Non-leptonic and semi-leptonic decays of Ω_c

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Outline

- Introduction
- Kinematics of $\frac{1}{2}^+ \to \frac{3}{2}^+$
- Model Calculation of Form Factors
- Numerical Results
- Summary

Introduction

Introduction

- **The reasons we study charmed baryons weak decays**

  - To understand physics around m_c
  - To study the CP violation

The CP violations in charmed meson have been already measured

R. Aaij et al. [LHCb], Phys. Rev. Lett. 108, 111602 (2012)

CP violations in charmed baryons are interested

- **The experiments we have recently**

LHCb's results of doubly charmed baryons weak decays

R. Aaij et al. [LHCb], JHEP 05, 038 (2022)

R. Aaij et al. [LHCb], Phys. Rev. Lett. 121, no.16, 162002 (2018)

R. Aaij et al. [LHCb], JHEP 12, 107 (2021)

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BesIII&Belle research on singly charmed baryons weak decays

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 129 no.23, 231803 (2022)

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 128 no.14, 142001 (2022)

J. X. Cui et al. [Belle], [arXiv:2312.02580[hep-ex]]

S. X. Li et al. [Belle], Phys. Rev. D 107, 032003 (2023)

S. S. Tang et al. [Belle], Phys. Rev. D 107, no.3, 032005 (2023)

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- **The motivations we study Ω_c weak decays**

  - Ω_c is the lightest sextet baryon because of its' special flavor component
  - Charmed baryon weak decays

Introduction

A brief overview about the reasearch on Ω_c

	Experiments	Theories
Early	D. Cronin-Hennessy et al. [CLEO], Phys. Rev. Lett 86, 3730-3734 (2001) R. Ammar et al. [CLEO], Phys. Rev. Lett. 89, 171803 (2002) B. Aubert et al. [BaBar], [arXiv:hep-ex/0507011[hep-ex]]	H. Y. Cheng, Phys. Rev. D 56, 2799-2811 (1997) J. G. Korner and M. Kramer, Z. Phys. C 55,659-670 (1992) Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 3836-3844 (1992)
Recently	R. Aaij et al. [LHCb], [arXiv:2308.08512[hep-ex]] X. Han et al. [Belle], JHEP 01 (2023), 055 Y. B. Li et al. [Belle], Phys. Rev. D 105 (2022) no.9, L091101 J. Yelton et al. [Belle], Phys. Rev. D 97 (2018) no.3 032001	C. W. Liu, [arXiv:2308.07754] K. L. Wang, Q. F. Lü, J. J. Xie and X. H. Zhong, Phys. Rev. D 107, no.3, 034015 (2023) T. M. Aliev, S. Bilmis and M. Savci, Phys. Rev. D 106, no.7, 074022 (2022) Y. K. Hsiao, L. Yang, C. C. Lih and S. Y. Tsai, Eur. Phys. J. C 80, no.11, 1066 (2020) S. Hu, G. Meng and F. Xu, Phys. Rev. D 101, no.9 094033 (2020)

Introduction

We have some experimental results of Ω_c weak decays

$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- e^+ \nu_e)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} = 1.98 \pm 0.13 \pm 0.08$$

$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} = 1.94 \pm 0.18 \pm 0.10$$

Y. B. Li et al. [Belle], Phys. Rev. D 105 (2022) no.9, L091101

$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- \rho^+)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} > 1.3$$

J. Yelton et al. [Belle], Phys. Rev. D 97 (2018) no.3 032001

$$\frac{\mathcal{B}(\Omega_c^0 \to \Xi^- \pi^+)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} = 0.1581 \pm 0.0087 \pm 0.0043 \pm 0.0016$$

$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- K^+)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} = 0.0608 \pm 0.0051 \pm 0.0040$$

R. Aaij et al. [LHCb], [arXiv:2308.08512[hep-ex]]

They are all branching fractions ratios with branching fraction of $\Omega_c^0 \to \Omega^- \pi^+$

we couldn't obtain theorical results from QCD
obviously we need model calculation

Introduction

Characteristic of Ω_c weak decays in model calculations

Branching fraction of $\Omega_c^0 \to \Omega^- \pi^+$ (denominator of ratio)

Difficulty: It is a decay of $1/2^+$ to $3/2^+$, need to determine the wave functions

Simplicity: Only receive factorizable contributions

Branching fraction of $\Omega_c^0 \to B_f\ M(l\nu)$ (numerator of ratio)

Difficulty: Need to consider both factorizable and nonfactorizable contributions.

