Electron scattering from nucleon and nuclei

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Lecture 1 : electron scattering in general

What is electron scattering?
what can we learn by electron scattering?
what have been revealed so far for stable nuclei?

Lecture 2 : selected topics related to NUSYS

electron scattering for

- proton charge radius
- exotic nuclei
- neutron distribution in nuclei

A series of lectures on electron scattering at Beihang Univ., Aug. 3, 4

Electron scattering from nucleon and nuclei

Introduction to electron scattering

what is electron scattering?
what can we learn from electron scattering?

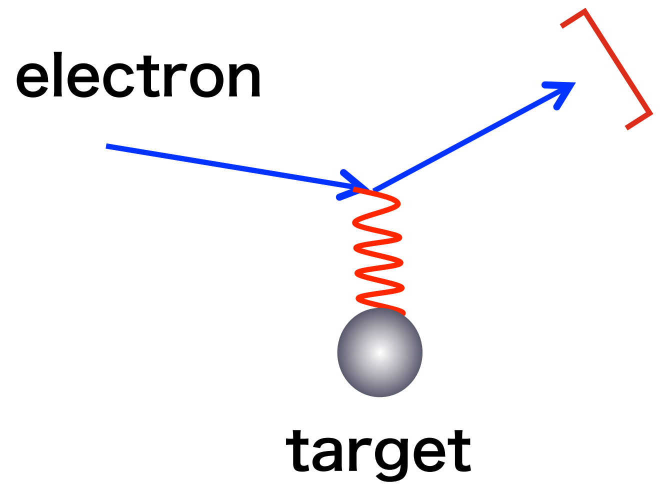
Selected topics of electron scattering for nucleon and nuclei

exotic nuclei
proton charge radius (+ neutron charge radius)
neutron distribution in nuclei

email me if you would like to have. (suda@lns.tohoku.ac.jp)

https://www.dropbox.com/scl/fo/irxbkzkd5pffbkrh5s234/h?rlkey=nf4xb3prcrg2pch65ck8ry1aj&dl=0

Electron scattering has consistently played an essential role to reveal detailed structures of nucleon and nuclei



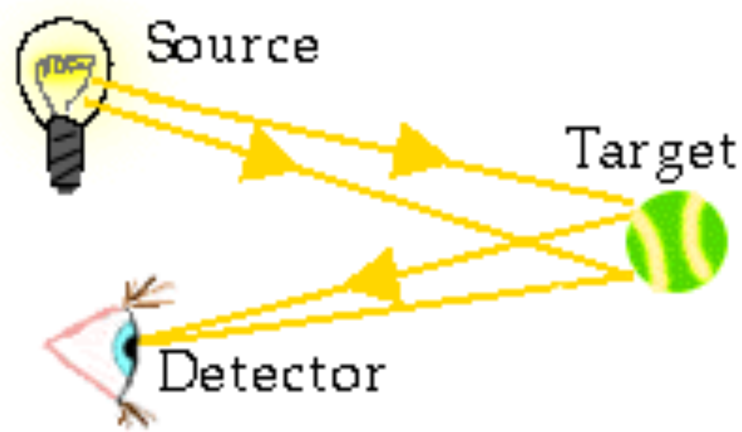
one detects only scattered electrons

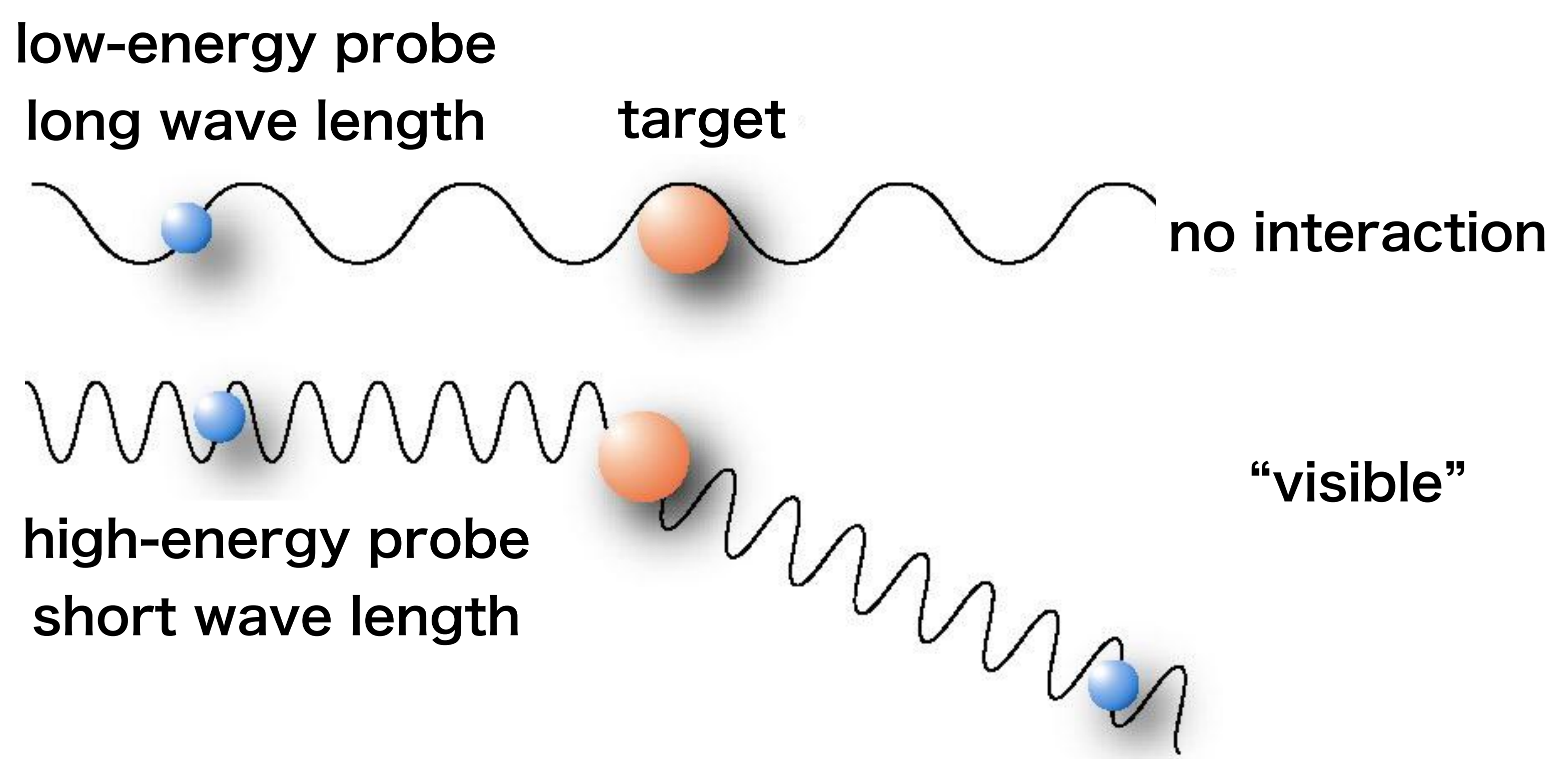
→ very "simple" measurements

1. elementary particle - structure-less -
2. electro-weak interaction - best understood -
3. "relatively" weak - probing deep inside of the target -

Electrons do not experience the nuclear strong force.

Electron beam energy for nuclear physics ?





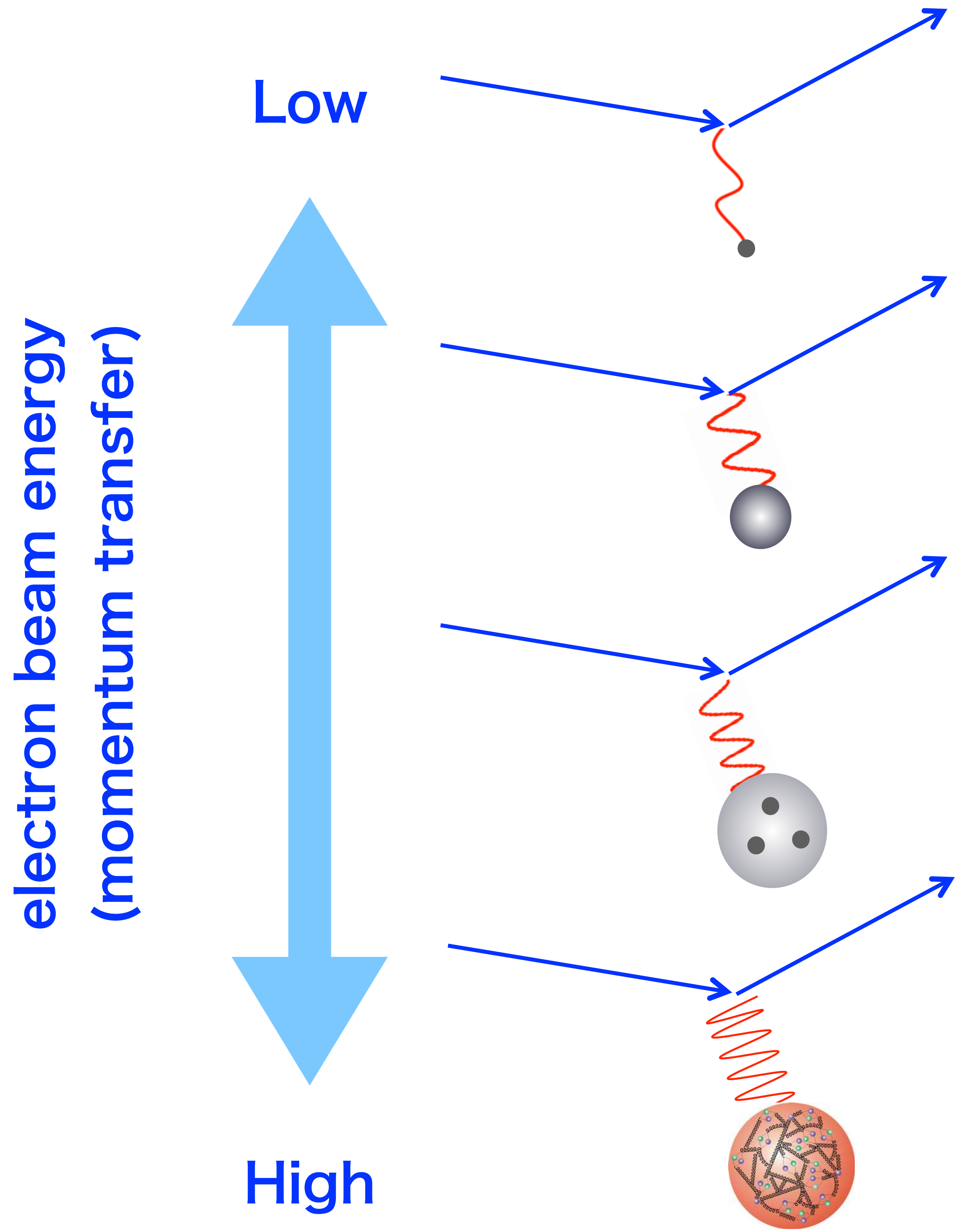
de Broglie wave length of electron ~ target size (~fm)

$$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} = \frac{\hbar}{p} = \frac{\hbar c}{pc}$$

$$= \frac{197[\mathrm{MeV}\cdot\mathrm{fm}]}{pc[\mathrm{MeV/c}\cdot\mathrm{c}]} \sim 1\mathrm{fm}$$

$$p \sim 200\ \mathrm{MeV/c}$$

Proton as an example



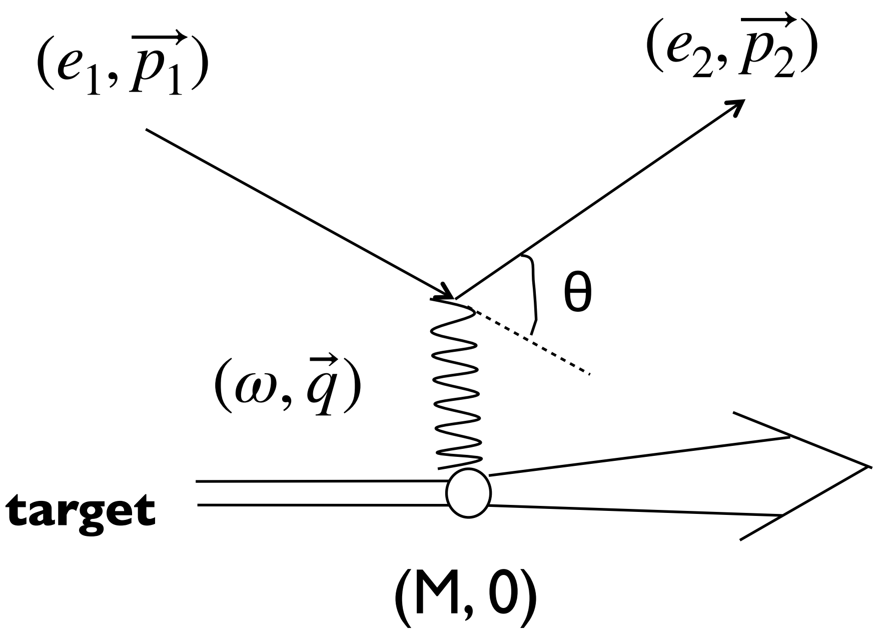
$$\lambda \gg r_p$$

$$\lambda \sim r_p$$

$$\lambda < r_p$$

$$\lambda \ll r_p$$

Lab. frame (taget at rest)



kinematical valuables under URL

$$\omega = e_1 - e_2 \quad \text{energy transfer}$$

$$\vec{q} = \vec{e_1} - \vec{e_2} \quad \text{momentum transfer}$$

$$Q^2 = \omega^2 - \vec{q}^2 = 4e_1 e_2 \sin^2\frac{\theta}{2} \quad \text{4-momentum transfer}$$

$$e^2 = \vec{p_e}^2 + m_e^2$$

URL (Ultra-relativistic limit) : $e >> m_e$ ⇨ $|\vec{p}_{1,2}| = e_{1,2}$

Mott cross section

$\beta \sim 1 \qquad e \sim p$

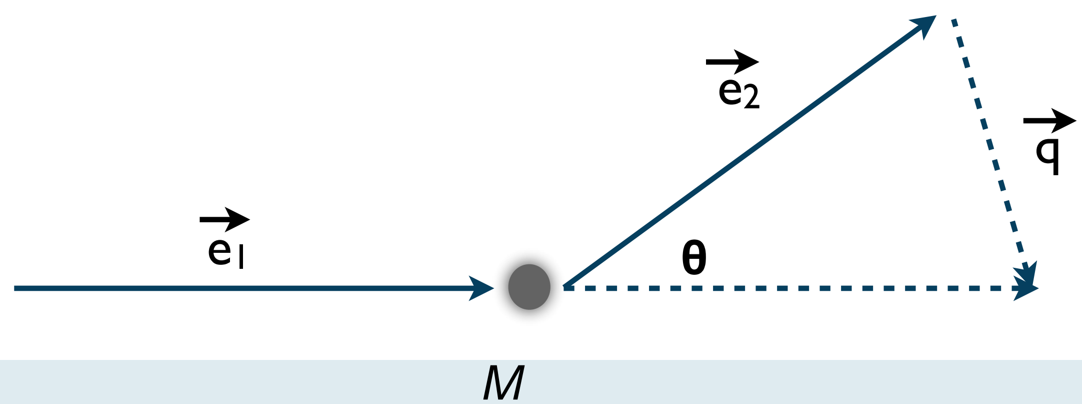
$$\frac{d\sigma_{Mott}}{d\Omega} = \frac{Z^2\alpha^2}{4e^2}\frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}$$

Rutherford cross section

$\beta \ll 1 \qquad e = \dfrac{p^2}{2m}$

$$\frac{d\sigma_{Rutherford}}{d\Omega} = \frac{Z^2\alpha^2}{16e^2}\frac{1}{\sin^4\frac{\theta}{2}}$$

Elastic scattering off atomic nucleus



kinematics

$$e_2 = \frac{e_1}{1 + \frac{2e_1}{M}\sin^2\frac{\theta}{2}}$$

$$M \gg e_1 \Rightarrow e_2 \sim e_1$$

$$|e_1| = |e_2| \quad (= e)$$

$$\omega = e_1 - e_2 = 0$$

$$|q| = 2e\sin(\theta/2)$$

$$q = e_1 (= e_2)$$

$$(\theta = 60^\circ)$$

cross section (under one-photon-exchange approximation)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{Mott}}{d\Omega}|F_c(q)|^2$$

$$F_c(\vec{q}) = \int \rho(\vec{r})e^{-i\vec{q}\cdot\vec{r}}d\vec{r}$$

form factor

including target structure information

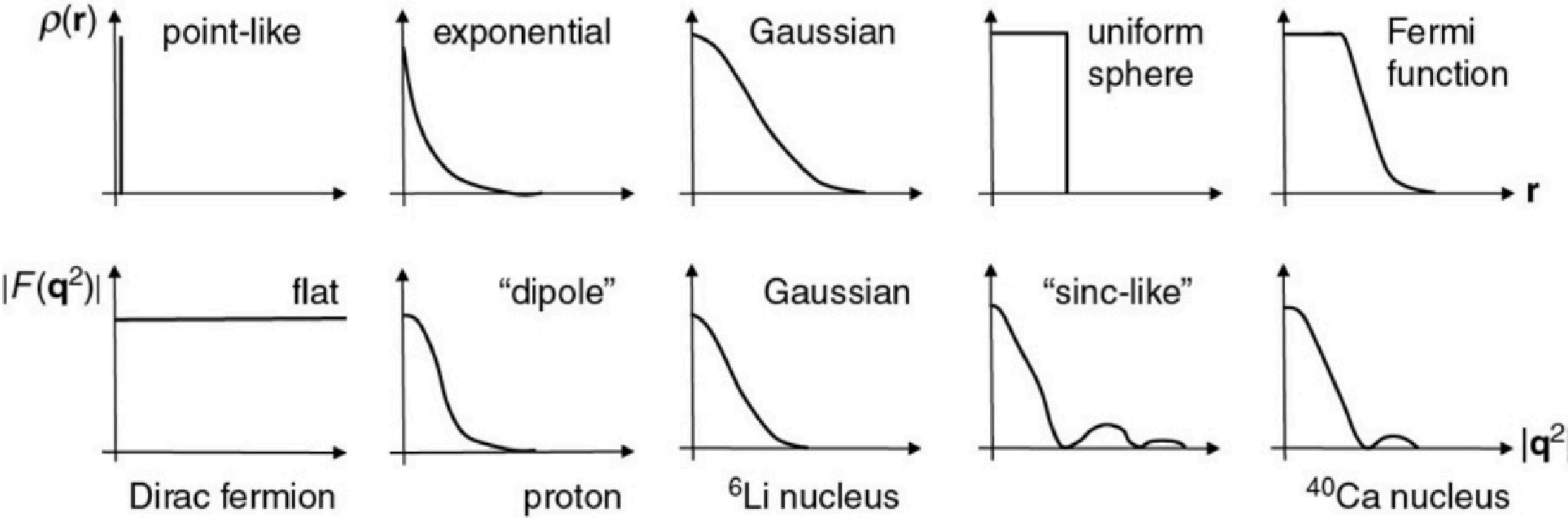
(=1 if the target is a point particle having the charge Z)

