Next-to-leading order QCD corrections to the form factors of $B \to S$ decays

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- Motivation
- $B \to S$ form factors with one loop corrections
- Numerical analysis
- Summary

Motivation

- The semileptonic decays of B meson: CKM matrix elements, NP.
- The internal structures of the scalar mesons: $a_0^+(1450)$, $\bar{K}_0^*(1430)$, $K_0^{*+}(1430)$, $f_0(1500)$.

Calculation method:
light-front approach, LQCD, QCDSR, pQCD, **LCSR** $\cdots$

- light-front approach: C.-H. Chen, C.-Q. Geng, C.-C. Lih and C.-C. Liu, 0703106
- QCDSR approach: M.-Z. Yang, 0509103;
  T.M. Aliev, K. Azizi and M. Savci,0710.1508
- pQCD approach: R.-H. Li, C.-D. Lü, W. Wang and X.-X. Wang, 0811.2648
- LCSR approach
  - light-meson LCSR: Y.-M. Wang, M.J. Aslam, C.-D. Lü, 0804.2204; Y.-J. Sun, Z.-H. Li and T. Huang, 1011.3901; Z.-G. Wang, 1409.6449
  - ***B*-meson LCSR**: R. Khosravi, 2203.09997

Form factors in LCSR

Definition of B-meson transition form factors

$$\langle S(p)\,|\bar{q}'\,i\sigma_{\mu\nu}\gamma_5 q^\nu\, b|\,B(p_B)\rangle = -i\left[2q^2 p_\mu - (m_B^2 - m_S^2 - q^2)q_\mu\right]\frac{f_{BS}^T(q^2)}{m_B + m_S},$$

$$\langle S(p)\,|\bar{q}'\gamma_\mu\gamma_5 b|\,B(p_B)\rangle = -i\left[(p_B + p - \frac{m_B^2 - m_S^2}{q^2}q)_\mu f_{BS}^A(q^2) + \frac{m_B^2 - m_S^2}{q^2}q_\mu f_{BS}^P(q^2)\right].$$

***B*-meson LCSR**: A. Khodjamirian, T. Mannel and N. Ofen, 0504091, 0611193;

F. De Fazio, T. Feldmann and T. Hurth, 0504088, 0711.3999.

$$\Pi_\mu^{A,T}(n\cdot p,\bar{n}\cdot p) = \int d^4x e^{ip\cdot x}\left\langle 0\left|T\left\{\bar{q}(x)q'(x),\bar{q}'(0)\Gamma_\mu b(0)\right\}\right|B(p_B)\right\rangle$$
$$= \begin{cases} \Pi^A(n\cdot p,\bar{n}\cdot p)n_\mu + \tilde{\Pi}^A(n\cdot p,\bar{n}\cdot p)\bar{n}_\mu, & \Gamma_\mu = \gamma_\mu\gamma_5 \\ \Pi^T(n\cdot p,\bar{n}\cdot p)\left[\bar{n}\cdot q n_\mu - n\cdot q\bar{n}_\mu\right], & \Gamma_\mu = i\sigma_{\mu\nu}q^\nu\gamma_5 \end{cases}.$$

$$n\cdot p \sim \mathcal{O}(m_b)\,,\quad |\,\bar{n}\cdot p\,| \sim \mathcal{O}(\Lambda)\,.$$

Form factors in LCSR

- Quark level: perturbative QCD calculation
  $\bar{n} \cdot p < 0$, Light-cone OPE: $x^2 \to 0$
- Hadronic level: insert complete set $\sum_n |\, n\rangle\langle n\,|$

$$\Pi_\mu(n \cdot p, \bar{n} \cdot p) = \int d^4x \; e^{ip\cdot x} \langle 0| T\left\{ \bar{q}(x) q'(x), \; \bar{q}'(0)\, \Gamma_\mu \, b(0) \right\} |\bar{B}(p_B)\rangle$$