Simplicity: Some of them are the decays of $1/2^+$ to $1/2^+$

Our work: current algebra + nonrelativistic quark model

Kinematics of $\frac{1}{2}^+ \to \frac{3}{2}^+$

• Hamiltonian

General expression: $H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left(c_1 \ O_1 \ + c_2 \ O_2 \ \right) + H.c.$

Amplitudes

Form factors

$$\langle \ B_f \ (p_f)|V_\mu|B_i(p_i)\rangle = \bar{u}_f^\nu[\bar{f}_1(q^2)g_{\nu\mu} + \bar{f}_2(q^2)p_{1\nu}\gamma_\mu + \bar{f}_3(q^2)p_{1\nu}p_{2\mu}]\gamma_5$$

$$\langle \ B_f \ (p_f)|A_\mu|B_i(p_i)\rangle = \bar{u}_f^\nu[\bar{g}_1(q^2)g_{\nu\mu} + \bar{g}_2(q^2)p_{1\nu}\gamma_\mu + \bar{g}_3(q^2)p_{1\nu}p_{2\mu}]$$

Quark models | Lattice QCD | Other methods

Rarita-Schwinger vector spinor

$$u_1^\mu = (u_1^0, \vec{u}_1) = (0, \vec{\epsilon}_1 u_\uparrow)$$

$$u_2^\mu = (u_2^0, \vec{u}_2) = \left(\sqrt{\frac{2}{3}}\frac{|\vec{p}|}{m}u_\uparrow, \frac{1}{\sqrt{3}}\vec{\epsilon}_1 u_\downarrow - \sqrt{\frac{2}{3}}\frac{E}{m}\vec{\epsilon}_3 u_\uparrow\right)$$

$$u_3^\mu = (u_3^0, \vec{u}_3) = \left(\sqrt{\frac{2}{3}}\frac{|\vec{p}|}{m}u_\downarrow, \frac{1}{\sqrt{3}}\vec{\epsilon}_2 u_\uparrow - \sqrt{\frac{2}{3}}\frac{E}{m}\vec{\epsilon}_3 u_\downarrow\right)$$

$$u_4^\mu = (u_4^0, \vec{u}_4) = (0, \vec{\epsilon}_2 u_\downarrow)$$

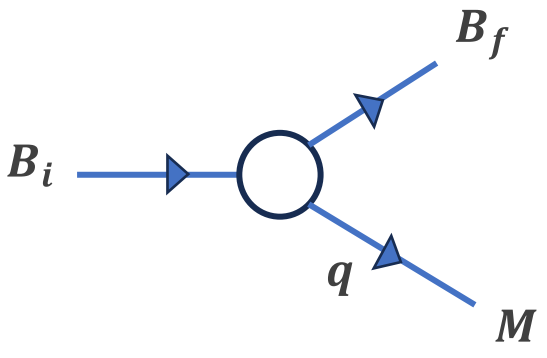
D. Lurie, Particles and Fields (Interscience, New York, 1969)

Kinematics

- **Observable**

Non-leptonic weak decay

$$\Gamma(\mathcal{B}_i \to \mathcal{B}_f M) = \frac{p_c}{16\pi m_i^2} H_M, \quad (M = P, V)$$



P

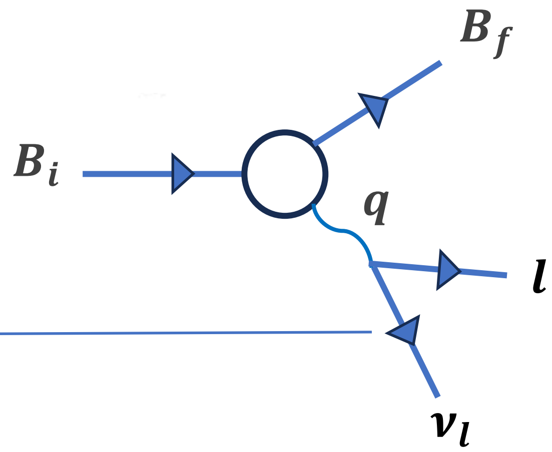
$$H_P = \frac{Q_+ Q_-}{3m_f^2}\left(Q_+|C|^2 + Q_-|D|^2\right)$$

