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Hadron FCNC decays with invisible particles in light of recent data

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Based on: [1] Phys.Rev.D 106 (2022) 11, 115007 (arXiv: 2206.01575), [2] Phys.Lett.B 839 (2023) 137811 (arXiv: 2212.04699), [3] JHEP 11 (2023) 205 (arXiv: 2306.05333).

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Collaborator

Figure 1: Prof. Jusak Tandean and Dr. Geng Li

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Motivation

Motivation

- Searching for the dark matter candidates on colliders;
- Flavor-Changing Neutral Current (FCNC) processes of long-lived hadrons: $c \to u$, $s \to d$, $b \to d(s)$;
- In the SM, undetected neutrinos contribute to missing energy (MS);
- Experiments like Belle (Belle II), BarBar, BES III, LHCb, etc., and future colliders like STCF, CEPC, FCC-ee, etc., detect invisible decays via missing energy/momentum;
- In light of recent data, the Belle II first report an anomaly of $2.8\,\sigma$ in $B^+ \to K^+ \bar{\nu}\nu$ on August 2023.

Motivation

For $s \to d$ transition,

$$\mathcal{B}(K_L \to \pi^0 \bar{\nu}\nu)_{\mathrm{KOTO}} < 3.0 \times 10^{-9} \text{ at } 90\% \text{ C.L.}$$

$$\mathcal{B}(K_L \to \pi^0 \bar{\nu}\nu)_{\mathrm{SM}} = (3.4 \pm 0.6) \times 10^{-11}$$

$$\mathcal{B}(K^+ \to \pi^+ \bar{\nu}\nu)_{\mathrm{NA62}} = (11.0^{+4.0}_{-3.5}(\mathrm{stat}) \pm 0.3(\mathrm{syst})) \times 10^{-11} \text{ at } 68\% \text{ C.L.}$$

$$\mathcal{B}(K^+ \to \pi^+ \bar{\nu}\nu)_{\mathrm{E949}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$$

$$\mathcal{B}(K^+ \to \pi^+ \bar{\nu}\nu)_{\mathrm{SM}} = (8.4 \pm 1.0) \times 10^{-11}$$

For $c \to u$ transition,

$$\mathcal{B}(D^0 \to \not{E})_{\mathrm{Belle}} < 9.4 \times 10^{-5}$$

$$\mathcal{B}(D^0 \to \pi^0 \bar{\nu}\nu)_{\mathrm{BES\,III}} < 2.1 \times 10^{-4} \text{ at } 90\% \text{ C.L.}$$

$$\mathcal{B}(\Lambda_c^+ \to p\gamma')_{\mathrm{BES\,III}} < 8.0 \times 10^{-5}$$

$\mathcal{B}(D^0 \to \bar{\nu}\nu)_{\mathrm{SM}} \sim 0$ (Helicity suppress)

$\mathcal{B}(D^0 \to \pi^0 \bar{\nu}\nu)_{\mathrm{SM}} \sim 10^{-17}$ (GIM mechanism suppress)

$\mathcal{B}(\Lambda_c^+ \to p \bar{\nu}\nu)_{\mathrm{SM}} \sim 10^{-17}$ (GIM mechanism suppress)

For $b \to d$ transition, the Belle II first report an anomaly of $2.8\,\sigma$ in $B^+ \to K^+\bar{\nu}\nu$ on August 2023.

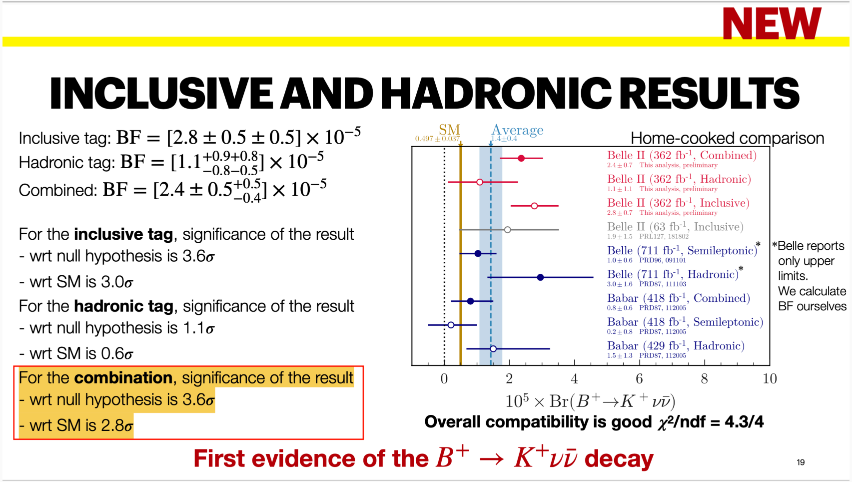


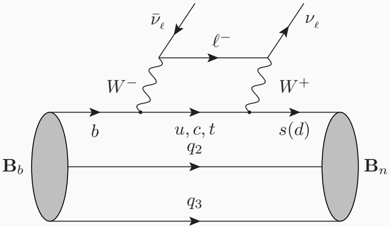
Figure 2: Recent report by Belle II.

Bottomed hadron with $\not{E}$

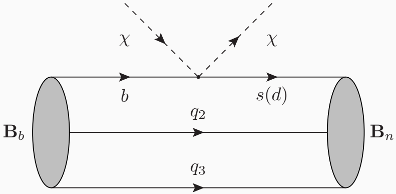
Experimental bounds

Table 1: The branching ratios ($\mathcal{B}$) (in units of 10^{-6}) of B decays involving missing energy.

Experimental bound	SM prediction	Invisible particles bound
$\mathcal{B}(B^\pm \to K^\pm \not{E}) < 16$	$\mathcal{B}(B^\pm \to K^\pm \nu\bar{\nu}) = 4.73 \pm 0.56$	$\mathcal{B}(B^\pm \to K^\pm \chi\chi) < 11.8$
$\mathcal{B}(B^\pm \to \pi^\pm \not{E}) < 14$	$\mathcal{B}(B^\pm \to \pi^\pm \nu\bar{\nu}) = 8.12 \pm 0.01$	$\mathcal{B}(B^\pm \to \pi^\pm \chi\chi) < 5.89$
$\mathcal{B}(B^\pm \to K^{*\pm} \not{E}) < 40$	$\mathcal{B}(B^\pm \to K^{*\pm} \nu\bar{\nu}) = 8.93 \pm 1.07$	$\mathcal{B}(B^\pm \to K^{*\pm} \chi\chi) < 32.1$
$\mathcal{B}(B^\pm \to \rho^\pm \not{E}) < 30$	$\mathcal{B}(B^\pm \to \rho^\pm \nu\bar{\nu}) = 0.48 \pm 0.18$	$\mathcal{B}(B^\pm \to \rho^\pm \chi\chi) < 29.7$



(a) Within the Standard Model



(b) Beyond the Standard Model

Figure 3: Feynman diagrams of bottomed baryon FCNC decays with missing energy.

SM expectations

The FCNC decay processes of bottomed baryons with missing energy are described as

$$\mathcal{L}_{\bar{\nu}\nu} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{\ell=e,\mu,\tau} \sum_{q=u,c,t} V_{bq} V_{sq} X^\ell(x_q)(\bar{s}_L \gamma^\mu b_L)(\bar{\nu}_{\ell L} \gamma_\mu \nu_{\ell L}),$$

The loop is calculated by Inami-Lim function

$$X^\ell(x_q) = \frac{x_q}{8}\left[\frac{x_q+2}{x_q-1} + \frac{3(x_q-2)}{(x_q-1)^2}\ln x_q\right],$$

where G_F represents the Fermi coupling constant, α corresponds to the fine structure constant, θ_W stands for the Weinberg angle, V_{ij} are the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, and $x_q = m_q^2/M_W^2$ with m_q (M_W) being the mass of the quark (W-boson).

The transition amplitude is given by

$$\langle \mathbf{B}_n \bar{\nu}\nu | \mathcal{L}_{\bar{\nu}\nu} | \mathbf{B}_b \rangle = \frac{\sqrt{2} G_F \alpha}{4\pi \sin^2\theta_W} V_{bt} V_{st} X^\ell(x_t) \langle \mathbf{B}_n | \bar{s}\gamma^\mu (1-\gamma^5) b | \mathbf{B}_b \rangle \times \bar{u}_{\nu_\ell} \gamma_\mu (1-\gamma^5) v_{\nu_\ell}.$$

Baryonic amplitude

The baryonic transition matrix elements can be parameterized by the form factors (FFs) of $f_i^{V,A}$ $(i=1,2,3)$, f^S and f^P, defined by

$$\langle \mathbf{B}_n(P_f, s_f)|(\bar{q}_f\gamma_\mu q)|\mathbf{B}_b(P, s)\rangle = \bar{u}_{\mathbf{B}_n}(P_f, s_f)\left[\gamma_\mu f_1^V(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M}f_2^V(q^2) + \frac{q^\mu}{M}f_3^V(q^2)\right]u_{\mathbf{B}_b}(P, s),$$

$$\langle \mathbf{B}_n(P_f, s_f)|(\bar{q}_f q)|\mathbf{B}_b(P, s)\rangle = \bar{u}_{\mathbf{B}_n}(P_f, s_f)f^S(q^2)u_{\mathbf{B}_b}(P, s),$$

$$\langle \mathbf{B}_n(P_f, s_f)|(\bar{q}_f\gamma_\mu\gamma^5 q)|\mathbf{B}_b(P, s)\rangle = \bar{u}_{\mathbf{B}_n}(P_f, s_f)\left[\gamma_\mu f_1^A(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M}f_2^A(q^2) + \frac{q^\mu}{M}f_3^A(q^2)\right]\gamma^5 u_{\mathbf{B}_b}(P, s),$$

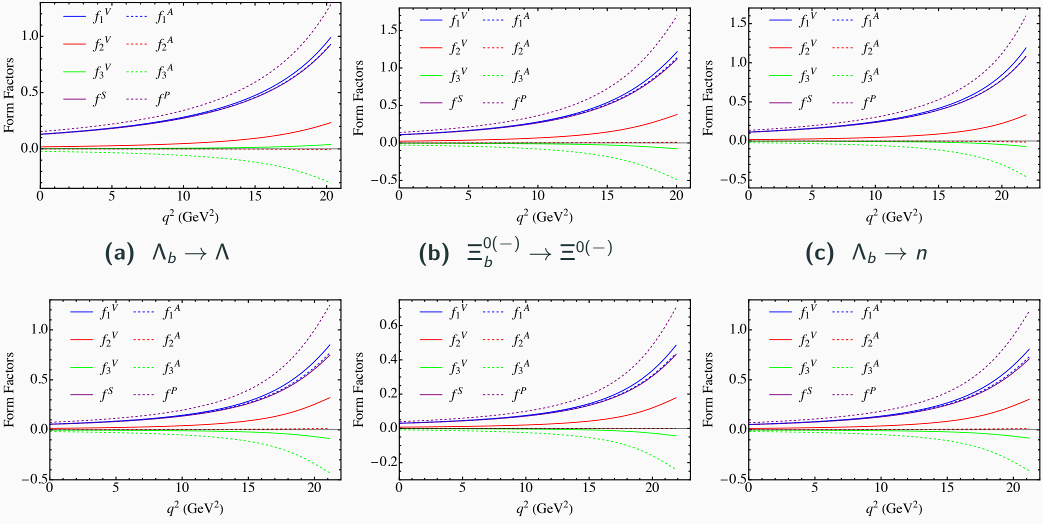
$$\langle \mathbf{B}_n(P_f, s_f)|(\bar{q}_f\gamma^5 q)|\mathbf{B}_b(P, s)\rangle = \bar{u}_{\mathbf{B}_n}(P_f, s_f)f^P(q^2)\gamma^5 u_{\mathbf{B}_b}(P, s),$$

By integrating the three-body phase space, we obtain the decay branching ratio to be

$$\mathcal{B}(\mathbf{B}_b \to \mathbf{B}_n\bar{\nu}\nu) = \frac{1}{512\pi^3 M^3\Gamma_{\mathbf{B}_b}}\int \frac{dq^2}{q^2}\lambda^{1/2}(M^2, q^2, M_f^2)\lambda^{1/2}(q^2, m_1^2, m_2^2)\int d\cos\theta \sum|\mathcal{M}|^2,$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Kallen function.

