

The promising $B \rightarrow K^* \nu \bar{\nu}$ in view of SMEFT

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Outline

1 Belle-II measurement vs SM prediction

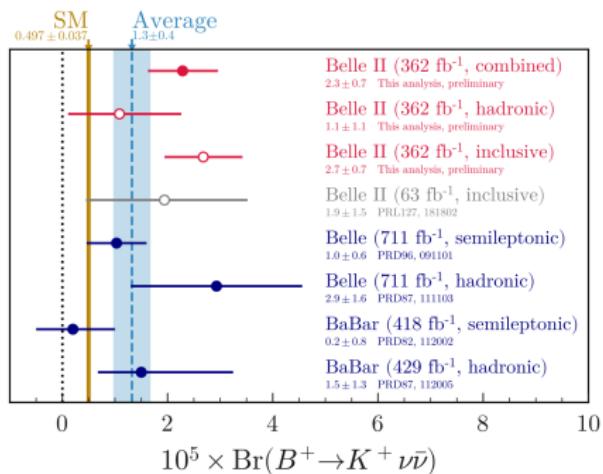
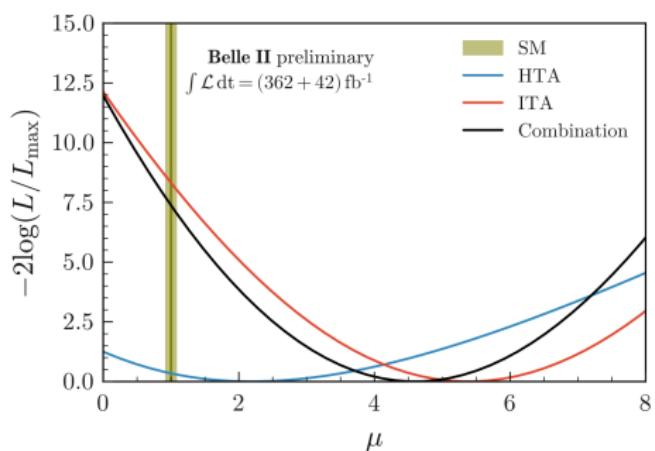
2 Effective field theory

3 Matching to SMEFT

4 Results



Combination and Comparison with other measurements



Significance of Belle-II observation is 3.6σ and the result is within 2.8σ vs SM

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{exp}} = (2.3 \pm 0.7) \times 10^{-5}$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}} = (0.497 \pm 0.037) \times 10^{-5}$$

W. G. Parrott, C. Bouchard, and C. T. H. Davies, Phys. Rev. D 107, 014511 (2023)



New Physics

- Heavy NP particles (missing energies are carried by SM neutrinos)
 - ▶ Leptoquark
 - ▶ 2HDM
 - ▶ ...
- Light NP particles (missing energies are carried by new light particles)
 - ▶ Right-handed neutrinos
 - ▶ Axion/ axion-like particles
 - ▶ Dark matter
 - ▶ ...



Effective Field Theory

- Low-energy effective field theory (LEFT)

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QED+QCD}} + \mathcal{L}_{\not{\mu}}^{(3)} + \sum_i \sum_{d \geq 5} L_i^{(d)} \mathcal{O}_i^{(d)}$$

- ▶ Symmetry: $SU(3)_C \times U(1)_{em}$
- ▶ Scale: $\mu < \Lambda_{EW}$
- ▶ Degrees of freedom: SM particles except W, Z, t, h
- ▶ Power counting: $L_i^{(d)} \sim 1/v^{d-4}$

- Standard Model effective field theory (SMEFT)

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \sum_{d \geq 5} C_i^{(d)} \mathcal{Q}_i^{(d)}$$

- ▶ Symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- ▶ Scale: $\mu \in [\Lambda_{EW}, \Lambda], \Lambda \gg \Lambda_{EW}$
- ▶ Degrees of freedom: SM particles
- ▶ Power counting: $C_i^{(d)} \sim 1/\Lambda^{d-4}$

- EFT extension with right-handed neutrino

- ▶ LEFT+right-handed neutrino $N_R \rightarrow \text{LNEFT}$
- ▶ SMEFT+right-handed neutrino $N_R \rightarrow \text{SMNEFT}$



$b \rightarrow s\nu\bar{\nu}$ transition

- Effective Hamiltonian (assuming lepton flavor is conserved and no ν_R),

$$C_{V,LL}^{\nu_\ell, \text{SM}} = -6.32, \quad C_{V,RL}^{\nu_\ell, \text{SM}} = 0$$

$$\mathcal{H}_{\text{eff}} = -\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} \left[C_{V,LL}^{\nu_\ell} \mathcal{O}_{V,LL}^{\nu_\ell} + C_{V,RL}^{\nu_\ell} \mathcal{O}_{V,RL}^{\nu_\ell} \right] + \text{h.c.}$$

$$\mathcal{O}_{V,LL}^{\nu_\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell), \quad \mathcal{O}_{V,RL}^{\nu_\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)$$

- $B^+ \rightarrow K^+ \nu \bar{\nu}$ and $B^0 \rightarrow K^{*0} \nu_\ell \bar{\nu}_\ell$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu_\ell \bar{\nu}_\ell) = 3.46 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} + C_{V,RL}^{\nu_\ell} \right|^2$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu_\ell \bar{\nu}_\ell) = 6.84 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} - C_{V,RL}^{\nu_\ell} \right|^2 + 1.36 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} + C_{V,RL}^{\nu_\ell} \right|^2$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{exp}} < 1.8 \times 10^{-5} \text{ vs } \mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}} = (9.2 \pm 1.0) \times 10^{-6}$$

