Status of V_{xb} measurements from Belle (II)



曹 山谷 (lu.cao@desy.de)

全国第二十届重味物理和CP破坏研讨会 2023年12月16日

















- Decay chain: $\mathbf{B}^0 \rightarrow \mathbf{D}^{*+} \ell \nu$, $\mathbf{D}^{*+} \rightarrow \mathbf{D}^0 \pi^+_{slow}$, $\mathbf{D}^0 \rightarrow \mathbf{K}^- \pi^+$
- Untagged strategy (higher efficiency than tagged)
- Select energetic signal lepton $p^{CM} > 1.2 \text{ GeV}$
- Measured total \mathscr{B} and differential spectra: recoil parameter w, and angles $\cos\theta_{\ell}$, $\cos\theta_{\nu}$, χ
- Extract |V_{cb}|, lepton angular asymmetry, D* **longitudinal polarization fractions**





<u>PRD 108, 092013 (2023)</u>









- parameterisations [Phys. Rev. D56, 6895)]
- BGL truncation based on nested hypothesis test [Phys. Rev. D100, 013005]

$$\chi^{2} = \sum_{i,j}^{34} \left(\frac{\Delta \Gamma_{i}^{\text{obs}}}{\Gamma^{\text{obs}}} - \frac{\Delta \Gamma_{i}^{\text{pre}}}{\Gamma^{\text{pre}}} \right) C_{ij}^{-1} \left(\frac{\Delta \Gamma_{j}^{\text{obs}}}{\Gamma^{\text{obs}}} - \frac{\Delta \Gamma_{j}^{\text{pre}}}{\Gamma^{\text{pre}}} \right) <= 8$$

$$+ \frac{(\Gamma^{\text{obs}} - \Gamma^{\text{pre}})^{2}}{\sigma_{\Gamma}^{2}}, \quad <= \text{normalization}$$

$$|V_{cb}|_{\text{BGL}} = (40.57 \pm 0.000) + (40.13 \pm 0.000)$$

• Fit differential shapes on w, $\cos\theta_{\ell}$, $\cos\theta_{\nu}$, χ with **Caprini-Lellouch-Neubert (CLN)** [Nucl. Phys. B530, 153] & **Boyd-Grinstein-Lebed (BGL)**

Traditional way

shape

$$|V_{cb}|\eta_{\rm EW}\mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}} \left(\frac{|\tilde{b}_0|}{P_f(0)\phi_f(0)}\right)$$

$0.31 \pm 0.95 \pm 0.58) \times 10^{-3}$

$0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$

stat. syst. LQCD uncertainty (dominated by slow pion on F(1) tracking eff. leptonID)









- parameterisations [Phys. Rev. D56, 6895)]
- BGL truncation based on nested hypothesis test [Phys. Rev. D100, 013005]
- Inclusion of LQCD constraints [MILC: EPJC 82, 1141 (2022)] at beyond zero-recoil (w = [1.03, 1.10, 1.17]) in two scenarios

$$\chi^{2} = \sum_{i,j}^{34} \left(\frac{\Delta \Gamma_{i}^{\text{obs}}}{\Gamma^{\text{obs}}} - \frac{\Delta \Gamma_{i}^{\text{pre}}}{\Gamma^{\text{pre}}} \right) C_{ij}^{-1} \left(\frac{\Delta \Gamma_{j}^{\text{obs}}}{\Gamma^{\text{obs}}} - \frac{\Delta \Gamma_{j}^{\text{pre}}}{\Gamma^{\text{pre}}} \right) + \frac{(\Gamma^{\text{obs}} - \Gamma^{\text{pre}})^{2}}{\sigma_{\Gamma}^{2}} + \sum_{ij} (F_{i}^{\text{LQCD}} - F_{i}^{\text{exp}}) C_{ij}^{-1} (F_{j}^{\text{LQCD}} - F_{j}^{\text{exp}}) C_{ij}^{-1} (F_{j}^{\text{LQCD}} - F_{j}^{-1}) C_{ij}^{-1} (F_{j}^{\text{LQCD}} - F_{j}^{-1}) C_{ij}^{-1} (F_{j}^{\text{LQCD}} - F_{j}^{-1}) C_{ij}^{-1} (F_{j}^{\text{LQCD}} - F_{j}^{-1}) C_{ij}^$$