V

$$H_V = \Big\{ 2\left[Q_+|C_1|^2 + Q_-|D_1|^2\right] + \frac{2}{3}\left[Q_+\left|C_1 - \frac{Q_-}{m_f}C_2\right|^2 + Q_-\left|D_1 - \frac{Q_+}{m_f}D_2\right|^2\right]$$
$$+ \frac{1}{3m_f^2 q^2}\left[Q_+\left|(M_+M_- - q^2)C_1 + Q_-M_+C_2 + \frac{Q_+Q_-}{2}C_3\right|^2\right.$$
$$\left. + Q_-\left|(M_+M_- - q^2)D_1 - Q_+M_-D_2 + \frac{Q_+Q_-}{2}D_3\right|^2\right]\Big\}$$

Semi-leptonic weak decay

$$\Gamma(\mathcal{B}_i \to \mathcal{B}_f \ell^+ \nu_\ell) = \frac{1}{192\pi^3 m_i^2}\int_{m_\ell^2}^{(m_i - m_f)^2} \frac{(q^2 - m_\ell^2)^2 p_c}{q^2} H_\ell \, dq^2$$



H_ℓ

$$M_\pm = m_i \pm m_f$$

$$Q_\pm = M_\pm^2 - q^2$$

Kinematics

- **Form factors expressions**

Take $\bar{f}_1$ as an example

Form factor definition
+
Vector-spinor/spinor expression

$$\langle \mathcal{B}_f(3/2)|V_x|\mathcal{B}_i(1/2)\rangle = \frac{|\vec{p}_f|}{\sqrt{2}}\frac{1}{2m_f}\bar{f}_1$$

Here we take x component of the vector current, and spin of B_i and B_f to be $\frac{1}{2}$ and $\frac{3}{2}$

$$\bar{f}_1 = \sqrt{2}N(\sigma_z)|_{(\frac{1}{2},\frac{1}{2})}X$$

To obtain the final result we need model calculation

Matrix element calculation

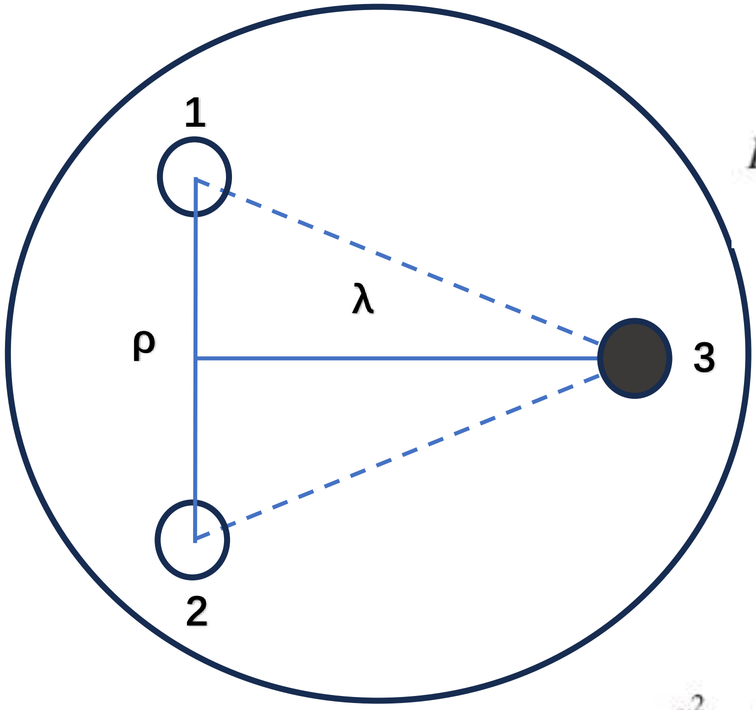
$$\langle \mathcal{B}_f(3/2)|V_x|\mathcal{B}_i(1/2)\rangle = \frac{|\vec{p}_f|}{2m_f}N(\sigma_x)|_{(\frac{3}{2},\frac{1}{2})}X$$

Such result depends on the calculations of both spin-flavor part and spatial part .

