Strong decays of $P_\psi^N(4312)^+$ within the BS framework

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Outline

1. Introduction
2. $P_\psi^N(4312)^+$ as the $\bar{D}\Sigma_c$ molecular state
3. Strong decays $P_\psi^N(4312)^+ \to J/\psi(\eta_c)p$ and $\bar{D}^{(*)0}\Lambda_c^+$
4. Numerical results and discussions

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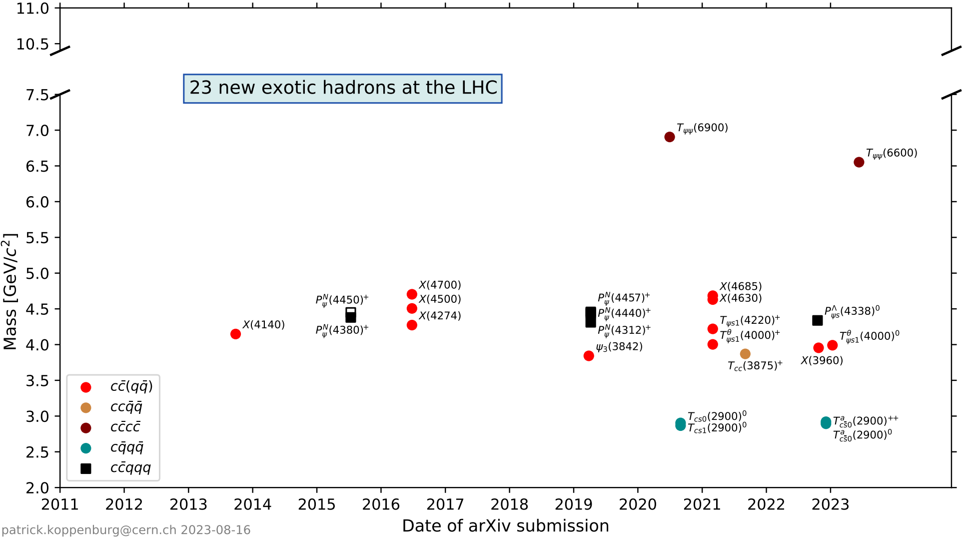
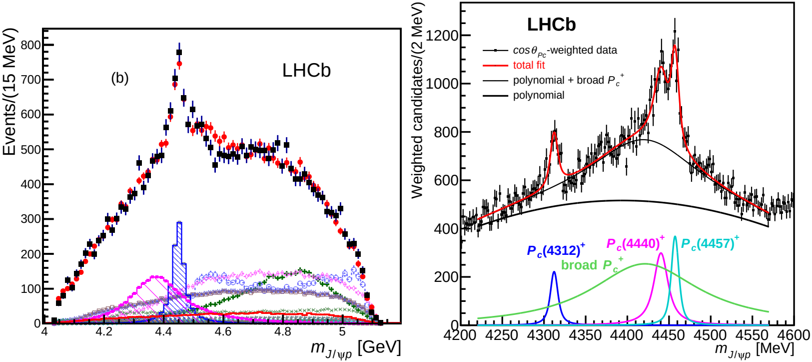
Exotic hadrons at LHC



Pentaquark with 2 charms

- In 2015 LHCb detected 2 pentaquark states with quark content $c\bar{c}qqq$, $P_c(4380)$ and $P_c(4450)$, PRL115, 072001 (2015).
- In 2019 LHCb discovered $P_c(4312)^+$ and resolved the previous $P_c(4450)$ as two peaks $P_c(4440)^+$ and $P_c(4457)^+$, PRL122, 222001 (2019).