- No-go scenario with a single parameter, e.g., $C_{V,LL}^{\nu_\ell} \neq 0, C_{V,RL}^{\nu_\ell} = 0$

$$\begin{cases} \frac{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})}{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})} = \frac{8.20 \times 10^{-8} |C_{V,LL}^{\nu_\ell}|^2}{3.46 \times 10^{-8} |C_{V,LL}^{\nu_\ell}|^2} \simeq 2.37 \\ \mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = (2.3 \pm 0.7) \times 10^{-5} \end{cases}$$

$$\Rightarrow \mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) = (5.5 \pm 1.7) \times 10^{-5} > 1.8 \times 10^{-5}$$



Matching to SMEFT

$b \rightarrow s\ell^+\ell^-$

$$\mathcal{O}_{V,LL}^\ell$$

←

$$[\mathcal{O}_{\ell q}^{(1)}]_{\ell\ell 23}$$

→

$$\mathcal{O}_{V,LL}^{\nu_\ell}$$

$$\mathcal{O}_{V,LL}^\ell$$

←

$$[\mathcal{O}_{\ell q}^{(3)}]_{\ell\ell 23}$$

→

$$-\mathcal{O}_{V,LL}^{\nu_\ell}$$

$$\mathcal{O}_{V,RL}^\ell$$

←

$$[\mathcal{O}_{\ell d}]_{\ell\ell 23}$$

→

$$\mathcal{O}_{V,RL}^{\nu_\ell}$$

$$\mathcal{O}_{V,LR}^\ell$$

←

$$[\mathcal{O}_{qe}]_{23\ell\ell}$$

$$\mathcal{O}_{V,RR}^\ell$$

←

$$[\mathcal{O}_{ed}]_{\ell\ell 23}$$

$$[\mathcal{O}_{\ell q}^{(3)}]_{\ell\ell 23} \rightarrow \sum_i 2V_{is}(\bar{\ell}\gamma_\mu P_L \nu_\ell)(\bar{u}_i \gamma_\mu P_L b)$$

where $(Q_i = [(V^\dagger u_L)_i, d_{Li}])$

$$\mathcal{O}_{V,LL}^\ell = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \ell),$$

$$[\mathcal{O}_{\ell q}^{(1)}]_{prst} = (\bar{L}_p \gamma^\mu L_r)(\bar{Q}_s \gamma_\mu Q_t),$$

$$\mathcal{O}_{V,LL}^{\nu_\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)$$

$$\mathcal{O}_{V,RL}^\ell = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu P_L \ell),$$

$$[\mathcal{O}_{\ell q}^{(3)}]_{prst} = (\bar{L}_p \gamma^\mu \tau^I L_r)(\bar{Q}_s \gamma_\mu \tau^I Q_t),$$

$$\mathcal{O}_{V,RL}^{\nu_\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)$$

$$\mathcal{O}_{V,LR}^\ell = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_R \ell),$$

$$[\mathcal{O}_{\ell d}]_{prst} = (\bar{L}_p \gamma^\mu L_r)(\bar{d}_{Rs} \gamma_\mu d_{Rt}),$$

$$\mathcal{O}_{V,RR}^\ell = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu P_R \ell),$$

$$[\mathcal{O}_{qe}]_{prst} = (\bar{Q}_p \gamma^\mu Q_r)(\bar{e}_{Rs} \gamma_\mu e_{Rt}),$$

$$[\mathcal{O}_{ed}]_{prst} = (\bar{e}_{Rp} \gamma^\mu e_{Rr})(\bar{d}_{Rs} \gamma_\mu d_{Rt}),$$



$b \rightarrow s\ell^+\ell^-$ and $b \rightarrow u_i\ell\nu_\ell$

- $b \rightarrow s\ell^+\ell^-$

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} \sum_i \left[C_i^\ell \mathcal{O}_i^\ell + C_i^{\ell'} \mathcal{O}_i^{\ell'} \right] + \text{h.c.}$$

where $(\mathcal{O}_{9,10}^\ell = \mathcal{O}_{V,LR}^\ell \pm \mathcal{O}_{V,LL}^\ell, \quad \mathcal{O}_{9,10}^{\ell'} = \mathcal{O}_{V,RR}^\ell \pm \mathcal{O}_{V,RL}^\ell)$

$$\begin{aligned}\mathcal{O}_9^\ell &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), & \mathcal{O}_9^{\ell'} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell), \\ \mathcal{O}_{10}^\ell &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell), & \mathcal{O}_{10}^{\ell'} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),\end{aligned}$$

Observables: $B_s \rightarrow \ell^+\ell^-$, $R_{K(*)}$, $\Lambda_b \rightarrow \Lambda\ell^+\ell^- \dots$

- $b \rightarrow u_i\ell\nu_\ell$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ib} \left[C_{V,LL}^{\ell u_i} \mathcal{O}_{V,LL}^{\ell u_i} + C_{V,RL}^{\ell u_i} \mathcal{O}_{V,RL}^{\ell u_i} \right]$$

where

$$\mathcal{O}_{V,LL}^{\ell u_i} = (\bar{c}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu_\ell), \quad \mathcal{O}_{V,RL}^{\ell u_i} = (\bar{c}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu P_L \nu_\ell) \quad (1)$$

Observables: $R_{D(*)}$, $B_c^+ \rightarrow \tau^+\nu_\tau$, $B^+ \rightarrow \tau^+\nu_\tau \dots$



