江西师范大学 JIANGXI NORMAL UNIVERSITY

# Studying $D \to A(A \to VP)\ell\nu$ decays with the SU(3) flavor symmetry

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**Work done in collaboration with Ru-Min Wang, Yi Qiao et al.**

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# Contents

# 1. Motivation

✓Some $B(D\to P/V\ell v)$ have been well measured, but $B(D\to A\ell v)$ and $B(D\to VP\ell v)$ have not been well measured.

| | |
|---|---|
| $\mathcal{B}(D^0\to K_1^-(1270)e^+\nu_e)=(1.01\pm0.18)\times10^{-3}$ | BESIII(2102.10850),PDG |
| $\mathcal{B}(D^+\to \overline{K}_1^0(1270)e^+\nu_e)=(2.30^{+0.40}_{-0.42})\times10^{-3}$ | BESIII(1907.11370) |
| $\mathcal{B}(D^+\to b_1^0(1235)e^+\nu_e,\ b_1^0(1235)\to\omega\pi^0)\le1.75\times10^{-4}$ | PDG |
| $\mathcal{B}(D^0\to b_1^-(1235)e^+\nu_e,\ b_1^-(1235)\to\omega\pi^-)\le1.12\times10^{-4}$ | PDG |

✓A large number of charmed hadrons are produced at the BESIII, LHCb and Belle-II.

✓Theoretical calculations of the form factors

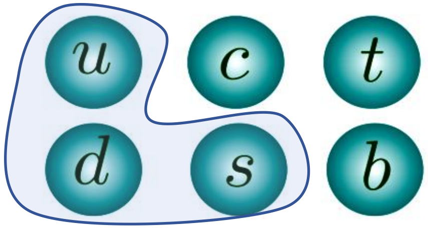
☺ $D\to P/V\ell^+v_l$

☹ $D\to A\ell^+v_l$ ,$D\to VP\ell^+v_l$

✓Symmetries (for an example, SU(3) flavor symmetry) provide very important information for particle physics.

# 1. Motivation

✓**SU(3) flavor symmetry approach:**




✓ Irreducible representation approach

✓ Topological diagram approach

✓**Advantage:** Independent of the detailed dynamics.

(Don't need calculate the form factors)

✓**Disadvantage:** it can not determine the sizes of the amplitudes by itself.

# Meson decays with the SU(3) flavor symmetry

| decays \ baryons | Charmed mensons | Bottom mensons |
|---|---|---|
| $B/D \to M_1 M_2$ | ① Hai -Yang Cheng et. al., PRD 86, 014014(2012); PRD 93, 114010(2016)<br>② Xiao-Gang He,Wei Wang et. al., EPJC 80, 359(2020) ; CPC 42, 103108(2018).<br>③ D. Pirtskhalava, PLB 742, 81 (2012).<br>④ ...... | ① Si -Hong Zhou et. al., PRD 92094016(2015); EPJC 77(2017) 2, 125;PRD 92, 094016(2015).<br>② Hai -Yang Cheng et. al., JHEP 09, 024(2011);PRD 91, 014011(2015).<br>③ Xiao-Gang He,Wei Wang et. al., EPJC 80, 359(2020) ; CPC 42, 103108(2018); PRD 93, 114002(2016); JHEP 08, 065(2013); PRD 69, 074002(2004); PRD 64, 034002(2001); EPJC 9, 443(1999); PRL 75, 1703(1995).<br>④ ...... |
| $B/D \to Ml\nu$<br>$B/D \to M_1 M_2 l\nu$ | $D \to P/V/Sl\nu$ NPB995,116349(2023)<br>$D \to PPl\nu$ PRD107,056022(2023)<br>$D \to VPl\nu$ working | |

# Mensons $q\bar{q}$

# $M^i_j$ $i,j = 1,2,3$ for u, d, s.

$$D_i = \left(D^0(c\bar{u}),\ D^+(c\bar{d}),\ D_s^+(c\bar{s})\right)$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{3}} + \frac{f_8}{\sqrt{6}} & a_1^+ & K_{1A}^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{3}} + \frac{f_8}{\sqrt{6}} & K_{1A}^0 \\ K_{1A}^- & \overline{K}_{1A}^0 & \frac{f_1}{\sqrt{3}} - \frac{2f_8}{\sqrt{6}} \end{pmatrix}$$

$J^{PC} = 1^{++}$

$$B = \begin{pmatrix} \frac{b_1^0}{\sqrt{2}} + \frac{h_1}{\sqrt{3}} + \frac{h_8}{\sqrt{6}} & b_1^+ & K_{1B}^+ \\ b_1^- & -\frac{b_1^0}{\sqrt{2}} + \frac{h_1}{\sqrt{3}} + \frac{h_8}{\sqrt{6}} & K_{1B}^0 \\ K_{1B}^- & \overline{K}_{1B}^0 & \frac{h_1}{\sqrt{3}} - \frac{2h_8}{\sqrt{6}} \end{pmatrix}$$

$J^{PC} = 1^{+-}$

$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix}$$

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_{3P1} & \sin\theta_{3P1} \\ -\sin\theta_{3P1} & \cos\theta_{3P1} \end{pmatrix} \begin{pmatrix} f_1 \\ f_8 \end{pmatrix}$$

$$\begin{pmatrix} h_1(1170) \\ h_1(1415) \end{pmatrix} = \begin{pmatrix} \cos\theta_{1P1} & \sin\theta_{1P1} \\ -\sin\theta_{1P1} & \cos\theta_{1P1} \end{pmatrix} \begin{pmatrix} h_1 \\ h_8 \end{pmatrix}$$

# Contents

# 2.1 Amplitudes of $D \to A \ell^+ \nu_\ell$

$$\mathcal{H}_{eff}(c \to q \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq} \bar{q} \gamma^\mu (1-\gamma_5) c \, \bar{\nu}_\ell \gamma_\mu (1-\gamma_5) \ell,$$

$$\mathcal{M}(D \to A \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq} \sum_{mn} g_{mn} L_m^{\lambda_\ell \lambda_\nu} H_n^{\lambda_A},$$

$$L_m^{\lambda_\ell \lambda_\nu} = \epsilon_\alpha(m) \bar{\nu}_\ell \gamma^\alpha (1-\gamma_5) \ell,$$

$$H_{\lambda_A n}^{A} = V_{cq} \epsilon_\beta(n) \langle A(p_A, \epsilon) | \bar{q} \gamma^\beta (1-\gamma_5) c | D(p_A) \rangle,$$

**form factors**

**SU(3) flavor symmetry**

$$\frac{dBr(D \to A \ell^+ \nu_l)}{dq^2} = \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda_A}\, G_F^2 \, |V_{cq}|^2 \, \tau_{Dq}}{384\, \pi^3 \, m_{Dq}^3} \times |H_{D \to A}|^2$$

JPG 10, 105006(2019)

## 2.1 Amplitudes of $D \to A \ell^+ \nu_\ell$

$$H(D \to A\, l^+ \nu_l) = c_0^M D_i M^i_{\ j} H^j \ ,$$

$M = A/B$ $\quad c_i^A \quad c_i^B$

$$\Delta H(D \to A l^+ \nu_l) = c_1^M D_a W^a_{\ i} M^i_{\ j} H^j + c_2^M D_i M^i_{\ a} W^a_{\ j} H^j,$$

$c_i^M$ : non-perturbative coefficients

$H^j$ : CKM matrix elements

SU(3) flavor breaking coefficient matrix $W = (W^i_j) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

# 2.1 Amplitudes of $D\to A\ell^+\nu_\ell$

TABLE II: The hadronic helicity amplitudes for the $D\to A\ell^+\nu$ decays including both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions. $A_1\equiv c_0^A+c_1^A-2c_2^A$, $A_2\equiv c_0^A-2c_1^A-2c_2^A$, $A_3\equiv c_0^A+c_1^A+c_2^A$, $A_4\equiv c_0^A-2c_1^A+c_2^A$. $B_1\equiv c_0^B+c_1^B-2c_2^A$, $B_2\equiv c_0^B-2c_1^B-2c_2^B$, $B_3\equiv c_0^B+c_1^B+c_2^B$, $B_4\equiv c_0^B-2c_1^B+c_2^B$. $A_1=A_2=A_3=A_4=c_0^A$, $B_1=B_2=B_3=B_4=c_0^B$ if neglecting the SU(3) flavor breaking $c_1^A$, $c_2^A$ and $c_1^B$, $c_2^B$ terms.

$A=c_0^A,\ B=c_0^B$

| Hadronic helicity amplitudes | SU(3) flavor amplitudes |
|---|---|
| $H(D^0\to K_1^-(1270)\ell^+\nu_\ell)$ | $(sin\theta_{K_1}A_1+cos\theta_{K_1}B_1)\ V_{cs}^*$ |
| $H(D^0\to K_1^-(1400)\ell^+\nu_\ell)$ | $(cos\theta_{K_1}A_1-sin\theta_{K_1}B_1)\ V_{cs}^*$ |
| $H(D^+\to \overline{K}_1^0(1270)\ell^+\nu_\ell)$ | $(sin\theta_{K_1}A_1+cos\theta_{K_1}B_1)\ V_{cs}^*$ |
| $H(D^+\to \overline{K}_1^0(1400)\ell^+\nu_\ell)$ | $(cos\theta_{K_1}A_1-sin\theta_{K_1}B_1)\ V_{cs}^*$ |
| $H(D_s^+\to f_1^0(1285)\ell^+\nu_\ell)$ | $(\frac{1}{\sqrt{3}}cos\theta_{3P1}-\sqrt{\frac{2}{3}}sin\theta_{3P1})\ A_2\ V_{cs}^*$ |
| $H(D_s^+\to f_1^0(1420)\ell^+\nu_\ell)$ | $(-\frac{1}{\sqrt{3}}sin\theta_{3P1}-\sqrt{\frac{2}{3}}cos\theta_{3P1})\ A_2\ V_{cs}^*$ |
| $H(D_s^+\to h_1^0(1170)\ell^+\nu_\ell)$ | $(\frac{1}{\sqrt{3}}cos\theta_{1P1}-\sqrt{\frac{2}{3}}sin\theta_{1P1})\ B_2\ V_{cs}^*$ |
| $H(D_s^+\to h_1^0(1415)\ell^+\nu_\ell)$ | $(-\frac{1}{\sqrt{3}}sin\theta_{1P1}-\sqrt{\frac{2}{3}}cos\theta_{1P1})\ B_2\ V_{cs}^*$ |
| $H(D^0\to a_1^-(1260)\ell^+\nu_\ell)$ | $A_3\ V_{cd}^*$ |
| $H(D^0\to b_1^-(1235)\ell^+\nu_\ell)$ | $B_3\ V_{cd}^*$ |
| $H(D^+\to a_1^0(1260)\ell^+\nu_\ell)$ | $-\frac{1}{\sqrt{2}}\ A_3\ V_{cd}^*$ |
| $H(D^+\to b_1^0(1235)\ell^+\nu_\ell)$ | $-\frac{1}{\sqrt{2}}\ B_3\ V_{cd}^*$ |
| $H(D^+\to f_1^0(1285)\ell^+\nu_\ell)$ | $(\frac{1}{\sqrt{3}}cos\theta_{3P1}+\frac{1}{\sqrt{6}}sin\theta_{3P1})\ A_3\ V_{cd}^*$ |
| $H(D^+\to f_1^0(1420)\ell^+\nu_\ell)$ | $(-\frac{1}{\sqrt{3}}sin\theta_{3P1}+\frac{1}{\sqrt{6}}cos\theta_{3P1})\ A_3\ V_{cd}^*$ |
| $H(D^+\to h_1^0(1170)\ell^+\nu_\ell)$ | $(\frac{1}{\sqrt{3}}cos\theta_{1P1}+\frac{1}{\sqrt{6}}sin\theta_{1P1})\ B_3\ V_{cd}^*$ |
| $H(D^+\to h_1^0(1415)\ell^+\nu_\ell)$ | $(-\frac{1}{\sqrt{3}}sin\theta_{1P1}+\frac{1}{\sqrt{6}}cos\theta_{1P1})\ B_3\ V_{cd}^*$ |
| $H(D_s^+\to K_1^0(1270)\ell^+\nu_\ell)$ | $(sin\theta_{K_1}A_4+cos\theta_{K_1}B_4)\ V_{cd}^*$ |
| $H(D_s^+\to K_1^0(1400)\ell^+\nu_\ell)$ | $(cos\theta_{K_1}A_4-sin\theta_{K_1}B_4)\ V_{cd}^*$ |