$$\Uparrow$$

$$|S\rangle\langle S|$$

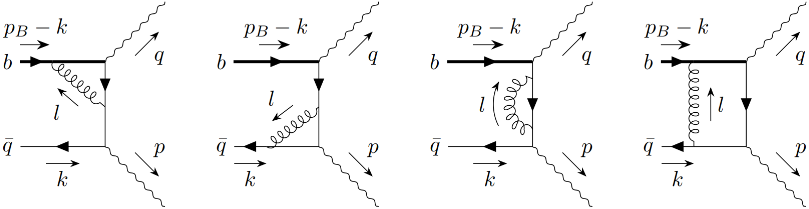
Standard sum rules technique

- Dispersion relation: $\Pi(p^2) = \frac{1}{\pi} \int_0^\infty d\omega \frac{\mathrm{Im}\Pi(\omega)}{\omega - \bar{n}\cdot p}$
- Quark-hadron duality ansatz: $\mathrm{Im}\Pi^h(s) \to \mathrm{Im}\Pi^{pert}(s), \; \omega_s = s_0/n \cdot p$
- Borel transformation: $\bar{n} \cdot p \to \omega_M = M^2/n \cdot p$

$$f^+_{BS}(q^2) = \frac{\tilde{f}_B \, m_B}{m_S \bar{f}_S} e^{m_S^2/(n\cdot p\, \omega_M)} \int_0^{\omega_s} d\omega \, e^{-\omega/\omega_M} \phi_B^+(\omega) + \mathcal{O}(\alpha_s)$$

NLO correction

One loop correction diagrams



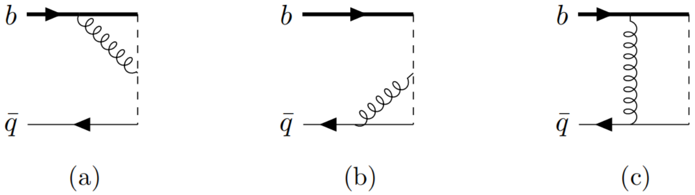
(a) (b) (c) (d)

- **Method of regions**: M. Beneke and V.A. Smirnov, 9711391
  hard, hard-collinear and soft regions have leading power contribution

  soft region $= \phi^{(1)} \otimes T^{(0)}$

NLO correction

One-loop diagrams for B meson DA



- Factorization: hard scale $[\mathcal{O}(m_b)]$, hard-collinear scale $[\mathcal{O}(\sqrt{m_b\Lambda})]$ and soft scale $[\mathcal{O}(\Lambda)]$

$$\Phi_{b\bar{q}}^{(0)} \otimes T_\mu^{(1)} = \left[\Pi_{\mu,\mathrm{weak}}^{(1),\mathrm{h}} + \left(\Pi_{\mu,\mathrm{bwf}}^{(1)} - \Phi_{b\bar{q},\mathrm{bwf}}^{(1)} \otimes T_\mu^{(0)}\right)\right] + \left[\Pi_{\mu,\mathrm{weak}}^{(1),\mathrm{hc}} + \Pi_{\mu,\mathrm{scalar}}^{(1),\mathrm{hc}} + \Pi_{\mu,\mathrm{wfc}}^{(1),\mathrm{hc}}\right]$$

$$\Pi = \tilde{f}_B(\mu)\, m_B\, C(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B(\omega,\mu)$$

NLO correction

The scale dependence of the correlation functions at the one-loop order

$$\frac{d}{d\ln\mu}\Pi + \gamma_S \Pi = \mathcal{O}\left(\alpha_s^2\right)$$

- Resummation: factorization scale $\mu \sim \mathcal{O}(\sqrt{m_b\Lambda}) \sim 1.5 GeV$,
  sum logs in C.

$$\tilde{C}^{(+)}(n\cdot p,\mu) = U_1\left(n\cdot p,\mu_{h1},\mu\right)\tilde{C}^{(+)}\left(n\cdot p,\mu_{h1}\right)$$
$$\tilde{f}_B(\mu) = U_2\left(\mu_{h2},\mu\right)\tilde{f}_B\left(\mu_{h2}\right)$$