RGE and Wilson coefficient matching relation

- Small RG running effects between μ_b and μ_{EW}

$$\mu \frac{dC_{V,XY}^\ell}{d\mu} = \pm 12q_d q_e \frac{\alpha_{em}}{4\pi} C_{V,XY}^\ell, \quad (+ \text{ for } XY = LL, RR, - \text{ for } XY = LR, RL)$$

$$\mu \frac{dC_{V,XY}^{\ell u_i}}{d\mu} = \pm 6q_u q_e \frac{\alpha_{em}}{4\pi} C_{V,XY}^{\ell u_i}, \quad (+ \text{ for } XY = LL, - \text{ for } XY = RL)$$

$$\mu \frac{dC_{V,XY}^{\nu_\ell}}{d\mu} = 0, \quad (\text{for } XY = LL, RL)$$

- Matching to SMEFT ($\ell = e, \mu, \tau$)

$$\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^\ell - C_{10}^\ell) = [C_{\ell q}^{(1)}]_{\ell\ell 23} + [C_{\ell q}^{(3)}]_{\ell\ell 23},$$

$$\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^\ell + C_{10}^\ell) = [C_{qe}]_{23\ell\ell},$$

$$\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^{\ell'} + C_{10}^{\ell'}) = [C_{ed}]_{\ell\ell 23},$$

$$\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^{\ell'} - C_{10}^{\ell'}) = [C_{ed}]_{\ell\ell 23},$$

$$\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} C_{V,LL}^{\nu_\ell} = [C_{\ell q}^{(1)}]_{\ell\ell 23} - [C_{\ell q}^{(3)}]_{\ell\ell 23},$$

$$\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} C_{V,RL}^{\nu_\ell} = [C_{\ell d}]_{\ell\ell 23},$$

$$\frac{4G_F}{\sqrt{2}} \frac{V_{ib}}{2V_{is}} C_{V,LL}^{\ell u_i} = [C_{\ell q}^{(3)}]_{\ell\ell 23},$$



Global fits of new physics in $b \rightarrow s$ after the $R_K^{(*)}$ 2022 release

Qiaoyi Wen, Fanrong Xu, Phys.Rev.D 108 (2023) 9, 095038

effective Hamiltonian: $\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i) + h.c.$

$C_i^{(r)\ell} = C_i^{(r)\ell;SM} + \Delta C_i^{(r)\ell;NP} = C_i^{(r)\ell;SM} + \Delta C_i^{(r)\ell}$

high energy information

QCDF

$\mathcal{O}_7 = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu},$
 $\mathcal{O}_8 = \frac{g_s m_b}{e^2} (\bar{s}\sigma_{\mu\nu} T^\mu P_L b) G_a^{\mu\nu},$
 $\mathcal{O}_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell),$
 $\mathcal{O}_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$
 $\mathcal{O}_S = m_b (\bar{s}P_R b)(\bar{\ell}\ell),$
 $\mathcal{O}_P = m_b (\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell),$
 $\mathcal{O}'_7 = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu},$
 $\mathcal{O}'_8 = \frac{g_s m_b}{e^2} (\bar{s}\sigma_{\mu\nu} T^\mu P_L b) G_a^{\mu\nu},$
 $\mathcal{O}'_9 = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell),$
 $\mathcal{O}'_{10} = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$
 $\mathcal{O}'_S = m_b (\bar{s}P_L b)(\bar{\ell}\ell),$
 $\mathcal{O}'_P = m_b (\bar{s}P_L b)(\bar{\ell}\gamma_5 \ell).$

decay amplitude:

	C_5	C_6	C_7^{eff}	C_8^{eff}	C_9	C_{10}
LL	-0.5093	1.0256	-0.0050	-0.0686	0.0005	0.0010
NLL	-0.3001	1.0080	-0.0047	-0.0827	0.0003	0.0009
NNLL	-	-	-	-	-	-

in leading logarithmic (LL), next-to-leading logarithmic (NLL) and terms listed in Table II are used.