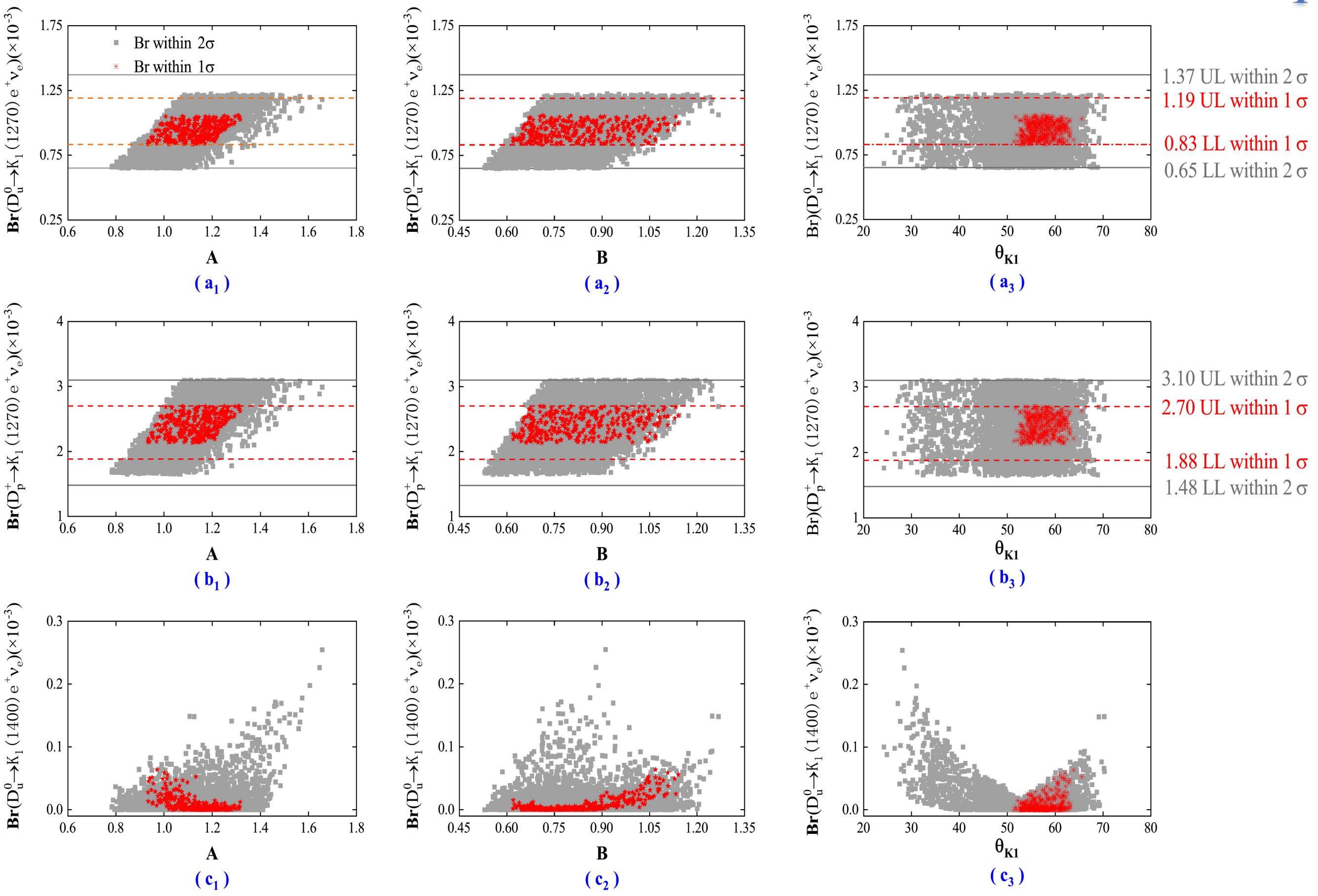
# 2.2 Numerical results of $D \to A e^+ \nu_e$

| Branching ratios | Experimental data | Predictions with $1\sigma$ | Predictions with $2\sigma$ | CLFQM 1707.02851 | LCSR 2102.12241 |
|---|---|---|---|---|---|
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e)(\times 10^{-3})$ | $1.01 \pm 0.18$ | $0.94 \pm 0.11$ | $0.94 \pm 0.29$ | | |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e)(\times 10^{-3})$ | | $0.03 \pm 0.03$ | $0.13 \pm 0.13$ | | |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e)(\times 10^{-3})$ | $2.30^{+0.40}_{-0.42}$ | $2.42 \pm 0.28$ | $2.38 \pm 0.72$ | $3.2 \pm 0.40$ | |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e)(\times 10^{-3})$ | | $0.08 \pm 0.08$ | $0.32 \pm 0.32$ | $\{0.005, 0.02\}$ | |
| $\mathcal{B}(D_s^+ \to f_1^0(1285)e^+\nu_e)(\times 10^{-3})$ | | $0.63 \pm 0.55$ | $0.95 \pm 0.95$ | $\{0.06, 0.36\}$ | |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)e^+\nu_e)(\times 10^{-3})$ | | $0.28 \pm 0.28$ | $0.48 \pm 0.48$ | $0.25 \pm 0.05$ | |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e)(\times 10^{-3})$ | | $0.17 \pm 0.17$ | $0.35 \pm 0.35$ | $\{0, 0.197\}$ | |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e)(\times 10^{-3})$ | | $0.33 \pm 0.20$ | $0.36 \pm 0.28$ | $0.64 \pm 0.07$ | |
| $\mathcal{B}(D^0 \to a_1^-(1260)e^+\nu_e)(\times 10^{-5})$ | | $3.84 \pm 1.70$ | $5.40 \pm 4.12$ | | 6.90 |
| $\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e)(\times 10^{-5})$ | | $2.47 \pm 1.36$ | $2.84 \pm 2.02$ | | 4.85 |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e)(\times 10^{-5})$ | | $4.90 \pm 2.17$ | $6.87 \pm 5.22$ | | 9.38 |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e)(\times 10^{-5})$ | | $3.17 \pm 1.75$ | $3.60 \pm 2.55$ | $7.4 \pm 0.70$ | 6.58 |
| $\mathcal{B}(D^+ \to f_1^0(1285)e^+\nu_e)(\times 10^{-5})$ | | $2.12 \pm 2.12$ | $3.60 \pm 3.60$ | $3.7 \pm 0.80$ | |
| $\mathcal{B}(D^+ \to f_1^0(1420)e^+\nu_e)(\times 10^{-5})$ | | $1.11 \pm 0.97$ | $1.68 \pm 1.68$ | $\{0.02, 0.14\}$ | |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e)(\times 10^{-5})$ | | $3.81 \pm 2.32$ | $4.12 \pm 3.17$ | $14 \pm 1.50$ | |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e)(\times 10^{-5})$ | | $0.19 \pm 0.19$ | $0.42 \pm 0.42$ | $\{0, 0.02\}$ | |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e)(\times 10^{-5})$ | | $8.73 \pm 1.05$ | $8.64 \pm 2.82$ | $17 \pm 2.00$ | |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e)(\times 10^{-5})$ | | $0.34 \pm 0.34$ | $1.33 \pm 1.33$ | $\{0.05, 0.14\}$ | |

# 2.2 Numerical results of $D \to A \mu^+ \nu_\mu$

| Branching ratios | Experimental data | Predictions with $1\sigma$ | Predictions with $2\sigma$ | 1707.02851 | 2102.12241 |
|---|---|---|---|---|---|
| $\mathcal{B}(D^0 \to K_1^-(1270)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.69 \pm 0.08$ | $0.69 \pm 0.21$ | | |
| $\mathcal{B}(D^0 \to K_1^-(1400)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.02 \pm 0.02$ | $0.08 \pm 0.08$ | | |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)\mu^+\nu_\mu)(\times 10^{-3})$ | | $1.78 \pm 0.20$ | $1.75 \pm 0.54$ | $2.6 \pm 0.30$ | |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.05 \pm 0.05$ | $0.21 \pm 0.21$ | $\{0.004, 0.017\}$ | |
| $\mathcal{B}(D_s^+ \to f_1^0(1285)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.49 \pm 0.43$ | $0.74 \pm 0.74$ | $\{0.052, 0.306\}$ | |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.19 \pm 0.19$ | $0.33 \pm 0.33$ | $0.21 \pm 0.05$ | |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.14 \pm 0.14$ | $0.29 \pm 0.29$ | $\{0, 0.174\}$ | |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)\mu^+\nu_\mu)(\times 10^{-3})$ | | $0.23 \pm 0.14$ | $0.26 \pm 0.20$ | $0.54 \pm 0.06$ | |
| $\mathcal{B}(D^0 \to a_1^-(1260)\mu^+\nu_\mu)(\times 10^{-5})$ | | $2.89 \pm 1.34$ | $4.18 \pm 3.28$ | | 6.27 |
| $\mathcal{B}(D^0 \to b_1^-(1235)\mu^+\nu_\mu)(\times 10^{-5})$ | | $1.85 \pm 1.02$ | $2.13 \pm 1.52$ | | 4.40 |
| $\mathcal{B}(D^+ \to a_1^0(1260)\mu^+\nu_\mu)(\times 10^{-5})$ | | $3.70 \pm 1.71$ | $5.33 \pm 4.16$ | | 8.52 |
| $\mathcal{B}(D^+ \to b_1^0(1235)\mu^+\nu_\mu)(\times 10^{-5})$ | | $2.38 \pm 1.31$ | $2.71 \pm 1.92$ | $6.4 \pm 0.6$ | 6.00 |
| $\mathcal{B}(D^+ \to f_1^0(1285)\mu^+\nu_\mu)(\times 10^{-5})$ | | $1.53 \pm 1.53$ | $2.60 \pm 2.60$ | $3.2 \pm 0.6$ | |
| $\mathcal{B}(D^+ \to f_1^0(1420)\mu^+\nu_\mu)(\times 10^{-5})$ | | $0.67 \pm 0.58$ | $1.01 \pm 1.01$ | $\{0.02, 0.12\}$ | |
| $\mathcal{B}(D^+ \to h_1^0(1170)\mu^+\nu_\mu)(\times 10^{-5})$ | | $2.98 \pm 1.81$ | $3.21 \pm 2.48$ | $12.2 \pm 1.3$ | |
| $\mathcal{B}(D^+ \to h_1^0(1415)\mu^+\nu_\mu)(\times 10^{-5})$ | | $0.12 \pm 0.12$ | $0.26 \pm 0.26$ | $\{0, 0.02\}$ | |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)\mu^+\nu_\mu)(\times 10^{-5})$ | | $6.84 \pm 0.83$ | $6.77 \pm 2.20$ | $15 \pm 2$ | |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)\mu^+\nu_\mu)(\times 10^{-5})$ | | $0.24 \pm 0.24$ | $0.95 \pm 0.95$ | $\{0.05, 0.12\}$ | |