Form Factors from MBM



(a) $\Lambda_b \to \Lambda$ **(b)** $\Xi_b^{0(-)} \to \Xi^{0(-)}$ **(c)** $\Lambda_b \to n$

(d) $\Xi_b^- \to \Sigma^-$ **(e)** $\Xi_b^0 \to \Lambda$ **(f)** $\Xi_b^0 \to \Sigma^0$

Figure 4: Form factors as function of q^2

SM expectations

For the $b \to s$ transition, the decay branching ratios associated with $\bar{\nu}\nu$ are as follows:

$$\mathcal{B}(\Lambda_b \to \Lambda\bar{\nu}\nu) = 5.52^{+0.28}_{-0.28} \times 10^{-6},$$
$$\mathcal{B}(\Xi_b^{0(-)} \to \Xi^{0(-)}\bar{\nu}\nu) = 7.80^{+0.71}_{-0.67} \times 10^{-6}.$$

Here, due to the $\mathrm{SU}(3)$ flavor symmetry, the branching ratios of Ξ_b^0 and Ξ_b^- are considered approximately to be equal.

Similarly, for the $b \to d$ transition we have that

$$\mathcal{B}(\Lambda_b \to n\bar{\nu}\nu) = 2.76^{+0.17}_{-0.16} \times 10^{-7},$$
$$\mathcal{B}(\Xi_b^- \to \Sigma^-\bar{\nu}\nu) = 2.65^{+0.29}_{-0.26} \times 10^{-7},$$
$$\mathcal{B}(\Xi_b^0 \to \Sigma^0\bar{\nu}\nu) = 1.24^{+0.13}_{-0.12} \times 10^{-7},$$
$$\mathcal{B}(\Xi_b^0 \to \Lambda\bar{\nu}\nu) = 3.88^{+0.37}_{-0.40} \times 10^{-8},$$

Effective Lagrangian with invisible particles

Under the low energy scale, the model-independent effective Lagrangian is given by

$$\mathcal{L}_{eff} = \sum_{i=1}^{6} g_{mi} Q_i,$$

where g_{fi} are the phenomenological coupling constants. There are 6 independent dimension-six effective operators

$$Q_1 = (\bar{q}_f q)(\chi\chi), \qquad Q_2 = (\bar{q}_f \gamma^5 q)(\chi\chi), \qquad Q_3 = (\bar{q}_f q)(\chi\gamma^5\chi),$$
$$Q_4 = (\bar{q}_f \gamma^5 q)(\chi\gamma^5\chi), \quad Q_5 = (\bar{q}_f \gamma_\mu q)(\chi\gamma^\mu\gamma^5\chi), \quad Q_6 = (\bar{q}_f \gamma_\mu \gamma^5 q)(\chi\gamma^\mu\gamma^5\chi),$$

where the invisible particles of χ have been assumed to be the Majorana type. Since $\chi\gamma^\mu\chi = 0$ and $\chi\sigma^{\mu\nu}\chi = 0$, there is no contribution from the vector or tensor current.

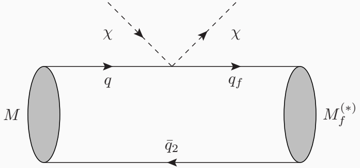


Figure 5: Feynman diagram of bottomed meson FCNC decays with invisible particles.

Decay amplitude

For the $0^- \to 0^-$ meson decays of $M^- \to M_f^- \chi\chi$, only operators $Q_{1,3,5}$ give the contributions. The amplitudes of the $0^- \to 0^-$ decays can be simplified as

$$\begin{aligned} \langle M_f^- \chi\chi|\mathcal{L}_{eff}|M^-\rangle = & \, 2g_{m1}\langle M_f^-|(\bar{q}_f q)|M^-\rangle \bar{u}_\chi v_\chi + 2g_{m3}\langle M_f^-|(\bar{q}_f q)|M^-\rangle \bar{u}_\chi \gamma^5 v_\chi \\ & + 2g_{m5}\langle M_f^-|(\bar{q}_f \gamma_\mu q)|M^-\rangle \bar{u}_\chi \gamma^\mu \gamma^5 v_\chi , \end{aligned}$$

For the $0^- \to 1^-$ meson decays of $M^- \to M_f^{*-} \chi\chi$ only operators $Q_{2,4,5,6}$ give the contributions. The amplitudes of the $0^- \to 1^-$ decays can be simplified as

$$\begin{aligned} \langle M_f^{*-} \chi\chi|\mathcal{L}_{eff}|M^-\rangle = & \, 2g_{m2}\langle M_f^{*-}|(\bar{q}_f \gamma^5 q)|M^-\rangle \bar{u}_\chi v_\chi + 2g_{m4}\langle M_f^{*-}|(\bar{q}_f \gamma^5 q)|M^-\rangle \bar{u}_\chi \gamma^5 v_\chi \\ & + 2g_{m5}\langle M_f^{*-}|(\bar{q}_f \gamma_\mu q)|M^-\rangle \bar{u}_\chi \gamma^\mu \gamma^5 v_\chi + 2g_{m6}\langle M_f^{*-}|(\bar{q}_f \gamma_\mu \gamma^5 q)|M^-\rangle \bar{u}_\chi \gamma^\mu \gamma^5 v_\chi . \end{aligned}$$

For baryonic decay, all operators should be considered. The decay amplitude can be expressed as

$$\begin{aligned} \langle \mathbf{B}_n \chi\chi|\mathcal{L}_{eff}|\mathbf{B}_b\rangle = & \, 2g_{m1}\langle \mathbf{B}_n|(\bar{q}_f q)|\mathbf{B}_b\rangle \bar{u}_\chi v_\chi + 2g_{m2}\langle \mathbf{B}_n|(\bar{q}_f \gamma^5 q)|\mathbf{B}_b\rangle \bar{u}_\chi v_\chi \\ & + 2g_{m3}\langle \mathbf{B}_n|(\bar{q}_f q)|\mathbf{B}_b\rangle \bar{u}_\chi \gamma^5 v_\chi + 2g_{m4}\langle \mathbf{B}_n|(\bar{q}_f \gamma^5 q)|\mathbf{B}_b\rangle \bar{u}_\chi \gamma^5 v_\chi \\ & + 2g_{m5}\langle \mathbf{B}_n|(\bar{q}_f \gamma_\mu q)|\mathbf{B}_b\rangle \bar{u}_\chi \gamma^\mu \gamma^5 v_\chi + 2g_{m6}\langle \mathbf{B}_n|(\bar{q}_f \gamma_\mu \gamma^5 q)|\mathbf{B}_b\rangle \bar{u}_\chi \gamma^\mu \gamma^5 v_\chi . \end{aligned}$$

Mesonic form factors

The hadronic transition matrix elements can be expressed as

$$\langle M_f^-|(\bar{q}_f q)|M^-\rangle = \frac{M^2 - M_f^2}{m_q - m_{q_f}} f_0(q^2),$$

$$\langle M_f^-|(\bar{q}_f \gamma_\mu q)|M^-\rangle = (P+P_f)_\mu f_+(q^2) + (P-P_f)_\mu \frac{M^2 - M_f^2}{q^2}\left[f_0(q^2) - f_+(q^2)\right],$$

$$\langle M_f^-|(\bar{q}_f \sigma_{\mu\nu} q)|M^-\rangle = i\left[P_\mu (P-P_f)_\nu - P_\nu (P-P_f)_\mu\right] \frac{2}{M+M_f} f_T(q^2),$$

and

$$\langle M_f^{*-}|(\bar{q}_f \gamma^5 q)|M^-\rangle = -i\left[\epsilon\cdot(P-P_f)\right] \frac{2M_f}{m_q + m_{q_f}} A_0(q^2),$$

$$\langle M_f^{*-}|(\bar{q}_f \gamma_\mu \gamma^5 q)|M^-\rangle = i\Big\{ \epsilon_\mu (M+M_f) A_1(q^2) - (P+P_f)_\mu \frac{\epsilon\cdot(P-P_f)}{M+M_f} A_2(q^2)$$
$$- (P-P_f)_\mu \left[\epsilon\cdot(P-P_f)\right] \frac{2M_f}{q^2}\left[A_3(q^2) - A_0(q^2)\right] \Big\},$$

$$\langle M_f^{*-}|(\bar{q}_f \gamma_\mu q)|M^-\rangle = \varepsilon_{\mu\nu\rho\sigma}\epsilon^\nu P^\rho (P-P_f)^\sigma \frac{2}{M+M_f} V(q^2),$$

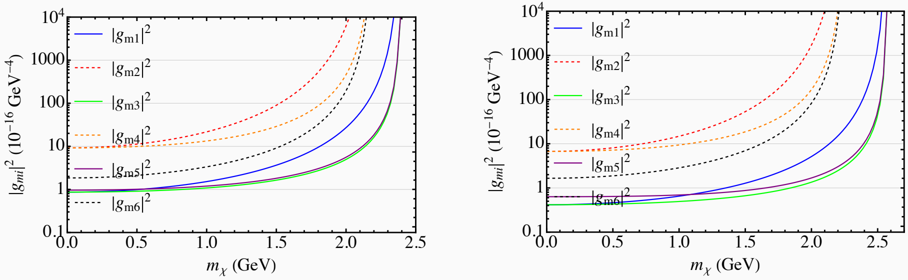
where f_j $(j = 0, +, T)$, A_k $(k = 0-3)$ and V are the FFs, which are evaluated from the method of the LCSR, and ϵ is the polarization vector of the final meson with the convention of $\varepsilon^{0123} = 1$.

Coupling constants

In our calculation, we assume that only one operator contributes to the process at a time. The upper limits of the coupling constants g_{mi} can be obtained from Table 1, given by

$$\mathcal{B}(M \to M_f^{(*)} \not{E})_{\rm exp} - \mathcal{B}(M \to M_f^{(*)} \bar{\nu}\nu)_{\rm SM} \geq \mathcal{B}(M \to M_f^{(*)} \chi\chi)_{Q_i} = \frac{|g_{mi}|^2 \widetilde{\Gamma}_{ii}}{\Gamma_{M_B}},$$

The upper limits of $|g_{mi}|^2$ on $(bs\chi\chi)$ and $(bd\chi\chi)$ vertices are shown as functions of m_χ in Fig. 6 with m_χ is the mass of χ.

