Cut structures and an observable singularity in the three-body threshold dynamics: the T_{cc}^{+} case

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Outline

- Three-body threshold effect in hadron-hadron scattering
- Complex scaling method
- Complex scaled Lippman-Schwinger equation (CSLSE)
- Application to the recently observed T_{cc}^{+}

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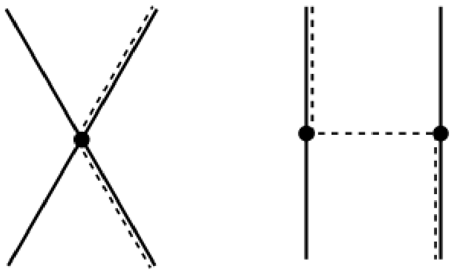
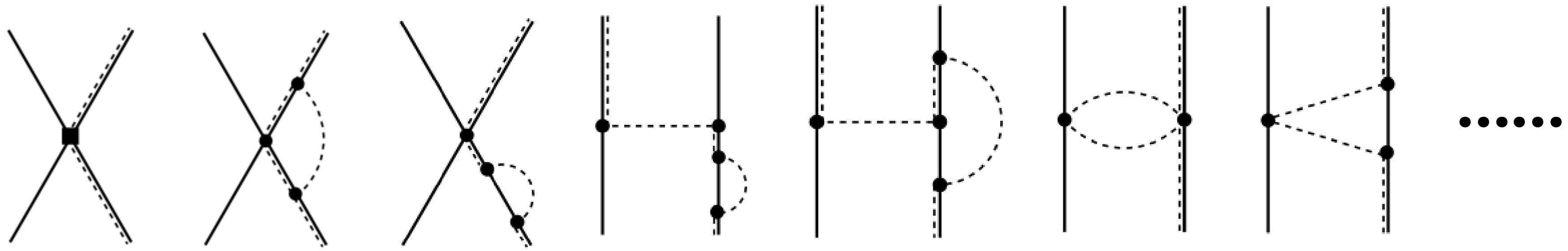
Three-body threshold effect in hadron-hadron scattering

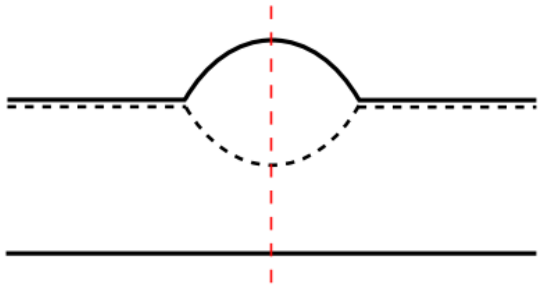
Leading order

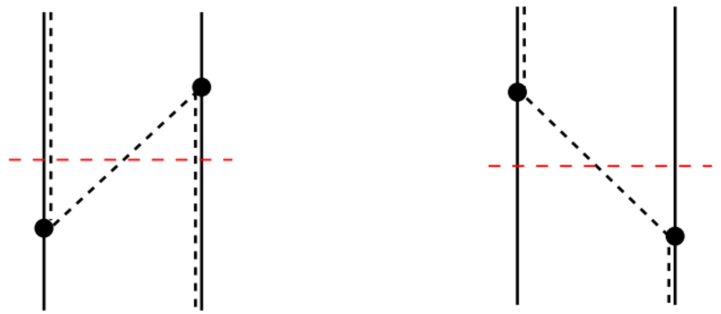


Next − to − leading order



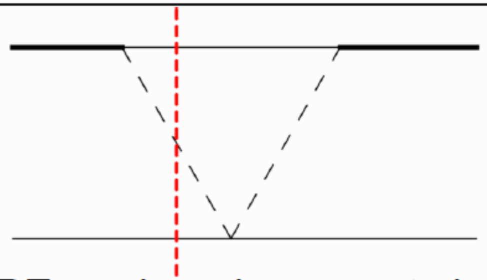
Three-body dynamics





(a) (b)

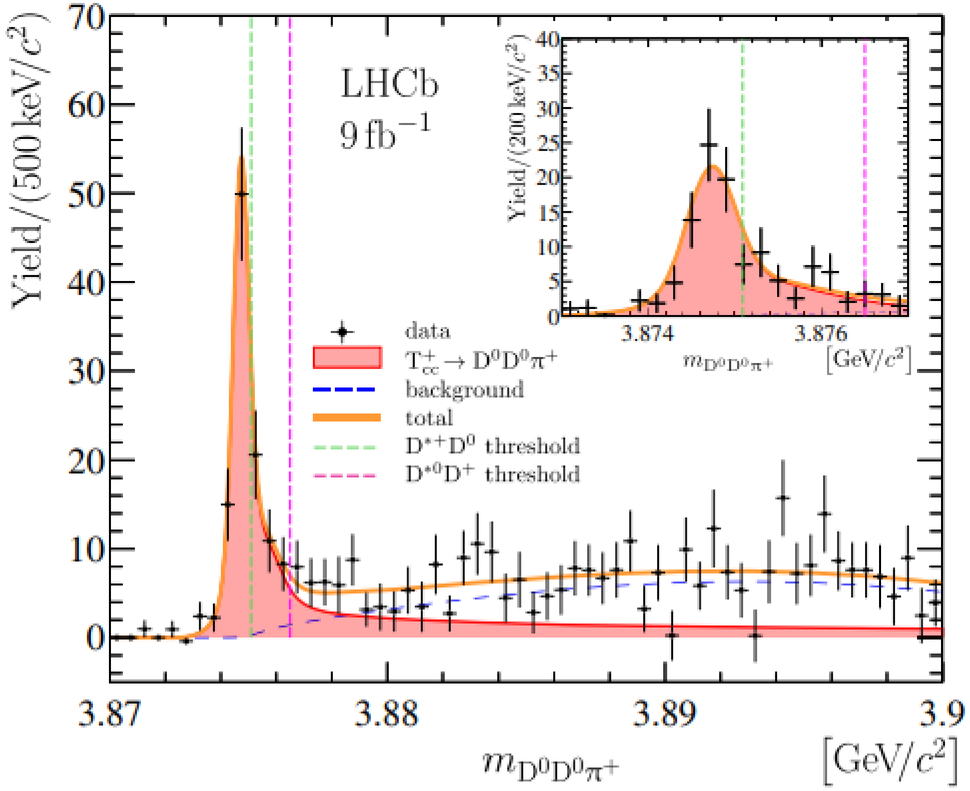
One-pion exchange (OPE)



In TPE, an imaginary part shows up naturally in the loop integral.