Form factors and charge distributions

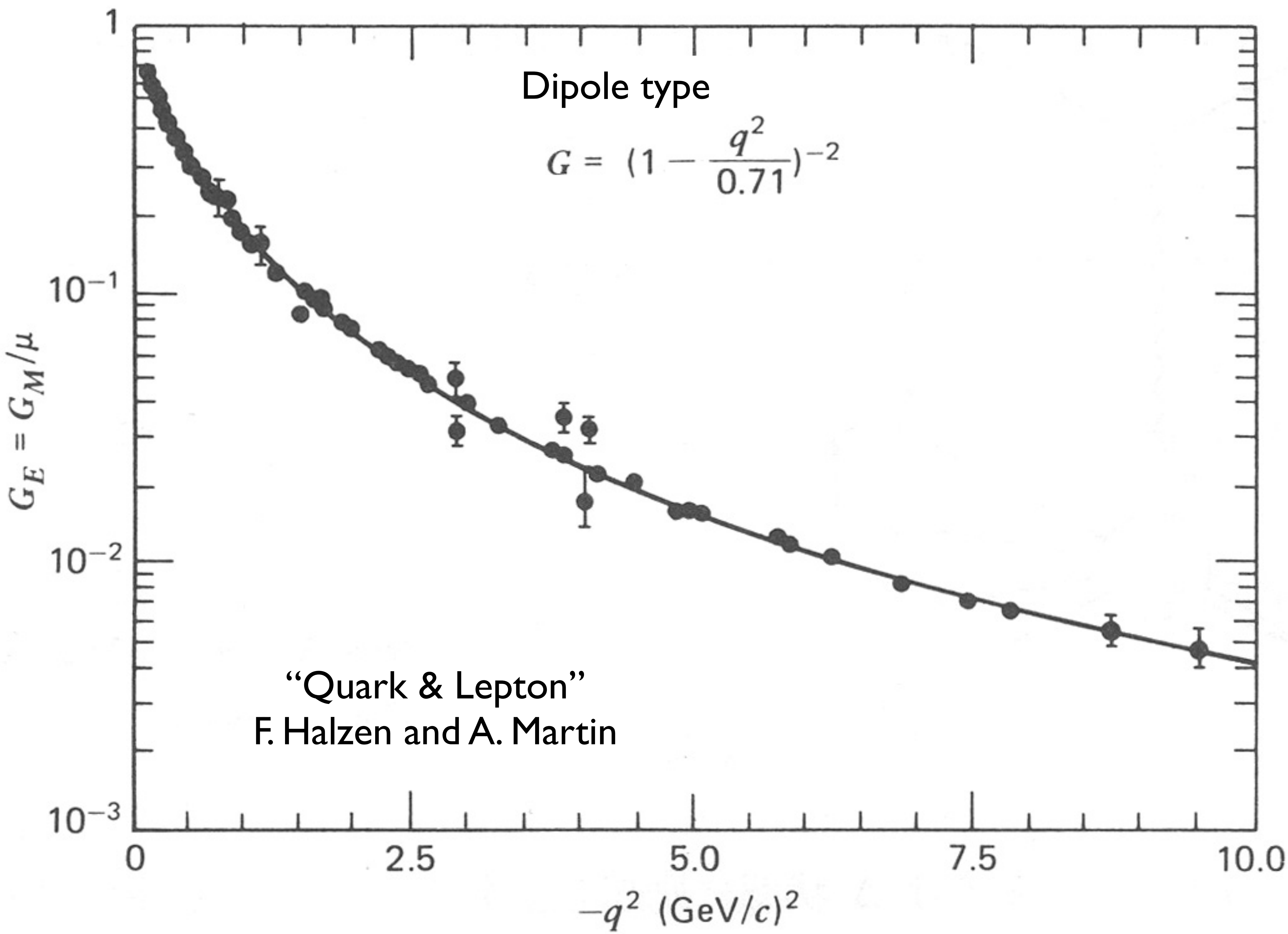
$$F_c(\vec{q}) = \int \rho(\vec{r}) e^{-i\vec{q}\cdot\vec{r}} d\vec{r}$$

for spherically symmetric distributions

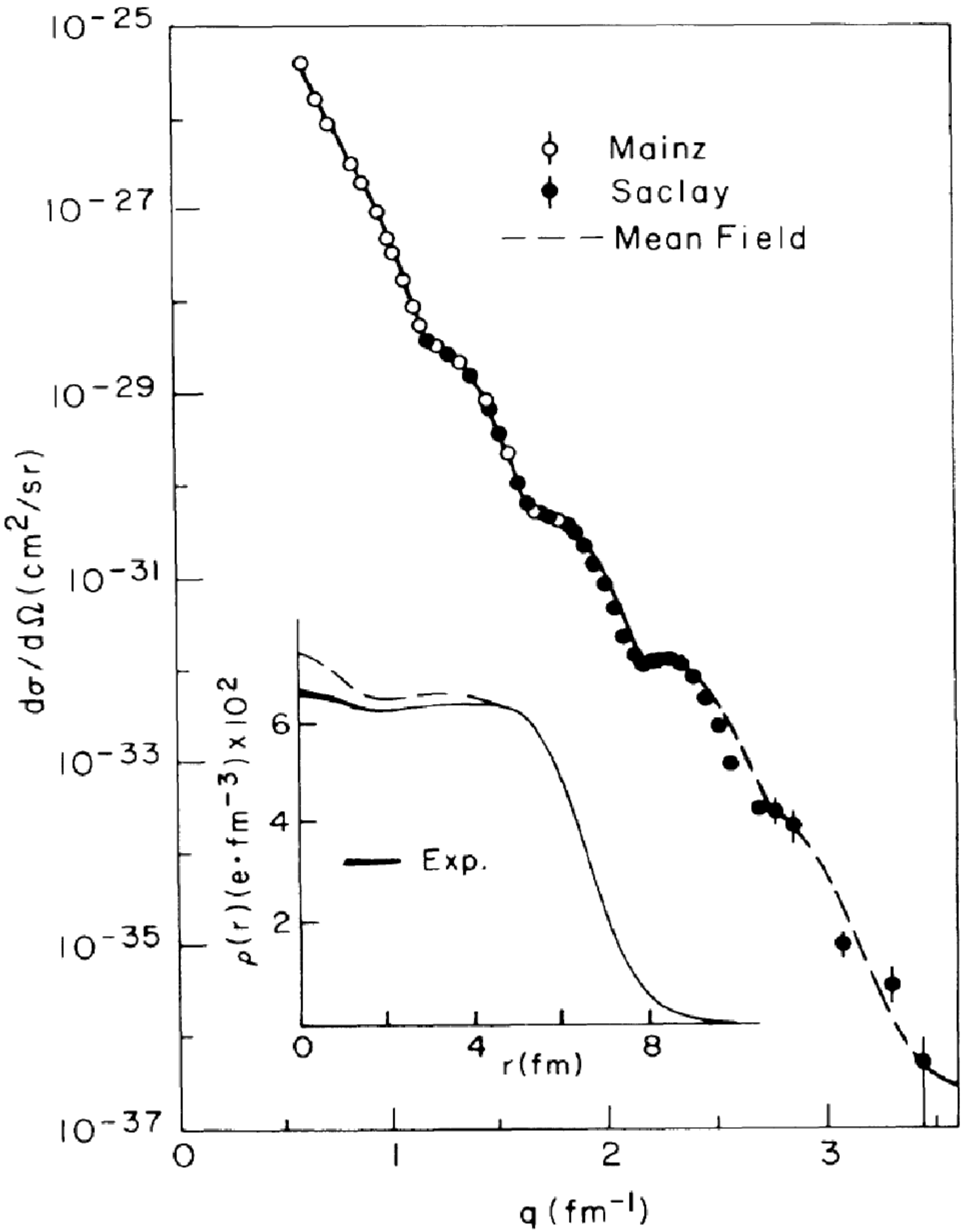
$$F_c(q) = 4\pi \int_0^\infty \rho(r) \frac{\sin(qr)}{qr} r^2 \, dr$$



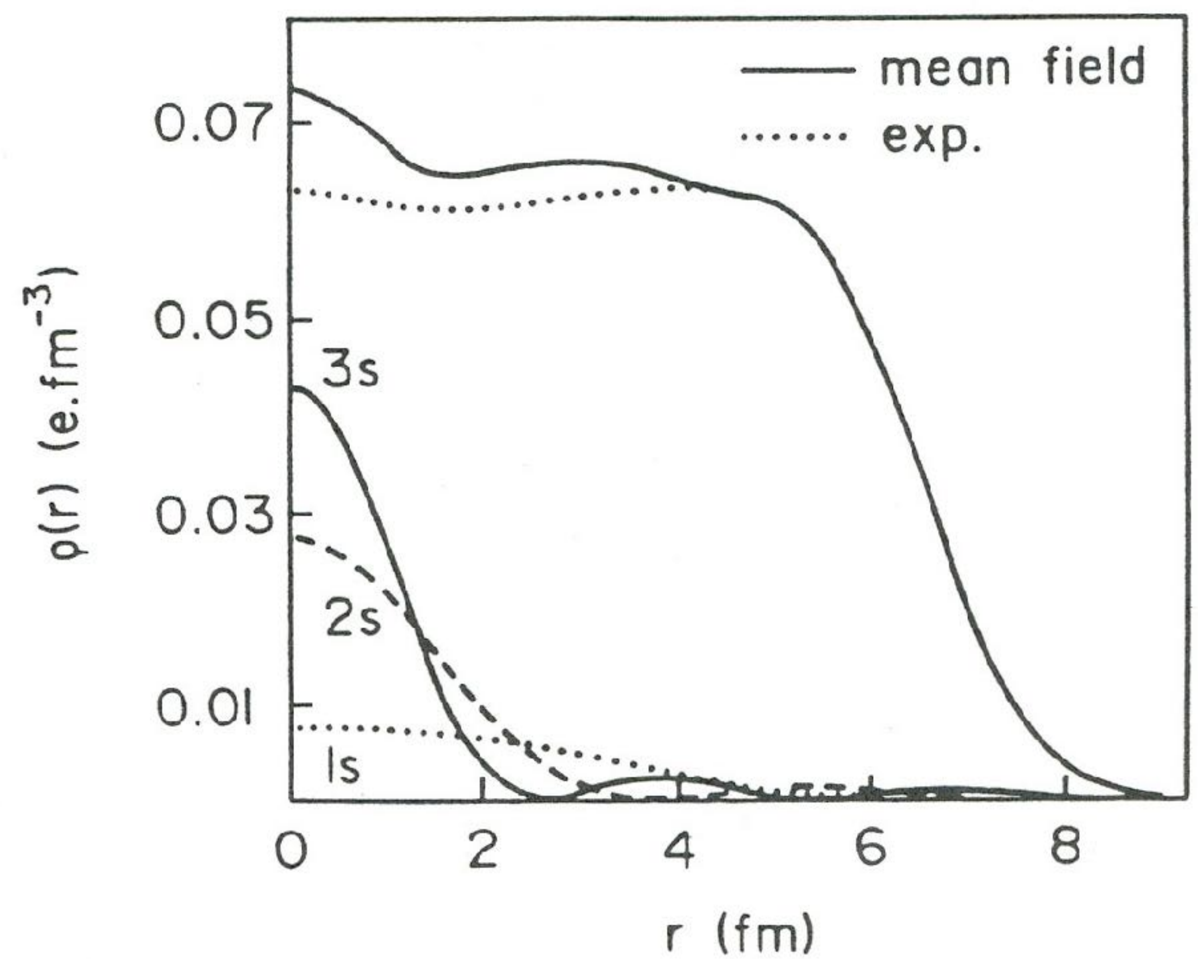
proton (charge) form factor



"Quark & Lepton"
F. Halzen and A. Martin



J. M. Cavedon et al. PRL 58 (1987) 195 -198.



Phys. Rev. Lett. 38(1977)153.

“Nuclear Structures” in textbooks

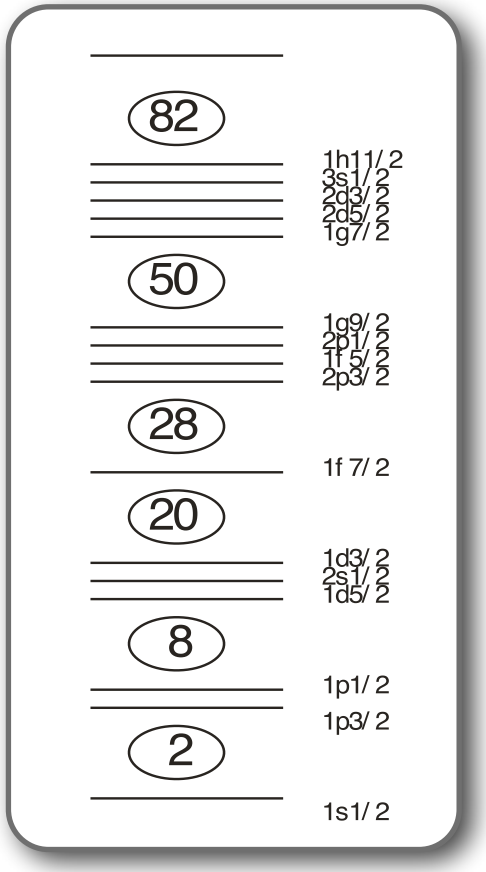
Magic numbers and stability

2, 8, 20, 28, 50, 82, 126

doubly magic nuclei

^{4}He, ^{16}O, ^{40,48}Ca, ^{208}Pb


M.G.Mayer and J.Jensen



Density distributions

electron scattering

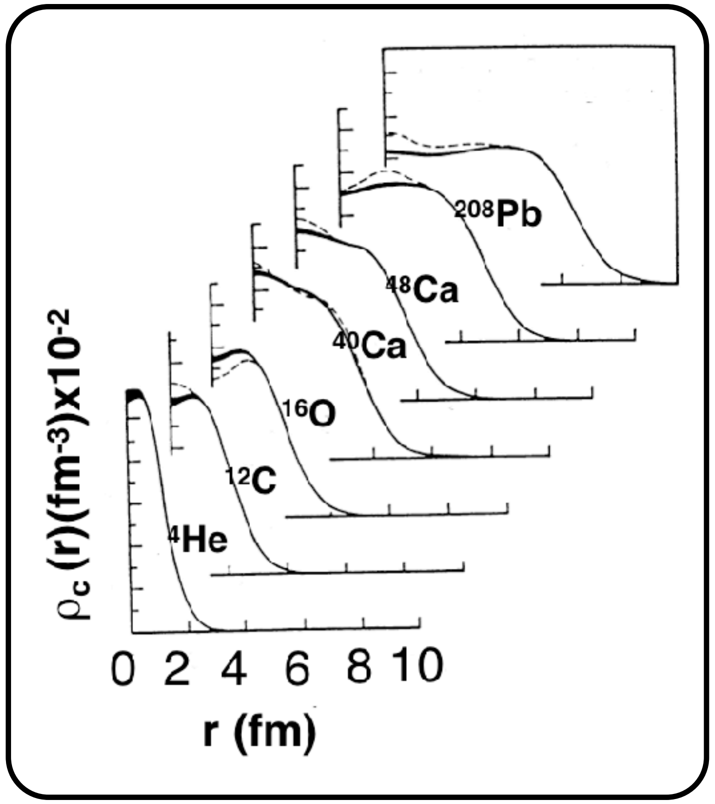
1） **radius and mass number** $\mathrm{r} = \mathrm{r_0 A^{1/3}}$

2） **density saturation**

3） **surface diffuseness ~ const.**

4） **$\rho_p(r) \sim \rho_n(r)$**

R. Hofstadter



kinematical valuables

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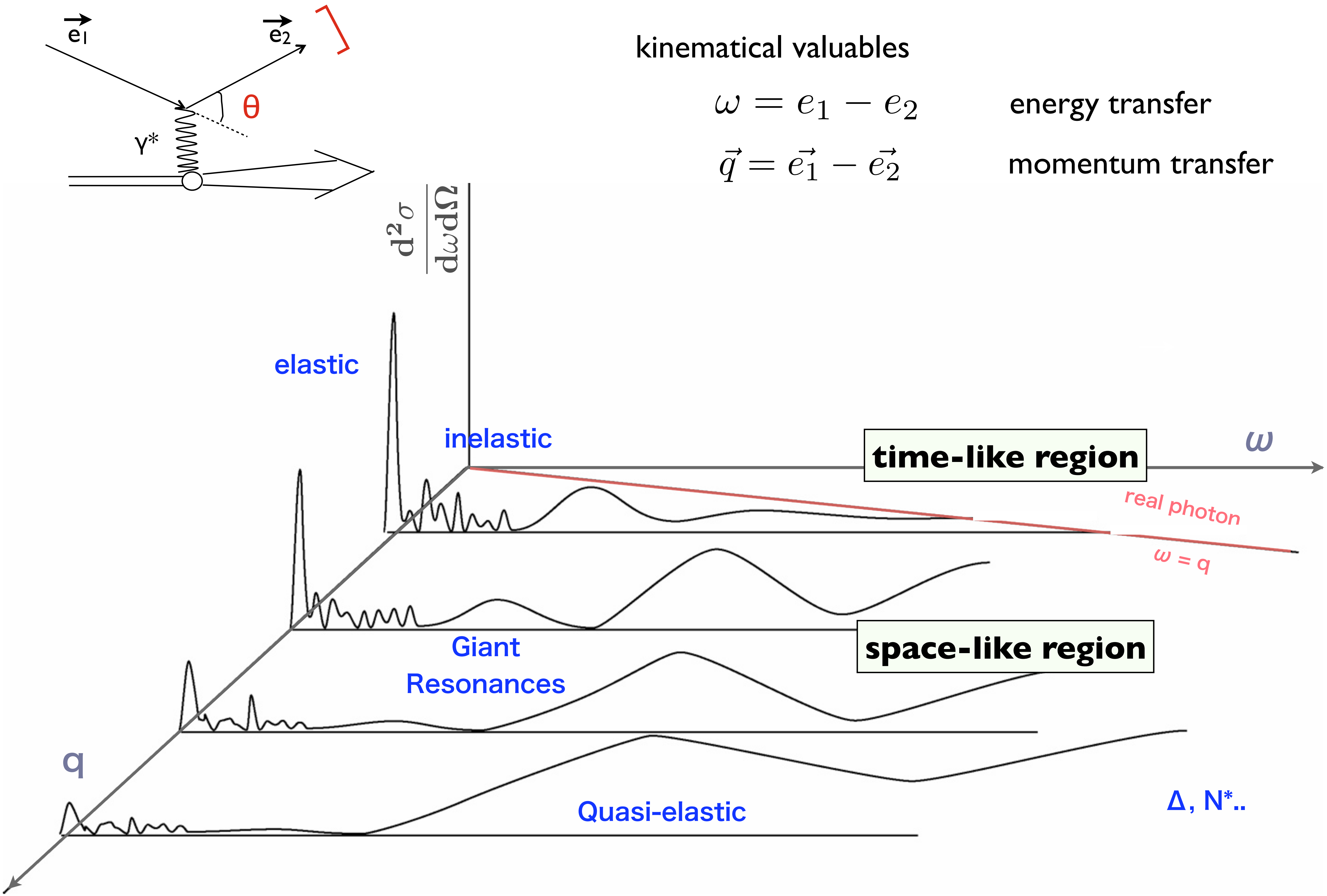


