$\Xi_c - \Xi_c'$ mixing from Lattice QCD

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Phys.Lett.B 841 (2023) 137941 and arXiv:2309.05432

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第二十届重味物理和 CP 破坏研讨会

OUTLINE

- Motivation
- $\Xi_c - \Xi_c'$ mixing from two-point correlation functions
- An improved method to extract $\Xi_c - \Xi_c'$ mixing from three-point correlation functions
- Summary and outlook

Motivation

Experimental studies

$B_{\text{Belle}}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%,$ Phys. Rev. Lett. **127** no. 12, (2021)

$B_{\text{ALICE}}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.48 \pm 0.25 \pm 0.40 \pm 0.72)\%.$ Phys. Rev. Lett. **127** no. 27, (2021)

2022 Review of Particle Physics (using only the Belle measurement as input)

$B_{\text{PDG}}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.04 \pm 0.24)\%.$ PTEP **2022** (2022) 083C01.

Theoretical studies

$B_{\text{Lattice}}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.38 \pm 0.30 \pm 0.32)\%$ Zhang et al., (2022)

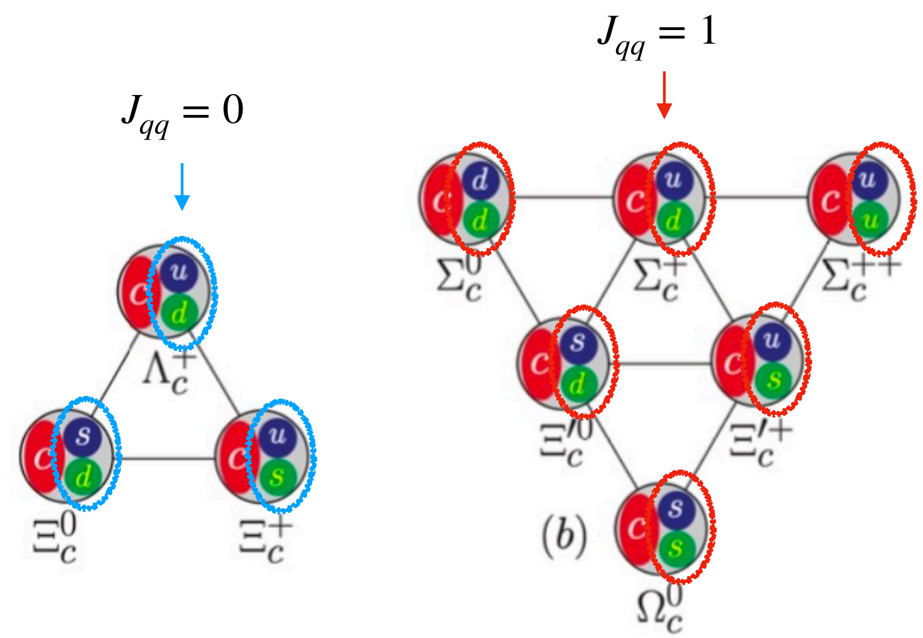
Method	$B(\Xi_c^0 \to \Xi^- e^+ \nu_e)$	
light-front quark model	$(3.49 \pm 0.95)\%$	Geng, Liu, and Tsai, 2021
QCD sum rules	$(3.4 \pm 1.7)\%$	Zhao, 2021
rel. quark model	$2.38\ \%^*$	Faustov and Galkin, 2019
SU(3)	$(3.0 \pm 0.3)\%^*$	Geng et al., 2019
light-front quark model	$1.35\ \%^*$	Zhao, 2018
SU(3)	$(4.87 \pm 1.74)\%^*$	Geng et al., 2018
SU(3)	$(11.9 \pm 1.6)\%^*$	Geng et al., 2017
light-cone QCD sum rules	$(7.26 \pm 2.54)\%^*$	Azizi, Sarac, and Sundu, 2012
QCD sum rules	$2.4\ \%^*$	Liu and Huang, 2010

Big discrepancy!

A competitive explanation is through the $\Xi_c - \Xi_c'$ mixing

$\Xi_c - \Xi_c'$ mixing

- In **heavy quark limit**, heavy baryon with one **charm quark** can be classified according to angular momentum J_{qq} of **light quark system**.



Triplet   Sextet

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{+\prime}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{0\prime}}{\sqrt{2}} \\ \frac{\Xi_c^{+\prime}}{\sqrt{2}} & \frac{\Xi_c^{0\prime}}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}$$

- Mixing effect can be described by a 2 × 2 matrix

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi_c'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^{6}\rangle \end{pmatrix}$$

Energy eigenstates   **Flavor eigenstates**

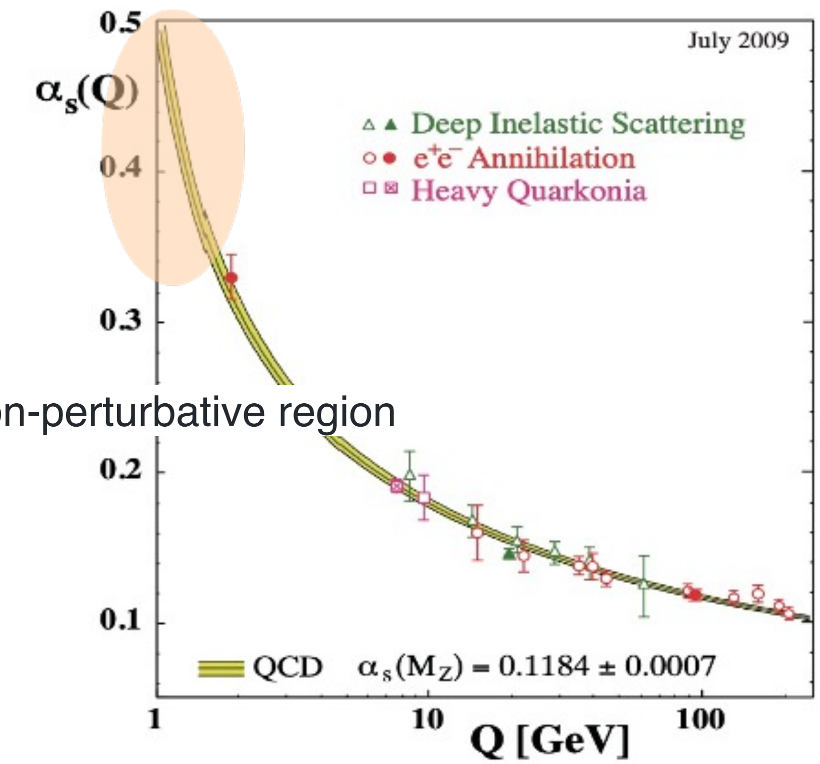
$\Xi_c - \Xi_c'$ mixing

- **Some results from various methods**

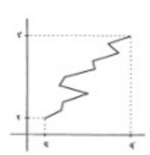
Sum rule	$5.5° \pm 1.8°$	*Phys. Rev. D 83, 016008 (2011);*
HQET	$8.12° \pm 0.80°$	*Nucl. Phys. A 1008, 122139, (2021);*
Quark model	$16.27° \pm 2.30°$	*Phys. Rev. D 105, 096011 (2022);*
	$24.66° \pm 0.90°$	*Phys. Lett. B 838, 137736, (2023);* *Phys. Lett. B 839, 137831, (2023);*
Lattice QCD	**Negligibly small**	*Phys. Rev. D 90, 094507, (2014).*

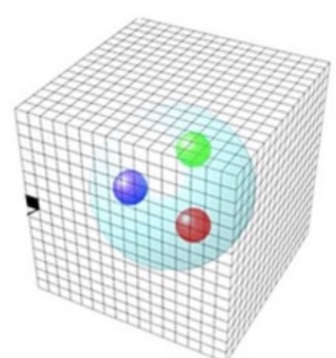
Lattice QCD

- ⭐ QCD is the theory to describe the strong interaction；
- ⭐ When the energy scale is lowered, the interaction becomes stronger, and QCD enters into the non-perturbative region.
- ⭐ Lattice QCD(Wilson,1974): Non-perturbative methods based on the first principles.