The importance of the three-body dynamics

$$\delta m_{\text{pole}} = -360 \pm 40\ \text{keV},\ \ \Gamma_{\text{pole}} = 48 \pm 2\ \text{keV}$$



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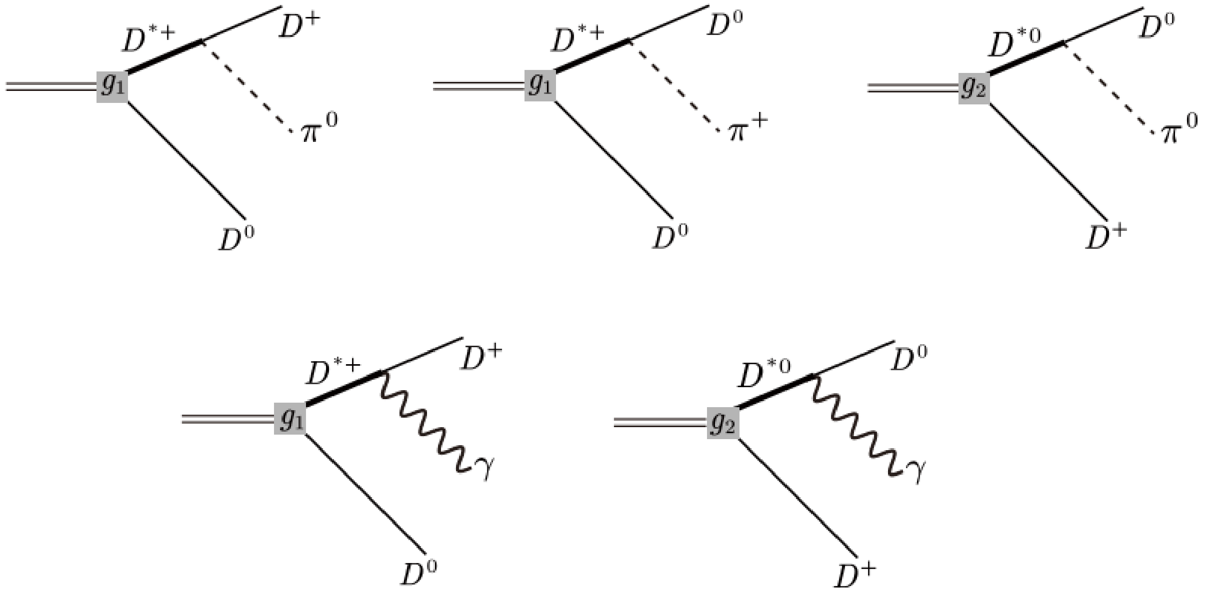
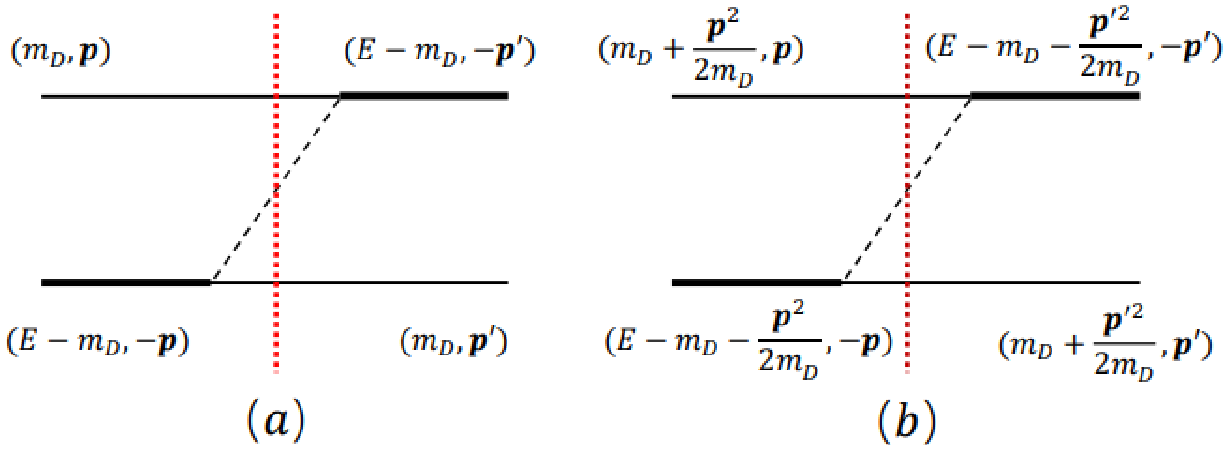


$$|T_{cc}^+\rangle = \cos\theta|D^{*+}D^0, \phi_1\rangle + \sin\theta|D^{*0}D^+, \phi_2\rangle,$$

The revised OPE potential involving three-body threshold dynamics



$$V_{\rm OPE}^{I=0} = -\frac{g^2}{8f_\pi^2}\frac{(p^2 + p'^2 - 2pp'z)(\varepsilon' \cdot \varepsilon)}{(E' + \delta)^2 - (p^2 + p'^2 - 2pp'z) - m_\pi^2 + i\epsilon}$$

$$\begin{aligned} V_{\rm OPE}^{I=0}(p, p') &\propto ((k_0^2/(2\mu) + \delta + m_\pi)((k_0^2/(2\mu) + \delta - m_\pi))^{-1} \quad (8) \\ &\propto ((E - m_D - m_D - m_\pi)(E - m_D - m_D + m_\pi))^{-1}, \end{aligned}$$

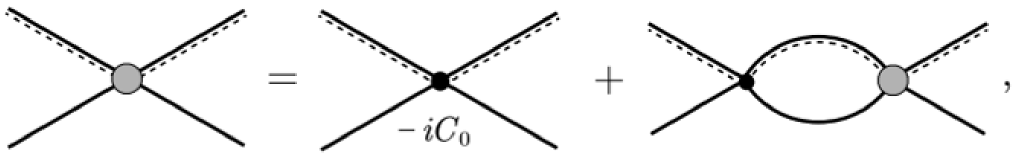
Three-body effect: energy-dependent potential

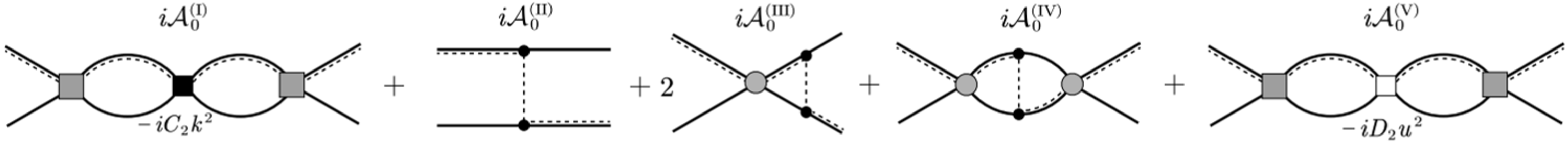
$$\frac{i}{q^2 - m_\pi^2 + i\epsilon} \longrightarrow \frac{i}{(E + \delta)^2 - \boldsymbol{q}^2 - m_\pi^2 + i\epsilon}$$

Partial-wave potential

$$V_S^{I=0}(p, p') = 4\pi C_t + \int_{-1}^{1} dz 2\pi V_{\rm OPE}^{I=0}(p, p', z).$$

The re-summation in the dynamical equation-
Lippmann-Schwinger equation (LSE) and Schrödinger equation





- Schrödinger equation

$$\frac{\boldsymbol{k}^2}{2m}\phi(\boldsymbol{k}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} V(\boldsymbol{k},\boldsymbol{p})\phi(\boldsymbol{p}) = E\phi(\boldsymbol{k})$$

- Lippmann-Schwinger equation

$$T(\boldsymbol{k}',\boldsymbol{k};E) = V(\boldsymbol{k}',\boldsymbol{k}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{V(\boldsymbol{k}',\boldsymbol{p})T(\boldsymbol{p},\boldsymbol{k};E)}{E-\boldsymbol{p}^2/2\mu + i\epsilon}$$