- Based on the work arXiv:2301.02094;
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Strong decays of $P_{\psi}^{N}(4312)^+$ to $J/\psi(\eta_c)p$ and $\bar{D}^{(*)}\Lambda_c$ within the Bethe-Salpeter framework

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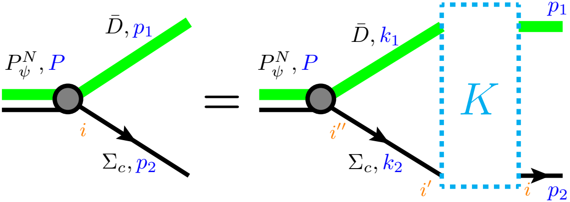
BSE of $J^P = 0^-$ and $\frac{1}{2}^+$ constiuents

- Bethe-Salpeter equation for a meson and a baryon reads

$$\Gamma(P,q,r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4}(-i)K(k,q)[S(k_2)\Gamma(P,k,r)D(k_1)].$$



1. $iK(k,q) \sim iK(k_\perp - q_\perp)$, effective meson-baryon interaction kernel;
2. $S(k_2)$ and $D(k_1)$, propagator of the baryon and the meson;
3. BS wave function $\psi(P,k) \equiv S(k_2)\Gamma(P,k,r)D(k_1)$;
4. Salpeter wave function $\varphi(q_\perp) \equiv -\mathrm{i}\int \frac{\mathrm{d}q_P}{2\pi}\psi(q)$.

Instataneous approximation

- Under the instataneous approximation, the BSE can be rewritten as

$$M\varphi = (w_1 + w_2)H_2(p_{2\perp})\varphi + \frac{1}{2w_1}\gamma_0\Gamma(q_\perp).$$

- Vertex $\Gamma(q_\perp)$ is expressed as the integral of the Salpeter wave function,

$$\Gamma(q_\perp) = \int \frac{\mathrm{d}^3 k_\perp}{(2\pi)^3} K(k_\perp - q_\perp)\varphi(k_\perp).$$

- Salpeter wave function

$$\begin{aligned}\varphi(P, q_\perp, r) &= \left(f_1 + f_2 \frac{q_\perp}{q}\right)\gamma^5 u(P, r)\\ &= 2\sqrt{\pi}\left[f_1 Y_0^0 + \frac{1}{\sqrt{3}} f_2 \left(Y_1^1\gamma^- + Y_1^{-1}\gamma^+ - Y_1^0\gamma^3\right)\right]\gamma^5 u(P, r).\end{aligned}$$

- Normalization condition

$$\int \frac{\mathrm{d}^3 q_\perp}{(2\pi)^3} 2w_1 \left(f_1^2 + f_2^2\right) = 1.$$

Interaction kernel

- Interaction kernels are calculated from the constituent particles scattering based on the one-boson exchange.
- $P_\psi^N(4312)^+$ (minimal quark content $c\bar{c}uud$) is taken as the $\bar{D}\Sigma_c$ molecular state with isospin $I=\frac{1}{2}$ and $I_3=+\frac{1}{2}$

$$|\tfrac{1}{2},\tfrac{1}{2}\rangle = \frac{\sqrt{2}}{\sqrt{3}}\,|\Sigma_c^{++}\rangle\,|D^-\rangle - \frac{1}{\sqrt{3}}\,|\Sigma_c^{+}\rangle\,|\bar{D}^0\rangle\,.$$

- For $P_\psi^N(4312)^+$, only the light scalar and vector meson contribute.
- Considering the isospin symmetry of strong interaction,

$$\langle\tfrac{1}{2},\tfrac{1}{2}|H_{\rm eff}|\tfrac{1}{2},\tfrac{1}{2}\rangle = \tfrac{3}{2}\,\langle\Sigma_c^{++}D^-|\,H_{\rm eff}\,|\Sigma_c^{++}D^-\rangle - \tfrac{1}{2}\,\langle\Sigma_c^{+}\bar{D}^0|H_{\rm eff}|\Sigma_c^{+}\bar{D}^0\rangle$$

Effective lagrangian within the HQET

- Involved Lagrangian describing the charmed anti-heavy-light meson and a light scalar and vector meson reads

$$\mathcal{L} = \sigma_1 \langle \bar{H}_{\bar{c}} \sigma H_{\bar{c}} \rangle - \rho_{\mathrm{V}1} \langle \bar{H}_{\bar{c}} v_\alpha V^\alpha H_{\bar{c}} \rangle - \rho_{\mathrm{T}1} \langle \bar{H}_{\bar{c}} \sigma^{\alpha\beta} (\partial_\alpha V_\beta - \partial_\beta V_\alpha) H_{\bar{c}} \rangle \,.$$