$\mu_{h1} \sim n\cdot p, \quad \mu_{h2} \sim m_b.$

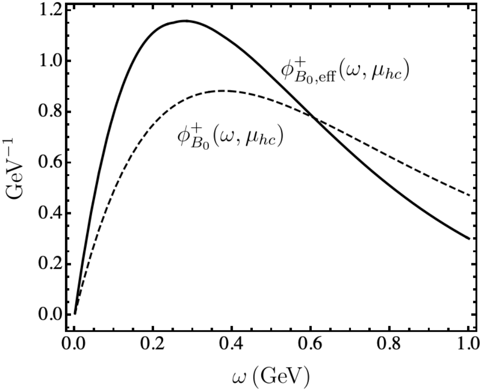
Final expressions

$$
\begin{aligned}
f_{BS}^{+}\left(q^{2}\right) =& \frac{m_B}{m_S \bar{f}_S} \mathrm{e}^{m_S^2/(n\cdot p \omega_M)} \left[U_2\left(\mu_{h_2},\mu\right)\tilde{f}_B\left(\mu_{h_2}\right)\right] \times \\
& \int_0^{\omega_s} d\omega' \mathrm{e}^{-\omega'/\omega_M} \left[U_1\left(n\cdot p,\mu_{h_1},\mu\right)\tilde{C}^{A,(+)}\left(n\cdot p,\mu_{h_1}\right)\phi_{B,\mathrm{eff}}^{+}\left(\omega',\mu\right)\right. \\
& \left. + \frac{n\cdot p - m_B}{m_B} C^{A,(+)}(n\cdot p,\mu)\phi_B^{+}\left(\omega',\mu\right)\right] \\
f_{BS}^{-}\left(q^{2}\right) =& \frac{n\cdot p}{m_S \bar{f}_S} \mathrm{e}^{m_S^2/(n\cdot p \omega_M)} \left[U_2\left(\mu_{h_2},\mu\right)\tilde{f}_B\left(\mu_{h_2}\right)\right] \times \\
& \int_0^{\omega_s} d\omega' \mathrm{e}^{-\omega'/\omega_M} \left[C^{A,(+)}(n\cdot p,\mu)\phi_B^{+}\left(\omega',\mu\right)\right] \\
f_{BS}^{T}\left(q^{2}\right) =& \frac{m_B + m_S}{2 m_S \bar{f}_S} \mathrm{e}^{m_S^2/(n\cdot p \omega_M)} \left[U_2\left(\mu_{h_2},\mu\right)\tilde{f}_B\left(\mu_{h_2}\right)\right] \times \\
& \left[U_1\left(n\cdot p,\mu_{h_1},\mu\right)C^{T,(+)}\left(n\cdot p,\mu_{h_1}\right)\right] \int_0^{\omega_s} d\omega' \mathrm{e}^{-\omega'/\omega_M} \phi_{B,\mathrm{eff}}^{+}\left(\omega',\mu\right)
\end{aligned}
$$

B-meson LCDA

Three-parameter model: M. Beneke, V.M. Braun, Y. Ji and Y.-B. Wei, 1804.04962

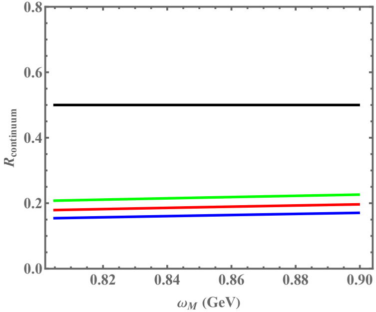
$$\phi_B^+(\omega,\mu) = U_\phi\left(\mu,\mu_0\right)\frac{1}{\omega^{p+1}}\frac{\Gamma(\beta)}{\Gamma(\alpha)}\mathcal{G}(\omega;0,2,1)$$