- **Two-body threshold unitary cut**
- **Three-body threshold cut**
- **Left-hand or right-hand cut**

Complex scaling method (CSM) in Schrödinger equation

Schrödinger equation

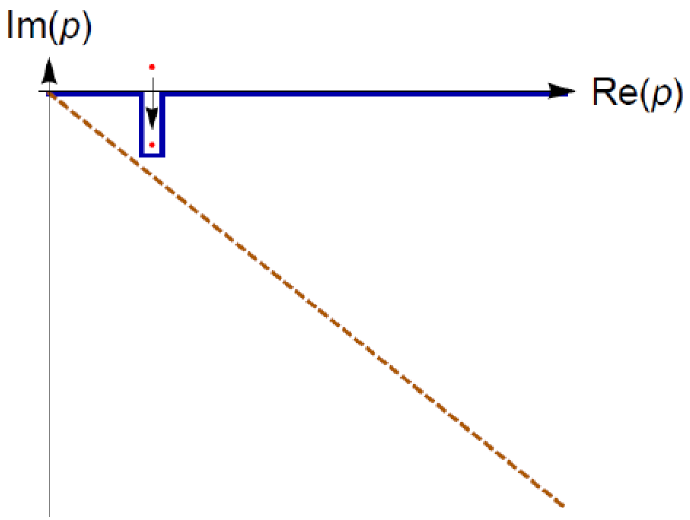
$$\frac{\boldsymbol{k}^2}{2m}\phi(\boldsymbol{k}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} V(\boldsymbol{k},\boldsymbol{p})\phi(\boldsymbol{p}) = E\phi(\boldsymbol{k})$$

- Analytical extension of the wave function

$$\phi(\boldsymbol{k}) = \frac{1}{E_R - \frac{\boldsymbol{k}^2}{2m}} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} V(\boldsymbol{k},\boldsymbol{p})\phi(\boldsymbol{p})$$

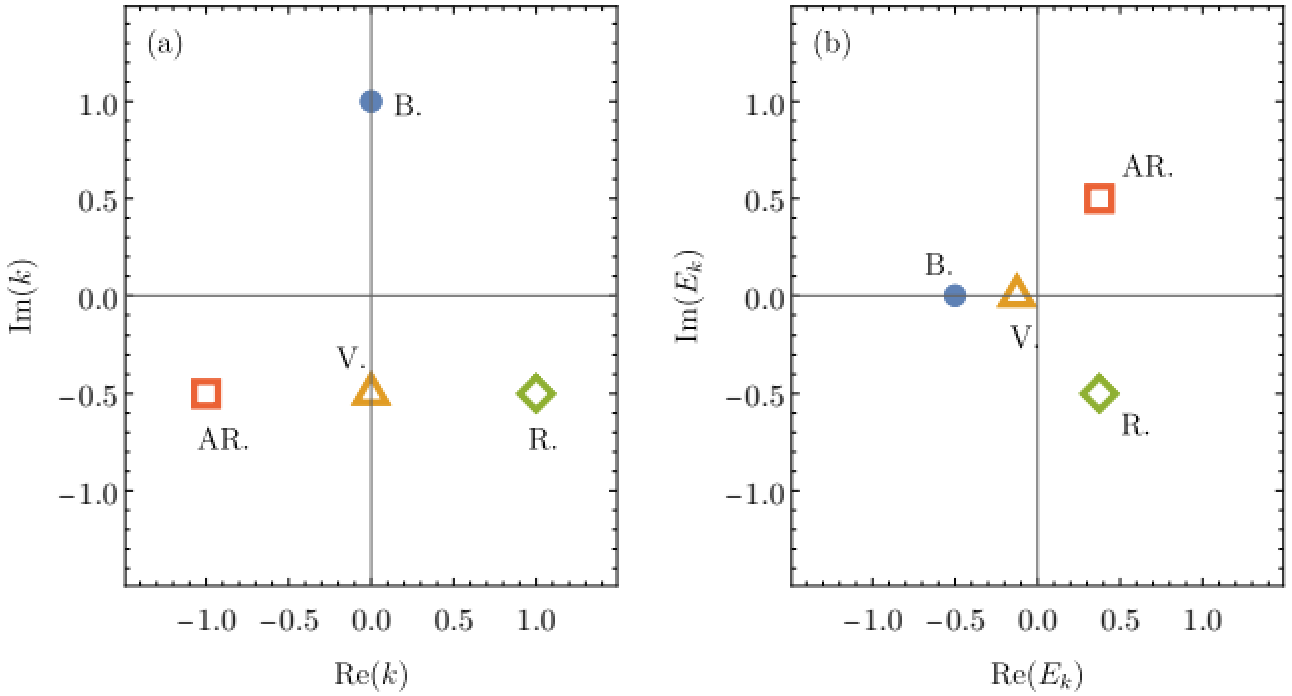
k can be anywhere on the complex plane

p is on the integral path



- $\phi(\boldsymbol{k})$ has two poles $k = \pm\sqrt{2mE_R}$

Different Riemann sheets in T matrix

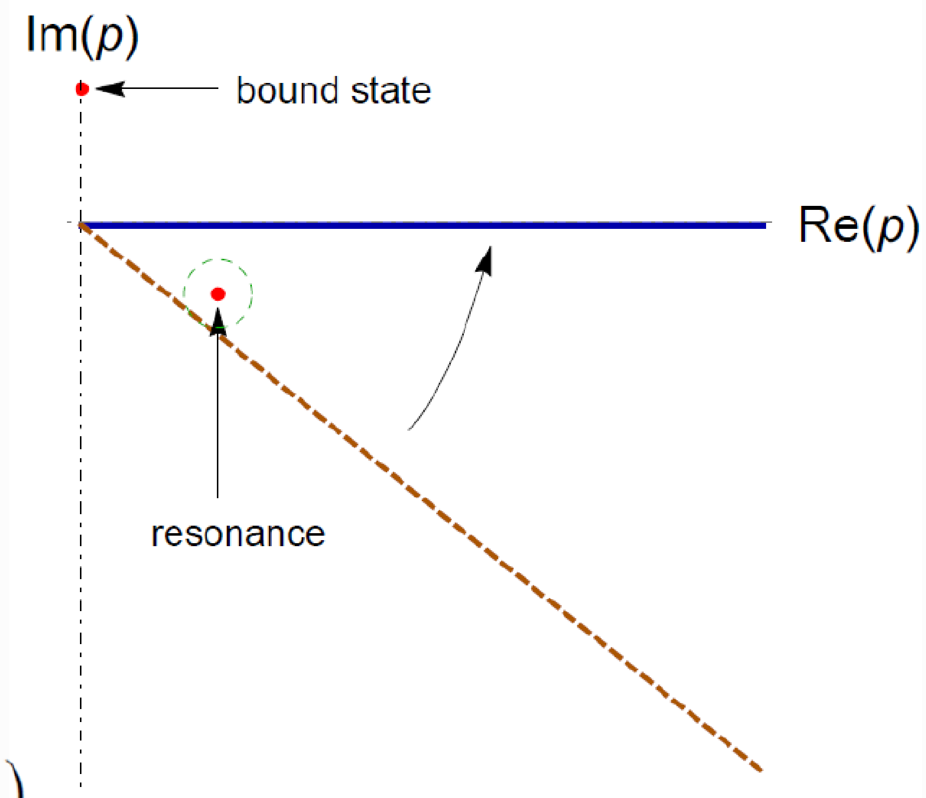


$$
\begin{array}{lll}
\text{bound states:} & k_B = i\gamma_b & \text{(RS-I)},\\
\text{virtual states:} & k_V = -i\gamma_v & \text{(RS-II)},\\
\text{resonances:} & k_R = \kappa_r - i\gamma_r & \text{(RS-II)},\\
\text{anti-resonances:} & k_{AR} = -\kappa_r - i\gamma_r & \text{(RS-II)}.
\end{array}
$$