Params	S-I'	S-II'	S-III'	S-IV'	S-I	S-II	S-III	S-IV	ADCMN [23]	AS [24]	HMMN [25]	GGJLCS [26]
Reduced χ^2	183.404(n-12)	197.556(n-12)	182.869(n-16)	176.807(n-20)	190.044(n-12)	177.891(n-12)	185.386(n-16)	178.953(n-20)	260.66(254-6)	179.1(183-20)	96.88/90	
$x_{\tau^0}^*/\text{d.o.f}$	= 0.970	= 1.045	= 0.988	= 0.977	= 0.995	= 0.931	= 0.991	= 0.978	= 1.05	= 1.1	= 1.08	
ΔC_1	-0.003+0.020	-0.001+0.015	...	0.001+0.018	-0.000+0.020	-0.001+0.015	...	-0.000+0.016	0.00+0.01	0.06+0.03	0.06+0.03	...
ΔC_7	0.017-0.039	0.020-0.035	...	0.020-0.014	0.017+0.004	0.020+0.014	...	0.023+0.004	+0.06+0.02	-0.01+0.01	0.21+0.01	...
ΔC_8	-0.788+0.995	-0.885+0.438	...	-0.773+0.431	-0.995+0.546	-0.921+0.443	...	-0.773+0.403	-0.80+0.06	-0.80+0.06	-0.80+0.06	...
ΔC_9	-0.073+1.080	-0.093+0.921	...	-0.089+0.996	-0.080+1.046	-0.076+0.893	...	-0.258+1.007	...	-0.30+1.20
ΔC_7^*	-0.806+0.237	-0.795+0.205	-1.068+0.161	-0.863+0.214	-0.752+0.261	-0.789+0.216	-1.054+0.171	-0.872+0.211	-1.08+0.18	-0.82+0.23	-1.14+0.18	-1.07+0.29
ΔC_8^*	0.194+0.608	0.056+0.338	-0.342+0.342	0.112+0.397	0.020+0.348	0.174+0.441	0.130+0.437	0.088+0.378	0.16+0.36	-0.10+0.34	0.05+0.32	0.32+0.31
ΔC_{10}^*	0.236-0.183	0.145-0.136	0.164-0.180	0.213-0.155	-0.019+0.236	0.163+0.160	0.112+0.166	0.171+0.173	0.15+0.13	0.14+0.23	0.21+0.20	0.21+0.14
ΔC_9^*	-0.096+0.251	-0.108+0.186	-0.115+0.200	-0.089+0.177	-0.118+0.206	-0.093+0.183	-0.115+0.221	-0.062+0.197	-0.18+0.20	-0.33+0.23	-0.03+0.19	-0.26+0.14
ΔC_7^{eff}	0.066+1.091	-0.004+1.102	-0.008+0.983	-0.043+0.642	0.023+1.064	0.060+1.108	-0.066+0.946	0.009+0.958	...	0.01+0.09	0.01+0.09	...
ΔC_8^{eff}	0.065+1.140	-0.003+1.103	-0.002+0.871	-0.059+0.644	0.014+1.166	0.061+1.122	-0.070+0.900	0.012+0.862	...	-0.01+0.05	0.01+0.05	...
ΔC_9^{eff}	0.167+1.225	-0.197+0.836	0.092+0.994	0.117+0.847	0.079+1.138	0.478+1.003	0.189+1.031	0.124+1.040	...	-0.04+0.02	0.04+0.02	...
ΔC_7^{eff}	0.053+1.169	0.891+0.729	0.010+1.083	0.040+0.854	-0.032+1.156	0.370+0.803	0.098+1.004	0.038+0.894	...	-0.04+0.02	0.04+0.02	...
ΔC_8^{eff}	...	-0.795+0.205	-1.753+0.781	-1.351+0.627	...	-0.789+0.219	-1.623+0.662	-1.511+0.700	...	-0.24+1.17	-0.50+1.90	...
ΔC_9^{eff}	...	-0.795+0.205	-1.753+0.781	-1.351+0.627	...	-0.789+0.219	-1.623+0.662	-1.511+0.700	...	-0.24+1.17	-0.50+1.90	...
$\Delta C_{10}^{\text{eff}}$	0.056-0.338	1.725+1.734	2.170+1.486	...	0.048+1.538	1.090+1.609	0.864+1.563	1.40+2.30
ΔC_{10}^*	0.145-0.136	0.108-0.661	0.058-0.661	...	0.163+0.160	0.555+0.570	0.383+0.623	...	-0.24+0.78	-0.0
ΔC_7^*	-0.108+0.186	0.600+0.208	0.655+0.094	0.655+0.094	-0.093+1.101	0.088+1.060	0.002+0.881	0.002+0.881	0.0	...
ΔC_8^*	-0.004+1.032	-0.719+0.961	-0.549+1.223	-0.549+1.223	0.060+1.330	-0.952+1.212	-0.806+1.900	-0.38+0.41
ΔC_9^*	0.003+1.103	-0.699+1.837	-0.550+1.618	-0.550+1.618	0.061+1.330	-1.051+2.251	-0.803+1.863	-0.36+0.30
ΔC_7^*	1.017+0.836	-1.592+0.552	-1.688+0.566	-0.976+0.566	0.478+0.899	-1.568+1.544	-1.837+1.930	-0.98+0.21
ΔC_8^*	0.891+0.812	-1.360+1.218	-1.431+1.212	-0.870+1.207	0.370+0.897	-1.477+1.025	-1.652+1.200	-0.95+0.29



Discussion

- Global fit results show that, for $\ell = e, \mu$ [Qiaoyi Wen, Fanrong Xu, Phys.Rev.D 108 \(2023\) 9, 095038](#)

$$C_9^\ell \neq 0, C_9^{\ell'} \sim C_{10}^\ell \sim C_{10}^{\ell'} \sim 0$$
$$\Rightarrow [C_{qe}]_{23\ell\ell} = [C_{\ell q}^{(1)}]_{\ell\ell 23} + [C_{\ell q}^{(3)}]_{\ell\ell 23}, [C_{ed}]_{\ell\ell 23} = [C_{\ell d}]_{\ell\ell 23} = 0, C_{V,RL}^{\nu_\ell} = 0$$

- For $\ell = \tau$, all WCs are free parameters, The SMEFT WCs that simultaneously contribute to both $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow s\nu\bar{\nu}$ are $[C_{\ell q}^{(1)}]_{3323}, [C_{\ell q}^{(3)}]_{3323}, [C_{\ell d}]_{3323}$:

$$\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^\tau - C_{10}^\tau) = [C_{\ell q}^{(1)}]_{3323} + [C_{\ell q}^{(3)}]_{3323}, \quad \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} (C_9^{\tau'} - C_{10}^{\tau'}) = [C_{\ell d}]_{3323},$$
$$\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} C_{V,LL}^{\nu_\tau} = [C_{\ell q}^{(1)}]_{3323} - [C_{\ell q}^{(3)}]_{3323}, \quad \frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} C_{V,RL}^{\nu_\tau} = [C_{\ell d}]_{3323}$$

- $[C_{\ell q}^{(3)}]_{3323}$ also contributes to $b \rightarrow u_i \tau \nu_\tau$

$$\frac{4G_F}{\sqrt{2}} \frac{V_{ib}}{2V_{is}} C_{V,LL}^{\tau u_i} = [C_{\ell q}^{(3)}]_{3323}$$



Wilson coefficients summary

Parameter	Process
$C_9^\ell = \frac{[C_{\ell q}^{(1)}]_{\ell\ell 23} + [C_{\ell q}^{(3)}]_{\ell\ell 23}}{\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi}},$	$(b \rightarrow s\ell^+\ell^-, \ell = e, \mu)$
$C_{V,LL}^{\nu_\ell} = \frac{[C_{\ell q}^{(1)}]_{\ell\ell 23} - [C_{\ell q}^{(3)}]_{\ell\ell 23}}{\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi}},$	$(b \rightarrow s\nu_\ell\bar{\nu}_\ell, \ell = e, \mu, \tau)$
$C_{V,RL}^{\nu_\tau} = \frac{[C_{\ell d}]_{3323}}{\frac{8G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi}},$	$(b \rightarrow s\nu_\tau\bar{\nu}_\tau)$
$C_{V,LL}^{\tau u_i} = \frac{[C_{\ell q}^{(3)}]_{3323}}{\frac{4G_F}{\sqrt{2}} \frac{V_{ib}}{2V_{is}}}, \quad C_{V,RL}^{\tau u_i}$	$(b \rightarrow u_i\tau\nu_\tau)$
$C_9^\tau - C_{10}^\tau = \frac{[C_{\ell q}^{(1)}]_{3323} + [C_{\ell q}^{(3)}]_{3323}}{\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi}},$	$(b \rightarrow s\tau^+\tau^-)$
$C_9^{\tau'} - C_{10}^{\tau'} = \frac{[C_{\ell d}]_{3323}}{\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi}},$	$(b \rightarrow s\tau^+\tau^-)$