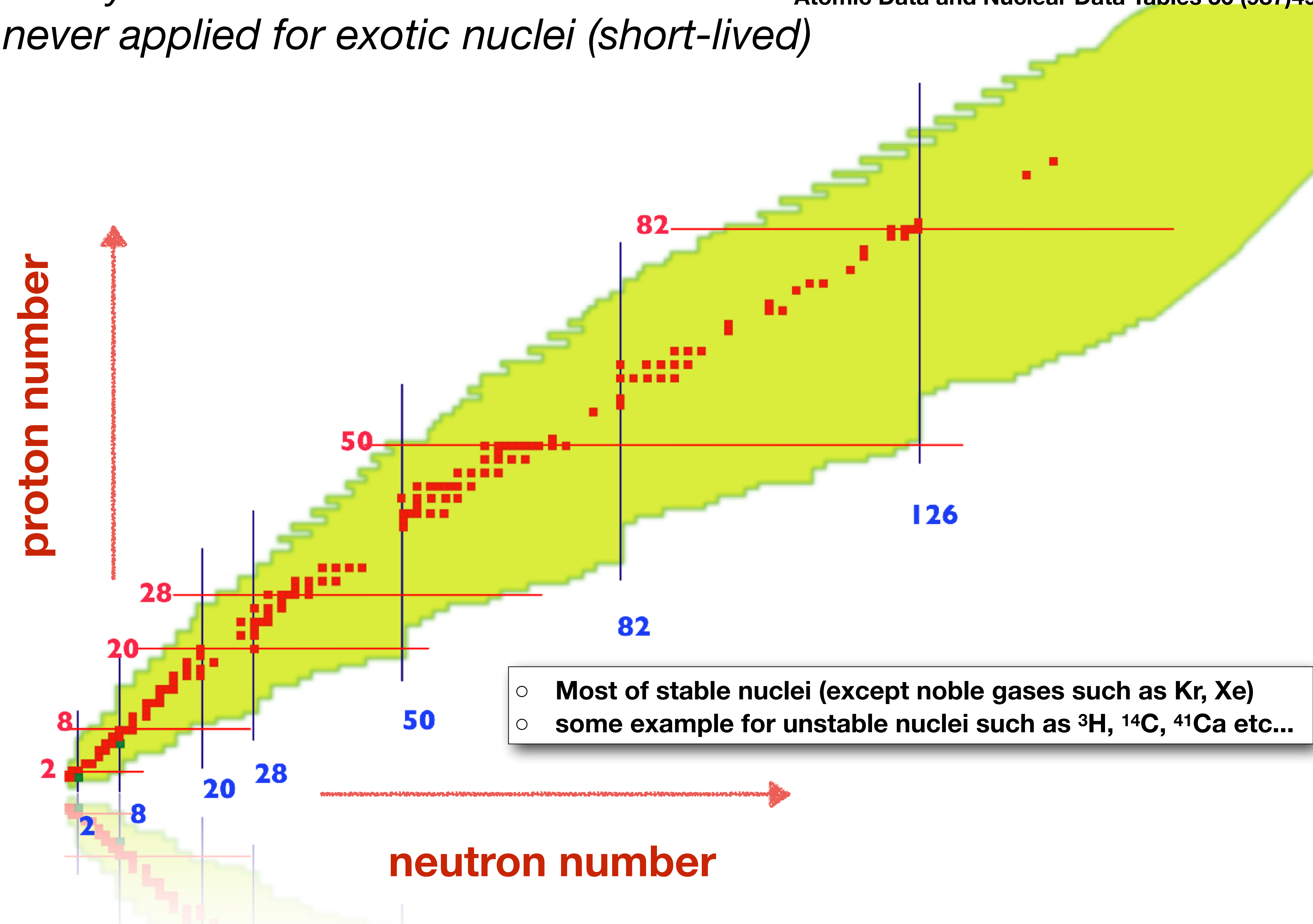
Elastic scattering

1) Charge density distributions
2) single-particle properties
3) deformed nuclei
4) valence neutron

nuclei ever studied by electron scattering

- *strictly limited to stable nuclei*
- *never applied for exotic nuclei (short-lived)*

H.deVries, C. deJager and C. deVries
Atomic Data and Nuclear Data Tables 36 (987)495



- Most of stable nuclei (except noble gases such as Kr, Xe)
- some example for unstable nuclei such as ^{3}H, ^{14}C, ^{41}Ca etc...

elastic scattering for 0+ nuclei

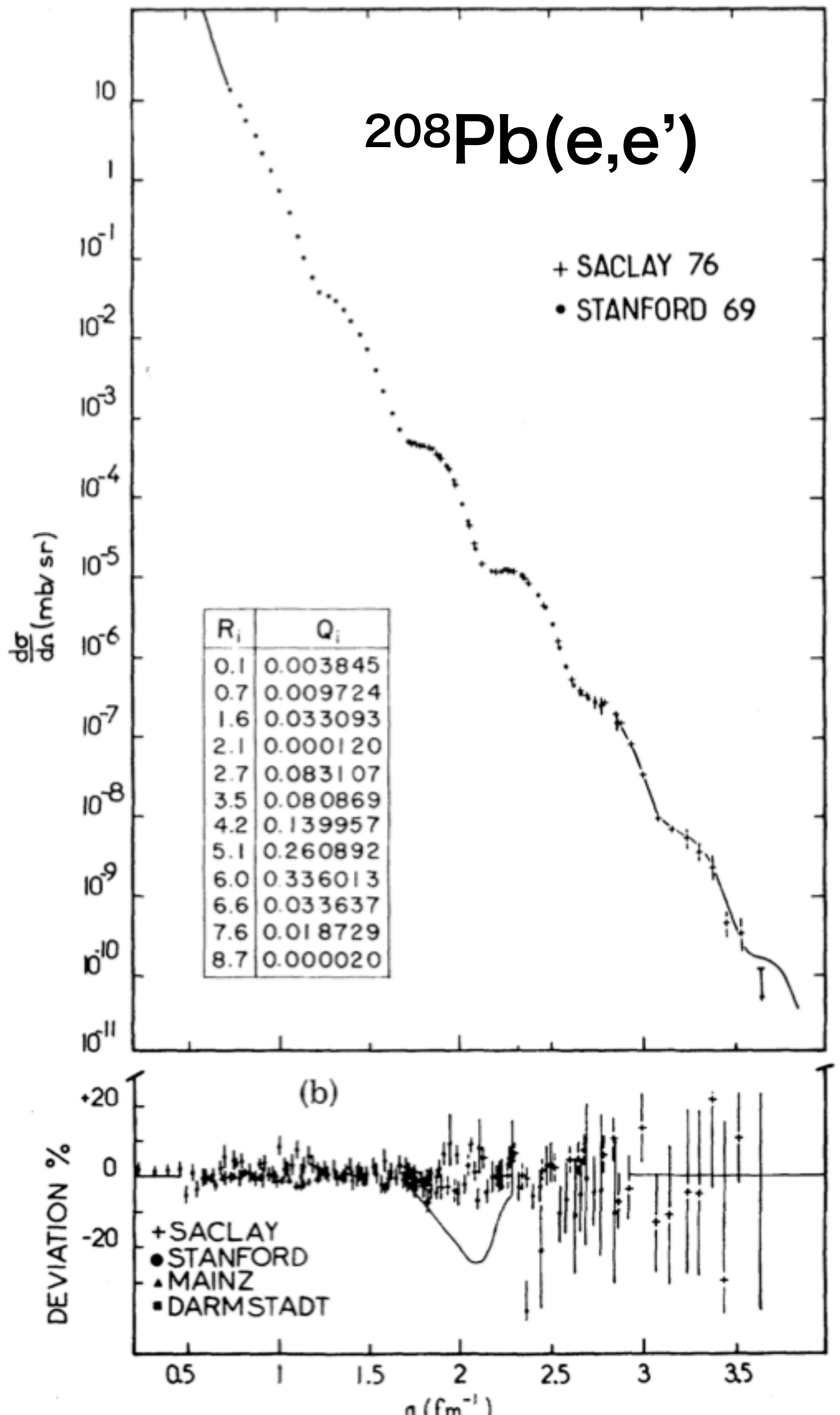
even-even nuclei

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{Mott}}{d\Omega} \, |F_c(q)|^2$$

$$F_c(q) = \int \rho(\vec{r}) \, e^{-i\vec{q}\cdot\vec{r}} \, d\vec{r}$$

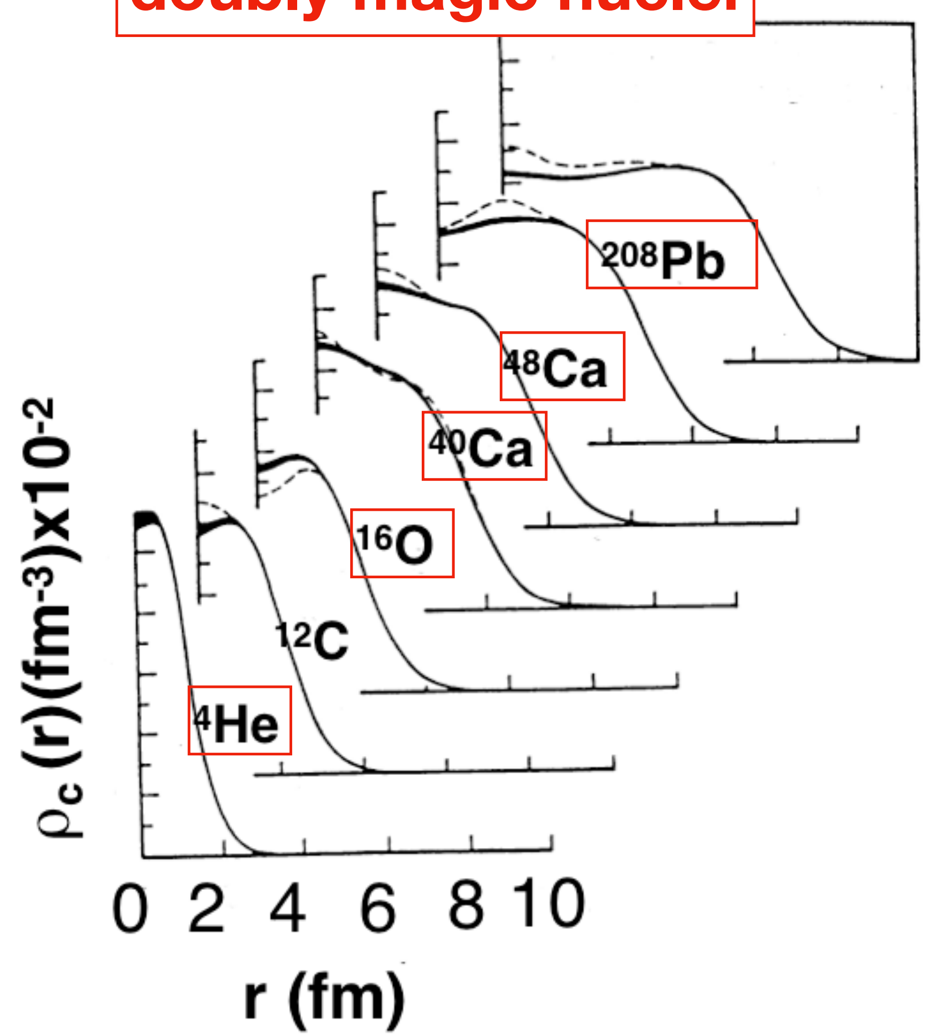
$$\Rightarrow \quad \rho(r) = \int F_c(q) \, e^{i\vec{q}\cdot\vec{r}} \, d\vec{q}$$

$$\Rightarrow \quad \rho(r) = \sum_{i=1}^{Z} |\phi_i(r)|^2$$



Phys. Rev. Lett. 38(1977)153.

doubly magic nuclei

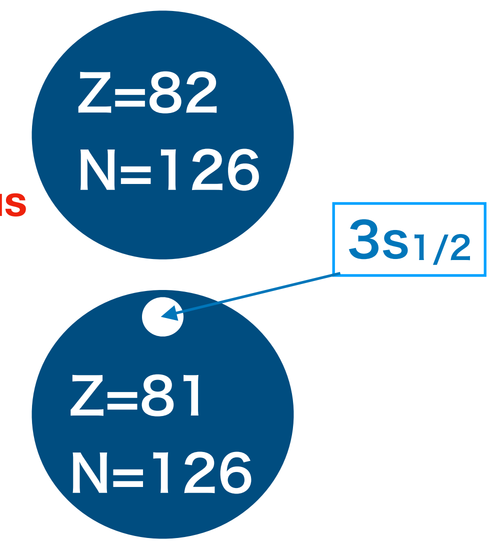


let's take ^{208}Pb and ^{207}Tl as examples

^{208}**Pb**

doubly magic nucleus

^{207}**Tl**



$$\rho_{^{208}Pb}(r) = \sum_{i=1}^{82} |\psi_i(r)|^2 = \int F_{^{208}Pb}(q)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{q}$$

$$\rho_{^{207}Tl}(r) = \sum_{i=1}^{81} |\psi_i(r)|^2 = \int F_{^{207}Tl}(q)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{q}$$

under a naive independent particle picture

$$|\psi_{3s_{1/2}}(r)|^2 = \sum_{i=1}^{82} |\psi_i(r)|^2 - \sum_{i=1}^{81} |\psi_i(r)|^2$$

$$= \rho_{^{208}Pb}(r) - \rho_{^{207}Tl}(r)$$

$$= \int \left[F_{^{208}Pb}(q) - F_{^{207}Tl}(q) \right] e^{i\vec{q}\cdot\vec{r}}\, d\vec{q}$$

elastic electron scattering
for ^{208}Pb and ^{207}Tl

only electromagnetic interaction involved !!

3s1/2 orbit from electron scattering

^{208}Pb - ^{207}Tl

unstable
$\tau \sim$ 5 min.

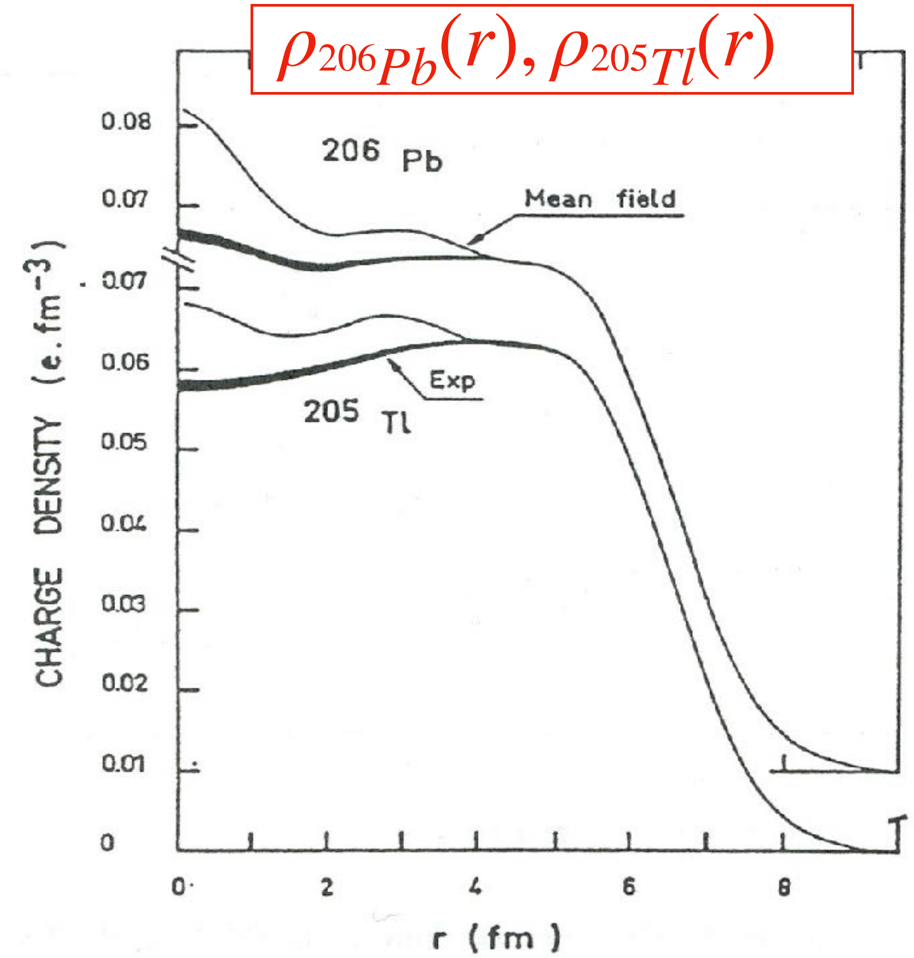
Po							
Bi	203	204	205	206	207	208	209
Pb	202	203	204	205	206	207	208
Tl	201	202	203	204	205	206	207
Hg	200	201	202	203	204	205	206
Au							