- $H_{\bar{c}}$ represents the field of the $(\bar{D}, \bar{D}^*)$ doublet

$$H_{\bar{c}} = \left(\bar{D}^{*\mu} \gamma_\mu + \mathrm{i} \bar{D} \gamma_5 \right) \frac{1 - \not{v}}{2},$$

where $\bar{D} = (\bar{D}^0, D^-, D_s^-)$.

- V denotes the 3×3 matrix consisting of the 9 light vector meson fields

$$V = \begin{bmatrix} \frac{(\rho^0 + \omega)}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{(\rho^0 - \omega)}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{bmatrix} .$$

- Effective Lagrangian of the heavy-light baryon and light vector mesons

$$\mathcal{L}_{B_6 B_6 V} = \rho_{\mathrm{V}2} \langle \bar{S}_\mu v_\alpha V^\alpha S^\mu \rangle + \mathrm{i}\rho_{\mathrm{T}2} \langle \bar{S}_\mu (\partial_\mu V_\nu - \partial_\nu V_\mu) S_\nu \rangle + \sigma_2 \langle \bar{S}_\mu \sigma S^\mu \rangle .$$

- Baryon spin doublet

$$S_\mu = -\frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma^5 B_6 + B^*_{6\mu},$$

- Systematic baryon sextet B_6 in 3×3 matrix

$$B_6 = \begin{bmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^{+} & \frac{1}{\sqrt{2}}\Xi_c^{\prime +} \\ \frac{1}{\sqrt{2}}\Sigma_c^{+} & \Sigma_c^{0} & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} \\ \frac{1}{\sqrt{2}}\Xi_c^{\prime +} & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} & \Omega_c^{0} \end{bmatrix}.$$

Interaction kernel under one-boson exchange

- Under the one-boson exchange, the calculated interaction kernel for $\bar{D}\Sigma_c$ in isospin-$\frac{1}{2}$

$$K(s_\perp) = F^2(s_\perp^2)\left(V_1 + V_2 \frac{\not{s}_\perp}{|\boldsymbol{s}|}\right),$$

- The potential V_1 and V_2

$$V_1 = -2\sigma_1\sigma_2 M_D \frac{1}{E_\sigma^2} + \rho_{\mathrm{V}1}\rho_{\mathrm{V}2} M_D \left(\frac{1}{E_\rho^2} - \frac{1}{2E_\omega^2}\right),$$

$$V_2 = -\frac{1}{3}\rho_{\mathrm{V}1}\rho_{\mathrm{T}2} M_D |\boldsymbol{s}| \left(\frac{2}{E_\rho^2} - \frac{1}{E_\omega^2}\right),$$

- Propagator-type regulator function

$$F(\boldsymbol{s}^2) = \frac{m_\Lambda^2}{\boldsymbol{s}^2 + m_\Lambda^2},$$

where $m_\Lambda = 0.87\,\mathrm{GeV}$ is the only introduced cutoff parameter and the value is fix by the hadron mass.

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Interaction between $J/\psi(\eta_c)$ and $D^{(*)}$

- Assuming symmetry of independent heavy quark spin, the effective coupling between the S-wave charmonia and the heavy-light mesons reads

$$\mathcal{L}_2 = g_2 \mathrm{Tr}\,(R\bar{H}_{\bar{c}}\overleftrightarrow{\not\partial}\bar{H}_c) + \mathrm{H.c.}.$$

