重味四夸克态$QQqq$的质量和衰变

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Related papers:
S.Q. Luo, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C 77, 709 (2017)
J. Wu, X. Liu, Y.R. Liu, S.L. Zhu, Phys. Rev. D 99, 014037 (2019)
J.B. Cheng, S.Y. Li, Y.R. Liu, Y.N. Liu, Z.G. Si, T. Yao, Phys. Phys. D 101, 114017 (2020)
J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Chin. Phys. C 45, 043102 (2021)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J.Wu, to be submitted

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3. Doubly heavy tetraquark states $QQ\overline{qq}$

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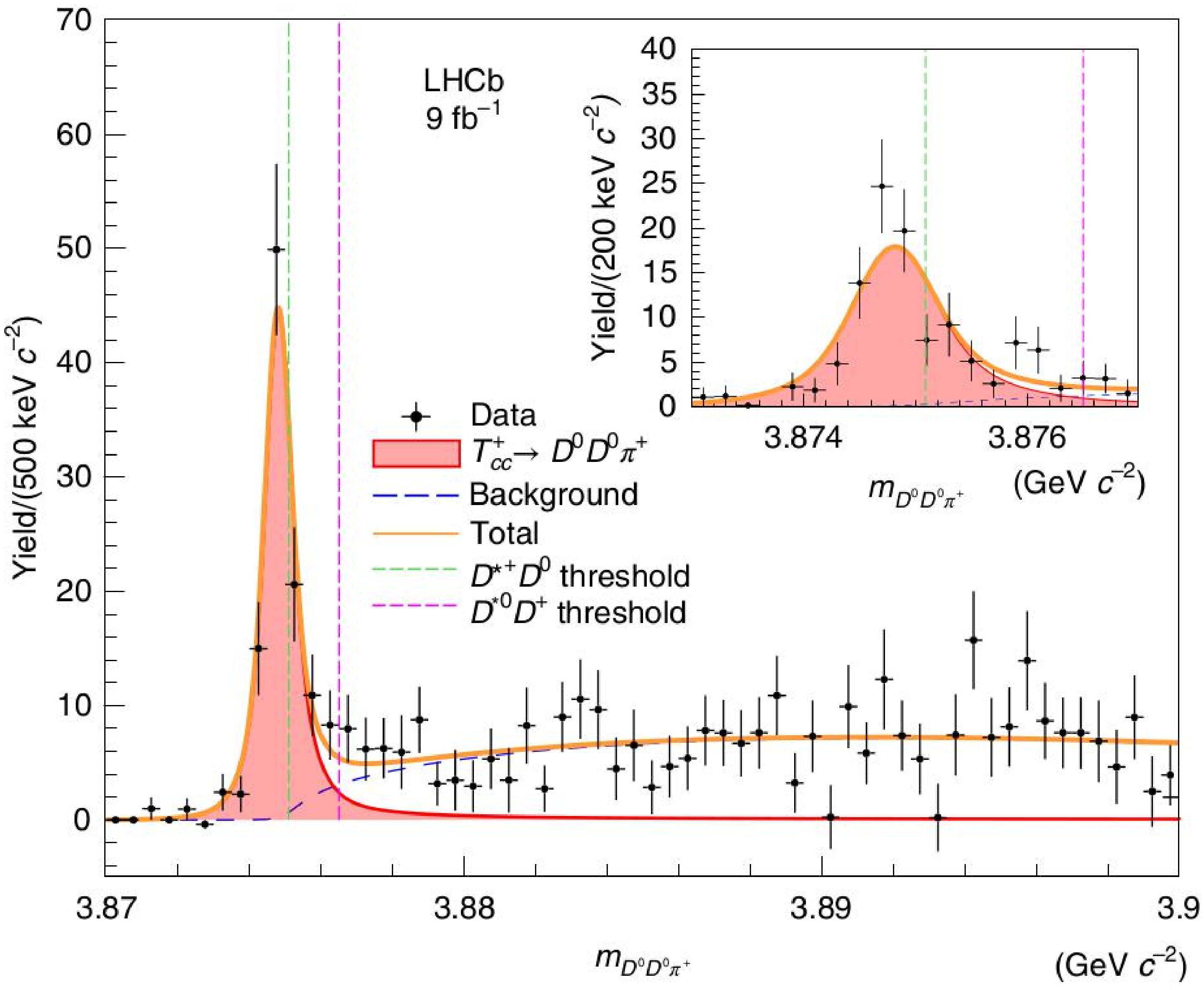
Introduction



LHCb, Nature Phys. 18, 751 (2022):

$$m_{D^{*+}} + m_{D^0} = 3875.1 \text{ MeV}$$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{BW} = 273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

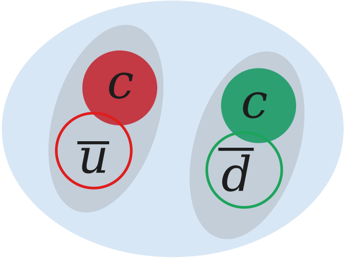
LHCb, Nature Commun. 13,3351 (2022):