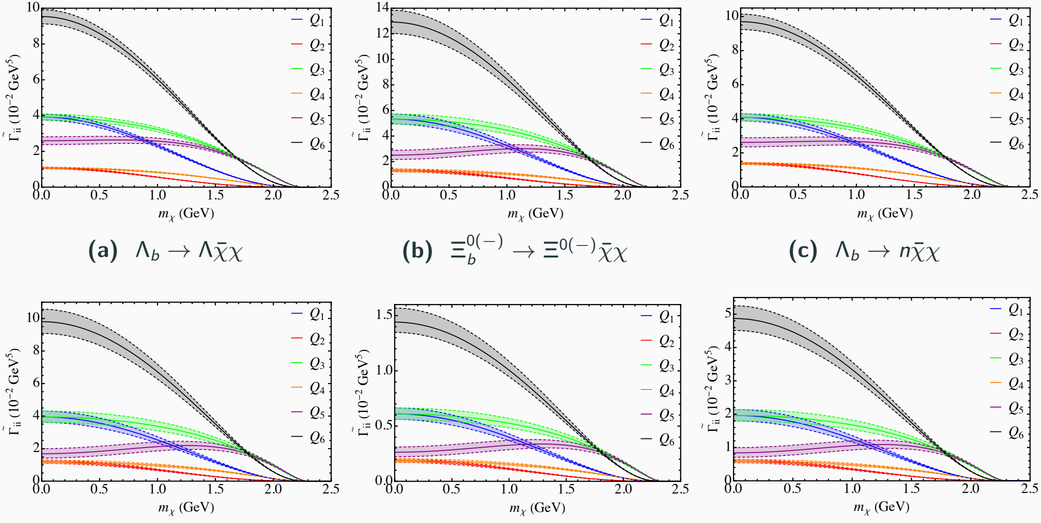


(a) $(bs\chi\chi)$ vertex

(b) $(bd\chi\chi)$ vertex

Figure 6: Upper limits of $|g_{mi}|^2$ as functions of m_χ

Partial width

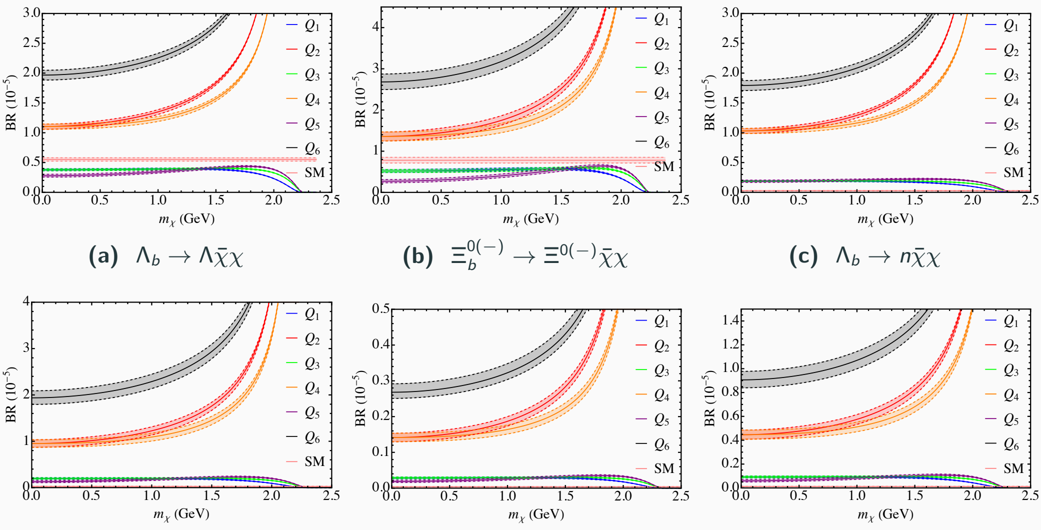


(a) $\Lambda_b \to \Lambda\bar{\chi}\chi$ **(b)** $\Xi_b^{0(-)} \to \Xi^{0(-)}\bar{\chi}\chi$ **(c)** $\Lambda_b \to n\bar{\chi}\chi$

(d) $\Xi_b^- \to \Sigma^-\bar{\chi}\chi$ **(e)** $\Xi_b^0 \to \Lambda\bar{\chi}\chi$ **(f)** $\Xi_b^0 \to \Sigma^0\bar{\chi}\chi$

Figure 7: $\widetilde{\Gamma}_{ij}$ as functions of m_χ, where the shadows represent the errors estimated by varying the bag radius within $\pm 5\%$

Prediction of the branching ratios



(a) $\Lambda_b \to \Lambda\bar{\chi}\chi$ **(b)** $\Xi_b^{0(-)} \to \Xi^{0(-)}\bar{\chi}\chi$ **(c)** $\Lambda_b \to n\bar{\chi}\chi$

(d) $\Xi_b^- \to \Sigma^-\bar{\chi}\chi$ **(e)** $\Xi_b^0 \to \Lambda\bar{\chi}\chi$ **(f)** $\Xi_b^0 \to \Sigma^0\bar{\chi}\chi$

Figure 8: The upper limits of the $\mathcal{BR}$ as function of m_χ in bottomed baryon FCNC decays

Branching ratios

Table 2: Upper limits of $\mathcal{B}(\mathbf{B}_b \to \mathbf{B}_n \chi\chi)$ when $m_\chi \to 0$ GeV (in units of 10^{-5})

Operator	$\Lambda_b \to \Lambda\chi\chi$	$\Xi_b^{0(-)} \to \Xi^{0(-)}\chi\chi$	$\Lambda_b \to n\chi\chi$	$\Xi_b^- \to \Sigma^-\chi\chi$	$\Xi_b^0 \to \Sigma^0\chi\chi$	$\Xi_b^0 \to \Lambda\chi\chi$
Q_1	0.38	0.52	0.19	0.20	0.092	0.029
Q_2	1.1	1.4	1.0	0.95	0.45	0.14
Q_3	0.38	0.52	0.19	0.20	0.092	0.029
Q_4	1.1	1.4	1.0	0.95	0.45	0.14
Q_5	0.28	0.27	0.19	0.13	0.060	0.019
Q_6	2.0	2.7	1.8	1.9	0.91	0.27
SM	$\Lambda_b \to \Lambda\bar{\nu}\nu$	$\Xi_b^{0(-)} \to \Xi^{0(-)}\bar{\nu}\nu$	$\Lambda_b \to n\bar{\nu}\nu$	$\Xi_b^- \to \Sigma^-\bar{\nu}\nu$	$\Xi_b^0 \to \Sigma^0\bar{\nu}\nu$	$\Xi_b^0 \to \Lambda\bar{\nu}\nu$
	0.55	0.78	0.028	0.027	0.012	0.0039

Charmed hadron with $\not{E}$

Predictions of the standard model

Before dealing with the SM case, we consider the more general, effective couplings of invisible spin-1/2 Dirac fermion fields $\mathtt{f}$ and $\mathtt{f}'$ to vector and axialvector $c \to u$ currents described by

$$\mathcal{L}_{\mathtt{ff}'} = -\bar{u}\gamma^\mu c\,\bar{\mathtt{f}}\gamma_\mu\big(\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'} + \gamma_5 \mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}\big)\mathtt{f}' - \bar{u}\gamma^\mu\gamma_5 c\,\bar{\mathtt{f}}\gamma_\mu\big(\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'} + \gamma_5 \tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}\big)\mathtt{f}'\,,$$

where the constants $\mathtt{C}^{\mathtt{V,A}}_{\mathtt{ff}'}$ and $\tilde{\mathrm{C}}^{\mathrm{V,A}}_{\mathtt{ff}'}$ may be complex. It will induce $D^0 \to \gamma\mathtt{f}\bar{\mathtt{f}}'$, $\mathbb{D} \to \mathbb{P}\mathtt{f}\bar{\mathtt{f}}', \mathbb{V}\mathtt{f}\bar{\mathtt{f}}'$, and $\Lambda_c^+ \to p\mathtt{f}\bar{\mathtt{f}}'$ if kinematically allowed. We then arrive at the (differential) rates

$$\Gamma_{D^0\to\mathtt{f}\bar{\mathtt{f}}'} = \frac{\lambda^{1/2}\big(m^2_{D^0}, m^2_{\mathtt{f}}, m^2_{\mathtt{f}'}\big)}{8\pi\, m^3_{D^0}}\Big[|\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2\big(m^2_{D^0} - \tilde{\mu}^2_+\big)\tilde{\mu}^2_- + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2\big(m^2_{D^0} - \tilde{\mu}^2_-\big)\tilde{\mu}^2_+\Big] f^2_D\,,$$

$$\frac{d\Gamma_{D^0\to\gamma\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{\alpha_{\mathrm{e}}\,\tilde{\lambda}^{1/2}_{\mathtt{ff}'}\,\big(m^2_{D^0} - \hat{s}\big)^3}{192\pi^2\, m^5_{D^0}\,\hat{s}^2}\Big\{\Big[\Big(|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2 F^2_V + |\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2 F^2_A\Big)(3\hat{s} - \tilde{s}_+)\tilde{s}_- + \Big(|\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2 F^2_V + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2 F^2_A\Big)(3\hat{s} - \tilde{s}_-)\tilde{s}_+\Big]\Big\}\,,$$

$$\frac{d\Gamma_{\mathbb{D}\to\mathbb{P}\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{4\,\tilde{\lambda}^{1/2}_{\mathbb{DP}}\,\tilde{\lambda}^{1/2}_{\mathtt{ff}'}}{3(8\pi m_{\mathbb{D}}\,\hat{s})^3}\Big\{\Big[|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2(3\hat{s} - \tilde{s}_+)\tilde{s}_- + |\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2(3\hat{s} - \tilde{s}_-)\tilde{s}_+\Big]\tilde{\lambda}_{\mathbb{DP}}\, F^2_+ + 3\Big(|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2\,\tilde{\mu}^2_-\,\tilde{s}_+ + |\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2\,\tilde{\mu}^2_+\,\tilde{s}_-\Big) F^2_0\,\mathtt{m}^2_+\mathtt{m}^2_-\Big\}\,,$$