Model Calculation of Form Factors

Model Calculation of Form Factors

- **Nonrelativistic quark model**



Hamiltonian

$$H=\sum_{i=1}^{3}\frac{\mathbf{p}_i^2}{2m_i}+\frac{1}{2}K\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2, \quad \Rightarrow \quad H=\frac{\mathbf{p}^2}{2M}+\frac{\mathbf{p}_\rho^2}{2m_\rho}+\frac{\mathbf{p}_\lambda^2}{2m_\lambda}+\frac{1}{2}m_\rho\omega_\rho^2\boldsymbol{\rho}^2+\frac{1}{2}m_\lambda\omega_\lambda^2\boldsymbol{\lambda}^2$$

$$\boldsymbol{R}_c=\frac{m_1\boldsymbol{r}_1+m_2\boldsymbol{r}_2+m_3\boldsymbol{r}_3}{m_1+m_2+m_3},$$

$$\boldsymbol{\rho}=\boldsymbol{r}_1-\boldsymbol{r}_2,$$

$$\boldsymbol{\lambda}=\frac{m_1\boldsymbol{r}_1+m_2\boldsymbol{r}_2}{m_1+m_2}-\boldsymbol{r}_3.$$

$$m_\lambda=\frac{m_3(m_1+m_2)}{m_1+m_2+m_3}$$

$$m_\rho=\frac{m_1m_2}{m_1+m_2}$$

Harmonic oscillator strengths

$$\alpha_\rho^2=m_\rho\omega_\rho=\sqrt{\frac{3Km_1m_2}{2(m_1+m_2)}} \qquad \alpha_\lambda^2=m_\lambda\omega_\lambda=\sqrt{\frac{2Km_3(m_1+m_2)}{m_1+m_2+m_3}} \qquad \alpha_\lambda=\left[\frac{4m_3(m_1+m_2)^2}{3m_1m_2(m_1+m_2+m_3)}\right]^{\frac{1}{4}}\alpha_\rho$$

Baryon spectroscopy

F. E. Close and E. S. Swanson, Phys. Rev. D 72, 094004 (2005)

• Model calculations (Take $\bar{f}_1$ as an example)

Wave function

$$|\mathcal{B}(\boldsymbol{P}_c)_{J,M}\rangle = \sum_{S_z,M_L;c_i} \langle L, M_L; S, S_z|J, M\rangle \int d\boldsymbol{p}_\rho d\boldsymbol{p}_\lambda \chi^{S,S_z}_{s1,s2,s3} \Psi_{N,L,M_L}(\boldsymbol{P}_c, \boldsymbol{p}_\rho, \boldsymbol{p}_\lambda) \frac{\epsilon_{c_1c_2c_3}}{\sqrt{6}} \phi_{i_1,i_2,i_3} b_1^\dagger b_2^\dagger b_3^\dagger |0\rangle \ , \qquad \langle \mathcal{B}(\boldsymbol{P}'_c)_{J,M}|\mathcal{B}(\boldsymbol{P}_c)_{J,M}\rangle = \delta^3(\boldsymbol{P}'_c - \boldsymbol{P}_c)$$

$$\Psi_{LM_Ln_\rho l_\rho n_\lambda l_\lambda}(\boldsymbol{P}, \boldsymbol{p}_\rho, \boldsymbol{p}_\lambda) = \delta^3(\boldsymbol{P} - \boldsymbol{P}_c) \sum \langle LM_L|l_\rho m, l_\lambda M_L - m\rangle \psi_{n_\rho l_\rho m}(\boldsymbol{p}_\rho)\psi_{n_\lambda l_\lambda (M_L-m)}(\boldsymbol{p}_\lambda)$$

$$\psi_{nLm}(\boldsymbol{p}) = (i)^l(-1)^n\left[\frac{2n!}{(n+L+\frac{1}{2})!}\right]^{\frac{1}{2}} \frac{1}{\alpha^{L+\frac{3}{2}}} e^{-\frac{\boldsymbol{p}^2}{2\alpha^2}} L_n^{L+\frac{1}{2}}\left(\frac{\boldsymbol{p}^2}{\alpha^2}\right)\mathcal{Y}_{Lm}(\boldsymbol{p})$$