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

Minimal quark content: $cc\bar{u}\bar{d}$

Introduction: theory studies



DD^* molecule

Li et al, Chin. Phys. Lett. 38, 092001 (2021);
Ren, Wu, Zhu, Adv. High Energy Phys. 2022, 9103031 (2022);
Ling et al, PLB 826, 136897 (2022);
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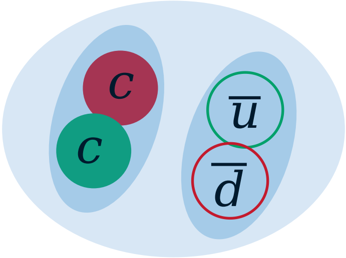
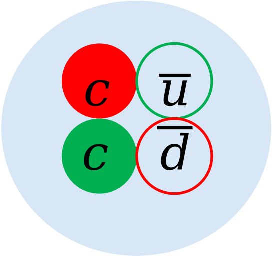


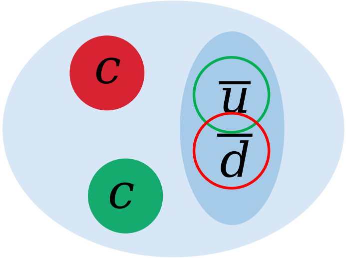
Diquark-antidiquark:

Agaev, Azizi, Sundu, NPB 975, 115650 (2022);
T.W. Wu, Y.L. Ma, PRD 107, L071501 (2023).



Compact tetraquark:

Chen, Yang, CPC 46, 054103 (2022);
Meng, PLB 846, 138221 (2023);
Ma et al, 2309.17068
He, Harada, Zou, PRD 108, 054025 (2023);
Liu, Zhang, Jia, PRD 108, 054019 (2023);
Wang et al, EPJC 82, 721 (2022);
…



Antidiquark-(heavy quark)-(heavy quark):

Kim, Oka, Suzuki, PRD 105, 074021 (2022)

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Formalism: the color-magnetic interaction (CMI) model

Short-range

- Mass splittings for S-wave states: [Rujula, Georgi, Glashow, PRD12, 147 (1975)]

$$H = \sum_i m_i + \sum_i T_i + V_{eff},$$
$$V_{eff} = \sum_{i<j}\left[A(r_{ij})\lambda_i\cdot\lambda_j + B\frac{\delta^3(r_{ij})}{m_i m_j}\lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j\right].$$

$$\langle\lambda_i\cdot\lambda_j\rangle = \begin{cases} -\frac{16}{3}, & (q\bar{q}) \\ -\frac{8}{3}, & (qq) \end{cases}$$

$$H = \sum_i m_i^{eff} + H_{eff},$$
$$H_{eff} = -\sum_{i<j} C_{ij}\lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j.$$

$$M = \sum_i m_i + E_{\text{CMI}}$$

m_i (m_i^{eff}) is effctive quark mass, which contians effects from kinetic energy, color confinement , and so on.

H_{CMI} (H_{eff}) is color-magnetic interaction. $\boldsymbol{E_{CMI}}$ is eigenvalue of $\boldsymbol{H_{CMI}}$

(or $H = M_0 + T + V_{SS} + V_{conf}$)

(or $H = \sum m_Q + H_{SS}^{(QQ)} + H_{SS}^{(Q\bar{Q})} + H_{SL} + H_{LL}$)

Formalism: CMI model (symmetry analysis)

- Although problems:

  Dynamics (no);

  Effective quark masses (system to system);

  Effective coupling constants (conventional → multiquark);

  Estimated masses (uncertainty).
- Simple for estimation of rough positions of multiquark states
- CMI model for mass splittings can catch basic features of spectra
- Research methods in this model:

  (1) Construct flavor-color-spin wave functions;

  (2) Mixing between different color-spin structures.

Hadron	CMI	Hadron	CMI	Parameter(MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn}=18.4$
Σ	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{ns}$	$C_{ns}=12.4$
Ξ^0	$\frac{8}{3}(C_{ss}-4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss}+C_{ns})$	
Ω	$8C_{ss}$			$C_{ss}=6.5$
Λ	$-8C_{nn}$			
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}}=6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}}=6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}}=2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}}=2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}}=5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}}=2.9$
Σ_c	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}$	$C_{cn}=4.0$
Ξ_c'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{cn}-\frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{cn}+\frac{8}{3}C_{cs}$	$C_{cs}=4.8$
Σ_b	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{bn}$	$C_{bn}=1.3$
Ξ_b'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{bn}-\frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{bn}+\frac{8}{3}C_{bs}$	$C_{bs}=1.2$

$$H=\sum_i m_i^{eff}-\sum_{i<j}C_{ij}\lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j$$

$$m_n = 361.8 \text{ MeV},$$
$$m_s = 540.4 \text{ MeV},$$
$$m_c = 1724.8 \text{ MeV},$$
$$m_b = 5052.9 \text{ MeV}.$$
$$(n=u,d)$$

TABLE III. Comparison for hadron masses between experimental data and theoretical estimation. All the values are in units of MeV.

Hadron	Theory	Experiment	Deviation	Hadron	Theory	Experiment	Deviation
D	1975.9	1864.8	111.1	D^*	2121.0	2007.0	114.0
D_s	2154.5	1968.3	186.2	D_s^*	2299.5	2112.1	187.4
η_c	3361.0	2983.6	377.4	J/ψ	3474.1	3096.9	377.2
Σ_c	2452.9	2454.0	1.1	Σ_c^*	2516.9	2518.4	−1.5
Ω_c	2796.2	2695.2	101.0	Ω_c^*	2845.3	2765.9	79.4
Ξ_c	2525.9	2471.0	54.9	Ξ_c'	2612.3	2577.9	34.4
Ξ_c^*	2680.6	2645.9	34.7				

- Our scheme for multiquarks:

$$M = \sum_i m_i + E_{\text{CMI}} \quad \longrightarrow \quad \boxed{M = [M_{ref} - (E_{\text{CMI}})_{ref}] + E_{\text{CMI}}}$$

(1) Important color-mixing (base independent results)

(2) M= $\sum_i m_i + E_{CMI}$ as upper limit

(3) Hadron-hadron threshold as a reference scale

(4) M= $M_{ref} - (E_{CMI})_{ref} + E_{CMI}$, find lower limit

and roughly reasonable masses

$cs\bar{c}\bar{s}$, $QQ\bar{Q}\bar{Q}$, $qq\bar{Q}\bar{Q}$, $Qq\bar{Q}\bar{q}$, $QQ\bar{Q}\bar{q}$;
$c\bar{c}qqq$, $QQqq\bar{q}$, $QQQq\bar{q}$.