Predictions of the standard model

$$
\begin{aligned}
\frac{d\Gamma_{\mathbb{D}\to\mathbb{V}\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{4\,\tilde{\lambda}_{\mathbb{D}\mathbb{V}}^{3/2}\,\tilde{\lambda}_{\mathtt{ff}'}^{1/2}}{(8\pi\, m_{\mathbb{D}}\,\hat{s})^3}\,
&\Bigg\{\Big[|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_+)\widetilde{s}_- + |\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_-)\widetilde{s}_+\Big]\frac{2\hat{s}V^2}{3\,\widetilde{\mathtt{m}}_+^2} \\
&+\left[\frac{A_1^2\,\widetilde{\mathtt{m}}_+^2}{6m_{\mathbb{V}}^2}\left(\frac{1}{2}+\frac{6m_{\mathbb{V}}^2\hat{s}}{\tilde{\lambda}_{\mathbb{D}\mathbb{V}}}\right)+\frac{\tilde{\lambda}_{\mathbb{D}\mathbb{V}}A_2^2}{12\,\widetilde{\mathtt{m}}_+^2\,m_{\mathbb{V}}^2}+\frac{\hat{s}-\widetilde{\mathtt{m}}_+\widetilde{\mathtt{m}}_-}{6m_{\mathbb{V}}^2}A_1A_2\right] \\
&\quad\times\Big[|\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_+)\widetilde{s}_- + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_-)\widetilde{s}_+\Big] \\
&+\left(|\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2\widetilde{\mu}_-^2\,\widetilde{s}_+ + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2\widetilde{\mu}_+^2\,\widetilde{s}_-\right)A_0^2\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Gamma_{\Lambda_c^+\to p\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{4\,\tilde{\lambda}_{\Lambda_c p}^{1/2}\,\tilde{\lambda}_{\mathtt{ff}'}^{1/2}}{3(8\pi\, m_{\Lambda_c}\,\hat{s})^3}\,
&\Big\{\Big[|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_+)\widetilde{s}_- + |\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_-)\widetilde{s}_+\Big]\left(2f_\perp^2\,\hat{s}+f_+^2\mathtt{M}_+^2\right)\hat{\sigma}_- \\
&+3\left(|\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'}|^2\,\widetilde{\mu}_-^2\,\widetilde{s}_+ + |\mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}|^2\,\widetilde{\mu}_+^2\,\widetilde{s}_-\right)\hat{\sigma}_+f_0^2\,\mathtt{M}_-^2 \\
&+\Big[|\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_+)\widetilde{s}_- + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2(3\hat{s}-\widetilde{s}_-)\widetilde{s}_+\Big]\left(2g_\perp^2\,\hat{s}+g_+^2\,\mathtt{M}_-^2\right)\hat{\sigma}_+ \\
&+3\left(|\tilde{\mathrm{C}}^{\mathrm{V}}_{\mathtt{ff}'}|^2\,\widetilde{\mu}_-^2\,\widetilde{s}_+ + |\tilde{\mathrm{C}}^{\mathrm{A}}_{\mathtt{ff}'}|^2\,\widetilde{\mu}_+^2\,\widetilde{s}_-\right)\hat{\sigma}_-g_0^2\,\mathtt{M}_+^2\Big\},
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\mu}_\pm &= m_{\mathtt{f}} \pm m_{\mathtt{f}'}\,, & \mathtt{m}_\pm &= m_{\mathbb{D}} \pm m_{\mathbb{P}}\,, & \mathtt{M}_\pm &= m_{\Lambda_c} \pm m_p\,, \\
\widetilde{s}_\pm &= \hat{s} - \widetilde{\mu}_\pm^2\,, & \widetilde{\mathtt{m}}_\pm &= m_{\mathbb{D}} \pm m_{\mathbb{V}}\,, & \hat{\sigma}_\pm &= \mathtt{M}_\pm^2 - \hat{s}\,.
\end{aligned}
$$

Predictions of the standard model

In the SM, the FCNC decays with neutrinos arising from loop diagrams by the effective Hamiltonian

$$\mathcal{H}^{\rm SM}_{c\to u\nu\bar{\nu}} = \frac{\alpha_{\rm e} G_{\rm F}}{\sqrt{2}\,\pi\,\sin^2\theta_W} \sum_{\ell=e,\mu,\tau} \sum_{q=d,s,b} \hat{\lambda}_q\, \bar{D}(r_q, r_\ell)\; \bar{u}\gamma^\eta P_L c \overline{\nu_\ell}\gamma_\eta P_L \nu_\ell\,,$$

where $r_f = m_f^2/m_W^2$ and with the loop function

$$\bar{D}(x,y) = \frac{x(4-y)^2}{8(1-y)^2}\,\frac{y\ln y}{x-y} + \frac{x(4-x)^2}{8(1-x)^2}\,\frac{x\ln x}{y-x} + \frac{4-2x+x^2}{8(1-x)^2}x\ln x - \frac{4+2x+5y-2xy}{8(1-x)(1-y)}x\,.$$

Accordingly, with $\mathtt{f} = \mathtt{f}' = \nu_\ell$ and $m_{\mathtt{f}} = m_{\mathtt{f}'} = 0$,

$$\mathtt{C}^{\rm V}_{\mathtt{ff}'} = -\mathtt{C}^{\rm A}_{\mathtt{ff}'} = -\tilde{\rm C}^{\rm V}_{\mathtt{ff}'} = \tilde{\rm C}^{\rm A}_{\mathtt{ff}'} = \sum_{q=d,s,b} \frac{\alpha_{\rm e} G_{\rm F} \hat{\lambda}_q \bar{D}(r_q, r_\ell)}{4\sqrt{2}\,\pi\,\sin^2\theta_W}\,.$$

We obtained

$$\mathcal{B}(D^0\to\nu\bar{\nu}) = 0\,,\qquad \mathcal{B}(D^0\to\gamma\nu\bar{\nu}) = 1.8\times10^{-19}\,,$$

$$\mathcal{B}(D^0\to\pi^0\nu\bar{\nu}) = 2.5\times10^{-17}\,,\qquad \mathcal{B}(D^0\to\rho^0\nu\bar{\nu}) = 1.1\times10^{-17}\,,$$

$$\mathcal{B}(D^+\to\pi^+\nu\bar{\nu}) = 1.3\times10^{-16}\,,\qquad \mathcal{B}(D^+\to\rho^+\nu\bar{\nu}) = 5.9\times10^{-17}\,,$$

$$\mathcal{B}(D_s^+\to K^+\nu\bar{\nu}) = 4.5\times10^{-17}\,,\qquad \mathcal{B}(D_s^+\to K^{*+}\nu\bar{\nu}) = 3.3\times10^{-17}\,,$$

$$\mathcal{B}(\Lambda_c^+\to p\nu\bar{\nu}) = 7.3\times10^{-17}\,,\qquad \mathcal{B}(\Xi_c^+\to \Sigma^+\nu\bar{\nu}) = 1.1\times10^{-16}\,,$$

$$\mathcal{B}(\Xi_c^0\to\Sigma^0\nu\bar{\nu}) = 1.8\times10^{-17}\,,\qquad \mathcal{B}(\Xi_c^0\to\Lambda\nu\bar{\nu}) = 6.5\times10^{-18}\,.$$

Prediction of invisible fermions

The allowed $|\mathrm{K}_{\mathrm{NN'}}|$ range for each $(m_{\mathrm{N}}, m_{\mathrm{N'}})$ pair is below the lowest curve.

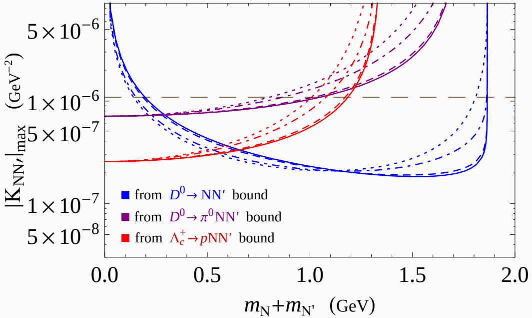


Figure 9: The upper limits on $|\mathrm{K}_{\mathrm{NN'}}|$ versus $m_{\mathrm{N}} + m_{\mathrm{N'}}$, with $\mathrm{N} = \mathrm{N}_2$ and $\mathrm{N'} = \mathrm{N}_1$, implied by the $D^0 \to \mathrm{N}\bar{\mathrm{N}}'$ (blue), $D^0 \to \pi^0\mathrm{N}\bar{\mathrm{N}}'$ (purple), and $\Lambda_c^+ \to p\mathrm{N}\bar{\mathrm{N}}'$ (red) bounds for $m_{\mathrm{N'}}/m_{\mathrm{N}} = 0.001$ (dotted curves), 0.1 (dash-dotted curves), 0.5 (dashed curves), 1 (solid curves). The horizontal brown dashed line marks $|\mathrm{K}_{\mathrm{NN'}}| < 1.1\ \mathrm{TeV}^{-2}$ inferred from collider and perturbativity restrictions.

FCNC charm decay with invisible singlet fermions

The leptoquark $\bar{S}_1$ transforms as $(\bar{3}, 1, -2/3)$ under the SM gauge groups $\mathrm{SU}(3)_{\rm color} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. We write the Lagrangian for the renormalizable interaction of $\bar{S}_1$ with $\mathtt{N}_{1,2,3}$ and the quarks as

$$\mathcal{L}_{\rm LQ} = \bar{\mathtt{Y}}_{jl} \overline{\mathcal{U}_j^{\rm c}} P_R \mathtt{N}_l \bar{S}_1 + {\rm H.c.}\,,$$

where $\bar{\mathtt{Y}}_{jl}$ are generally complex elements of the LQ Yukawa matrix $\bar{\mathtt{Y}}$, $\mathcal{U}_{1,2,3} = (u, c, t)$.

The effective Lagrangian for $c \to u\mathtt{N}\bar{\mathtt{N}}'$

$$\mathcal{L}_{\mathtt{ff}'} = -\bar{u}\gamma^\mu c\, \bar{\mathtt{f}}\gamma_\mu \left(\mathtt{C}^{\mathtt{V}}_{\mathtt{ff}'} + \gamma_5 \mathtt{C}^{\mathtt{A}}_{\mathtt{ff}'}\right)\mathtt{f}' - \bar{u}\gamma^\mu\gamma_5 c\, \bar{\mathtt{f}}\gamma_\mu \left(\tilde{\rm C}^{\rm V}_{\mathtt{ff}'} + \gamma_5 \tilde{\rm C}^{\rm A}_{\mathtt{ff}'}\right)\mathtt{f}'\,,$$

with the coefficients being given by

$$\mathtt{C}^{\mathtt{V}}_{\mathtt{N}_j\mathtt{N}_l} = \mathtt{C}^{\mathtt{A}}_{\mathtt{N}_j\mathtt{N}_l} = \tilde{\rm C}^{\rm V}_{\mathtt{N}_j\mathtt{N}_l} = \tilde{\rm C}^{\rm A}_{\mathtt{N}_j\mathtt{N}_l} = -\frac{\bar{\mathtt{Y}}^*_{1j}\bar{\mathtt{Y}}_{2l}}{8m^2_{\bar{S}_1}}\,.$$

To evade the stringent restrictions from D^0-$\bar{D}^0$ mixing, we choose one of the simplest examples,

$$\bar{\mathtt{Y}} = \begin{pmatrix} 0 & \bar{\mathtt{y}}_{u2} & 0 \\ \bar{\mathtt{y}}_{c1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

in which case only $c \to u\mathtt{N}_2\bar{\mathtt{N}}_1$ can occur with

$$\mathtt{C}^{\mathtt{V}}_{\mathtt{N}_2\mathtt{N}_1} = \mathtt{C}^{\mathtt{A}}_{\mathtt{N}_2\mathtt{N}_1} = \tilde{\rm C}^{\rm V}_{\mathtt{N}_2\mathtt{N}_1} = \tilde{\rm C}^{\rm A}_{\mathtt{N}_2\mathtt{N}_1} = -\frac{\bar{\mathtt{y}}^*_{u2}\bar{\mathtt{y}}_{c1}}{8m^2_{\bar{S}_1}} \equiv {\rm K}_{\mathtt{NN}'}\,,$$

Prediction of invisible fermions

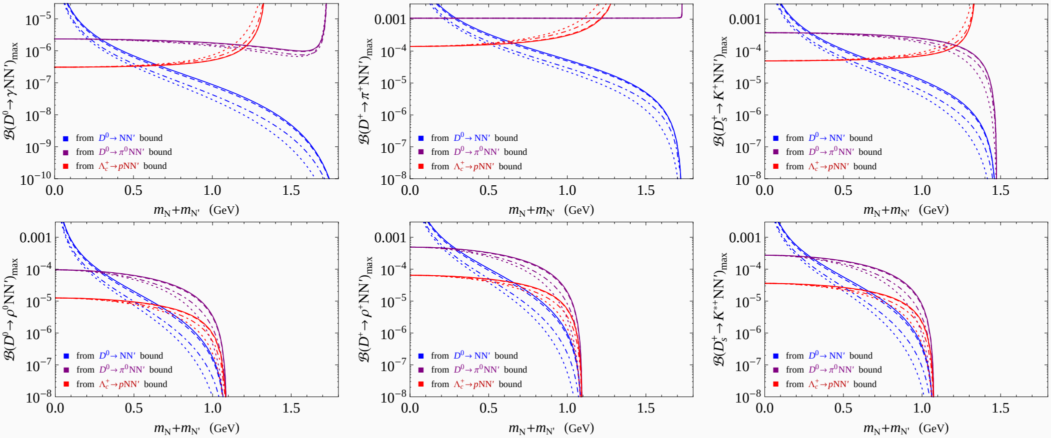


Figure 10: Maximal branching fractions of $D^0 \to (\gamma, \rho^0)\mathrm{N}\bar{\mathrm{N}}'$, $D^+ \to (\pi^+, \rho^+)\mathrm{N}\bar{\mathrm{N}}'$, and $D_s^+ \to (K^+, K^{*+})\mathrm{N}\bar{\mathrm{N}}'$ translated from the $|\mathrm{K}_{\mathrm{NN}'}|_{\max}$ values in Fig. 9 inferred from the $D^0 \to \mathrm{N}\bar{\mathrm{N}}'$ (blue), $D^0 \to \pi^0\mathrm{N}\bar{\mathrm{N}}'$ (purple), and $\Lambda_c^+ \to p\mathrm{N}\bar{\mathrm{N}}'$ (red) bounds.