• Fit differential shapes on w, $\cos\theta_{\ell}$, $\cos\theta_{\nu}$, χ with **Caprini-Lellouch-Neubert (CLN)** [Nucl. Phys. B530, 153] & **Boyd-Grinstein-Lebed (BGL)**

BGL	Constraints on $h_{A_1}(w)$	Constraints on $h_{A_1}(w), R_1(w), R_2(w)$	
$a_0 \times 10^3$ $b_0 \times 10^3$	21.7 ± 1.3 13.19 ± 0.24	$\begin{array}{c} 25.6 \pm 0.8 \\ 13.61 \pm 0.23 \end{array}$	
$b_1 \times 10^3$ $c_1 \times 10^3$ $ V_1 \times 10^3$	-6 ± 6 -0.9 ± 0.7 40 3 ± 1 2	2 ± 6 -0.0 ± 0.7 38 3 ± 1.1	
χ^2/ndf p value	39/33 21%	75/39 0.04%	V _{cb} shifts include ful
			constraints

Similar tension seen in recent Belle (2023) measurement [PRD 108, 012002 (2023)]

⇒ Both found large disagreements wrt LQCD results on R₂







- Lepton-flavor-universality tested with separate results on \mathscr{B} of e & mu modes
- All in good agreement with SM expectations

$$R_{e/\mu} = 0.998 \pm 0.009 \pm 0.020$$
 stat. syst.

Test on forward-backward asymmetry

$$\begin{split} \mathcal{A}_{\mathrm{FB}} &= \frac{\int_{0}^{1} \mathrm{d} \cos \theta_{\ell} \mathrm{d} \Gamma / \mathrm{d} \cos \theta_{\ell} - \int_{-1}^{0} \mathrm{d} \cos \theta_{\ell} \mathrm{d} \Gamma / \mathrm{d} \cos \theta_{\ell}}{\int_{0}^{1} \mathrm{d} \cos \theta_{\ell} \mathrm{d} \Gamma / \mathrm{d} \cos \theta_{\ell} + \int_{-1}^{0} \mathrm{d} \cos \theta_{\ell} \mathrm{d} \Gamma / \mathrm{d} \cos \theta_{\ell}} \\ \mathcal{A}_{\mathrm{FB}}^{e} &= 0.228 \pm 0.012 \pm 0.018, \\ \mathcal{A}_{\mathrm{FB}}^{\mu} &= 0.211 \pm 0.011 \pm 0.021, \\ \end{split}$$
$$\begin{split} \Delta \mathcal{A}_{\mathrm{FB}} &= \left(-17 \pm 16 \pm 16\right) \times 10^{-3} \end{split}$$



<u>PRD 108, 092013 (2023)</u>



Test on D* longitudinal polarization fraction

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta_V} = \frac{3}{2} \left(F_L \cos^2\theta_V + \frac{1 - F_L}{2} \sin^2\theta_V \right)$$

 $F_L^e = 0.520 \pm 0.005 \pm 0.005$ $F_L^{\mu} = 0.527 \pm 0.005 \pm 0.005$ $\Delta F_L = 0.006 \pm 0.007 \pm 0.005$





Angular Coefficients of $B \rightarrow D^* \ell \nu$

- Full Belle data set of 711 fb⁻¹ for $B^{\pm,0}$, $\ell = e, \mu$
- Hadronic tagging and background subtracted via fitting $M_{
 m miss}^2$
- Instead of binning in w, $\cos\theta_{\ell}$, $\cos\theta_{\nu}$, χ , measure total 12 angular **coefficients** J_i in four bins of w => 4D differential decay rate
 - Angles provide information on, e.g. forward-backward asymmetry, longitudinal polarization fraction
 - "S" observables sensitive to new physics ($J_{7,8,9} = 0$ in SM)
- Extract |V_{cb}| with external constraint on normalisation + LQCD beyond zero-recoil
- Can also test **LFU** via $\Delta = J_i^e J_i^\mu$