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Formalism: CMI model

- When *ref* =hadron-hadron threshold, [Luo et al, EPJC 77, 709 (2017)]

  (1) six $J^P = 1^+$ bound $QQ\overline{qq}$ tetraquark states:

$$(cc\bar{u}\bar{d})^{I=0},\ (cc\bar{n}\bar{s})^{I=1/2},$$
$$(bb\bar{u}\bar{d})^{I=0},\ (bb\bar{n}\bar{s})^{I=1/2},$$
$$(bc\bar{u}\bar{d})^{I=0},\ (bc\bar{n}\bar{s})^{I=1/2};$$

  (2) two $J^P = 0^+$ bound $QQ\overline{qq}$ tetraquark states:

$$(bc\bar{u}\bar{d})^{I=0},\ (bc\bar{n}\bar{s})^{I=1/2}.$$

- With heavy diquark-antiquark symmetry (HDAS), two $J^P = 1^+$ bound $QQ\overline{qq}$ tetraquark states: [Cheng et al, CPC 45, 043102 (2021)]

$$(bb\bar{u}\bar{d})^{I=0},\ (bb\bar{n}\bar{s})^{I=1/2}.$$

$$QQq \overset{HDAS}{\Longleftrightarrow} \overline{QQ}qq$$

Why not a reference multiquark?

Formalism: CMI model (mass splitting model)

Li et al., PRD 108, 056015 (2023)

Assumption: $X(4140)$ observed in $J/\psi\phi$ as the lowest 1^{++} $c\bar{c}s\bar{s}$ tetraquark

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

where $\Delta_{ij} = m_i - m_j$ denotes the effective quark mass gap between i quark and j quark

$$m_{cc\bar{n}\bar{n}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - 2\Delta_{sn},$$
$$m_{cc\bar{n}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - \Delta_{sn},$$
$$m_{cc\bar{s}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI},$$
$$m_{bc\bar{n}\bar{n}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - 2\Delta_{sn} + \Delta_{bc},$$
$$m_{bc\bar{n}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - \Delta_{sn} + \Delta_{bc},$$
$$m_{bc\bar{s}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} + \Delta_{bc},$$
$$m_{bb\bar{n}\bar{n}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - 2\Delta_{sn} + 2\Delta_{bc},$$
$$m_{bb\bar{n}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} - \Delta_{sn} + 2\Delta_{bc},$$
$$m_{bb\bar{s}\bar{s}} = m_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} + 2\Delta_{bc}.$$

Wu et al., PRD 99, 014037 (2019);
Cheng et al., PRD 101, 114017 (2020)

TABLE IV. Quark mass differences (units: MeV) determined with various hadrons. The values from the extracted effective quark masses are $m_s - m_n = 178.6$ MeV and $m_b - m_c = 3328.2$ MeV.

Hadron	Hadron	$(m_s - m_n)$	Hadron	Hadron	$(m_b - m_c)$
D_s	D	103.5	B	D	3340.9
B_s	B	90.8	B_s	D_s	3328.2
Σ	N	187.1	η_b	η_c	3188.4
Λ	N	177.4	Λ_b	Λ_c	3333.1
Ω_c	Σ_c	158.8	Σ_b	Σ_c	3328.5
Ω_b	Σ_b	147.9	Ξ_b	Ξ_c	3326.2
Ξ_c	Λ_c	133.4	Ω_b	Ω_c	3315.7
Ξ_c	Σ_c	119.5			
Ξ_b	Λ_b	126.9			
Ξ_b	Σ_b	117.6			

C_{ij}	n	s	c	b	$C_{i\bar{j}}$	$\bar{n}$	$\bar{s}$	$\bar{c}$	$\bar{b}$
n	18.3	12.1	4.0	1.3	n	29.8	18.7	6.6	2.1
s		6.5	4.3	1.3	s		9.8	6.7	2.3
c			3.5	2.0	c			5.3	3.3
b				1.9	b				2.9

Formalism: rearrangement decay

Combine information from spectrum and decay!