Prediction of invisible fermions

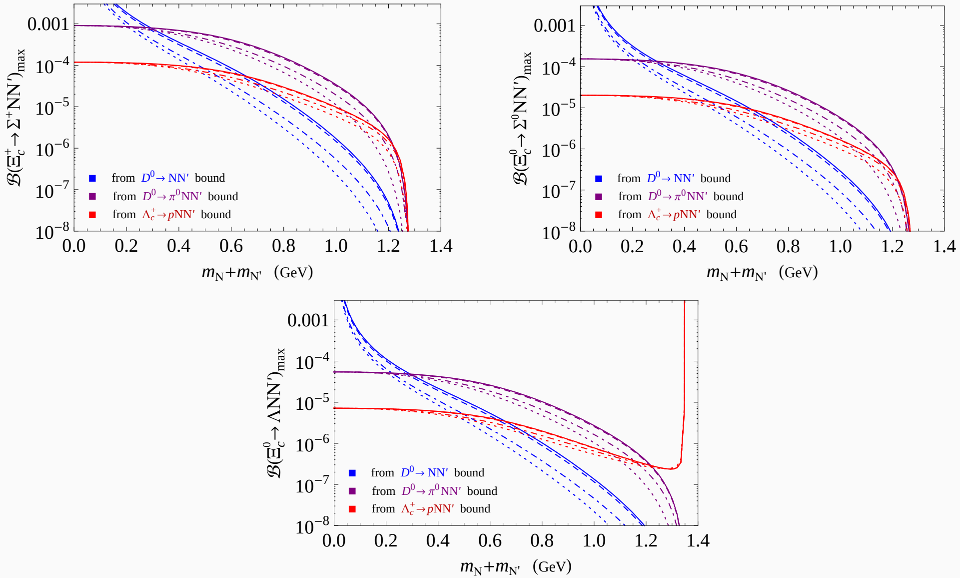


Figure 11: The same as Fig. 9 but for $\Xi_c^{+,0} \to \Sigma^{+,0} N\bar{N}'$ and $\Xi_c^0 \to \Lambda N\bar{N}'$.

Prediction of invisible fermions

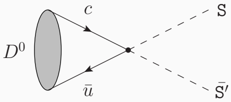
Table 3: The upper limits on the branching fractions, in units of 10^{-5}, of various charmed-hadron decays induced by the $c \to u\mathrm{N}\bar{\mathrm{N}}'$ interactions and evaluated with the lowest $|\mathrm{K}_{\mathrm{NN}'}|_{\max}$ for $m_{\mathrm{N}'} = m_{\mathrm{N}} = 0$ and $m_{\mathrm{N}'} = 0.1\, m_{\mathrm{N}} = 0.05$ GeV if the $\Lambda_c^+ \to p\mathrm{N}\bar{\mathrm{N}}'$ bound is absent and, in brackets, if it is included and the strongest.

Decay modes	$m_{\mathrm{N}'} = m_{\mathrm{N}} = 0$	$m_{\mathrm{N}'} = 0.1\, m_{\mathrm{N}} = 0.05$ GeV
$D^0 \to \mathrm{N}\bar{\mathrm{N}}'$	-	9.4 [Input]
$D^0 \to \gamma\mathrm{N}\bar{\mathrm{N}}'$	0.15 (0.020)	0.021
$D^0 \to \pi^0\mathrm{N}\bar{\mathrm{N}}'$	21 [Input] (2.8)	3.1
$D^+ \to \pi^+\mathrm{N}\bar{\mathrm{N}}'$	107 (14)	16
$D_s^+ \to K^+\mathrm{N}\bar{\mathrm{N}}'$	38 (4.9)	4.9
$D^0 \to \rho^0\mathrm{N}\bar{\mathrm{N}}'$	9.6 (1.3)	0.68
$D^+ \to \rho^+\mathrm{N}\bar{\mathrm{N}}'$	49 (6.4)	3.5
$D_s^+ \to K^{*+}\mathrm{N}\bar{\mathrm{N}}'$	27 (3.6)	1.9
$\Lambda_c^+ \to p\mathrm{N}\bar{\mathrm{N}}'$	61 (8.0 [Input])	7.2
$\Xi_c^+ \to \Sigma^+\mathrm{N}\bar{\mathrm{N}}'$	91 (12)	5.4
$\Xi_c^0 \to \Sigma^0\mathrm{N}\bar{\mathrm{N}}'$	15 (2.0)	0.91
$\Xi_c^0 \to \Lambda\mathrm{N}\bar{\mathrm{N}}'$	5.5 (0.71)	0.34

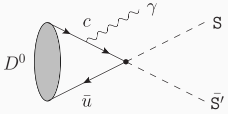
Effective Lagrangian with invisible scalars

The leading-order low-energy effective $|\Delta C| = 1$ operators containing invisible bosons are

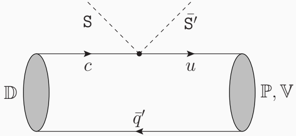
$$\mathcal{L}_{\mathrm{SS}'} = -\left(\kappa^{\mathrm{V}}_{\mathrm{SS}'}\overline{u}\gamma_\mu c + \kappa^{\mathrm{A}}_{\mathrm{SS}'}\overline{u}\gamma_\mu\gamma_5 c\right) i\left(\mathrm{S}^\dagger\partial^\mu\mathrm{S}' - \partial^\mu\mathrm{S}^\dagger\mathrm{S}'\right) - \left(\kappa^{\mathrm{S}}_{\mathrm{SS}'}\overline{u}c + \kappa^{\mathrm{P}}_{\mathrm{SS}'}\overline{u}\gamma_5 c\right) m_c\,\mathrm{S}^\dagger\mathrm{S}' + \mathrm{H.c.}\,,$$



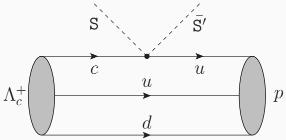
(a) Fully invisible decay



(b) Semi-invisible radiative decay



(c) Semi-invisible mesonic decay



(d) Semi-invisible baryonic decay

Figure 12: Diagrams of FCNC charmed-hadron decays with two invisible light spin-0 bosons.

(a) Fully invisible decay

The amplitude for the invisible channel $D^0 \to \mathrm{S}\bar{\mathrm{S}}'$ can be expressed as

$$\mathcal{M}_{D^0\to \mathrm{S}\bar{\mathrm{S}}'} = \kappa^{\mathrm{A}}_{\mathrm{SS}'} \langle 0|\bar{u}\gamma^\mu\gamma_5 c|D^0\rangle (\mathrm{k}-\mathrm{k}')_\mu + \kappa^{\mathrm{P}}_{\mathrm{SS}'}\, m_c \langle 0|\bar{u}\gamma_5 c|D^0\rangle\,,$$

with the mesonic matrix elements

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 c|D^0\rangle = -if_D\, p^\mu_{D^0}\,, \qquad \langle 0|\bar{u}\gamma_5 c|D^0\rangle = \frac{if_D\, m^2_{D^0}}{m_u+m_c}\,,$$

There are no contributions of $\kappa^{\mathrm{S}}_{\mathrm{SS}'}$ and $\kappa^{\mathrm{V}}_{\mathrm{SS}'}$ because $\langle 0|\bar{u}\gamma^\mu c|D^0\rangle = \langle 0|\bar{u}c|D^0\rangle = 0$.
Neglecting m_u compared to m_c then leads to

$$\mathcal{M}_{D^0\to \mathrm{S}\bar{\mathrm{S}}'} = i\left[\kappa^{\mathrm{A}}_{\mathrm{SS}'}\left(m^2_{\mathrm{S}'}-m^2_{\mathrm{S}}\right) + \kappa^{\mathrm{P}}_{\mathrm{SS}'} m^2_{D^0}\right] f_D\,.$$

$$\Gamma_{D^0\to \mathrm{S}\bar{\mathrm{S}}'} = \frac{\lambda^{1/2}\left(m^2_{D^0}, m^2_{\mathrm{S}}, m^2_{\mathrm{S}'}\right)}{16\pi m^3_{D^0}} \left|\kappa^{\mathrm{A}}_{\mathrm{SS}'}\left(m^2_{\mathrm{S}'}-m^2_{\mathrm{S}}\right) + \kappa^{\mathrm{P}}_{\mathrm{SS}'} m^2_{D^0}\right|^2 f_D^2\,,$$

which contains the Källén function

$$\lambda(x,y,z) = x^2+y^2+z^2-2(xy+xz+yz)\,.$$

Evidently, $D^0 \to \mathrm{S}\bar{\mathrm{S}}'$ can in general probe $\kappa^{\mathrm{P}}_{\mathrm{SS}'}$ and $\kappa^{\mathrm{A}}_{\mathrm{SS}'}$, which accompany the parity-odd quark bilinears in $\mathcal{L}_{\mathrm{SS}'}$, but the sensitivity to $\kappa^{\mathrm{A}}_{\mathrm{SS}'}$ will be lost if $m_{\mathrm{S}'} = m_{\mathrm{S}}$.

(b) Semi-invisible radiative decay

The amplitude for $D^0 \to \gamma \mathrm{S}\bar{\mathrm{S}}'$ is

$$\mathcal{M}_{D^0\to\gamma \mathrm{S}\bar{\mathrm{S}}'} = \kappa^{\mathrm{V}}_{\mathrm{SS}'} \langle\gamma|\bar{u}\gamma_\mu c|D^0\rangle (\mathrm{k}-\mathrm{k}')^\mu + \kappa^{\mathrm{A}}_{\mathrm{SS}'} \langle\gamma|\bar{u}\gamma^\mu\gamma_5 c|D^0\rangle (\mathrm{k}-\mathrm{k}')_\mu \,,$$

where $\mathrm{k}^{(\prime)}$ designates the momentum of $\mathrm{S}^{(\prime)}$ and

$$\langle\gamma|\bar{u}\gamma_\mu c|D^0\rangle = \frac{eF_V}{m_{D^0}} \epsilon_{\mu\zeta\eta\theta}\, \varepsilon^{\zeta *}_\gamma\, p^\eta_{D^0}\, p^\theta_\gamma \,, \quad \langle\gamma|\bar{u}\gamma^\mu\gamma_5 c|D^0\rangle = \frac{ieF_A}{m_{D^0}} \left(p_\gamma \cdot p_{D^0}\, \varepsilon^{\mu *}_\gamma - \varepsilon^*_\gamma \cdot p_{D^0}\, p^\mu_\gamma\right) ,$$

Since $\langle\gamma|\bar{u}c|D^0\rangle = \langle\gamma|\bar{u}\gamma_5 c|D^0\rangle = 0,$ there are no $\kappa^{\mathrm{S,P}}_{\mathrm{SS}'}$ terms.