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^*\ell\bar{\nu}_{\ell})}{\mathrm{d}w\,\mathrm{d}\cos\theta_{\ell}\,\mathrm{d}\cos\theta_{\mathrm{V}}\,\mathrm{d}\chi} = \frac{2G_{\mathrm{F}}^2\eta_{\mathrm{EW}}^2|V_{\mathrm{cb}}|^2m_B^4m_{\mathrm{D}^*}}{2\pi^4} \times \left(J_{1s}\sin^2\theta_{\mathrm{V}} + J_{1c}\cos^2\theta_{\mathrm{V}}\right) + (J_{2s}\sin^2\theta_{\mathrm{V}} + J_{2c}\cos^2\theta_{\mathrm{V}})\cos^2\theta_{\ell} + J_{3}\sin^2\theta_{\mathrm{V}}\sin^2\theta_{\mathrm{V$$





arXiv: 2310.20286



 $1^2 \, heta_\ell \cos 2\chi$ $(J_{6s}\sin^2\theta_{\rm V}+J_{6c}\cos^2\theta_{\rm V})\cos\theta_{\ell}$ $J_9 \sin^2 \theta_{\rm V} \sin^2 \theta_\ell \sin 2\chi \Big) \,.$



Angular Coefficients of $B \rightarrow D^* \ell \nu$

- Obtained $|V_{cb}|$ is in **agreement** with the fit of the one-dimensional partial rates determined from the same data set
- Also **agrees** with the latest & most precise determinations of $|V_{cb}|$ from **inclusive mode**







arXiv: 2310.20286



Angular Coefficients of $B \rightarrow D^* \ell \nu$





Plans for Future V_{cb} Measurements at Belle II

- Belle II Physics Week Vcb workshop @ Nov 2023 [indico link]
- Experimentalists + Theorists: discussed crucial proposals on V_{cb} in 3 years & beyond
- Will wrap into a KEK report (in preparation)

Exclusive

- Latest exclusive $|V_{cb}|$ in Belle's angular analysis is in agreement with inclusive determination
- Two out of three **LQCD** groups's results on $B \rightarrow D^* \ell \nu$ form factors showing tensions with both experiments and HQE
- Measure angular coefficients / helicity amplitude with external constraints on normalization (only shape measurement, hadronic tagged)
- Separate measurement for **branching fraction** (preferably untagged)
- Simultaneous analysis of $B \rightarrow D, D^* \ell \nu$ provide better sensitivity

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Inclusive

- Latest inclusive $|V_{cb}|$ extraction from full (q^2, E_l^B, M_X) moments is consistent with previous incl. determinations
- LQCD effort of inclusive calculation started
- Measure all kinematic moments simultaneously for future
- Semi-inclusive measurements are needed to improve the understanding of "gap" modes
- Need to evaluate **QED effects** (provide additional results w/o FSR correction)
- Provide differential inclusive spectra to support LQCD study





First Simultaneous Determination of $V_{ub}^{\text{incl.}}$ & $V_{ub}^{\text{excl.}}$

- Using **full Belle** dataset of 711 fb⁻¹
- Hadronic tagging with Neutral Networks
- Use BDT to suppress backgrounds with 11 training features, e.g. $M_{\rm miss}^2$, #K±, #Ks, etc.
- Reconstruction strategy inherited from recent Belle's $B \rightarrow X_{\mu} \ell \nu$ measurements (phase space region $E_{\ell}^B > 1$ GeV)
 - $\Delta \mathcal{B}, |V_{ub}| @ PRD 104, 012008 (2021)$
 - Differential spectra @ PRL 127, 261801 (2021)