- S-wave charmonium doublet

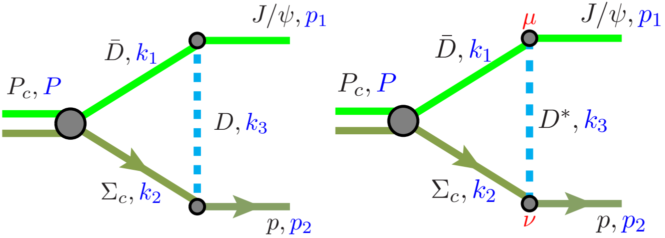
$$R = \frac{1+\not v}{2}(\psi^\mu\gamma_\mu + \mathrm{i}\eta_c\gamma_5)\frac{1-\not v}{2}.$$

- Expand to obtain the Lagrangians

$$\begin{aligned}
\mathcal{L}_2 = &+ g_{\psi DD}\psi^{\dagger\mu}\bar{D}\partial_\mu D \\
&- \mathrm{i}g_{\psi DD^*}\frac{1}{M_\psi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\psi^\dagger_\nu(\bar{D}\partial_\alpha D^*_\beta + \bar{D}^*_\alpha\partial_\beta D) \\
&+ g_{\psi D^*D^*}\psi^{\dagger\mu}(\bar{D}^{*\nu}\partial_\nu D^*_\mu - \partial_\nu\bar{D}^*_\mu D^{*\nu} + 2\partial_\mu\bar{D}^{*\nu}D^*_\nu) \\
&+ g_{DD^*\eta_c}\eta_c^\dagger(\partial_\mu\bar{D}D^{*\mu} - \bar{D}^{*\mu}\partial_\mu D) \\
&+ \mathrm{i}g_{D^*D^*\eta_c}\frac{1}{M_{\eta_c}}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\eta_c^\dagger\bar{D}^*_\nu\partial_\alpha D^*_\beta + \mathrm{H.c.}.
\end{aligned}$$

Amplitude for $P_\psi^N(4312)^+ \to J/\psi p$

- $P_\psi^N(4312)^+$ as the $\bar{D}\Sigma_c$ molecular state can decay to $J/\psi p$ by exchanging either a D or a D^* virtual meson.



- Invariant amplitude for $P_\psi^N(4312)^+ \to J/\psi p$ by exchanging a D

$$\mathcal{A}_1 = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \bar{u}_2(-\mathrm{i}g_{ND\Sigma_c})\gamma^5 \left[S(k_2)\Gamma(P,k,r)D(k_1)\right] (g_{\psi DD})D(k_3)(\mathrm{i})e_1^{*\alpha} k_{1\alpha},$$

$$T_{1\alpha}u(P,r) = \gamma_5 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[S(k_2)\Gamma(P,k,r)D(k_1)\right] D(k_3)k_{1\alpha}$$
$$= (s_{1P}\gamma_\alpha + s_{2P}\hat{P}_\alpha)u(P,r).$$

- Amplitude for $P_\psi^N(4312)^+ \to J/\psi p$ by exchanging a D^*

$$\mathcal{A}_2 = (g_{\Sigma_c ND^*} g_{\psi DD^*}) e_1^{*\alpha} \bar{u}_2 \gamma^\mu (-\mathrm{i}) \frac{1}{M_1} \epsilon_{P_1 \alpha\beta\mu} T_2^\beta u(P,r),$$

$$T_2^\beta u(P,r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[S(k_2)\Gamma(P,k,r)D(k_1) \right] D(k_3) k_1^\beta .$$

- Perform integral over k_P to obtain

$$T_2^\beta u(P,r) = \int \frac{\mathrm{d}^3 k_\perp}{(2\pi)^3} \frac{1}{2w_3} \left(a_1^\beta \varphi^+ + a_2^\beta \varphi^- \right).$$

- Perform the numerial integral to express the amplitude by form factor

$$\mathcal{A}_2 = (g_{\psi DD^*} g_{\Sigma_c ND^*}) e_1^{*\alpha} \bar{u}_2 (s_{1V}\gamma_\alpha + s_{2V}\hat{P}_\alpha) u(P,r).$$

The following identity of the Levi-Civita symbol is used

$$\mathrm{i}\gamma_\mu \epsilon^{\mu\alpha\beta\nu} = \gamma^5 (\gamma^\alpha\gamma^\beta\gamma^\nu - \gamma^\alpha g^{\beta\nu} + \gamma^\beta g^{\alpha\nu} - \gamma^\nu g^{\alpha\beta}).$$

