QED corrections in leptonic B meson decays

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Based on:

Complete analysis on QED corrections to $B \to \tau^+ \tau^-$ *JHEP* 10 (2023) 073

in collaboration with Y.K. Huang, Y.L. Shen, X.C.Zhao

QED corrections to $B \to \tau \nu$ at Subleading power *in progress*

Outline

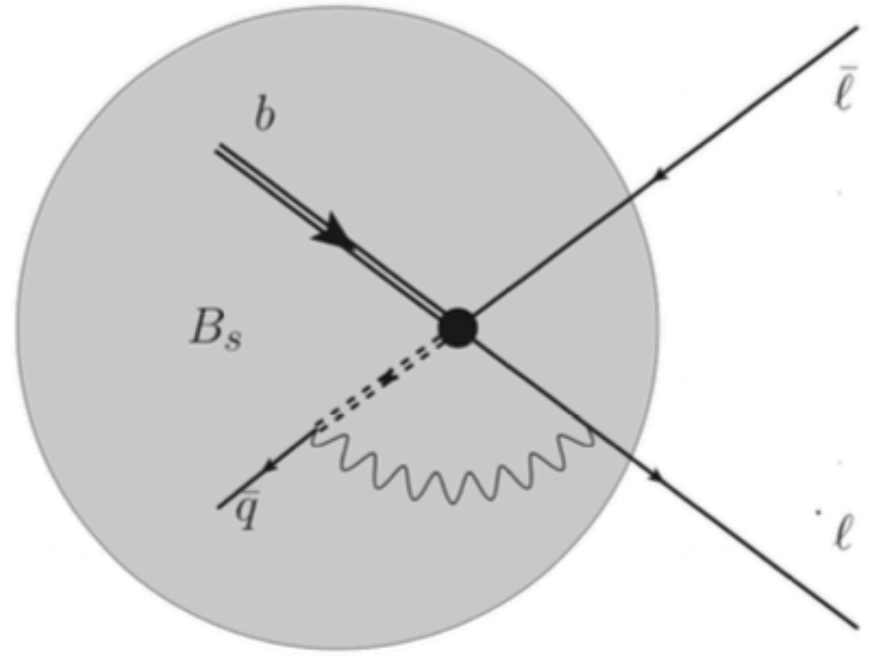
- Motivation for precision flavor physics
- QED corrections in QCD bound-states
- QED corrections to $B_q \to \tau^+ \tau^-, B_q \to \tau \nu_\tau$ in SCET

  ($B_q \to \tau^+ \tau^-$ at Leading Power and $B_q \to \tau \nu_\tau$ at NLP)
- Summary

Why do we need to know the QED corrections in flavor physics?

- Large logarithmic $\ln(m_b^2/m_\ell^2)$ enhancements can mimic lepton-flavor universality violation
- Expected precision of measurements may require the inclusion of QED corrections or at least a proof that no effects above 1% exist.

with QED



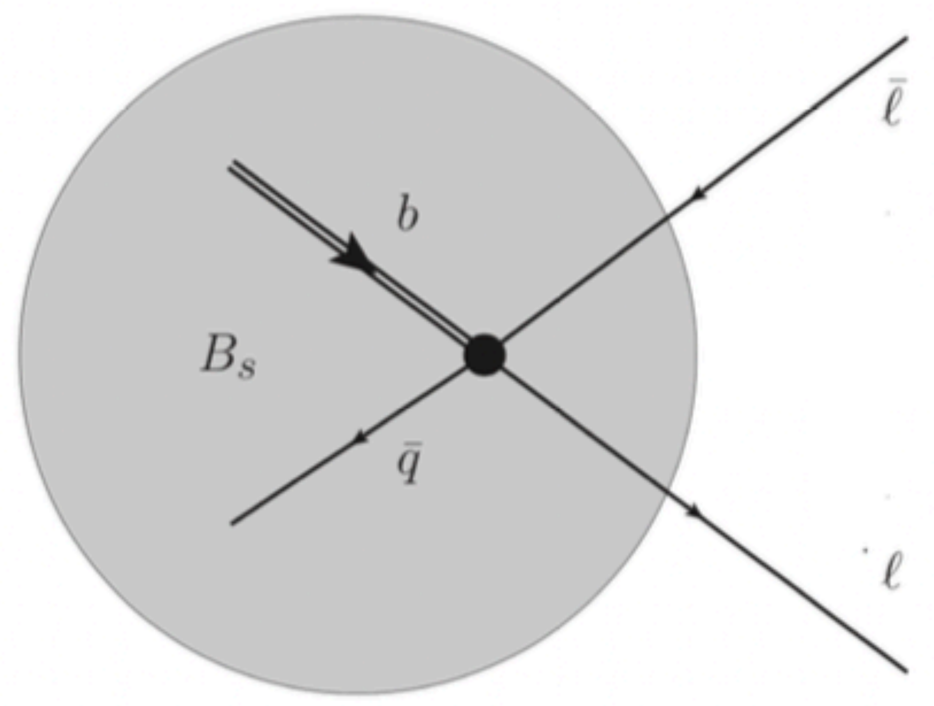
e.g. power-enhanced effects from QED correction in $B_q \to \mu^+\mu^-$

[M.Beneke et ac., Phys.Rev.Lett.120(2018)1]

a dynamical enhancement by a power of $m_b/\Lambda_{\rm QCD}$ and by large logarithms $\ln m_b\Lambda_{\rm QCD}/m_\mu^2 \to 1\,\%$ of $\mathrm{Br}(B_q \to \mu^+\mu^-)$ $>$ previous estimates of NLO QED effects $\alpha_{\rm em}/\pi \sim 0.3\,\%$

QED corrections in QCD bound-states

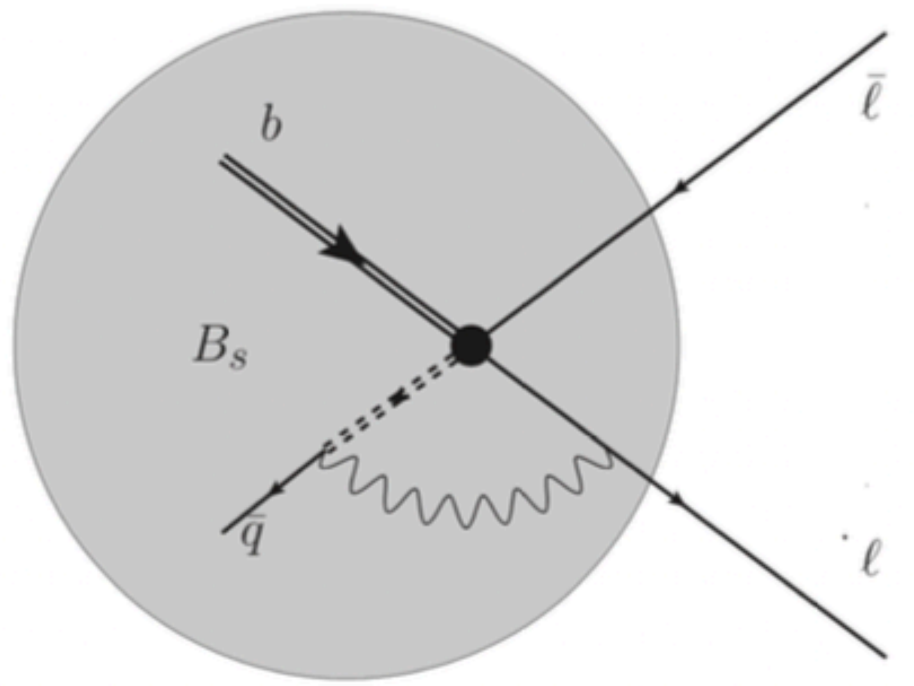
without QED



QCD contained in the meson decay constant for purely leptonic final state, in the absence of QED

$$\left\langle 0 \left| \bar{q}(0)\, \gamma^\mu\, \gamma_5\, b(0) \right| \bar{B}_q(p) \right\rangle = i f_{B_q}\, p^\mu$$

with QED



- virtual photons can resolve the structure of B meson
- virtual photons can couple to initial and final states

Non-local time ordered products have to be evaluated when QED effects are included

$$\left\langle 0 \left| \int d^4x\, e^{iqx}\, T\left\{ j_{\mathrm{QED}}(x), \mathscr{L}_{\Delta B=1}(0) \right\} \right| \bar{B}_q \right\rangle$$

This can be done for QED bound-states but QCD is non-perturbative at low scales.

Scales in the problem

$B_q \to \ell^+\ell^-$ **is a muti-scale problem , we need the EFTs**

- Hard scale m_b
- Hard-collinear scale $\sqrt{m_b \Lambda_{\text{QCD}}}$
- Soft scale Λ_{QCD}
- Collinear scale $m_\mu \sim \Lambda_{\text{QCD}}$ for $\ell = \mu$;

  Hard-collinear scale $m_\tau \sim \sqrt{m_b \Lambda_{\text{QCD}}}$ for $\ell = \tau$;

