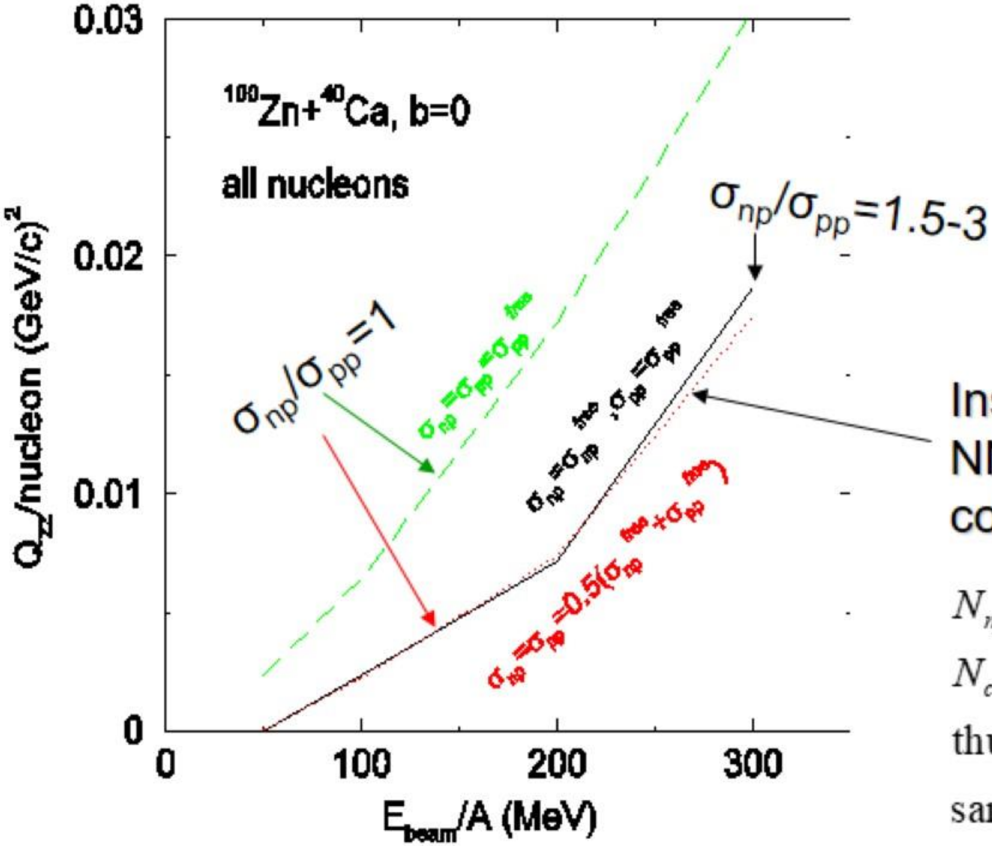
$\sigma_{medium} / \sigma_{free}$ in neutron-rich matter at zero temperature

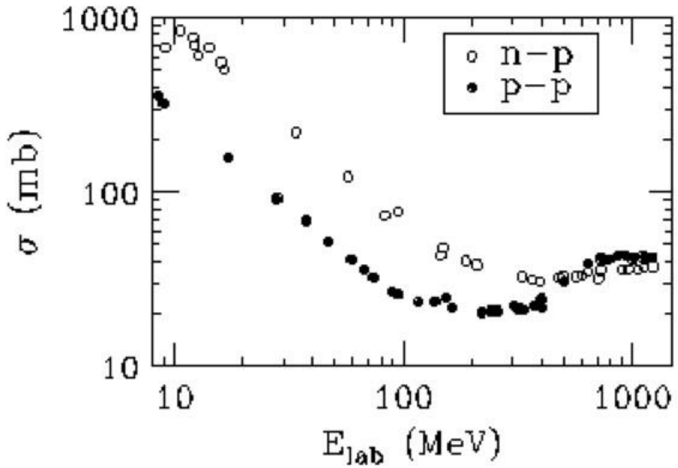


How to determine experimentally the isospin-dependence of in-medium NN xsections

Traditional measures of stopping power using global observables, such as, quadrupole moment, LMT, ERAT, etc, are sensitive to the values of the in-medium NN xsections, but they are ambiguous for extracting the isospin-dependence of the NN xsections.