- 2-channel example
- $k_1 = \sqrt{2\mu_1(E - E_{th1})}$ $k_2 = \sqrt{2\mu_2(E - E_{th2})}$
- Sheet I: $\mathrm{Im}(k_1) > 0$ $\mathrm{Im}(k_2) > 0$
  bound state
- Sheet II: $\mathrm{Im}(k_1) < 0$ $\mathrm{Im}(k_2) < 0$
  quasibound state (Feshbach-type resonance)
- Sheet III: $\mathrm{Im}(k_1) < 0$ $\mathrm{Im}(k_2) < 0$
  resonance
- Sheet IV: $\mathrm{Im}(k_1) < 0$ $\mathrm{Im}(k_2) < 0$
  “threshold cusp”

Complex scaling method (CSM) in Schrödinger equation

- 1st Riemann sheet: integrate along the real axis
- 2nd Riemann sheet: the residue of the pole must be include
- Or change the integral path



- E.g. complex scaling method:

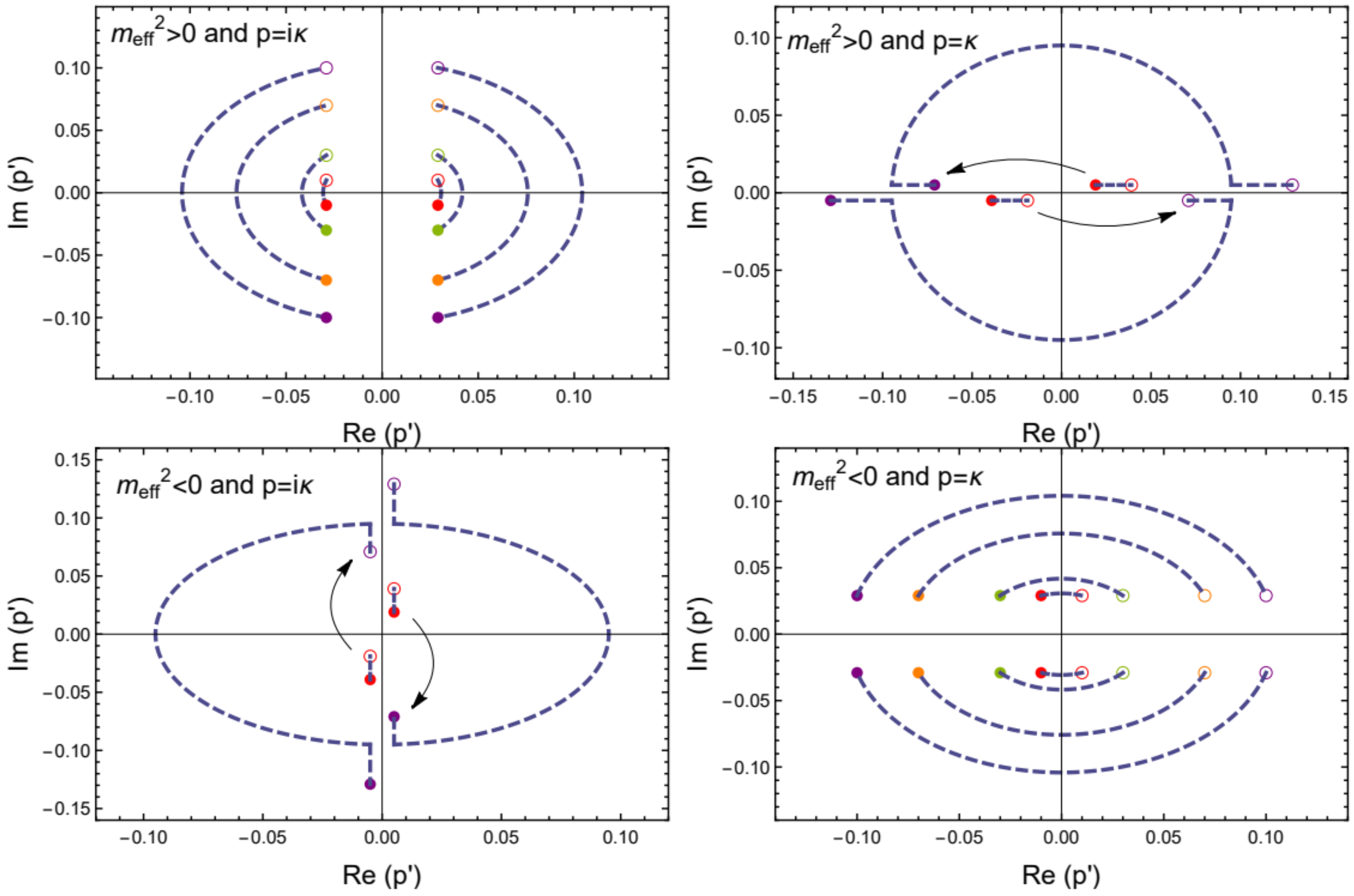
$$E\tilde{\phi}_l(p) = \frac{p^2 e^{-2i\theta}}{2m}\tilde{\phi}_l(p) + \int \frac{p'^2 e^{-3i\theta} dp'}{(2\pi)^3} V_{l,l'}(pe^{-i\theta}, p'e^{-i\theta})\tilde{\phi}_{l'}(p')$$

- Avoid the branch cut in the potential

It is okay for searching for pole position!