Weak EFT

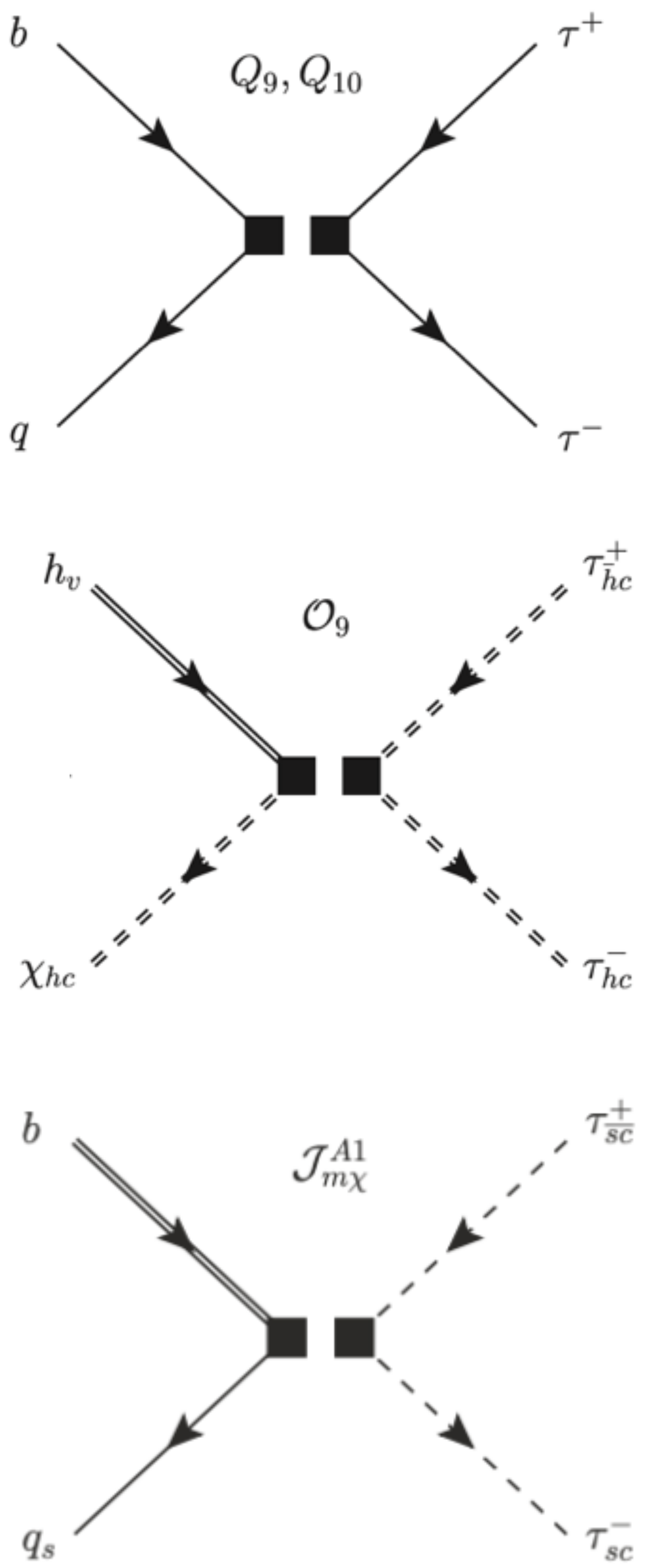
↓

$\text{HQET} \otimes \text{SCET}_\text{I}$

↓

$\text{HQET} \otimes \text{bHLET}$

heavy leptonic filed become to a soft-collinear (sc) field in boost HLET after integrating m_τ

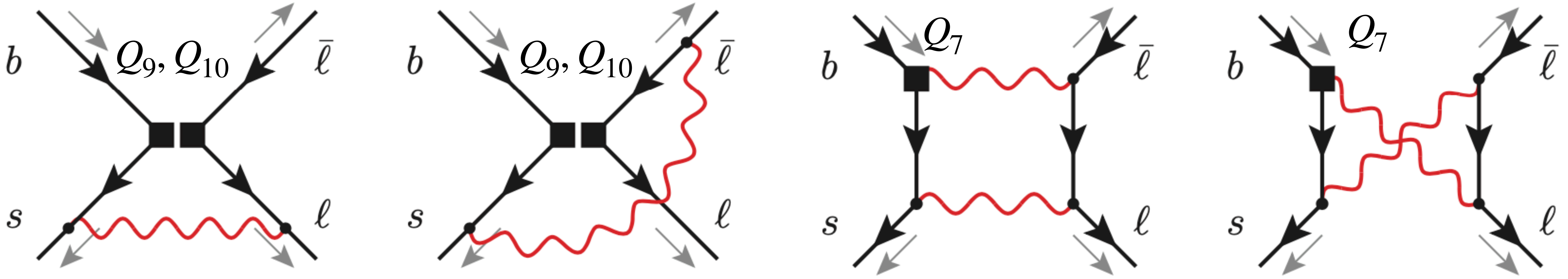


We focus on $B_q \to \tau^+\tau^-$

The extension form $B_q \to \mu^+\mu^-$ **to** $B_q \to \tau^+\tau^-$ **is non-trival**

Diagrams in the Weak EFT

Power-enhanced (Non-local SCET operator)



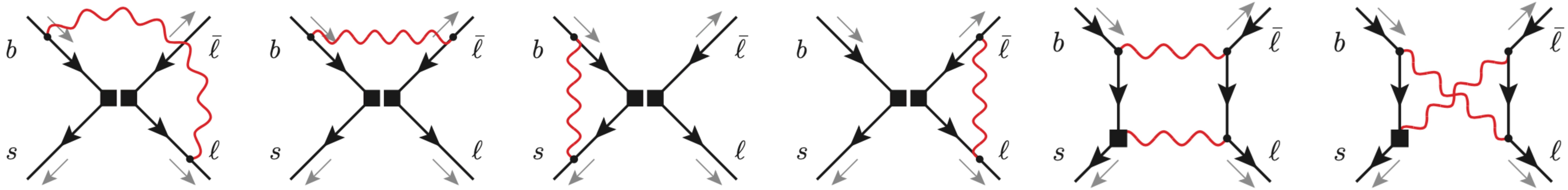
Weak EFT operators

$$Q_9 = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{q}\gamma^\mu P_L b\right)\sum_\ell \bar{\ell}\gamma_\mu \ell$$

$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{q}\gamma^\mu P_L b\right)\sum_\ell \bar{\ell}\gamma_\mu\gamma_5 \ell$$

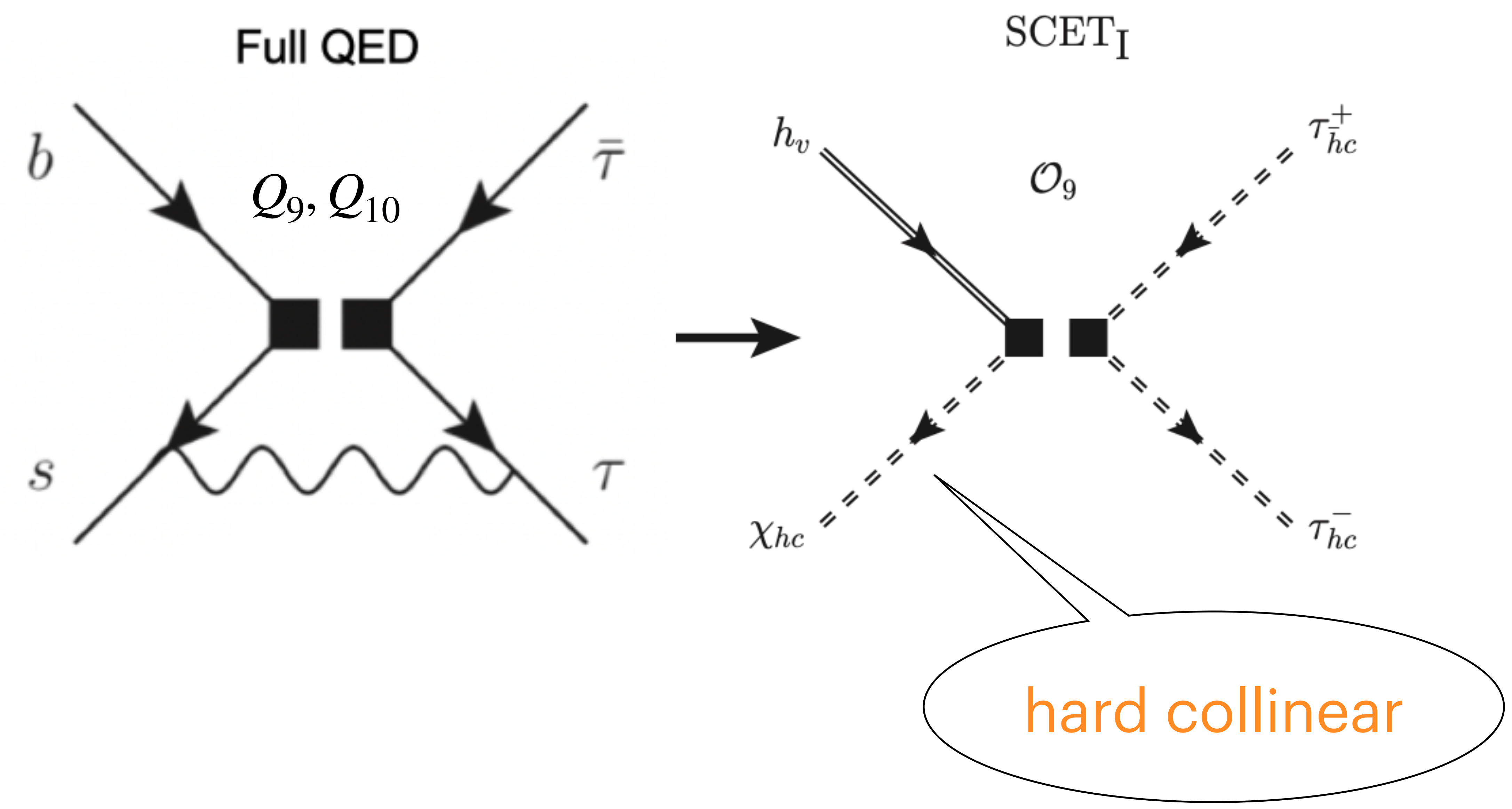
$$Q_7 = \frac{e}{(4\pi)^2}\bar{m}_b\left[\bar{q}\sigma^{\mu\nu}P_R b\right]F_{\mu\nu}$$

Not power-enhanced (local SCET operator)



Hard functions

We only consider Power-enhanced contribution



Non-local operator

$$\widetilde{O}_9(s,t) = g^{\perp}_{\mu\nu}\left[\bar{\chi}_C\left(sn_+\right)\gamma^{\mu}_{\perp}P_L h_v(0)\right]\left[\bar{\ell}_C\left(tn_+\right)\gamma^{\nu}_{\perp}\ell_{\bar{C}}(0)\right]$$

The Fourier-transformed SCET operators

$$O_9(u) = n_+p_C\int\frac{dr}{2\pi}e^{-iu(n_+p_C)\,r}\,\widetilde{O}_9(0,r)\,,\qquad u \equiv \frac{n_+p_\ell}{n_+p_C}$$