Observables summary

$$\mathcal{B}(B^+ \rightarrow K^+ \nu_\ell \bar{\nu}_\ell) = 3.46 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} + C_{V,RL}^{\nu_\ell} \right|^2$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu_\ell \bar{\nu}_\ell) = 6.84 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} - C_{V,RL}^{\nu_\ell} \right|^2 + 1.36 \times 10^{-8} \left| C_{V,LL}^{\nu_\ell} + C_{V,RL}^{\nu_\ell} \right|^2$$

$$R_D = R_D^{\text{SM}} \left[1 + 1.5 \text{Re} \left(C_{V,RL}^{\tau c} + C_{V,LL}^{\tau c} \right) + 1.0 \left| C_{V,RL}^{\tau c} + C_{V,LL}^{\tau c} \right|^2 \right]$$

$$R_{D^*} = R_{D^*}^{\text{SM}} \left[1 + 0.12 \text{Re} \left(C_{V,RL}^{\tau c} - C_{V,LL}^{\tau c} \right) + 0.05 \left| C_{V,RL}^{\tau c} - C_{V,LL}^{\tau c} \right|^2 \right]$$

$$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) = 2.32 \times 10^{-2} \left| (1 + C_{V,LL}^{\tau c}) - C_{V,RL}^{\tau c} \right|^2$$

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = 8.88 \times 10^{-5} \left| (1 + C_{V,LL}^{\tau u}) - C_{V,RL}^{\tau u} \right|^2$$

$$\mathcal{B}(B_s \rightarrow \tau^+ \tau^-) = 1.41 \times 10^{-8} |m_\tau (C_{10}^\tau - C_{10}'^\tau)|^2$$



Inputs

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = (2.3 \pm 0.7) \times 10^{-5},$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) < 1.8 \times 10^{-5} \text{ (90\% CL)},$$

$$\mathcal{B}(B_s \rightarrow \tau^+ \tau^-) < 6.8 \times 10^{-3},$$

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.25 \pm 0.39) \times 10^{-4},$$

$$R_D = 0.357 \pm 0.029,$$

$$R_{D^*} = 0.284 \pm 0.012$$

$$C_9^\mu = -0.789_{-0.210}^{+0.198}$$

Belle II 2311.14647

Belle Phys.Rev.D 97, 09902 (2018)

LHCb Phys.Rev.Lett 118 (2017) 14647

Belle Phys.Rev.D 92, 051102

HFLAV Phys.Rev.D 107 052008

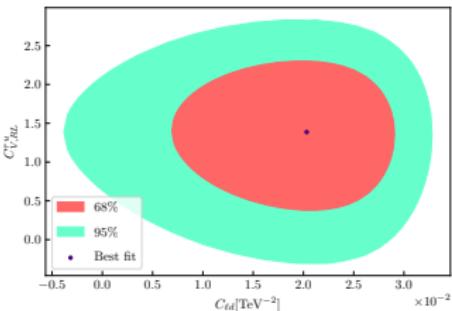
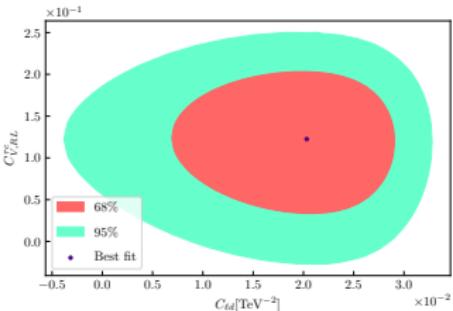
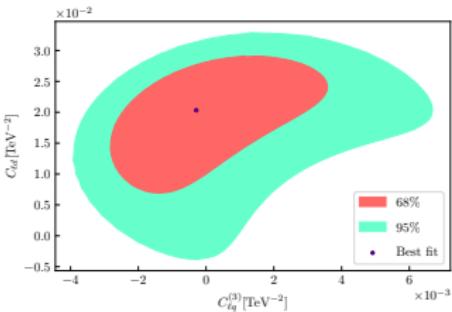
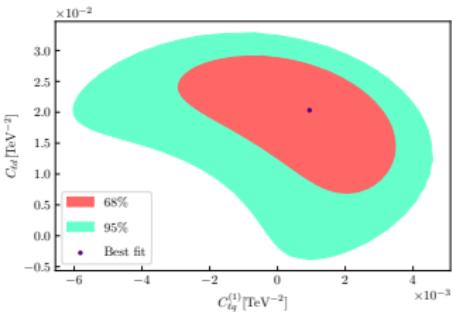
HFLAV Phys.Rev.D 107 052008

