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正德厚生 篤學敏行

矢量底粲介子衰变的有效理论研究与极化分析

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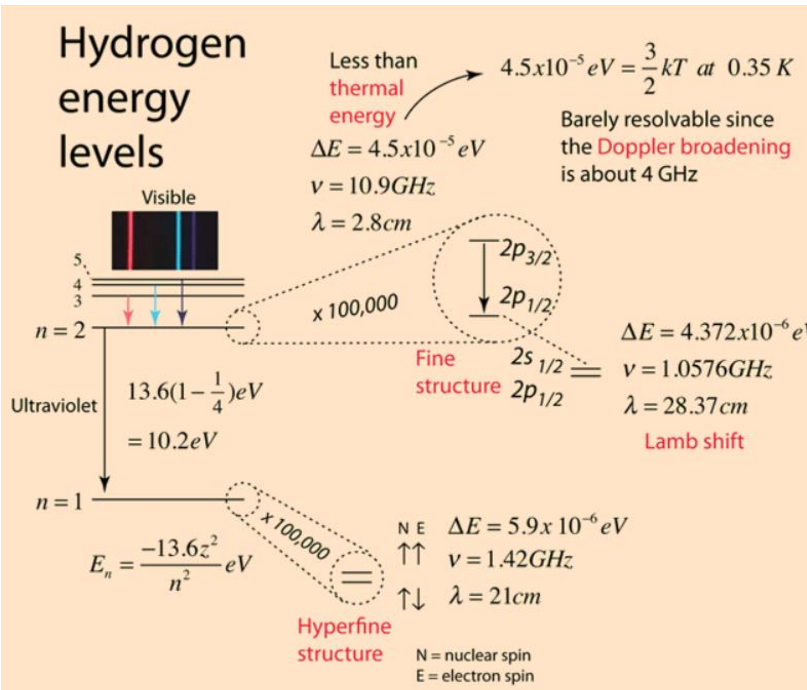
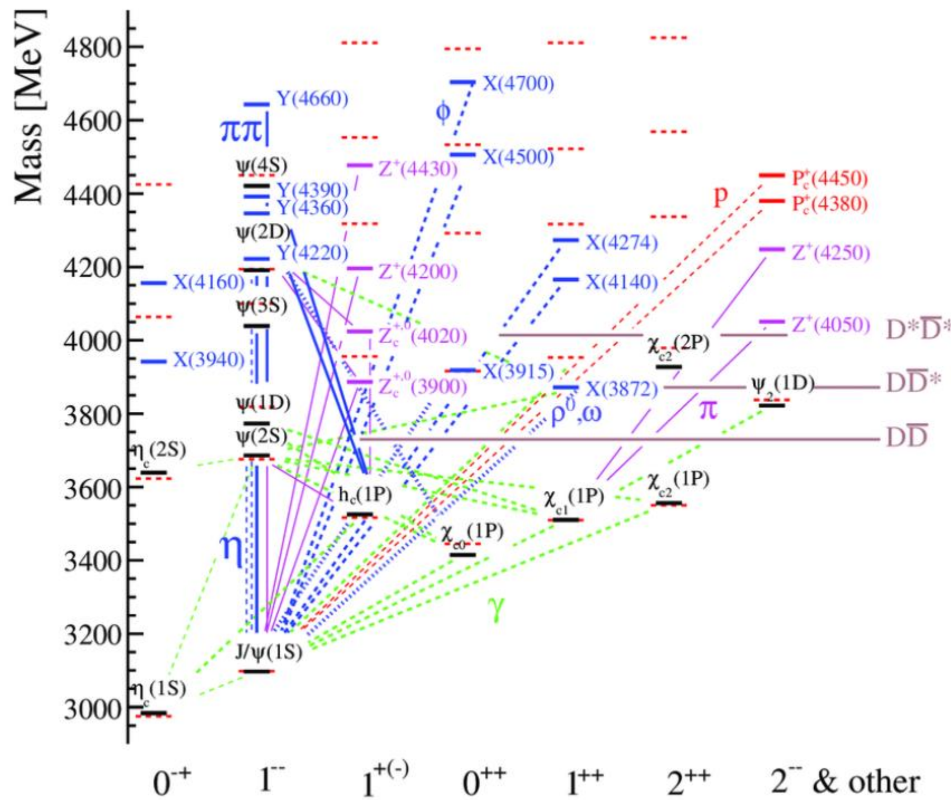
1、Research Background

Spectroscopy at angstrom scale and femto-scale



Hydrogen Spectrum
QM->QED



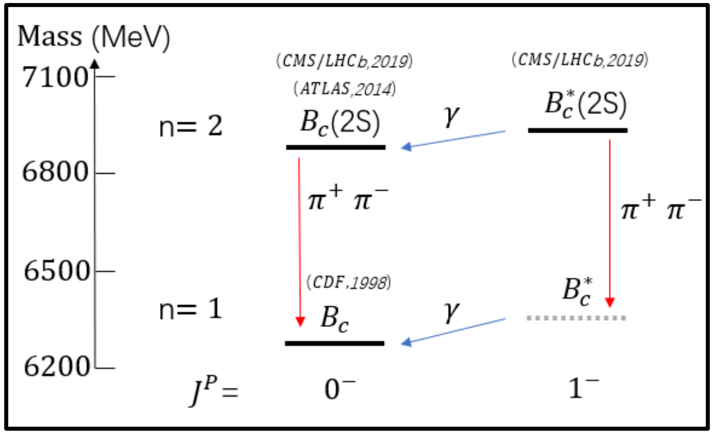
Charm region Spectrum
QCD

Conventional Meson Spectroscopy

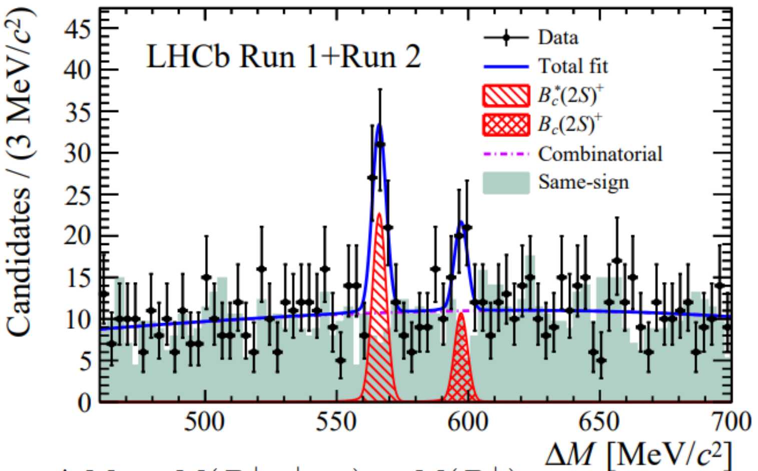
Quark-antiquark S-wave meson puzzle

$[u\bar{b}]$ B^+, B^{*+}	$[u\bar{c}]$ $\bar{D}^0, \bar{D}^{*0}$	$[u\bar{s}]$ K^+, K^{*+}	$[u\bar{d}]$ π^+, ρ^+	$[u\bar{u}]$ π^0, η, η' ρ^0, ω, ϕ
$[d\bar{b}]$ B^0, B^{*0}	$[d\bar{c}]$ D^-, D^{*-}	$[d\bar{s}]$ K^0, K^{*0}	$[d\bar{d}]$ π^0, η, η' ρ^0, ω, ϕ	
$[s\bar{b}]$ B_s^0, B_s^{*0}	$[s\bar{c}]$ D_s^-, D_s^{*-}	$[s\bar{s}]$ η, η' ω, ϕ		
$[c\bar{b}]$ B_c^+, B_c^{*+}	$[c\bar{c}]$ $\eta_c, J/\psi$			
$[b\bar{b}]$ η_b, Υ				