Select the current and spin components

$$\boldsymbol{p}_{\rho i} = \boldsymbol{p}_{\rho f}; \quad \boldsymbol{p}_{\lambda i} = \boldsymbol{p}_{\lambda f} + \frac{2m_l}{m_i}\boldsymbol{p}_f$$

$$\begin{aligned}\langle \mathcal{B}_f(3/2)|V_x|\mathcal{B}_i(1/2)\rangle &= \int d\boldsymbol{p}_{\rho i} d\boldsymbol{p}_{\lambda i} d\boldsymbol{p}_{\rho f} d\boldsymbol{p}_{\lambda f} \Psi_f^{*3/2}\Psi_i^{1/2}\langle q_3'q_2'q|\bar{q}\gamma_1 Q|Qq_2q_3\rangle \\ &= \int d\boldsymbol{p}_{\rho i} d\boldsymbol{p}_{\lambda i} d\boldsymbol{p}_{\rho f} d\boldsymbol{p}_{\lambda f} \Psi_f^{*3/2}\Psi_i^{1/2}\langle q_3'q_2'|q_2q_3\rangle\langle q|\bar{q}\gamma_1 Q|Q\rangle \\ &= \frac{|\boldsymbol{p}_f|}{2\sqrt{2}m_f}\bar{f}_1\end{aligned}$$

Delta function

$$\Rightarrow \int d\boldsymbol{p}_{\rho i} d\boldsymbol{p}_{\lambda i} \Psi^*_{(\boldsymbol{p}_{\rho i},\boldsymbol{p}_{\lambda i})}\left(\frac{\boldsymbol{p}_q}{2m_q} - \frac{\boldsymbol{p}_Q}{2m_Q}\right)\Psi_{(\boldsymbol{p}_{\rho f},\boldsymbol{p}_{\lambda f})} N(\sigma_x)|_{(\frac{3}{2},\frac{1}{2})} = \frac{|\boldsymbol{p}_f|}{2\sqrt{2}m_f}\bar{f}_1$$