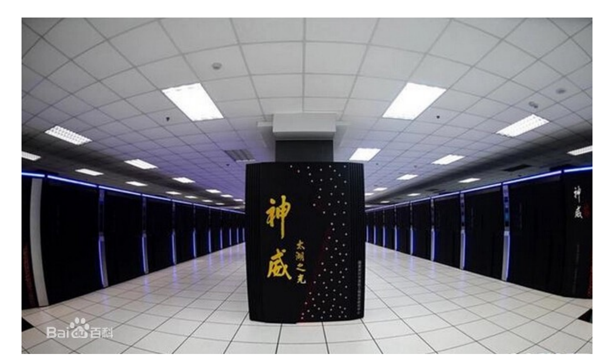


K. G. Wilson



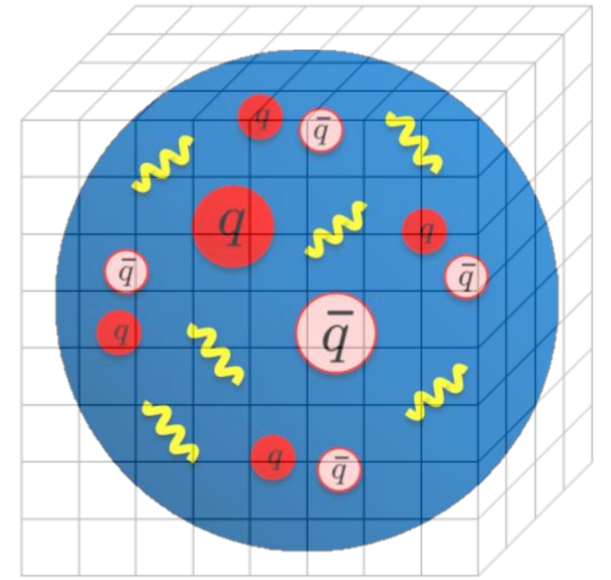

Lattice QCD

$> 2^{1000000}$


Super Computer

Lattice simulation

$$S_E^{latt} = \underbrace{\sum \frac{6}{g^2} ReTr(1 - U_P)}_{\text{wilson gauge action}} + \sum_q \underbrace{\bar{q}(D_\mu^{lat}\gamma_\mu + am_q)q}_{\text{lattice fermion action}}$$



- **Charm quark on discrete lattice:**

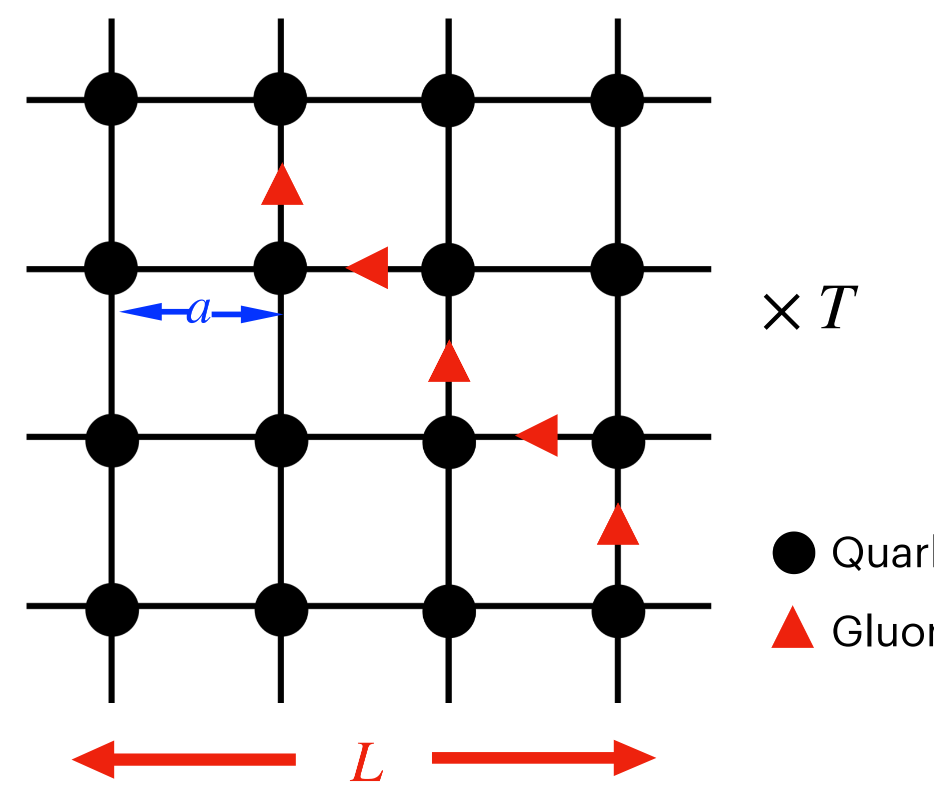
consider both IR and UV effects:

$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

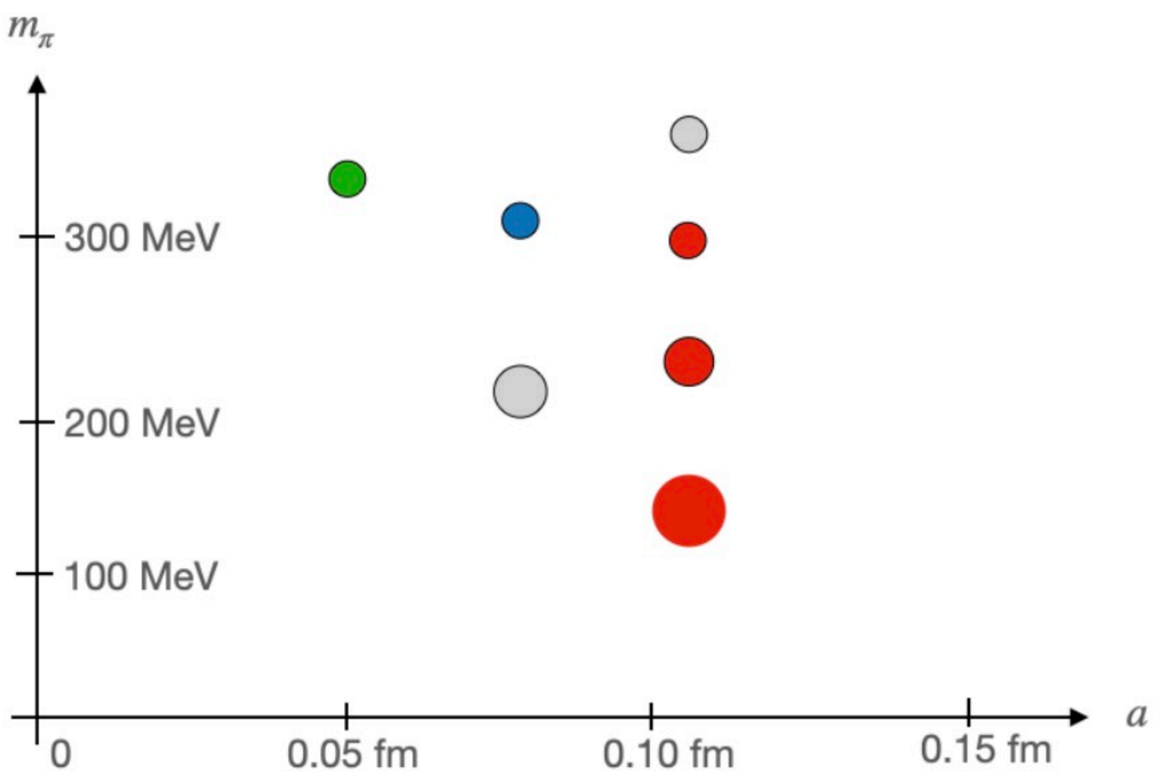
for $m_\pi = m_\pi^{\text{phy}} \sim 140\text{MeV}$**, and** $m_c \simeq 1.3\text{GeV}$,

$$L \gtrsim 5.6\text{fm}$$

$$a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$$



CLQCD ensembles



$\Xi_c - \Xi_c'$ mixing from Lattice QCD

H.Liu et at., Phys.Lett.B 841 (2023) 137941

An improved method to determine the $\Xi_c - \Xi_c'$ mixing

H. Liu et al., arXiv:hep-ph/2309.05432

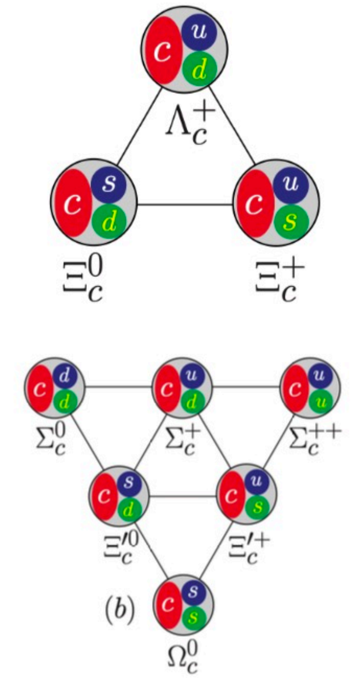
Ensemble	β	$L^3 \times T$	a (fm)	m_l^{b}	m_s^{b}	m_c^{b}	m_π	N_{meas}
C11P14L		$48^3 \times 96$		-0.2825	-0.2310	0.4800	135	203×48
C11P22M	6.20	$32^3 \times 64$	0.108	-0.2790	-0.2310	0.4800	222	451×20
C11P29S		$24^3 \times 72$		-0.2770	-0.2315	0.4780	284	432×26
C08P30S	6.41	$32^3 \times 96$	0.080	-0.2295	-0.2010	0.2326	297	653×26
C06P30S	6.72	$48^3 \times 144$	0.055	-0.1850	-0.1687	0.0770	312	136×80