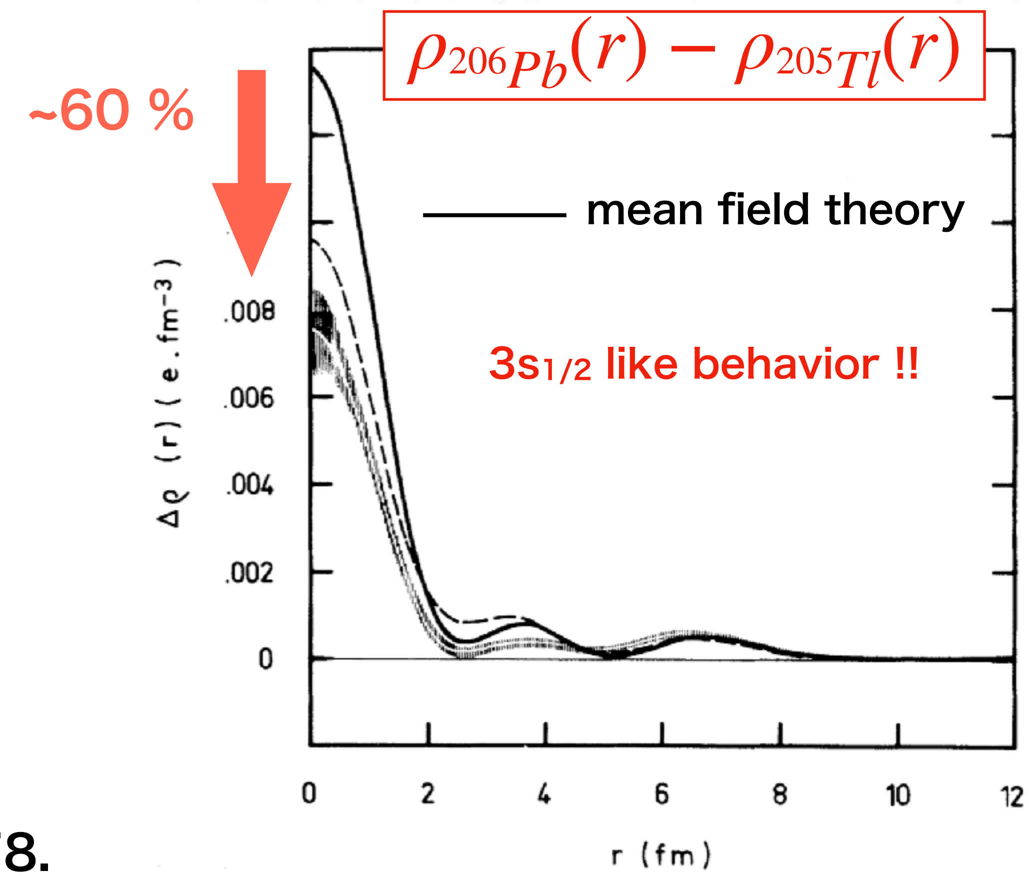
Z = 82

N = 126

instead

^{206}Pb - ^{205}Tl

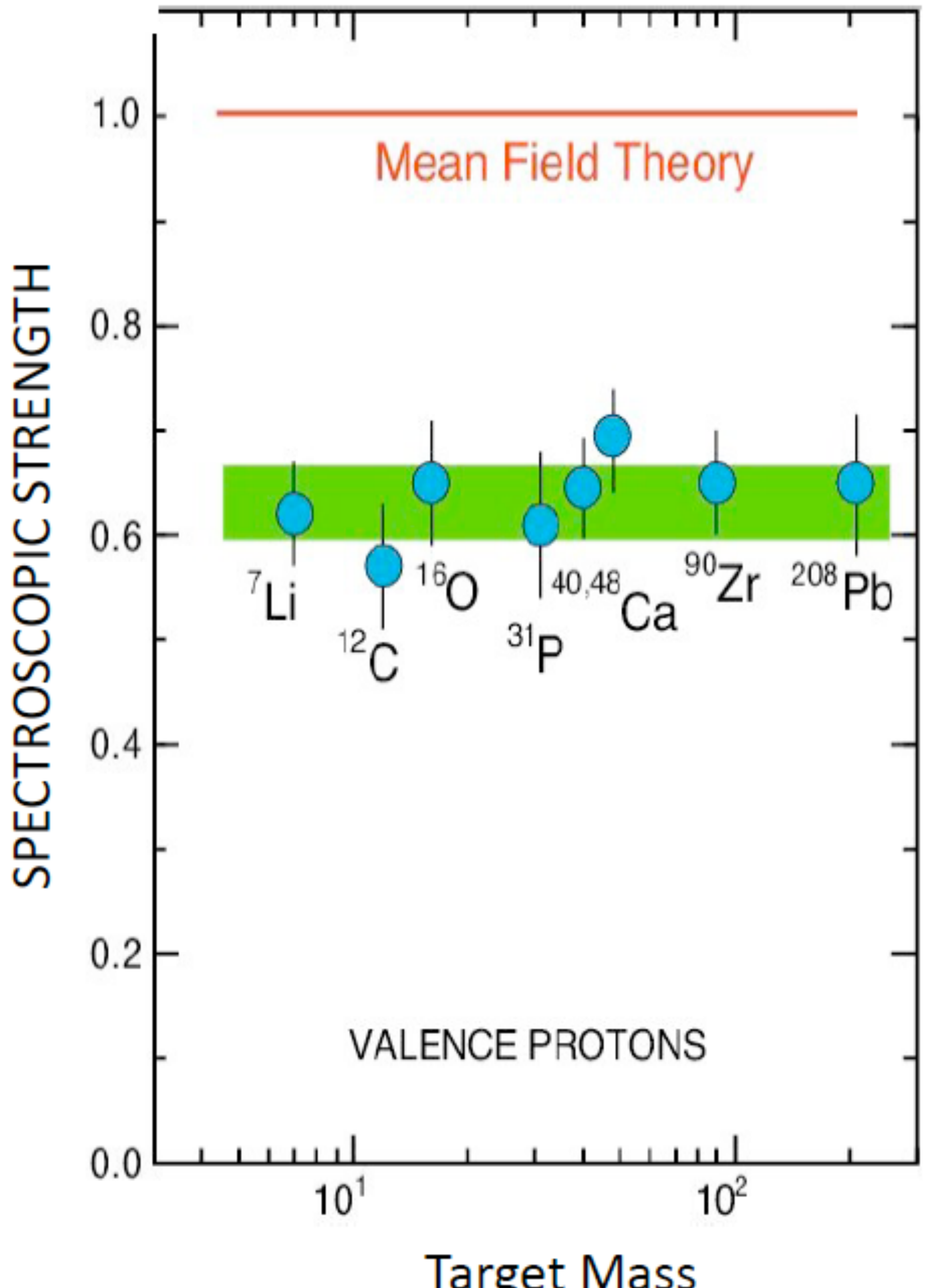


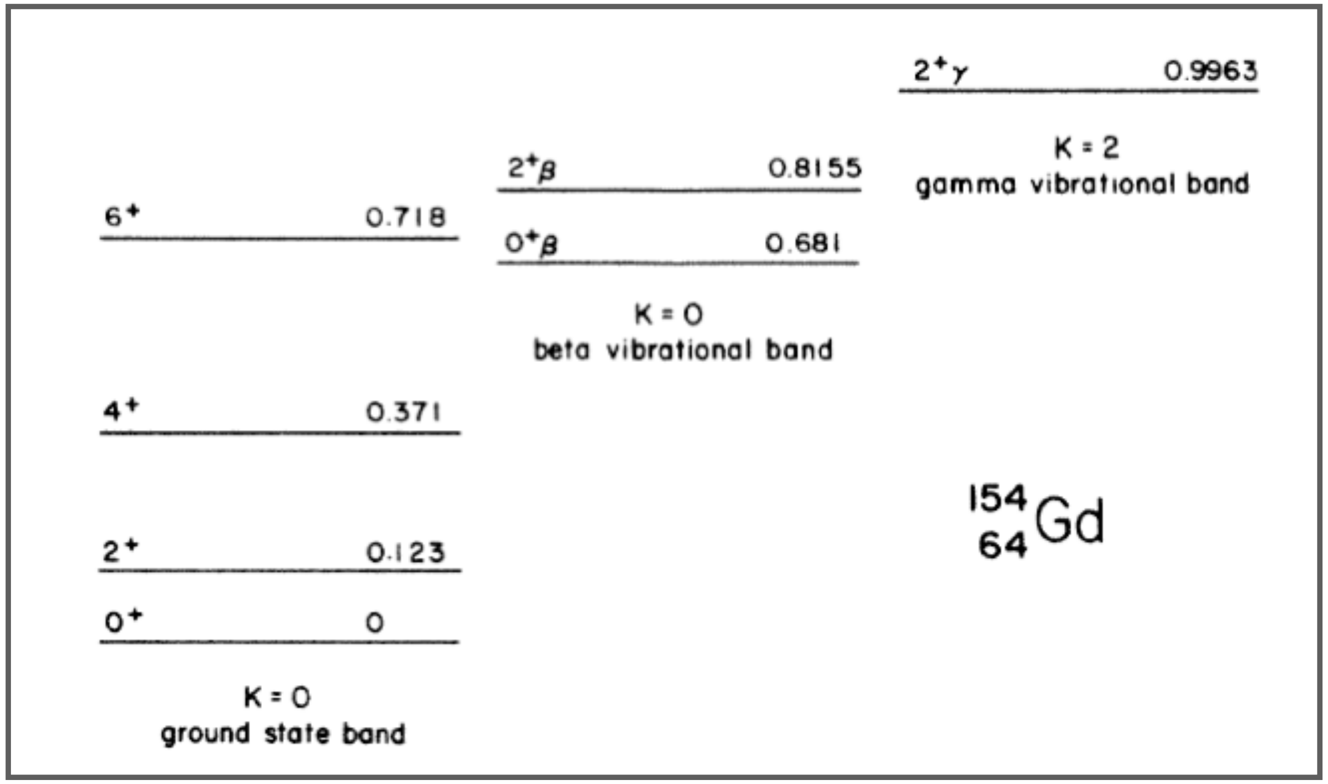


Phys. Rev. Lett. 49 (1982)978.

spectroscopic factors deduced from quasi-elastic (e,e'p)

60-70 % of the occupation expected from the independent particle model





elastic + inelastic electron scattering

Transition form factors (0+-> 2+, 4+, 6+)

➡ their transition densities

$$\rho_\lambda(r) =< \lambda||\hat{\rho}||0 >= \frac{1}{\sqrt{2I+1}} \int \rho(r,\Omega) Y_{\lambda 0}(\Omega) d\Omega$$

$$\rho(r,\Omega) = \sum_\lambda \rho_\lambda(r) Y_{L0}(\Omega)$$

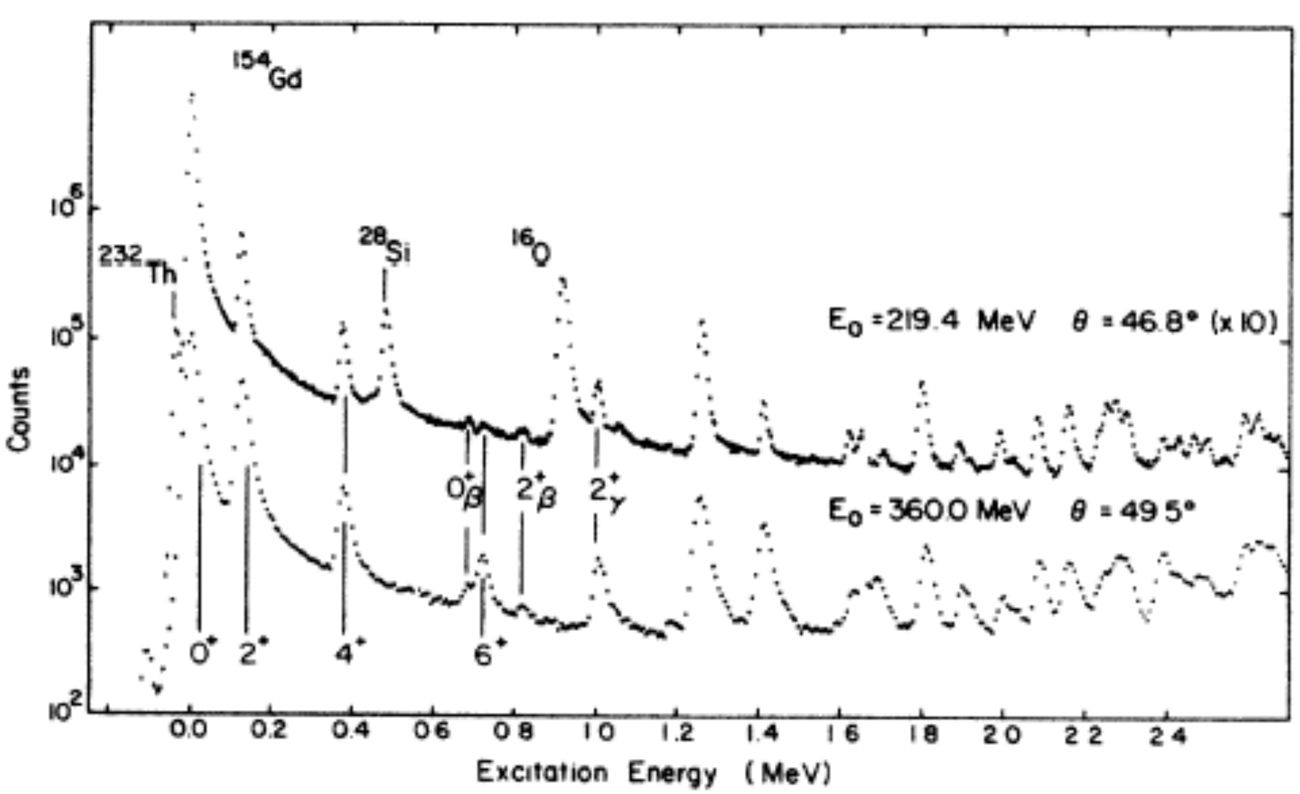


FIG. 5. Energy loss spectra of electrons scattered inelastically from ^{154}Gd at two incident energies. Spectrometer momentum resolution was 6×10^{-5}.

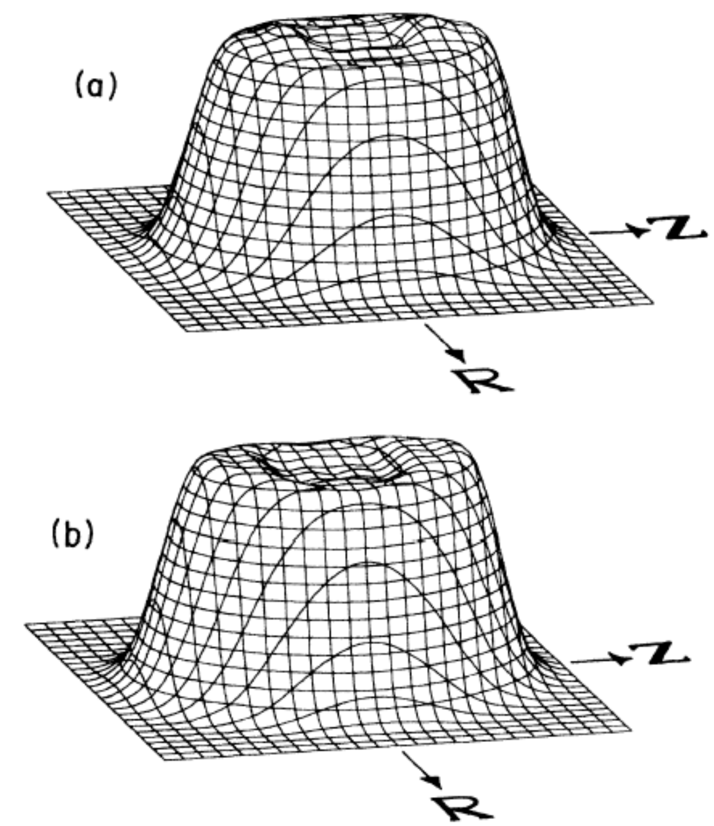


FIG. 3. Deformed charge density distributions in the body reference frame for (a) the calculated density, and (b) the density reconstructed from the experimental results. The symmetry axis and a perpendicular axis are denoted by Z and R, respectively.

Phys. Rev. C33(1986)1905.

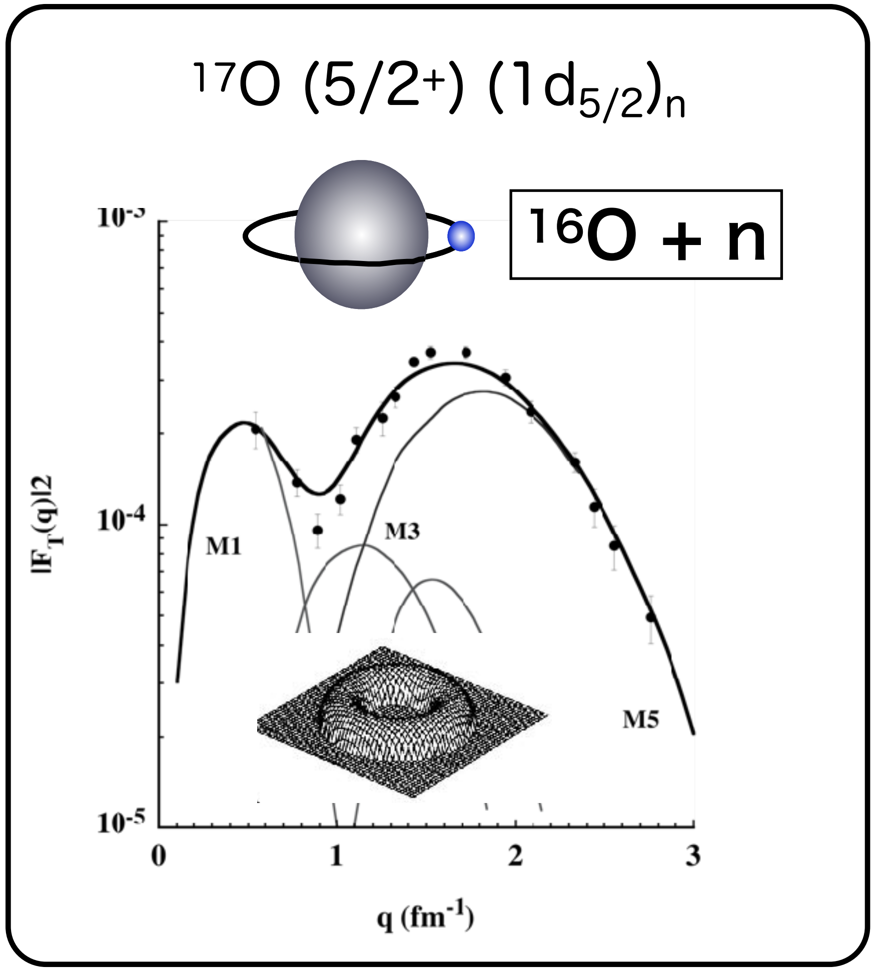
$$\frac{d\sigma_{Mott}}{d\Omega} = 0 \qquad \frac{d\sigma_{Mott}}{d\Omega} = \frac{Z^2\alpha^2}{4e^2}\frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}$$

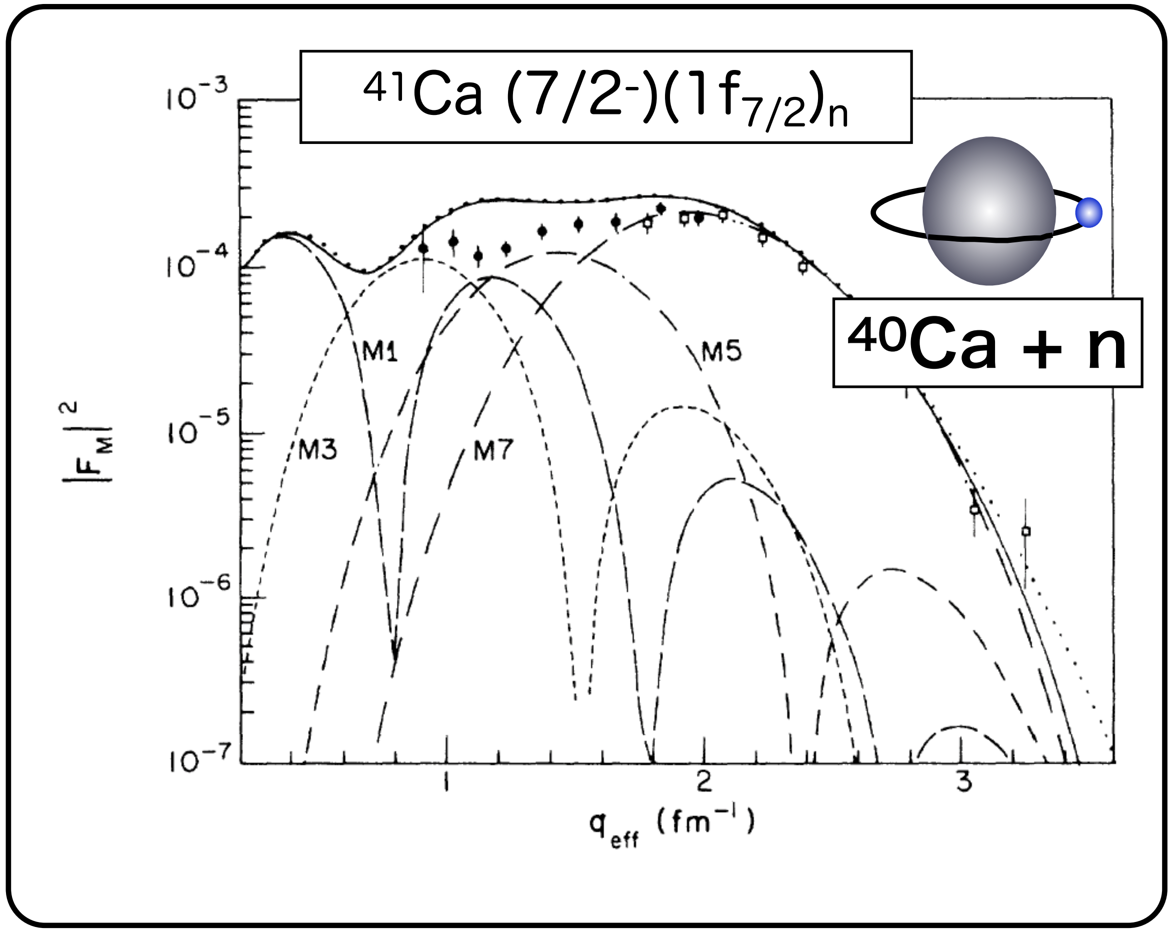
$$\frac{d\sigma}{d\Omega}(\theta = 180^\circ) \propto \frac{\cancel{\cos^2\frac{\theta}{2}}}{\sin^4\frac{\theta}{2}} \times \frac{\sin^2\frac{\theta}{2}}{\cancel{\cos^2\frac{\theta}{2}}} |<J_0||T_J^M(q)||J_0>|^2$$