Spin-Flavor

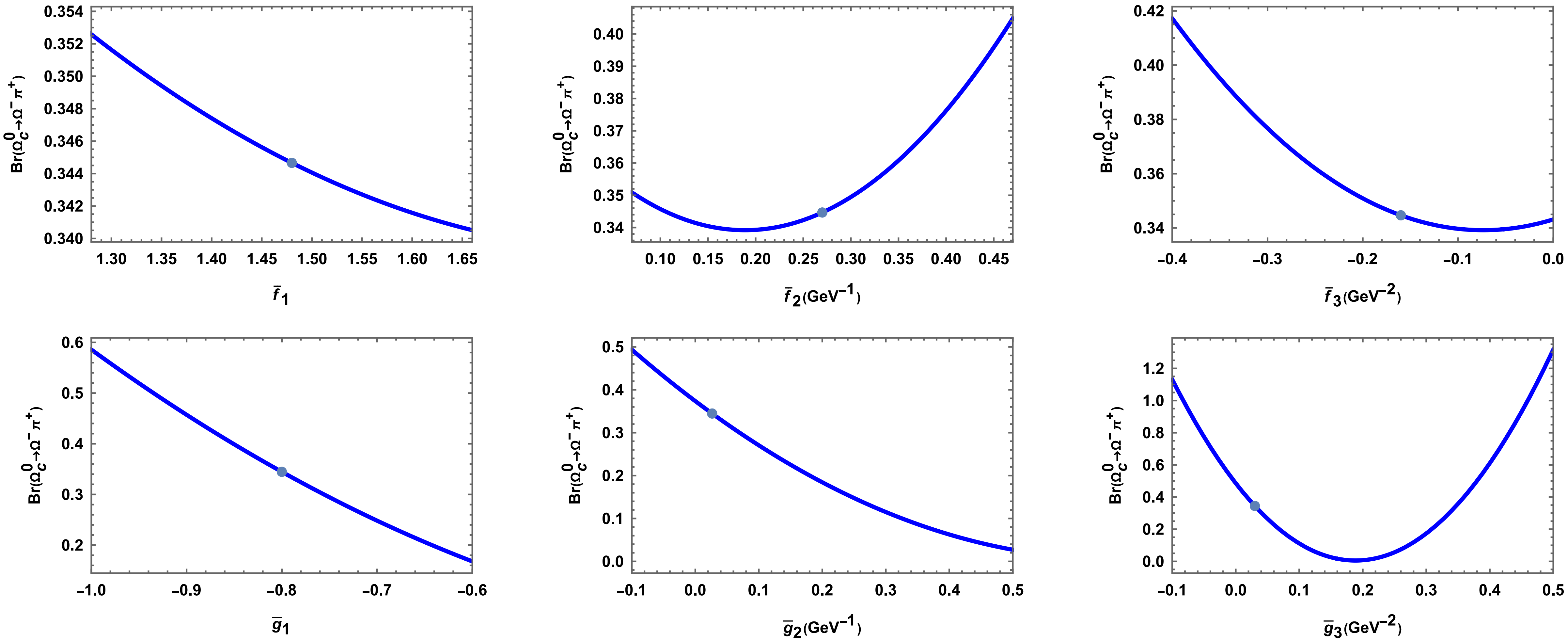
Result

$$\bar{f}_1 = \frac{2}{\sqrt{3}}\left(\frac{2\alpha_{\lambda i}\alpha_{\lambda f}}{\alpha_{\lambda i}^2 + \alpha_{\lambda f}^2}\right)^{3/2}\left[1 + \frac{2m_l\alpha_{\lambda f}^2}{m_q(\alpha_{\lambda i}^2 + \alpha_{\lambda f}^2)} + \frac{2m_l\alpha_{\lambda i}^2}{m_Q(\alpha_{\lambda i}^2 + \alpha_{\lambda f}^2)}\right]$$