$\Xi_c - \Xi_c'$ mixing from LQCD

- **Baryonic operators of $SU(3)_F$ eigenstates**

$$O^{\bar{3}}_{SU(3)} = \epsilon^{abc}(\underbrace{q^{Ta}C\gamma_5 s^b}_{J=0})P_+ c^c,$$

$$O^{6}_{SU(3)} = \epsilon^{abc}(\underbrace{q^{Ta}C\vec{\gamma}s^b}_{J=1})\cdot\vec{\gamma}\gamma_5 P_+ c^c$$



- **Build the 2x2 correlation function matrix of lattice calculation**

$$\mathcal{C}(t,t_0) = \sum_{\vec{x}} \begin{pmatrix} \langle O^{\bar{3}}_p(\vec{x},t)\bar{O}^{\bar{3}}_w(\vec{0},t_0)\rangle & \langle O^{\bar{3}}_p(\vec{x},t)\bar{O}^{6}_w(\vec{0},t_0)\rangle \\ \langle O^{6}_p(\vec{x},t)\bar{O}^{\bar{3}}_w(\vec{0},t_0)\rangle & \langle O^{6}_p(\vec{x},t)\bar{O}^{6}_w(\vec{0},t_0)\rangle \end{pmatrix}$$

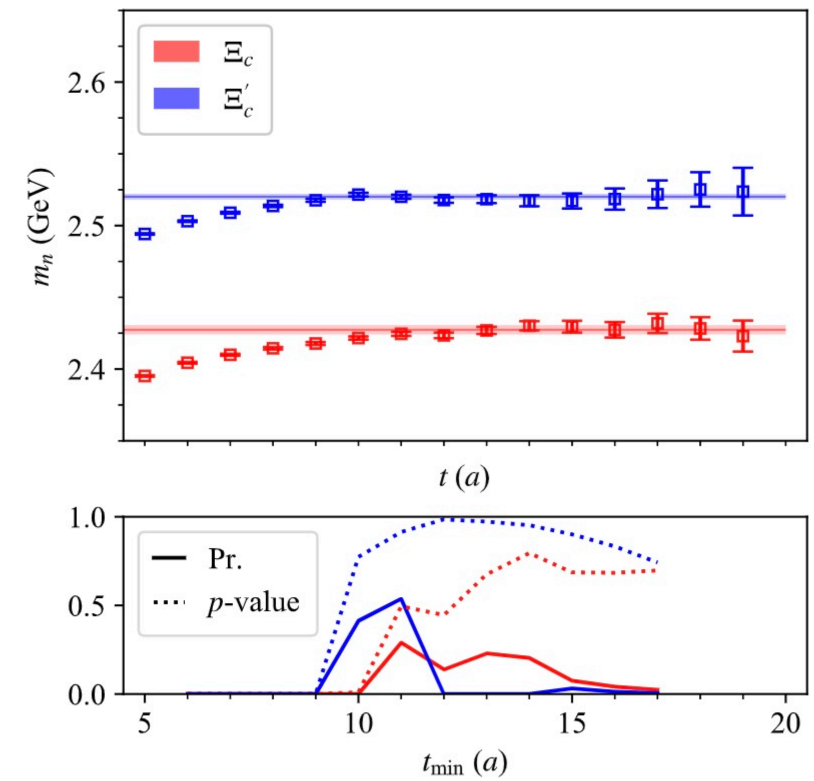
Mixing angle from generalized eigenvalue problem(GEVP)

- **Solving the generalized eigenvalue problem:**

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

- **Mass from eigenvalues**

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

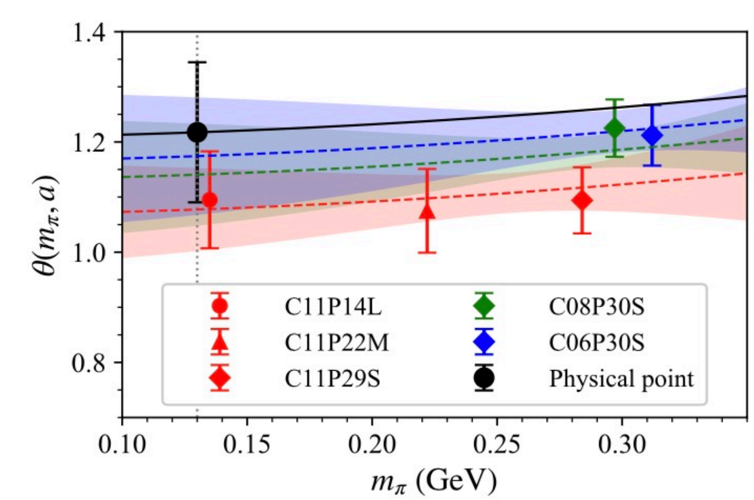


Model average method

- **Mixing angle from eigenvectors**

$$v_1 = \sqrt{1 + \frac{A_p^2 \cot^2\theta}{B_p^2}} \begin{pmatrix} \frac{A_p}{B_p}\cot\theta \\ 1 \end{pmatrix},$$

$$v_2 = \sqrt{1 + \frac{A_p^2 \tan^2\theta}{B_p^2}} \begin{pmatrix} -\frac{A_p}{B_p}\tan\theta \\ 1 \end{pmatrix}$$



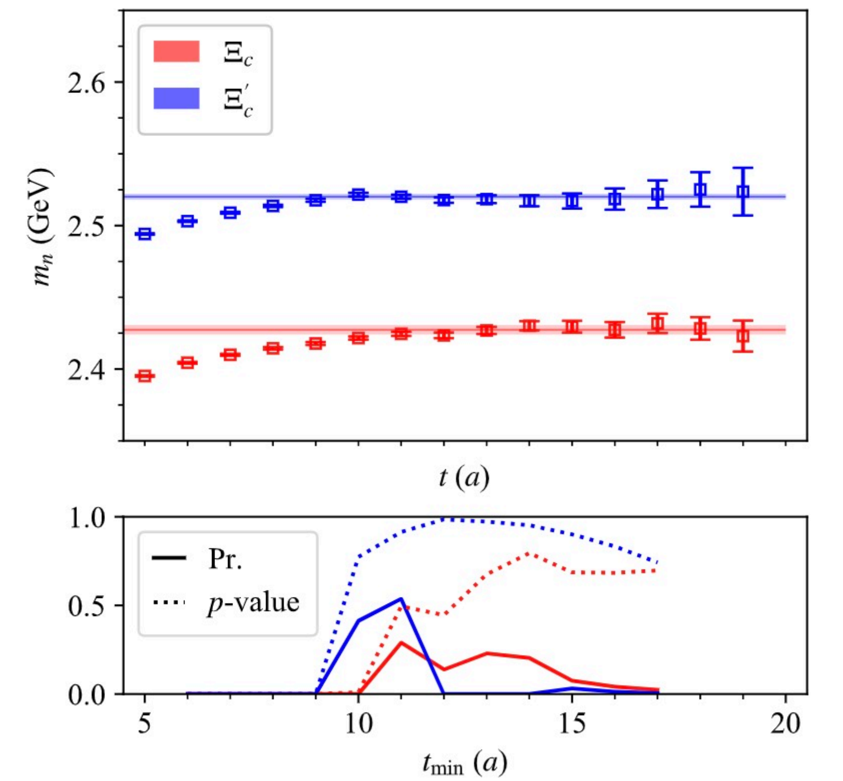
Mixing angle from generalized eigenvalue problem(GEVP)

- **Solving the generalized eigenvalue problem:**

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

- **Mass from eigenvalues** $\mathcal{C}(t_r)^{-1}\mathcal{C}(t)v_n(t) = \lambda_n(t)v_n(t)$