More information:T matrix, cross section, line shape, etc,--- LSE

The cut structures of three-body threshold dynamics



Half-on-shell amplitude

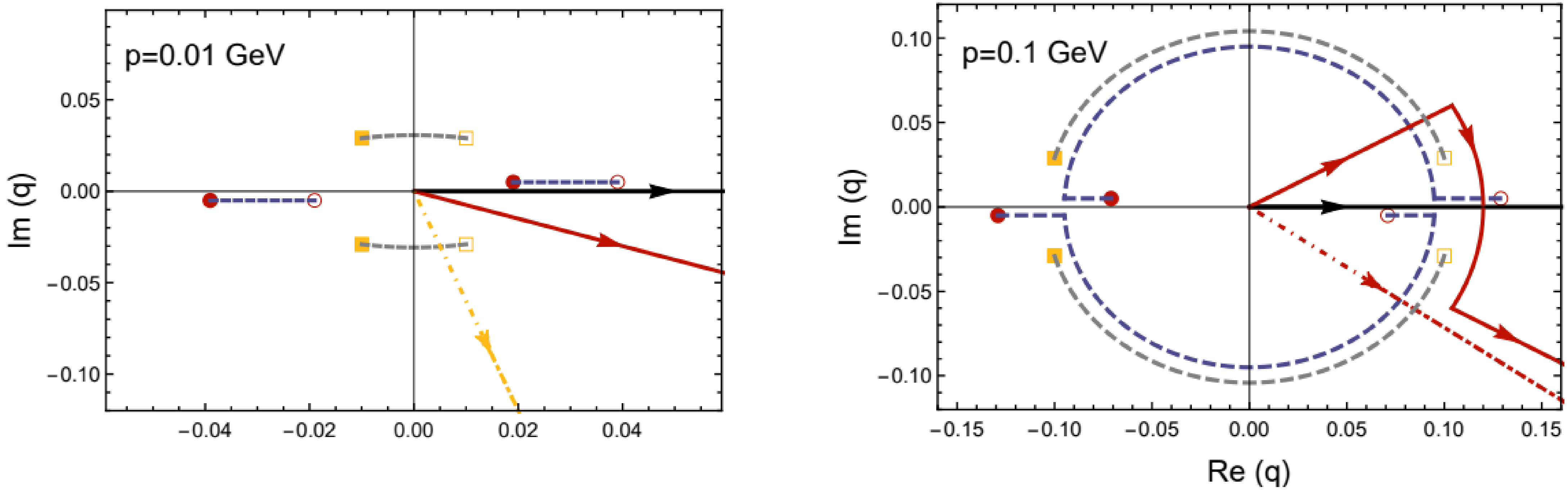
Critical point $\kappa = |m_{eff}|.$

$$m_{eff}^2 = (E' + \delta)^2 - m_\pi^2.$$

FIG. 2: The cut structures of the half-on-shell scattering amplitudes from the OPE potential involving the three-body dynamics under different cases. The red, green, orange and purple singularities correspond to κ = 0.01, 0.03, 0.07 and 0.1 GeV, respectively and the dashed line denotes the branch cuts.

Complex scaled Lippmann-Schwinger equation (CSLSE)

$$T(p,p',k_0) = V(p,p',k_0) + \int_0^\infty \frac{dq q^2}{(2\pi)^3} V(p,q,k_0) \times G(q,k_0) T(q,p',k_0),$$



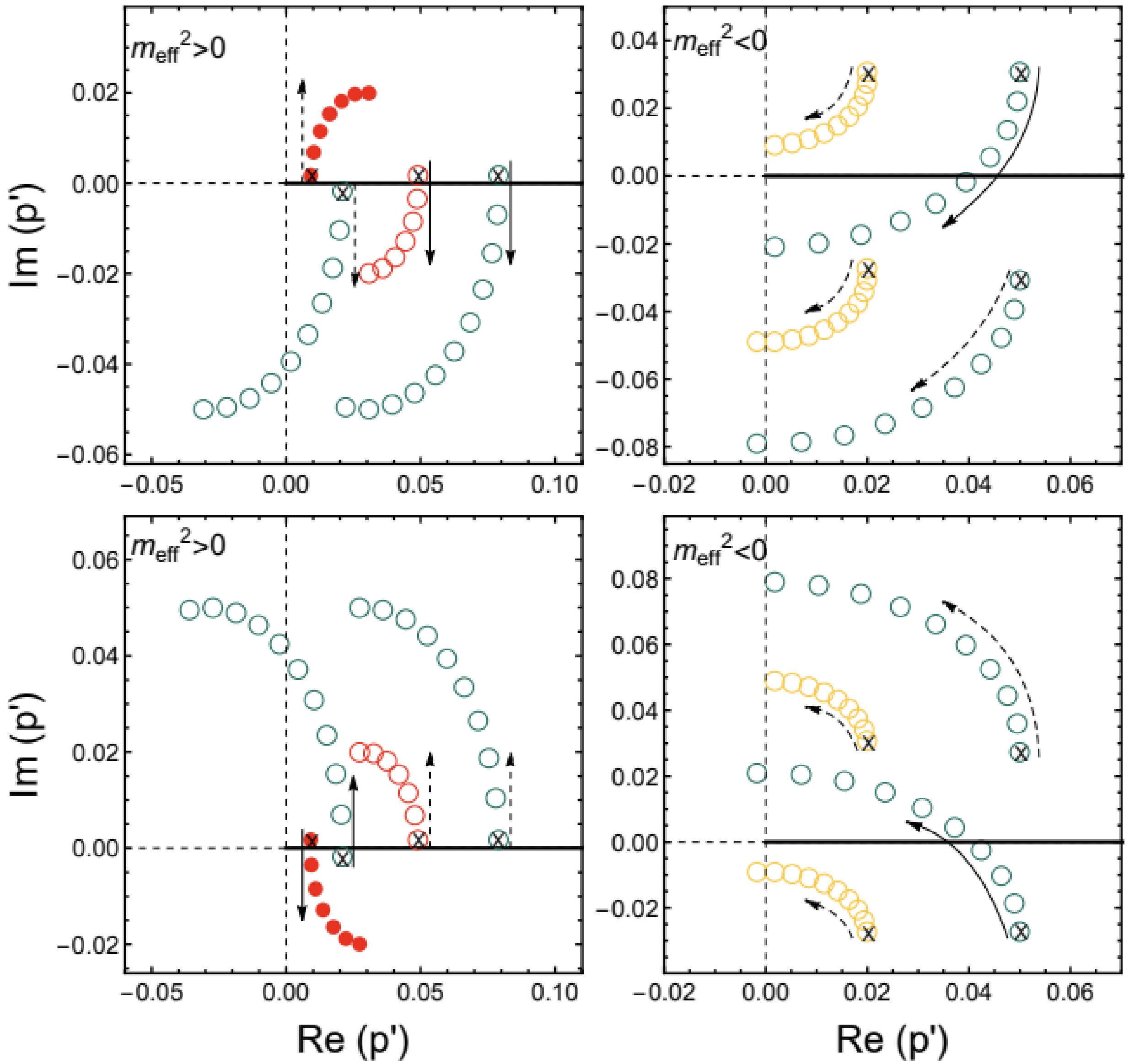
Complex scaled Lippmann-Schwinger equation (CSLSE)

Proper analytical extension of LSE

$$T(\kappa e^{-i\phi}, p', k_0) = V(\kappa e^{-i\phi}, p', k_0) + \int_0^\infty \frac{d^3\mathbf{q}}{(2\pi)^3} e^{-3i\phi} V(\kappa e^{-i\phi}, qe^{-i\phi}, k_0) G_\gamma(qe^{-i\phi}, k_0) T(qe^{-i\phi}, p', k_0), \quad (11)$$