The Bc family is not complete



Beauty-charm family

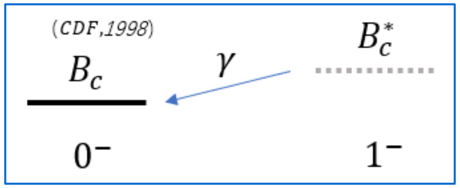


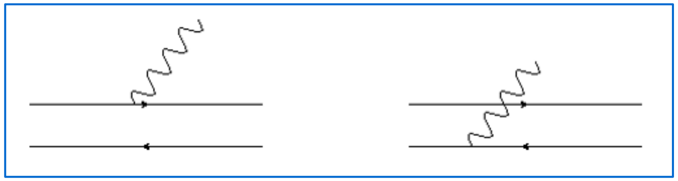
$\Delta M = M(B_c^+\pi^+\pi^-) - M(B_c^+)$

Vector Bc*(1⁻) Electromagnetic and Weak Decays

- **Hyperfine splitting between Bc* and Bc: ~ 60MeV**

- **Bc* major (99.99%) electromagnetic decays to ground Bc**



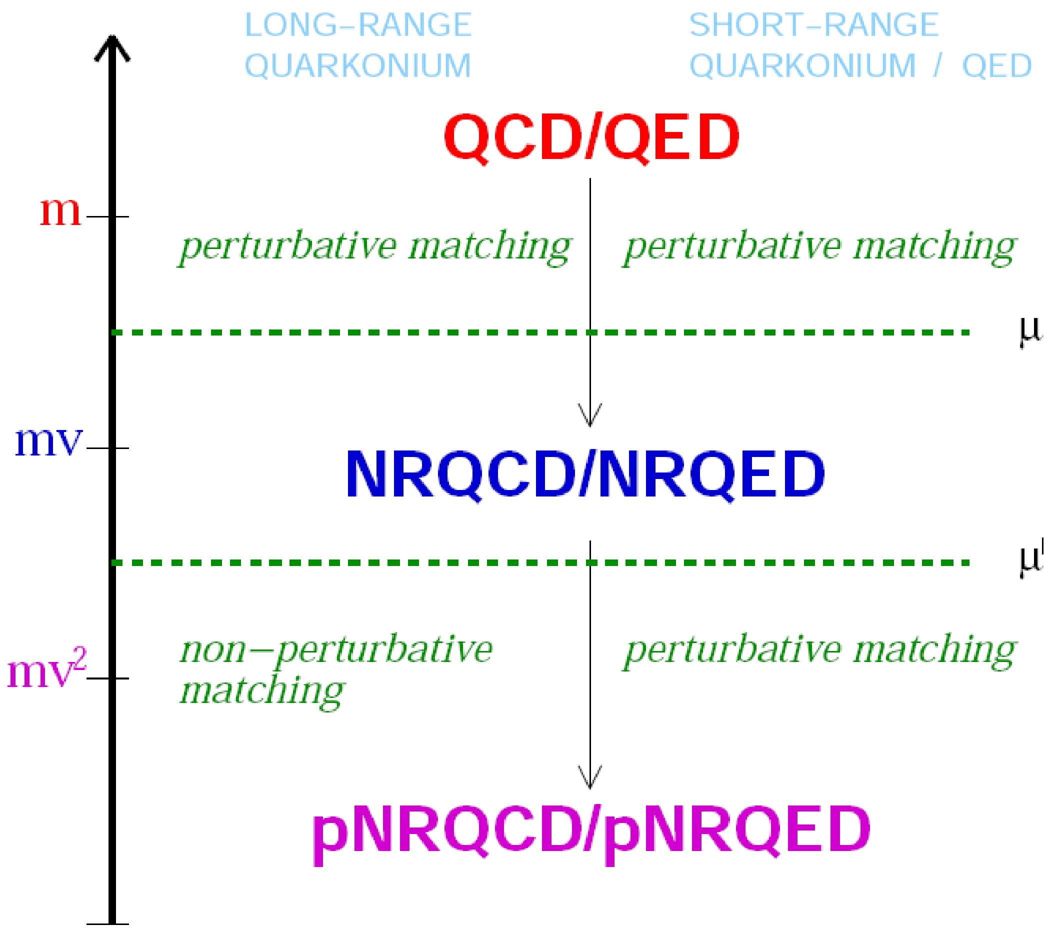


- ***However, 60MeV photon is hard to detect at LHC environment; current $e^+ e^-$ colliders can not create two beauty and charm pairs***

- ***Solid theoretical analysis and new observables are required***

2、EFT Frameworks for Bc* Decays

- **Nonrelativistic Effective Theory in QCD/QED**



$v^2 \approx 0.1$ for the Υ

$\alpha_s(mv) \sim v$

Bodwin-Braaten-Lapage 1995

Pineda-Soto-Brambilla-Vairo 2000

NRQCD/pNRQCD/γpNRQCD in unequal mass case

- **QCD Lagrangian**

$$\mathcal{L}_{\rm QCD} = \sum_{q=u,d,s,\dots} \overline{\psi}_{qi}(x)\left[(i\gamma_\mu D^\mu)_{ij} - m_q\delta_{ij}\right]\Psi_{qj}(x) - \frac{1}{4}F^a_{\mu\nu}(x)F^{\mu\nu\, a}(x)\ ,$$

- **Rewrite heavy quark field and do the NR expansion**

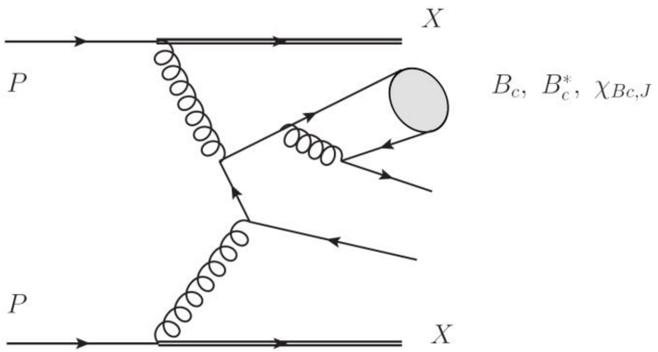
$$\Psi = e^{-iMt}\tilde{\Psi} = e^{-iMt}\begin{pmatrix}\psi\\ \chi\end{pmatrix}, \qquad \Psi' = e^{iM't}\tilde{\Psi} = e^{iM't}\begin{pmatrix}\psi'\\ \chi'\end{pmatrix},$$

- **Obtain NRQCD Lagrangian**

$$\begin{aligned}\mathcal{L}_{NRQCD} =& \psi^\dagger\left(iD_t - \frac{1}{2M}(i\mathbf{D})^2\right)\psi + \frac{c_F}{2M}\psi^\dagger\boldsymbol{\sigma}\cdot g\mathbf{B}\psi\\ &+ \frac{c_D}{8M^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi^\dagger + \frac{c_S}{8M^2}\psi^\dagger(i\boldsymbol{\sigma}\cdot\mathbf{D}\times g\mathbf{E} - i\boldsymbol{\sigma}\cdot g\mathbf{E}\times\mathbf{D})\psi^\dagger\\ &+ \frac{c_4}{8M^3}\psi'^\dagger\left(\mathbf{D}^2\right)^2\psi' + \mathcal{O}\left(1/M^3\right)\\ &+ [\psi\to i\sigma^2\chi'*, A_\mu\to -A_\mu^T, M\to M'] + \mathcal{L}_{light}].\end{aligned}$$

Vector Bc*(1⁻) Production

➢ Production at LHC



$\sigma_{B_c(1^1S_0)}$	49.8
$\sigma_{B_c^*(1^3S_1)}$	121.