A simple decay scheme:

We assume that the Hamiltonian is a constant $H_{decay} = \mathcal{C}$ and the sum of two-body rearrangement decay widths is equal to the measured width: $\Gamma_{sum} \approx \Gamma_{total}$

$J=2$	$\phi_1\chi_1 = \lvert(Q_1Q_2)^1_{6_c}(\bar{q}_3\bar{q}_4)^1_{\bar{6}_c}\rangle^2_{1_c}\delta_{12}\delta^S_{34}$	$\phi_2\chi_1 = \lvert(Q_1Q_2)^1_{\bar{3}_c}(\bar{q}_3\bar{q}_4)^1_{3_c}\rangle^2_{1_c}\delta^A_{34}$
$J=1$	$\phi_1\chi_2 = \lvert(Q_1Q_2)^1_{6_c}(\bar{q}_3\bar{q}_4)^1_{\bar{6}_c}\rangle^1_{1_c}\delta_{12}\delta^S_{34}$	$\phi_2\chi_2 = \lvert(Q_1Q_2)^1_{\bar{3}_c}(\bar{q}_3\bar{q}_4)^1_{3_c}\rangle^1_{1_c}\delta^A_{34}$
	$\phi_1\chi_4 = \lvert(Q_1Q_2)^1_{6_c}(\bar{q}_3\bar{q}_4)^0_{\bar{6}_c}\rangle^1_{1_c}\delta_{12}\delta^A_{34}$	$\phi_2\chi_4 = \lvert(Q_1Q_2)^1_{\bar{3}_c}(\bar{q}_{3_c}\bar{q}_4)^0_3\rangle^1_{1_c}\delta^S_{34}$
	$\phi_1\chi_5 = \lvert(Q_1Q_2)^0_{6_c}(\bar{q}_3\bar{q}_4)^1_{\bar{6}_c}\rangle^1_{1_c}\delta^S_{34}$	$\phi_2\chi_5 = \lvert(Q_1Q_2)^0_{\bar{3}_c}(\bar{q}_{3_c}\bar{q}_4)^1_3\rangle^1_{1_c}\delta_{12}\delta^A_{34}$
$J=0$	$\phi_1\chi_3 = \lvert(Q_1Q_2)^1_{6_c}(\bar{q}_3\bar{q}_4)^1_{\bar{6}_c}\rangle^0_{1_c}\delta_{12}\delta^S_{34}$	$\phi_2\chi_3 = \lvert(Q_1Q_2)^1_{\bar{3}_c}(\bar{q}_3\bar{q}_4)^1_{3_c}\rangle^0_{1_c}\delta^A_{34}$
	$\phi_1\chi_6 = \lvert(Q_1Q_2)^0_{6_c}(\bar{q}_3\bar{q}_4)^0_{\bar{6}_c}\rangle^0_{1_c}\delta^A_{34}$	$\phi_2\chi_6 = \lvert(Q_1Q_2)^0_{\bar{3}_c}(\bar{q}_3\bar{q}_4)^0_{3_c}\rangle^0_{1_c}\delta_{12}\delta^S_{34}$

$$(Q_1Q_2)_{1c}(\bar{q}_3\bar{q}_4)_{1c} \to (Q_1\bar{q}_3)_{1c} + (Q_2\bar{q}_4)_{1c},$$
$$(Q_1Q_2)_{1c}(\bar{q}_3\bar{q}_4)_{1c} \to (Q_1\bar{q}_4)_{1c} + (Q_2\bar{q}_3)_{1c}.$$

$$|\mathcal{M}|^2 = \mathcal{C}^2|\sum_{ij} x_i y_j|^2,$$

$$\Psi_{tetra} = \sum_i x_i(Q_1Q_2\bar{q}_3\bar{q}_4),$$

$$\Psi_{final} = \sum_i y_i(Q_1Q_2\bar{q}_3\bar{q}_4).$$

$$\mathcal{C} = 7282.15 \text{ MeV from } X(4140)$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M^2_{Q Q\bar{q}\bar{q}}}$$

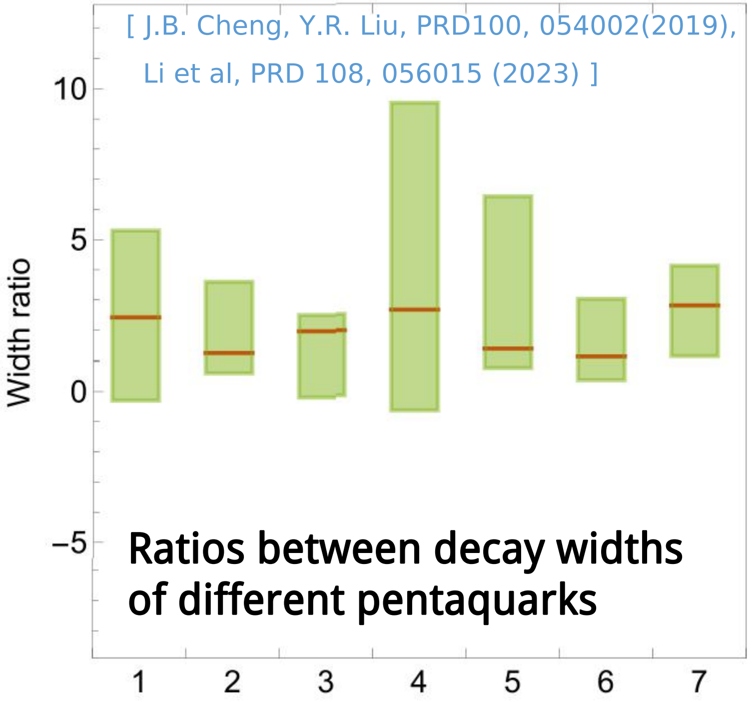
Example of formalism: Pc states

$(nnn)_{8_c}(c\bar{c})_{8_c}$-$(nnn)_{1_c}(c\bar{c})_{1_c}$

Assume Pc(4312) as the second lowest

$I(J^P)=\frac{1}{2}(\frac{3}{2}^-)\ nnnc\bar{c}$

[J.B. Cheng, Y.R. Liu, PRD100, 054002(2019), Li et al, PRD 108, 056015 (2023)]