# 2.2 Numerical results of $D\to K_1 e^+\nu_e$




| | |
|---|---|
| $H(D^0\to K_1^-(1270)\ell^+\nu_\ell)$ | $(sin\theta_{K_1}A_1+cos\theta_{K_1}B_1)\,V_{cs}^*$ |
| $H(D^0\to K_1^-(1400)\ell^+\nu_\ell)$ | $(cos\theta_{K_1}A_1-sin\theta_{K_1}B_1)\,V_{cs}^*$ |
| $H(D^+\to \overline{K}_1^0(1270)\ell^+\nu_\ell)$ | $(sin\theta_{K_1}A_1+cos\theta_{K_1}B_1)\,V_{cs}^*$ |
| $H(D^+\to \overline{K}_1^0(1400)\ell^+\nu_\ell)$ | $(cos\theta_{K_1}A_1-sin\theta_{K_1}B_1)\,V_{cs}^*$ |

Our results of $\theta_{K_1}$:

[51°,65°] within 1σ errors

[24°,70°] within 2σ errors

Previous ranges of $\theta_{K_1}$:

[35°,55°] PRD 56 1368(1997)

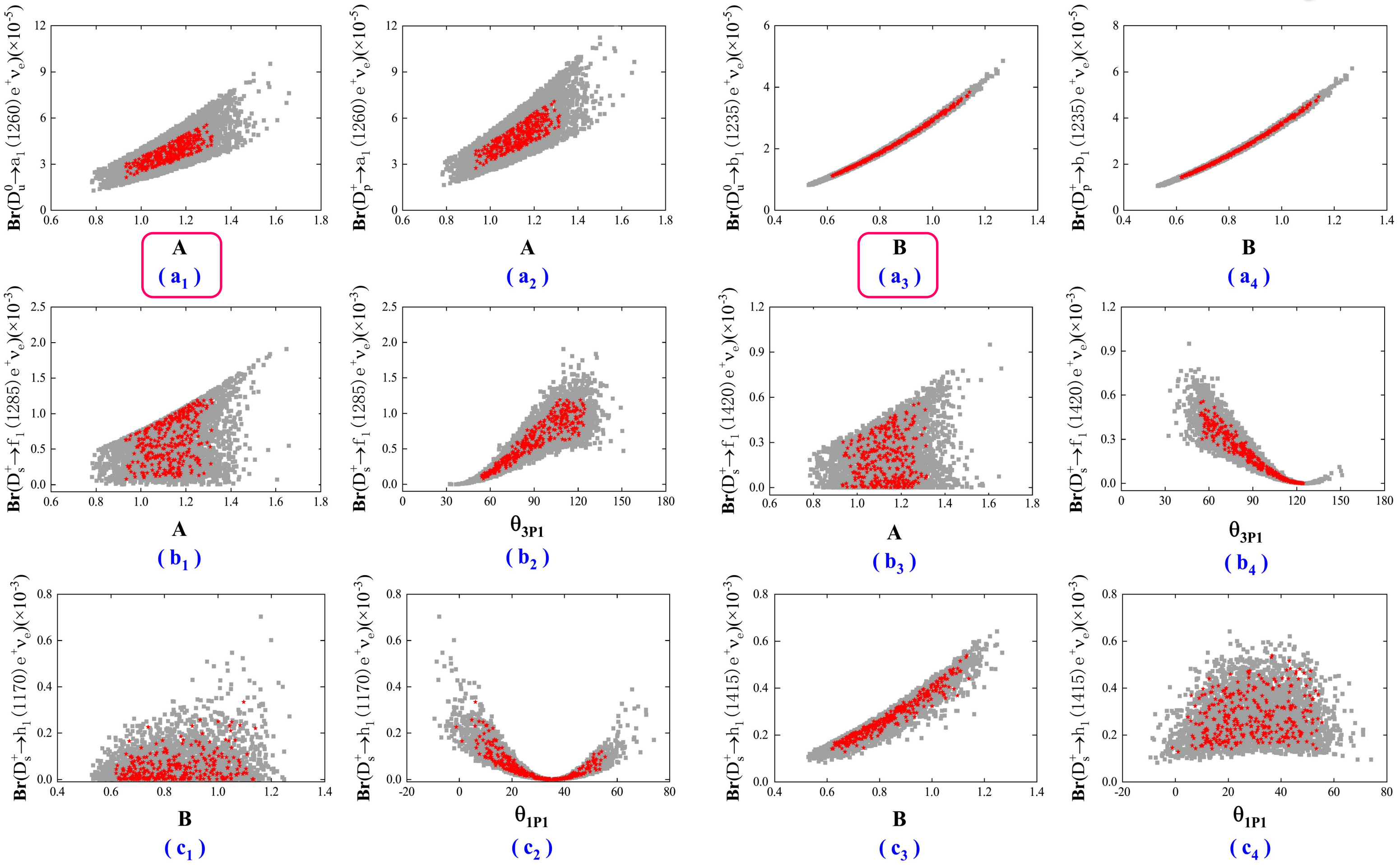
[35°,45°] arxiv: 1110.2249

[33°,57°] PRD 47 1252(1993)

[33°,57°] arxiv: 1311.2370

(34±13)° PRD 77 094023(2008)

# 2.2 Numerical results of $D \to A e^+ \nu_e$




Our results of $\theta_{3P1}$:

[55°,125°] within 1σ errors

[30°,145°] within 2σ errors

Our results of $\theta_{1P1}$:

[8°,58°] within 1σ errors

[-10°,75°] within 2σ errors

# Contents

# 3.1 Amplitudes of A→VP

$$\mathrm{A}(A\rightarrow VP) = c_0{}^{M} A^i{}_j M^j{}_k N^k{}_i \;, \qquad M = A/B \qquad c_i^A \qquad c_i^B$$

$$\Delta\mathrm{A}(A\rightarrow VP) = c_1{}^{M} A^a{}_j W^i{}_a M^j{}_k N^k{}_i + c_2{}^{M} A^i{}_j M^a{}_k W^j{}_a N^k{}_i + c_3{}^{M} A^i{}_j M^j{}_k N^a{}_i W^a{}_k \,,$$

$$\Gamma_{M\rightarrow VP} = \frac{P_{cA}}{8\pi M_A^2} \overline{\sum} |A_{A\rightarrow VP}|^2$$

PRD 70, 094006 (2004)

$$\Gamma_{M\rightarrow VP} = \frac{1}{\pi^2} \int dM_{AO}^2 dM_{VO}^2 Im\left\{\frac{1}{M_{AO}^2 - M_A^2 + iM_A\Gamma_A}\right\} Im\left\{\frac{1}{M_{VO}^2 - M_V^2 + iM_V\Gamma_V}\right\} \cdot \Gamma_{MVP}(M_{AO}, M_{AO})\Theta(M_{AO} - M_{VO} - M_P)$$

# 3.1 Amplitudes of A→VP

The $K_1(1270)/K_1(1400) \to VP$ decay amplitudes. $F_1 = c_0^A - c_1^A + c_2^A$, $D_1 = c_0^B - c_1^B + c_2^B$. $F_2 = c_0^A - c_1^A - 2c_2^A$, $D_2 = c_0^B - c_1^B - 2c_2^B$.

| Hadronic helicity amplitudes | SU(3) flavor amplitudes |
|---|---|
| $A(K_1^-(1270) \to K^{*-}\pi^0)$ | $\frac{1}{\sqrt{2}}(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(K_1^-(1270) \to K^{*-}\eta)$ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}}) + (F_2 sin\theta_{K1} - D_2 cos\theta_{K1})(\frac{2cos\theta_P}{\sqrt{6}} + \frac{sin\theta_P}{\sqrt{3}})$ |
| ~~$A(K_1^-(1270) \to K^{*-}\eta')$~~ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}}) + (F_2 sin\theta_{K1} - D_2 cos\theta_{K1})(\frac{2sin\theta_P}{\sqrt{6}} - \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(K_1^-(1270) \to \rho^0 K^-)$ | $\frac{1}{\sqrt{2}}(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(K_1^-(1270) \to \omega K^-)$ | $\frac{1}{\sqrt{2}}(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(K_1^-(1270) \to K^{*0}\pi^-)$ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(K_1^-(1270) \to \rho^- \overline{K}^0)$ | $(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| ~~$A(K_1^-(1270) \to \phi K^-)$~~ | $(F_2 sin\theta_{K1} + D_2 cos\theta_{K1})$ |
| $A(K_1^-(1400) \to K^{*-}\pi^0)$ | $\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})$ |
| $A(K_1^-(1400) \to K^{*-}\eta)$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}}) + (F_2 cos\theta_{K1} + D_2 sin\theta_{K1})(\frac{2cos\theta_P}{\sqrt{6}} + \frac{sin\theta_P}{\sqrt{3}})$ |
| $A(K_1^-(1400) \to K^{*-}\eta')$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}}) + (F_2 cos\theta_{K1} + D_2 sin\theta_{K1})(\frac{2sin\theta_P}{\sqrt{6}} - \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(K_1^-(1400) \to \rho^0 K^-)$ | $-\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(K_1^-(1400) \to \omega K^-)$ | $-\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(K_1^-(1400) \to \overline{K}^{*0}\pi^-)$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})$ |
| $A(K_1^-(1400) \to \rho^- \overline{K}^0)$ | $-(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(K_1^-(1400) \to \phi K^-)$ | $(F_2 cos\theta_{K1} - D_2 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to K^{*-}\pi^+)$ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to \rho^+ K^-)$ | $(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to \overline{K}^{*0}\pi^0)$ | $-\frac{1}{\sqrt{2}}(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to \overline{K}^{*0}\eta)$ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}}) + (F_2 sin\theta_{K1} - D_2 cos\theta_{K1})(\frac{2cos\theta_P}{\sqrt{6}} + \frac{sin\theta_P}{\sqrt{3}})$ |
| ~~$A(\overline{K}_1^0(1270) \to \overline{K}^{*0}\eta')$~~ | $(F_1 sin\theta_{K1} + D_1 cos\theta_{K1})(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}}) + (F_2 sin\theta_{K1} - D_2 cos\theta_{K1})(\frac{2sin\theta_P}{\sqrt{6}} - \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(\overline{K}_1^0(1270) \to \rho^0 \overline{K}^0)$ | $-\frac{1}{\sqrt{2}}(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to \omega \overline{K}^0)$ | $\frac{1}{\sqrt{2}}(-F_1 sin\theta_{K1} + D_1 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1270) \to \phi \overline{K}^0)$ | $(F_2 sin\theta_{K1} + D_2 cos\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to K^{*-}\pi^+)$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to \rho^+ K^-)$ | $-(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to \overline{K}^{*0}\pi^0)$ | $-\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to \overline{K}^{*0}\eta)$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}}) + (F_2 cos\theta_{K1} + D_2 sin\theta_{K1})(\frac{2cos\theta_P}{\sqrt{6}} + \frac{sin\theta_P}{\sqrt{3}})$ |
| $A(\overline{K}_1^0(1400) \to \overline{K}^{*0}\eta')$ | $(F_1 cos\theta_{K1} - D_1 sin\theta_{K1})(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}}) + (F_2 cos\theta_{K1} + D_2 sin\theta_{K1})(\frac{2sin\theta_P}{\sqrt{6}} - \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(\overline{K}_1^0(1400) \to \rho^0 \overline{K}^0)$ | $\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to \omega \overline{K}^0)$ | $-\frac{1}{\sqrt{2}}(F_1 cos\theta_{K1} + D_1 sin\theta_{K1})$ |
| $A(\overline{K}_1^0(1400) \to \phi \overline{K}^0)$ | $(F_2 cos\theta_{K1} - D_2 sin\theta_{K1})$ |