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$



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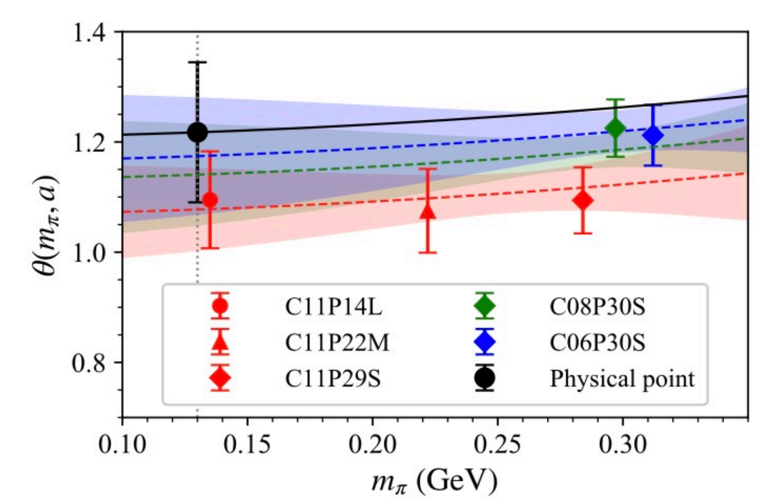

Mixing angle from generalized eigenvalue problem(GEVP)

- **Solving the generalized eigenvalue problem:**

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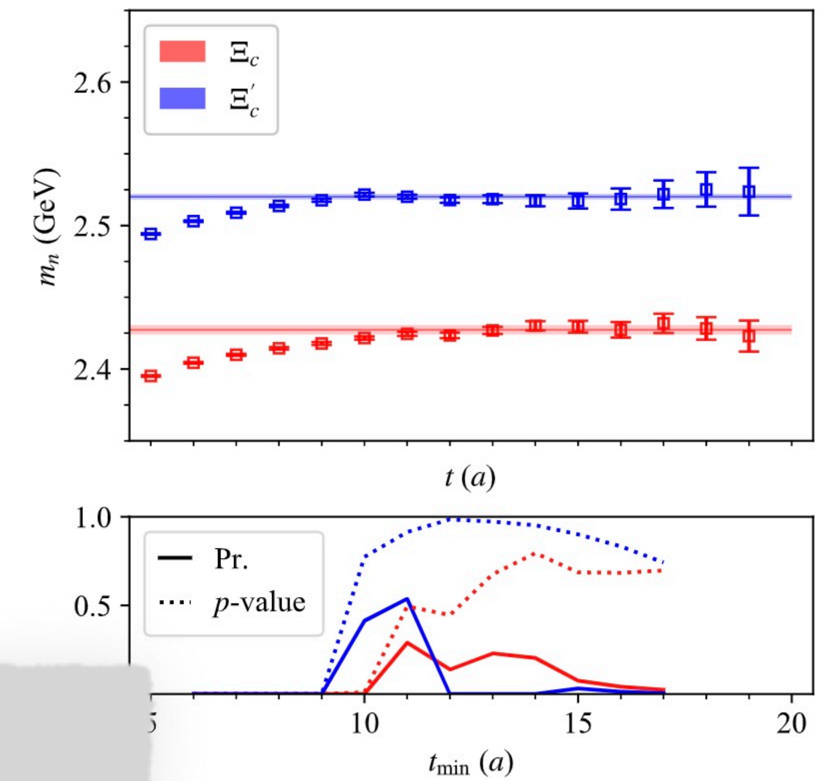


Model average method

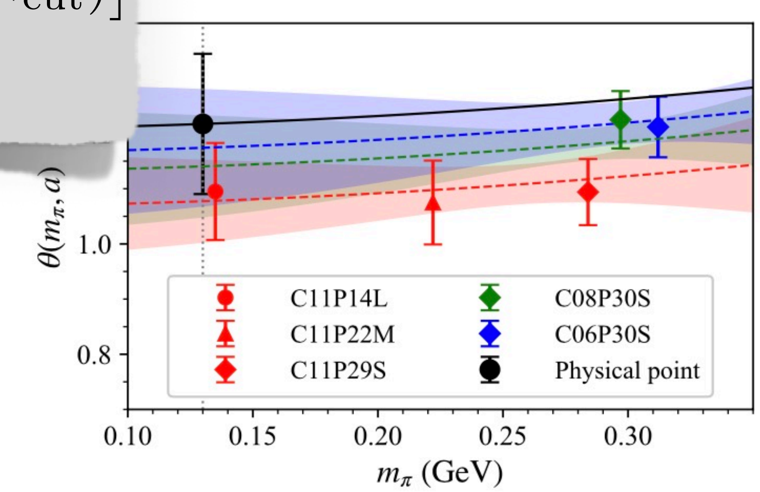
$$\langle a\rangle = \sum_M \langle a\rangle_M \mathrm{Pr}(M|D),$$

$$\mathrm{Pr}(M|D) \approx \exp\Big[-\frac{1}{2}(\chi^2_{\rm aug}(\mathbf{a}^*) + 2\mathrm{k} + 2\mathrm{N}_{\rm cut})\Big]$$

- **Mixing angle from e**

$$v_1 = \sqrt{1 + \frac{A_p^2\cot^2\theta}{B_p^2}}\begin{pmatrix}\frac{A_p}{B_p}\cot\theta\\ 1\end{pmatrix},$$

$$v_2 = \sqrt{1 + \frac{A_p^2\tan^2\theta}{B_p^2}}\begin{pmatrix}-\frac{A_p}{B_p}\tan\theta\\ 1\end{pmatrix}$$



Mixing angle from generalized eigenvalue problem(GEVP)

- **Solving the generalized eigenvalue problem:**

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

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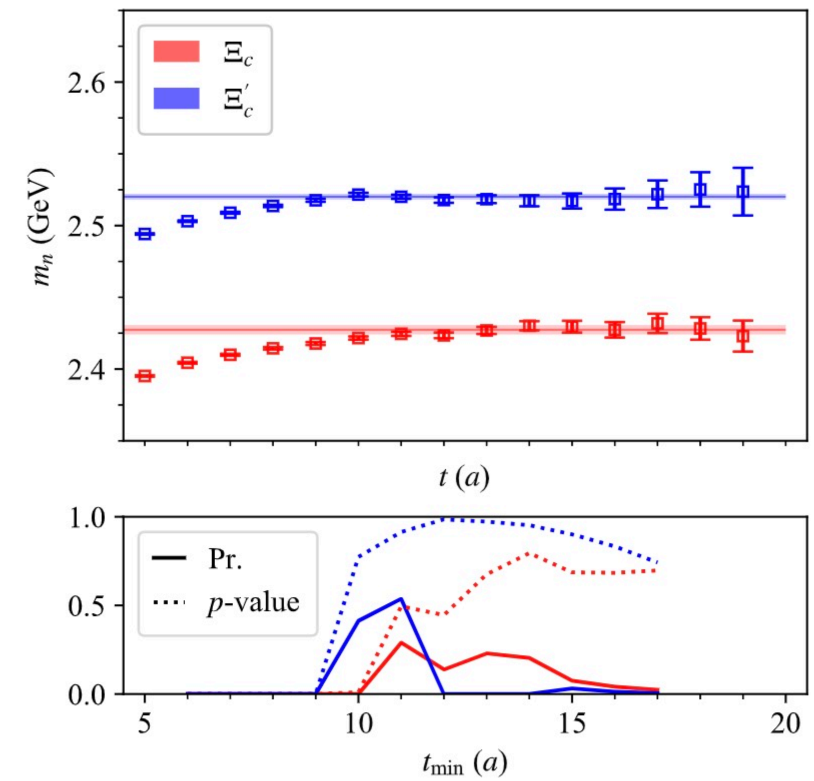