Ratios between decay widths of different pentaquarks

Theoretical states

$$\Gamma(\tilde{P}_c(4421)^+):\Gamma(\tilde{P}_c(4461)^+)=2.42,$$
$$\Gamma(\tilde{P}_c(4421)^+):\Gamma(\tilde{P}_c(4312)^+)=1.24,$$
$$\Gamma(\tilde{P}_c(4312)^+):\Gamma(\tilde{P}_c(4461)^+)=1.96,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4461)^+)=2.64,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4312)^+)=1.35,$$
$$\Gamma(\tilde{P}_c(4324)^+):\Gamma(\tilde{P}_c(4421)^+)=1.09.$$

Experimental states

$$\Gamma(P_c(4440)^+):\Gamma(P_c(4457)^+)=3.2^{+2.1}_{-3.5},$$
$$\Gamma(P_c(4440)^+):\Gamma(P_c(4312)^+)=2.1^{+1.5}_{-1.5},$$
$$\Gamma(P_c(4312)^+):\Gamma(P_c(4457)^+)=1.5^{+1.0}_{-1.7},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4457)^+)=4.5^{+5.0}_{-5.2},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4312)^+)=3.0^{+3.4}_{-2.3},$$
$$\Gamma(P_c(4337)^+):\Gamma(P_c(4440)^+)=1.4^{+1.6}_{-1.1}.$$

$P_c(4457)^+$, $P_c(4440)^+$, $P_c(4337)^+$ can be regarded as the J=3/2, J=1/2, and J=1/2 pentaquark states, respectively.

For $P_c(4457)^+$ $\Gamma(\Sigma_c^*\bar{D}):\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi)=2.3:4.0:1.0$

For $P_c(4440)^+$ $\Gamma(\Lambda_c\bar{D}^*):\Gamma(\Sigma_c\bar{D}):\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi):\Gamma(N\eta_c)=45.5:3.0:3.0:7.5:1.0$

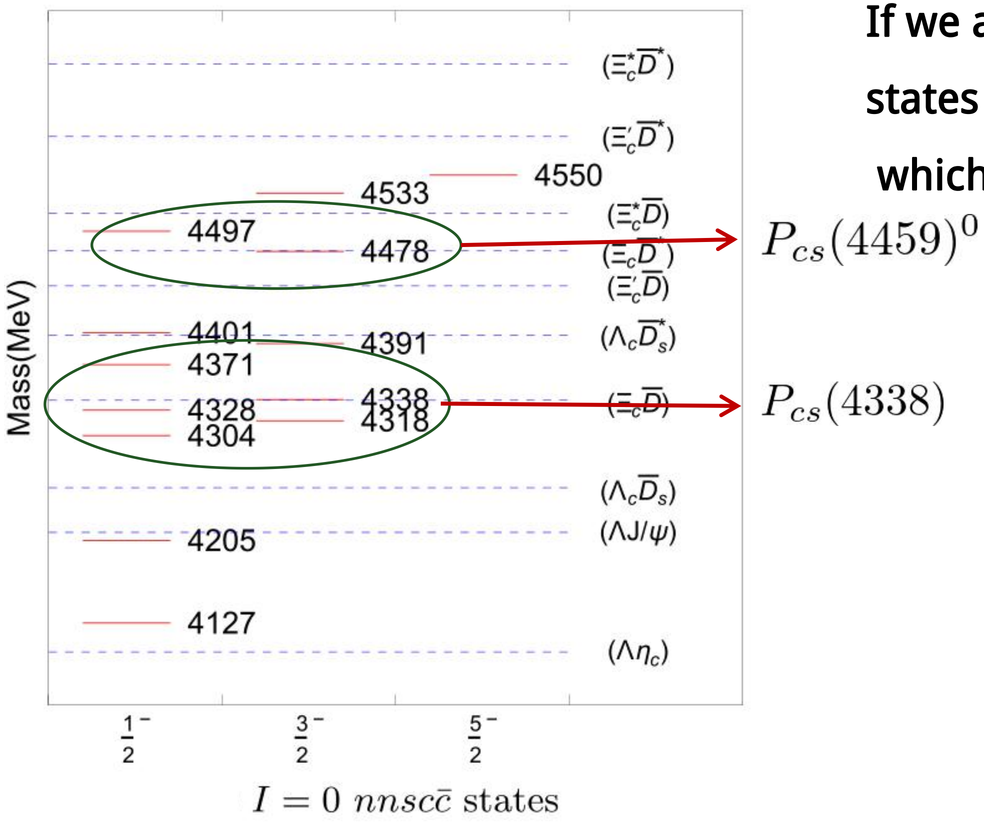
For $P_c(4312)^+$ $\Gamma(NJ/\psi):\Gamma(\Lambda_c\bar{D}^*)=1.1$

For $P_c(4337)^+$ $\Gamma(\Lambda_c\bar{D}):\Gamma(NJ/\psi)=1.3$

Predictions

Example of formalism: Pcs states

$(nns)_{8_c}(c\bar{c})_{8_c}$-$(nns)_{1_c}(c\bar{c})_{1_c}$



If we assign the $P_{cs}(4459)^0$, $P_{cs}(4338)^0$ to be J=3/2 pentaquark states $\tilde{P}_{cs}(4478)$, $\tilde{P}_{cs}(4338)$, respectively, $\Gamma(\tilde{P}_{cs}(4478)):\Gamma(\tilde{P}_{cs}(4338)) \sim 0.12$ which is contradicted with the experimantal value.