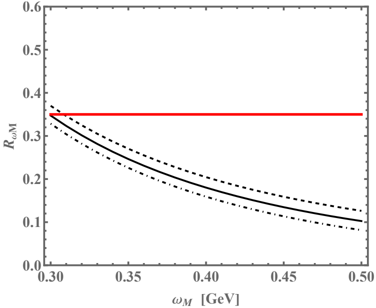


Borel parameter and effective threshold

- The continuum contributions need to be less than 50%.
- Form factors are insensitive to the variation of the Borel mass:

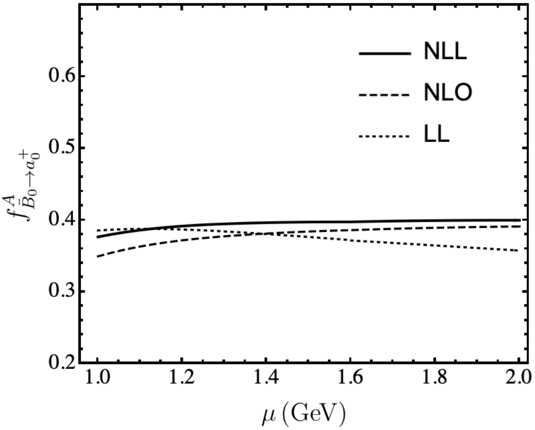
$$\frac{\partial \ln f_{BS}^{A,P,T}}{\partial \ln \omega_M} \leq 35\%.$$



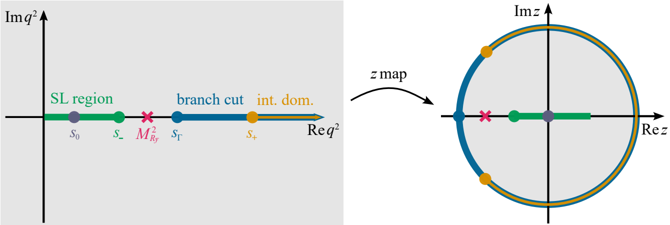


Numerical results

NLL reduces the scale dependence of the result



Fit



BCL parameterization:

$$f_{BS}^{A,T} = \frac{1}{1 - q^2/m^2_{B_{q1}}} \sum_{k=0}^{k=N-1} b_k^{A,T} \left[z\left(q^2, t_0\right)^k - (-1)^{k-N} \frac{k}{N} z\left(q^2, t_0\right)^N \right]$$

$$f_{BS}^{P} = \frac{1}{1 - q^2/m^2_{B_q}} \sum_{k=0}^{k=N-1} b_k^{P} z\left(q^2, t_0\right)^k$$

Fit

Dispersion relation of the form factors:

$$f_{BS}^{A}\left(q^{2}\right)=\frac{g_{B_{q1}BS}f_{B_{q1}}}{2m_{B_{q1}}\left(1-q^{2}/m_{B_{q1}}^{2}\right)}+\frac{1}{\pi}\int_{s_A}^{\infty}ds\frac{\mathrm{Im}\,f_{BS}^{A}(s)}{s-q^{2}}$$

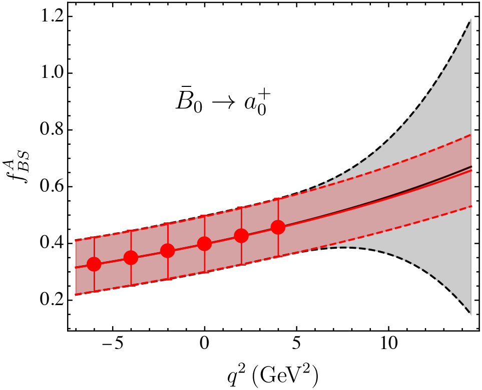
The constraint from the strong coupling constant:

$$g_{B_{q1}BS}=\frac{2m_{B_{q1}}}{f_{B_{q1}}}\lim_{q^{2}\to m_{B_{q1}}^{2}}\left[\left(1-q^{2}/m_{B_{q1}}^{2}\right)f_{BS}^{A}\left(q^{2}\right)\right]$$

From LCSR, $\mathrm{g}_{B_{q1}Ba_0^+}^{(\mathrm{tw2,LO})}=0.0683129\pm0.0146775.$

Fit

Red area: adding the strong coupling constant constraint.
Grey area: data points given only by LCSR.