An example: quadrupole moment Q_{ZZ}





Insensitive to the isospin-dependence of the NN xsection because the total no. of NN collisions are about the same:

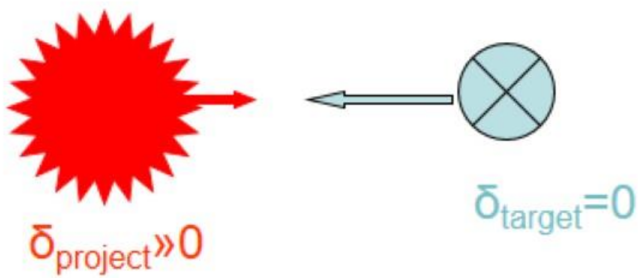
$$N_{np}/(N_{nn}+N_{pp}) \approx 1-2\delta_1\delta_2$$

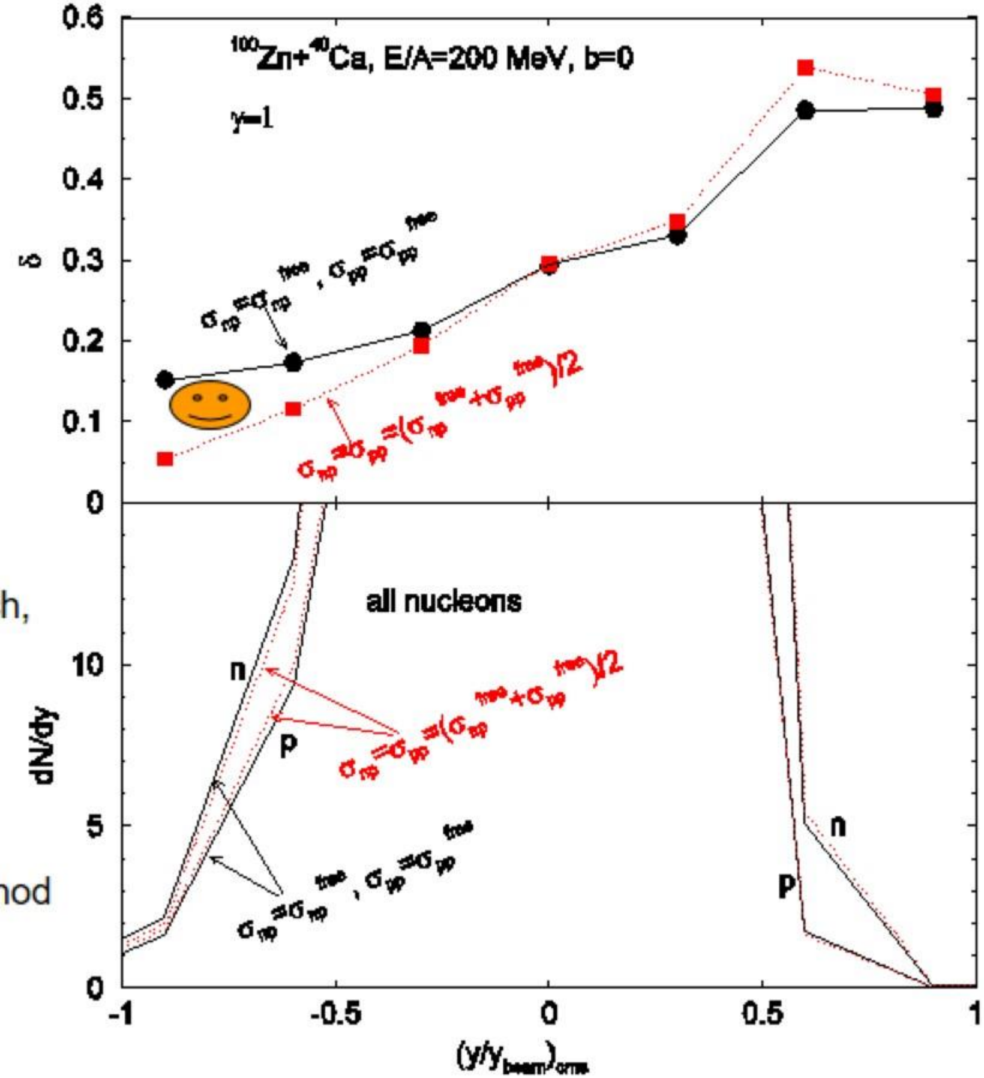
$$N_{coll} \propto N_{np}\sigma_{np} + (N_{nn}+N_{pp})\sigma_{pp} \approx N_{np}(\sigma_{np}+\sigma_{pp})$$

thus the total number of NN collisions is about the same with $\sigma_{\rm np} = \sigma_{np}^{free}$, $\sigma_{\rm pp} = \sigma_{pp}^{free}$ $(\sigma_{\rm np}/\sigma_{\rm pp} \approx 1.5 \sim 3)$

or $\sigma_{np} = \sigma_{pp} = \frac{1}{2}(\sigma_{np}^{free} + \sigma_{pp}^{free})$ $(\sigma_{\rm np}/\sigma_{\rm pp} = 1)$