Can fully assign each final state particle to either the tag side or signal side

 \Rightarrow Allows to reconstruct X_{μ}

Reconstructed kinematic variables

Hadronic system X:

$$p_X = \sum_i (\sqrt{m_\pi^2 + |\mathbf{p_i}|^2}, \mathbf{p_i}) + \sum_i (E_i, \mathbf{k_i})$$

Missing mass squared:

$$M_{\rm miss}^2 = (p_{\Upsilon(4S)} - p_{\rm tag} - p_X - p_\ell)^2$$

Leptonic system:

$$q^2 = (p_B - p_X)^2 = (p_\ell - p_\nu)^2$$







Signal Extraction of Incl. & Excl. $B \rightarrow X_{\mu} \ell \nu$

- Additional selections on thrust **T** of X system in c.m.s to **increase significance** of $B \to \pi \ell \nu$
- Extract signal in q^2 : $N_{\pi^{\pm}}$ for $B \to \pi \ell \nu$ and other $B \to X_{\mu} \ell \nu$ events
- Simultaneous determination of signal yields and $B \rightarrow \pi \ell \nu$ form factor (FF) parameters
- Systematic uncertainties included via **bin-wise Nuisance para.** θ of each template





PRL 131, 211801 (2023)







Signal Extraction of Incl. & Excl. $B \rightarrow X_u \ell \nu$

- Additional selections on thrust **T** of X system in c.m.s to **increase significance** of $B \to \pi \ell \nu$
- Extract signal in \mathbf{q}^2 : $\mathbb{N}_{\pi^{\pm}}$ for $B \to \pi \ell \nu$ and other $B \to X_u \ell \nu$ events
- Simultaneous determination of signal yields and $B \rightarrow \pi \ell \nu$ form factor (FF) parameters
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PRL 131, 211801 (2023)

$$(1 + \epsilon \cdot \theta) + \theta \rho_{\theta}^{-1} \theta^{T} + \chi_{\text{FF}}^{2}$$
Constraints on BCL parameters, input taken
LQCD / LQCD+exp fits in FLAG Review 2021

Differential decay rate
Acceptance & reco. effic
Forward-folding q²

 $\Rightarrow \pi^{0} \ell \nu$
all background
$$W_{ub}^{\text{excl.}} = \sqrt{\frac{\mathscr{B}(B \to \pi \ell)}{\tau_{B} \cdot \Gamma_{\text{FF}}}}$$

$$W_{ub}^{\text{excl.}} = \sqrt{\frac{\mathscr{B}(B \to \pi \ell)}{\tau_{B} \cdot \Gamma_{\text{FF}}}}$$













Results of Incl. & Excl. |V_{ub}|

- Various fit scenarios applied:
 - **Combined** or separate $B \to \pi^+ \ell \nu, B \to \pi^0 \ell \nu$ (isospin relation)
 - Input BCL constraint: LQCD + exp. or only LQCD [FLAG: Eur. Phys. J. C 82, 869 (2022)]



V_{ub} in combined scenario with **LQCD+exp** const.:

Excl. $(3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3}$

Incl. $(3.88 \pm 0.20_{\text{stat}} \pm 0.31_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3}$

Ratio 0.97 ± 0.12 ($\rho = 0.11$)

compatible with the world average within 1.2σ



PRL 131, 211801 (2023)







Global Fits

- Large model dependence in B meson shape function (SF) => need data-driven treatment
- Most information in differential spectra
- Combining different decay modes (same leading SF) in a global analysis is the future

$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2 \left|C_7^{\mathrm{incl}}
ight|^2 \int\!\mathrm{d}k\,\widehat{W}_{77}(E_{oldsymbol{\gamma}};k)\,\widehat{F}(m_B+d\Gamma_u) = |V_{ub}|^2 \int\!\mathrm{d}k\,\widehat{W}_u(p_X^-,p_X^+,E_\ell;k)\widehat{F}(p_X^+-k) + \cdot$$