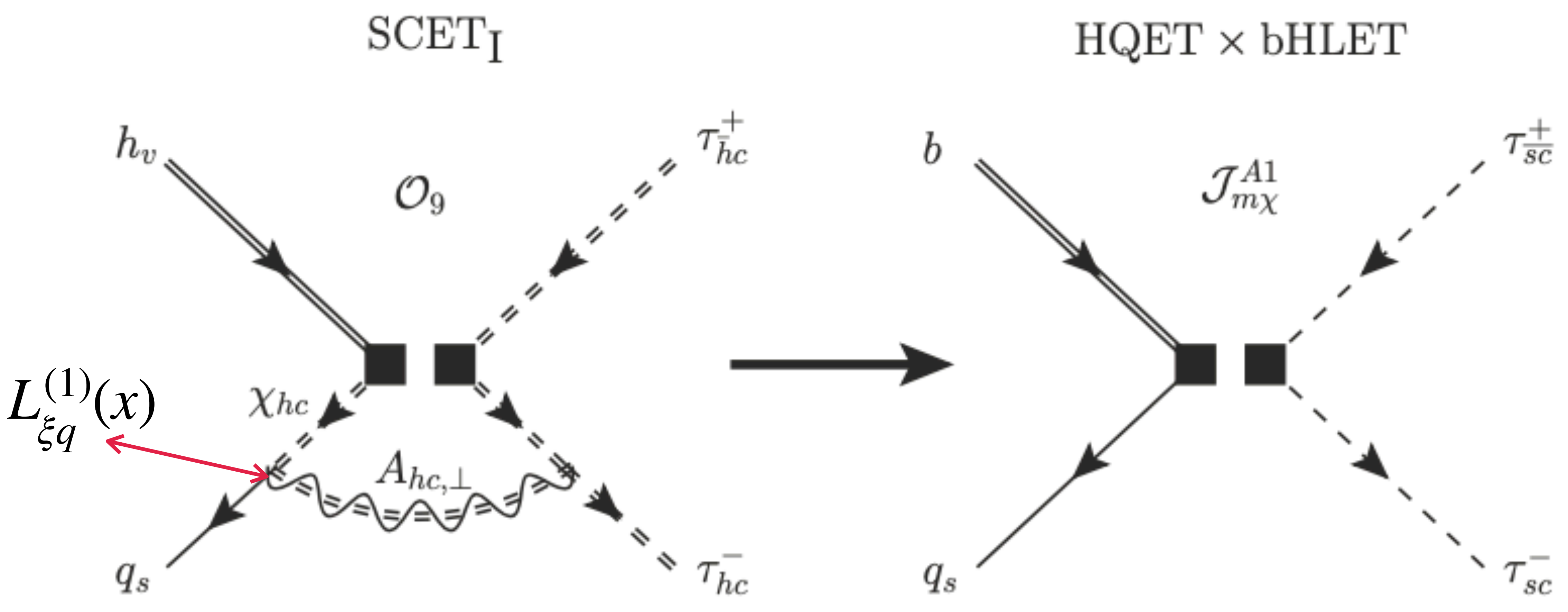
$$\sum_k C_k\left(\mu_b\right)\,Q_k = \int_0^1 du\,H_9\left(u,\mu_b\right)\,O_9(u)$$

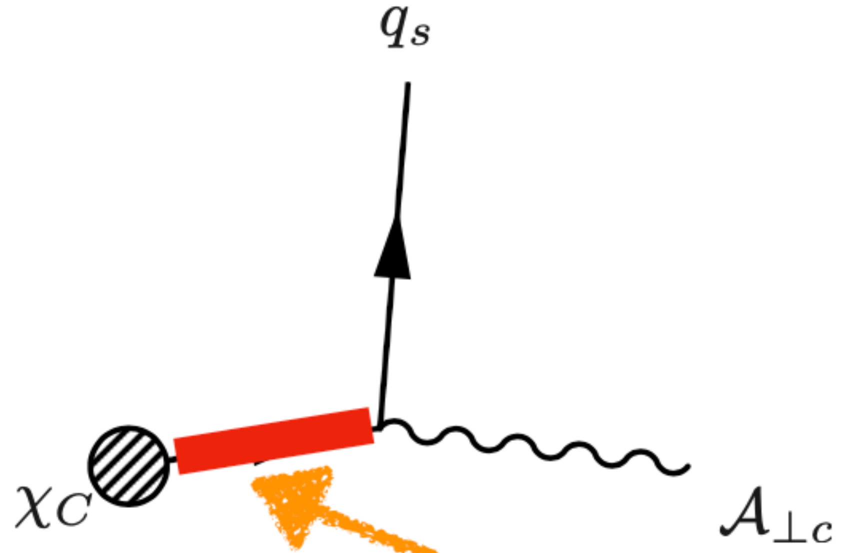
$$H_9\left(u,\mu_b\right) = \mathcal{N}\left[C^{(0)}_{9\,\mathrm{eff}}\left(u,\mu_b\right) + C^{(0)}_{10}\left(u,\mu_b\right) - \frac{2Q_\ell}{u}C^{\mathrm{eff}}_7\left(u,\mu_b\right)\right] + \mathcal{O}(\alpha_{\mathrm{em}})$$

No endpoint divergence

Hard-collinear functions

Hard-collinear quark becomes soft field





$$\bar{u}_s(\ell_q)\, \not\epsilon_\perp(k) \frac{\not n_+}{2} \frac{1}{n_- p_q}$$

power enhanced factor $\dfrac{1}{\Lambda_{\rm QCD}}$

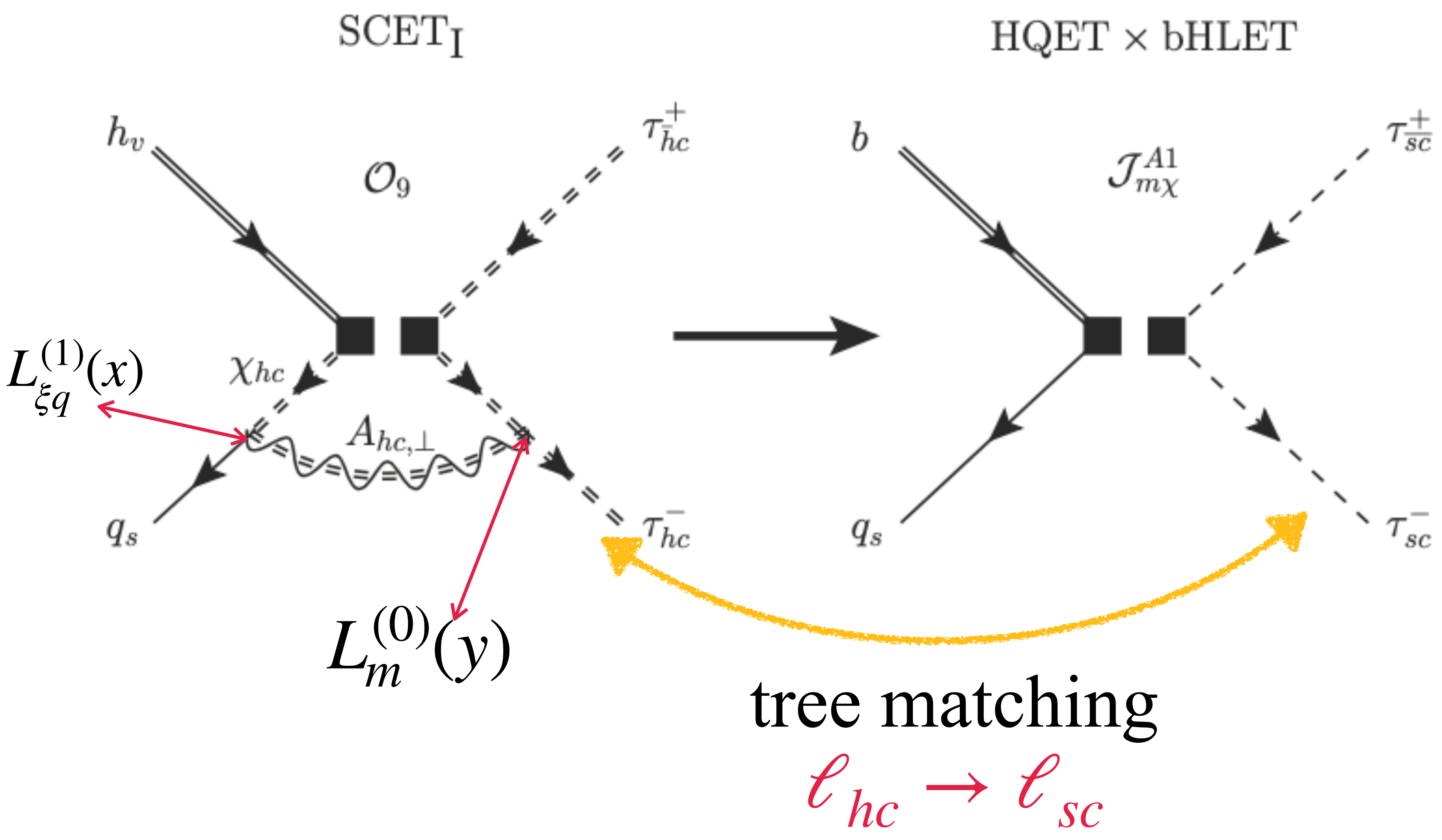
(1) To convert hard-collinear quark into a soft quark to get a non-vanishing overlap the B-meson state, we need power suppressed interaction

$$L^{(1)}_{\xi q}(x) = \bar{q}_s(x_-)[W_{\xi C} W_C]^\dagger(x)\, i\, D_{C\perp}\, \xi_C(x) + \mathrm{h.c.}$$

Small component ($n_- p_q$) of hard-collinear the same as soft momentum → Soft fields become delocalized along the light-cone