$P_\psi^N(4312)^+ \to \bar{D}^{*0}\Lambda_c^+$

- Strong decay of $P_\psi^N(4312) \to \bar{D}^{*0}\Lambda_c^+$ is similar with that to $J/\psi p$, just J/ψ replaced by $\bar{D}^{*0}$, p replaced by Λ_c^+, and the propagator $D^{(*)}$ replaced by the $\pi(\rho)$ respectively.
- Effective Lagrangians

$$\mathcal{L}_{\bar{D}\bar{D}^*\phi} = g_{\bar{D}\bar{D}^*\phi}(\bar{D}^{*\mu})^\dagger \partial_\mu \phi \bar{D},$$
$$\mathcal{L}_{\Sigma_c\Lambda_c\phi} = (-\mathrm{i}) g_{\Lambda_c\Sigma_c\phi} \bar{\Lambda}_c \gamma_5 \Sigma_c \phi,$$

- ϕ is the 3×3 traceless hermitian matrix of eight pseudo-scalar meson fields,

$$\phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix}.$$

- Amplitude by exchanging a π and a ρ respectively,

$$\mathcal{A}_4 = (g_{\Lambda_c\Sigma_c\phi} g_{D^*D\phi})(e_1^\alpha)^* \bar{u}_2 \left(s_{4P}\gamma_\alpha + s_{5P}\hat{P}_\alpha \right) u(P,r),$$
$$\mathcal{A}_5 = (g_{D^*D\rho} g_{\Lambda_c\Sigma_c\rho}) e_1^{*\alpha} \bar{u}_2 \left(s_{4V}\gamma_\alpha + s_{5V}\hat{P}_\alpha \right) u(P,r),$$

- Decay to $\eta_c p$ can only happen by exchanging a D^*

$$\mathcal{A}_3 = (g_{ND^*\Sigma_c} g_{\eta_c DD^*})\, \bar{u}_2 T_3 u(P,r).$$

- T_3 is expressed by the integral over BS vertex as

$$T_3 u(P,r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} (\gamma^\alpha P_1^\beta)[S(k_2)\Gamma(k,r)D(k_1)]D_{\alpha\beta}(k_3)$$
$$= \int \frac{\mathrm{d}^3 k_\perp}{(2\pi)^3} \frac{1}{2w_3} (\gamma^\alpha P_1^\beta) \sum_{i=1}^{3} \left[c_i d_{\alpha\beta}(y_i)\varphi^+ + c_{i+3} d_{\alpha\beta}(y_{i+3})\varphi^-\right]$$
$$= s_{3V}\gamma_5 u(P,r).$$

- Similar for $P_\psi^N(4312)^+ \to \bar{D}^0 \Lambda_c^+$ by exchanging a ρ

$$\mathcal{A}_6 = \mathrm{i}(g_{\Lambda_c \Sigma_c V} g_{\bar{D}\bar{D}V})\bar{u}_2 T_6 u(P,r)$$

$$T_6 u(P,r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} (\gamma^\alpha P_1^\beta)[S(k_2)\Gamma(k,r)D(k_1)]D_{\alpha\beta}(k_3) = s_{6V}\gamma_5 u(P,r)$$

Decay widths

- Amplitude for $P_\psi^N(4312)^+ \to \bar{D}^{*0}\Lambda_c^+$

$$\mathcal{A}[P_\psi^N(4312)^+ \to \bar{D}^{*0}\Lambda_c^+] = \mathcal{A}_4 + \mathcal{A}_5 = e_1^{*\alpha}\bar{u}_2\left(s_4\gamma_\alpha + s_5\hat{P}_\alpha\right)u(P,r).$$