- Fit parameters: $|V_{tb}V_{ts}^*|^2 m_b^2$, $|V_{ub}|^2$, $\widehat{F}(\lambda x) = \frac{1}{\lambda} \left[\sum_{n=0}^{\infty} c_n f_n(x)\right]^2$
 - Theory Input: $\widehat{W}_i(\ldots;k)$ computed to (N)NNL'+NNLO in 1S scheme •
 - Factorized shape function: •

$$S(\omega,\mu_\Lambda) = \int \mathrm{d}k\, \widehat{C}_0(\omega-k,\mu_\Lambda)\, \widehat{F}(k)$$

F(k) nonperturbative part

- Determines peak region
- Fit from data

 $C_0(\omega,\mu_\Lambda)$ perturbative part

• Generates perturbative tail with correct μ_{Λ} dependence

$B \rightarrow X_{s} \gamma$ Global Fit: PRL 127, 102001 (2021)





Global Fits Prospects on $B \rightarrow X_s \gamma + B \rightarrow X_u \ell \nu$

- Theory
 - NLL' + NLO
 - ignore subleading SFs •
- Toy study •
 - Generate mX, EI, and E_X from theory ٠
 - Smeared from uncertainties and • correlations inspired by BaBar hadronic tag analysis, Belle II hadronic tagging efficiency is much better by now
 - Target lumi: 1/ab, 5/ab
 - **Caveats:** •
 - No resolution effects considered
 - No theory uncertainties included (!)
 - Not done with Belle II MC







Credit: F. Bernlochner

 $E_\gamma ~[{
m GeV}]$



Global Analysis



Credit: F. Bernlochner







Search for $B \rightarrow \tau \nu$ at Belle II

- Sensitivity based on a data set of 362 fb⁻¹ studied with MC simulations
- Full analysis chain validated in pseudo-data
- With optimized signal selection: statistical uncertainty 37% (2.8σ from null hypothesis), systematic uncertainty ~13 %
- Results expected to be released in **2024**

Most discriminating variables for signal:

- E_{ECL}^{extra} , the extra energy not associated with the B_{tag} and B_{sig} (Rest of Event).
- $M_{miss}^2 = E_{miss}^2 p_{miss}^2$, squared magnitude of the four-momentum p_{miss} .

A data-driven correction is applied to both the variables to correct the distributions.





$M_{\rm miss}^2$ $5 10 M_{miss}^2 (GeV)^2$ 15 20





Expected Sensitivity of Leptonic V_{ub} on Belle II

- Expected event yields and precisions on the branching fraction and determined Vub
- Ultimate precision on |V_{ub}| at ~2.5% level from $B \rightarrow \tau \nu, B \rightarrow \mu \nu$



Summary

- Several new results on $|V_{xb}|$ measured recently at Belle and Belle II
- Useful discussions in Belle II Physics week trigger a clear plan for |V_{cb}| in 3 years
- Continuous efforts from experiment and theory are still needed
- Expected **upcoming** $|V_{xb}|$ results in 2024 spring:
 - Had. tagged $B \to \pi \ell \nu, B \to \rho \ell \nu$ lacksquare
 - Untagged simultaneous $B \to \pi \ell \nu, B \to \rho \ell \nu$ lacksquare
 - Had. tagged $B \rightarrow \tau \nu$
 - Had. tagged $B \rightarrow \mu\nu$ [Belle+Belle II]

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THANK YOU

	-			

Backup: Angular Coefficients of $B \rightarrow D^* \ell \nu$

$$\begin{split} A_{\rm FB} &= \frac{3}{2} \frac{(J_{6c} + 2J_{6s})}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ F_L(D^*) &= \frac{(3J_{1c} - J_{2c})}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ S_3 &= \frac{1}{\pi} \frac{4J_3}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ S_5 &= \frac{3J_5}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ S_7 &= \frac{3J_7}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ S_9 &= \frac{1}{\pi} \frac{4J_9}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ S_9 &= \frac{1}{\pi} \frac{4J_9}{3J_{1c} - J_{2c} + 2(3J_{1s} - J_{2s})}, \\ \end{split}$$

arXiv: 2310.20286

E VII. The fitted coefficients for the BGL fit described in the text.