Isospin tracer in the backward direction in radioactive beam+symmetric target reaction in inverse kinematics as a probe of the isospin-dependence of the σ^{medium}





Bao-An Li, Pawel Danielewicz and William G. Lynch,

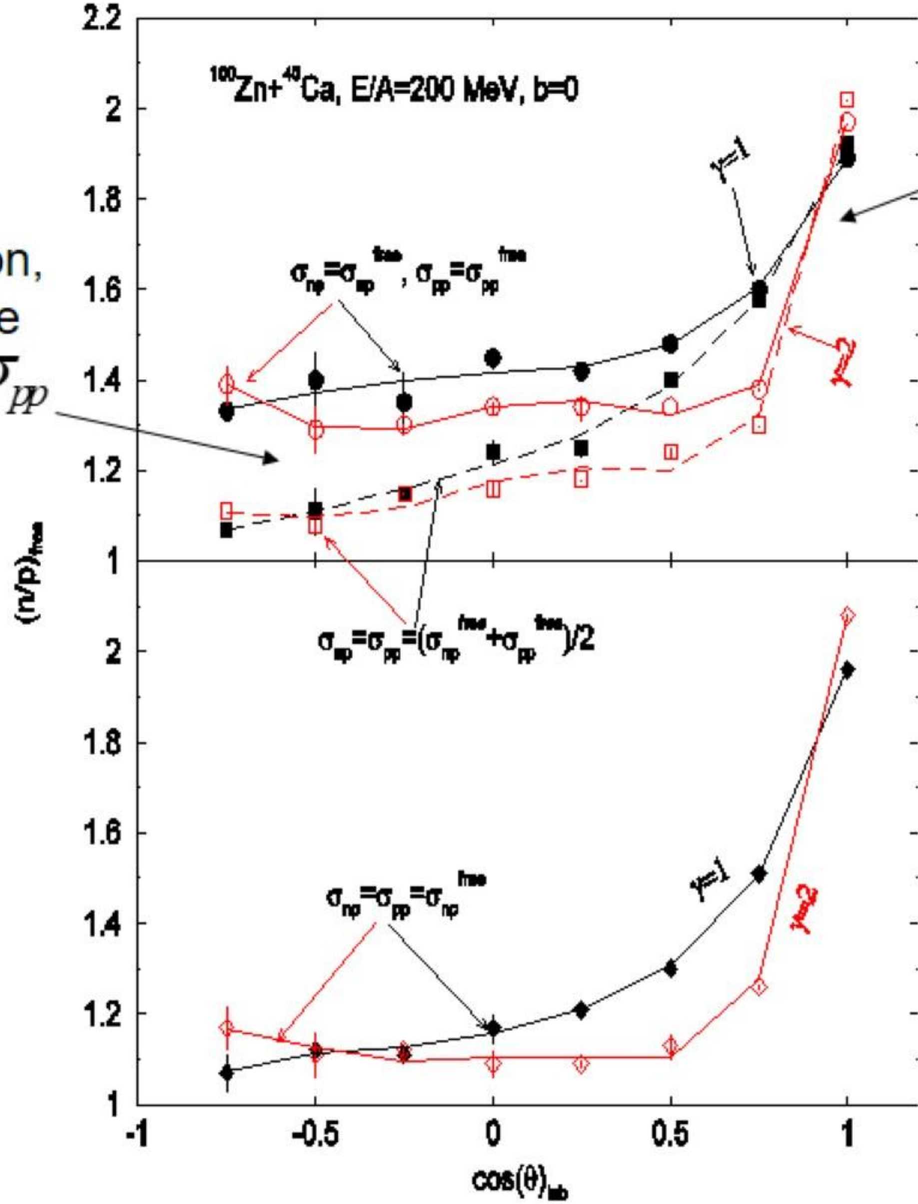
Phys. Rev. C**71**, 054603 (2005).