The $a_1(1260) \to VP$ decay amplitudes. $F_3 = c_0^A + 2c_1^A + c_2^A$, $F_4 = c_0^A + 2c_1^A - 2c_2^A$.

| Branching ratios | SU(3) flavor amplitudes |
|---|---|
| $A(a_1^0(1260) \to \rho^+\pi^-)$ | $\sqrt{2}F_3$ |
| $A(a_1^0(1260) \to \rho^-\pi^+)$ | $-\sqrt{2}F_3$ |
| $A(a_1^0(1260) \to K^{*0}\overline{K}^0)$ | $-\frac{1}{\sqrt{2}}F_4$ |
| $A(a_1^0(1260) \to \overline{K}^{*0}K^0)$ | $\frac{1}{\sqrt{2}}F_4$ |
| $A(a_1^0(1260) \to K^{*+}K^-)$ | $\frac{1}{\sqrt{2}}F_4$ |
| $A(a_1^0(1260) \to K^{*-}K^+)$ | $-\frac{1}{\sqrt{2}}F_4$ |
| $A(a_1^-(1260) \to \rho^-\pi^0)$ | $\sqrt{2}F_3$ |
| $A(a_1^-(1260) \to \rho^0\pi^-)$ | $-\sqrt{2}F_3$ |
| $A(a_1^-(1260) \to K^{*0}K^-)$ | $F_4$ |
| $A(a_1^-(1260) \to K^{*-}K^0)$ | $-F_4$ |

The $b_1(1235) \to VP$ decay amplitudes. $D_3 = c_0^B + 2c_1^B + c_2^B$, $D_4 = c_0^B + 2c_1^B - 2c_2^B$.

| Branching ratios | SU(3) flavor amplitudes |
|---|---|
| $A(b_1^0(1235) \to \rho^0\eta)$ | $2D_3(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}})$ |
| ~~$A(b_1^0(1235) \to \rho^0\eta')$~~ | $2D_3(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(b_1^0(1235) \to \omega\pi^0)$ | $\sqrt{2}D_3$ |
| $A(b_1^0(1235) \to K^{*0}\overline{K}^0)$ | $-\frac{1}{\sqrt{2}}D_4$ |
| $A(b_1^0(1235) \to \overline{K}^{*0}K^0)$ | $-\frac{1}{\sqrt{2}}D_4$ |
| $A(b_1^0(1235) \to K^{*+}K^-)$ | $\frac{1}{\sqrt{2}}D_4$ |
| $A(b_1^0(1235) \to K^{*-}K^+)$ | $\frac{1}{\sqrt{2}}D_4$ |
| $A(b_1^-(1235) \to \rho^-\eta)$ | $2D_3(\frac{cos\theta_P}{\sqrt{6}} - \frac{sin\theta_P}{\sqrt{3}})$ |
| ~~$A(b_1^-(1235) \to \rho^-\eta')$~~ | $2D_3(\frac{sin\theta_P}{\sqrt{6}} + \frac{cos\theta_P}{\sqrt{3}})$ |
| $A(b_1^-(1235) \to \omega\pi^-)$ | $\sqrt{2}D_3$ |
| $A(b_1^-(1235) \to K^{*0}K^-)$ | $D_4$ |
| $A(b_1^-(1235) \to K^{*-}K^0)$ | $D_4$ |

# 3.1 Amplitude of A→VP

The $f_1(1285) \to VP$ and $f_1(1420) \to VP$ decay amplitudes.

$F_4 = c_0^A + 2c_1^A - 2c_2^A$, $F_5 = c_0^A - 4c_1^A + c_2^A$.

| Branching ratios | SU(3) flavor amplitudes |
|---|---|
| $A(f_1(1285) \to K^{*+}K^-)$ | $\frac{cos\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) + \frac{sin\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1285) \to K^{*-}K^+)$ | $-\frac{cos\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) - \frac{sin\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1285) \to K^{*0}\overline{K}^0)$ | $\frac{cos\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) + \frac{sin\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1285) \to \overline{K}^{*0}K^0)$ | $-\frac{cos\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) - \frac{sin\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1420) \to K^{*+}K^-)$ | $-\frac{sin\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) + \frac{cos\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1420) \to K^{*-}K^+)$ | $\frac{sin\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) - \frac{cos\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1420) \to K^{*0}\overline{K}^0)$ | $-\frac{sin\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) + \frac{cos\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |
| $A(f_1(1420) \to \overline{K}^{*0}K^0)$ | $\frac{sin\theta_{3P1}}{\sqrt{3}}(F_4 - F_5) - \frac{cos\theta_{3P1}}{\sqrt{6}}(F_4 + 2F_5)$ |

The $h_1(1170) \to VP$ and $h_1(1400) \to VP$ decay amplitudes. $D_3 = c_0^B + 2c_1^B + c_2^B$, $D_4 = c_0^B + 2c_1^B - 2c_2^B$, $D_5 = c_0^B - 4c_1^B + c_2^B$, $D_6 = c_0^B - 4c_1^B - 2c_2^B$.

| Branching ratios | SU(3) flavor amplitudes |
|---|---|
| $A(h_1^0(1170) \to \rho^0\pi^0)$ | $\frac{2}{\sqrt{6}}D_3(\sqrt{2}cos\theta_{1P1} + sin\theta_{1P1})$ |
| $A(h_1^0(1170) \to \omega\eta)$ | $\frac{\sqrt{2}}{3}D_3(\sqrt{2}cos\theta_{1P1} + sin\theta_{1P1})(cos\theta_P - \sqrt{2}sin\theta_P)$ |
| ~~$A(h_1^0(1170) \to \omega\eta')$~~ | $\frac{\sqrt{2}}{3}D_3(\sqrt{2}cos\theta_{1P1} + sin\theta_{1P1})(sin\theta_P + \sqrt{2}cos\theta_P)$ |
| $A(h_1^0(1170) \to \rho^+\pi^-)$ | $\frac{2}{\sqrt{6}}D_3(\sqrt{2}cos\theta_{1P1} + sin\theta_{1P1})$ |
| $A(h_1^0(1170) \to \rho^-\pi^+)$ | $\frac{2}{\sqrt{6}}D_3(\sqrt{2}cos\theta_{1P1} + sin\theta_{1P1})$ |
| $A(h_1^0(1170) \to K^{*+}K^-)$ | $\frac{cos\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{sin\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1170) \to K^{*-}K^+)$ | $\frac{cos\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{sin\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1170) \to K^{*0}\overline{K}^0)$ | $\frac{cos\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{sin\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1170) \to \overline{K}^{*0}K^0)$ | $\frac{cos\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{sin\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1170) \to \phi\eta)$ | $\frac{2}{3}D_6(-cos\theta_{1P1} + \sqrt{2}sin\theta_{1P1})(\sqrt{2}cos\theta_P + sin\theta_P)$ |
| ~~$A(h_1^0(1170) \to \phi\eta')$~~ | $\frac{2}{3}D_6(-cos\theta_{1P1} + \sqrt{2}sin\theta_{1P1})(\sqrt{2}sin\theta_P - cos\theta_P)$ |
| $A(h_1^0(1415) \to \rho^0\pi^0)$ | $\frac{2}{\sqrt{6}}D_3(-\sqrt{2}sin\theta_{1P1} + cos\theta_{1P1})$ |
| $A(h_1^0(1415) \to \omega\eta)$ | $\frac{\sqrt{2}}{3}D_3(-\sqrt{2}sin\theta_{1P1} + cos\theta_{1P1})(cos\theta_P - \sqrt{2}sin\theta_P)$ |
| ~~$A(h_1^0(1415) \to \omega\eta')$~~ | $\frac{\sqrt{2}}{3}D_3(-\sqrt{2}sin\theta_{1P1} + cos\theta_{1P1})(sin\theta_P + \sqrt{2}cos\theta_P)$ |
| $A(h_1^0(1415) \to \rho^+\pi^-)$ | $\frac{2}{\sqrt{6}}D_3(-\sqrt{2}sin\theta_{1P1} + cos\theta_{1P1})$ |
| $A(h_1^0(1415) \to \rho^-\pi^+)$ | $\frac{2}{\sqrt{6}}D_3(-\sqrt{2}sin\theta_{1P1} + cos\theta_{1P1})$ |
| $A(h_1^0(1415) \to K^{*+}K^-)$ | $-\frac{sin\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{cos\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1415) \to K^{*-}K^+)$ | $-\frac{sin\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{cos\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1415) \to K^{*0}\overline{K}^0)$ | $-\frac{sin\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{cos\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1415) \to \overline{K}^{*0}K^0)$ | $-\frac{sin\theta_{1P1}}{\sqrt{3}}(D_4 + D_5) + \frac{cos\theta_{1P1}}{\sqrt{6}}(D_4 - 2D_5)$ |
| $A(h_1^0(1415) \to \phi\eta)$ | $\frac{2}{3}D_6(sin\theta_{1P1} + \sqrt{2}cos\theta_{1P1})(\sqrt{2}cos\theta_P + sin\theta_P)$ |
| ~~$A(h_1^0(1415) \to \phi\eta')$~~ | $\frac{2}{3}D_6(sin\theta_{1P1} + \sqrt{2}cos\theta_{1P1})(\sqrt{2}sin\theta_P - cos\theta_P)$ |