The cross section is in unit of nb.

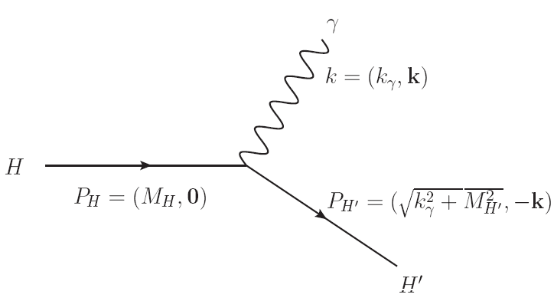
Chang-Wu,EPJC38,264(2004)

Larger cross section when consider DPS

➢ Other excited states hadronic transition.

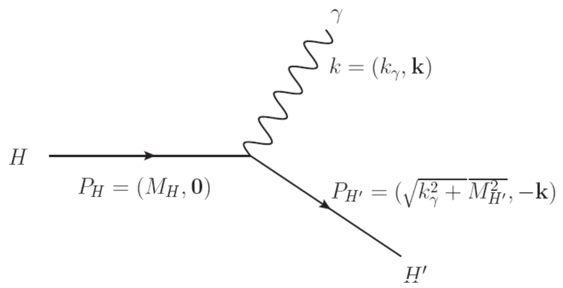
P-wave Bc states can electromagnetic decays to Bc* (1S) : E1 transition

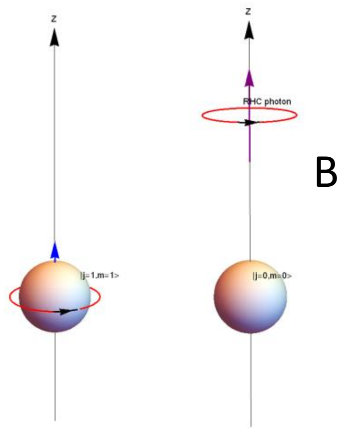
$$
\begin{aligned}
\mathcal{L}^{E1}_{\gamma\mathrm{pNRQCD}} = e\int d^3r\,\mathrm{Tr}\Big\{ & \frac{e_Q - e'_Q}{2}[V^{r\cdot E}S^\dagger \mathbf{r}\cdot\mathbf{E}^{em}S + V_O^{r\cdot E}O^\dagger\mathbf{r}\cdot\mathbf{E}^{em}O \\
& + \frac{1}{24}V^{(r\nabla)^2 r\cdot E}S^\dagger\mathbf{r}\cdot\left[(\mathbf{r}\boldsymbol{\nabla})^2\mathbf{E}^{em}\right]S] \\
& + (\frac{e_Q m'_Q - e'_Q m_Q}{2m_Q m'_Q})[i\frac{1}{4}V^{\nabla\cdot(r\times B)}S^\dagger\left\{\boldsymbol{\nabla}\cdot,\mathbf{r}\times\mathbf{B}^{em}\right\}S \\
& + i\frac{1}{12}V^{(r\nabla)\nabla_r\cdot(r\times B)}S^\dagger\left\{\boldsymbol{\nabla}_r,\mathbf{r}\times[(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em}]\right\}S \\
& + \frac{1}{4}V^{(r\nabla)\sigma\cdot B}\left[S^\dagger,\boldsymbol{\sigma}\right]\cdot(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em}S \\
& + \frac{1}{r}V^{r\cdot E/r}S^\dagger\mathbf{r}\cdot\mathbf{E}^{em}S] \\
& - i(\frac{e_Q m^2_{Q'} - e'_Q m^2_Q}{8m^2_Q m^2_{Q'}})V^{\sigma\cdot(E\times\nabla_r)}\left[S^\dagger,\boldsymbol{\sigma}\right]\cdot(\mathbf{E}^{em}\times\boldsymbol{\nabla}_r)S\Big\}
\end{aligned}
$$



Vector Bc*(1⁻) Electromagnetic Decay

- **Bc* (1S) major (99.99%) electromagnetic decays to Bc(1S): M1 transition**



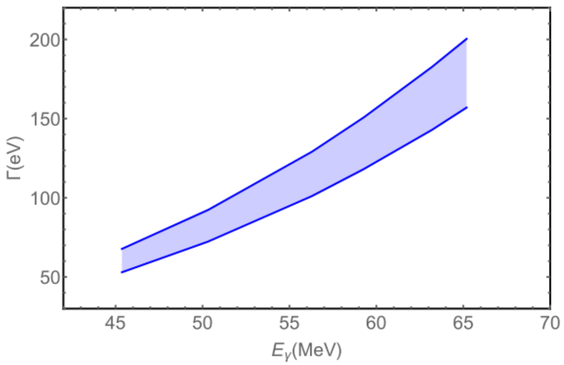


$$\Gamma_{J/\psi \to \eta_c \gamma} = (1.5 \pm 1.0)\ \mathrm{keV}.$$

Brambilla-jia-Vairo,PRD73,054005(2006)

$$\begin{aligned}
\mathcal{L}_{\gamma\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big[& e\frac{e_Q - e'_Q}{2} V_A^{\mathrm{em}} \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\mathrm{em}}\mathrm{S} \\
& + e\Big(\frac{e_Q m'_Q - e'_Q m_Q}{4 m_Q m'_Q}\Big)\Big[V_S^{\frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\}\mathrm{S} \\
& + \frac{1}{8} V_S^{(r\cdot\nabla)^2\frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \mathbf{r}^i\mathbf{r}^j \left(\boldsymbol{\nabla}^i\nabla^j\boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}}\right)\right\}\mathrm{S} \\
& + V_O^{\frac{\sigma\cdot B}{m}} \left\{\mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}}\right\}\mathrm{O}\Big] \\
& + e\Big(\frac{e_Q m_{Q'}^2 - e'_Q m_Q^2}{32 m_Q^2 m_{Q'}^2}\Big)\Big[4\frac{V_S^{\frac{\sigma\cdot B}{m^2}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\}\mathrm{S} \\
& + 4\frac{V_S^{\frac{\sigma\cdot(r\times r\times B)}{m^2}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\mathrm{em}})\right] \right\}\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla\times E}{m^2}} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}\times, \mathbf{E}^{\mathrm{em}}\right] \right]\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla_r\times r\cdot\nabla E}{m^2}} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i\left(\boldsymbol{\nabla}^i\mathbf{E}^{\mathrm{em}}\right)\right] \right]\mathrm{S}\Big] \\
& + e\Big(\frac{e_Q m_{Q'}^3 - e'_Q m_Q^3}{8 m_Q^3 m_{Q'}^3}\Big)\Big[V_S^{\frac{\nabla_r^2\sigma\cdot B}{m^3}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\}\nabla_r^2\,\mathrm{S} \\
& + V_S^{\frac{(\nabla r\cdot\sigma)(\nabla r\cdot B)}{m^3}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}^i\mathbf{B}^{\mathrm{em}j} \right\}\boldsymbol{\nabla}_r^i\nabla_r^j\,\mathrm{S}\Big]\Big],
\end{aligned}$$