Only magnetic term survives at 180°

Core + valence neutron : ^{17}O, ^{41}Ca

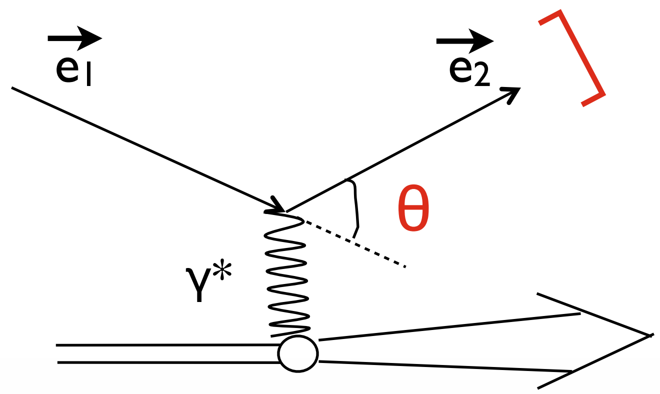
electron scatters off the valence neutron via its anomalous magnetic moment





T. Donnelly and I. Sick, Rev. Mod. Phys. 56 (1984) 461

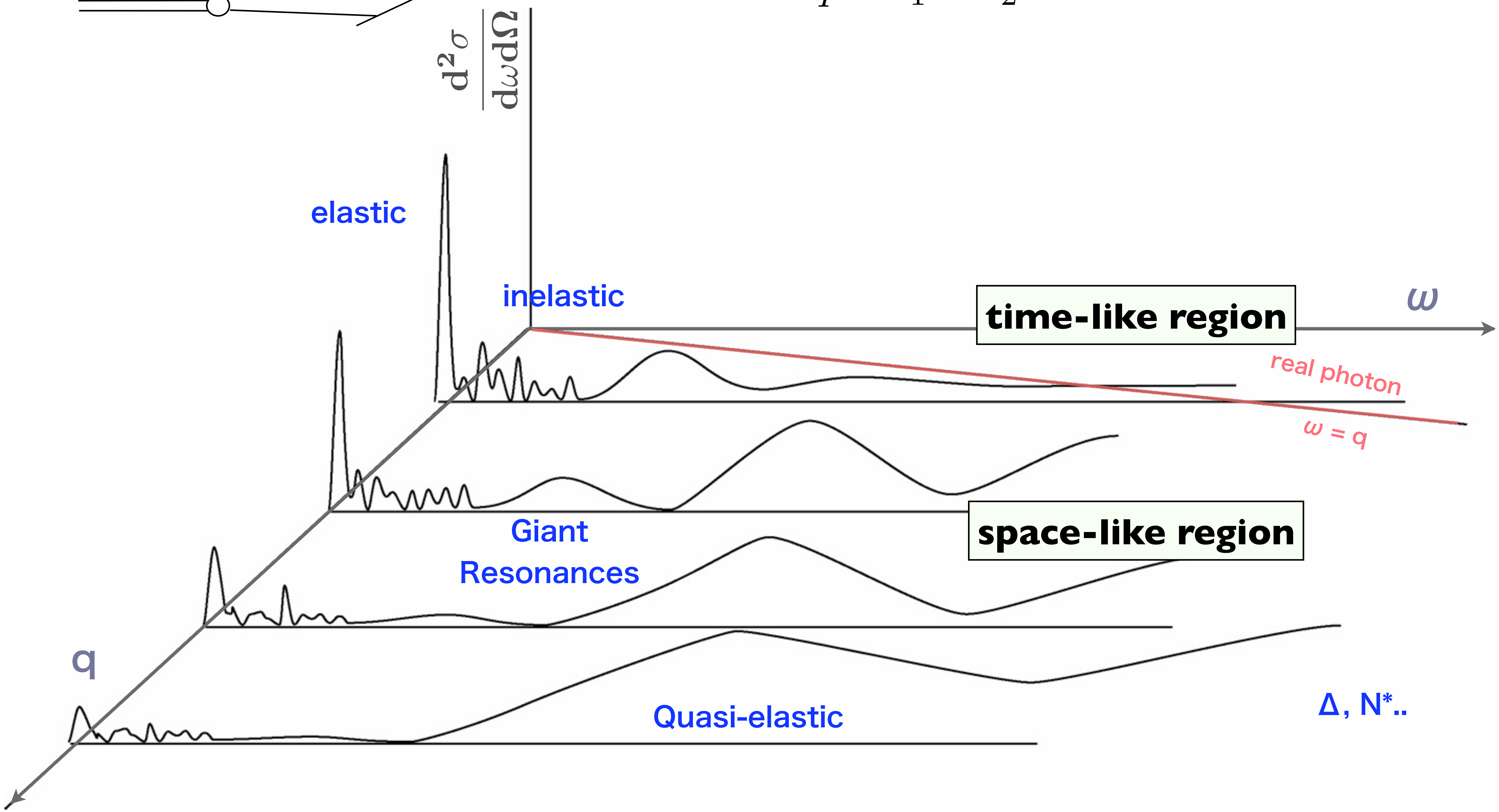
Nuclear Response



kinematical valuables

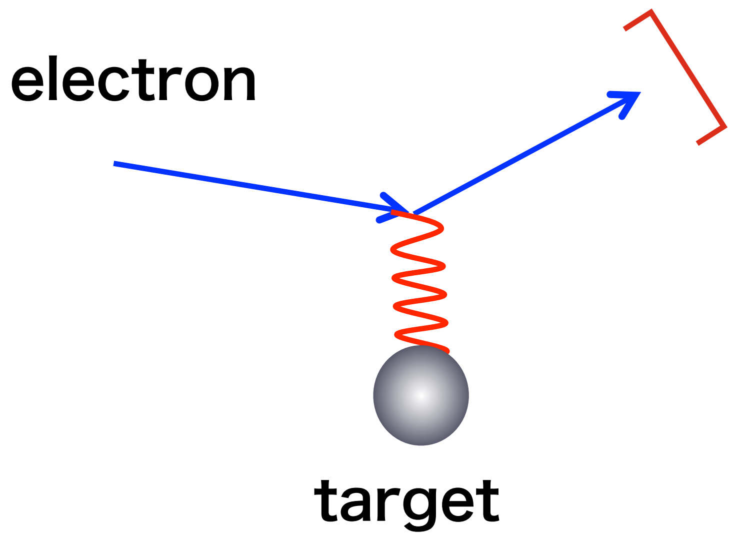
$$\omega = e_1 - e_2 \qquad \text{energy transfer}$$

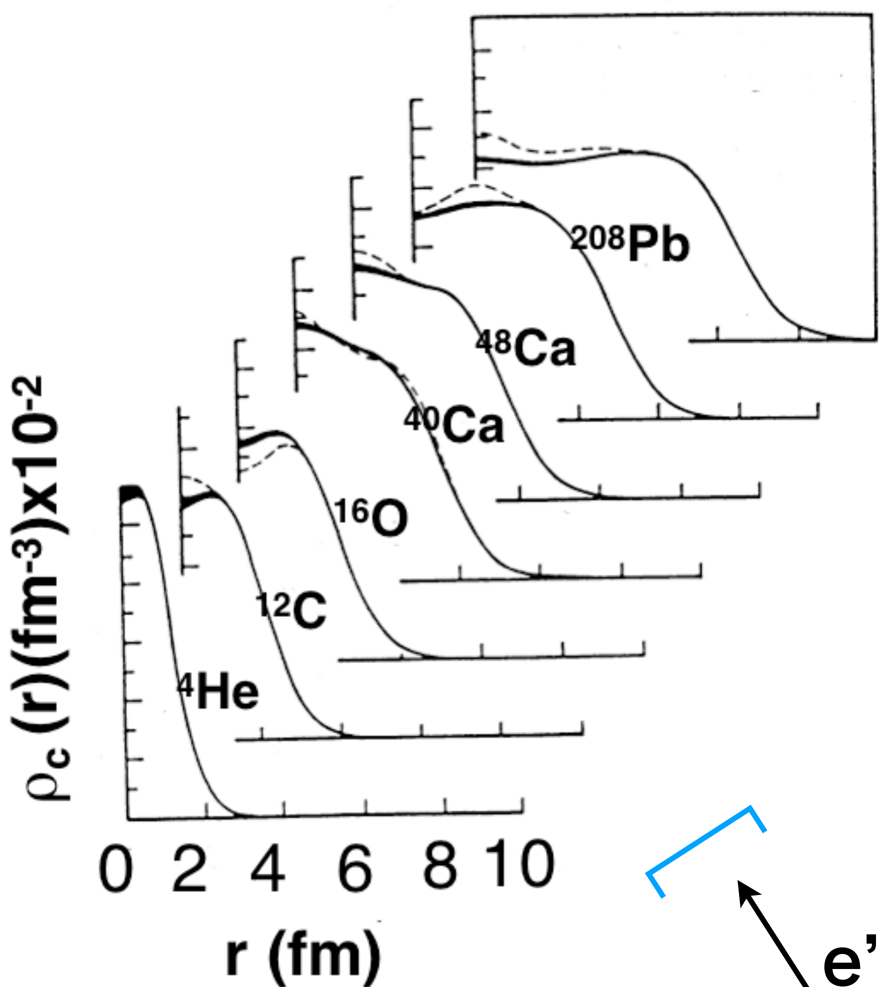
$$\vec{q} = \vec{e_1} - \vec{e_2} \qquad \text{momentum transfer}$$



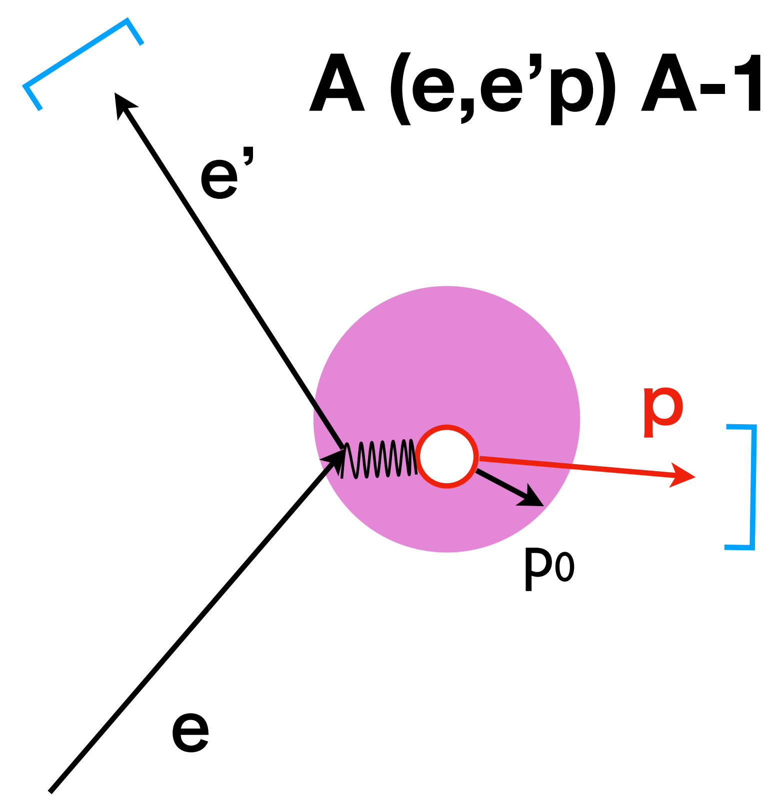
Quasi-elastic scattering

elastic scattering : e+A





quasi-elastic scattering



"elastic scattering" off proton in a nucleus

e+"p" -> e+"p" (the residual is a spectator)

one studies protons inside a nucleus (momentum, energy ...)

^{12}C(e,e'p)@NIKHEF

under PWIA

$$\frac{d^6\sigma}{dk_{e'}d\Omega_{e'}dp_p d\Omega_p} = K\frac{d\sigma_{ep}}{d\Omega}S(E_m,\vec{p}_m)$$

Ee = 285 ~ 481 MeV
q = 155 ~ 520 MeV/c

$$S(E_m,\vec{p}_m) = \rho(\vec{p}_m)n_\alpha\delta(Em - E_{tr})$$

$$\rho(\vec{p}_m)n_\alpha = \int S(E_m,\vec{p}_m)dE_m$$

1p3/2

1S1/2

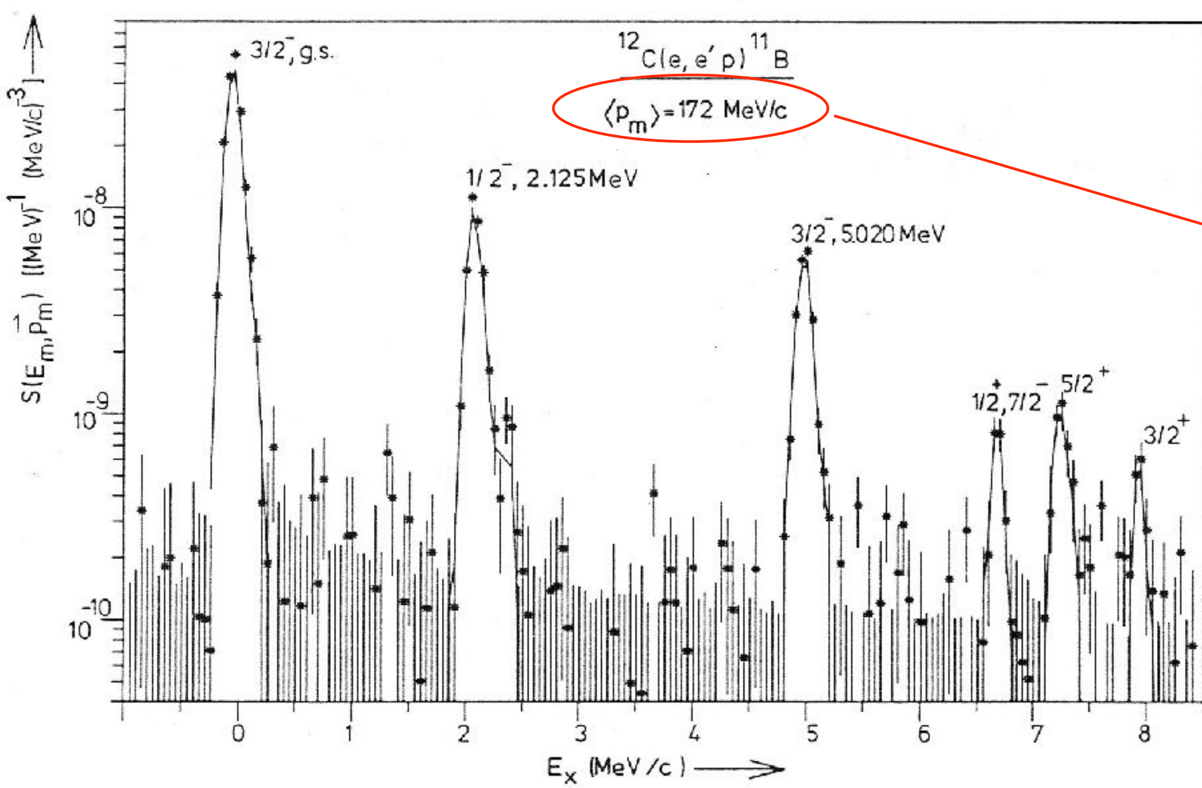
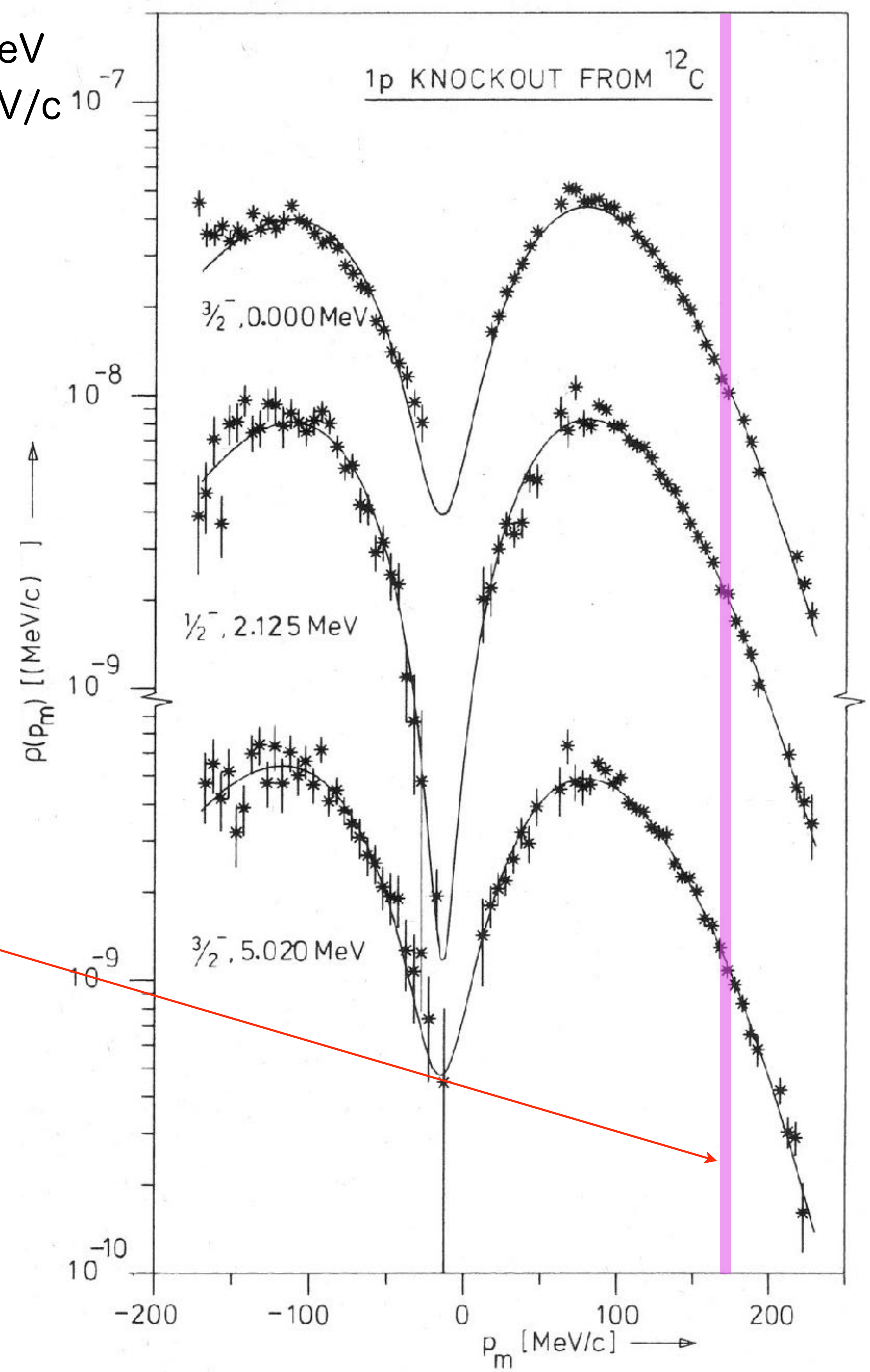


Fig. 1. Excitation-energy spectrum of the ^{12}C(e, e'p)^{11}B reaction at E_0 = 284.5 MeV. The central value of the missing momentum is 172 MeV/c.