Q. Wen and F. Xu Phys.Rev.D 108 (2023) 9, 095038



Results

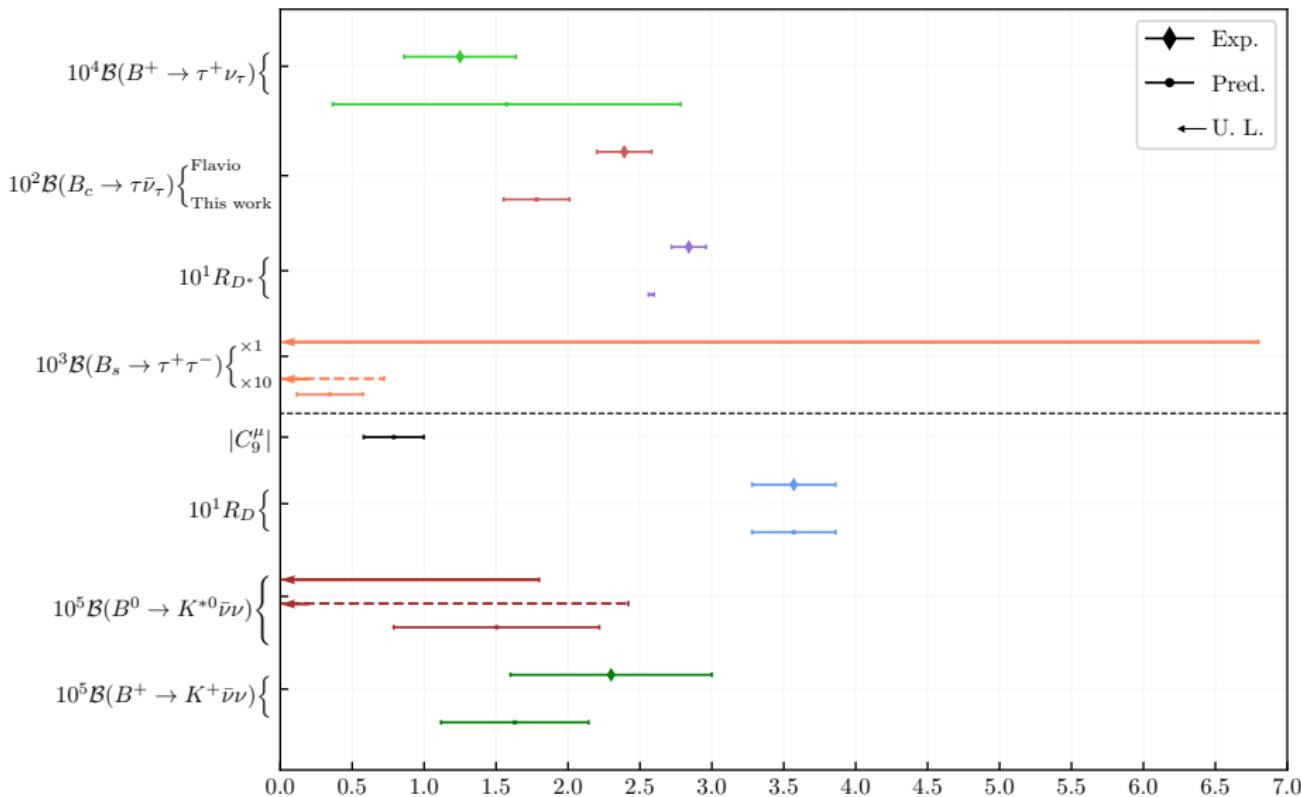
WC (GeV^{-2})	Value
$[C_{\ell q}^{(1)}]_{2223}$	$(0.9 \pm 2.2) \times 10^{-3}$
$[C_{\ell q}^{(3)}]_{2223}$	$(-0.3 \pm 2.2) \times 10^{-3}$
$[C_{\ell d}]_{3323}$	$(2 \pm 0.8) \times 10^{-2}$
$C_{V,RL}^{\tau c}$	0.12 ± 0.06
$C_{V,RL}^{\tau u}$	1.4 ± 0.7



Remake predictions

Observable	prediction	measurement
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	$(1.63 \pm 0.51) \times 10^{-5}$	$(2.3 \pm 0.7) \times 10^{-5}$
$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})$	$(1.50 \pm 0.72) \times 10^{-5}$	$< 1.8 \times 10^{-5}$
$\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)$	$(3.45 \pm 2.34) \times 10^{-5}$	$< 6.8 \times 10^{-3}$
$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)$	$(1.57 \pm 1.21) \times 10^{-4}$	$(1.25 \pm 0.39) \times 10^{-4}$
$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$	0.018 ± 0.02	0.024 ± 0.02
R_D	0.357 ± 0.029	0.357 ± 0.029
R_{D^*}	0.258 ± 0.002	0.284 ± 0.012
C_9^μ	-0.789 ± 0.210	$-0.789^{+0.198}_{-0.210}$





Summary

- We have used SMEFT to link $B^+ \rightarrow K^+\nu\bar{\nu}$ and $B^0 \rightarrow K^{*0}\nu\bar{\nu}$ to C_9^μ , $R_{D(*)}$, $B_s \rightarrow \tau^+\tau^-$, and $B_{u,c}^+ \rightarrow \tau^+\nu_\tau$, the best fit values of parameters are obtained by using the maximum likelihood function, we also remake predictions on these observables by using the fit results;
- All observables except R_{D*} are consistent with current experimental bounds; our prediction on $B_c^+ \rightarrow \tau^+\nu_\tau$ also deviate from the result of Flavio;
- The value of $|C_9^\mu|$ almost overlaps with the one of our previous work;
- The upper limit of $B^0 \rightarrow K^{*0}\nu\bar{\nu}$ is slightly larger than the one of current experiment;
- Our prediction on $B_s \rightarrow \tau^+\tau^-$ is about two order of magnitude more stringent than the current upper limit, which may be checked by the future experiment, e.g., CEPC.