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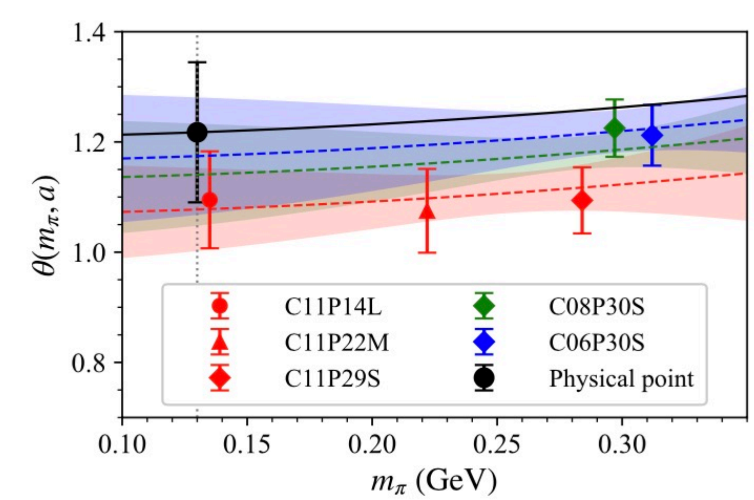
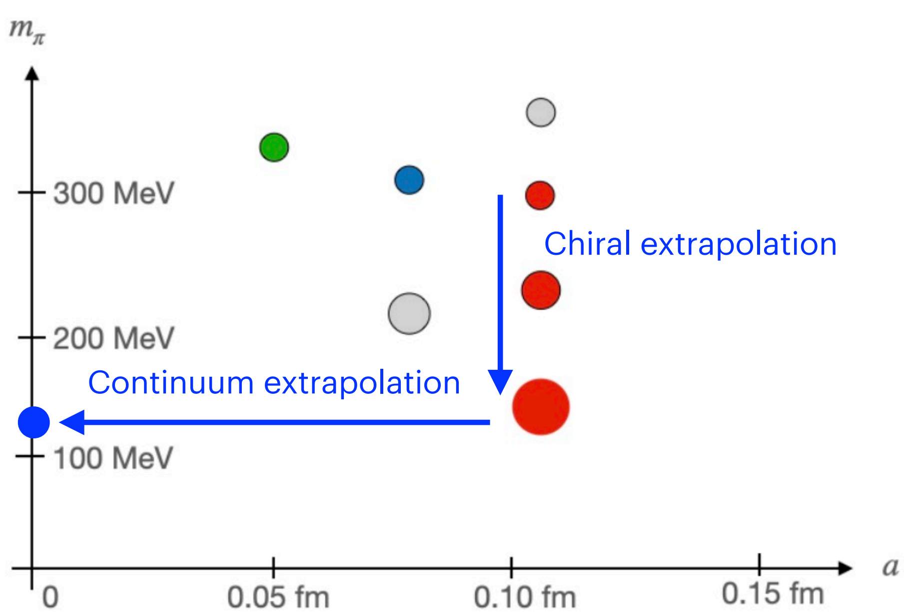
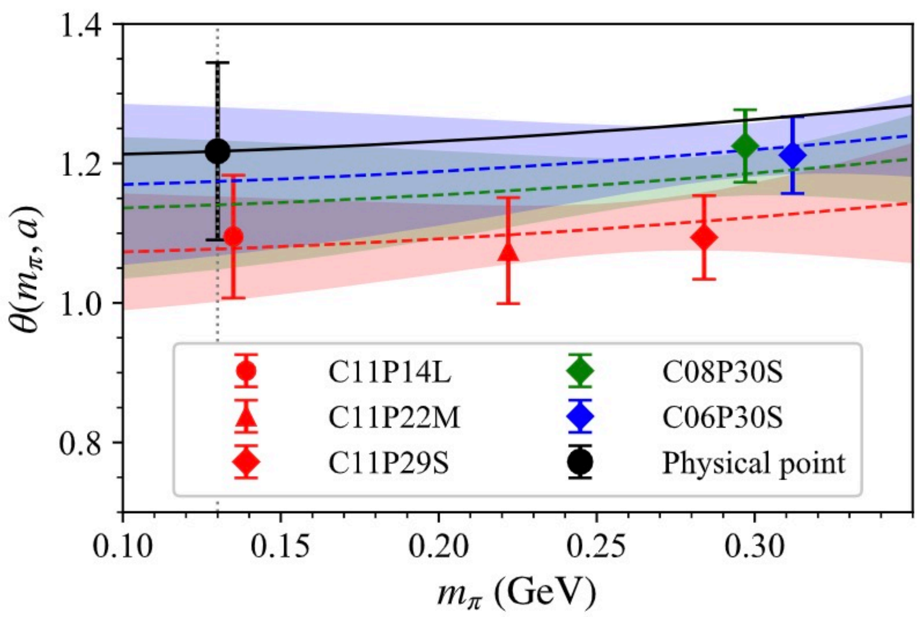


Chiral and continuum extrapolation



- **Extrapolation formula:**

$$\theta(m_\pi, a) = \theta_{\rm phy} + c_1\left(m_\pi^2 - m_{\pi,\rm phy}^2\right) + c_2 a^2,$$
$$m_n(m_\pi, a) = m_{n,\rm phy} + c_1\left(m_\pi^2 - m_{\pi,\rm phy}^2\right) + c_2 a^2$$



$$\theta = (1.22 \pm 0.13 \pm 0.01)^\circ$$

Mixing angle from correlated joint fit

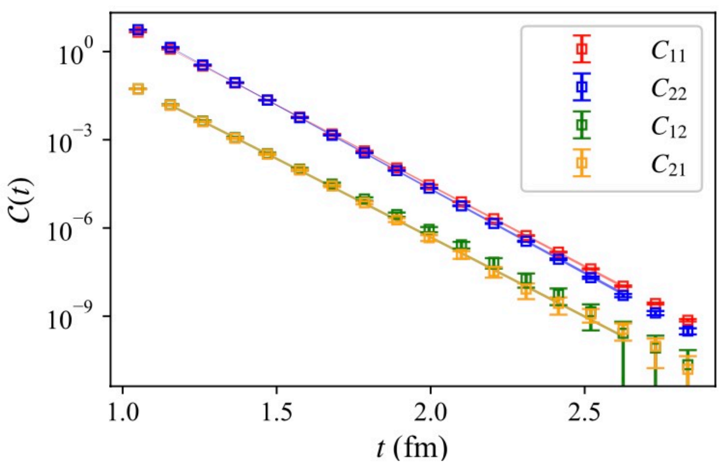
- **Insert the mass eigenstates and consider the excited state contributions are greatly suppressed, parametrization form of correlation function matrix elements:**

$$C_{11}(t,t_0) = A_p A_w^\dagger \left[\frac{\cos^2\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\sin^2\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right]$$

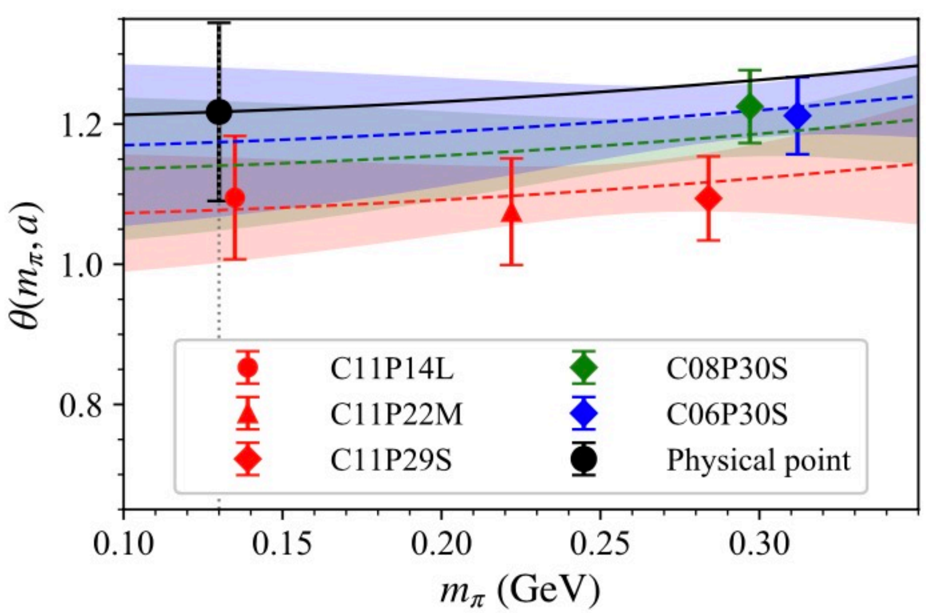
$$C_{12}(t,t_0) = A_p B_w^\dagger \left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

$$C_{21}(t,t_0) = B_p A_w^\dagger \left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

$$C_{22}(t,t_0) = B_p B_w^\dagger \left[\frac{\sin^2\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\cos^2\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right]$$



- **One can extract the mixing parameters from a joint analysis of both diagonal and non-diagonal terms.**



	m_{Ξ_c} (GeV)	$m_{\Xi_c'}$ (GeV)	θ (°)	χ^2/d.o.f	fit range (fm)
C11P14L	2.4256(19)	2.5196(22)	1.083(30)	0.96	1.19 − 2.81
C11P22M	2.4380(27)	2.5351(30)	0.988(49)	1.0	1.19 − 2.92
C11P29S	2.4587(27)	2.5536(29)	1.002(50)	1.1	1.19 − 3.24
C08P30S	2.4753(21)	2.5809(26)	1.080(42)	0.95	1.20 − 2.40
C06P30S	2.4695(37)	2.5815(48)	1.021(67)	1.2	1.32 − 2.40
Extrapolated	$2.4380(68)_{\mathrm{stat}}(403)_{\mathrm{syst}}$	$2.5562(74)_{\mathrm{stat}}(422)_{\mathrm{syst}}$	$1.20(9)_{\mathrm{stat}}(2)_{\mathrm{syst}}$		
Exp. data [19]	$2.46794^{+0.00017}_{-0.00020}$	2.5784 ± 0.0005	—		

m_c dependence

☆In HQET, the mixing would vanish in the heavy-quark limit.