	Value	Corre	lation	L						
	2.68 ± 0.06	1.00	0.03	-0.11	0.17	0.00	-0.02	0.08	-0.04	-0.19
	-2.65 ± 2.35	0.03	1.00	-0.67	-0.01	0.17	-0.03	0.09	-0.03	-0.10
	-145.18 ± 87.35	-0.11	-0.67	1.00	0.03	-0.02	-0.03	0.01	-0.05	-0.03
	1.30 ± 0.01	0.17	-0.01	0.03	1.00	-0.02	0.02	0.02	-0.03	-0.50
	1.30 ± 0.66	0.00	0.17	-0.02	-0.02	1.00	-0.66	0.59	-0.46	-0.28
	-11.50 ± 22.40	-0.02	-0.03	-0.03	0.02	-0.66	1.00	-0.45	0.40	-0.04
	-0.25 ± 0.16	0.08	0.09	0.01	0.02	0.59	-0.45	1.00	-0.82	-0.32
	0.78 ± 3.29	-0.04	-0.03	-0.05	-0.03	-0.46	0.40	-0.82	1.00	0.05
•	40.97 ± 0.67	-0.19	-0.10	-0.03	-0.50	-0.28	-0.04	-0.32	0.05	1.00

Backup: Angular Coefficients of $B \rightarrow D^* \ell \nu$

arXiv: 2310.20286 preliminary

Backup: First Simultaneous Determination of Incl. & Excl. Vub

- Various fit scenarios applied:
 - **Combined** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$ (isospin relation) \bullet
 - Input BCL constraint: LQCD + exp. or only LQCD [FLAG: Eur. Phys. J. C 82, 869 (2022)]

Points: subtract other $B \to X_{\mu} \ell \nu$ and background in data, and apply unfolding + eff. correction

PRL 131, 211801 (2023)

Fitted BCL parameters (LQCD + exp.)

	$ V_{ub} \times 10^3$	a_0^+	a_1^+	a_{2}^{+}	a_0^0	a_1^0
Central	3.777	0.414	-0.493	-0.297	0.500	-1.426
Uncertainty	0.309	0.014	0.053	0.180	0.023	0.054
$ V_{ub} $	1.000	-0.452	-0.168	0.232	-0.109	-0.105
a_0^+		1.000	0.151	-0.451	0.259	0.142
a_1^+			1.000	-0.798	-0.096	0.214
a_2^+				1.000	0.012	-0.097
a_0^0					1.000	-0.451
a_1^0						1.000

Backup: First Simultaneous Determination of Incl. & Excl. Vub

Leading Systematic Uncertainties

	Relative Syst. Uncertainty
de	$\mathscr{B}(B \to \pi \ell \nu)$
	4.1%
	3.5%
	1.2%
de	$\Delta \mathscr{B}(B \to X_u \ell \nu)$
	10.9%
	5.3%
	2.8%

Hybrid Model of $B \to X_{\mu} \ell \nu$

Hybrid MC is a **combination** of **resonances** (exclusive decays) and **non-resonant** contribution in the inclusive $B \to X_{\mu} \ell \nu$ decays

EvtGen simulation: \bullet

(1) exclusive modes $B \to (\pi, \rho, \omega, \eta^{(\prime)}) \ell \nu$ with latest WA form factors & branching fractions

(2) fully inclusive $B \to X_{\mu} \ell \nu$ (only non-resonant shapes, e.g. BLNP, GGOU)

Calculate hybrid weights to mix resonance & non-res. in **3D** \bullet binning of (q^2, E_{ℓ}^B, M_X) to recover total $\mathscr{B}(B \to X_{\mu} \ell \nu)$ in each bin

$$H_i = \frac{R_i}{R_i} + \omega_i N_i$$

Systematic uncertainties include the impact from exclusive FFs & BRs, total $\mathscr{B}(B \to X_{\mu} \ell \nu)$, inclusive models

PRD 104, 012008 (2021)