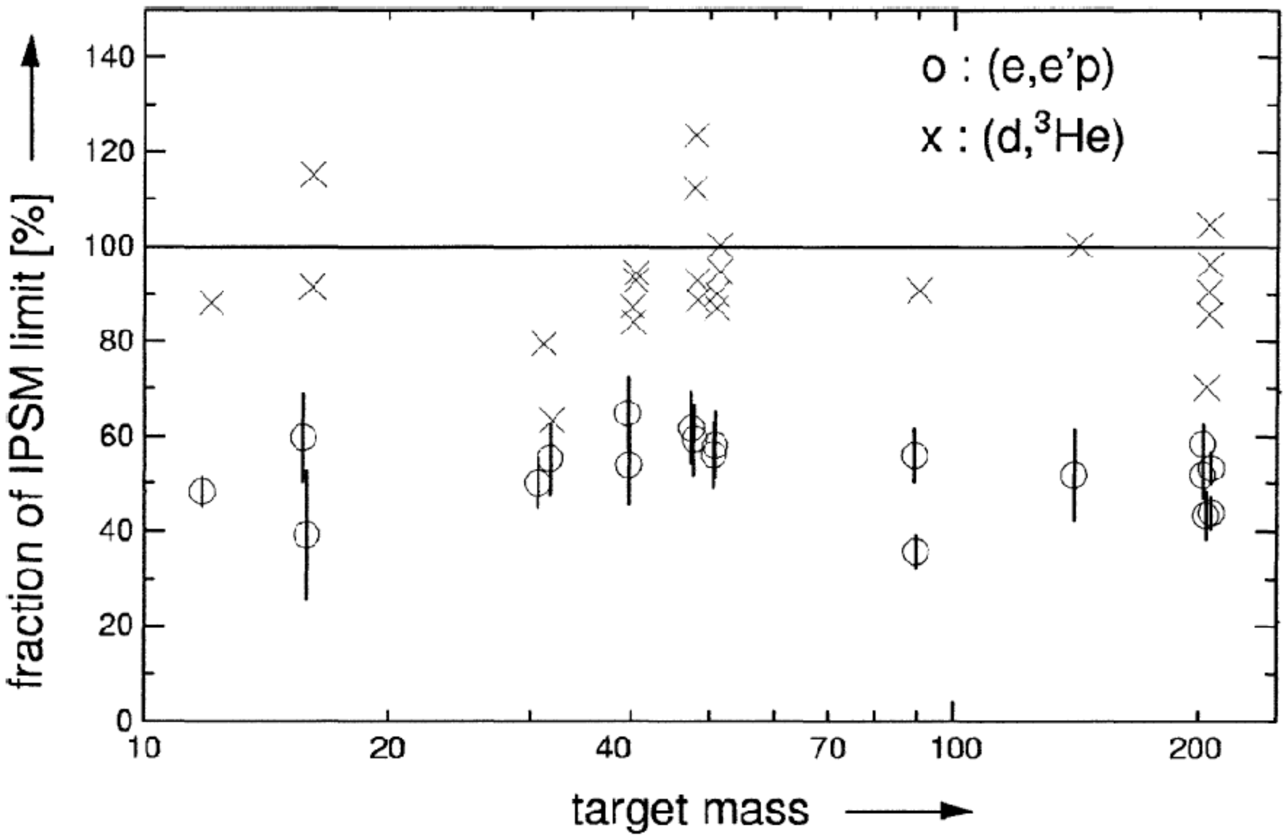
Nucl. Phys. A480(1988)547.

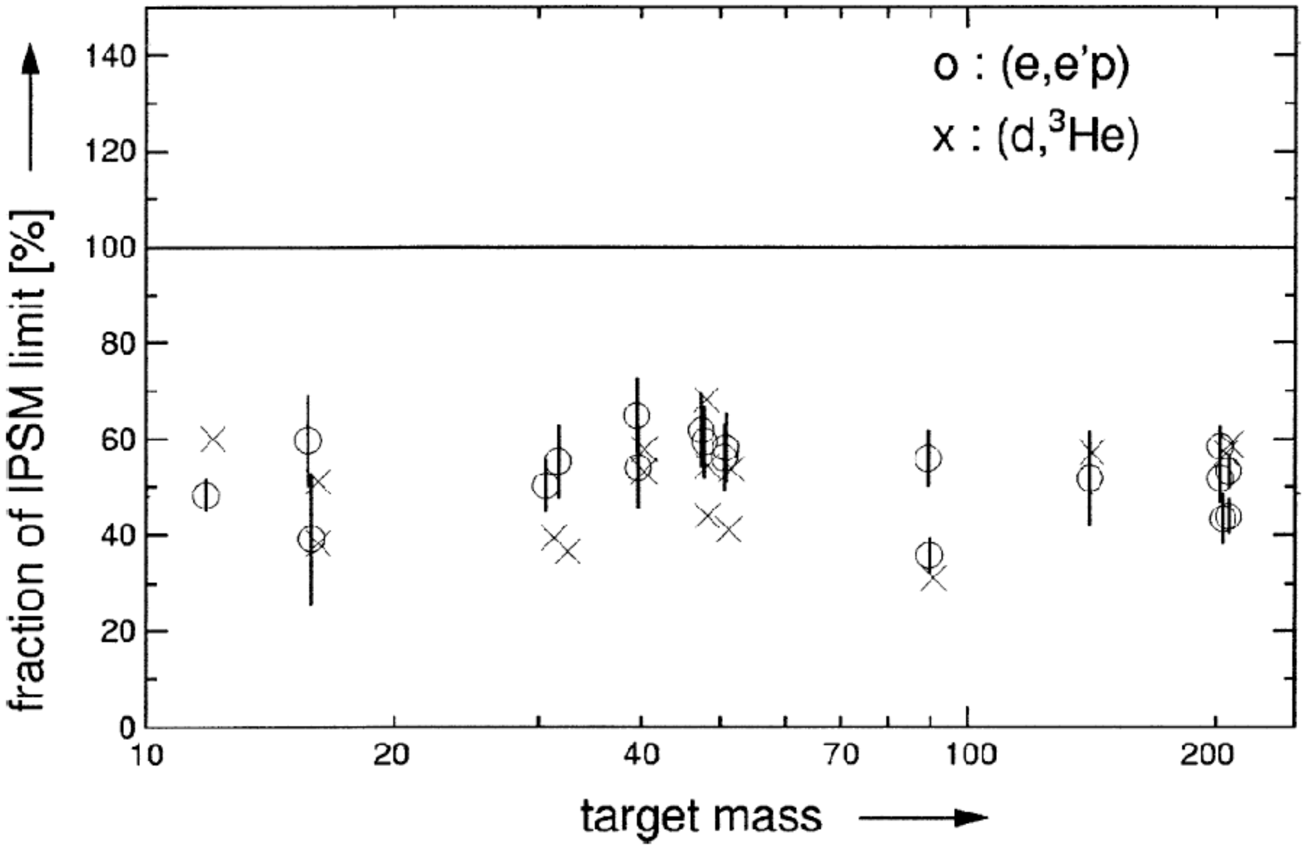
S-factors by (e,e'), (e,e'p) and (d,^{3}He)

$$\frac{d^6\sigma}{dk_{e'}d\Omega_{e'}dp_p d\Omega_p} = K\frac{d\sigma_{ep}}{d\Omega}S(E_m,\vec{p}_m)$$

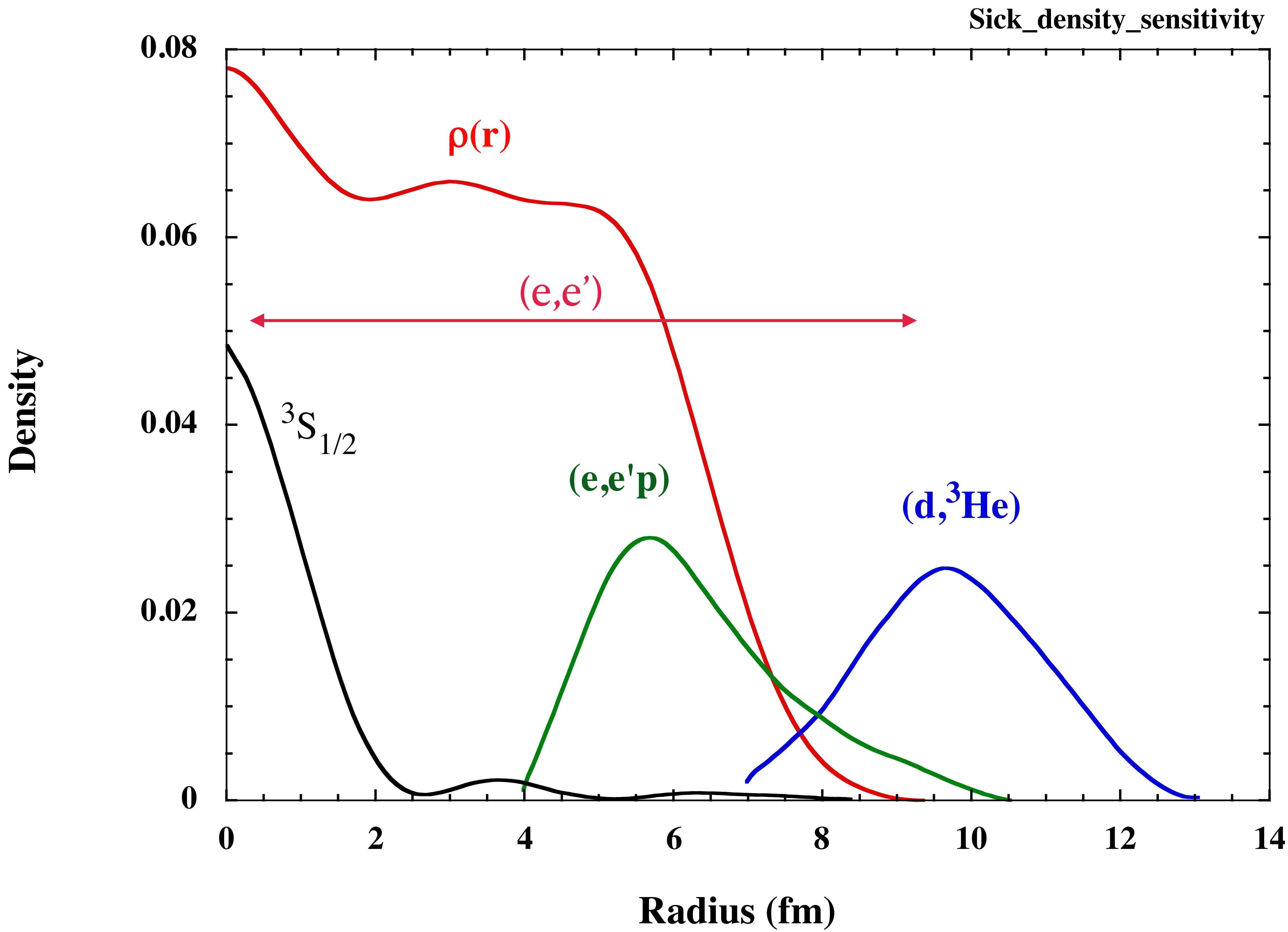
$$S(E_m,\vec{p}_m) = \rho(\vec{p}_m)n_\alpha\delta(Em - E_{tr})$$

$$\rho(\vec{p}_m)n_\alpha = \int S(E_m,\vec{p}_m)dE_m$$



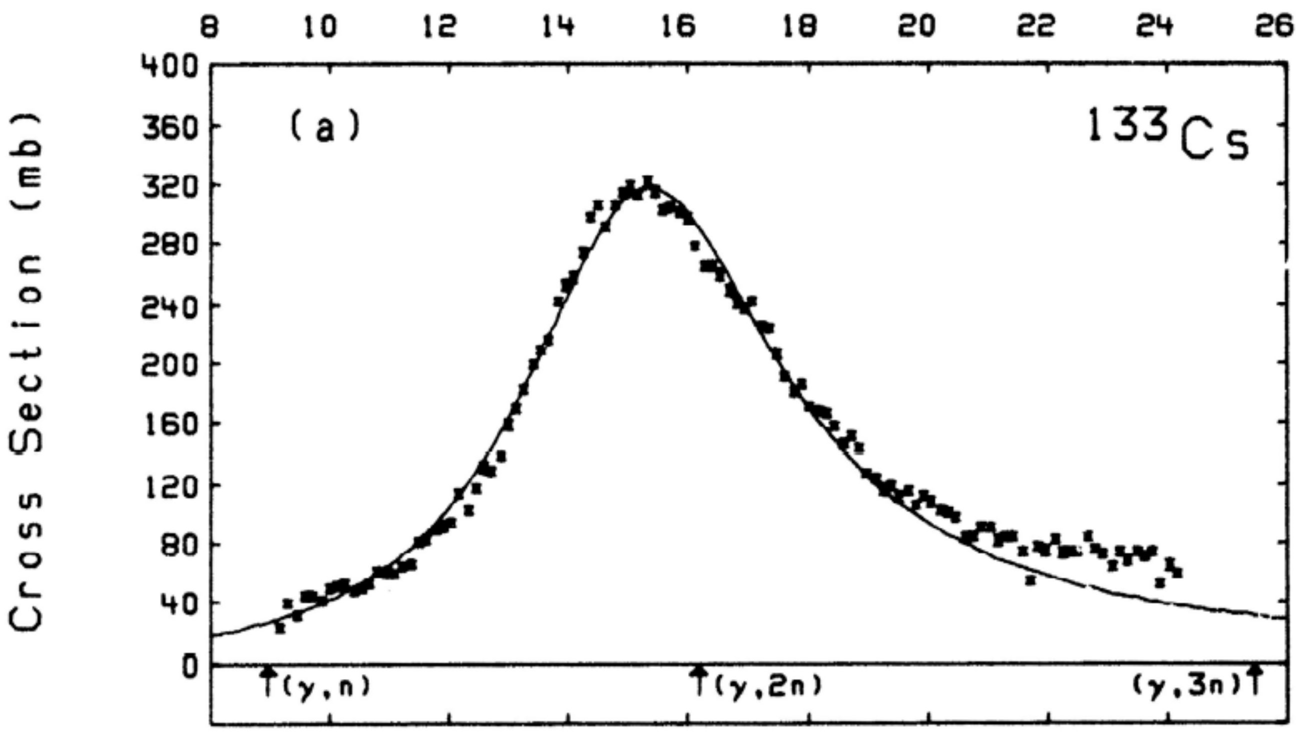


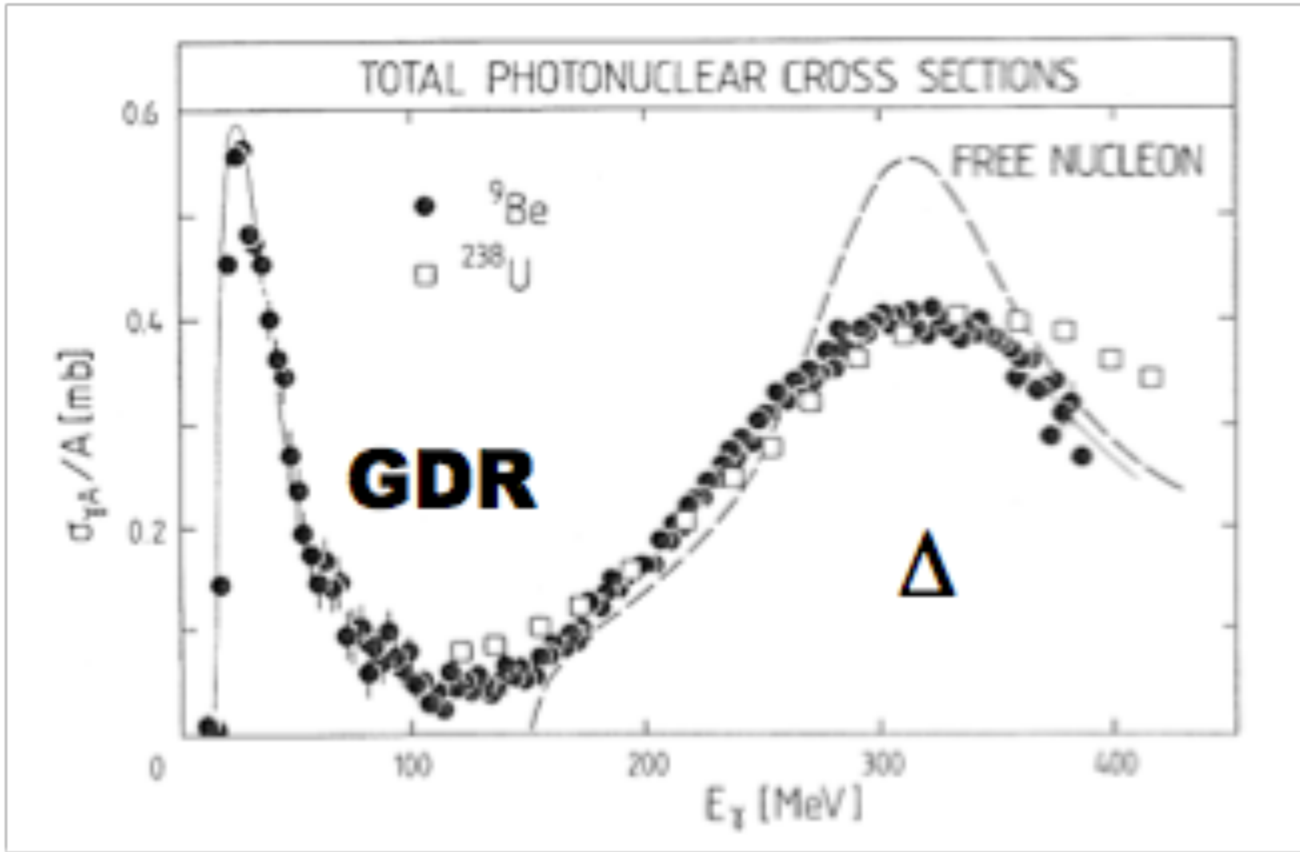
Nucl. Phys. A679(2001)267.



Nucl. Phys. A679(2001)267.