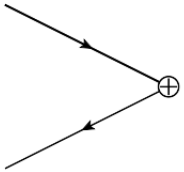
- **We generalize the pNRQCD to unequal mass case and obtain the effective Lagrangian**



Bc* weak decay and its decay constant in EFT

- **Bc* decay constants in QCD**

$$\left\langle 0 \left| \bar{b}\gamma^{\mu} c \right| B_c^*(P,\varepsilon) \right\rangle = f_{B_c^*}^{v} m_{B_c^*} \varepsilon^{\mu},$$



(a) LO

- **Bc* decay constants in NRQCD**

mathing coefficients   NRQCD LDMEs

$$f_{B_c^*}^{v} = \sqrt{\frac{2}{m_{B_c^*}}} \underline{C_v(m_b, m_c, \mu_f)} \underline{\left\langle 0 \left| \chi_b^{\dagger} \sigma \cdot \varepsilon \psi_c \right| B_c^*(\mathbf{P}) \right\rangle (\mu_f)} + \mathcal{O}(v^2)$$

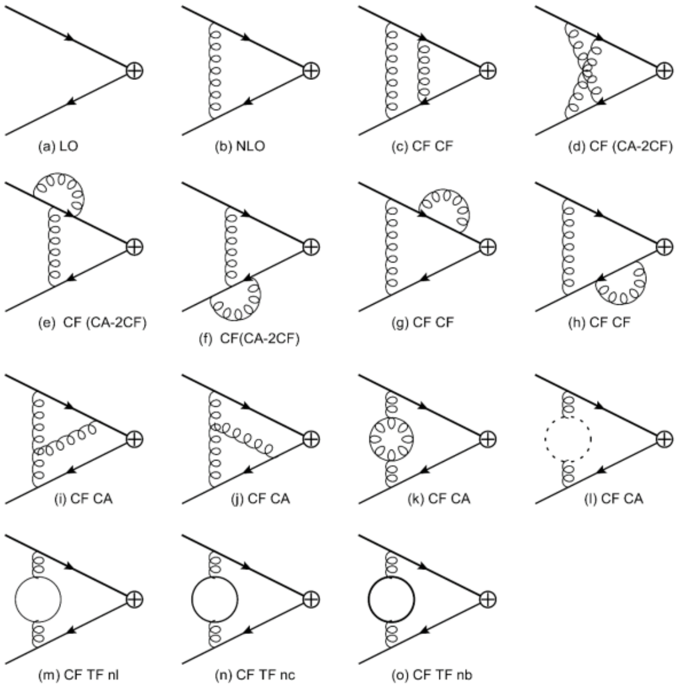
- **Matching Formulae**

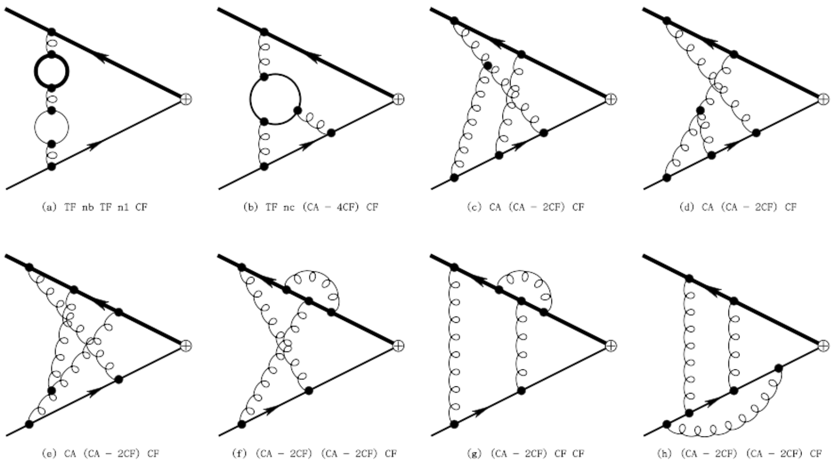
Braaten-Fleming,PRD52,181(1995);
Lee-Sang-Kim,JHEP01,113(2011)

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

$\tilde{Z}_J$:NRQCD $\overline{\mathrm{MS}}$ current renormalization constants

Typical diagrams up to three-loop





LO:1, NLO:1, NNLO:11, N^3LO:268

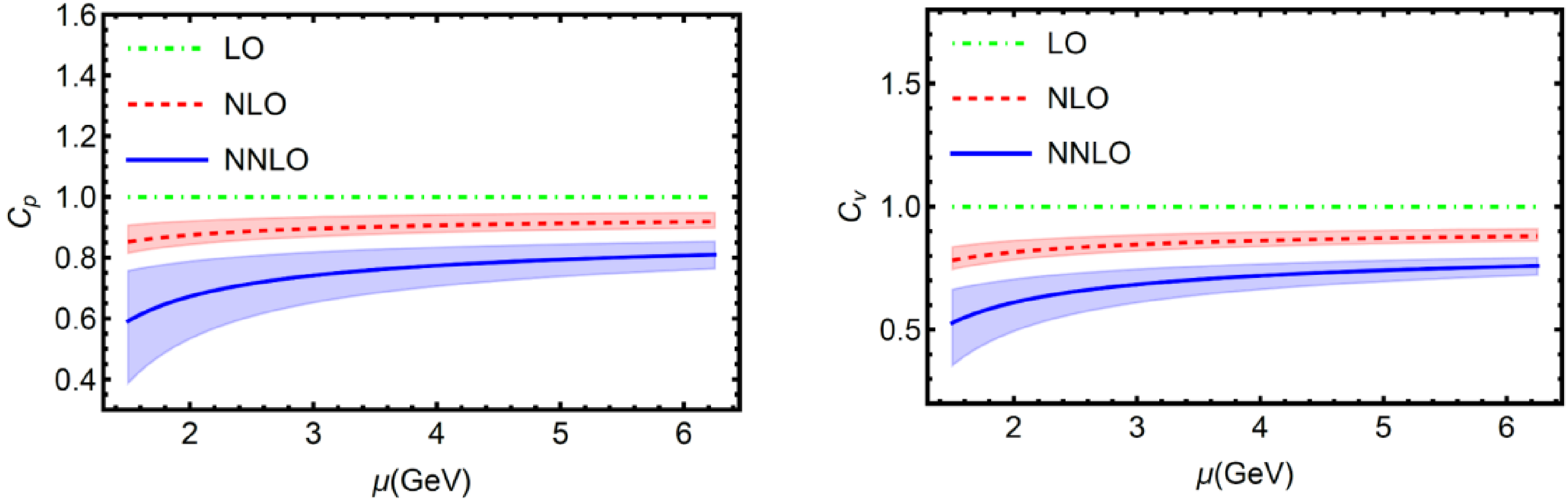
Two-loop results

➢ Matching coefficients at two loop

$\mu_f \in [1.5,1.2,1]$GeV, $\mu \in [6.25,4.75,3]$GeV, $m_b \in [5.25,4.75,4.25]$GeV, $m_c \in [2,1.5,1]$GeV