$$F_1=F_2=F_3=F_4=F_5=F=c_0^A$$
$$D_1=D_2=D_3=D_4=D_5=D_6=D=c_0^B$$

# 3.2 Numerical results of $K_1 \to VP$

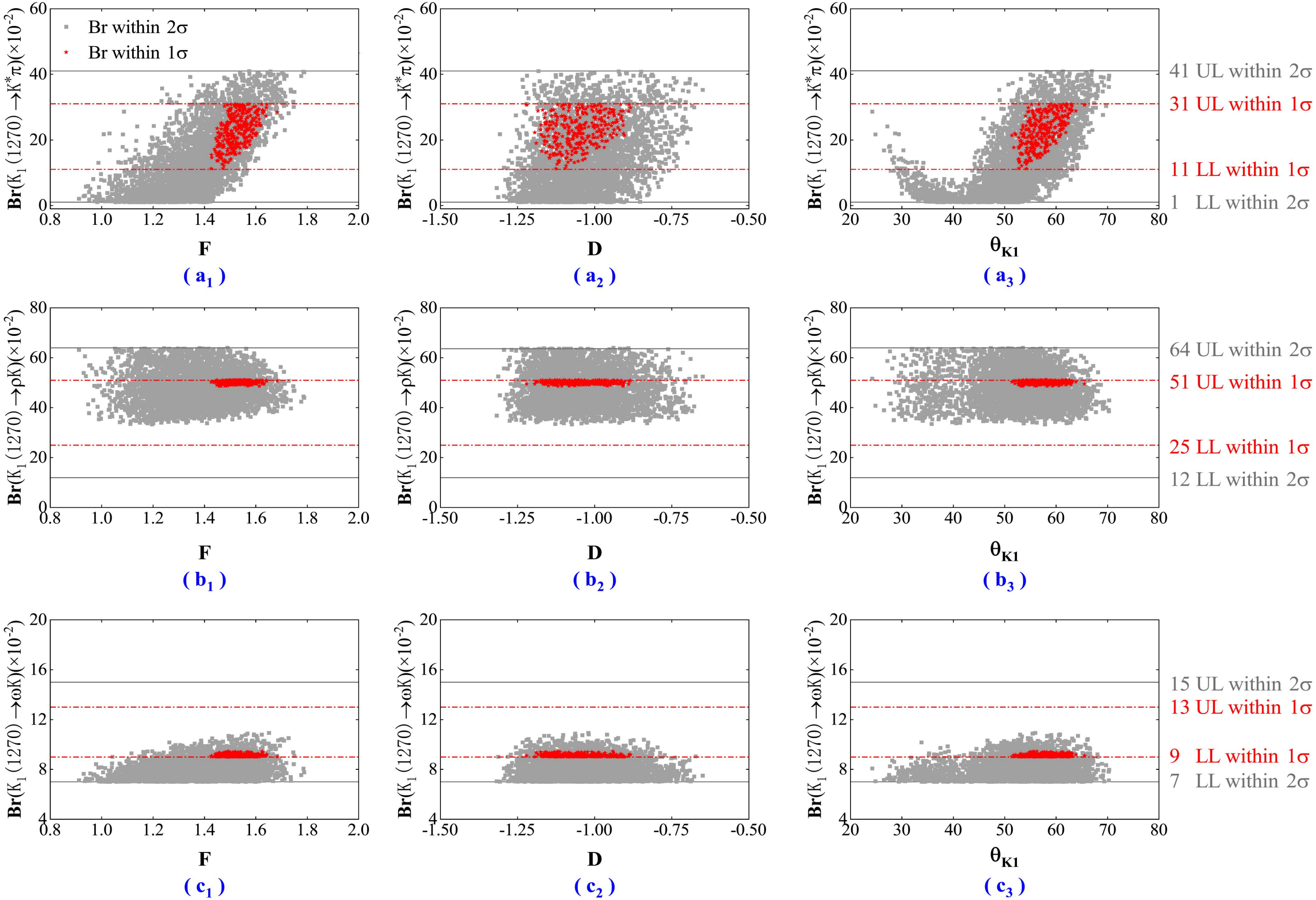
TABLE X: Branching raitos of $K_1 \to VP$ decays.

| Branching ratios | Experimental data | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|---|
| $\mathcal{B}(K_1^-(1270) \to K^{*-}\pi^0)(\times 10^{-2})$ | | $7.10 \pm 3.32$ | $7.07 \pm 6.73$ |
| $\mathcal{B}(K_1^-(1270) \to K^{*-}\eta)(\times 10^{-2})$ | | $0.50 \pm 0.15$ | $0.69 \pm 0.68$ |
| $\mathcal{B}(K_1^-(1270) \to \rho^0 K^-)(\times 10^{-2})$ | | $16.84 \pm 0.43$ | $16.53 \pm 5.32$ |
| $\mathcal{B}(K_1^-(1270) \to \omega K^-)(\times 10^{-2})$ | | $9.54 \pm 0.23$ | $9.25 \pm 2.03$ |
| $\mathcal{B}(K_1^-(1270) \to \overline{K}^{*0}\pi^-)(\times 10^{-2})$ | | $13.95 \pm 6.51$ | $13.86 \pm 13.20$ |
| $\mathcal{B}(K_1^-(1270) \to \rho^-\overline{K}^0)(\times 10^{-2})$ | | $32.67 \pm 0.81$ | $31.85 \pm 10.01$ |
| $\mathcal{B}(K_1^-(1270) \to \phi K^-)(\times 10^{-6})$ | | | $44.34 \pm 44.34$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to K^{*-}\pi^+)(\times 10^{-2})$ | | $14.12 \pm 6.60$ | $14.05 \pm 13.38$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \rho^+ K^-)(\times 10^{-2})$ | | $33.87 \pm 0.85$ | $33.13 \pm 10.56$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \overline{K}^{*0}\pi^0)(\times 10^{-2})$ | | $7.02 \pm 3.28$ | $6.98 \pm 6.65$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \overline{K}^{*0}\eta)(\times 10^{-2})$ | | $0.42 \pm 0.14$ | $0.61 \pm 0.61$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \rho^0\overline{K}^0)(\times 10^{-2})$ | | $16.24 \pm 0.40$ | $15.91 \pm 5.06$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \omega\overline{K}^0)(\times 10^{-2})$ | | $8.88 \pm 0.22$ | $8.55 \pm 2.01$ |
| $\mathcal{B}(\overline{K}_1^0(1270) \to \phi\overline{K}^0)(\times 10^{-6})$ | | | $24.33 \pm 24.33$ |
| $\mathcal{B}(K_1(1270) \to K^*\pi)(\times 10^{-2})$ | $21 \pm 10$ | $21.09 \pm 9.85$ | $20.98 \pm 19.98$ |
| $\mathcal{B}(K_1(1270) \to \rho K)(\times 10^{-2})$ | $38 \pm 13$ | $49.77 \pm 1.21$ | $48.61 \pm 15.37$ |
| $\mathcal{B}(K_1(1270) \to \omega K)(\times 10^{-2})$ | $11 \pm 2$ | $9.22 \pm 0.22$ | $8.96 \pm 1.96$ |
| $\mathcal{B}(K_1(1270) \to K^*\eta)(\times 10^{-2})$ | | $0.46 \pm 0.15$ | $0.65 \pm 0.65$ |
| $\mathcal{B}(K_1(1270) \to \phi K)(\times 10^{-6})$ | | | $34.34 \pm 34.34$ |

TABLE X: Branching raitos of $K_1 \to VP$ decays.

| Branching ratios | Experimental data | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|---|
| $\mathcal{B}(K_1^-(1400) \to K^{*-}\pi^0)(\times 10^{-2})$ | | $30.49 \pm 0.93$ | $29.69 \pm 2.19$ |
| $\mathcal{B}(K_1^-(1400) \to K^{*-}\eta)(\times 10^{-2})$ | | $2.72 \pm 1.75$ | $3.11 \pm 2.66$ |
| $\mathcal{B}(K_1^-(1400) \to \rho^0 K^-)(\times 10^{-2})$ | | $0.85 \pm 0.85$ | $1.50 \pm 1.50$ |
| $\mathcal{B}(K_1^-(1400) \to \omega K^-)(\times 10^{-2})$ | | $0.82 \pm 0.82$ | $1.44 \pm 1.44$ |
| $\mathcal{B}(K_1^-(1400) \to \overline{K}^{*0}\pi^-)(\times 10^{-2})$ | | $60.21 \pm 1.80$ | $58.68 \pm 4.36$ |
| $\mathcal{B}(K_1^-(1400) \to \rho^-\overline{K}^0)(\times 10^{-2})$ | | $1.67 \pm 1.67$ | $2.95 \pm 2.95$ |
| $\mathcal{B}(K_1^-(1400) \to \phi K^-)(\times 10^{-2})$ | | $3.80 \pm 0.61$ | $3.69 \pm 1.48$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to K^{*-}\pi^+)(\times 10^{-2})$ | | $60.77 \pm 1.84$ | $59.18 \pm 4.37$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \rho^+ K^-)(\times 10^{-2})$ | | $1.70 \pm 1.70$ | $3.01 \pm 3.01$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \overline{K}^{*0}\pi^0)(\times 10^{-2})$ | | $30.21 \pm 0.90$ | $29.44 \pm 2.19$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \overline{K}^{*0}\eta)(\times 10^{-2})$ | | $2.58 \pm 1.66$ | $2.96 \pm 2.53$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \rho^0\overline{K}^0)(\times 10^{-2})$ | | $0.84 \pm 0.84$ | $1.47 \pm 1.47$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \omega\overline{K}^0)(\times 10^{-2})$ | | $0.81 \pm 0.81$ | $1.42 \pm 1.42$ |
| $\mathcal{B}(\overline{K}_1^0(1400) \to \phi\overline{K}^0)(\times 10^{-2})$ | | $3.63 \pm 0.59$ | $3.53 \pm 1.43$ |
| $\mathcal{B}(K_1(1400) \to K^*\pi)(\times 10^{-2})$ | $94 \pm 6$ | $90.84 \pm 2.73$ | $88.53 \pm 6.53$ |
| $\mathcal{B}(K_1(1400) \to \rho K)(\times 10^{-2})$ | $3 \pm 3$ | $2.53 \pm 2.53$ | $4.47 \pm 4.47$ |
| $\mathcal{B}(K_1(1400) \to \omega K)(\times 10^{-2})$ | $1 \pm 1$ | $0.81 \pm 0.81$ | $1.43 \pm 1.43$ |
| $\mathcal{B}(K_1(1400) \to \eta K^*)(\times 10^{-2})$ | | $2.65 \pm 1.71$ | $3.04 \pm 2.60$ |
| $\mathcal{B}(K_1(1400) \to K\phi)(\times 10^{-2})$ | *seen* | $3.71 \pm 0.60$ | $3.61 \pm 1.45$ |

# 3.2 Numerical results of $K_1(1270)\to VP$




Our results of $\theta_{K_1}$:

[51°,65°] within 1σ errors

[24°,70°] within 2σ errors

$K_1(1270)\to K^*\pi$

$K_1(1270)\to K^*\eta$

Our results of F：

[1.43,1.68] within 1σ errors

[0.91,1.79] within 2σ errors

Our results of D：

[-1.22,-0.88] within 1σ errors

[-1.32,-0.65] within 2σ errors

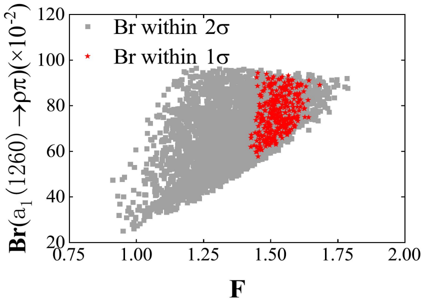
# 3.2 Numerical results of $a_1/b_1 \to VP$

TABLE XI: Branching raitos of $a_1/b_1 \to VP$ decays.