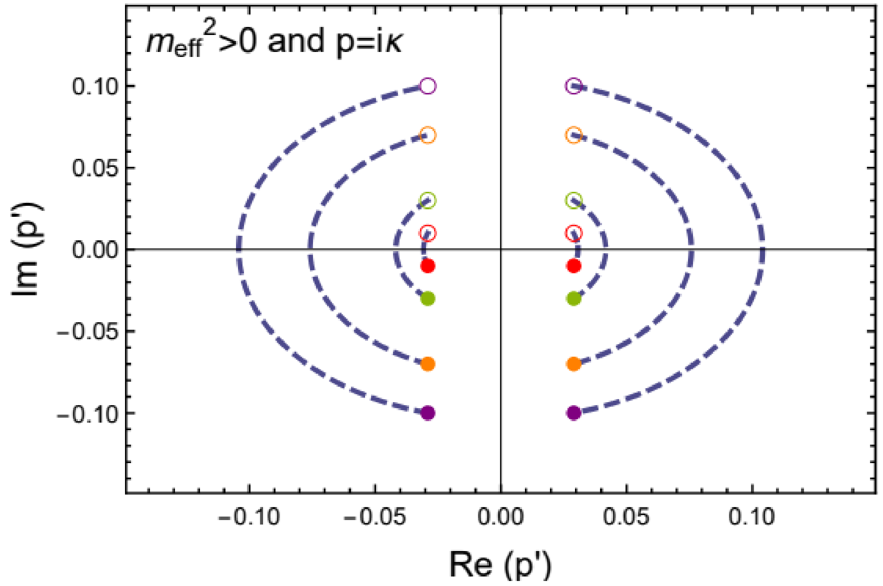
$$T(\kappa e^{i\phi}, p', k_0) = V(\kappa e^{i\phi}, p', k_0) + \int_0^\infty \frac{d^3\mathbf{q}}{(2\pi)^3} e^{-3i\phi} V(\kappa e^{i\phi}, qe^{-i\phi}, k_0) G_\gamma(qe^{-i\phi}, k_0) T(qe^{-i\phi}, p', k_0). \quad (12)$$

Critical point $\kappa = |m_{eff}|$.



04 Application to the recently observed T_{cc}^{+}

The solved on-shell T matrix with the energy below DD^* threshold



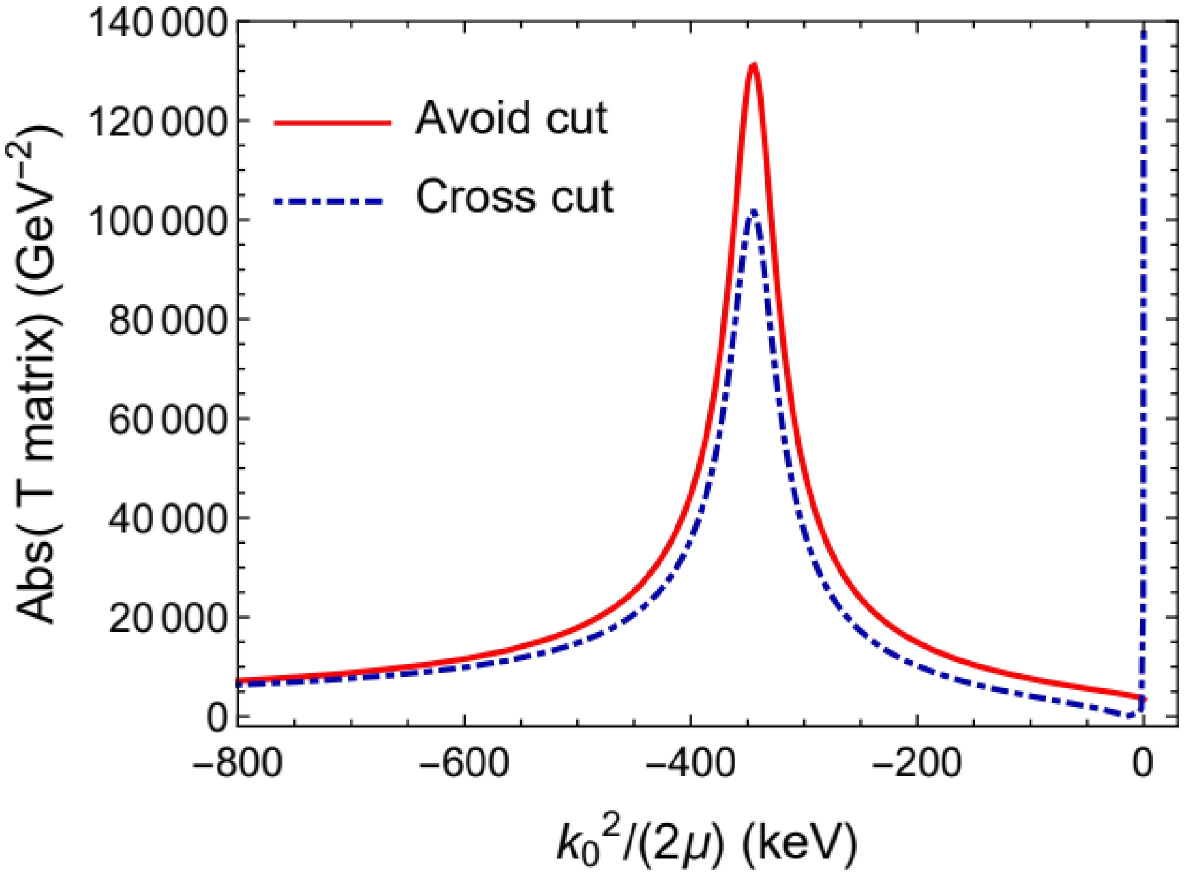


FIG. 5: The physical on-shell T matrix of the isoscalar DD^* scattering below the threshold.

An observable singularity structure in addition to T_{cc}^+

The solved on-shell T matrix with the energy above DD^* threshold (physical region!)