Backup

In the flavor basis with diagonal down-type quark Yukawa matrix (CKM matrix element in the upper component: $Q_i = [(V^\dagger u_L)_i, d_{Li}]^T$)

$$\begin{aligned} [\mathcal{O}_{\ell q}^{(3)}]_{prst} &= (\bar{L}_p \gamma^\mu \tau^I L_r) (\bar{Q}_s \gamma_\mu \tau^I Q_t) \\ &= \left[(\bar{\nu}_{Lp}, \bar{\ell}_{Lp}) \gamma^\mu \tau^I \begin{pmatrix} \nu_{Lr} \\ \ell_{Lr} \end{pmatrix} \right] \left[(\bar{u}_{Li} V_{is}, \bar{d}_{Ls}) \gamma^\mu \tau^I \begin{pmatrix} V_{tj}^\dagger u_{Lj} \\ d_{Lt} \end{pmatrix} \right] \\ &= V_{is} (V_{jt})^\dagger [(\bar{\nu}_{Lp} \gamma^\mu \nu_{Lr}) (\bar{u}_{Li} \gamma_\mu u_{Lj}) - (\bar{\ell}_{Lp} \gamma^\mu \ell_{Lr}) (\bar{u}_{Li} \gamma_\mu u_{Lj})] \\ &\quad + 2V_{is} (\bar{\ell}_{Lp} \gamma^\mu \nu_{Lr}) (\bar{u}_{Li} \gamma_\mu d_{Lt}) + 2(V_{jt})^\dagger (\bar{\nu}_{Lp} \gamma^\mu \ell_{Lr}) (\bar{d}_{Ls} \gamma_\mu u_{Lj}) \\ &\quad + (\bar{\ell}_{Lp} \gamma^\mu \ell_{Lr}) (\bar{d}_{Ls} \gamma_\mu d_{Lt}) - (\bar{\nu}_{Lp} \gamma^\mu \nu_{Lr}) (\bar{d}_{Ls} \gamma_\mu d_{Lt}) \\ \Rightarrow [\mathcal{O}_{\ell q}^{(3)}]_{3323} &= V_{is} (V_{jb})^\dagger [(\bar{\nu}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{u}_{Li} \gamma_\mu u_{Lj}) - (\bar{\tau}_{\tau L} \gamma^\mu \tau_L) (\bar{u}_{Li} \gamma_\mu u_{Lj})] \\ &\quad + 2V_{is} (\bar{\tau}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{u}_{Li} \gamma_\mu b_L) + 2(V_{jb})^\dagger (\bar{\nu}_{\tau L} \gamma^\mu \tau_L) (\bar{s}_L \gamma_\mu u_{Lj}) \\ &\quad + (\bar{\tau}_{\tau L} \gamma^\mu \tau_L) (\bar{s}_L \gamma_\mu b_L) - (\bar{\nu}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{s}_L \gamma_\mu b_L) \end{aligned}$$

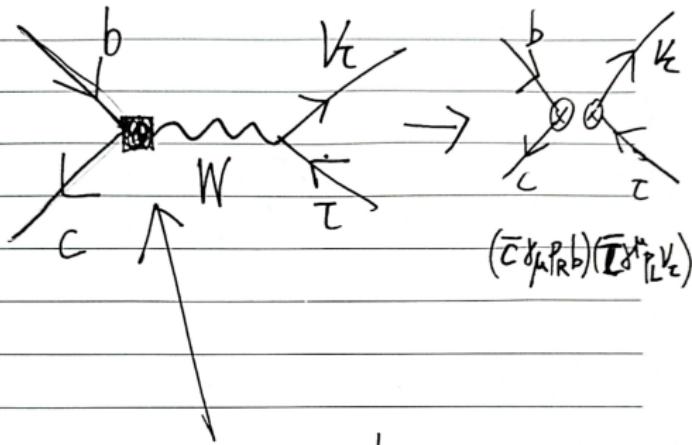
with $2V_{is} (\bar{\tau}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{u}_{Li} \gamma_\mu b_L) = 2V_{us} (\bar{\tau}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{u}_L \gamma_\mu b_L) + 2V_{cs} (\bar{\tau}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{c}_L \gamma_\mu b_L)$
 $+ 2V_{ts} (\bar{\tau}_{\tau L} \gamma^\mu \nu_{\tau L}) (\bar{t}_L \gamma_\mu b_L)$

$$[Q_{Hud}]_{pr} = i(\tilde{H}^\dagger D_\mu H)(\bar{u}_{Rp} \gamma^\mu d_{Rr}) \rightarrow [\mathcal{O}_{RL}^\ell]_{pr} = (\bar{u}_p \gamma_\mu P_R d_r) (\bar{\ell} \gamma^\mu P_L \nu_\ell)$$





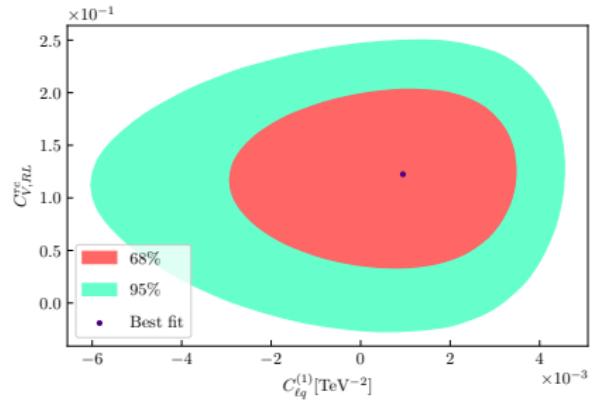
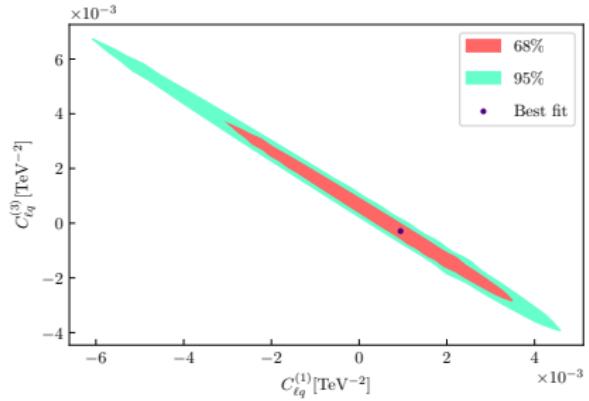
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$$(\bar{c} \gamma_\mu p_R b)(\bar{\ell} \gamma^\mu p_L \nu_e)$$

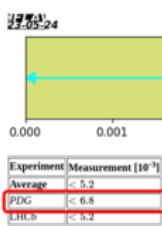
$$C_{Had} = (\bar{H}^\dagger D_\mu H)(\bar{u}_R \gamma_\mu d_R)$$





$(2.3 \pm 0.7) \times 10^{-5}$ for the $B^+ \rightarrow K^+ \nu \bar{\nu}$ decay branching
 $\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) \leq 1.8 \times 10^{-5}$ (90% c.l.).