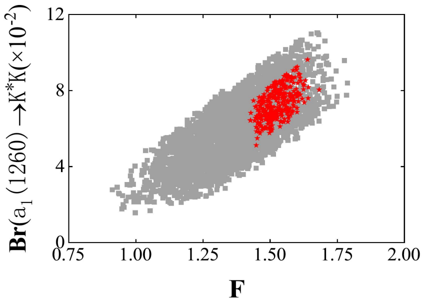
| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(a_1^0(1260) \to \rho^+\pi^-)(\times 10^{-2})$ | $37.97 \pm 9.17$ | $30.31 \pm 17.82$ |
| $\mathcal{B}(a_1^0(1260) \to \rho^-\pi^+)(\times 10^{-2})$ | $37.97 \pm 9.17$ | $30.31 \pm 17.82$ |
| $\mathcal{B}(a_1^0(1260) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $1.81 \pm 0.56$ | $1.54 \pm 1.17$ |
| $\mathcal{B}(a_1^0(1260) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $1.81 \pm 0.56$ | $1.54 \pm 1.17$ |
| $\mathcal{B}(a_1^0(1260) \to K^{*+}K^-)(\times 10^{-2})$ | $1.88 \pm 0.57$ | $1.61 \pm 1.20$ |
| $\mathcal{B}(a_1^0(1260) \to K^{*-}K^+)(\times 10^{-2})$ | $1.88 \pm 0.57$ | $1.61 \pm 1.20$ |
| $\mathcal{B}(a_1^-(1260) \to \rho^-\pi^0)(\times 10^{-2})$ | $38.15 \pm 9.21$ | $30.46 \pm 17.90$ |
| $\mathcal{B}(a_1^-(1260) \to \rho^0\pi^-)(\times 10^{-2})$ | $37.97 \pm 9.18$ | $30.36 \pm 17.90$ |
| $\mathcal{B}(a_1^-(1260) \to K^{*0}K^-)(\times 10^{-2})$ | $3.68 \pm 1.13$ | $3.14 \pm 2.36$ |
| $\mathcal{B}(a_1^-(1260) \to K^{*-}K^0)(\times 10^{-2})$ | $3.69 \pm 1.13$ | $3.16 \pm 2.37$ |

TABLE XI: Branching raitos of $a_1/b_1 \to VP$ decays.

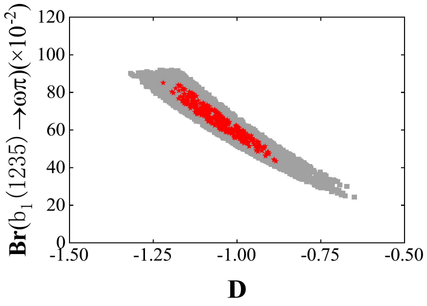
| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(b_1^0(1235) \to \rho^0\eta)(\times 10^{-2})$ | $5.05 \pm 1.79$ | $4.88 \pm 3.25$ |
| $\mathcal{B}(b_1^0(1235) \to \omega\pi^0)(\times 10^{-2})$ | $37.97 \pm 9.17$ | $58.16 \pm 33.83$ |
| $\mathcal{B}(b_1^0(1235) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $0.39 \pm 0.14$ | $0.39 \pm 0.28$ |
| $\mathcal{B}(b_1^0(1235) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $0.39 \pm 0.14$ | $0.39 \pm 0.28$ |
| $\mathcal{B}(b_1^0(1235) \to K^{*+}K^-)(\times 10^{-2})$ | $0.45 \pm 0.16$ | $0.45 \pm 0.31$ |
| $\mathcal{B}(b_1^0(1235) \to K^{*-}K^+)(\times 10^{-2})$ | $0.45 \pm 0.16$ | $0.45 \pm 0.31$ |
| $\mathcal{B}(b_1^-(1235) \to \rho^-\eta)(\times 10^{-2})$ | $5.11 \pm 1.83$ | $4.88 \pm 3.24$ |
| $\mathcal{B}(b_1^-(1235) \to \omega\pi^-)(\times 10^{-2})$ | $64.04 \pm 20.80$ | $57.90 \pm 33.68$ |
| $\mathcal{B}(b_1^-(1235) \to K^{*0}K^-)(\times 10^{-2})$ | $0.84 \pm 0.30$ | $0.83 \pm 0.58$ |
| $\mathcal{B}(b_1^-(1235) \to K^{*-}K^0)(\times 10^{-2})$ | $0.85 \pm 0.30$ | $0.85 \pm 0.59$ |



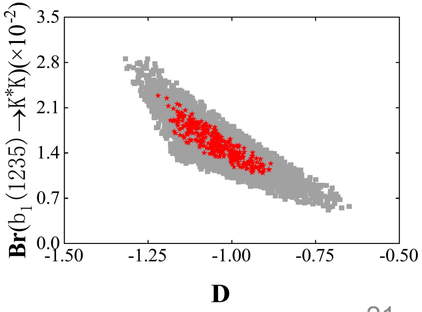
( $a_1$ )



( $a_2$ )



( $a_3$ )



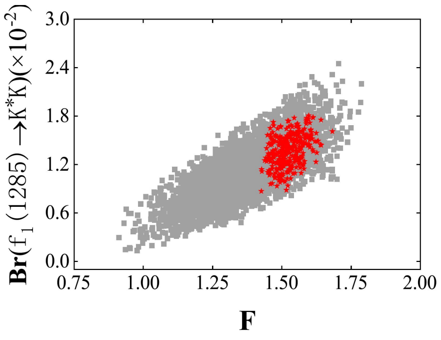
( $a_4$ )

# 3.2 Numerical results of $f_1 \to VP$

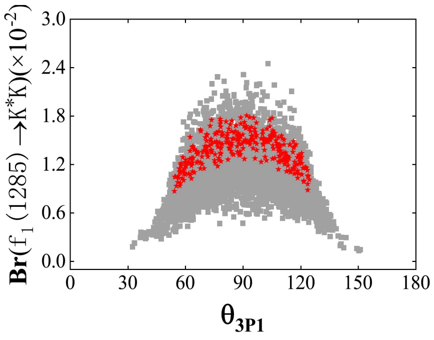
TABLE XII: Branching raitos of $f_1^0 \to VP$ decays

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(f_1^0(1285) \to K^{*+}K^-)(\times 10^{-2})$ | $0.54 \pm 0.19$ | $0.54 \pm 0.49$ |
| $\mathcal{B}(f_1^0(1285) \to K^{*-}K^+)(\times 10^{-2})$ | $0.54 \pm 0.19$ | $0.54 \pm 0.49$ |
| $\mathcal{B}(f_1^0(1285) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $0.13 \pm 0.05$ | $0.12 \pm 0.11$ |
| $\mathcal{B}(f_1^0(1285) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $0.13 \pm 0.05$ | $0.12 \pm 0.11$ |
| $\mathcal{B}(f_1^0(1285) \to KK^*)(\times 10^{-2})$ | $1.34 \pm 0.47$ | $1.29 \pm 1.16$ |
| $\mathcal{B}(f_1^0(1420) \to K^{*+}K^-)(\times 10^{-2})$ | $13.11 \pm 13.11$ | $13.25 \pm 13.25$ |
| $\mathcal{B}(f_1^0(1420) \to K^{*-}K^+)(\times 10^{-2})$ | $13.11 \pm 13.11$ | $13.25 \pm 13.25$ |
| $\mathcal{B}(f_1^0(1420) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $11.73 \pm 11.73$ | $11.84 \pm 11.84$ |
| $\mathcal{B}(f_1^0(1420) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $11.73 \pm 11.73$ | $11.84 \pm 11.84$ |
| $\mathcal{B}(f_1^0(1420) \to KK^*)(\times 10^{-2})$ | $49.68 \pm 49.68$ | $50.00 \pm 50.00$ |

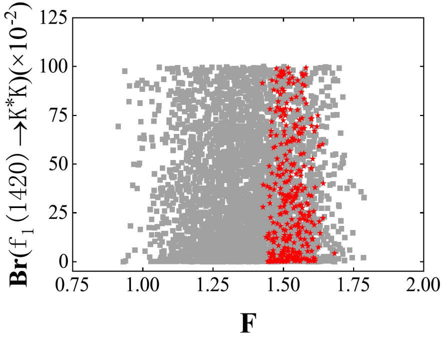
Our results of $\theta_{3P1}$: [55°,125°] within 1σ errors, [30°,145°] within 2σ errors



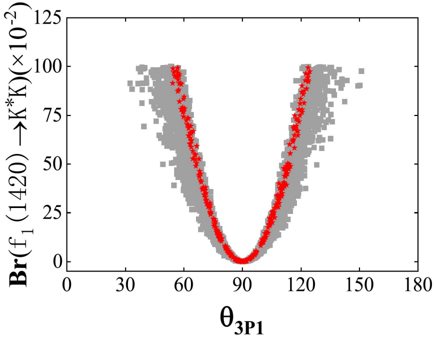
( $b_1$ )



( $b_2$ )



( $b_3$ )

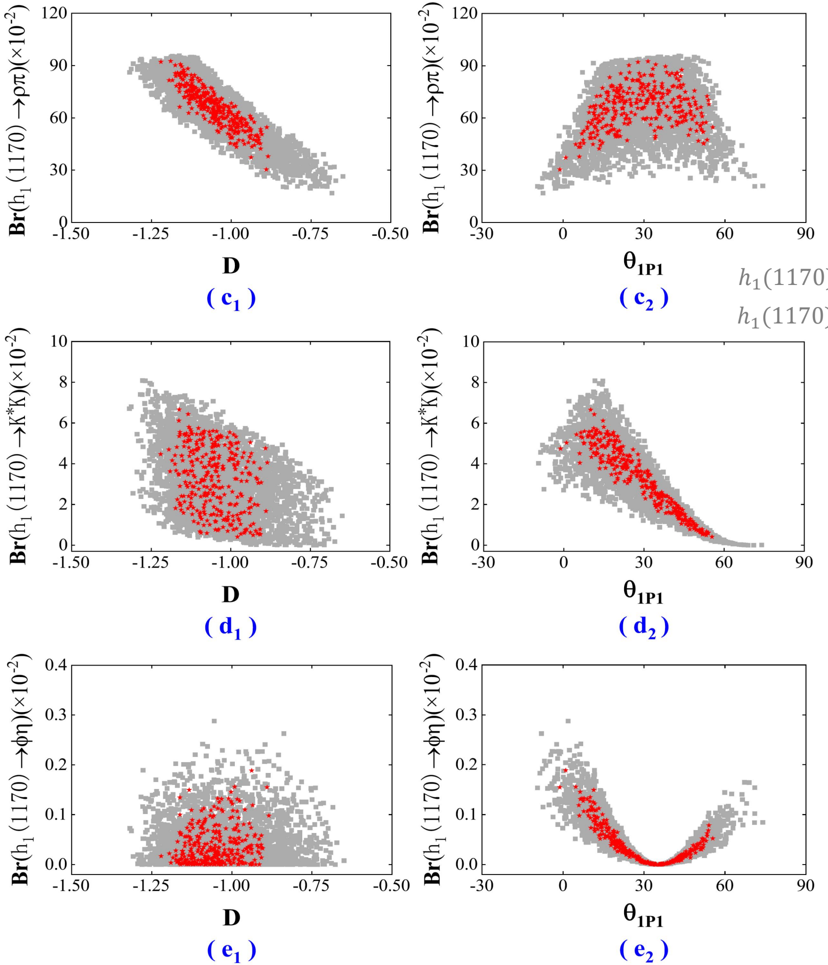


( $b_4$ )