Evaluating the absolute square of the amplitude times the three-body phase space yields the differential rate

$$\frac{d\Gamma_{D^0\to\gamma \mathrm{S}\bar{\mathrm{S}}'}}{d\hat{s}} = \frac{\alpha_{\mathrm{e}}\, \lambda^{3/2}\left(\hat{s}, m^2_{\mathrm{S}}, m^2_{\mathrm{S}'}\right)}{384\pi^2\, m^5_{D^0}\, \hat{s}^2} \left(m^2_{D^0} - \hat{s}\right)^3 \left(|\kappa^{\mathrm{V}}_{\mathrm{SS}'}|^2 F^2_V + |\kappa^{\mathrm{A}}_{\mathrm{SS}'}|^2 F^2_A\right) ,$$

which is to be integrated over $(m_{\mathrm{S}} + m_{\mathrm{S}'})^2 \le \hat{s} \le m^2_{D^0}.$ Thus, the invisible scalars' mass range covered by this mode is $0 \le m_{\mathrm{S}} + m_{\mathrm{S}'} < m_{D^0},$ the same as that in the $D^0 \to \mathrm{S}\bar{\mathrm{S}}'$ case.

(c) Semi-invisible mesonic decays

Initial mesons $\mathbb{D} = D^0, D^+, D_s^+$, and the final mesons $\mathbb{P} = \pi^0, \pi^+, K^+$ or $\mathbb{V} = \rho^0, \rho^+, K^{*+}$.

The amplitudes are

$$\mathcal{M}_{\mathbb{D}\to\mathbb{P}\mathtt{S}\bar{\mathtt{S}}'} = \kappa^{\rm V}_{\mathtt{SS}'} \langle\mathbb{P}|\bar{u}\gamma^\mu c|\mathbb{D}\rangle(\mathtt{k}-\mathtt{k}')_\mu + \kappa^{\rm S}_{\mathtt{SS}'}\, m_c \langle\mathbb{P}|\bar{u}c|\mathbb{D}\rangle\,,$$

$$\mathcal{M}_{\mathbb{D}\to\mathbb{V}\mathtt{S}\bar{\mathtt{S}}'} = \kappa^{\rm V}_{\mathtt{SS}'} \langle\mathbb{V}|\bar{u}\gamma_\mu c|\mathbb{D}\rangle(\mathtt{k}-\mathtt{k}')^\mu + \kappa^{\rm A}_{\mathtt{SS}'} \langle\mathbb{V}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle(\mathtt{k}-\mathtt{k}')_\mu + \kappa^{\rm P}_{\mathtt{SS}'}\, m_c \langle\mathbb{V}|\bar{u}\gamma_5 c|\mathbb{D}\rangle\,,$$

which involve the momentum $\mathtt{k}^{(\prime)}$ of $\mathtt{S}^{(\prime)}$ and the mesonic matrix elements

$$\langle\mathbb{P}|\bar{u}\gamma^\mu c|\mathbb{D}\rangle = \left(p^\mu_{\mathbb{D}} + p^\mu_{\mathbb{P}}\right) f_+ + \left(p^\mu_{\mathbb{D}} - p^\mu_{\mathbb{P}}\right)\left(f_0 - f_+\right)\frac{m^2_{\mathbb{D}} - m^2_{\mathbb{P}}}{q^2_{\mathbb{DP}}}\,, \qquad \langle\mathbb{P}|\bar{u}c|\mathbb{D}\rangle = \frac{m^2_{\mathbb{D}} - m^2_{\mathbb{P}}}{m_c - m_u} f_0\,,$$

$$\langle\mathbb{V}|\bar{u}\gamma_\mu c|\mathbb{D}\rangle = \frac{2V}{m_{\mathbb{D}} + m_{\mathbb{V}}}\,\epsilon_{\mu\beta\eta\theta}\,\varepsilon^{\beta*}_{\mathbb{V}}\,p^\eta_{\mathbb{V}}\,p^\theta_{\mathbb{D}}\,,$$

$$\langle\mathbb{V}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle = i(m_{\mathbb{D}} + m_{\mathbb{V}})\varepsilon^{\mu*}_{\mathbb{V}} A_1 - \left[\frac{p^\mu_{\mathbb{D}} + p^\mu_{\mathbb{V}}}{m_{\mathbb{D}} + m_{\mathbb{V}}} A_2 + \frac{p^\mu_{\mathbb{D}} - p^\mu_{\mathbb{V}}}{q^2_{\mathbb{DV}}}(A_3 - A_0) 2m_{\mathbb{V}}\right] i\varepsilon^*_{\mathbb{V}}\cdot p_{\mathbb{D}}\,,$$

$$\langle\mathbb{V}|\bar{u}\gamma_5 c|\mathbb{D}\rangle = \frac{-2iA_0\, m_{\mathbb{V}}}{m_c + m_u}\,\varepsilon^*_{\mathbb{V}}\cdot p_{\mathbb{D}}\,,$$

where f_+ and f_0 [V, A_0, A_1, and A_2] are form factors.

(c) Semi-invisible mesonic decays

Accordingly, from the absolute squares of the amplitudes, we arrive at

$$\frac{d\Gamma_{\mathbb{D}\to\mathbb{P}\mathrm{S}\bar{\mathrm{S}}'}}{d\hat{s}} = \frac{2\tilde{\lambda}_{\mathbb{DP}}^{1/2}\tilde{\lambda}_{\mathrm{SS}'}^{1/2}}{(8\pi m_{\mathbb{D}}\hat{s})^3}\left[\frac{1}{3}|\kappa_{\mathrm{SS}'}^{\mathrm{V}}|^2\tilde{\lambda}_{\mathbb{DP}}\tilde{\lambda}_{\mathrm{SS}'}f_+^2 + \left|\kappa_{\mathrm{SS}'}^{\mathrm{V}}\left(m_{\mathrm{S}}^2 - m_{\mathrm{S}'}^2\right) + \kappa_{\mathrm{SS}'}^{\mathrm{S}}\hat{s}\right|^2\left(m_{\mathbb{D}}^2 - m_{\mathbb{P}}^2\right)^2 f_0^2\right],$$

$$\frac{d\Gamma_{\mathbb{D}\to\mathbb{V}\mathrm{S}\bar{\mathrm{S}}'}}{d\hat{s}} = \frac{\tilde{\lambda}_{\mathbb{DV}}^{3/2}\tilde{\lambda}_{\mathrm{SS}'}^{3/2}}{(8\pi m_{\mathbb{D}}\hat{s})^3}\left\{\frac{|\kappa_{\mathrm{SS}'}^{\mathrm{A}}|^2}{6\,m_{\mathbb{V}}^2}\left[\left(1+\frac{12\,m_{\mathbb{V}}^2\hat{s}}{\tilde{\lambda}_{\mathbb{DV}}}\right)A_1^2\,\widetilde{\mathtt{m}}_+^2 + 2(\hat{s}-\widetilde{\mathtt{m}}_+\widetilde{\mathtt{m}}_-)A_1A_2 + \frac{\tilde{\lambda}_{\mathbb{DV}}A_2^2}{\widetilde{\mathtt{m}}_+^2}\right] + \frac{2A_0^2}{\tilde{\lambda}_{\mathrm{SS}'}}\left|\kappa_{\mathrm{SS}'}^{\mathrm{A}}\left(m_{\mathrm{S}'}^2 - m_{\mathrm{S}}^2\right) + \kappa_{\mathrm{SS}'}^{\mathrm{P}}\,\hat{s}\right|^2 + \frac{4|\kappa_{\mathrm{SS}'}^{\mathrm{V}}|^2\hat{s}V^2}{3\,\widetilde{\mathtt{m}}_+^2}\right\},$$

to be integrated over $(m_{\mathrm{S}} + m_{\mathrm{S}'})^2 \le \hat{s} = (\mathtt{k}+\mathtt{k}')^2 \le (m_{\mathbb{D}} - m_{\mathbb{P},\mathbb{V}})^2$, respectively, with

$$\tilde{\lambda}_{\mathtt{X}Y} = \lambda\left(m_{\mathtt{X}}^2, m_{\mathtt{Y}}^2, \hat{s}\right), \qquad \widetilde{\mathtt{m}}_\pm = m_{\mathbb{D}} \pm m_{\mathbb{V}}\,.$$

$\mathbb{D}\to\mathbb{P}\mathrm{S}\bar{\mathrm{S}}'$ can probe not only $\kappa_{\mathrm{SS}'}^{\mathrm{V}}$ but also $\kappa_{\mathrm{SS}'}^{\mathrm{S}}$, which is inaccessible to $D^0\to\mathrm{S}\bar{\mathrm{S}}', \gamma\mathrm{S}\bar{\mathrm{S}}'$ as well as to $\mathbb{D}\to\mathbb{V}\mathrm{S}\bar{\mathrm{S}}'$. However, the latter is sensitive to the other three parameters, $\kappa_{\mathrm{SS}'}^{\mathrm{V,P,A}}$.

(d) Semi-invisible baryonic decays

Of interest here are $\Lambda_c^+ \to p\mathrm{S}\bar{\mathrm{S}}'$ and $\Xi_c^{+,0} \to \Sigma^{+,0}\mathrm{S}\bar{\mathrm{S}}'$ plus $\Xi_c^0 \to \Lambda\mathrm{S}\bar{\mathrm{S}}'$,

$$\begin{aligned}\mathcal{M}_{\Lambda_c^+\to p\mathrm{S}\bar{\mathrm{S}}'} &= \kappa^{\mathrm{V}}_{\mathrm{SS}'}\,\langle p|\bar{u}\gamma^\mu c|\Lambda_c^+\rangle(\mathtt{k}-\mathtt{k}')_\mu + \kappa^{\mathrm{A}}_{\mathrm{SS}'}\,\langle p|\bar{u}\gamma^\mu\gamma_5 c|\Lambda_c^+\rangle(\mathtt{k}-\mathtt{k}')_\mu \\ &\quad + \kappa^{\mathrm{S}}_{\mathrm{SS}'}\, m_c\,\langle p|\bar{u}c|\Lambda_c^+\rangle + \kappa^{\mathrm{P}}_{\mathrm{SS}'}\, m_c\,\langle p|\bar{u}\gamma_5 c|\Lambda_c^+\rangle\,,\end{aligned}$$

where $\mathtt{k}^{(\prime)}$ is again the momentum of $\mathrm{S}^{(\prime)}$.