Hard-collinear functions

Hard-collinear quark becomes soft field



(2) The hard-collinear photon, $A_{C\perp}$ from $D_{C\perp}$, would be followed by the fusion

$$\bar{\ell}_C + A_{\perp C} \to m_\tau \bar{\ell}_C$$

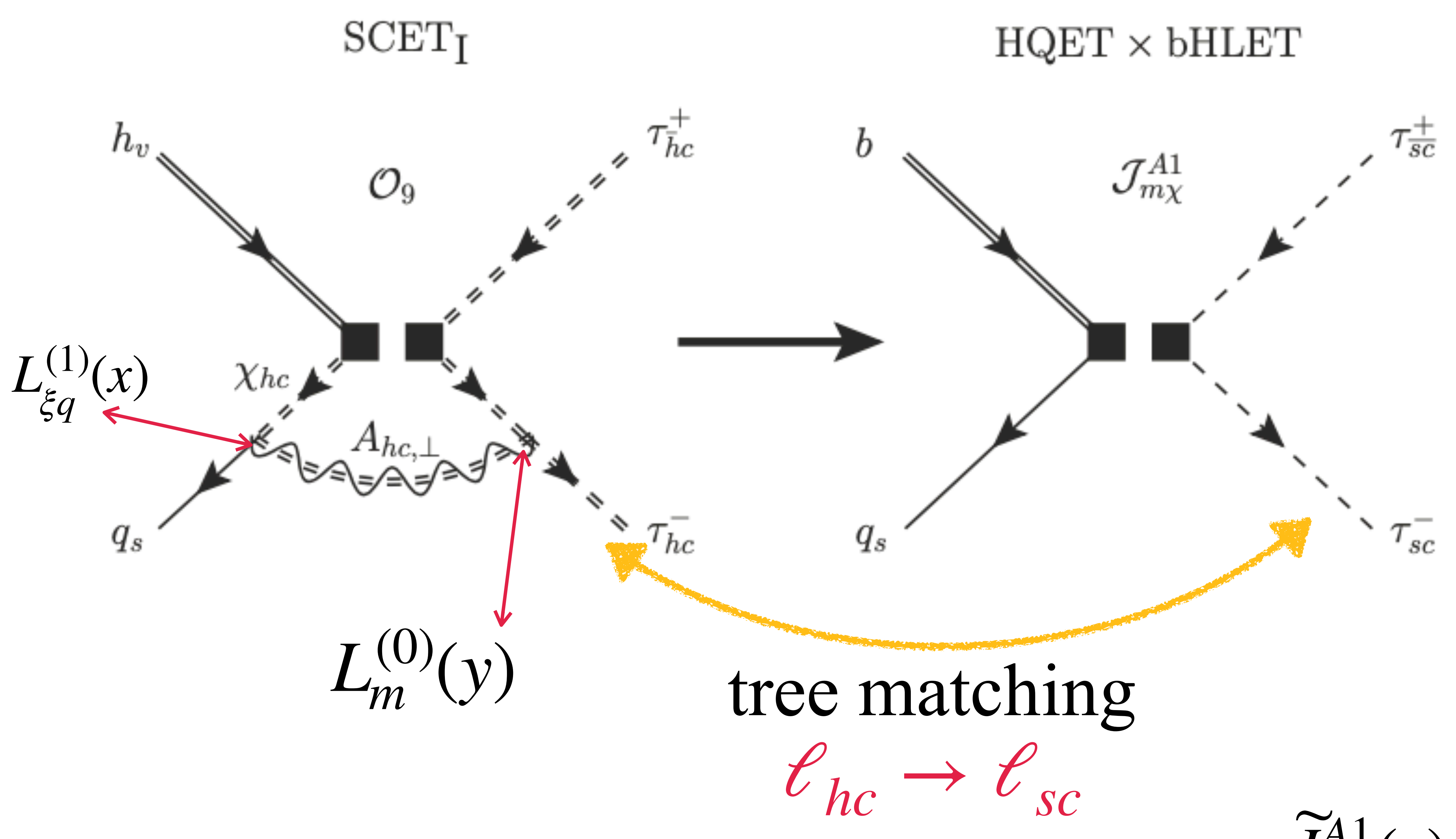
through the leading power Lagrangian

$$L^{(0)}_m(y) = m_\tau \bar{\ell}_C \left[i\, D_{C\perp}, \frac{1}{in_+ D_C} \right] \frac{\not{n}_+}{2} \ell_C$$

(3) heavy tau filed become to a soft-collinear (sc) field in boost HLET after integrating m_τ

Hard-collinear functions

Match $\mathrm{SCET_I}$ onto $\mathrm{HQET}\times\mathrm{bHLET}$



Therefore, we will match the time-ordered product of the $\mathrm{SECT_I}$ operators $O_9(u)$ with $L^{(1)}_{\xi q}(x)$ and $L^{(0)}_m(y)$,

$$\left\langle \ell(p_\ell)\,\bar{\ell}\,(p_{\bar{\ell}}) \,\middle|\, \int d^4x \int d^4y\, T\left\{O_9(u), L^{(1)}_{\xi q}(x), L^{(0)}_m(y)\right\} \middle|\, b\,(p_b)\,q\left(\ell_q\right)\right\rangle$$

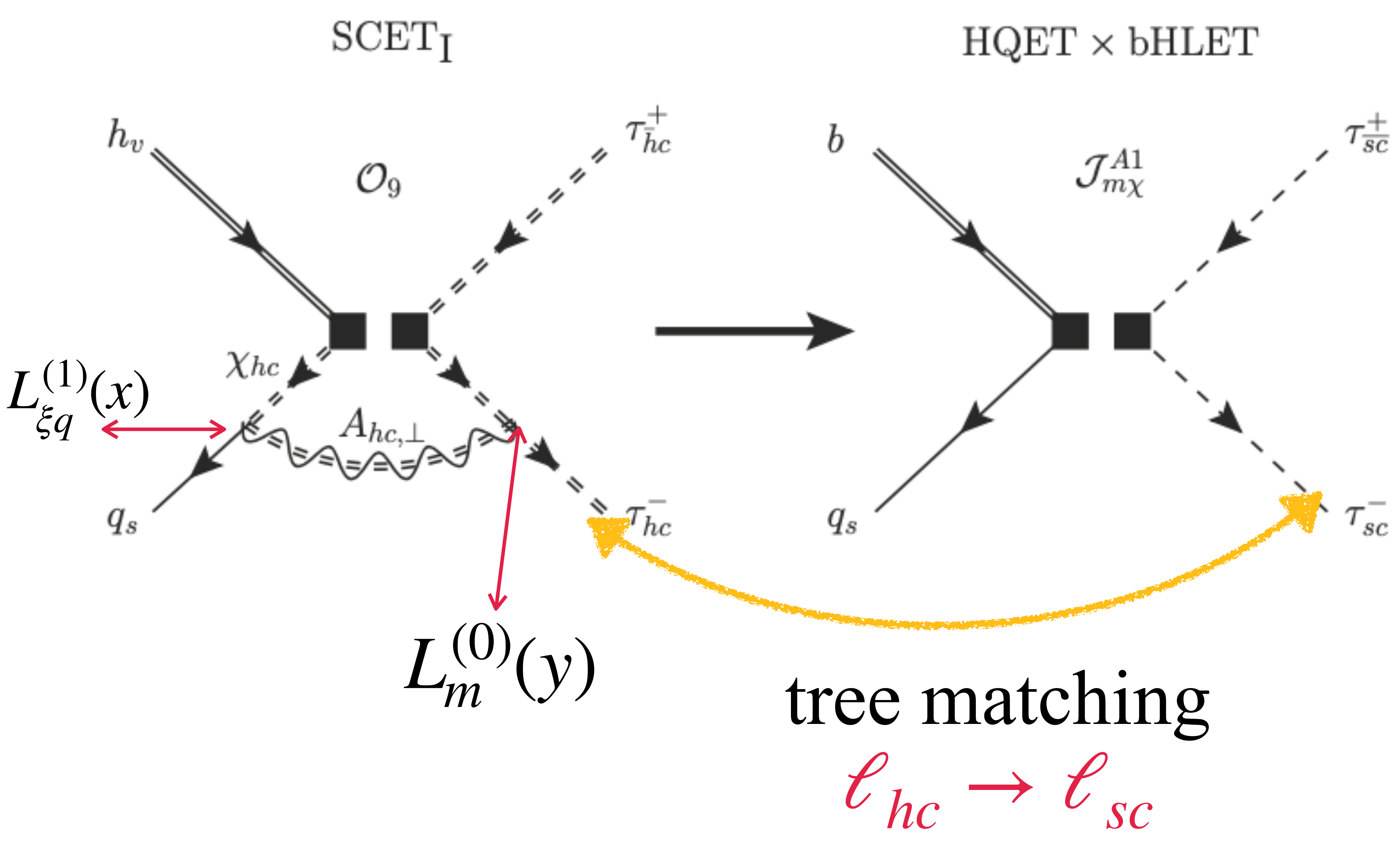
to matrix element of operator in $\mathrm{HQET}\times\mathrm{bHLET}$,

$$\widetilde{J}^{A1}_{m\chi}(v) = \left[\bar{q}_s(vn_-)\,Y(vn_-,0)\,\frac{\not{n}_-}{2}\,P_L\,h_v(0)\right]\left[Y_+^\dagger Y_-\right](0)\,\left[\bar{\ell}_{sc}(0)\,(4P_R)\,\ell_{\overline{sc}}(0)\right]$$