# 3.2 Numerical results of $h_1 \to VP$

TABLE XIII: Branching raitos of $h_1^0 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(h_1^0(1170) \to \rho^0\pi^0)(\times 10^{-2})$ | $20.54 \pm 10.40$ | $18.82 \pm 13.18$ |
| $\mathcal{B}(h_1^0(1170) \to \omega\eta)(\times 10^{-2})$ | $1.92 \pm 0.94$ | $2.08 \pm 1.66$ |
| $\mathcal{B}(h_1^0(1170) \to \rho^+\pi^-)(\times 10^{-2})$ | $20.43 \pm 10.34$ | $18.70 \pm 13.10$ |
| $\mathcal{B}(h_1^0(1170) \to \rho^-\pi^+)(\times 10^{-2})$ | $20.43 \pm 10.34$ | $18.70 \pm 13.10$ |
| $\mathcal{B}(h_1^0(1170) \to K^{*+}K^-)(\times 10^{-2})$ | $0.90 \pm 0.80$ | $1.04 \pm 1.04$ |
| $\mathcal{B}(h_1^0(1170) \to K^{*-}K^+)(\times 10^{-2})$ | $0.90 \pm 0.80$ | $1.04 \pm 1.04$ |
| $\mathcal{B}(h_1^0(1170) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $0.86 \pm 0.76$ | $0.99 \pm 0.99$ |
| $\mathcal{B}(h_1^0(1170) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $0.86 \pm 0.76$ | $0.99 \pm 0.99$ |
| $\mathcal{B}(h_1^0(1170) \to \phi\eta)(\times 10^{-2})$ | $0.09 \pm 0.09$ | $0.14 \pm 0.14$ |
| $\mathcal{B}(h_1^0(1415) \to \rho^0\pi^0)(\times 10^{-2})$ | $13.04 \pm 13.04$ | $15.15 \pm 15.15$ |
| $\mathcal{B}(h_1^0(1415) \to \omega\eta)(\times 10^{-2})$ | $3.43 \pm 3.43$ | $4.03 \pm 4.03$ |
| $\mathcal{B}(h_1^0(1415) \to \rho^+\pi^-)(\times 10^{-2})$ | $13.01 \pm 13.01$ | $15.14 \pm 15.14$ |
| $\mathcal{B}(h_1^0(1415) \to \rho^-\pi^+)(\times 10^{-2})$ | $13.01 \pm 13.01$ | $15.14 \pm 15.14$ |
| $\mathcal{B}(h_1^0(1415) \to K^{*+}K^-)(\times 10^{-2})$ | $11.52 \pm 10.20$ | $13.37 \pm 13.17$ |
| $\mathcal{B}(h_1^0(1415) \to K^{*-}K^+)(\times 10^{-2})$ | $11.52 \pm 10.20$ | $13.37 \pm 13.17$ |
| $\mathcal{B}(h_1^0(1415) \to K^{*0}\overline{K}^0)(\times 10^{-2})$ | $10.03 \pm 8.88$ | $11.59 \pm 11.45$ |
| $\mathcal{B}(h_1^0(1415) \to \overline{K}^{*0}K^0)(\times 10^{-2})$ | $10.03 \pm 8.88$ | $11.59 \pm 11.45$ |
| $\mathcal{B}(h_1^0(1415) \to \phi\eta)(\times 10^{-2})$ | $1.06 \pm 0.69$ | $1.32 \pm 1.27$ |




$h_1(1170) \to \rho\pi$

$h_1(1170) \to \omega\eta$

Our results of $\theta_{1P1}$: [8°,58°] within 1σ errors, [-10°,75°] within 2σ errors

# Contents

# 4.2 Numerical results of D→A(A→VP)ℓν

$$\mathcal{B}(D \to A\ell^+\nu_\ell, A \to VP) = \mathcal{B}(D \to A\ell^+\nu_\ell) \times \mathcal{B}(A \to VP)$$

TABLE XIV: Branching ratios of the $D_{(s)} \to K_1 e^+\nu_e,\ K_1 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to K^{*-}\pi^0)(\times 10^{-5})$ | $7.03 \pm 3.76$ | $8.26 \pm 8.02$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to \overline{K}^{*0}\pi^-)(\times 10^{-5})$ | $13.79 \pm 7.36$ | $16.23 \pm 15.77$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to \rho^0 K^-)(\times 10^{-5})$ | $15.99 \pm 2.08$ | $16.61 \pm 9.16$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to \rho^- \overline{K}^0)(\times 10^{-5})$ | $31.01 \pm 4.16$ | $32.24 \pm 17.65$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to \omega K^-)(\times 10^{-5})$ | $8.97 \pm 1.18$ | $8.98 \pm 4.21$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to K^{*-}\eta)(\times 10^{-6})$ | $4.82 \pm 1.92$ | $7.76 \pm 7.72$ |
| $\mathcal{B}(D^0 \to K_1^-(1270)e^+\nu_e,\ K_1^-(1270) \to \phi K^-)(\times 10^{-9})$ | | $55.06 \pm 55.06$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to K^{*-}\pi^+)(\times 10^{-4})$ | $3.58 \pm 1.92$ | $4.23 \pm 4.11$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \overline{K}^{*0}\pi^0)(\times 10^{-4})$ | $1.78 \pm 0.95$ | $2.10 \pm 2.04$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \rho^+ K^-)(\times 10^{-4})$ | $8.25 \pm 1.10$ | $8.58 \pm 4.74$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \rho^0 \overline{K}^0)(\times 10^{-4})$ | $3.96 \pm 0.51$ | $4.09 \pm 2.25$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \omega \overline{K}^0)(\times 10^{-5})$ | $21.45 \pm 2.83$ | $21.28 \pm 10.24$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \overline{K}^{*0}\eta)(\times 10^{-5})$ | $1.03 \pm 0.43$ | $1.80 \pm 1.80$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)e^+\nu_e,\ \overline{K}_1^0(1270) \to \phi \overline{K}^0)(\times 10^{-9})$ | | $67.45 \pm 67.45$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to K^{*-}\pi^+)(\times 10^{-6})$ | $12.82 \pm 6.79$ | $15.24 \pm 14.80$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \overline{K}^{*0}\pi^0)(\times 10^{-6})$ | $6.37 \pm 3.37$ | $7.58 \pm 7.37$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \rho^+ K^-)(\times 10^{-6})$ | $29.77 \pm 3.88$ | $31.19 \pm 17.12$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \rho^0 \overline{K}^0)(\times 10^{-6})$ | $14.25 \pm 1.83$ | $14.87 \pm 8.14$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \omega \overline{K}^0))(\times 10^{-6})$ | $7.69 \pm 1.02$ | $7.66 \pm 3.81$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \overline{K}^{*0}\eta)(\times 10^{-7})$ | $0.37 \pm 0.15$ | $0.66 \pm 0.66$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1270)e^+\nu_e,\ K_1^0(1270) \to \phi \overline{K}^0)(\times 10^{-10})$ | | $24.98 \pm 24.98$ |

TABLE XIV: Branching ratios of the $D_{(s)} \to K_1 e^+\nu_e,\ K_1 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to K^{*-}\pi^0)(\times 10^{-6})$ | $9.58 \pm 9.58$ | $35.76 \pm 35.76$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to \overline{K}^{*0}\pi^-)(\times 10^{-6})$ | $18.94 \pm 18.94$ | $70.64 \pm 70.64$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to \rho^0 K^-)(\times 10^{-7})$ | $4.46 \pm 4.46$ | $24.77 \pm 24.77$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to \rho^- \overline{K}^0)(\times 10^{-7})$ | $8.77 \pm 8.77$ | $48.59 \pm 48.59$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to \omega K^-)(\times 10^{-7})$ | $4.30 \pm 4.30$ | $23.89 \pm 23.89$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to K^{*-}\eta)(\times 10^{-7})$ | $6.17 \pm 6.17$ | $61.09 \pm 61.09$ |
| $\mathcal{B}(D^0 \to K_1^-(1400)e^+\nu_e,\ K_1^-(1400) \to \phi K^-)(\times 10^{-7})$ | $12.31 \pm 12.31$ | $44.26 \pm 44.26$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to K^{*-}\pi^+(\times 10^{-5})$ | $4.92 \pm 4.92$ | $18.20 \pm 18.20$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \overline{K}^{*0}\pi^0)(\times 10^{-5})$ | $2.45 \pm 2.45$ | $9.05 \pm 9.05$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \rho^+ K^-)(\times 10^{-6})$ | $2.30 \pm 2.30$ | $12.63 \pm 12.63$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \rho^0 \overline{K}^0)(\times 10^{-7})$ | $11.32 \pm 11.32$ | $62.15 \pm 62.15$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \omega \overline{K}^0)(\times 10^{-7})$ | $10.92 \pm 10.92$ | $59.89 \pm 59.89$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \overline{K}^{*0}\eta)(\times 10^{-6})$ | $1.51 \pm 1.51$ | $14.84 \pm 14.84$ |
| $\mathcal{B}(D^+ \to \overline{K}_1^0(1400)e^+\nu_e,\ \overline{K}_1^0(1400) \to \phi \overline{K}^0)(\times 10^{-6})$ | $3.03 \pm 3.03$ | $10.83 \pm 10.83$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to K^{*-}\pi^+)(\times 10^{-6})$ | $2.02 \pm 2.02$ | $7.45 \pm 7.45$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \overline{K}^{*0}\pi^0)(\times 10^{-7})$ | $10.03 \pm 10.03$ | $37.05 \pm 37.05$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \rho^+ K^-)(\times 10^{-8})$ | $9.32 \pm 9.32$ | $51.69 \pm 51.69$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \rho^0 \overline{K}^0)(\times 10^{-8})$ | $4.59 \pm 4.59$ | $25.44 \pm 25.44$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \omega \overline{K}^0)(\times 10^{-8})$ | $4.43 \pm 4.43$ | $24.52 \pm 24.52$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \overline{K}^{*0}\eta)(\times 10^{-8})$ | $6.16 \pm 6.16$ | $61.07 \pm 61.07$ |
| $\mathcal{B}(D_s^+ \to K_1^0(1400)e^+\nu_e,\ K_1^0(1400) \to \phi \overline{K}^0))(\times 10^{-8})$ | $12.43 \pm 12.43$ | $44.33 \pm 44.33$ |