- s_4 and s_5 are related to the coupling constants

$$s_4 = g_{\bar{D}\phi\bar{D}^*}g_{\Sigma_c\phi\Lambda_c}s_{4P} + g_{\bar{D}V\bar{D}^*}g_{\Sigma_cV\Lambda_c}s_{4V},$$
$$s_5 = g_{\bar{D}\phi\bar{D}^*}g_{\Sigma_c\phi\Lambda_c}s_{5P} + g_{\bar{D}V\bar{D}^*}g_{\Sigma_cV\Lambda_c}s_{5V}.$$

- Partial decay width of $P_\psi^N(4312)^+$ to $J/\psi(\eta_c)p$ or $\bar{D}^{(*)0}\Lambda_c^+$

$$\Gamma[P_\psi^N(4312)^+ \to M_{P(V)}B] = \frac{|\boldsymbol{P}_1|}{8\pi M^2}C_{\mathrm{I}}\frac{1}{2}\sum_{r,r_1,r_2}|\mathcal{A}|^2.$$

where C_{I} denotes the isospin factor.

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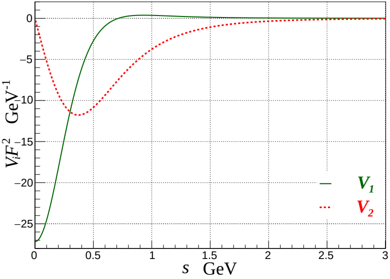
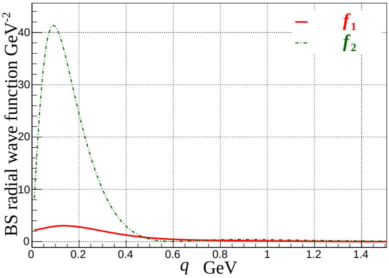
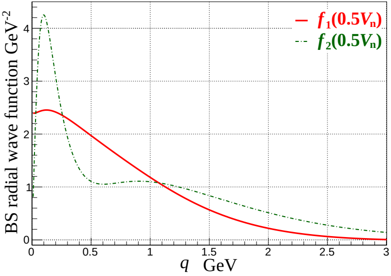
Potential and radial wave functions



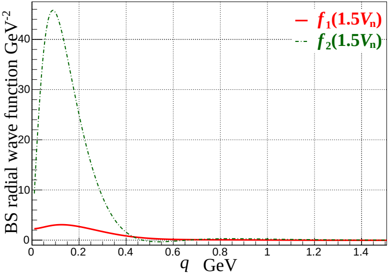
(a) $V_n F^2$ $(n = 1, 2)$



(b) *Wave function*



(c) $f_{1(2)}$ *in* $0.5V_n$



(d) $f_{1(2)}$ *in* $1.5V_n$

Mass and form factors

- Obtain bound state mass $M_{\bar{D}\Sigma_c} = 4.312\,\text{GeV}$ in $I = \frac{1}{2}$ with $m_\Lambda = 0.87\,\text{GeV}$.
- Our results do not support the existance of any radially excited states, which is robust even under the $\pm 50\%$ change of $V_{1(2)}$.
- Obtained numerical form factors for decays to $J/\psi p$ and $\bar{D}^{*0}\Lambda_c^+$,

  $s_{1P} = 1.1 \times 10^{-3}$, $s_{2P} = 1.9 \times 10^{-3}$, $s_{1V} = -4.1 \times 10^{-3}$, $s_{2V} = 7.6 \times 10^{-3}$;
  $s_{4P} = 7.5 \times 10^{-3}$, $s_{5P} = 6.0 \times 10^{-3}$, $s_{4V} = -8.7 \times 10^{-3}$, $s_{5V} = 1.2 \times 10^{-2}$.

  For decays to $\eta_c p$ and $\bar{D}^0\Lambda_c^+$, the obtained form factors s_{3V} and s_{6V} are

$$s_{3V} = 5.1 \times 10^{-3}, \;\; s_{6V} = 3.6 \times 10^{-2}.$$

Decay widths and comparisons

- Comparison of decay widths for $P_{\psi}^{N}(4312)^{+}$ to $J/\psi(\eta_c)p$ and $\bar{D}^{(*)0}\Lambda_c^{+}$ with other works in units of MeV.
- Theoretical uncertainties are induced by varying the coupling constants by $\pm 5\%$ in the effective Lagrangian.
- The sum of the 4 decay widths are ~ 9.1 MeV which amounts to $\sim 93\%$ of the total width $\Gamma_{\text{Tot}} = 9.8 \pm 2.7^{+3.7}_{-4.5}$ MeV [1].