Additional QED soft Wilson lines

Hard-collinear functions

Integrate out intermediate hard-coll. scale



At tree level, the hard collinear function $J_m(u, \omega)$

$$J_m^{(0)}(u, \omega; \mu = \mu_{hc}) = \frac{\alpha}{4\pi} Q_\ell Q_s m_\ell \frac{\bar{u}}{\omega} \ln\left(1 + \frac{u}{\bar{u}} \frac{\omega m_b}{m_\ell^2}\right) \theta(u)\, \theta(\bar{u}) \,,$$

where $\omega = n\_p_q \sim \Lambda_{QCD}$

power enhancement factor $1/\lambda^2$

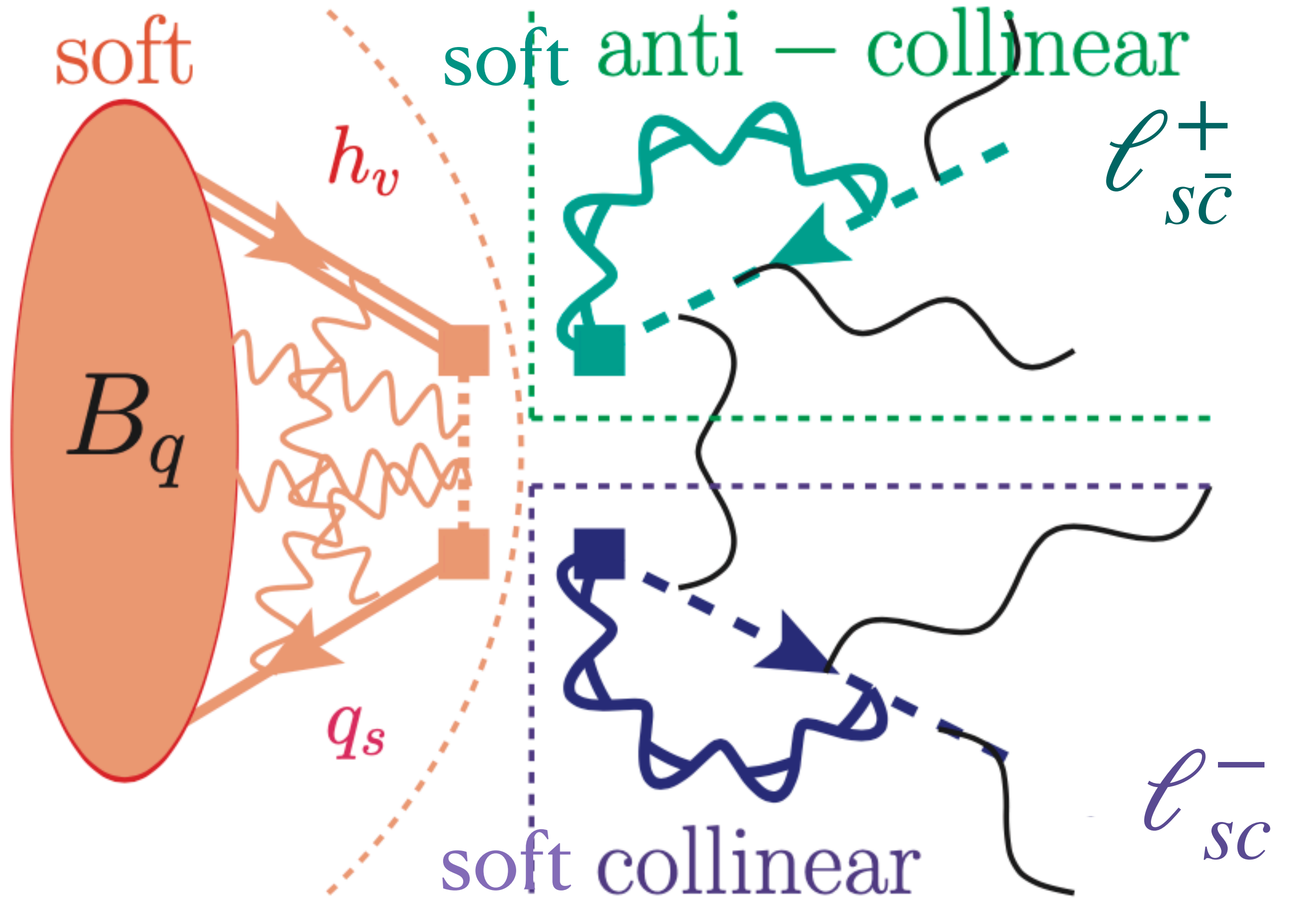
logarithms

Factorization

After two-step matching starting from QED onto $\mathrm{SCET_I}$, and successively onto $\mathrm{HQET} \times \mathrm{bHLET}$,

the amplitude factorized as

$$A_9 \sim \int_0^1 du\, 2\, H_9(u) \int_0^\infty d\omega\, J_m(u;\omega)\, \langle \ell^+ \ell^- | \widetilde{J}^{A1}_{m\chi} | \bar{B}_q \rangle$$



Renormalized (anti-) soft-coll. on-shell matrix elements

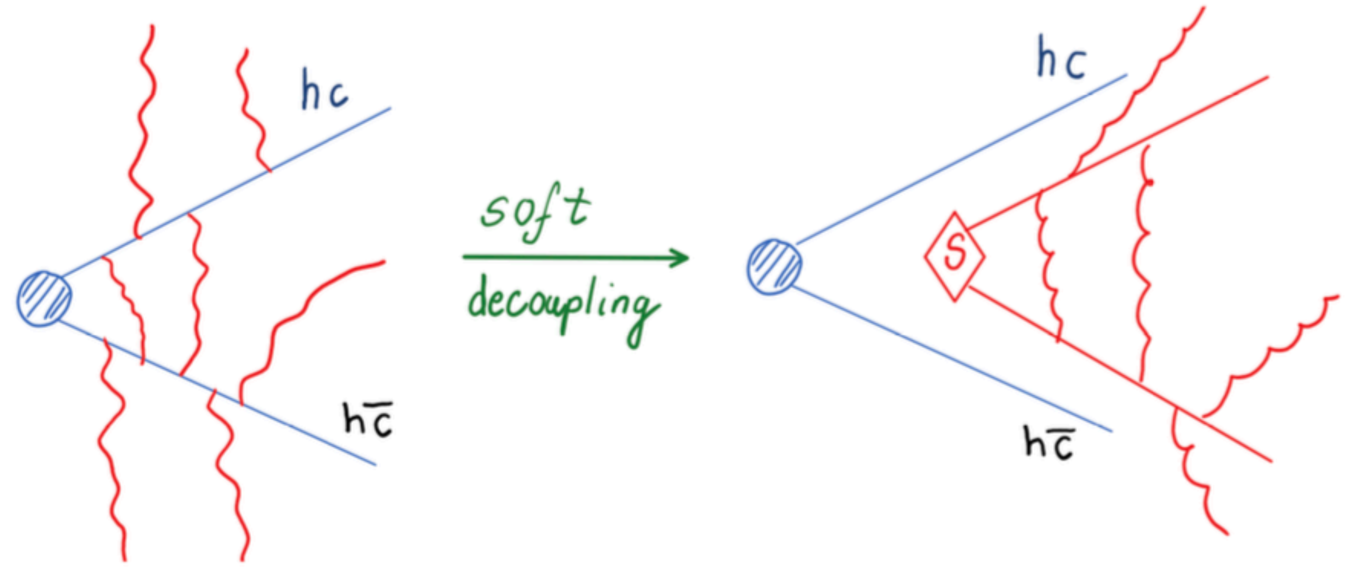
$$\left\langle \ell^-(p_\ell) \middle| [\text{ soft coll. }] \middle| 0 \right\rangle = Z_\ell \bar{u}_{sc}(p_\ell),$$

$$\left\langle \ell^+(p_{\bar{\ell}}) \middle| [\text{soft anticoll. }] \middle| 0 \right\rangle = Z_{\bar{\ell}} v_{s\bar{c}}(p_{\bar{\ell}})$$

$$\widetilde{J}^{A1}_{m\chi}(v) = \left[\bar{q}_s(vn_-)\, Y(vn_-,0)\, \frac{\not{n}_-}{2} P_L h_v(0)\right] \left[Y_+^\dagger Y_-\right](0) \left[\bar{\ell}_{sc}(0)(4P_R)\,\ell_{\overline{sc}}(0)\right]$$

Modified B-meson LCDA

$$\phi_+(\omega) \sim \left\langle 0 \middle| \bar{q}_s(vn_-)\, Y(vn_-,0)\ \not{n}_-\gamma_5 h_v(0) \left[Y_+^\dagger Y_-\right](0) \middle| \bar{B}_q(p) \right\rangle$$



soft function becomes process dependent!

(Depends on charges of the final state leptons)

Resumed amplitude and Numerical prediction

For LL accuracy, we can use standard LCDA and evolve it with QED corrections in the cusp anomalous dimension

$$\left\langle 0 \left| \bar{q}_s(vn_-)\, Y(vn_-,0)\ \not{n}_-\gamma_5 h_v(0)\left[Y_+^\dagger Y_-\right](0) \right| \bar{B}_q(p) \right\rangle \sim U_s^{\rm QED}\left(\mu_{hc},\mu_s;\omega\right) F_{B_q}\left(\mu_{hc}\right)\ \phi_+\left(\omega;\mu_{hc}\right)$$

This is justified for power-enhanced corrections since they are already α suppressed

The resumed result to LL

$$i\,A_9 = T_+\left(\mu_{hc}\right) m_{B_q} F_{B_q} \int_0^1 du\,\bar{u} \int_0^\infty \frac{d\omega}{\omega} U_h(\mu_b,\mu_{hc})\, U_\ell(\mu_{hc},\mu_{sc})\, U_s^{\rm QED}\left(\mu_{hc},\mu_s;\omega\right)$$

$$2\,H_9\left(u;\mu_b\right) \phi_+\left(\omega;\mu_{hc}\right) \ln\left(1+\frac{u}{\bar{u}}\frac{n_+p_\ell-\omega}{m_\ell^2}\right)$$

$T_+(\mu_{hc})$: kinematical dependence

m_{B_q}/ω: Power enhanced factor

hard function (U_h, H_9)

soft-LCDA (F_{B_q}, U_ℓ, $U_s^{\rm QED}$, ϕ_+)