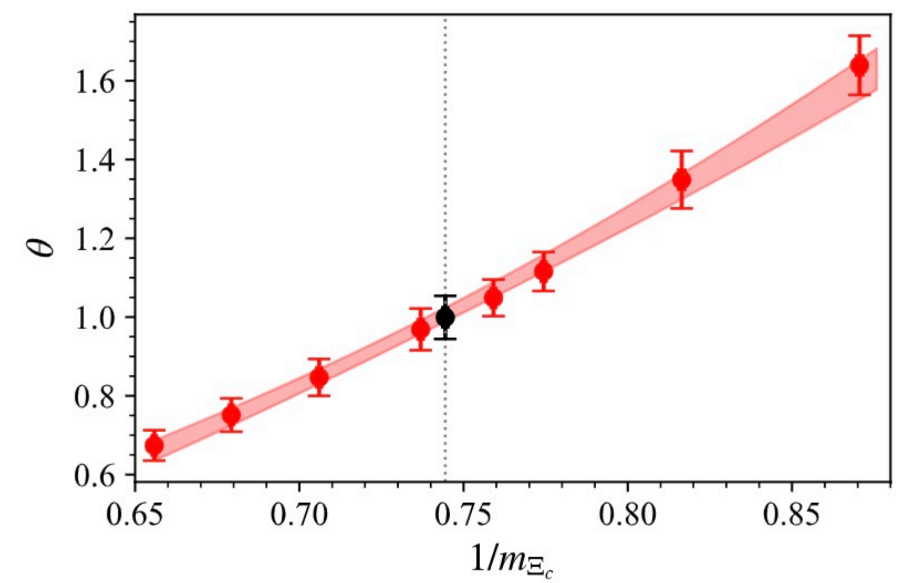
☆Valence quark mass in tunable in lattice QCD.

◆Solve the baryon masses and mixing angle from different charm quark masses;

◆Then fit the m_c dependence of θ:

$$\theta = \frac{B_1}{m_{\Xi_c}} + \frac{B_2}{m_{\Xi_c}^2}$$

$B_1 = -2.78(52)\mathrm{GeV}$
$B_2 = 12.9(1.3)\mathrm{GeV}^2$
$\chi^2/\mathrm{d.o.f} = 0.11.$



m_c^{b}	0.2	0.3	0.4	0.44	**0.478**	0.5	0.6	0.7	0.8
m_{Ξ_c} (GeV)	2.0987(25)	2.2380(28)	2.3594(26)	2.4069(26)	**2.4587(27)**	2.4793(29)	2.5878(30)	2.6898(30)	2.7859(31)
$m_{\Xi_c'}$ (GeV)	2.1834(24)	2.3249(29)	2.4514(24)	2.4999(24)	**2.5536(29)**	2.5718(29)	2.6823(29)	2.7859(30)	2.8835(30)
θ (°)	1.639(75)	1.349(73)	1.116(49)	1.049(46)	**1.002(50)**	0.969(53)	0.847(47)	0.751(42)	0.674(39)

An improved method of $\Xi_c - \Xi_c'$ mixing

◆ The QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - M)\psi$$

with

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$\Delta\mathcal{L} = -\bar{s}(m_s - m_u)s.$$

◆ Therefore, the Hamiltonian is correspondingly derived as

$$H = \int d^3\vec{x} \left[\frac{\partial\mathcal{L}}{\partial\dot{\psi}(\vec{x})}\dot{\psi}(\vec{x}) + \frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}(\vec{x})}\dot{\bar{\psi}}(\vec{x}) - \mathcal{L} \right]$$
$$\equiv H_0 + \Delta H,$$

with

$$\Delta H = (m_s - m_u)\int d^3\vec{x}\bar{s}s(\vec{x}).$$

SU(3) flavor symmetry breaking

Energy(mass) eigenstates and Flavor eigenstates

$$M_E(\vec{p}) \equiv \int \frac{d^3\vec{p'}}{(2\pi)^3} \times \begin{pmatrix} \langle \Xi_c(\vec{p})|H|\Xi_c(\vec{p'})\rangle & \langle \Xi_c(\vec{p})|H|\Xi'_c(\vec{p'})\rangle \\ \langle \Xi'_c(\vec{p})|H|\Xi_c(\vec{p'})\rangle & \langle \Xi'_c(\vec{p})|H|\Xi'_c(\vec{p'})\rangle \end{pmatrix}$$

$$M_E(\vec{p}=0) \equiv \begin{pmatrix} 2m^2_{\Xi_c} & 0 \\ 0 & 2m^2_{\Xi'_c} \end{pmatrix}.$$

Rotation

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi'_c\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^{6}\rangle \end{pmatrix}$$

$$\begin{aligned} M_{F,11} &= 2\cos^2\theta m^2_{\Xi_c} + 2\sin^2\theta m^2_{\Xi'_c}, \\ M_{F,12} &= 2\cos\theta\sin\theta(m^2_{\Xi_c} - m^2_{\Xi'_c}), \\ M_{F,21} &= 2\cos\theta\sin\theta(m^2_{\Xi_c} - m^2_{\Xi'_c}), \\ M_{F,22} &= 2\sin^2\theta m^2_{\Xi_c} + 2\cos^2\theta m^2_{\Xi'_c}, \end{aligned}$$

$$\begin{aligned} M_F(\vec{p}) &\equiv \int \frac{d^3\vec{p'}}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p})|H|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{\bar{3}}(\vec{p})|H|\Xi_c^{6}(\vec{p'})\rangle \\ \langle \Xi_c^{6}(\vec{p})|H|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{6}(\vec{p})|H|\Xi_c^{6}(\vec{p'})\rangle \end{pmatrix} \\ &= \int \frac{d^3\vec{p'}}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p})|(H_0+\Delta H)|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{\bar{3}}(\vec{p})|\Delta H|\Xi_c^{6}(\vec{p'})\rangle \\ \langle \Xi_c^{6}(\vec{p})|\Delta H|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{6}(\vec{p})|(H_0+\Delta H)|\Xi_c^{6}(\vec{p'})\rangle \end{pmatrix}. \end{aligned}$$

$|\Xi_c^{\bar{3}/6}\rangle$ are eigenstate of H_0 under the $SU(3)_F$ symmetry

$$M_F\ (\vec{p}=0) = \begin{pmatrix} 2m^2_{\Xi_c^{\bar{3}}} & 0 \\ 0 & 2m^2_{\Xi_c^{6}} \end{pmatrix} + (m_s - m_u) \times \begin{pmatrix} \langle \Xi_c^{\bar{3}}|\bar{s}s(\vec{x}=0)|\Xi_c^{\bar{3}}\rangle & \langle \Xi_c^{\bar{3}}|\bar{s}s(\vec{x}=0)|\Xi_c^{6}\rangle \\ \langle \Xi_c^{6}|\bar{s}s(\vec{x}=0)|\Xi_c^{\bar{3}}\rangle & \langle \Xi_c^{6}|\bar{s}s(\vec{x}=0)|\Xi_c^{6}\rangle \end{pmatrix}$$

$$M_{F,11} = 2m^2_{\Xi_c^{\bar{3}}} + (m_s - m_u)M_{\bar{s}s}^{\bar{3}-\bar{3}}$$

$$M_{F,22} = 2m^2_{\Xi_c^{6}} + (m_s - m_u)M_{\bar{s}s}^{6-6}$$

$$M_{F,12} = (m_s - m_u)M_{\bar{s}s}^{\bar{3}-6}$$

$$M_{F,21} = (m_s - m_u)M_{\bar{s}s}^{6-\bar{3}}$$

Energy(mass) eigenstates and Flavor eigenstates

$$M_E(\vec{p}) \equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \times \begin{pmatrix} \langle \Xi_c(\vec{p})|H|\Xi_c(\vec{p}')\rangle & \langle \Xi_c(\vec{p})|H|\Xi'_c(\vec{p}')\rangle \\ \langle \Xi'_c(\vec{p})|H|\Xi_c(\vec{p}')\rangle & \langle \Xi'_c(\vec{p})|H|\Xi'_c(\vec{p}')\rangle \end{pmatrix}$$

$$M_E(\vec{p}=0) \equiv \begin{pmatrix} 2m^2_{\Xi_c} & 0 \\ 0 & 2m^2_{\Xi'_c} \end{pmatrix}.$$