The initial n and p density profiles in ^{100}Zn were calculated using the Hartree-Fock-Bogoliubov method by J. Dobaczewski, Acta. Phys. Polon. B30, 1647 (1999)

Disentangle effects of the symmetry energy and in-medium NN xsections using reactions induced by radioactive beams in inverse kinematics

In backward region, it is more sensitive to the ratio σ_{np}/σ_{pp}

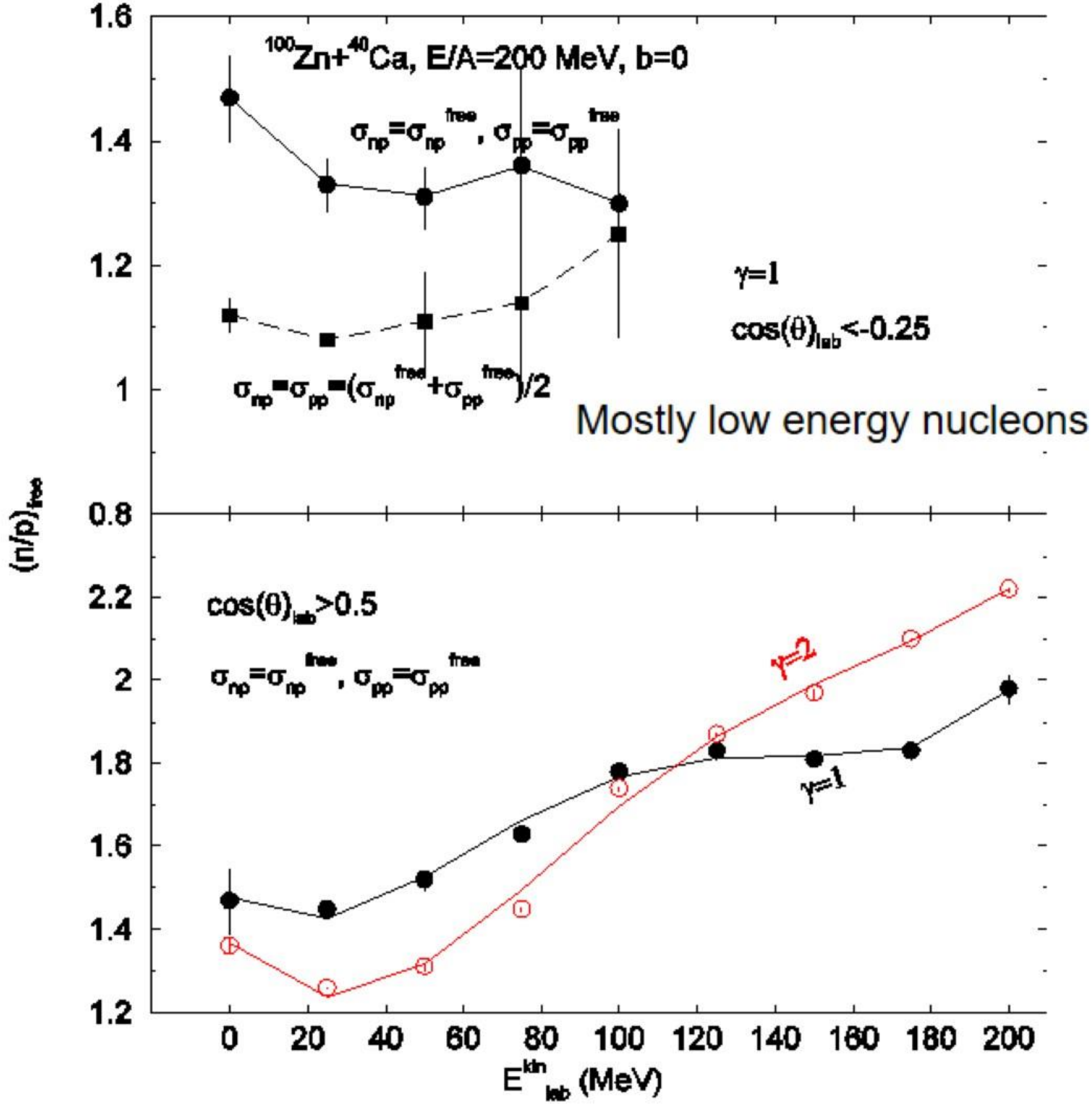
~15% effect



In forward region, the $(n/p)_{free}$ is more sensitive to the $E_{sym}(\rho)$

$$E_{sym}(\rho) = 30(\rho/\rho_0)^{\gamma}$$

Disentangle the isospin dependence of in-medium NN cross sections from that of the nuclear mean field