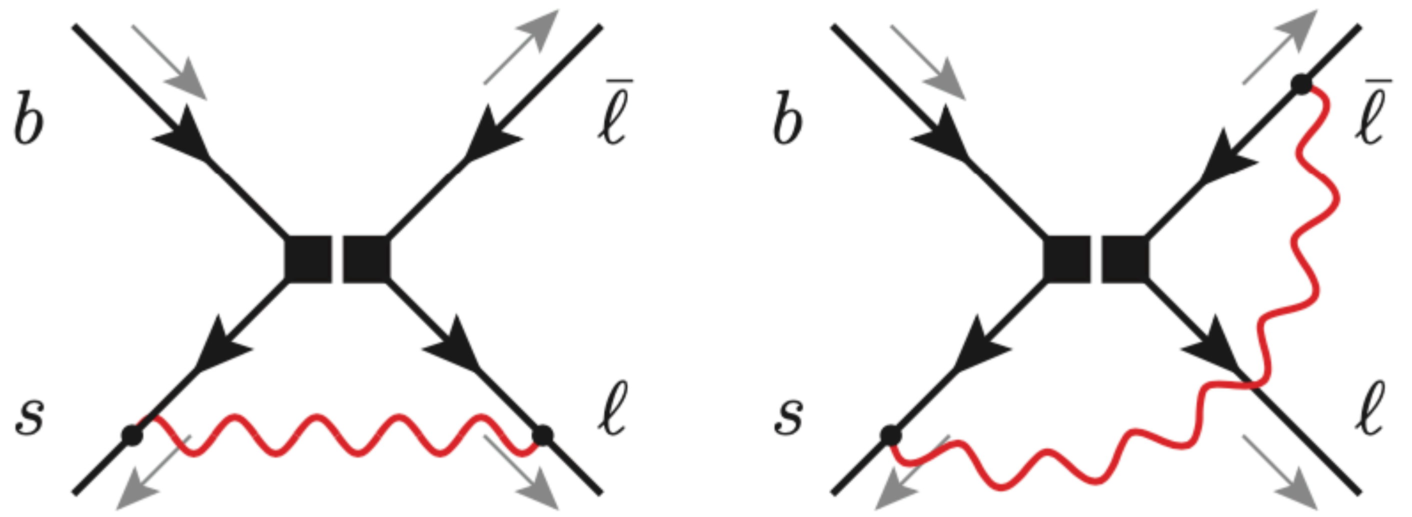
hard-collinear function (the logarithm)

Numerical prediction: Complete NLO+LL QED virtual correction changes the branching fraction less than 1 % , due to not large logarithms for tau

QED effects in $B \to \tau \nu$ at Subleading power

Power enhanced effects in $B \to \tau \nu$ at Leading Power is zero

$B \to \ell^+ \ell^-$



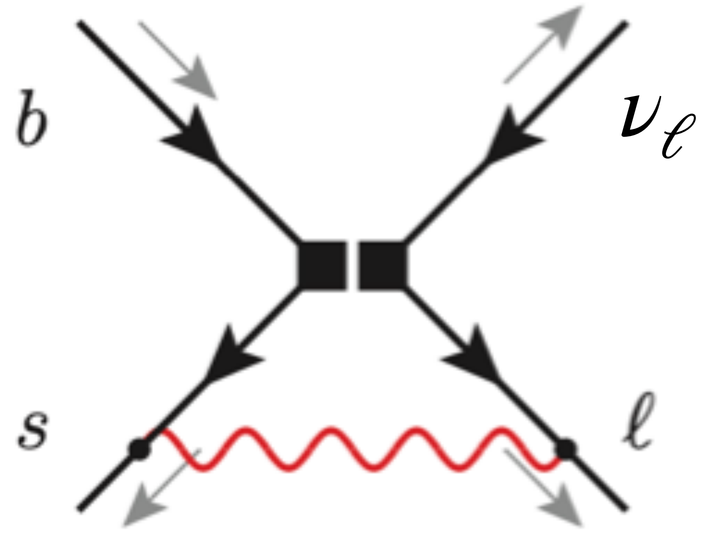
$$Q_9 = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{q}\, \gamma^\mu P_L b\right) \sum_\ell \bar{\ell}\, \gamma_\mu \ell$$

$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{q}\, \gamma^\mu P_L b\right) \sum_\ell \bar{\ell}\, \gamma_\mu \gamma_5 \ell$$

Lead power

$$A = (H_9 + H_{10}) \otimes J_m \otimes S$$

$B \to \ell \nu$



$$Q = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{q}\, \gamma^\mu P_L b\right) \sum_\ell \bar{\ell}\, \gamma_\mu (1 - \gamma_5) \ell$$

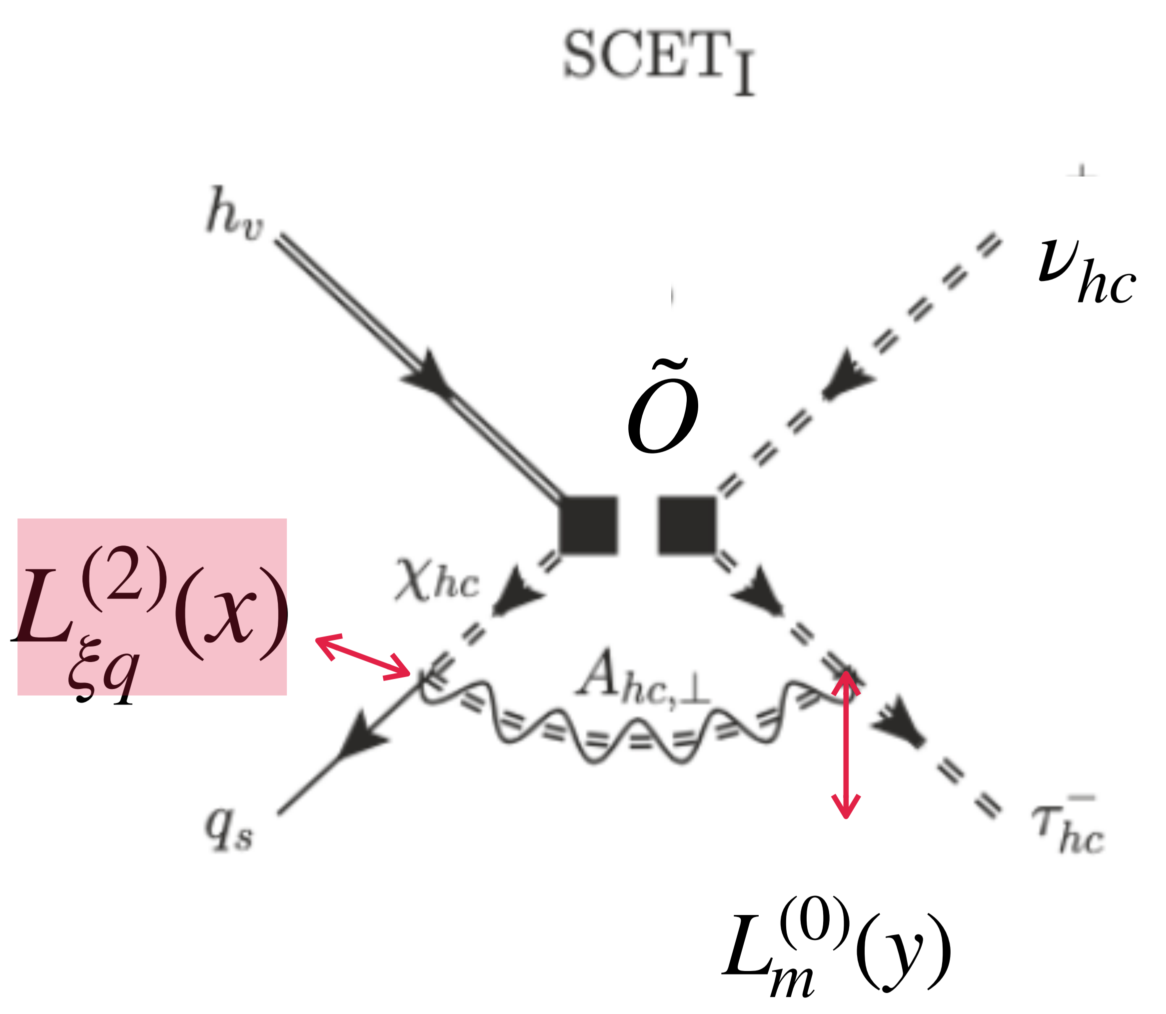
$$A = C\, Q$$

$H_9 = C$ and $H_{10} = -C$

$$A = (C - C) \otimes J_m \otimes S \sim 0$$

hard-collinear function at NLP

endpoint divergence ?



$$\tilde{O}(s,t) = \left[\bar{\chi}_C(sn_+)\,\gamma_\perp^\mu P_L h_v(0)\right]\left[\bar{\ell}_C(tn_+)\,\gamma_\perp^\nu(1-\gamma_5)\,\ell_{\bar{C}}(0)\right]$$

$$L_m^{(0)}(y) = m_\tau \bar{\ell}_C \left[i\ D_{C\perp}, \frac{1}{in_+D_C}\right]\frac{\not{n}_+}{2}\ell_C$$

$$L_{\xi q}^{(2)}(x) = \bar{q}_s(x_-)\left[W_{\xi C}W_C\right]^\dagger(x)\left(i\,n\_D + i\ D_\perp(i\,n_+D)^{-1} i\ D_\perp\right)\frac{\not{n}_+}{2}\xi_C(x)$$
$$+\bar{q}_s(x_-)\,D_s^\mu\, x_{\perp\mu}\left[W_{\xi C}W_C\right]^\dagger(x)\,i\ D_\perp\,\xi_C(x) + \mathrm{h.c.}$$

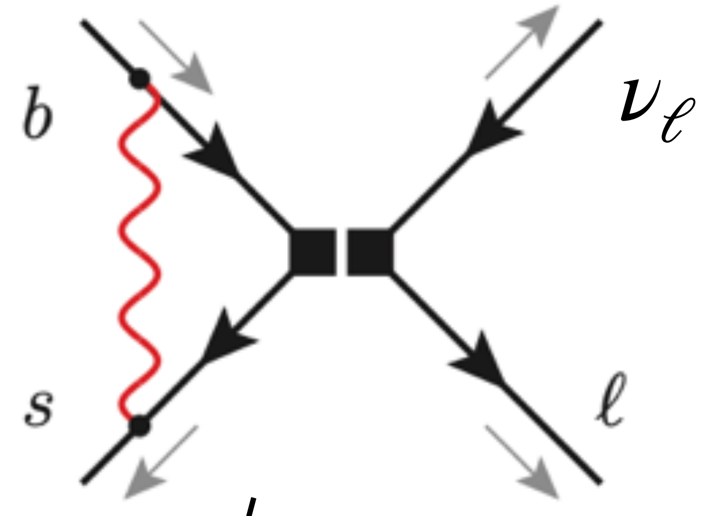
HQET $\times$ bHLET operator $\widetilde{J}^{A1}_{m\chi} = \left[\bar{v}_s(p_q)\frac{\not{n}}{2}(1-\gamma_5)\,u_h(p_b)\right]\left[\bar{u}_{sc}(p_\ell)(1-\gamma_5)\,v_{s\bar{c}}(p_\nu)\right]$

$$J_m^{\mathrm{NLP}}(u,\omega;\mu) = \frac{\alpha}{4\pi}Q_\ell Q_s m_\ell\, u\,\bar{u}\left[\frac{1}{\epsilon} - \frac{\bar{u}\,\bar{n}\cdot p_\ell}{u\,\omega}\ln\left(1+\frac{u\,n\cdot p_\ell\,\omega}{\bar{u}\,m_\ell^2}\right) - \ln\left(\frac{\bar{u}^2 m_b\,\bar{n}\cdot p_\ell + u\,\bar{u}\,m_b\,\omega}{\mu^2}\right)\right]\theta(u)\theta(\bar{u})$$