Other possible assignments:

$$\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4371)^0) = 0.15,$$
$$\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4328)^0) = 0.56,$$
$$\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4318)^0) = 2.57,$$
$$\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4304)^0) = 0.17,$$
$$\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4371)^0) = 0.72,$$
$$\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4338)^0) = 0.61,$$
$$\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4328)^0) = 2.78,$$
$$\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4318)^0) = 12.71,$$
$$\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4304)^0) = 0.83.$$

Theoretical widths are much smaller than the measured results.

$J^P = \frac{1}{2}^-$

Both $\boldsymbol{P_{cs}(4338)^0}$ and $\boldsymbol{P_{cs}(4459)^0}$ can be regarded as $\frac{1}{2}^-$ pentaquark states, respectively.

For $P_{cs}(4338)$, $\Gamma(\Lambda J/\Psi):\Gamma(\Lambda_c\bar{D}_s) = 3.0$

For $P_{cs}(4459)^0$, $\Gamma(\Lambda_c\bar{D}_s^*):\Gamma(\Xi_c\bar{D}^*):\Gamma(\Lambda J/\Psi) = 2.3:1.1:1.0$

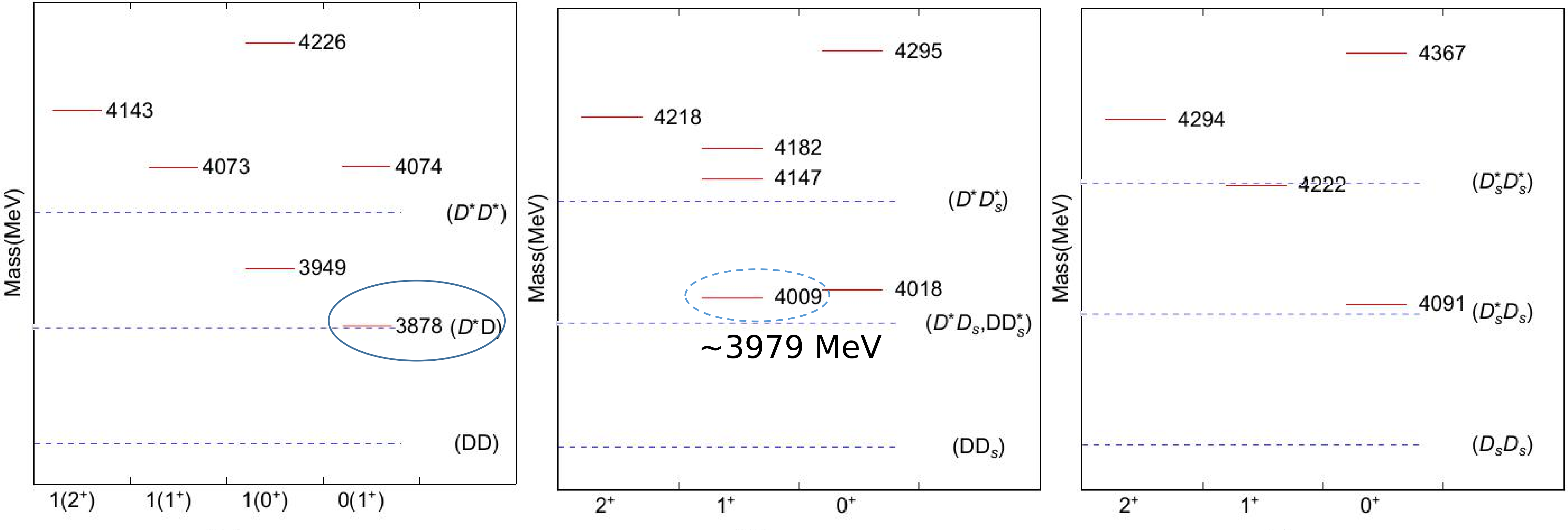
Predictions

The J=5/2 state, the lighest J=3/2 state, and the lighest J=1/2 state are narrow.