$$\mathcal{F}(p,p') = \exp[-\frac{p^2}{\Lambda^2} - \frac{p'^2}{\Lambda^2}].$$

$$\frac{k_0^2}{2\mu} = M - m_{threshold} \simeq 100\ \text{keV}$$

The right-hand cut from the revised OPE potential involving three-body dynamics

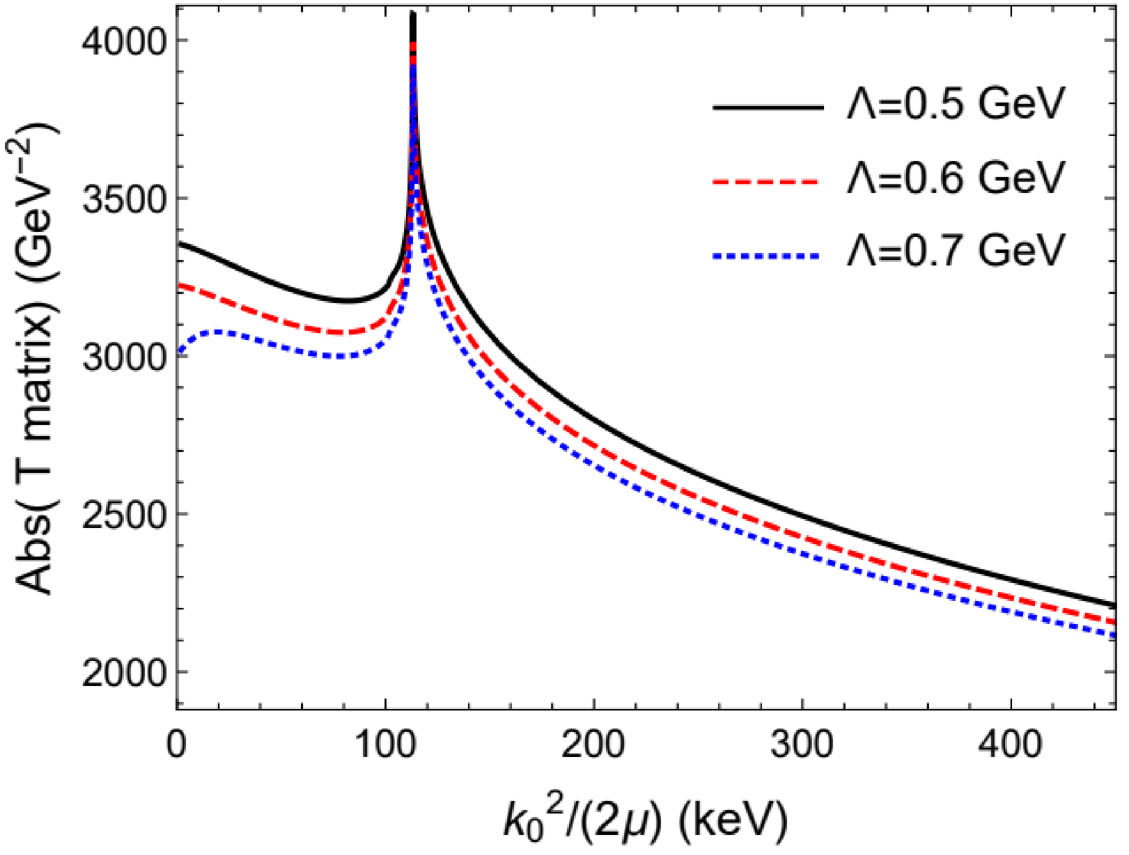
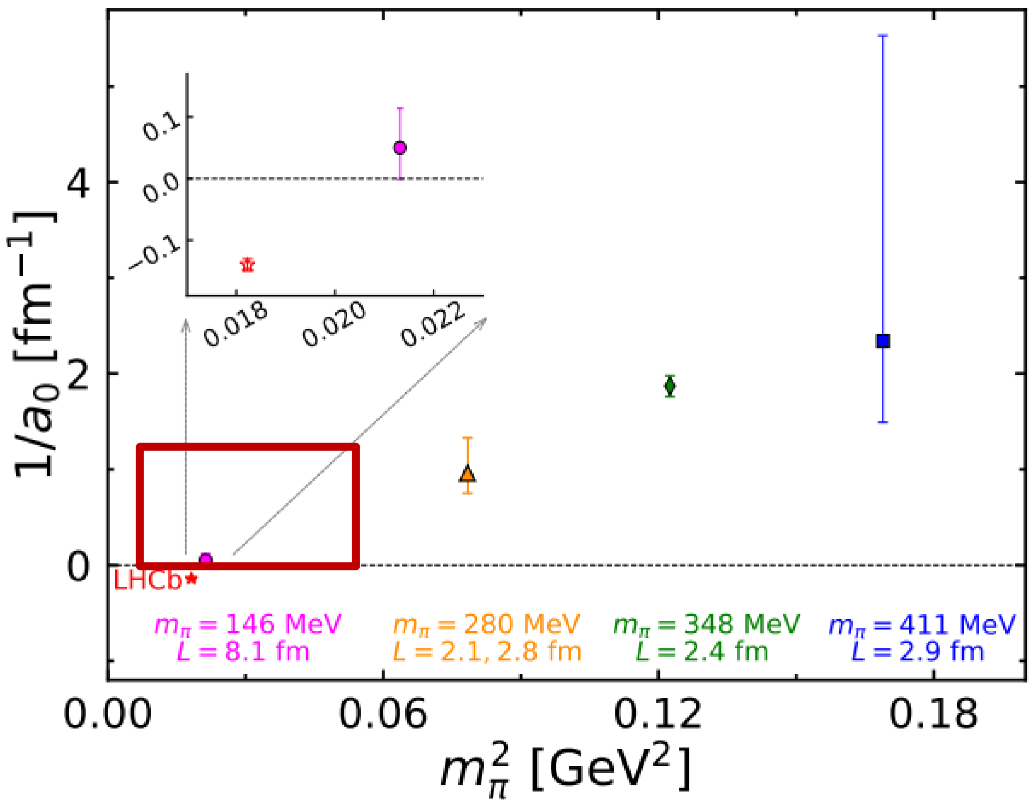
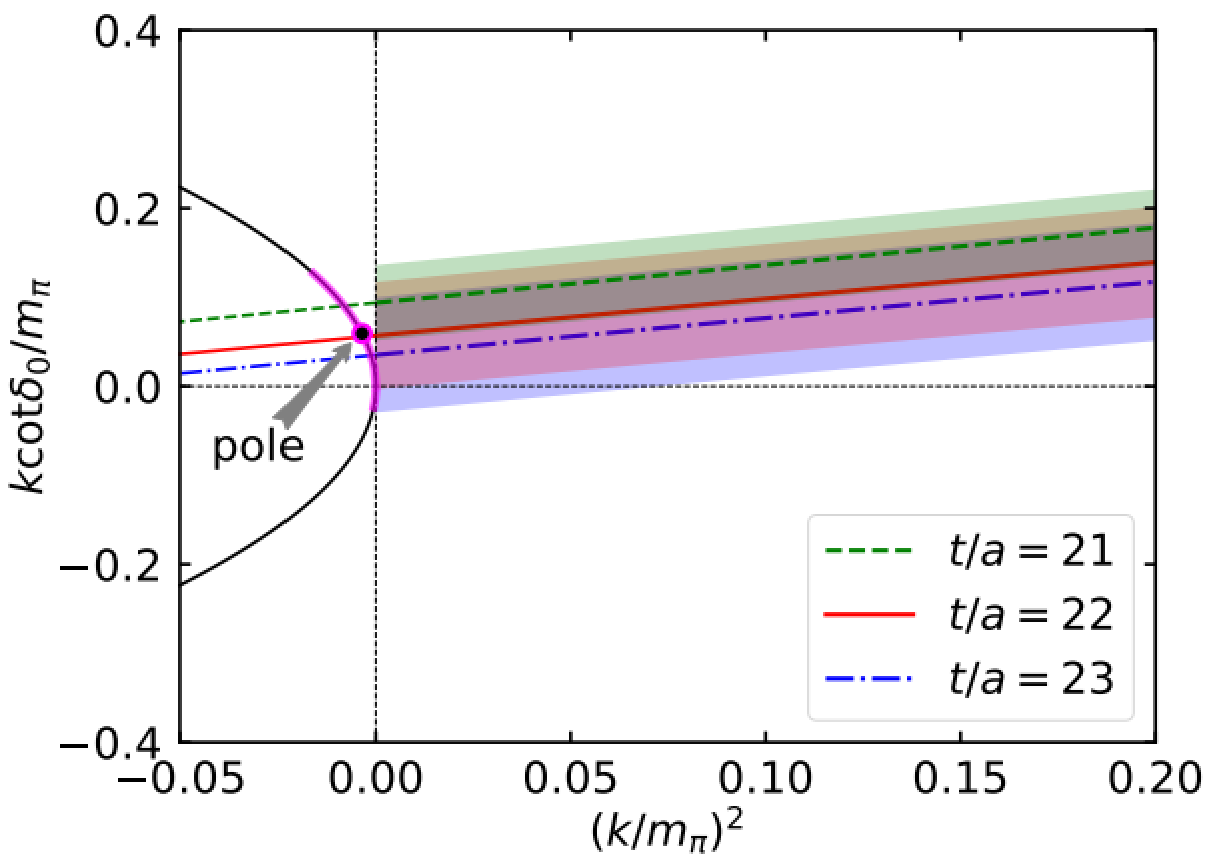


FIG. 6: The physical on-shell T matrix of the isoscalar DD^* scattering above the threshold.