The baryonic matrix elements are expressible as

$$\begin{aligned}\langle p|\bar{u}\gamma^\mu c|\Lambda_c^+\rangle &= \bar{u}_p\Bigg\{\Bigg[\gamma^\mu - \frac{\mathtt{M}_+\hat{p}^\mu - \mathtt{M}_-\hat{q}^\mu}{\mathtt{M}_+^2 - \hat{q}^2}\Bigg]\mathtt{F}_\perp + \Bigg[\hat{p}^\mu - \frac{\mathtt{M}_+\mathtt{M}_-\hat{q}^\mu}{\hat{q}^2}\Bigg]\frac{\mathtt{M}_+\,\mathtt{F}_+}{\mathtt{M}_+^2-\hat{q}^2} + \frac{\mathtt{M}_-\hat{q}^\mu}{\hat{q}^2}\mathtt{F}_0\Bigg\}u_{\Lambda_c}\,, \\ \langle p|\bar{u}\gamma^\mu\gamma_5 c|\Lambda_c^+\rangle &= \bar{u}_p\Bigg\{\Bigg[\gamma^\mu + \frac{\mathtt{M}_-\hat{p}^\mu - \mathtt{M}_+\hat{q}^\mu}{\mathtt{M}_-^2 - \hat{q}^2}\Bigg]\mathtt{G}_\perp - \Bigg[\hat{p}^\mu - \frac{\mathtt{M}_+\mathtt{M}_-\hat{q}^\mu}{\hat{q}^2}\Bigg]\frac{\mathtt{M}_-\,\mathtt{G}_+}{\mathtt{M}_-^2-\hat{q}^2} - \frac{\mathtt{M}_+\hat{q}^\mu}{\hat{q}^2}\mathtt{G}_0\Bigg\}\gamma_5 u_{\Lambda_c}\,, \\ \langle p|\bar{u}c|\Lambda_c^+\rangle &= \frac{\mathtt{M}_-\,\mathtt{F}_0}{m_c - m_u}\,\bar{u}_p u_{\Lambda_c}\,, \qquad \langle p|\bar{u}\gamma_5 c|\Lambda_c^+\rangle = \frac{\mathtt{M}_+\,\mathtt{G}_0}{m_c + m_u}\,\bar{u}_p\gamma_5 u_{\Lambda_c}\,,\end{aligned}$$

where u_p and u_{Λ_c} designate the Dirac spinors of the baryons, $\mathtt{F}_{\perp,+,0}$ and $\mathtt{G}_{\perp,+,0}$ symbolize form factors which depend on $\hat{s} = \hat{q}^2$,

$$\mathtt{M}_\pm = m_{\Lambda_c} \pm m_p\,, \qquad \hat{p} = p_{\Lambda_c} + p_p\,, \qquad \hat{q} = p_{\Lambda_c} - p_p\,.$$

(d) Semi-invisible baryonic decays

After averaging (summing) the absolute square of the amplitude over the initial (final) baryon polarizations and multiplying by the three-body phase space, we find the differential rate

$$\frac{d\Gamma_{\Lambda_c^+\to p\mathrm{S}\bar{\mathrm{S}}'}}{d\hat{s}} = \frac{2\tilde{\lambda}_{\Lambda_c p}^{1/2}\,\tilde{\lambda}_{\mathrm{SS}'}^{1/2}}{3(8\pi m_{\Lambda_c}\hat{s})^3}\left\{\left[|\kappa_{\mathrm{SS}'}^{\mathrm{V}}|^2\left(2\mathtt{F}_\perp^2\,\hat{s}+\mathtt{F}_+^2\mathtt{M}_+^2\right)\hat{\sigma}_- + |\kappa_{\mathrm{SS}'}^{\mathrm{A}}|^2\left(2\mathtt{G}_\perp^2\,\hat{s}+\mathtt{G}_+^2\mathtt{M}_-^2\right)\hat{\sigma}_+\right]\tilde{\lambda}_{\mathrm{SS}'} \right.$$
$$+\,3\left|\kappa_{\mathrm{SS}'}^{\mathrm{V}}\left(m_{\mathrm{S}}^2-m_{\mathrm{S}'}^2\right)+\kappa_{\mathrm{SS}'}^{\mathrm{S}}\hat{s}\right|^2\hat{\sigma}_+\,\mathtt{M}_-^2\,\mathtt{F}_0^2$$
$$\left. +\,3\left|\kappa_{\mathrm{SS}'}^{\mathrm{A}}\left(m_{\mathrm{S}'}^2-m_{\mathrm{S}}^2\right)+\kappa_{\mathrm{SS}'}^{\mathrm{P}}\hat{s}\right|^2\hat{\sigma}_-\,\mathtt{M}_+^2\,\mathtt{G}_0^2\right\},$$

where $\hat{\sigma}_\pm = \mathtt{M}_\pm^2 - \hat{s}$. It is to be integrated over $(m_{\mathrm{S}}+m_{\mathrm{S}'})^2 \le \hat{s} \le (m_{\Lambda_c}-m_p)^2$.

Numerical results for decays induced by $c \to u\mathrm{S}\bar{\mathrm{S}}'$

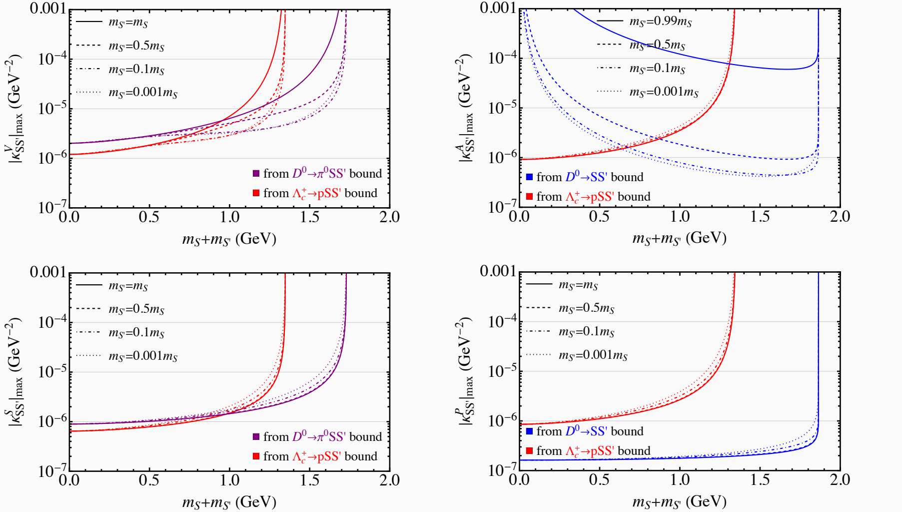


Figure 13: The upper limits on $|\kappa^{\mathrm{V}}_{\mathrm{SS}'}|$ (top left), $|\kappa^{\mathrm{A}}_{\mathrm{SS}'}|$ (top right), $|\kappa^{\mathrm{S}}_{\mathrm{SS}'}|$ (bottom left), and $|\kappa^{\mathrm{P}}_{\mathrm{SS}'}|$ (bottom right) versus $m_{\mathrm{S}} + m_{\mathrm{S}'}$ obtained from the $D^0 \to \mathrm{S}\bar{\mathrm{S}}'$ (blue), $D^0 \to \pi^0\mathrm{S}\bar{\mathrm{S}}'$ (purple), and $\Lambda_c^+ \to p\mathrm{S}\bar{\mathrm{S}}'$ (red) limits for various $m_{\mathrm{S}'}/m_{\mathrm{S}}$ values if only one of $\kappa^{\mathrm{V,A,S,P}}_{\mathrm{SS}'}$ is nonzero at a time.

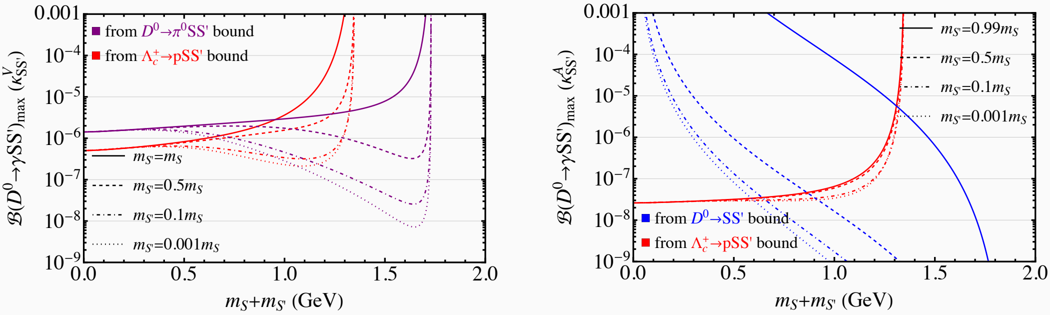


Figure 14: The maximal branching fraction of $D^0 \to \gamma S\bar{S}'$ due to $|\kappa^{V}_{SS'}|_{\max}$ (left) or $|\kappa^{A}_{SS'}|_{\max}$ (right) alone for various $m_{S'}/m_S$ choices.

Prediction of invisible scalars

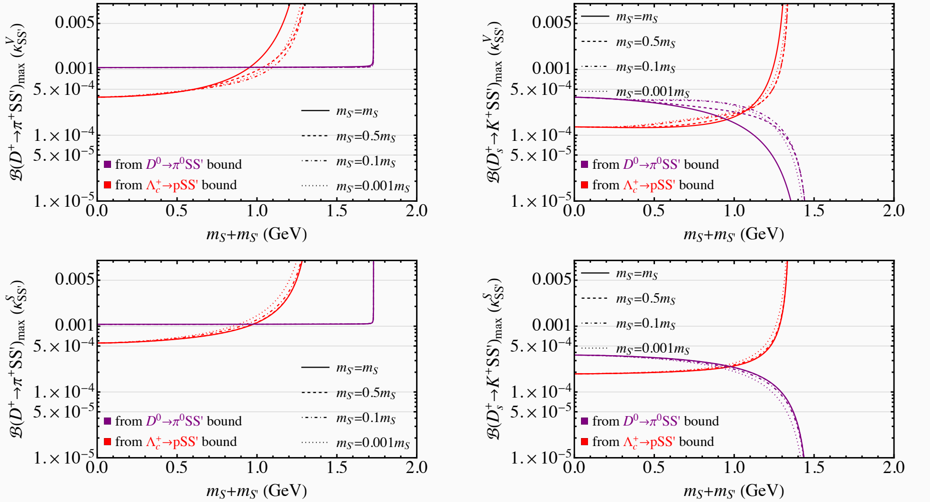


Figure 15: The maximal branching fractions of $D^+ \to \pi^+ S\bar{S}'$ (left column) and $D_s^+ \to K^+ S\bar{S}'$ (right column) due to $|\kappa^{V}_{SS'}|_{\max}$ (top row) or $|\kappa^{S}_{SS'}|_{\max}$ (bottom row) alone for different $m_{S'}/m_S$ values.