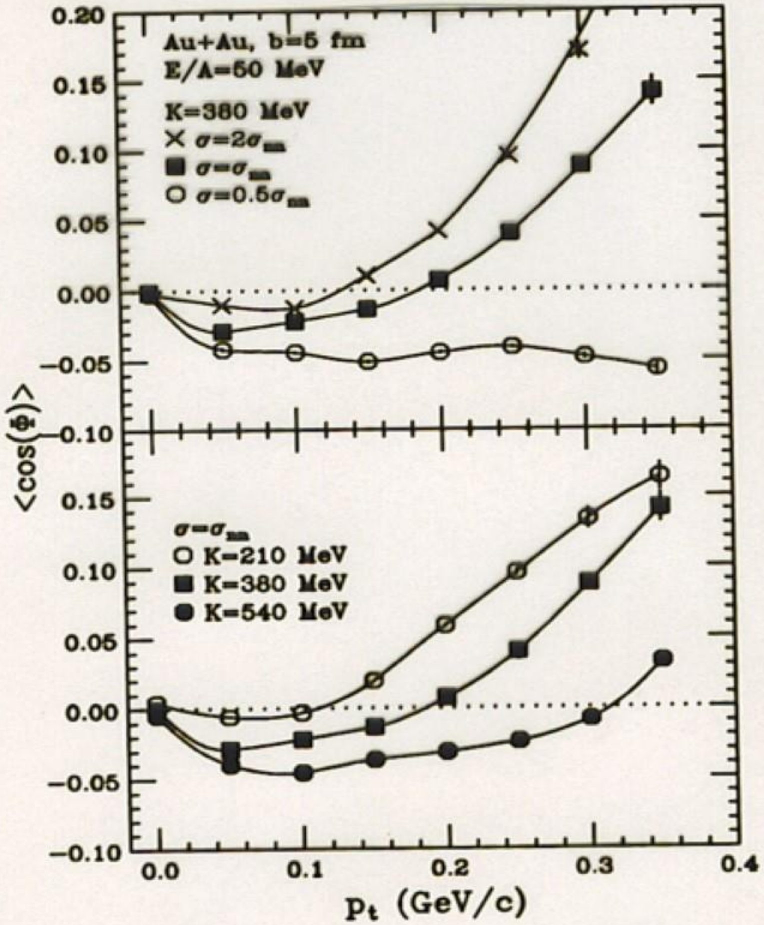


FIG. 3. Dependence of the differential flow on the in-medium cross section (upper) and compressibility (lower) for Au+Au at an impact parameter of 5 fm and a beam energy of 50 MeV/nucleon.

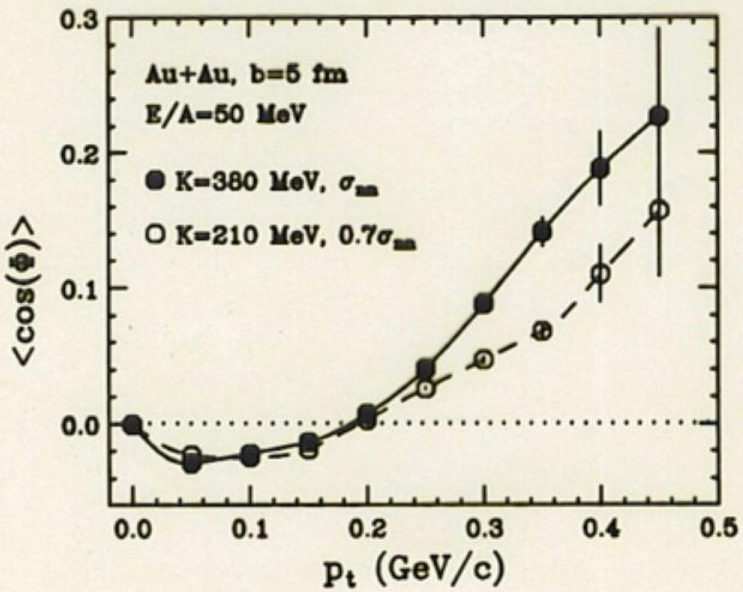


FIG. 2. Differential flow analysis for Au+Au at an impact parameter of 5 fm using two different parameter sets leading to the same balance energy of 50 MeV/nucleon.

Comparing the effective mass scaling model with DBHF in symmetric matter

F. Sammarruca and P. Krastev, nucl-th/0506081, Phys. Rev. C (2005) in press.