Phase shift of the DD^* scattering-Lattice QCD

HAL QCD approach



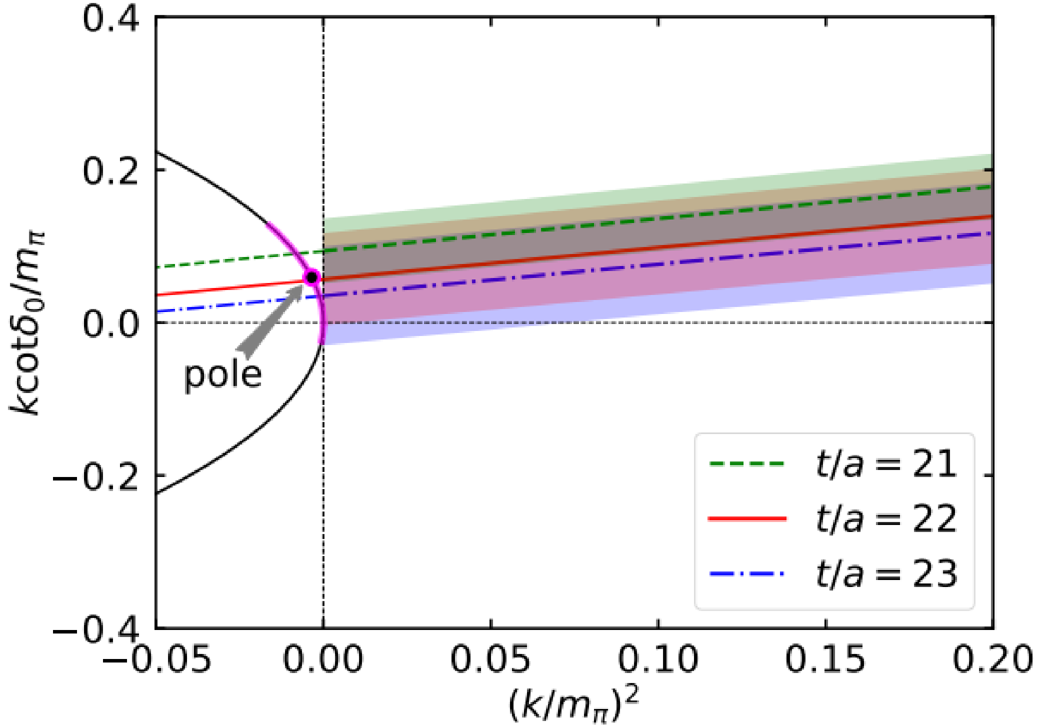


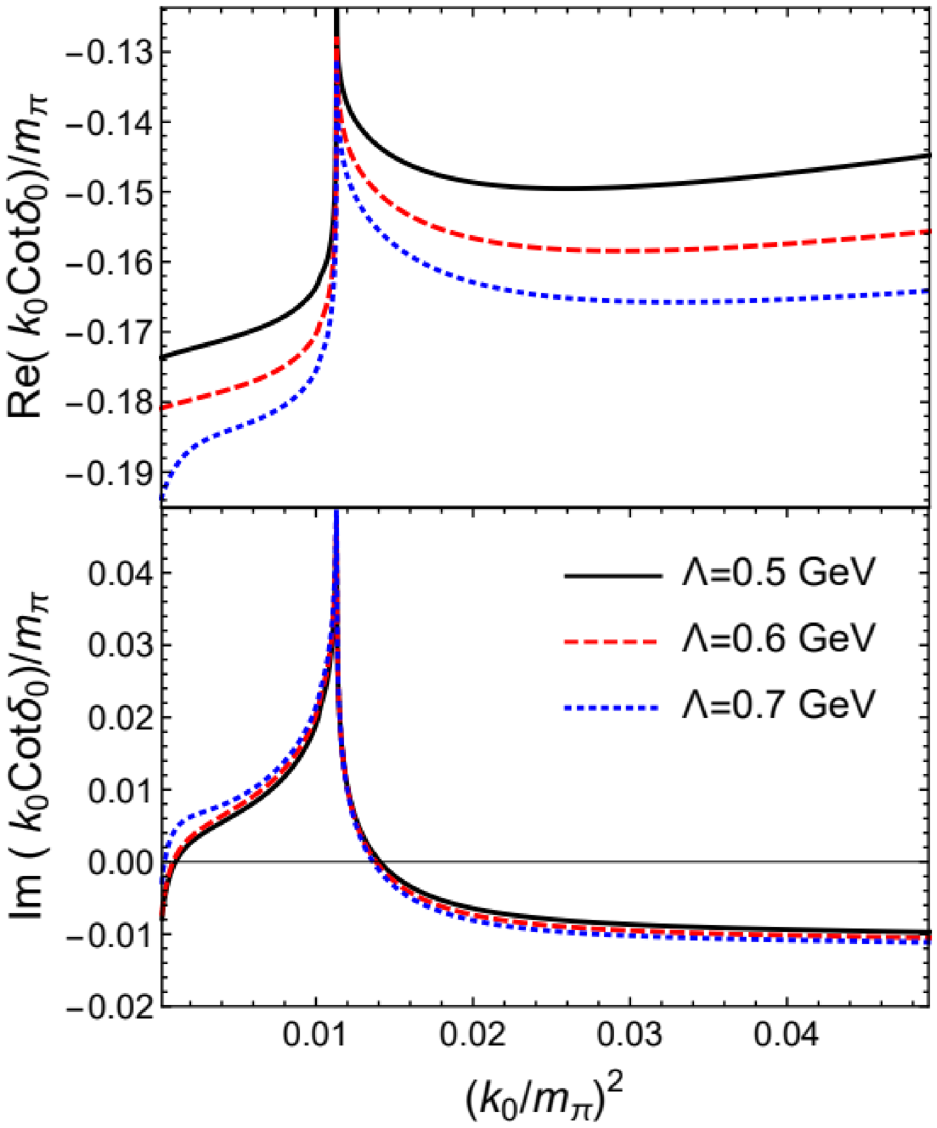
Effective-range expansion

$$k\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4),$$

Phase shift of physical DD^* scattering-rhc effect

$$k_0 \cot \delta_0 = -\frac{8\pi^2}{\mu} T^{-1}(k_0) + ik_0.$$





Summary

- We study the complicated cut structures of the three-body threshold dynamics from the OPE
- Based on cut structures, we developed a complex-scaled LSE to properly treat the interaction involving the three-body threshold dynamics
- Three-body threshold effect will induce an observable singularity in the physical on-shell T matrix, which can be tested in Lattice QCD

Thanks for your attention!