Prediction of invisible scalars

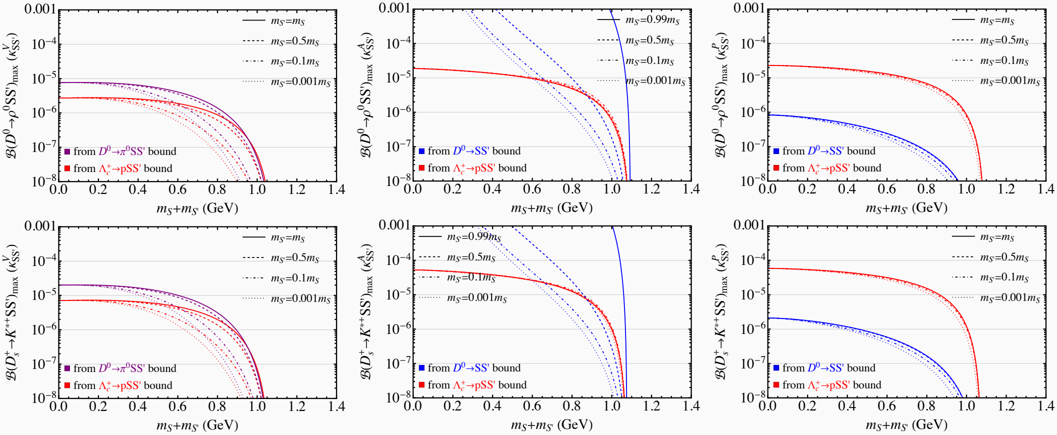


Figure 16: The maximal branching fractions of $D^0 \to \rho^0 S\bar{S}'$ and $D_s^+ \to K^{*+} S\bar{S}'$ due to $|\kappa^{V}_{SS'}|_{\max}$ or $|\kappa^{A}_{SS'}|_{\max}$ (middle row) or $|\kappa^{P}_{SS'}|_{\max}$ alone. The $D^+ \to \rho^+ S\bar{S}'$ curves, not displayed, are approximately $2\tau_{D^+}/\tau_{D^0} \sim 5$ times their $D^0 \to \rho^0 S\bar{S}'$ counterparts.

Prediction of invisible scalars

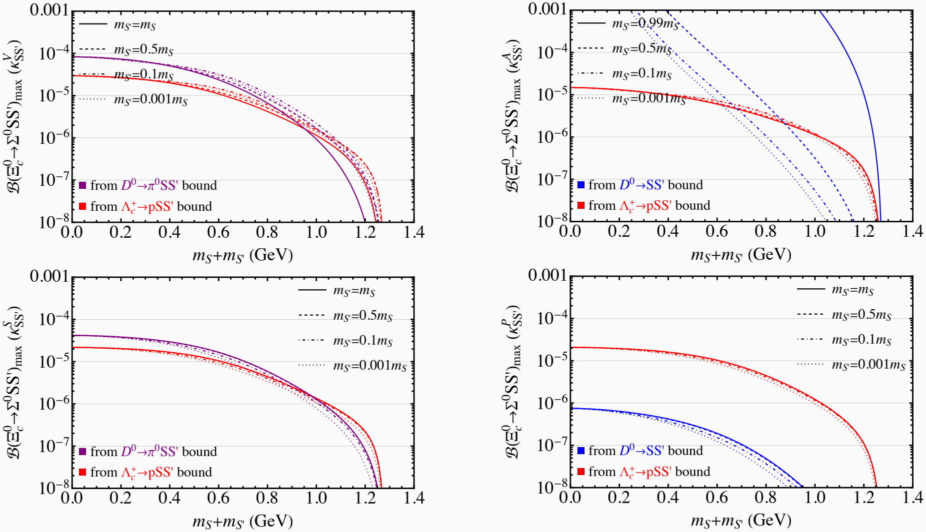


Figure 17: The maximal branching fractions of $\Xi_c^0 \to \Sigma^0 S\bar{S}'$ due to $|\kappa^{\rm V}_{SS'}|_{\rm max}$ or $|\kappa^{\rm A}_{SS'}|_{\rm max}$ or $|\kappa^{\rm S}_{SS'}|_{\rm max}$ or $|\kappa^{\rm P}_{SS'}|_{\rm max}$ alone. The $\Xi_c^+ \to \Sigma^+ S\bar{S}'$ curves, not shown, are approximately $2\tau_{\Xi_c^+}/\tau_{\Xi_c^0} \sim 6$ times their $\Xi_c^0 \to \Sigma^0 S\bar{S}'$ counterparts.

Prediction of invisible scalars

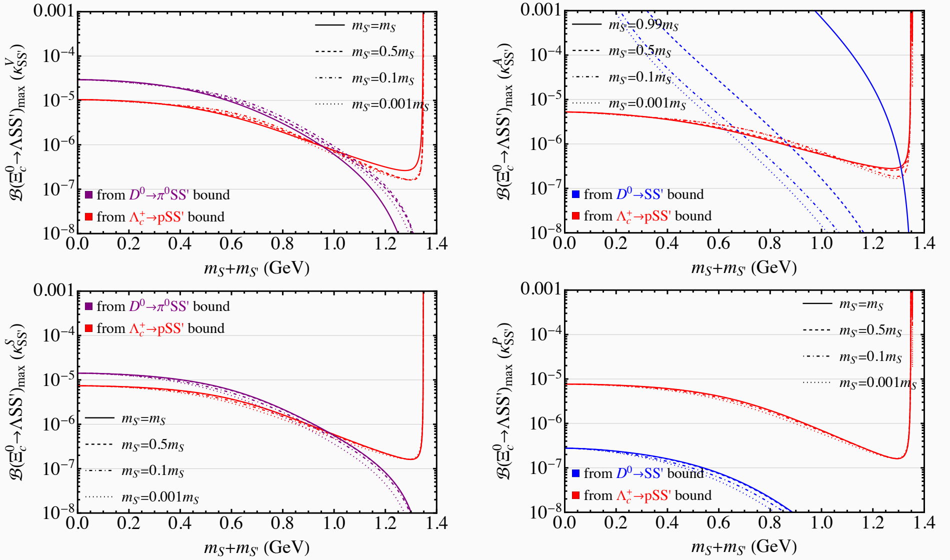


Figure 18: The maximal branching fractions of $\Xi_c^0 \to \Lambda S\bar{S}'$ due to $|\kappa^{\rm V}_{\rm SS'}|_{\rm max}$ or $|\kappa^{\rm A}_{\rm SS'}|_{\rm max}$ or $|\kappa^{\rm S}_{\rm SS'}|_{\rm max}$ or $|\kappa^{\rm P}_{\rm SS'}|_{\rm max}$ alone.

Prediction of invisible scalars

Table 4: The upper limits on branching fractions, in units of 10^{-5}, of various charmed-hadron decays induced by the $c \to u\mathrm{S}\bar{\mathrm{S}}'$ operators for $m_{\mathrm{S}'} = m_{\mathrm{S}} = 0$ if the $\Lambda_c^+ \to p\mathrm{S}\bar{\mathrm{S}}'$ bound is absent and, in brackets, if it is taken into account and the stronger. Only one of the coefficients $\kappa_{\mathrm{SS}'}^{\mathrm{V,A,S,P}}$ of the operators is taken to be nonzero at a time. A dash entry under $\kappa_{\mathrm{SS}'}^{\mathrm{X}} \neq 0$ means that $\kappa_{\mathrm{SS}'}^{\mathrm{X}}$ does not affect the decay.

Decay modes	$\kappa_{\mathrm{SS}'}^{\mathrm{V}} \neq 0$	$\kappa_{\mathrm{SS}'}^{\mathrm{A}} \neq 0$	$\kappa_{\mathrm{SS}'}^{\mathrm{S}} \neq 0$	$\kappa_{\mathrm{SS}'}^{\mathrm{P}} \neq 0$
$D^0 \to \mathrm{S}\bar{\mathrm{S}}'$	-	-	-	9.4 [Input]
$D^0 \to \gamma\mathrm{S}\bar{\mathrm{S}}'$	0.14 (0.050)	(0.0026)	-	-
$D^0 \to \pi^0\mathrm{S}\bar{\mathrm{S}}'$	21 [Input] (7.5)	-	21 [Input] (11)	-
$D^+ \to \pi^+\mathrm{S}\bar{\mathrm{S}}'$	107 (38)	-	107 (55)	-
$D_s^+ \to K^+\mathrm{S}\bar{\mathrm{S}}'$	38 (13)	-	36 (19)	-
$D^0 \to \rho^0\mathrm{S}\bar{\mathrm{S}}'$	0.74 (0.26)	(1.8)	-	0.081
$D^+ \to \rho^+\mathrm{S}\bar{\mathrm{S}}'$	3.8 (1.4)	(9.4)	-	0.42
$D_s^+ \to K^{*+}\mathrm{S}\bar{\mathrm{S}}'$	2.0 (0.71)	(5.3)	-	0.21
$\Lambda_c^+ \to p\mathrm{S}\bar{\mathrm{S}}'$	23 (8.0 [Input])	(8.0 [Input])	15 (8.0 [Input])	0.29
$\Xi_c^+ \to \Sigma^+\mathrm{S}\bar{\mathrm{S}}'$	49 (17)	(8.7)	25 (13)	0.44
$\Xi_c^0 \to \Sigma^0\mathrm{S}\bar{\mathrm{S}}'$	8.3 (2.9)	(1.5)	4.2 (2.2)	0.075
$\Xi_c^0 \to \Lambda\mathrm{S}\bar{\mathrm{S}}'$	2.9 (1.0)	(0.52)	1.4 (0.74)	0.028

Prediction of invisible scalars

Table 5: The same as Table I but for $m_{\mathrm{S}'} = 0.1\, m_{\mathrm{S}} = 0.05$ GeV.

Decay modes	$\kappa^{\mathrm{V}}_{\mathrm{SS}'} \neq 0$	$\kappa^{\mathrm{A}}_{\mathrm{SS}'} \neq 0$	$\kappa^{\mathrm{S}}_{\mathrm{SS}'} \neq 0$	$\kappa^{\mathrm{P}}_{\mathrm{SS}'} \neq 0$
$D^0 \to \mathrm{S}\bar{\mathrm{S}}'$	-	9.4 [Input] (3.5)	-	9.4 [Input]
$D^0 \to \gamma\mathrm{S}\bar{\mathrm{S}}'$	0.14 (0.063)	0.0081 (0.0030)	-	-
$D^0 \to \pi^0\mathrm{S}\bar{\mathrm{S}}'$	21 [Input] (9.3)	-	21 [Input] (13)	-
$D^+ \to \pi^+\mathrm{S}\bar{\mathrm{S}}'$	107 (47)	-	107 (68)	-
$D_s^+ \to K^+\mathrm{S}\bar{\mathrm{S}}'$	34 (15)	-	32 (20)	-
$D^0 \to \rho^0\mathrm{S}\bar{\mathrm{S}}'$	0.23 (0.10)	2.9 (1.1)	-	0.024
$D^+ \to \rho^+\mathrm{S}\bar{\mathrm{S}}'$	1.2 (0.55)	15 (5.6)	-	0.12
$D_s^+ \to K^{*+}\mathrm{S}\bar{\mathrm{S}}'$	0.62 (0.27)	8.1 (3.0)	-	0.060
$\Lambda_c^+ \to p\mathrm{S}\bar{\mathrm{S}}'$	18 (8.0 [Input])	22 (8.0 [Input])	13 (8.0 [Input])	0.14
$\Xi_c^+ \to \Sigma^+\mathrm{S}\bar{\mathrm{S}}'$	22 (9.9)	13 (4.7)	10 (6.5)	0.12
$\Xi_c^0 \to \Sigma^0\mathrm{S}\bar{\mathrm{S}}'$	3.8 (1.7)	2.2 (0.80)	1.7 (1.1)	0.020
$\Xi_c^0 \to \Lambda\mathrm{S}\bar{\mathrm{S}}'$	1.4 (0.61)	0.82 (0.30)	0.61 (0.39)	0.0078

Conclusion

Summary

- A window to detect new physics: highly suppressed SM background. Fully invisible decay; Semi-invisible radiative decay; Semi-invisible mesonic (bartonic) decay.
- Light invisible scalars and singlet fermions.
- The bounds of the coupling constants are extracted from very recent data. Based on these bounds, we predict the upper limits of other channels.
- Many of the predictions we have made [1, 2, 3, 4, 5, 6] are expected testable by the near-future collider experiments.

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Thanks! Questions?