Photonuclear reaction





1) Response functions (operators : well-known)

2) Sum Rules

TRK sum rule

$$\int_0^\infty \sigma(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2 \hbar}{M}\frac{NZ}{A}(1+\kappa) = 60\frac{NZ}{A}(1+\kappa)MeV \cdot mb$$

Bremmstrahlung sum rule

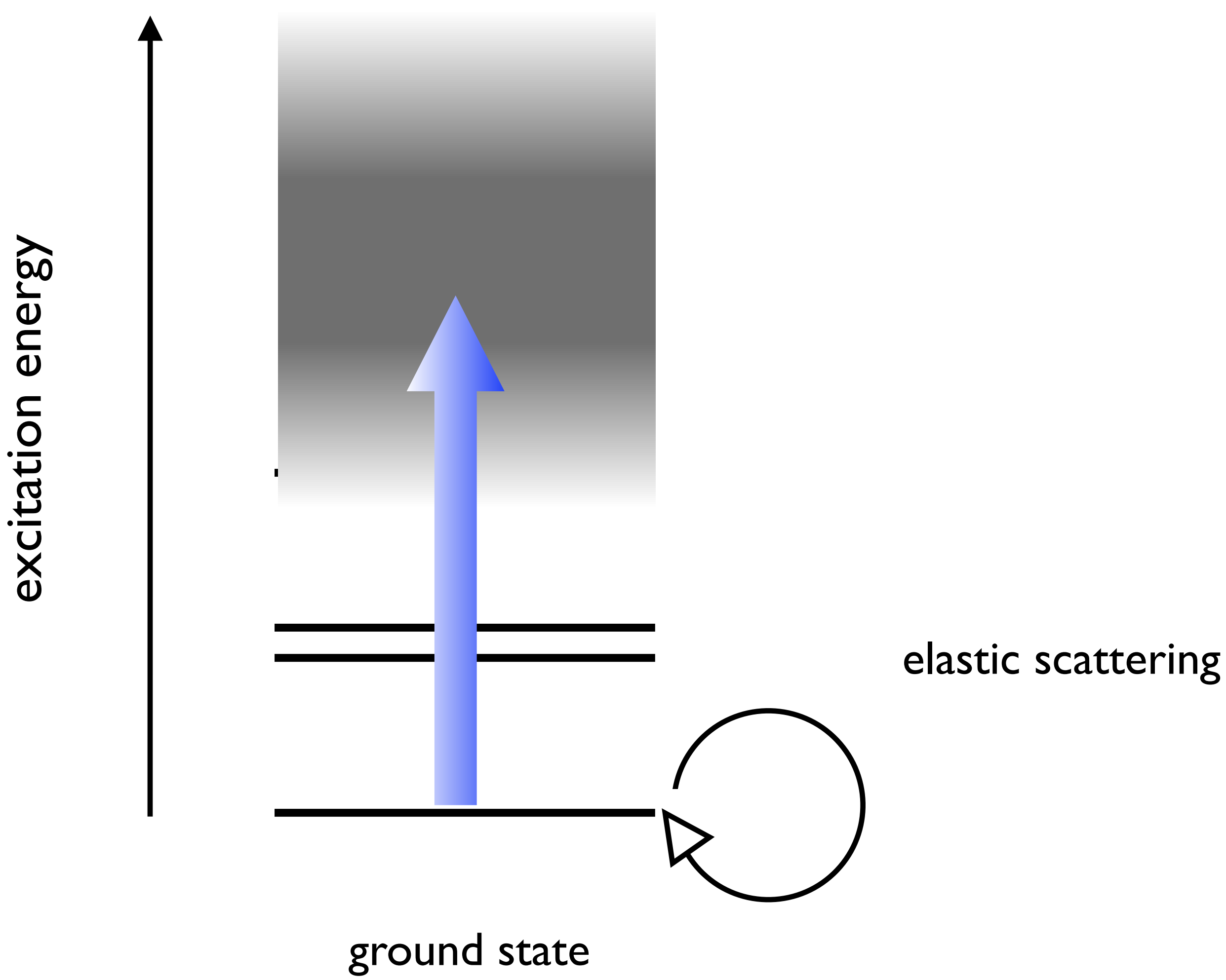
$$\int_0^\infty \frac{\sigma(E_\gamma)}{E_\gamma}dE_\gamma = \frac{4\pi^2 e^2}{3\hbar}\frac{NZ}{A-1} < r^2 >$$

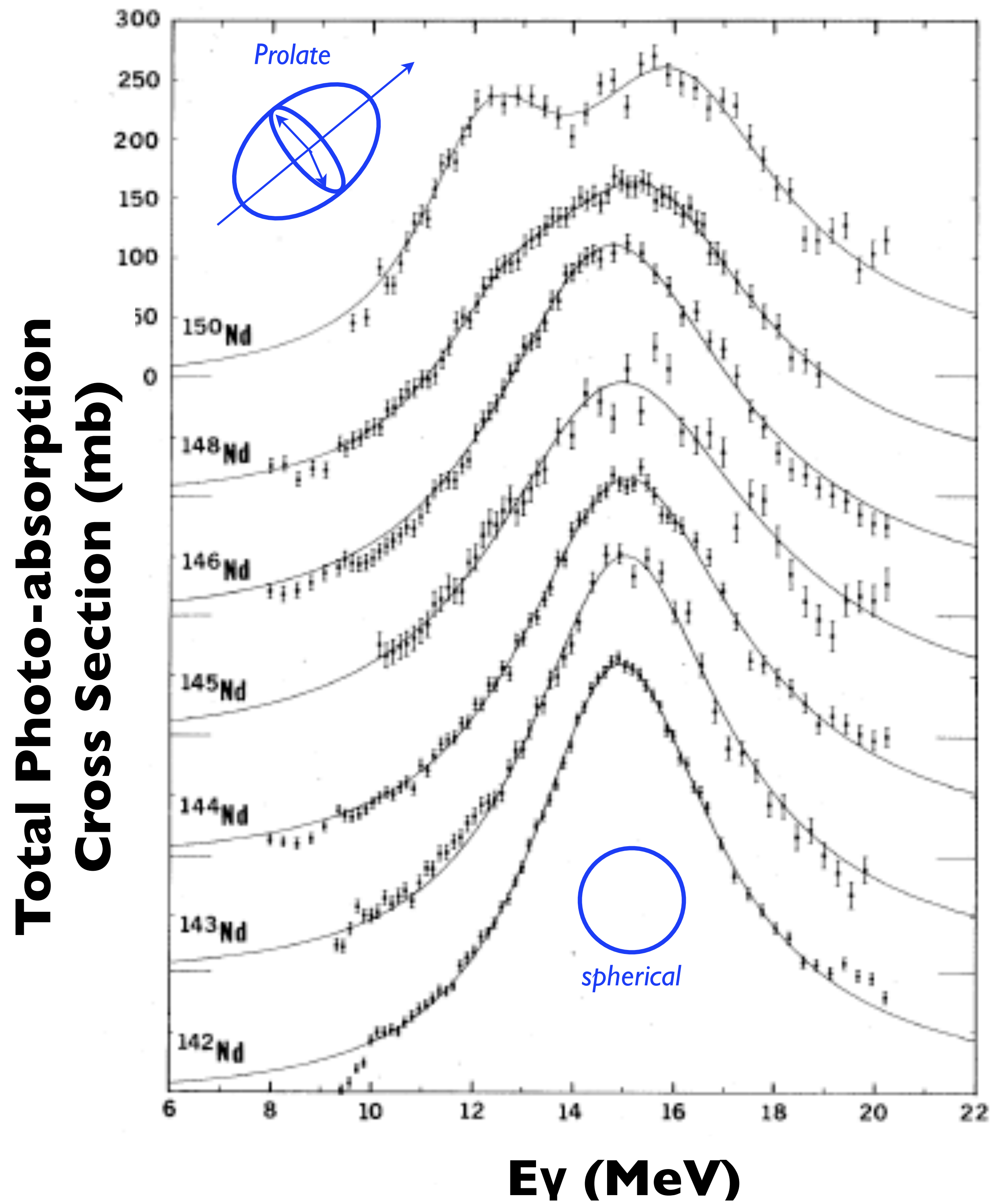
Migdal sum rule

$$\int_0^\infty \frac{\sigma(E_\gamma)}{E_\gamma^2}dE_\gamma = \frac{2\pi^2}{\hbar}P$$

P : polarizability

σ_{TOT} in GDR region

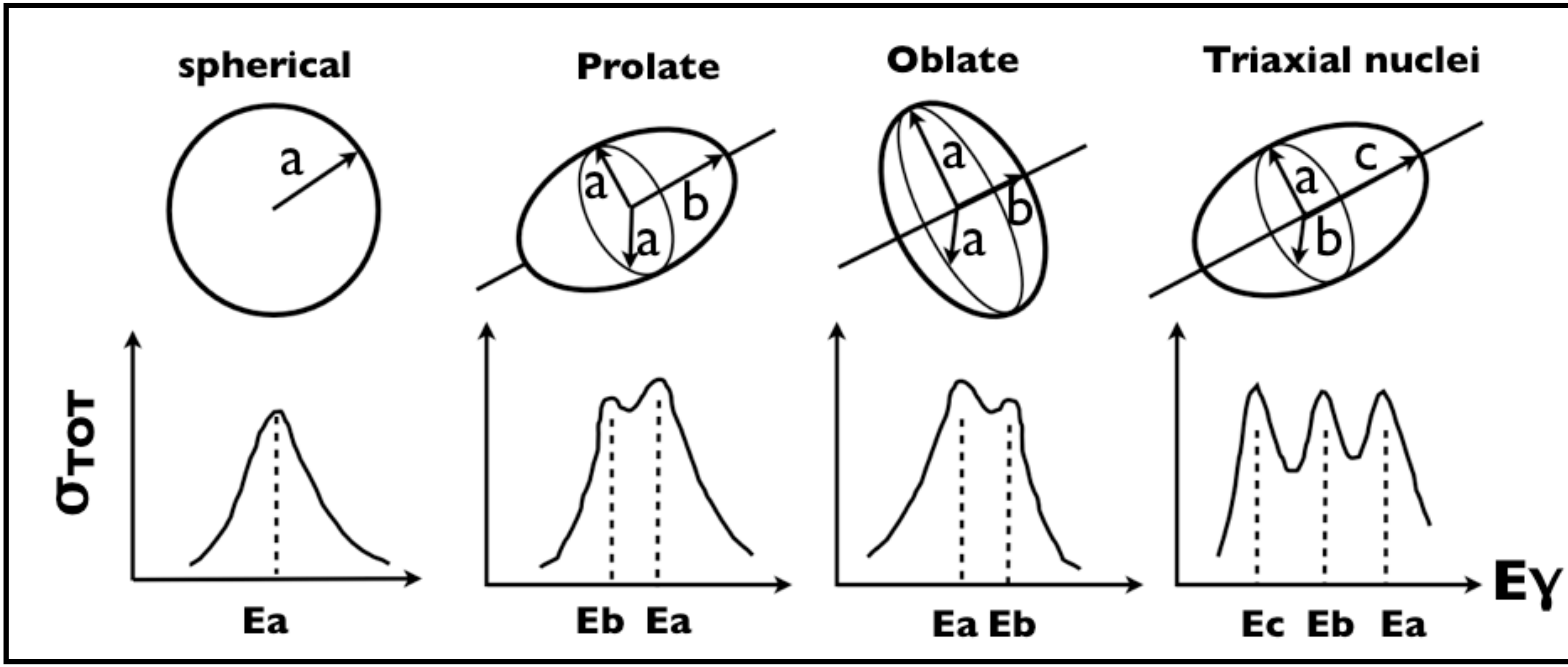




B. L. Berman and S. C. Fultz, Rev. of Mod. Phys. 47 (1975) 713

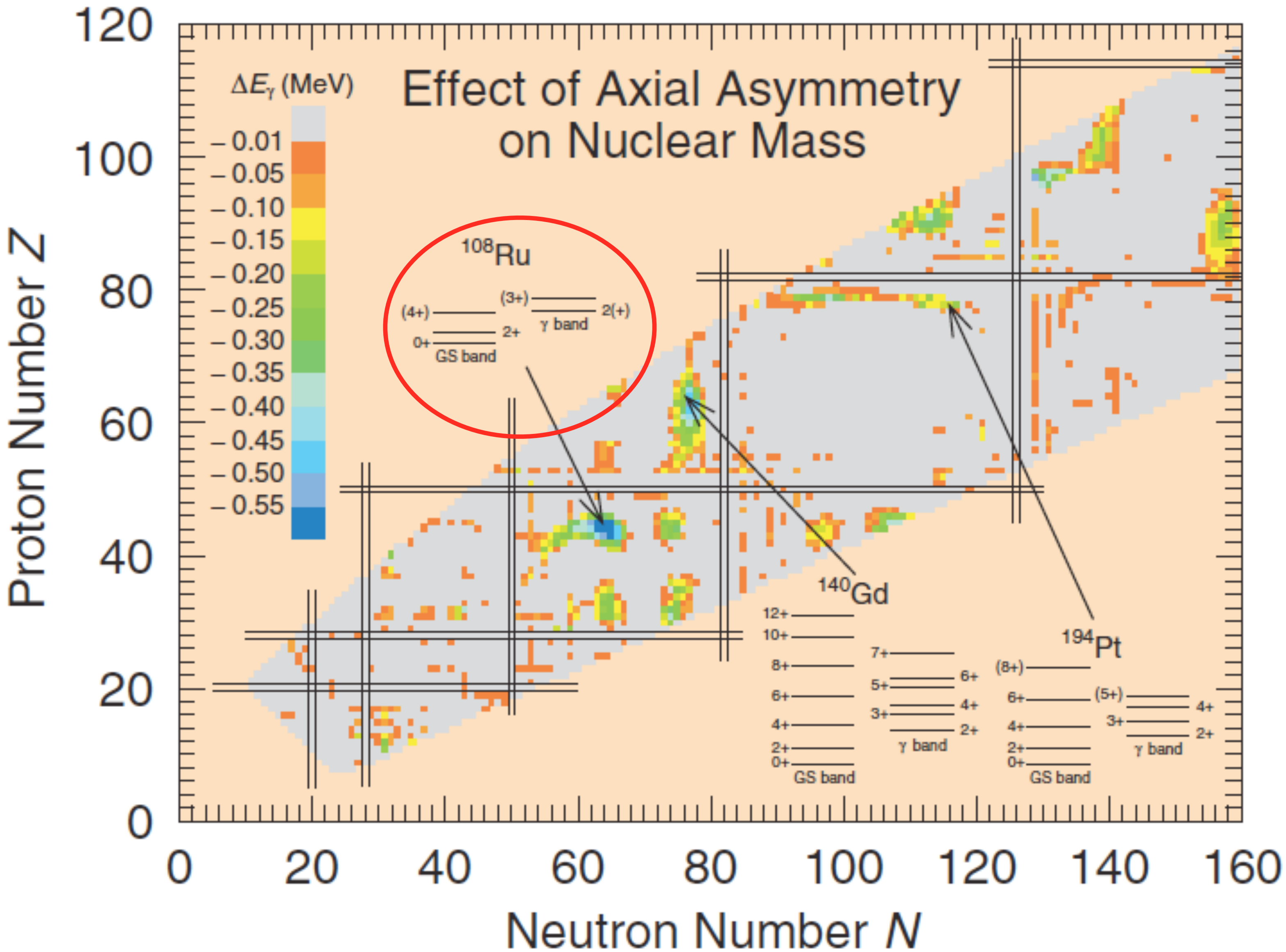
Total photo-absorption cross section covering GDR

$$\gamma + A \rightarrow X$$

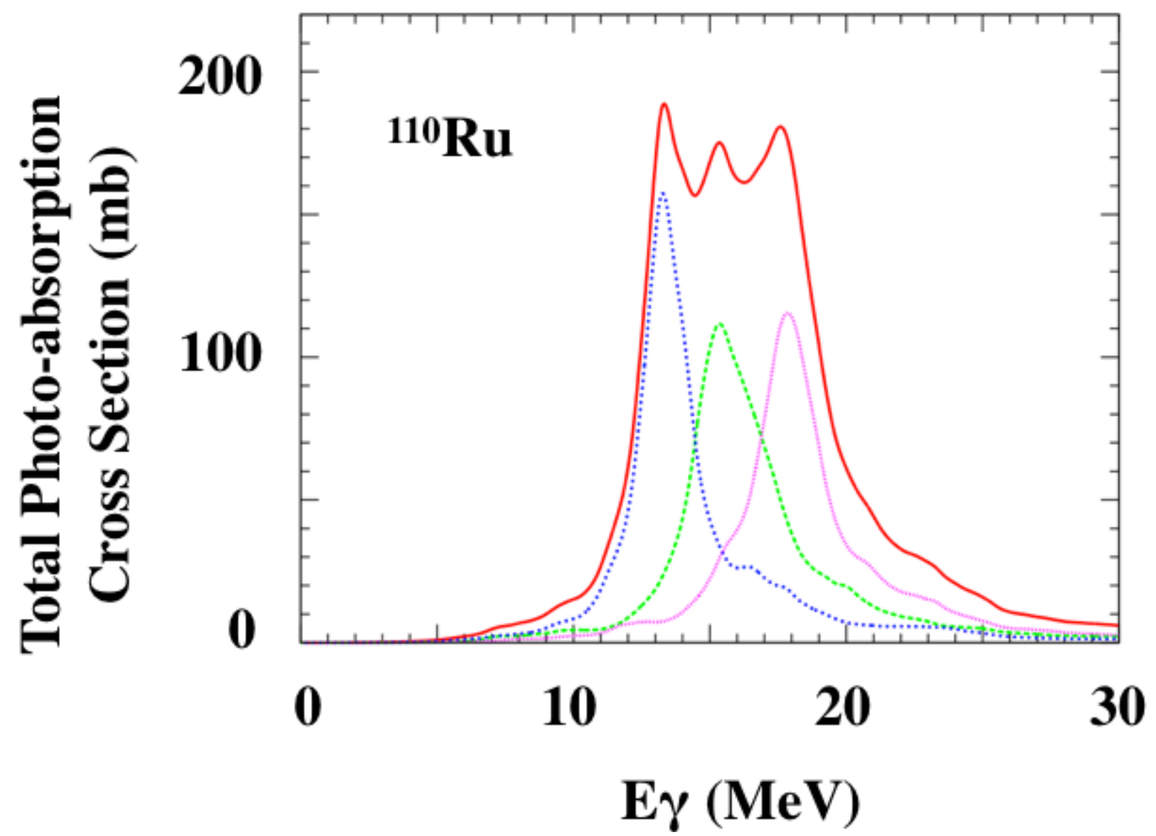


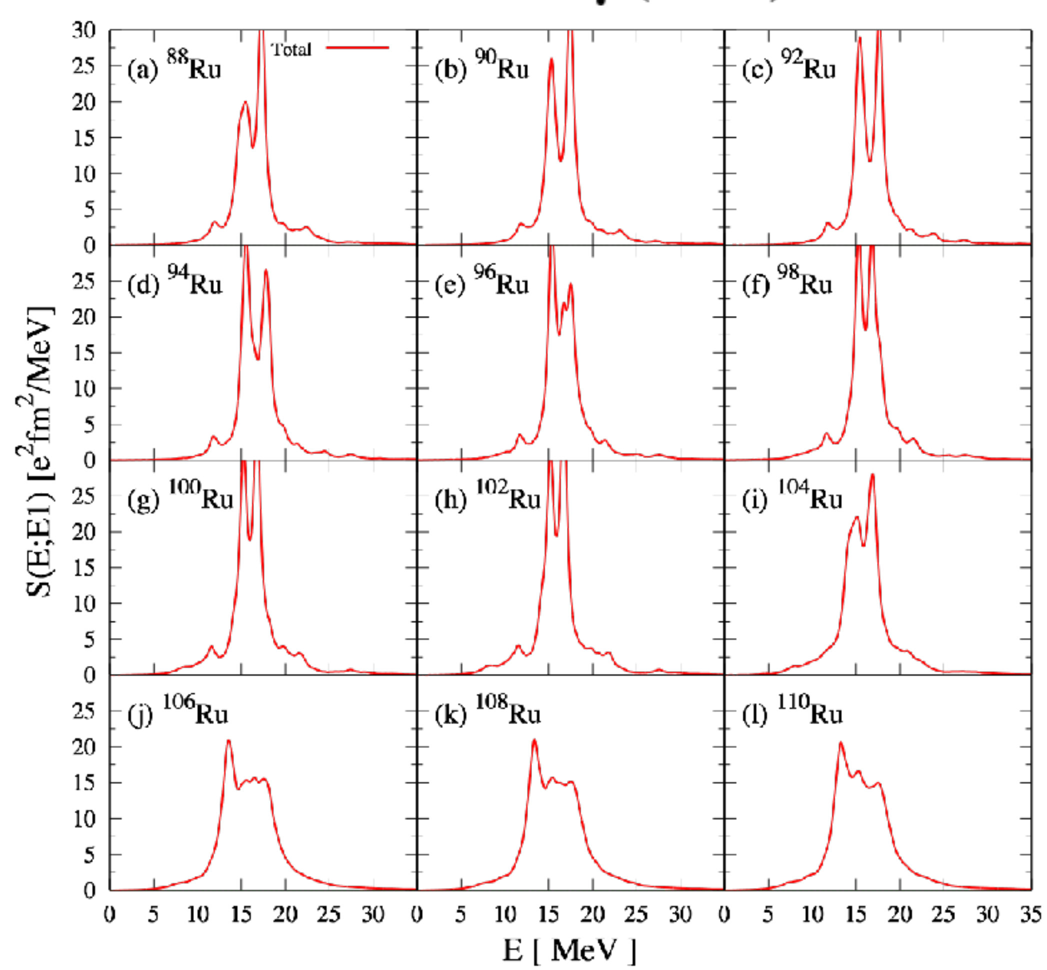
R. Bergere :Lecture Note in Physics, 61 (1971) 84