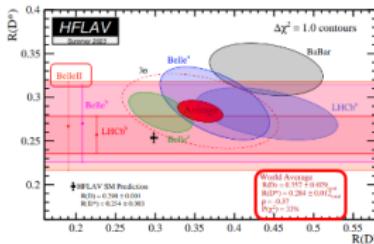
$\mathcal{B}(B_s^0 \rightarrow \tau^+ \tau^-)$



$$\Delta C_9^\mu = -0.752_{-0.265}^{+0.262} \quad -0.789_{-0.210}^{+0.198} \quad -1.054_{-0.171}^{+0.163} \quad -0.872_{-0.215}^{+0.215}$$

$\mathcal{B}(B^+ \rightarrow K^+ \bar{\nu}\nu) = (2.3 \pm 0.7) \times 10^{-5}$,
 $\mathcal{B}(B^0 \rightarrow K^{*0} \bar{\nu}\nu) < 1.8 \times 10^{-5}$ (90% C.L.),
 $\mathcal{B}(B_s \rightarrow \tau^+ \tau^-) < 6.8 \times 10^{-3}$,

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = [1.25 \pm 0.28(\text{stat.}) \pm 0.27(\text{syst.})] \times 10^{-4}$$



$\mathcal{B}(B^0 \rightarrow K^{*0} \bar{\nu}\nu) \leq 1.8 \times 10^{-5}$ (90% c.l.).

[13] J. Gygler et al. (Belle), Phys. Rev. D 96, 091101 (2017), [Addendum Phys. Rev. D 97, 099902 (2018)], 1702.03224.

$(2.3 \pm 0.7) \times 10^{-5}$ for the $B^+ \rightarrow K^+ \bar{\nu}\nu$ decay branching

Belle. 2311.14647

$\mathcal{B}(B_s \rightarrow \tau^+ \tau^-) < 6.8 \times 10^{-3}$ (95% C.L.)

LHCb. Phys.Rev.Lett. 118 (2017) 25, 251802

<https://hflav-eos.web.cern.ch/hflav-eos/rare/Apr2023/html/Bs/index.html>

$R_{D(*)}$

[/Users/mrtondo/WORK/HFLAV/RDs_summer23/rdrds_summer2023_preliminary_new.pdf_\(cern.ch\)](/Users/mrtondo/WORK/HFLAV/RDs_summer23/rdrds_summer2023_preliminary_new.pdf_(cern.ch))

<https://hflav-eos.web.cern.ch/hflav-eos/summer23/html/RDsDsstar/RDRDs.html>

HFLAV	$0.257 \pm 0.012 \pm 0.018$	Input	Accepted by PRD (arXiv:2305.01463)
Belle II	$0.267^{+0.024}_{-0.020}^{+0.020}_{-0.020}$	Input	Presented at Lepton Photon 2023 LeptonPhoton talk

Experiment	ADP	APD	Residual Correlation	Inputs	Remarks
Dalitz	$0.312 \pm 0.018 \pm 0.018$	$0.349 \pm 0.018 \pm 0.017$	$-0.01 \pm 0.01 \pm 0.01$	Input	Phys.Rev.Lett. 120 (2018) 091802 arXiv:1803.09696, Phys. Rev.D 98, 074012 (2018)
MES**	$0.211 \pm 0.018 \pm 0.015$	$0.375 \pm 0.018 \pm 0.014$	$-0.02 \pm 0.02 \pm 0.01$	Input	Phys.Rev.Lett. 120 (2018) 091803 arXiv:1803.02233, Phys. Rev.D 98, 074013 (2018)
MES**	$0.215 \pm 0.018 \pm 0.018$	$0.375 \pm 0.018 \pm 0.018$	$-0.02 \pm 0.02 \pm 0.02$	Input	Phys.Rev.Lett. 120 (2018) 091804 arXiv:1803.02234, Phys. Rev.D 98, 074014 (2018)
MES**	$0.211 \pm 0.018 \pm 0.014$	$0.337 \pm 0.018 \pm 0.011$	$-0.02 \pm 0.02 \pm 0.01$	Input	Phys.Rev.Lett. 120 (2018) 091805 arXiv:1803.02235, Phys. Rev.D 98, 074015 (2018)
LHCb	$0.265 \pm 0.020 \pm 0.018$	$0.341 \pm 0.020 \pm 0.018$	$-0.01 \pm 0.01 \pm 0.01$	Input	Phys.Rev.Lett. 120 (2018) 091806 arXiv:1803.02236, Phys. Rev.D 98, 074016 (2018)
LHCb	$0.261 \pm 0.018 \pm 0.018$	$0.341 \pm 0.018 \pm 0.018$	$-0.01 \pm 0.01 \pm 0.01$	Input	Phys.Rev.Lett. 120 (2018) 091807 arXiv:1803.02237, Phys. Rev.D 98, 074017 (2018)
Belle II	$0.267^{+0.024}_{-0.020}^{+0.020}_{-0.020}$	$0.341^{+0.020}_{-0.020}^{+0.020}_{-0.020}$	$-0.01 \pm 0.01 \pm 0.01$	Input	Accepted by PRD (arXiv:2305.01463)
Average	0.264 ± 0.012	0.337 ± 0.019	-0.01	Input	Accepted by PRD (arXiv:2305.01463)

$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = [1.25 \pm 0.28(\text{stat.}) \pm 0.27(\text{syst.})] \times 10^{-4}$.

Belle. Phys. Rev. D 92, 051102(R)

<https://pdglive.lbl.gov/BranchingRatio.action?pdgid=S041.184&home=MX0X045>