	LO	NLO	NNLO
C_p	1	$0.9117^{-0+0.0072+0.0061-0.0156}_{+0-0.0160-0.0064+0.0263}$	$0.7897^{-0.0310+0.0206+0.0119+0.0149}_{+0.0253-0.0482-0.0133-0.0141}$
C_v	1	$0.8697^{-0+0.0107+0.0061-0.0156}_{+0-0.0236-0.0064+0.0263}$	$0.7363^{-0.0234+0.0230+0.0106+0.0117}_{+0.0191-0.0526-0.0117-0.0121}$

μ dependence for matching coefficients



Three loop results for vector and pseudoscalar currents

➢ Matching coefficients for pseudoscalar current

$$\mathcal{C}(x_{\text{phys}}) = 1 - 1.62623\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right) - 6.51043\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^2 - 1520.59\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

Feng-Jia-Mo-Pan-Sang-Zhang, arXiv:2208.04302

➢ Matching coefficients for vector current

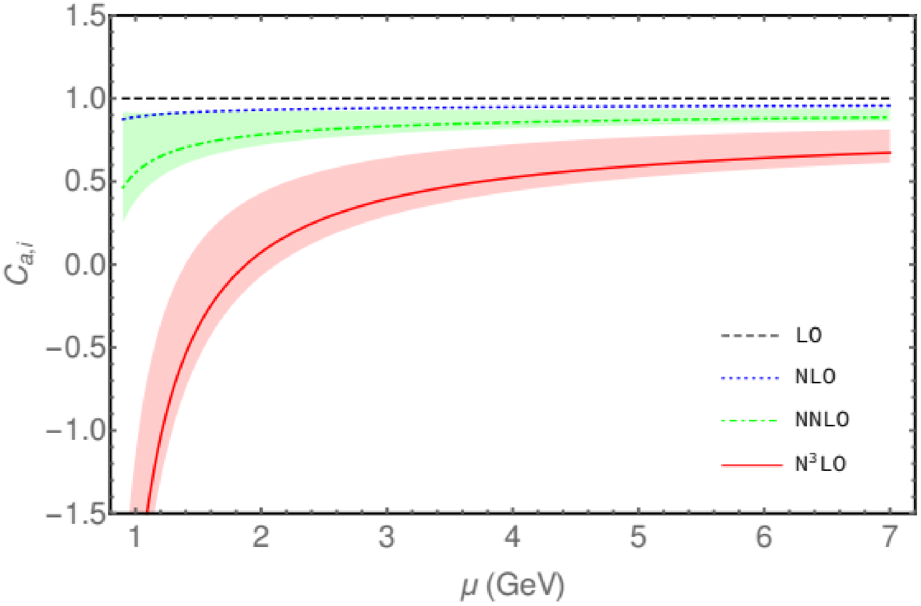
$$\mathcal{C} = 1 - 2.29\left(\frac{\alpha_s^{(n_l)}}{\pi}\right) - 35.44\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 - 1686.27\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4),$$

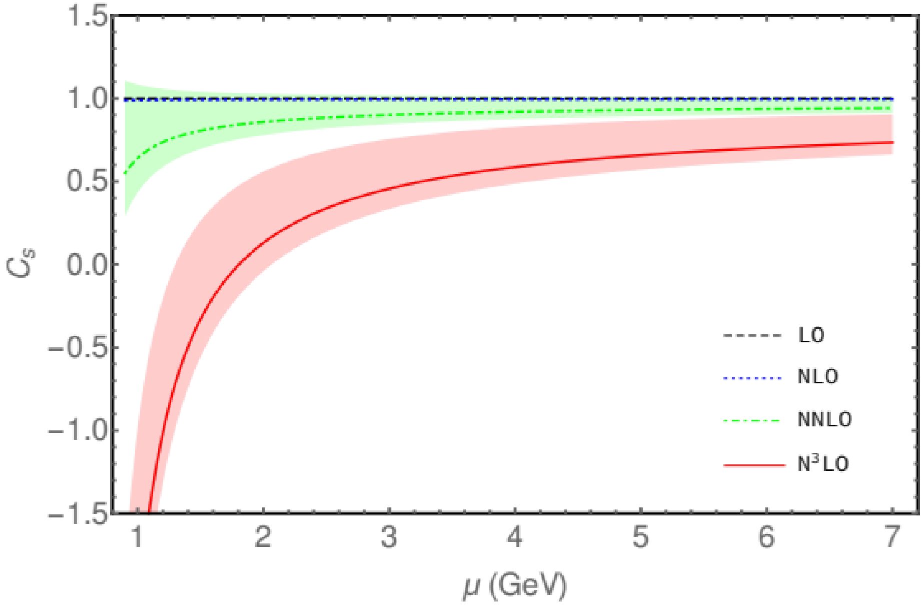
for $n_l = 3, n_c = 1, n_b = 0,$

Sang-Zhang-Zhou, arXiv:2210.02979

Results for axial-vector and scalar currents

- **Matching coefficients for axial-vector and scalar up to three loop**





Nonconvergence behaviors also in other two currents

Multi-loop integral calculated by AMFlow (Ma et al)

Sub-leading contribution

- **Relativistic corrections**

$$\langle 0|\overline{Q_1}\gamma^5 Q_2\,|Q_2\overline{Q_1}\rangle_{\text{QCD}}$$

$$= \sqrt{2M_H}\left[C_0^P\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\text{NRQCD}} + C_2^P\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\cdot\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\text{NRQCD}} + \cdots\right]$$

Employing EOM: $\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\,\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle = -2m_r E\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle.$

$$f_{B_c^*} = 2\sqrt{\frac{N_c}{m_{B_c^*}}}\left[\mathcal{C}_v + \frac{d_v E_{B_c^*}}{12}\left(\frac{8}{M} - \frac{3}{m_r}\right)\right]|\Psi_{B_c^*}(0)|,$$

$$f_{B_c} = 2\sqrt{\frac{N_c}{m_{B_c}}}\left[\mathcal{C}_p - \frac{d_p E_{B_c}}{4m_r}\right]|\Psi_{B_c}(0)|,$$

Wave function scale dependence

- Wave function at origin

For Power-law potential $$V(r) = Ar^a + C$$

Exact solution $$|\psi_\mu^n(0)|^2 = f(n,a)(A\mu)^{3/(2+a)}$$

Scale relation $$|\Psi_{B_c^*}(0)| = |\Psi_{J/\psi}(0)|^{1-y}\ |\Psi_\Upsilon(0)|^y,$$