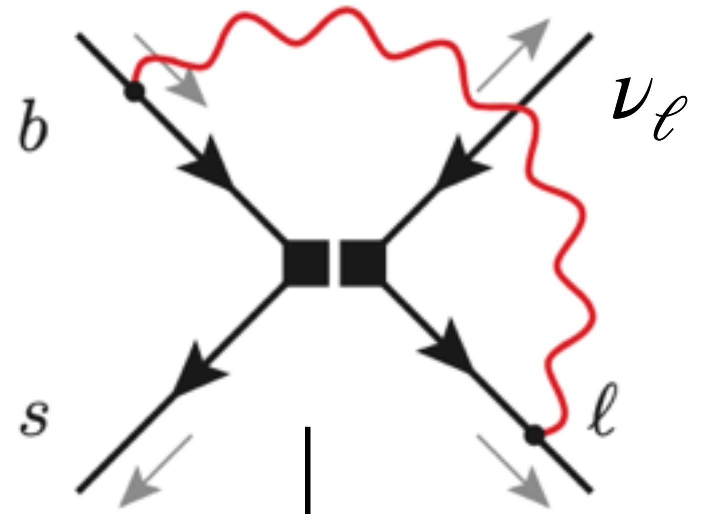
the convolution integrals of the hard and jet functions do not suffer from endpoint divergences.

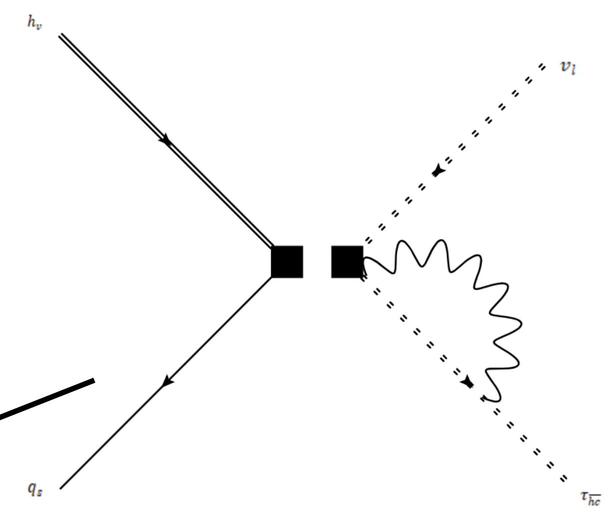
Local operator contribution

As the power-enhanced effect (non-local operator) is in NLP,
we also need to consider local contribution.



Hard function





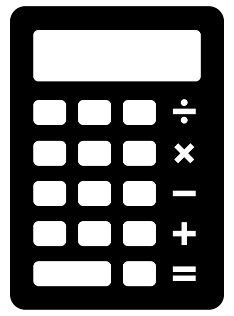
Hard collinear function

$$Q = \frac{\alpha_{\text{em}}}{4\pi} \left(\bar{q}\, \gamma^{\mu} P_L\, b\right) \sum_{\ell} \bar{\ell}\, \gamma_{\mu} (1-\gamma_5)\, \ell$$

$$\widetilde{J}^{A1}_{m\chi} = m_\ell\, [\bar{u}_s\, (1+\gamma_5)\, h_v]\, [\bar{\ell}_{sc}\, (1-\gamma_5)\, \ell_{s\bar{c}}]$$

HQET $\times$ bHLET

Resummation, Numerical calculation… in progress



Summary

1. Structure depended QED corrections can be calculated in SCET , HQET, bHLET

——convolution of hard function, jet function and QED specific B-meson LCDA

at NLO completely at leading power for $B \to \tau^+ \tau^-$ and $B \to \tau \nu$ at NLP

——QED interesting effect $\to$ power suppressed interaction $L_{\xi q}^{(1)}(x)$ lead to power enhanced correction $1/\Lambda_{\rm QCD}$

2. QED factorization more complicated than in QCD due to charged external states

——theoretically interesting, one cannot naively generalise QCD to QCD+QED

Thank you

Backup slides

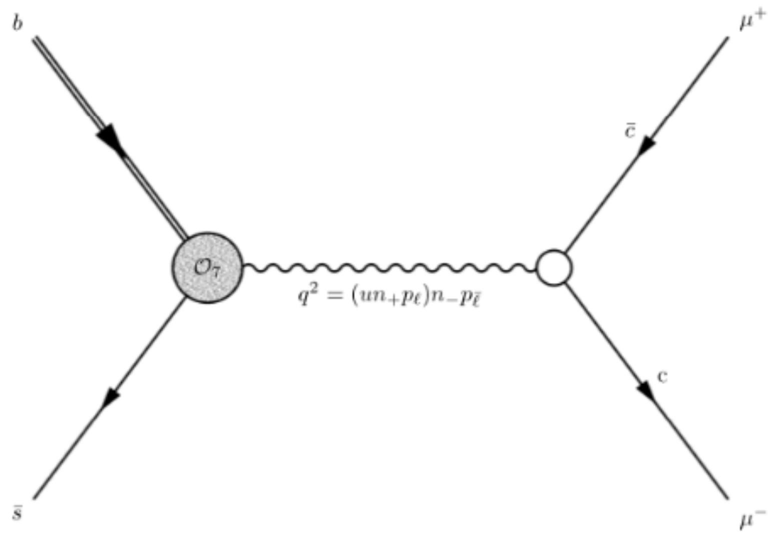
$$\begin{aligned}
\text{hard:}\quad & k_h^\mu = m_b\,(1,\,1,\,1) \sim (1,\,1,\,1)\,,\\
\text{hard-collinear:}\quad & k_{hc}^\mu = \left(m_b,\,\Lambda_{\rm QCD},\,\sqrt{m_b\,\Lambda_{\rm QCD}}\right) \sim (1,\,\lambda^2,\,\lambda)\,,\\
\text{anti-hard-collinear:}\quad & k_{\overline{hc}}^\mu = \left(\Lambda_{\rm QCD},\,m_b,\,\sqrt{m_b\,\Lambda_{\rm QCD}}\right) \sim (\lambda^2,\,1,\,\lambda)\,,\\
\text{soft:}\quad & k_s^\mu = (\Lambda_{\rm QCD},\,\Lambda_{\rm QCD},\,\Lambda_{\rm QCD}) \sim (\lambda^2,\,\lambda^2,\,\lambda^2)\,,\\
\text{soft-collinear:}\quad & k_{sc}^\mu = (1/b,\,b,\,1)\,\Lambda_{\rm QCD}, \sim (1/b,\,b,\,1)\,\lambda^2\,,\\
\text{anti-soft-collinear:}\quad & k_{sc}^\mu = (b,\,1/b,\,1)\,\Lambda_{\rm QCD}, \sim (b,\,1/b,\,1)\,\lambda^2\,,
\end{aligned}$$

$$\begin{aligned}
\text{soft heavy quark:}\quad & h_v \sim \lambda^3\,,\\
\text{hard-collinear light quark:}\quad & \chi_{hc} \sim \lambda\,,\\
\text{hard-collinear leptonic field:}\quad & \ell_{hc} \sim \lambda\,,\\
\text{soft light quark:}\quad & q_s \sim \lambda^3\,,\\
\text{soft-collinear leptonic field:}\quad & \ell_{sc} \sim \lambda^3\,,\\
\text{hard-collinear photon (gluon):}\quad & A_{hc}^\mu(G_{hc}^\mu) \sim (1,\,\lambda^2,\,\lambda)\,,\\
\text{soft photon (gluon):}\quad & A_s^\mu(G_s^\mu) \sim \lambda^2\,(1,\,1,\,1)\,.
\end{aligned}$$

$$Y(x,y) = \exp\left[i\,e\,Q_q\int_y^x dz_\mu\,A_s^\mu(z)\right]\mathcal{P}\,\exp\left[i\,g_s\int_y^x dz_\mu\,G_s^\mu(z)\right],$$

$$Y_\pm(x) = \exp\left[-i\,e\,Q_\ell\int_0^\infty ds\,n_\mp A_s\,(x+sn_\mp)\right].$$

- when the photon is hard, $Q_7 = \frac{2Q_\ell}{u}\,\mathcal{O}_9$



$$J_9(u;\omega;\mu=\mu_{hc}) \sim \frac{\alpha}{4\pi}\,Q_\ell Q_s\,\frac{\bar{u}}{\omega}\,\ln\left(\frac{\omega\,m_b}{m_\ell^2}\right)\,\theta(u)\theta(\bar{u})\,,$$

endpoint divergence $u \to 0$ in $B \to \mu\mu$.