Phenomenology

Processes	Methods	$\mathcal{BR}$	$\mathcal{A}_{\rm FB}$	$\mathcal{F}_{\rm H}$	$\mathcal{A}_{\lambda_\ell}$
$\bar{B}^0 \to a_0^+(1450)\mu\bar{\nu}_\mu$	This work	$1.00^{+0.43}_{-0.43}\times 10^{-4}$	$6.25^{+0.32}_{-0.32}\times 10^{-3}$	$1.61^{+0.13}_{-0.13}\times 10^{-2}$	$0.976^{+0.003}_{-0.003}$
	LCSR	$1.8^{+0.9}_{-0.6}\times 10^{-4}$			
	pQCD	$3.25^{+2.36}_{-1.36}\times 10^{-4}$			
$\bar{B}^0 \to a_0^+(1450)\tau\bar{\nu}_\tau$	This work	$3.0^{+1.2}_{-1.2}\times 10^{-5}$	$0.341^{+0.003}_{-0.003}$	$0.757^{+0.028}_{-0.028}$	$-0.158^{+0.085}_{-0.085}$
	LCSR	$6.3^{+3.4}_{-2.5}\times 10^{-5}$			
	pQCD	$1.32^{+0.97}_{-0.57}\times 10^{-4}$			
$\bar{B}^0 \to \bar{K}_0^*(1430)\nu_\ell\bar{\nu}_\ell$	This work	$2.50^{+0.97}_{-0.97}\times 10^{-6}$			
$\bar{B}_s \to K_0^{*+}(1430)\mu\bar{\nu}_\mu$	This work	$1.60^{+0.67}_{-0.67}\times 10^{-4}$	$5.81^{+0.31}_{-0.31}\times 10^{-3}$	$1.50^{+0.13}_{-0.13}\times 10^{-2}$	$0.978^{+0.003}_{-0.003}$
	LCSR	$1.3^{+1.2}_{-0.4}\times 10^{-4}$			
	QCDSR	$3.6^{+3.8}_{-2.4}\times 10^{-5}$			
	pQCD	$2.45^{+1.77}_{-1.05}\times 10^{-4}$			
$\bar{B}_s \to \bar{K}_0^{*+}(1430)\tau\bar{\nu}_\tau$	This work	$5.2^{+2.0}_{-2.0}\times 10^{-5}$	$0.334^{+0.004}_{-0.004}$	$0.737^{+0.030}_{-0.030}$	$-0.120^{+0.090}_{-0.090}$
	LCSR	$5.2^{+5.7}_{-1.8}\times 10^{-5}$			
	pQCD	$1.09^{+0.82}_{-0.47}\times 10^{-4}$			
$\bar{B}_s \to f_0(1500)\nu_\ell\bar{\nu}_\ell$	This work	$2.67^{+1.01}_{-1.01}\times 10^{-6}$			

LCSR: 0804.2204 QCDSR: 0509103 pQCD: 0811.2648

$$\mathcal{R}_S = \frac{\Gamma(B\to S\tau\bar{\nu}_\tau)}{\Gamma(B\to S\mu\bar{\nu}_\mu)},\qquad \mathcal{R}_{a_0(1450)} = 0.309\pm 0.032,$$

$$\mathcal{R}_{K_0^*(1430)} = 0.337\pm 0.032.$$

Summary

$B \to S$ form factors within B-meson LCSR

- Three-parameter model of B-meson LCDA
- NLL resummation at leading power
  - Method of regions $\to$ factorization formula of correlation function
  - RGE $\to$ NLL resummation
- z-series expansion: correlations, constraint of strong coupling constant
- phenomenology: $B \to S\ell\bar{\nu}$ and $B \to S\nu\bar{\nu}$

Thank you!