Rotation

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi'_c\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi^{\bar{3}}_c\rangle \\ |\Xi^{6}_c\rangle \end{pmatrix}$$

$$\begin{aligned} M_{F,11} &= 2\cos^2\theta m^2_{\Xi_c} + 2\sin^2\theta m^2_{\Xi'_c}, \\ M_{F,12} &= 2\cos\theta\sin\theta(m^2_{\Xi_c} - m^2_{\Xi'_c}), \\ M_{F,21} &= 2\cos\theta\sin\theta(m^2_{\Xi_c} - m^2_{\Xi'_c}), \\ M_{F,22} &= 2\sin^2\theta m^2_{\Xi_c} + 2\cos^2\theta m^2_{\Xi'_c}, \end{aligned}$$

$$\begin{aligned} M_F(\vec{p}) &\equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi^{\bar{3}}_c(\vec{p})|H|\Xi^{\bar{3}}_c(\vec{p}')\rangle & \langle \Xi^{\bar{3}}_c(\vec{p})|H|\Xi^{6}_c(\vec{p}')\rangle \\ \langle \Xi^{6}_c(\vec{p})|H|\Xi^{\bar{3}}_c(\vec{p}')\rangle & \langle \Xi^{6}_c(\vec{p})|H|\Xi^{6}_c(\vec{p}')\rangle \end{pmatrix} \\ &= \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi^{\bar{3}}_c(\vec{p})|(H_0+\Delta H)|\Xi^{\bar{3}}_c(\vec{p}')\rangle & \langle \Xi^{\bar{3}}_c(\vec{p})|\Delta H|\Xi^{6}_c(\vec{p}')\rangle \\ \langle \Xi^{6}_c(\vec{p})|\Delta H|\Xi^{\bar{3}}_c(\vec{p}')\rangle & \langle \Xi^{6}_c(\vec{p})|(H_0+\Delta H)|\Xi^{6}_c(\vec{p}')\rangle \end{pmatrix}. \end{aligned}$$

$|\Xi^{\bar{3}/6}_c\rangle$ are eigenstate of H_0 under the $SU(3)_F$ symmetry

$$M_F\ (\vec{p}=0) = \begin{pmatrix} 2m^2_{\Xi^{\bar{3}}_c} & 0 \\ 0 & 2m^2_{\Xi^{6}_c} \end{pmatrix} + (m_s - m_u) \times \begin{pmatrix} \langle \Xi^{\bar{3}}_c|\bar{s}s(\vec{x}=0)|\Xi^{\bar{3}}_c\rangle & \langle \Xi^{\bar{3}}_c|\bar{s}s(\vec{x}=0)|\Xi^{6}_c\rangle \\ \langle \Xi^{6}_c|\bar{s}s(\vec{x}=0)|\Xi^{\bar{3}}_c\rangle & \langle \Xi^{6}_c|\bar{s}s(\vec{x}=0)|\Xi^{6}_c\rangle \end{pmatrix}$$

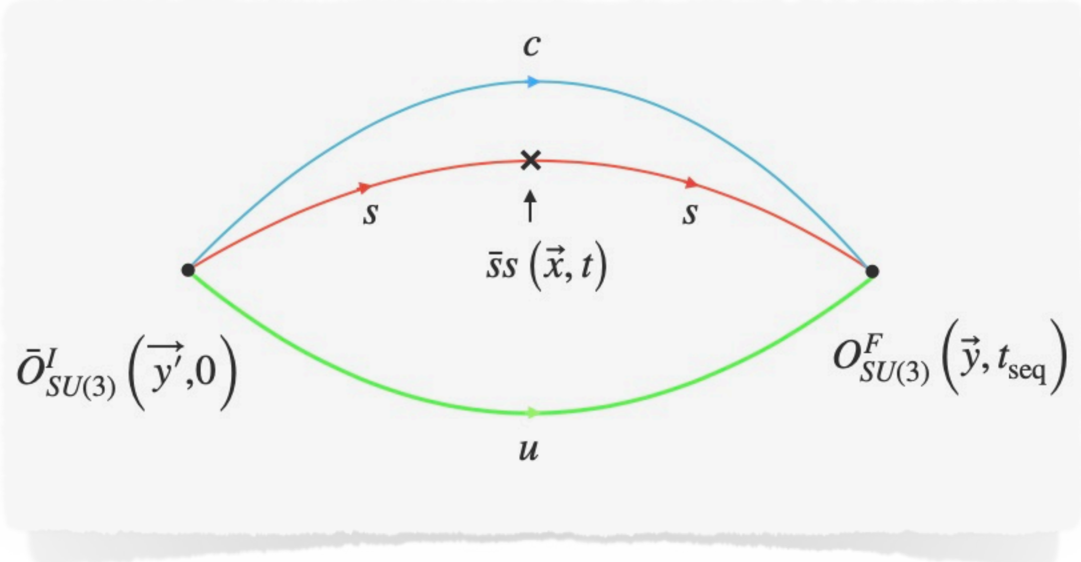
$$\begin{aligned} M_{F,11} &= 2m^2_{\Xi^{\bar{3}}_c} + (m_s - m_u)M^{\bar{3}-\bar{3}}_{\bar{s}s} \\ M_{F,22} &= 2m^2_{\Xi^{6}_c} + (m_s - m_u)M^{6-6}_{\bar{s}s} \\ M_{F,12} &= (m_s - m_u)M^{\bar{3}-6}_{\bar{s}s} \\ M_{F,21} &= (m_s - m_u)M^{6-\bar{3}}_{\bar{s}s} \end{aligned}$$

$$\sin 2\theta = \pm\frac{(m_s - m_u)M^{6-\bar{3}}_{\bar{s}s}}{m^2_{\Xi'_c} - m^2_{\Xi_c}},$$

Extraction of the matrix elements

- We want

$$M_{\bar{s}s}^{F-I} \equiv \langle \Xi_c^F(\vec{p}=0)|\bar{s}s(x=0)|\Xi_c^I(\vec{p}'=0)\rangle,$$



- We construct and simulate

$$O^{\bar{3}}_{SU(3)} = \epsilon^{abc}(q^{Ta}C\gamma_5 s^b)P_+c^c,$$
$$O^{6}_{SU(3)} = \epsilon^{abc}(q^{Ta}C\vec{\gamma}s^b)\cdot\vec{\gamma}\gamma_5P_+c^c.$$

Operators of SU(3) flavor eigenstates $\bar{3}$ and 6

$$C_3^{F-I}(t_{\rm seq},t) = \int \frac{d^3\vec{q}}{(2\pi)^3}\int d^3\vec{y}d^3\vec{y}'d^3\vec{x}e^{i\vec{q}\cdot\vec{x}}T_{\gamma'\gamma}\left\langle O^F_{\gamma,SU(3)}(\vec{y},t_{\rm seq})\bar{s}s(\vec{x},t)\bar{O}^I_{\gamma',SU(3)}(\vec{y}',0)\right\rangle,$$

Three-point function

$$C_2^{\bar{3}/6}(t) = \int d^3\vec{y}T'_{\gamma'\gamma}\langle O^{\bar{3}/6}_{\gamma,SU(3)}(\vec{y},t)\bar{O}^{\bar{3}/6}_{\gamma',SU(3)}(\vec{0},0)\rangle$$