Channel	This	[2]	[3]	[4]	[5]	[6]	[7]
$J/\psi p$	$0.17^{-0.04}_{+0.04}$	0.32 ± 0.08	$1.67^{+0.92}_{-0.56}$	$10^{-3} \sim 0.1$	0.033	$(3 \sim 8)$	$9.3^{+19.5}_{-9.3}$
$\eta_c p$	$0.085^{-0.016}_{+0.018}$	0.98 ± 0.25	$5.54^{+0.75}_{-0.50}$	$10^{-2} \sim 0.4$	0.066	–	$0.26^{-0.24}_{+0.55}$
$\bar{D}^{*0}\Lambda_c^{+}$	$8.8^{-1.6}_{+1.9}$	-	-	10.7	6.16	-	-
$\bar{D}^{0}\Lambda_c^{+}$	$0.026^{-0.05}_{+0.06}$	-	-	0.3	0	-	-

- The main difference between $J/\psi(\eta_c)p$ and $\bar{D}^{(*)}\Lambda_c$ channels stems from the much heavier exchanged particle D^* in the former one.
- Obtained partial decay widths are directly dependent on the coupling constants $g_{\psi DD}$, $g_{\psi DD^*}$, $g_{ND\Sigma_c}$, $g_{ND^*\Sigma_c}$, and $g_{DD^*\eta_c}$ in the relevant effective Lagrangian.
- A variation of $\pm 5\%$ in every coupling constant induces about $\sim 20\%$ relative uncertainties.
- Interpretation of $P_\psi^N(4312)^+$ as the $\bar{D}\Sigma_c$ molecular state with $J^P = \frac{1}{2}^-$ and isospin $I = \frac{1}{2}$ is favored by this work.
- $\bar{D}^{*0}\Lambda_c^+$ is a much more promising decay channel to be discovered in experiments.

谢谢

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Details of one form factor

- First perform the integral over k_P to obtain

$$T_1^\alpha u(P,r) = \gamma_5 \int \frac{\mathrm{d}k_\perp^3}{(2\pi)^3} \frac{1}{2w_3} (a_1^\alpha \varphi^+ + a_2^\alpha \varphi^-).$$

- Two coefficients a_1 and a_2 behaves as

$$a_1^\alpha = c_1 x_1^\alpha + c_2 x_2^\alpha + c_3 x_3^\alpha,$$
$$a_2^\alpha = c_4 x_4^\alpha + c_5 x_5^\alpha + c_6 x_6^\alpha,$$

where $x_i = k_1(k_P = k_{Pi})$ with $(i = 1, \cdots 6)$, and k_{Pi}s are defined as

$$k_{P1} = \zeta_1^+,\ k_{P2} = \zeta_2^+,\ k_{P3} = \zeta_3^+,\ k_{P4} = \zeta_1^-,\ k_{P5} = \zeta_2^-,\ k_{P6} = \zeta_3^-,$$
$$\zeta_1^\pm = -(\alpha_1 M \mp w_1),\ \zeta_2^\pm = (\alpha_2 M \mp w_2),\ \zeta_3^\pm = (E_1 - \alpha_1 M \pm w_3)$$

- The coefficients c_is $(i = 1, \cdots 6)$ are defined as

$$c_{1(4)} = \frac{1}{w_1 + w_3 \mp E_1},$$
$$c_{2(5)} = \frac{(-1)}{w_2 + w_3 \mp E_2},$$
$$c_{3(6)} = \frac{(w_1 + w_2 \mp M)}{(w_1 + w_3 \pm E_1)(w_2 + w_3 \mp E_2)}. \square$$