Exp: $\Gamma(P_{cs}(4459)^0):\Gamma(P_{cs}(4338)^0) = 2.5^{+1.6}_{-1.4}$

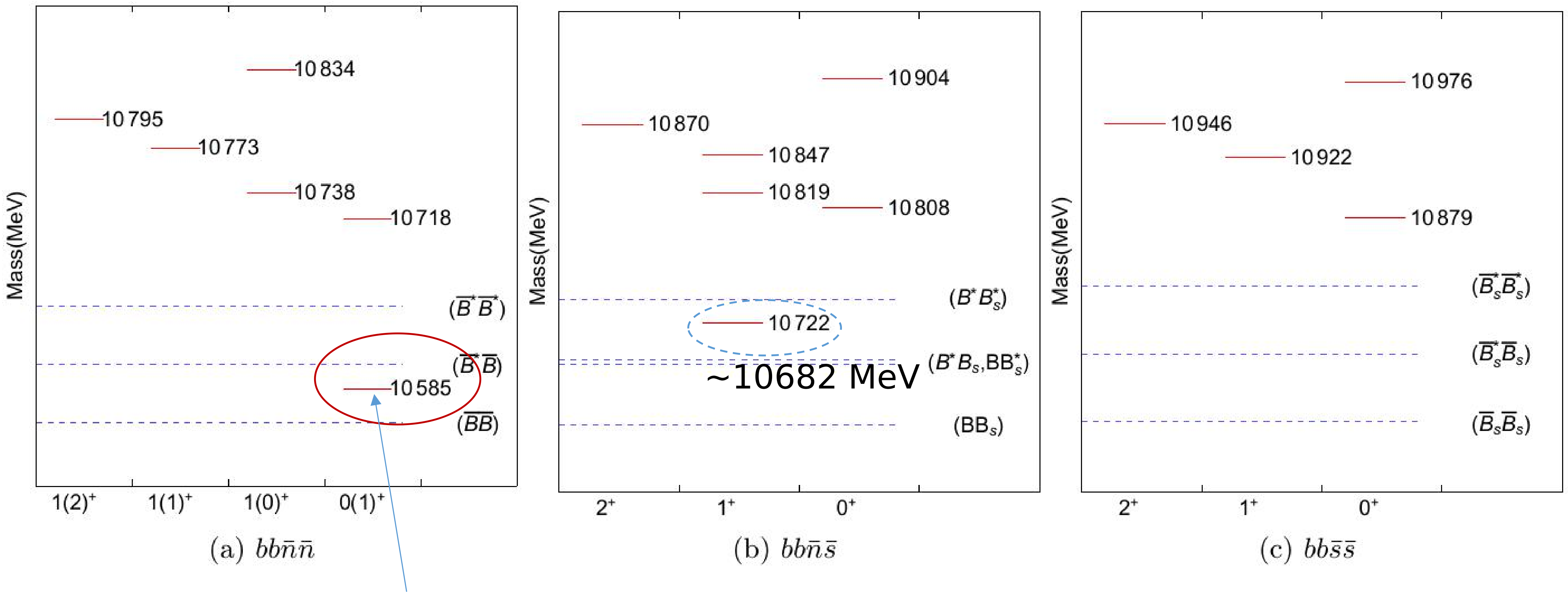
$QQ\overline{q}\overline{q}$ tetraquark states: spectrum

With $m_{X(4140)} = 4146.5$ MeV



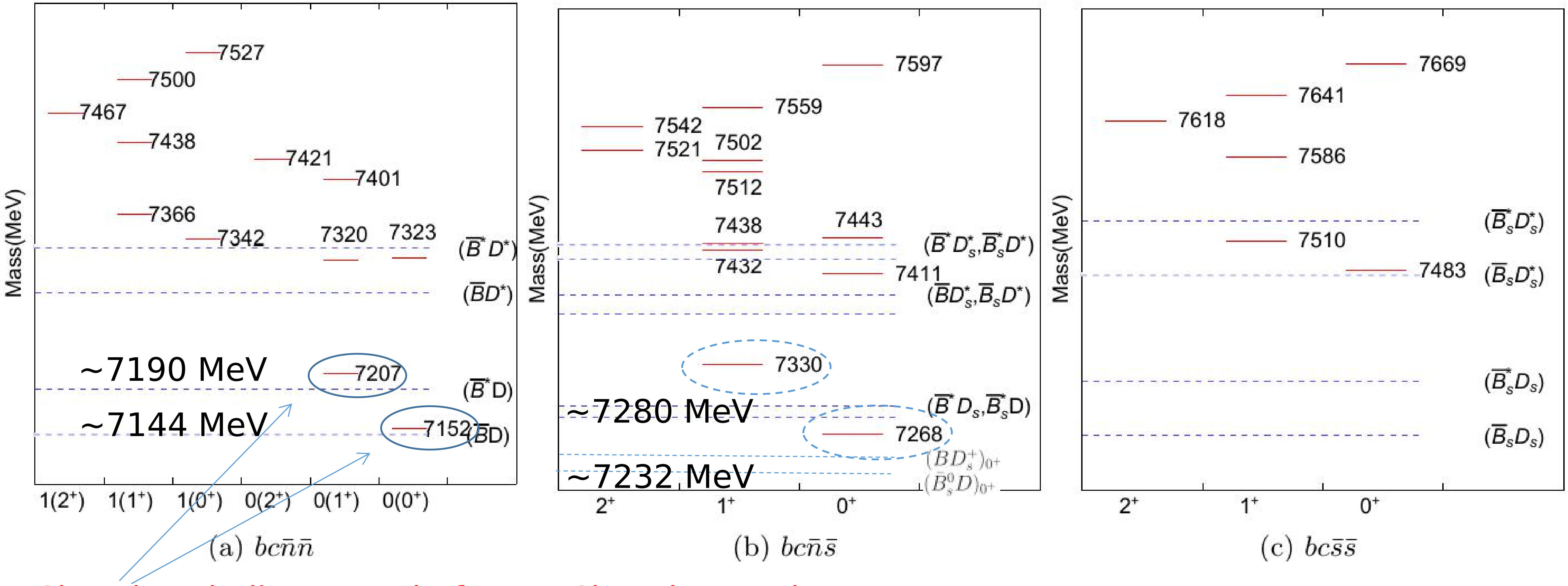
(a) $cc\bar{n}\bar{n}$ (b) $cc\bar{n}\bar{s}$ (c) $cc\bar{s}\bar{s}$

$QQ\bar{q}\bar{q}$ tetraquark states: spectrum



Almost all theoretical studies support this bound $bb\bar{u}\bar{d}$.

$QQ\bar{q}\bar{q}$ tetraquark states: spectrum



Situation similar to Tcc before LHCb's observation!

J.B. Cheng et al, CPC 45, 043102 (2021): 7167 MeV & 7223 MeV;
Karliner, Rosner, PRL 119, 202001 (2017): 11 MeV below BD;
Alexandrou et al, 2312.02925: shallow bound $bc\bar{u}\bar{d}$ with J=0 and 1.

Table 10. Stability of the double-heavy tetraquarks in various studies. The meanings of "S," "US," and "ND" are "stable," "unstable," and "not determined," respectively.