# 4.2 Numerical results of D→A(A→VP)ℓν

TABLE XV: Branching ratios of the $D_{(s)} \to a_1/b_1 e^+\nu_e,\ a_1/b_1 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D^0 \to a_1^-(1260)e^+\nu_e,\ a_1^-(1260) \to \rho^-\pi^0)(\times 10^{-5})$ | $1.61 \pm 0.79$ | $2.09 \pm 1.82$ |
| $\mathcal{B}(D^0 \to a_1^-(1260)e^+\nu_e,\ a_1^-(1260) \to \rho^0\pi^-)(\times 10^{-5})$ | $1.61 \pm 0.78$ | $2.08 \pm 1.81$ |
| $\mathcal{B}(D^0 \to a_1^-(1260)e^+\nu_e,\ a_1^-(1260) \to K^{*0}K^-)(\times 10^{-6})$ | $1.48 \pm 0.59$ | $1.34 \pm 1.07$ |
| $\mathcal{B}(D^0 \to a_1^-(1260)e^+\nu_e,\ a_1^-(1260) \to K^{*-}K^0)(\times 10^{-6})$ | $1.48 \pm 0.59$ | $1.34 \pm 1.07$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to \rho^+\pi^-)(\times 10^{-5})$ | $2.05 \pm 1.00$ | $2.64 \pm 2.29$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to \rho^-\pi^+)(\times 10^{-5})$ | $2.05 \pm 1.00$ | $2.64 \pm 2.29$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to K^{*0}\overline{K}^0)(\times 10^{-7})$ | $9.24 \pm 3.69$ | $8.47 \pm 6.76$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to \overline{K}^{*0}K^0)(\times 10^{-7})$ | $9.24 \pm 3.69$ | $8.47 \pm 6.76$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to K^{*+}K^-)(\times 10^{-7})$ | $9.61 \pm 3.81$ | $8.76 \pm 6.97$ |
| $\mathcal{B}(D^+ \to a_1^0(1260)e^+\nu_e,\ a_1^0(1260) \to K^{*-}K^+)(\times 10^{-7})$ | $9.61 \pm 3.81$ | $8.76 \pm 6.97$ |
| $\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e,\ b_1^-(1235) \to \omega\pi^-)(\times 10^{-6})$ | $16.53 \pm 11.10$ | $21.61 \pm 18.75$ $10^{-5}$ |
| $\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e,\ b_1^-(1235) \to \rho^-\eta)(\times 10^{-7})$ | $12.18 \pm 8.41$ | $16.69 \pm 14.74$ |
| $\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e,\ b_1^-(1235) \to K^{*0}K^-)(\times 10^{-7})$ | $2.10 \pm 1.44$ | $2.78 \pm 2.43$ |
| $\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e,\ b_1^-(1235) \to K^{*-}K^0)(\times 10^{-7})$ | $2.13 \pm 1.46$ | $2.82 \pm 2.46$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to \omega\pi^0)(\times 10^{-6})$ | $21.21 \pm 14.23$ | $27.49 \pm 23.82$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to \rho^0\eta)(\times 10^{-6})$ | $1.53 \pm 1.05$ | $2.14 \pm 1.89$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to K^{*0}\overline{K}^0)(\times 10^{-8})$ | $12.62 \pm 8.67$ | $16.78 \pm 14.75$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to \overline{K}^{*0}K^0)(\times 10^{-8})$ | $12.62 \pm 8.67$ | $16.78 \pm 14.75$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to K^{*+}K^-)(\times 10^{-8})$ | $14.50 \pm 9.90$ | $19.20 \pm 16.79$ |
| $\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to K^{*-}K^+)(\times 10^{-8})$ | $14.50 \pm 9.90$ | $19.20 \pm 16.79$ |

$$\mathcal{B}(D^+ \to b_1^0(1235)e^+\nu_e,\ b_1^0(1235) \to \omega\pi^0) \le 1.75 \times 10^{-4}$$

$$\mathcal{B}(D^0 \to b_1^-(1235)e^+\nu_e,\ b_1^-(1235) \to \omega\pi^-) \le 1.12 \times 10^{-4}$$

TABLE XVI: Branching ratios of the $D_{(s)} \to f_1^0 e^+\nu_e,\ f_1^0 \to VP$ decays..

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D_s^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*+}K^-)(\times 10^{-6})$ | $3.96 \pm 3.66$ | $5.13 \pm 5.13$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*-}K^+)(\times 10^{-6})$ | $3.96 \pm 3.66$ | $5.13 \pm 5.13$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*0}\overline{K}^0)(\times 10^{-7})$ | $9.45 \pm 8.75$ | $11.82 \pm 11.82$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to \overline{K}^{*0}K^0)(\times 10^{-7})$ | $9.45 \pm 8.75$ | $11.82 \pm 11.82$ |
| $\mathcal{B}(D^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*+}K^-)(\times 10^{-7})$ | $1.08 \pm 1.08$ | $1.60 \pm 1.60$ |
| $\mathcal{B}(D^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*-}K^+)(\times 10^{-8})$ | $10.82 \pm 10.82$ | $15.95 \pm 15.95$ |
| $\mathcal{B}(D^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to K^{*0}\overline{K}^0)(\times 10^{-8})$ | $2.29 \pm 2.29$ | $3.66 \pm 3.66$ |
| $\mathcal{B}(D^+ \to f_1^0(1285)e^+\nu_e,\ f_1^0(1285) \to \overline{K}^{*0}K^0)(\times 10^{-9})$ | $22.93 \pm 22.93$ | $36.57 \pm 36.57$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)e^+\nu_e,\ f_1^0(1420) \to K^{*+}K^-)(\times 10^{-5})$ | $7.16 \pm 7.16$ | $10.83 \pm 10.83$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to K^{*-}K^+)(\times 10^{-5})$ | $7.16 \pm 7.16$ | $10.83 \pm 10.83$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to K^{*0}\overline{K}^0)(\times 10^{-5})$ | $6.46 \pm 6.46$ | $9.67 \pm 9.67$ |
| $\mathcal{B}(D_s^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to \overline{K}^{*0}K^0)(\times 10^{-5})$ | $6.46 \pm 6.46$ | $9.67 \pm 9.67$ |
| $\mathcal{B}(D^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to K^{*+}K^-)(\times 10^{-7})$ | $24.63 \pm 24.63$ | $39.00 \pm 39.00$ |
| $\mathcal{B}(D^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to K^{*-}K^+)(\times 10^{-7})$ | $24.63 \pm 24.63$ | $39.00 \pm 39.00$ |
| $\mathcal{B}(D^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to K^{*0}\overline{K}^0)(\times 10^{-7})$ | $22.17 \pm 22.17$ | $35.24 \pm 35.24$ |
| $\mathcal{B}(D^+ \to f_1^0(1420)e^+\nu_e,\ f_1(1420) \to \overline{K}^{*0}K^0)(\times 10^{-7})$ | $22.17 \pm 22.17$ | $35.24 \pm 35.24$ |

# 4.1 Branching of D→A(A→VP)ℓν

$10^{-5}$

TABLE XVII: Branching ratios of the $D_{(s)} \to h_1^0 e + \nu_e,\ h_1^0 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^0\pi^0)(\times 10^{-5})$ | $2.57 \pm 2.57$ | $3.64 \pm 3.64$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^+\pi^-)(\times 10^{-5})$ | $2.55 \pm 2.55$ | $3.62 \pm 3.62$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^-\pi^+)(\times 10^{-5})$ | $2.55 \pm 2.55$ | $3.62 \pm 3.62$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*+}K^-)(\times 10^{-6})$ | $1.98 \pm 1.98$ | $4.20 \pm 4.20$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*-}K^+)(\times 10^{-6})$ | $1.98 \pm 1.98$ | $4.20 \pm 4.20$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*0}\overline{K}^0)(\times 10^{-6})$ | $1.88 \pm 1.88$ | $3.99 \pm 3.99$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \overline{K}^{*0}K^0)(\times 10^{-6})$ | $1.88 \pm 1.88$ | $3.99 \pm 3.99$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \omega\eta)(\times 10^{-6})$ | $2.49 \pm 2.49$ | $3.10 \pm 3.10$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^0\pi^0)(\times 10^{-6})$ | $8.84 \pm 7.09$ | $10.41 \pm 9.78$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^+\pi^-)(\times 10^{-6})$ | $8.79 \pm 7.04$ | $10.35 \pm 9.72$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \rho^-\pi^+)(\times 10^{-6})$ | $8.79 \pm 7.04$ | $10.35 \pm 9.72$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*+}K^-)(\times 10^{-7})$ | $3.84 \pm 3.63$ | $5.29 \pm 5.29$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*-}K^+)(\times 10^{-7})$ | $3.84 \pm 3.63$ | $5.29 \pm 5.29$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to K^{*0}\overline{K}^0)(\times 10^{-7})$ | $3.65 \pm 3.45$ | $5.05 \pm 5.05$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \overline{K}^{*0}K^0)(\times 10^{-7})$ | $3.65 \pm 3.45$ | $5.05 \pm 5.05$ |
| $\mathcal{B}(D^+ \to h_1^0(1170)e^+\nu_e,\ h_1^0(1170) \to \omega\eta)(\times 10^{-7})$ | $7.44 \pm 5.99$ | $11.86 \pm 11.38$ |

TABLE XVII: Branching ratios of the $D_{(s)} \to h_1^0 e + \nu_e,\ h_1^0 \to VP$ decays.

| Branching ratios | Predictions with $1\sigma$ | Predictions with $2\sigma$ |
|---|---|---|
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^0\pi^0)(\times 10^{-5})$ | $4.40 \pm 4.40$ | $6.42 \pm 6.42$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^+\pi^-)(\times 10^{-5})$ | $4.39 \pm 4.39$ | $6.40 \pm 6.40$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^-\pi^+)(\times 10^{-5})$ | $4.39 \pm 4.39$ | $6.40 \pm 6.40$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*+}K^-)(\times 10^{-5})$ | $5.27 \pm 5.08$ | $6.62 \pm 6.60$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*-}K^+)(\times 10^{-5})$ | $5.27 \pm 5.08$ | $6.62 \pm 6.60$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*0}\overline{K}^0)(\times 10^{-5})$ | $4.40 \pm 4.24$ | $5.83 \pm 5.82$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \overline{K}^{*0}K^0)(\times 10^{-5})$ | $4.40 \pm 4.24$ | $5.83 \pm 5.82$ |
| $\mathcal{B}(D_s^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \omega\eta)(\times 10^{-6})$ | $11.20 \pm 11.20$ | $15.18 \pm 15.18$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^0\pi^0)(\times 10^{-7})$ | $3.98 \pm 3.98$ | $11.79 \pm 11.79$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^+\pi^-)(\times 10^{-7})$ | $3.98 \pm 3.98$ | $11.75 \pm 11.75$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \rho^-\pi^+)(\times 10^{-7})$ | $3.98 \pm 3.98$ | $11.75 \pm 11.75$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*+}K^-)(\times 10^{-7})$ | $1.20 \pm 1.20$ | $2.35 \pm 2.35$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*-}K^+)(\times 10^{-7})$ | $1.20 \pm 1.20$ | $2.35 \pm 2.35$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to K^{*0}\overline{K}^0)(\times 10^{-7})$ | $1.02 \pm 1.02$ | $2.01 \pm 2.01$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \overline{K}^{*0}K^0)(\times 10^{-7})$ | $1.02 \pm 1.02$ | $2.01 \pm 2.01$ |
| $\mathcal{B}(D^+ \to h_1^0(1415)e^+\nu_e,\ h_1^0(1415) \to \omega\eta)(\times 10^{-7})$ | $1.01 \pm 1.01$ | $2.91 \pm 2.91$ |

# Conclusion

- ✓ Using SU(3) Flavor Symmetry approach to study $D \to A(A \to VP)\ell^+\nu_\ell$ decays.
- ✓ Predicted the not-yet-measured observables, and some of them are obtained for the first time.
- ✓ Some predictions of $D \to A(A \to VP)\ell^+\nu_\ell$ could be tested at BESIII, LHCb and Belle-II.

# Thank you for your attention !