-

$$J_m^{(0)}(u;\omega;\mu=\mu_{hc}) = \frac{\alpha}{4\pi}\,Q_\ell Q_s\,m_\ell\,\frac{\bar{u}}{\omega}\,\ln\left(1+\frac{u}{\bar{u}}\,\frac{\omega\,m_b}{m_\ell^2}\right)\,\theta(u)\theta(\bar{u})\,,$$

Numerical prediction: complete QED virtual correction

- The non-radiative branching fraction of $B_q \to \tau^+ \tau^-$ for central values of the parameters are

$$\mathrm{Br}^{(0)}(B_d \to \tau^+\tau^-) = \left(2.051_{\text{(LO)}} - 0.001_{\text{(NLO)}}\right) \times 10^{-8}$$
$$\mathrm{Br}^{(0)}(B_s \to \tau^+\tau^-) = \left(7.147_{\text{(LO)}} - 0.003_{\text{(NLO)}}\right) \times 10^{-7}$$

$\to$ Complete NLO+LL QED virtual correction (hard and hard collinear functions) changes the branching fraction by: $\sim 0.04\%$

$$J_m^{(0)}(u;\omega;\mu=\mu_{hc}) = \frac{\alpha}{4\pi} Q_\ell Q_s\, m_\ell\, \frac{\bar{u}}{\omega} \ln\left(1 + \frac{u}{\bar{u}} \frac{\omega\, m_b}{m_\ell^2}\right) \theta(u)\theta(\bar{u})\,, \qquad (30)$$

where $\omega = \bar{n} \cdot \ell_q \sim \Lambda_{QCD}$, power enhancement factor and large logarithms

- compared with $B_{d,s} \to \mu^+\mu^-$, power-enhanced correction ((hard) collinear functions) $\sim 0.4\%$, [Beneke, Bobeth, Szafron '17, '19]

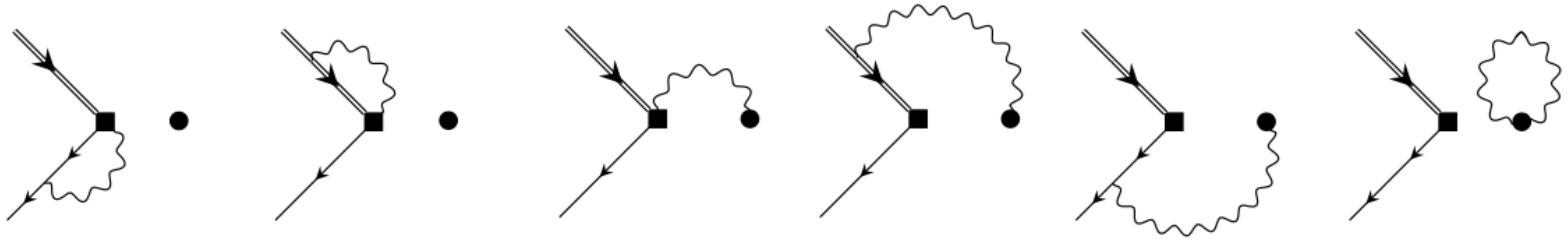
$$J_9(u;\omega;\mu=\mu_{hc}) \sim \frac{\alpha}{4\pi} Q_\ell Q_s\, \frac{\bar{u}}{\omega} \ln\left(\frac{\omega\, m_b}{m_\ell^2}\right) \theta(u)\theta(\bar{u})\,, \qquad (31)$$

QED effects in $B_q \to \tau\nu$ at Subleading power

$$\left\langle 0 \left| \bar{q}_s(vn_-)\, Y(vn_-,0)\ \not{n}_-\gamma_5 h_v(0)\left[Y_+^\dagger Y_-\right](0) \right| \bar{B}_q(p) \right\rangle \sim U_s^{\rm QED}\left(\mu_{hc},\mu_s;\omega\right) F_{B_q}\left(\mu_{hc}\right)\ \phi_+\left(\omega;\mu_{hc}\right)$$

This is justified for power-enhanced corrections since they are already α suppressed. What if we want to go beyond leading order in α or consider non–enhanced corrections?

Higher-order terms QCD and QED correction simultaneously are non-universal, non-local HQET matrix elements that have to be evaluated nonperturbatively. For example at one loop,

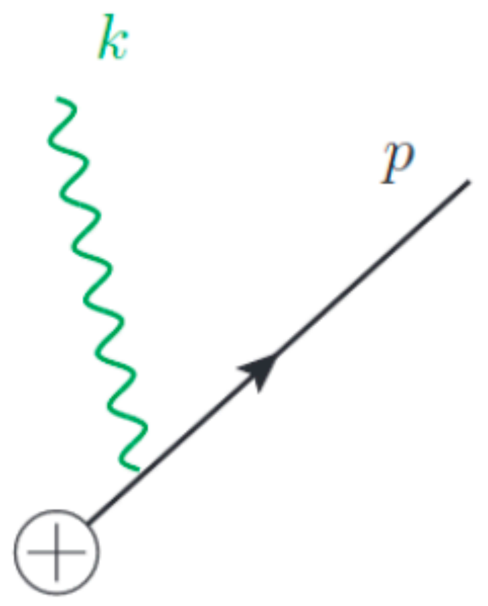


ΔE – cut on photon energy (e.g. due to detector resolution)

QED effects can be divided into two classes:

- Ultra-soft photons (under the assumption that $\Delta E \ll \Lambda_{\rm QCD}$)

  Based on eikonal approximation,



$$\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \frac{\varepsilon_\mu(k)p^\mu}{p\cdot k}\bar{u}(p)\,,$$

  note $k^\mu \ll p^\mu, m$

$$\delta_{\rm QED} \sim \frac{\alpha}{\pi}\ln^2\frac{m_B}{m_\ell}$$

  Large logarithmic enhancements can mimic lepton-flavor universality violation

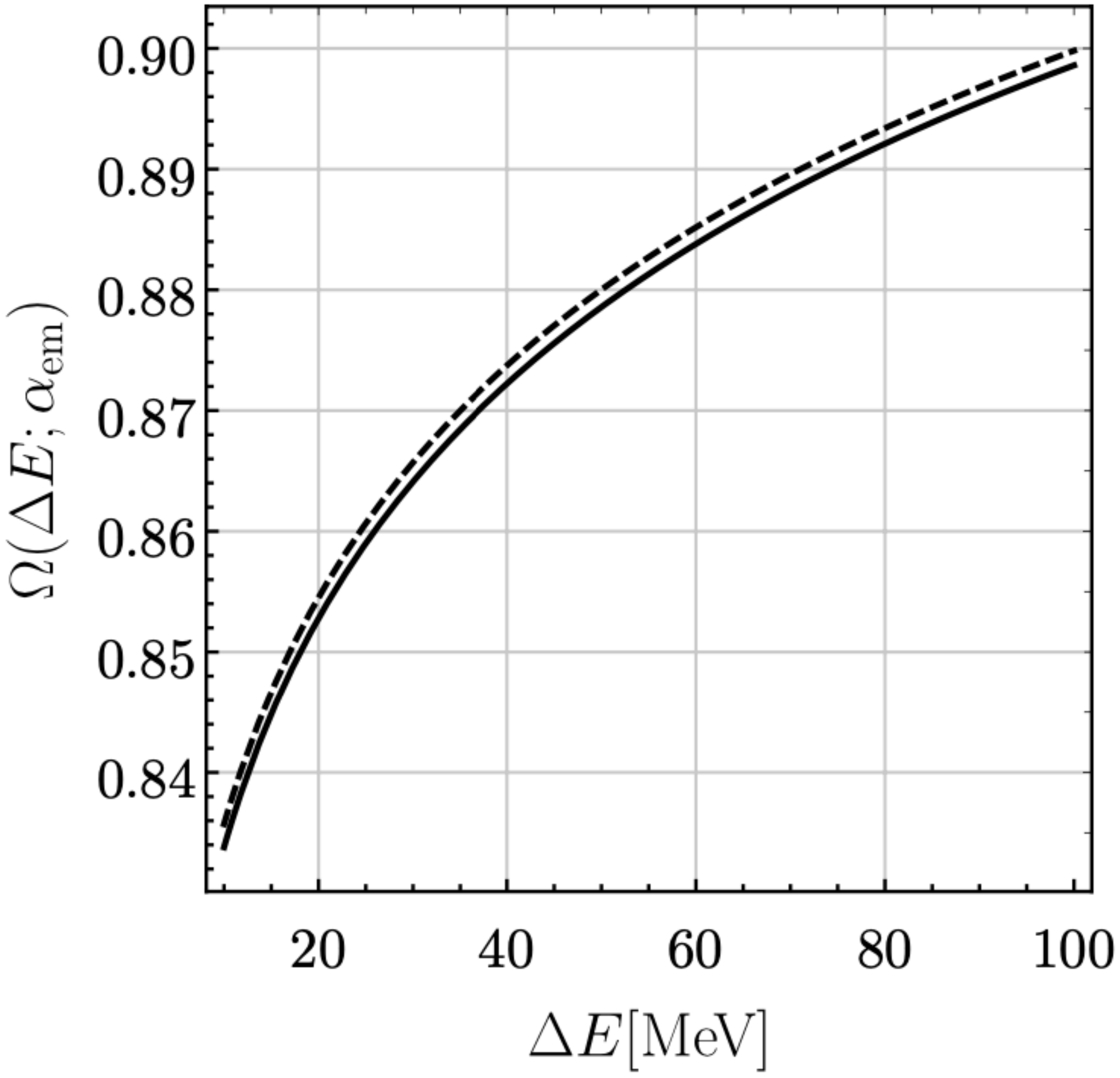
→ Ultra-soft photon corrections to $\bar{B}\to D\tau^-\bar{\nu}_\tau$ relative to $\bar{B}\to D\mu^-\bar{\nu}_\mu$ [S. de Boer, T. Kitahara, I. Nisandzic, 1803.05881] – relevant for lepton universality test $R(D)$

Universal for $B\to\mu^+\mu^-$ and $B\to\tau^+\tau^-$

$$\overline{\mathrm{Br}}_{q\mu}(\Delta E) \equiv \overline{\mathrm{Br}}^{(0)}_{q\mu} \times \Omega(\Delta E; \alpha_{\mathrm{em}}) \,,$$

with radiative factor

$$\Omega(\Delta E; \alpha_{\mathrm{em}}) \equiv \left(\frac{2\Delta E}{m_{B_q}}\right)^{-\frac{2\alpha_{\mathrm{em}}}{\pi}\left(1+\ln\frac{m_\mu^2}{m_{B_q}^2}\right)}$$



effective theory framework is set up. The dependence of the radiative factor Ω on $\Delta E = (m_{B_q}^2 - s_{\ell\bar{\ell}})^{1/2}$ is shown in Figure 6 for B_s mesons. One might consider $\Delta E \simeq 60$ MeV as