Triaxial nucleus and their $\sigma_{TOT}(E\gamma)$



P. Moller et al.,PRL97,162502 (2006)





S. Ebata : private communication

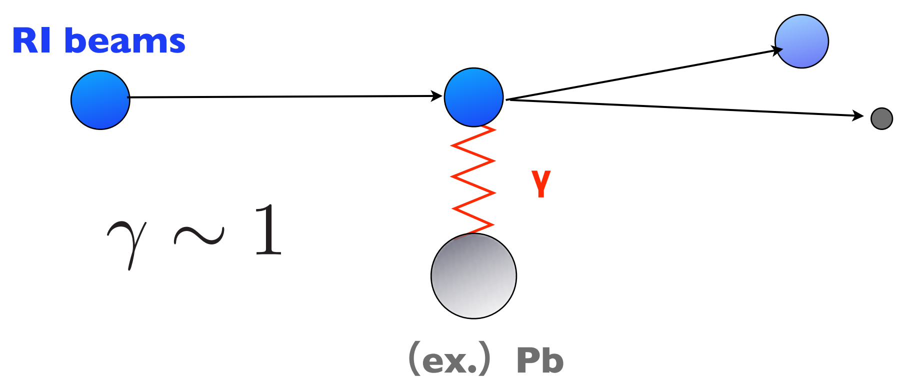
^{108}Ru 273 sec

^{110}Ru 15 sec

photonuclear reaction for exotic nuclei

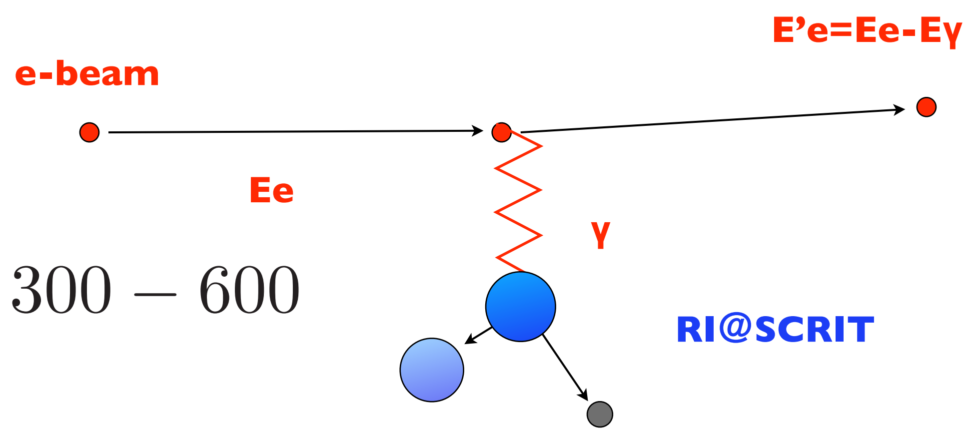
so far

only way : Coulomb excitation in heavy ion reaction



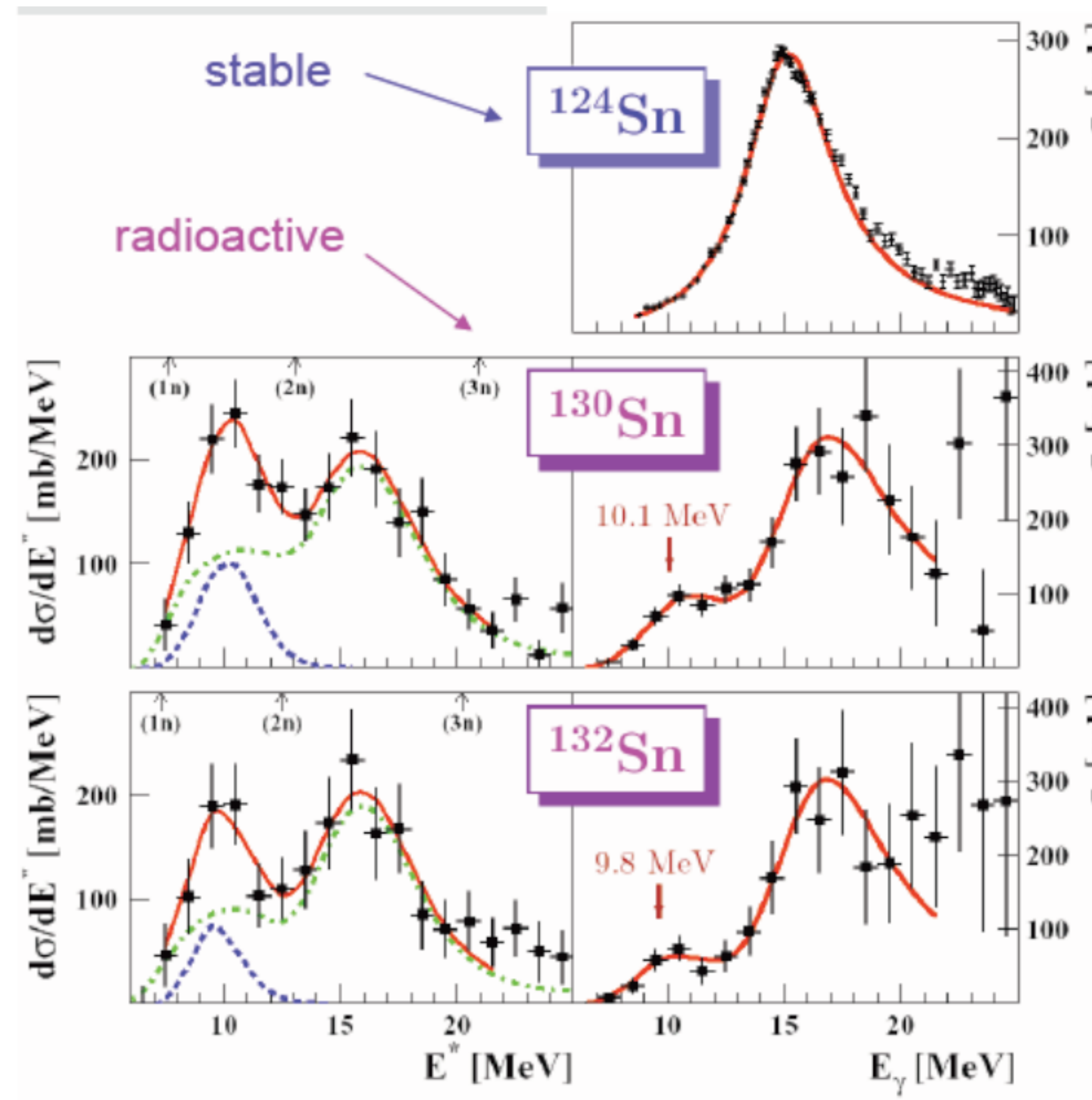
$\gamma \sim 1$

SCRIT facility



$\gamma \sim 300 - 600$

^{132}Sn+Pb -> ^{131}Sn + n + X @ GSI



purely EM probe
  well under control
  negligible multi-stop

ultra-forward
    electron scattering

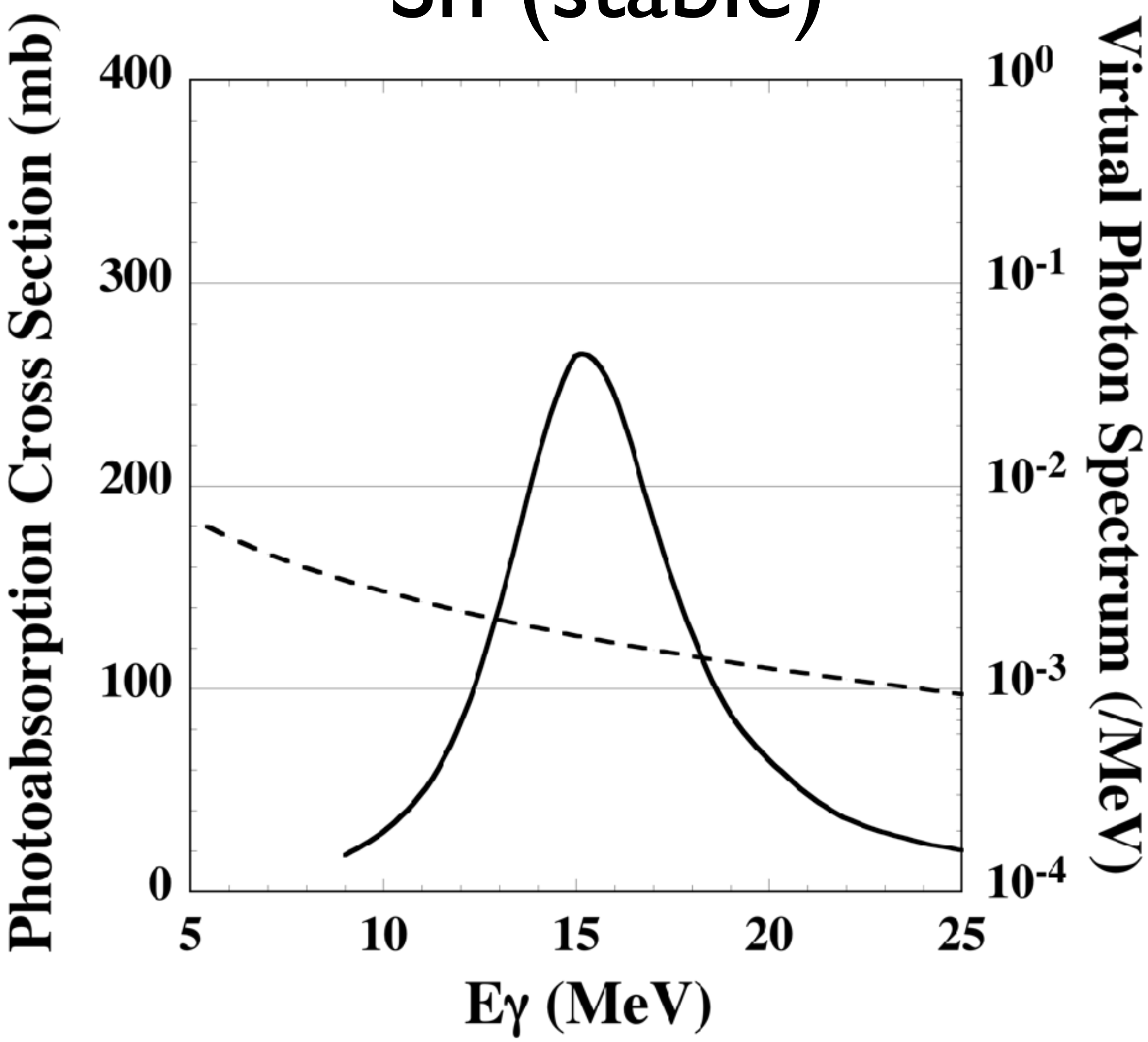
$$\frac{d^2\sigma}{dE_e d\Omega} = \overbrace{\sum \frac{d^2 N_e^{EL}(E, E_\gamma, \theta)}{dE_\gamma d\Omega}}^{\text{Virtual Photon flux}} \cdot \sigma_\gamma^{EL}(E_\gamma)$$

Expected reaction rate for $L = 10^{27}$ /cm²/s

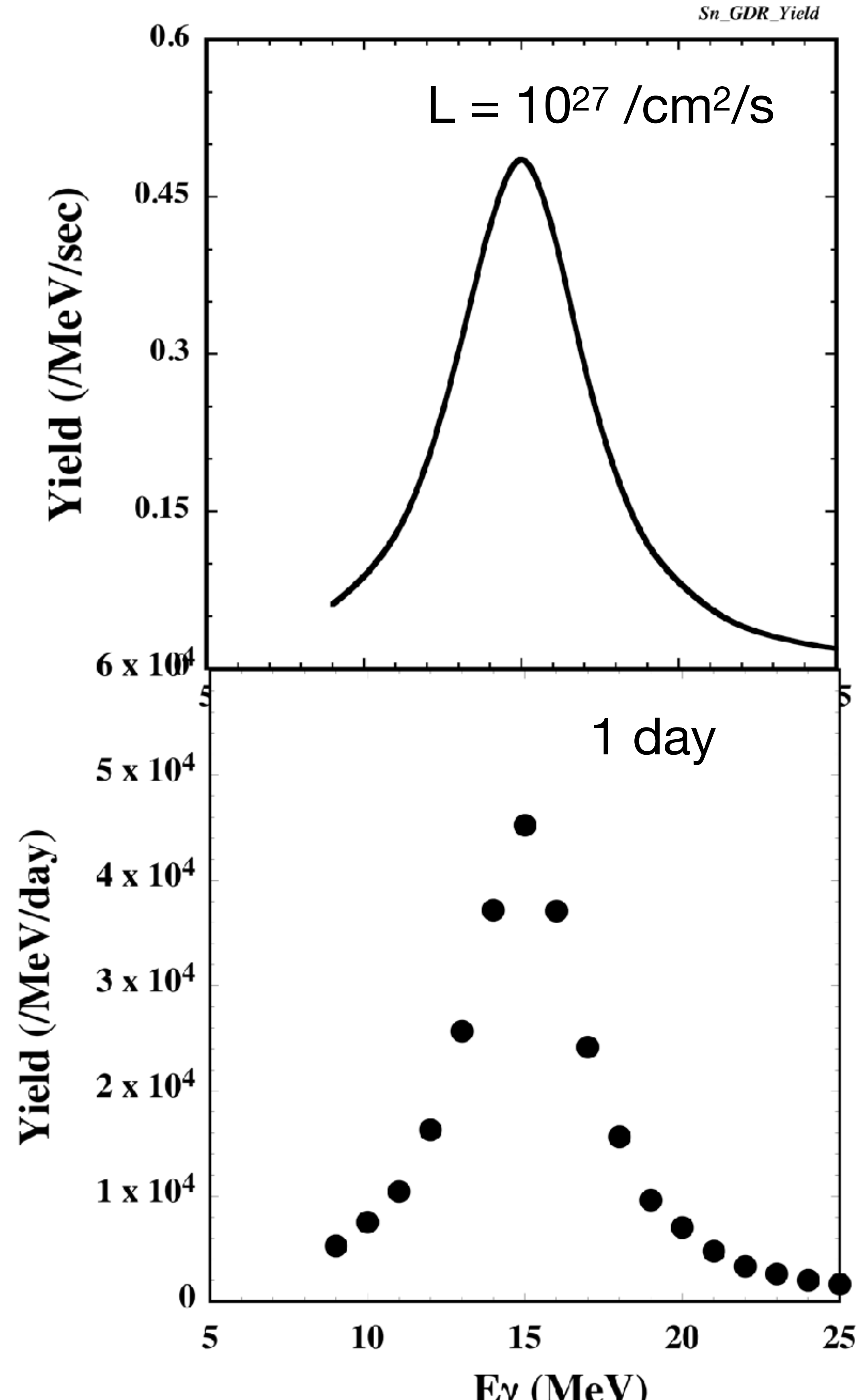
virtual photon theory

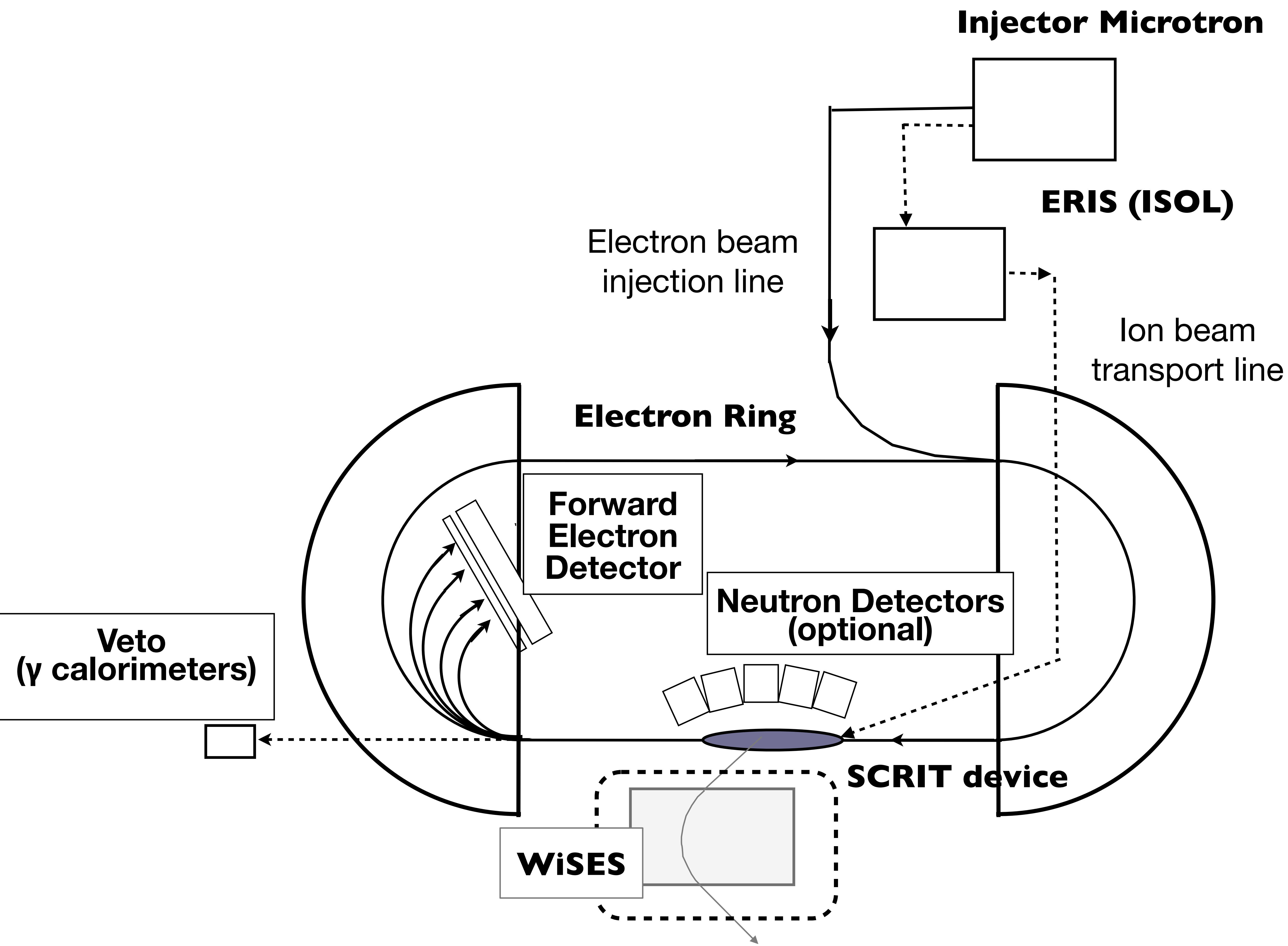
$$\frac{dN}{dE_\gamma} = L \cdot \int d\Omega \frac{d^2 N_e^{E1}(E, E_\gamma, \theta)}{dE_\gamma d\Omega} \cdot \sigma_\gamma^{E1}(E_\gamma)$$

^{120}Sn (stable)



P. Durgapal and D.S. Onley
Comp. Phys. Comm. 32 (1984) 291




Injector Microtron
ERIS (ISOL)
Electron beam
injection line
Ion beam
transport line
Electron Ring
Forward
Electron
Detector
Neutron Detectors
(optional)
Veto
(γ calorimeters)
SCRIT device
WiSES