$$y = y_c = \ln((1+m_c/m_b)/2)/\ln(m_c/m_b)$$

Collins-Imbo-King-Martell, PLB 393 (1997) 155–160

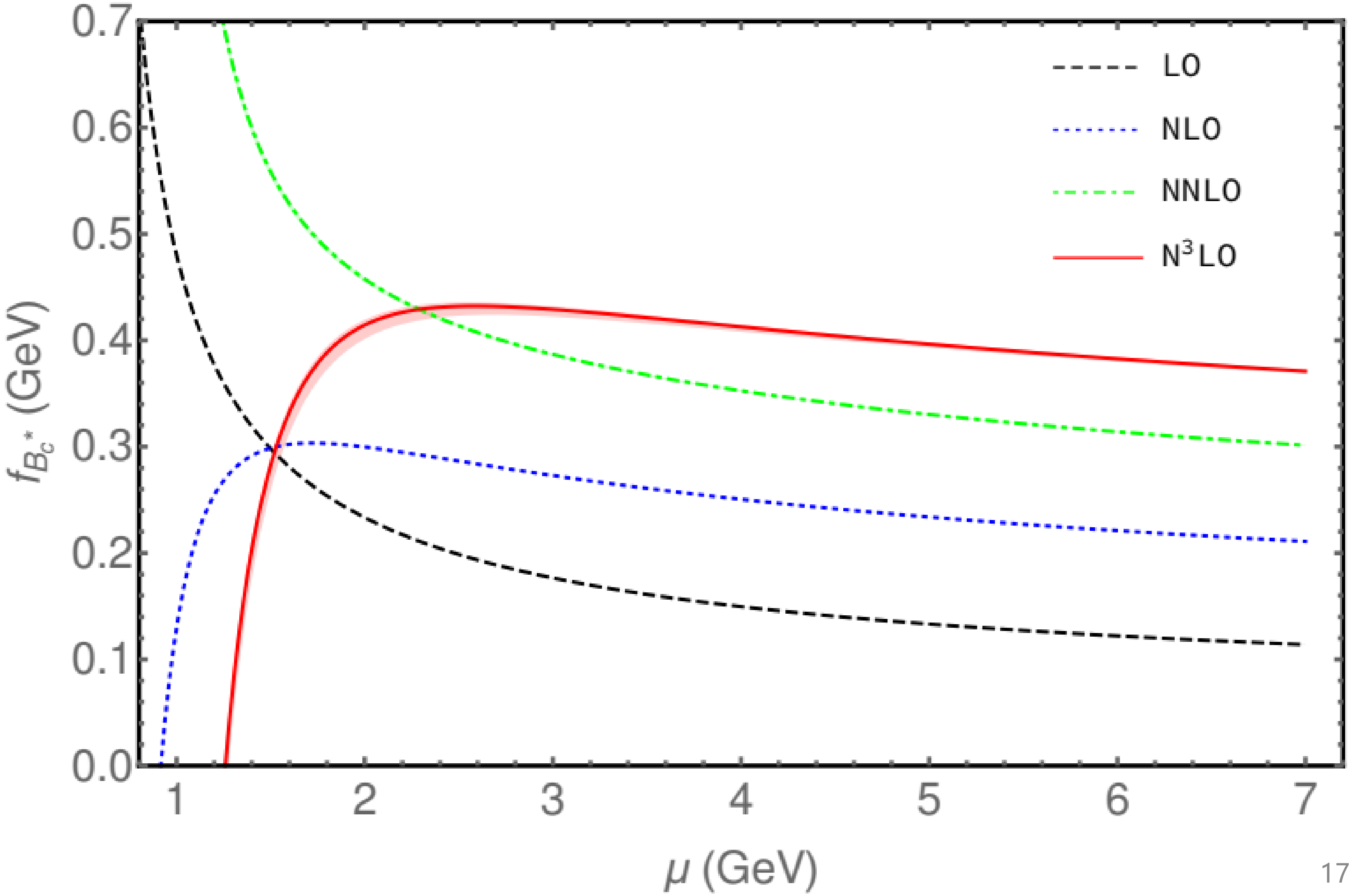
$$|\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2\left(1+\sum_{k=1}^{n} f_k a_s^k\right).$$

$$\left|\psi_1^{(0)}(0)\right|^2 = \frac{(m_b C_F\alpha_s)^3}{8\pi},$$

$$E_1^{(0)} = -\frac{1}{4}m_b(C_F\alpha_s)^2,$$

Beneke et al., PRL. 112, 151801 (2014)

Convergent vector Bc* decay constant



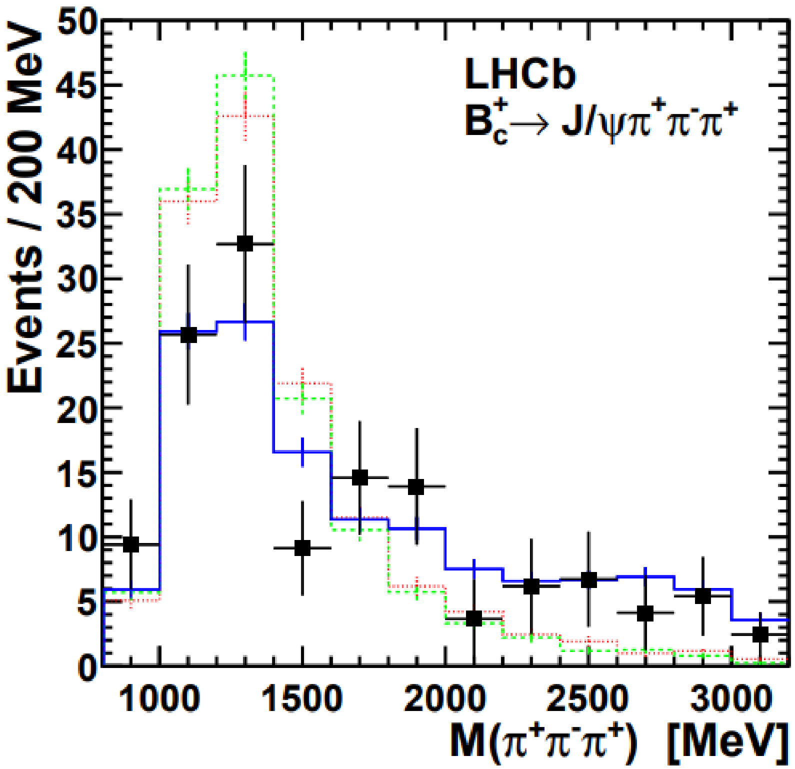
Leptonic decay branching ratios

Branching ratios	N^{3}LO
$\mathcal{B}(B_c^{*+} \to e^+\nu_e)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \mu^+\nu_\mu)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \tau^+\nu_\tau)$	$(3.40^{+0.25-0.06-1.19}_{-0.41+0.03+0.33}) \times 10^{-6}$
$\mathcal{B}(B_c^{+} \to e^+\nu_e)$	$(1.91^{+0.15-0.19-0.70}_{-0.23+0.12+0.22}) \times 10^{-9}$
$\mathcal{B}(B_c^{+} \to \mu^+\nu_\mu)$	$(8.18^{+0.63-0.83-2.99}_{-1.00+0.52+0.94}) \times 10^{-5}$
$\mathcal{B}(B_c^{+} \to \tau^+\nu_\tau)$	$(1.96^{+0.15-0.20-0.72}_{-0.24+0.12+0.23}) \times 10^{-2}$