Numerical Results

Numerical Results

- **Branching fraction results (model independent)**

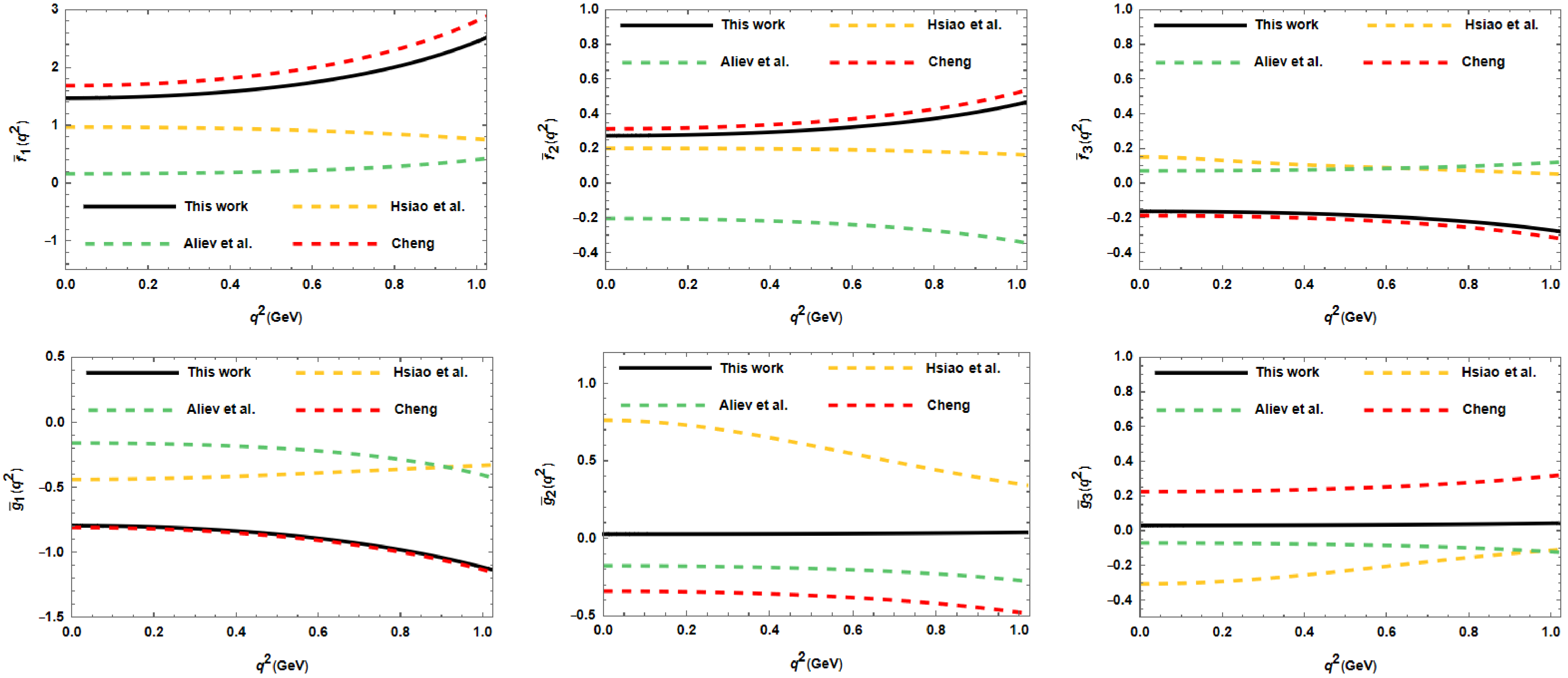


Dependences between branching fractions and form factors

S. Zeng, F. Xu, P. Y. Niu and H. Y. Cheng, Phys. Rev. D, no.3, 034009 (2023)

Numerical Results

- Form factors numerical results (q^2 dependent)



H. Y. Cheng, Phys. Rev. D 56, 2799-2811 (1997)

Y. K. Hsiao, L. Yang, C. C. Lih and S. Y. Tsai, Eur. Phys. J. C 80, no.11, 1066 (2020)

T. M. Aliev, S. Bilmis and M. Savci, Phys. Rev. D 106, no.7, 074022 (2022)

Numerical Results

- Numerical result

	This work	Cheng [1]	Wang et al.[2]	Gutsche et al.[3]	Hsiao et al.[4]	Aliev et al.[5]	Liu [6]	Experiments
Branching fractions								
$\Omega_c^0 \to \Omega^- \pi^+$	3.51%	4.19%	1.1%	0.2%	0.51%	2.9%	1.88%	
$\Omega_c^0 \to \Omega^- \rho^+$	18.39%	15.08%		1.9%	1.44±0.04%	6.3%		
$\Omega_c^0 \to \Xi^- \pi^+$	0.547%						0.94%	
$\Omega_c^0 \to \Omega^- e^+ \nu_e$	4.08%				0.54±0.02%	2.06%	2.54%	
$\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu$	3.83%				0.50±0.02%	1.96%		
Branching fraction ratios								
$R_{\Omega^- \rho^+}$	5.236	3.60		9.5	2.8±0.4	2.18		>1.3 [7]
$R_{\Xi^- \pi^+}$	0.158						0.5	0.158±0.011[8]
$R_{\Omega^- e^+ \nu_e}$	1.162				1.1±0.2	0.71	1.35	1.98±0.15[9]
$R_{\Omega^- \mu^+ \nu_\mu}$	1.091				1.059	0.68		1.94±0.21[9]
$R_{\Omega^- e^+ \nu_e}/R_{\Omega^- \mu^+ \nu_\mu}$	1.065				1.08	1.04		1.02±0.10[9]

[7] J. Yelton et al. [Belle], Phys. Rev. D 97 (2018) no.3, 032001 doi:10.1103/PhysRevD.97.032001
[8] R. Aaij et al. [LHCb], [arXiv:2308.08512 [hep-ex]].
[9] Y. B. Li et al. [Belle], Phys. Rev. D 105 (2022) no.9, L091101

[1] H. Y. Cheng, Phys. Rev. D 56, 2799-2811 (1997) [erratum: Phys. Rev. D 99, no.7, 079901 (2019)]
[2] K. L. Wang, Q. F. L¨u, J. J. Xie and X. H. Zhong, Phys. Rev. D 107, no.3, 034015 (2023)
[3] T. Gutsche, M. A. Ivanov, J. G. K¨orner and V. E. Lyubovitskij, Phys. Rev. D 98, no.7, 074011 (2018)
[4] Y. K. Hsiao, L. Yang, C. C. Lih and S. Y. Tsai, Eur. Phys. J. C 80, no.11, 1066 (2020)
[5]T. M. Aliev, S. Bilmis and M. Savci, Phys. Rev. D 106, no.7, 074022 (2022)
[6] C. W. Liu, [arXiv:2308.07754 [hep-ph]].

Summary

- Ω_c weak decays are important in many aspect.
- Experiments can provide Branching fraction ratio of Ω_c weak decays only, theoretical results are needed especially branching fraction of $\Omega_c^0 \to \Omega^- \pi^+$.
- $\Omega_c^0 \to \Omega^- \pi^+$ receive factorizable contribution only, NRQM could be a method to obtain its' form factor, which means obtain the observable .
- NRQM obtain good result fit the LHCb but could be improved.
- Since the results in numbers of models are quite different, Lattice QCD could be expected.

Thanks for your attention!