Two-point function

Inserting the hadronic states and keeping the lowest two

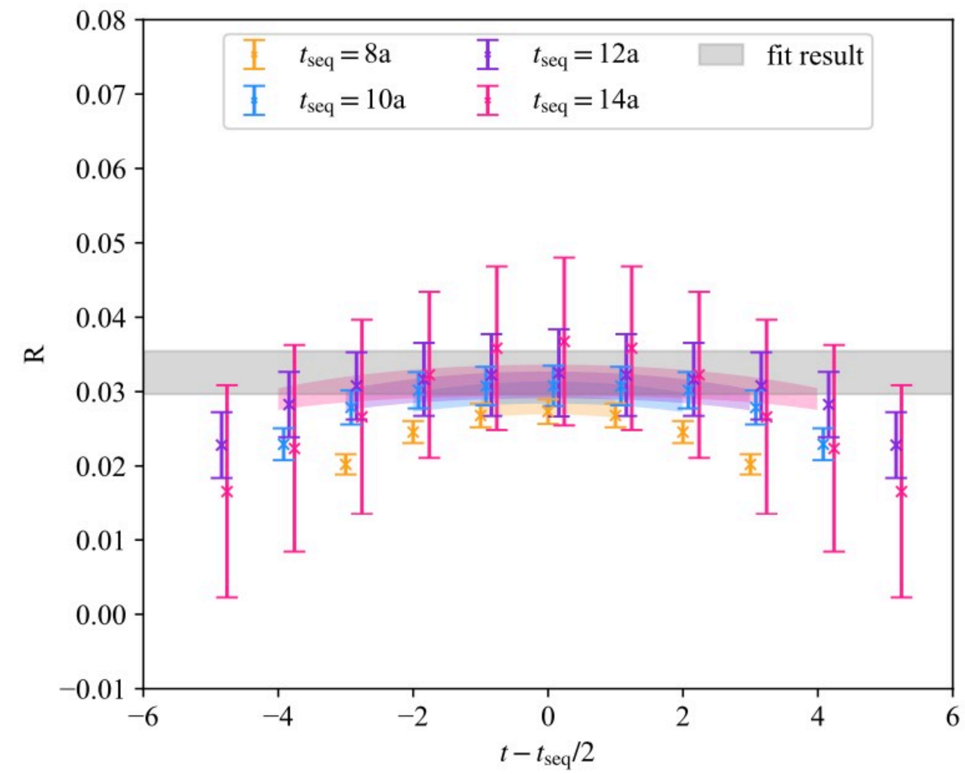
$$C_3^{F-I}(t_{\rm seq},t) = \frac{M_{\bar{s}s}^{F-I}}{\sqrt{4m_{\Xi_c^I}m_{\Xi_c^F}}}f_{\Xi_c^I}f_{\Xi_c^F}m^2_{\Xi_c^I}m^2_{\Xi_c^F}e^{-\left(m_{\Xi_c^I}-m_{\Xi_c^F}\right)t}e^{-m_{\Xi_c^F}t_{\rm seq}}\left(1+c_1e^{-\Delta m_{\Xi_c^I}t}\right)\left(1+c_2e^{-\Delta m_{\Xi_c^F}(t_{\rm seq}-t)}\right),$$

$$C_2^{\bar{3}/6}(t) = f^2_{\Xi_c^{\bar{3}/6}}m^4_{\Xi_c^{\bar{3}/6}}e^{-m_{\Xi_c^{\bar{3}/6}}t}(1+d_ie^{-\Delta m_{\Xi_c^{\bar{3}/6}}t})$$

Extraction of the matrix elements

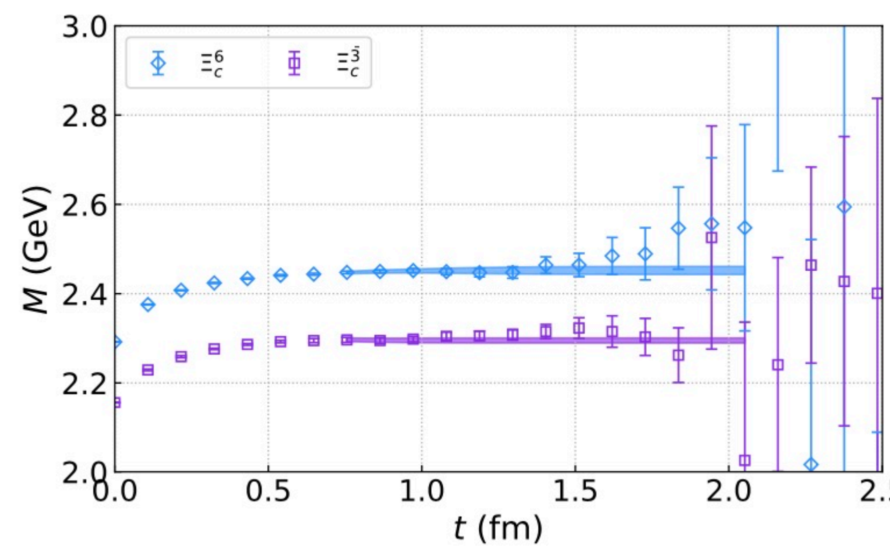
◆ **Combining the 3pt and 2pt, one can remove the dependence on the decay constants**

$$R=\sqrt{\frac{C_3^{FI}(t_{\rm seq},t)C_3^{FI}(t_{\rm seq},t_{\rm seq}-t)}{C_2^I(t_{\rm seq})C_2^F(t_{\rm seq})}}$$



R can be parameterized as

$$\begin{aligned} R &= \frac{|M_{\bar{s}s}^{F-I}|}{2\sqrt{m_{\Xi_c^I}m_{\Xi_c^F}}}\left(\frac{(1+c_1e^{-\Delta m_{\Xi_c^I}t})(1+c_1e^{-\Delta m_{\Xi_c^I}(t_{\rm seq}-t)})(1+c_2e^{-\Delta m_{\Xi_c^F}t})(1+c_2e^{-\Delta m_{\Xi_c^F}(t_{\rm seq}-t)})}{(1+d_1e^{-\Delta m_{\Xi_c^F}t_{\rm seq}})(1+d_2e^{-\Delta m_{\Xi_c^I}t_{\rm seq}})}\right)^{1/2} \\ &\simeq \frac{|M_{\bar{s}s}^{F-I}|}{2\sqrt{m_{\Xi_c^I}m_{\Xi_c^F}}}\left(\frac{(1+c_1e^{-\Delta m_{\Xi_c^I}t}+c_2e^{-\Delta m_{\Xi_c^F}(t_{\rm seq}-t)})(1+c_1e^{-\Delta m_{\Xi_c^I}(t_{\rm seq}-t)}+c_2e^{-\Delta m_{\Xi_c^F}t})}{(1+d_1e^{-\Delta m_{\Xi_c^I}t_{\rm seq}})(1+d_2e^{-\Delta m_{\Xi_c^F}t_{\rm seq}})}\right)^{1/2}, \end{aligned}$$



Numerical results of the mixing angle

$m_s - m_u \simeq 0.12\mathrm{GeV}$
from PDG

from the fitting
$|M^{6-\bar{3}}_{\bar{s}s}| = 0.155(14)\mathrm{GeV}$

$$\sin 2\theta = \pm \frac{(m_s - m_u) M^{6-\bar{3}}_{\bar{s}s}}{m^2_{\Xi'_c} - m^2_{\Xi_c}},$$

$m_{\Xi_c} = 2.468\mathrm{GeV}$
$m_{\Xi'_c} = 2.578\mathrm{GeV}$
from PDG.

	$\lvert M^{6-\bar{3}}_{\bar{s}s}\rvert$	$\lvert\theta\rvert$
C11P29S	0.155(14)GeV	$(0.97 \pm 0.08 \pm 0.25)^\circ$

statistical — from quark mass difference

The mixing angle is about 1° and consistent with the previous lattice investigation.

$\theta = 0.04^\circ$ **from QED correction(Z.F. Deng, Y.J. Shi, W. Wang, J. Zeng, arxiv: 2309.16386)**

SUMMARY

We have explored the $\Xi_c - \Xi_c'$ mixing by two different methods

- ☆ Calculate the two-point correlation matrix and adopt two independent methods to determine the mixing angle;
- ☆ Develop an improved method to explore the mixing which arises from the SU(3) flavor symmetry breaking effects.

Our numerical results are consistent and are not able to explain the large SU(3) symmetry breaking in semileptonic charmed baryon decays.

OUTLOOK

- ☆ The two methods can be used to other interesting examples such as the $K_1(1270)$ and $K_1(1400)$ mixing which exhibit effects on multiple decay channels…

Thank you!

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