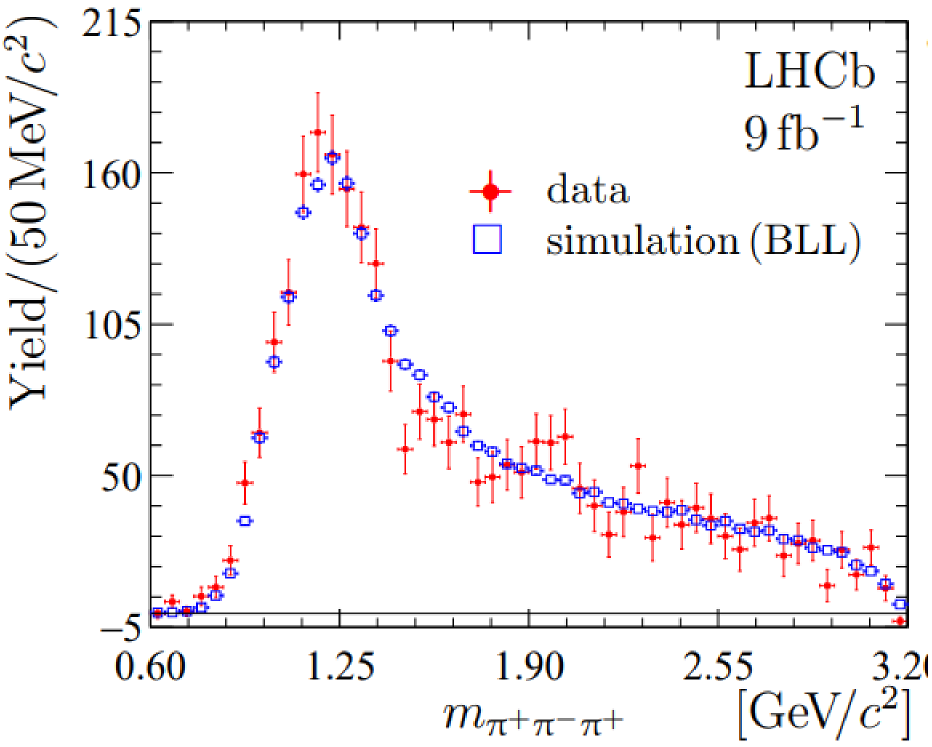
$$\Gamma(B_c^*(\lambda=\pm1) \to \ell\nu_\ell) = \frac{|V_{cb}|^2}{12\pi} G_F^2 f_{B_c^*}^2 \left(1 - \frac{m_\ell^2}{m_{B_c^*}^2}\right)^2 \times m_{B_c^*}^3,$$

$$\Gamma(B_c^*(\lambda=0) \to \ell\nu_\ell) = \frac{m_\ell^2 \Gamma(B_c^{*+}(\lambda=\pm1) \to \ell\nu_\ell)}{2m_{B_c^*}^2},$$

3、Polarization analysis of other Bc* decays



LHCb, arXiv:1204.0079



LHCb, arXiv:2111.03001

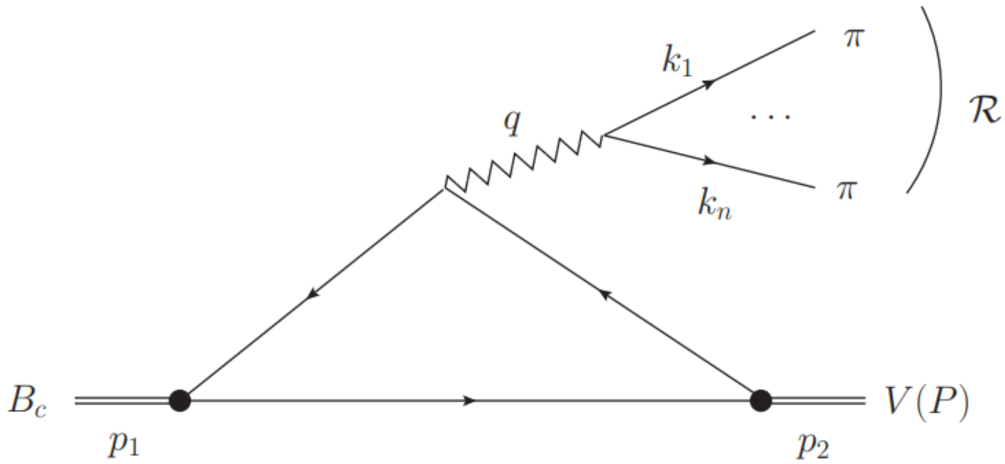
Around 10^5 Bc to Jpsi+X events

Invariant mass distribution in Bc* decays to J/psi +n h

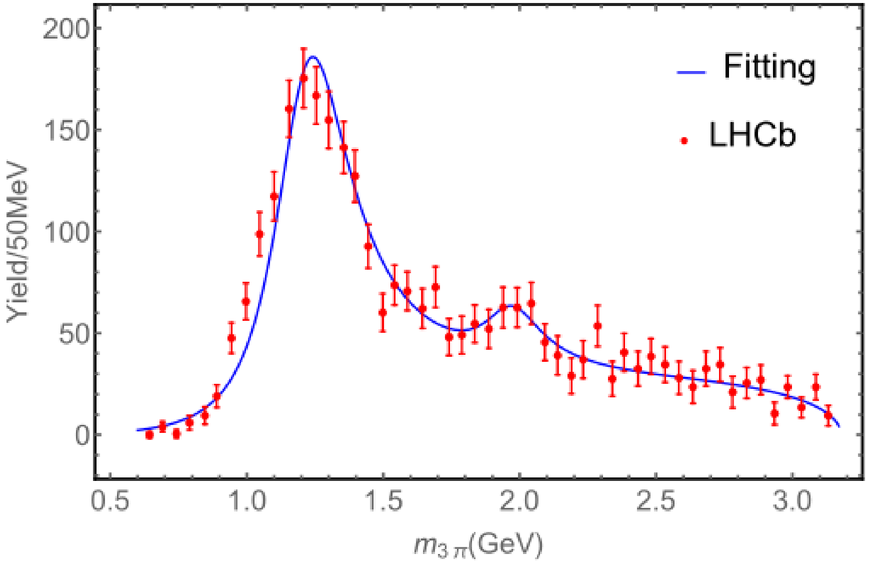
- **Helicity decomposition of weak decay width**

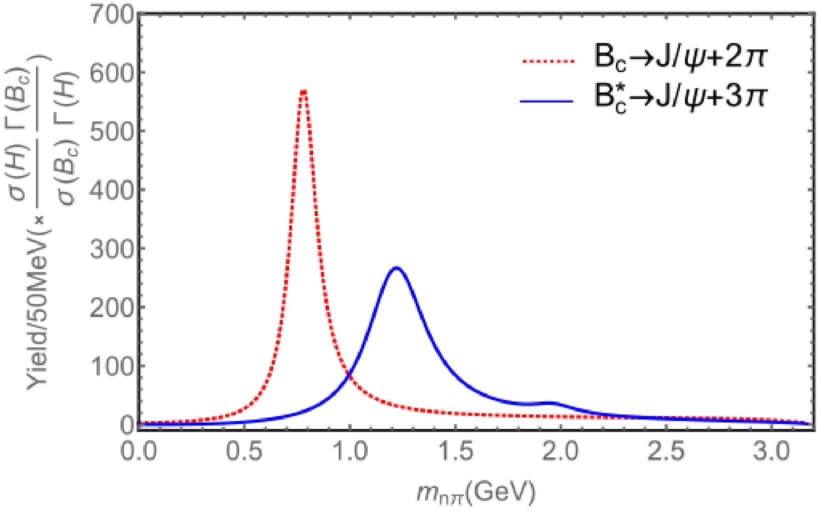
$$\frac{d\Gamma(B_c^{(*)} \to J/\psi + nh)}{dq^2} = \sum_{\lambda_i} \frac{|V_{cb}|^2 G_F^2 a_1^2 |\mathbf{p}'|}{32\pi M^2} \Gamma_{J_1\lambda_1 J_2 \lambda_2 \lambda_{nh}},$$

$$\Gamma_{11110} = 2\left[V_1^2\left((M-M')^2-q^2\right)\left((M'+M)^2-q^2\right) + \left(A_1\left(M^2-M'^2\right)+A_2 q^2\right)^2\right]\rho_T^{nh}(q^2),$$



Results of invariant mass distribution

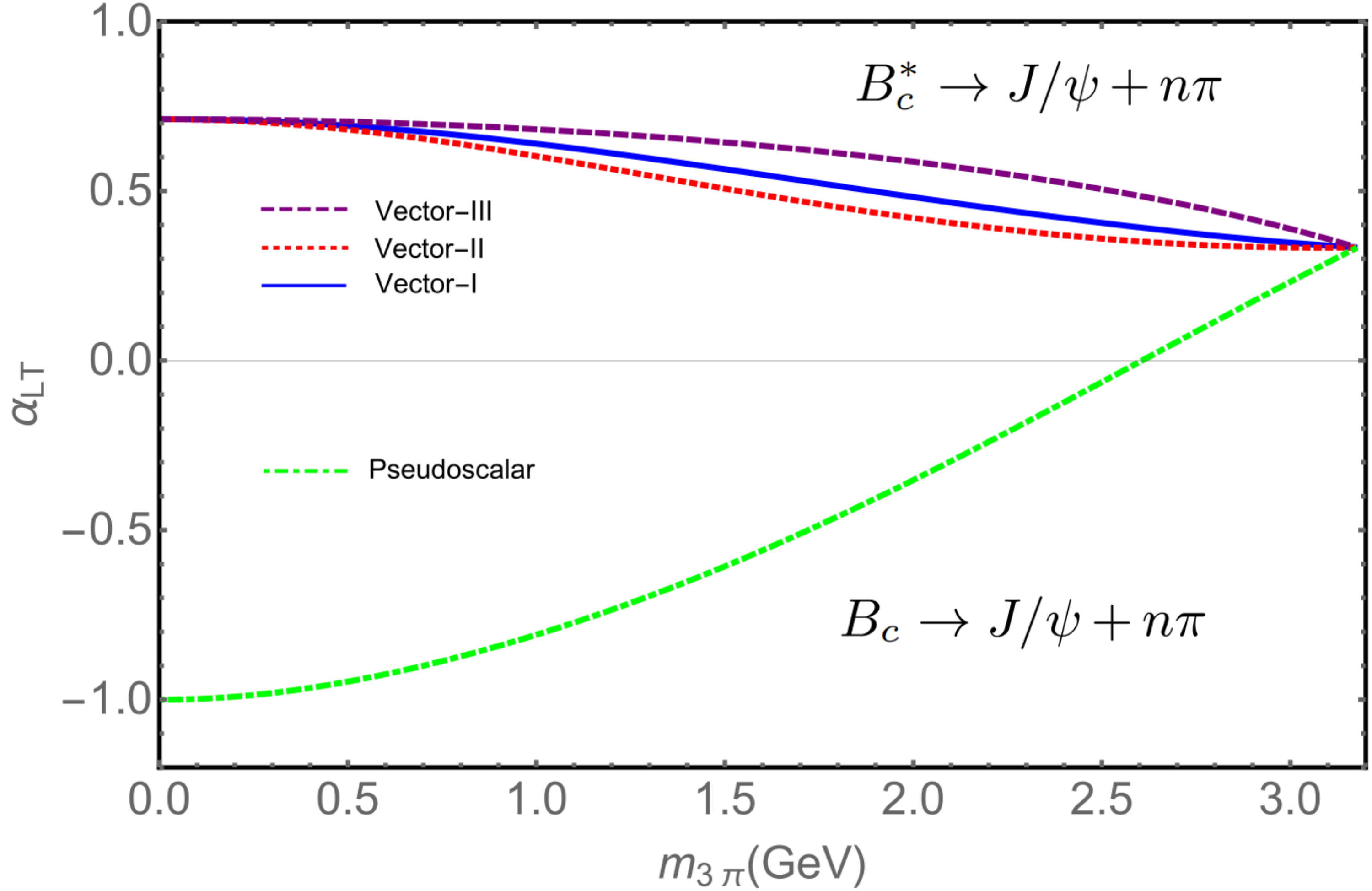




LHCb, arXiv:2111.03001

Polarization Asymmetry(A general law in V(P) to V transitions)

$$\alpha_{LT} = \sum_{\lambda_1,\lambda_{nh}} \frac{\Gamma_{J_1\lambda_1 11\lambda_{nh}} - \Gamma_{J_1\lambda_1 10\lambda_{nh}}}{\Gamma_{J_1\lambda_1 11\lambda_{nh}} + \Gamma_{J_1\lambda_1 10\lambda_{nh}}},$$



Summary and Outlook

Summary

- ✓ Bc* decay width is studied in QCD effective theory
- ✓ Convergent Bc* decay constant up to three-loop accuracy is obtained
- ✓ Distinguishing vector Bc* meson at LHC is possible by helicity decomposition

Outlook

- ➢ Nontrivial high-order calculation of wave function in un-equal mass cases
- ➢ Experimental analysis of Bc family at LHC/CEPC/Super-Z/EicC

Thank you a lot!

Hyperfine splitting

➢ Hyperfine splitting relation

$$(\Delta M)_{i\bar{j}} = 32\pi\alpha_s(2\mu_{i\bar{j}})|\Psi_{i\bar{j}}(0)|^2/9m_i m_j\ ,$$

$$\Delta M_{c\bar{b}} = \alpha_s(2m_r)x^{1-2q}\left(\frac{\Delta M_{c\bar{c}}}{\alpha_s(m_c)}\right)^{1-q}\left(\frac{\Delta M_{b\bar{b}}}{\alpha_s(m_b)}\right)^{q}.$$

$$\Delta M_{c\bar{b}(1S)} = 63.8^{+5.5}_{-8.4}(q)^{+1.2}_{-1.2}(exp)\ \mathrm{MeV},$$
$$\Delta M_{c\bar{b}(2S)} = 26.4^{+2.1}_{-3.3}(q)^{+1.5}_{-1.7}(exp)\ \mathrm{MeV},$$

$$\Delta M_{b\bar{b}(1S)} = 62.3 \pm 3.2\ \mathrm{MeV}$$
$$\Delta M_{b\bar{b}(2S)} = 24 \pm 4\ \mathrm{MeV}$$

CMS 2019

$$\Delta M_{c\bar{b}(2S)} = 29.1 \pm 1.5(stat) \pm 0.7(syst)\ \mathrm{MeV}$$

LHCb 2019

$$\Delta M_{c\bar{b}(2S)} = 31.0 \pm 1.4(stat) \pm 0.0(syst)\ \mathrm{MeV}$$

HPQCD lattice results

$$\Delta M_{c\bar{b}(1S)} = 54 \pm 4\ \mathrm{MeV}$$

Calculation procedure

- Feynman Diagrams & Amplitudes
  (Packages: FeynRules/FeynArts / QGraf)

- Feynman Amplitudes Simplification： Trace & Contraction
  (Packages: FeynCalc / FormCalc / FormLink)

- Feynman Integrals Reduction
  (Packages: Apart(Feng) / FIRE/Kira /…)

- Feynman Master Integrals Calculation:
  (Packages: AMFlow(Ma et al) / FIESTA /…)