Reference	$(cc\bar{n}\bar{n})$	$(cc\bar{n}\bar{s})$	$(cc\bar{s}\bar{s})$	$(bb\bar{n}\bar{n})$	$(bb\bar{n}\bar{s})$	$(bb\bar{s}\bar{s})$	$(bc\bar{n}\bar{n})$	$(bc\bar{n}\bar{s})$	$(bc\bar{s}\bar{s})$
This work	US	US	US	S	S	US	ND	US	US
[8]	S	S		S	S		S	US	
[11]	S	S	US	S	S	US	S	S	US
[16]	S			S					
[18]	S			S			S		
[19]	US			S			S		
[20]	US			S	S		US	US	
[24]	S			S			S		
[28]	S	US	US	S	S	US	S	US	US
[29]	S			S			S		
[30]	US	US	US	S	US	US	US	US	US
[31]	US	US	US	S	US	US	US	US	US
[32]			US			US			US
[33]	US	US	US	S	S	S			
[34]								S	S
[39]	US			S					
[44, 45]	US	US		S	S		S	US	
[47]							S		
[48]				S	S		US	US	
[63]	US			S			ND		
[69]							ND	US	
[83]	US	US	US	S	S	US	US	US	US
[84]	US	US	US	S	S	US	US	US	US

J.B. Cheng et al, CPC 45, 043102 (2021)

$$T_{cc} < 3965 \text{ MeV}$$

$$T_{bb} < 10627 \text{ MeV}$$

$$T_{bc} < 7199 \text{ MeV}$$

$QQ\overline{q}\overline{q}$ tetraquark states: spectrum

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

If assume $X(4274)$ observed in $J/\psi\phi$ as the high-mass 1^{++} $c\bar{c}s\bar{s}$ tetraquark, all $QQ\overline{q}\overline{q}$ masses become ~27 MeV smaller. Four bound states:

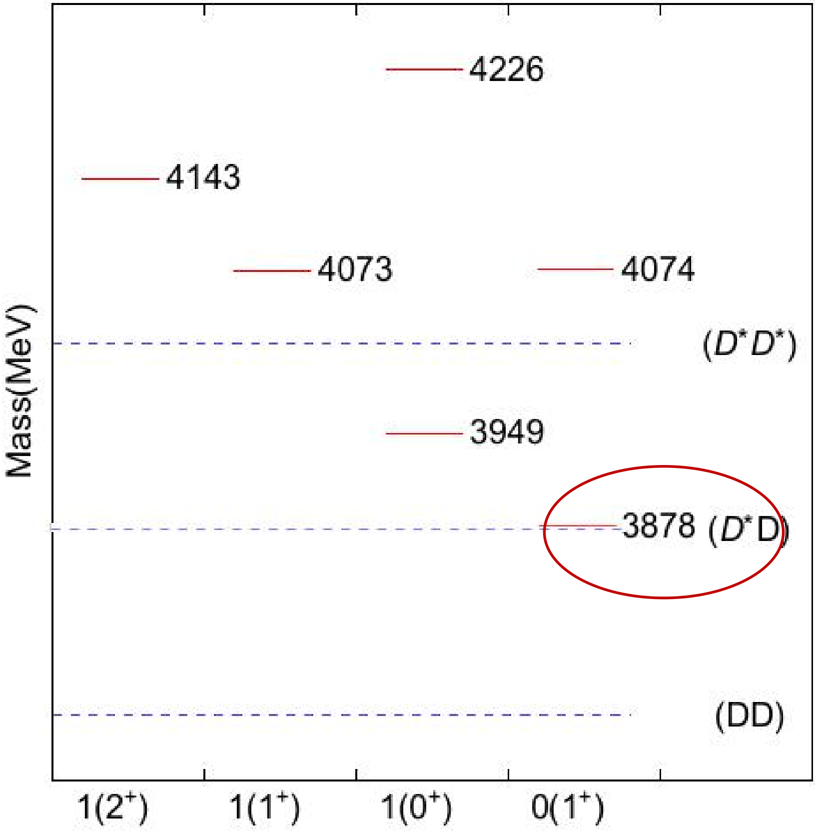
$I(J^P) = 0(1^+)$ $cc\bar{u}\bar{d}$ around 3851 MeV;

$I(J^P) = 0(1^+)$ $bb\bar{u}\bar{d}$ around 10559 MeV;

$I(J^P) = 0(1^+)$ $bc\bar{u}\bar{d}$ around 7181 MeV;

$I(J^P) = 0(0^+)$ $bc\bar{u}\bar{d}$ around 7125 MeV.

Lowest $I(J^P) = 0(1^+)\ cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$



Mass can also be expressed as:

Li, Liu, Liu, Si, Wu, EPJC 79, 87 (2019)

$$M = [M_{ref} - (E_{CMI})_{ref}] + \sum_{i<j} K_{ij} C_{ij}$$

$$K_{ij} = \lim_{\Delta C_{ij} \to 0} \frac{\Delta M}{\Delta C_{ij}} \to \frac{\partial M}{\partial C_{ij}}$$

	$cc\bar{n}\bar{n}$		
$I(J^P)$	K_{cc}	$K_{c\bar{n}}$	K_{nn}
$1(2^+)$	[2.7]	[5.3]	[2.7]
$1(1^+)$	[2.7]	[−5.3]	[2.7]
$1(0^+)$	[3.6 3.1]	[14.9 −25.5]	[3.6 3.1]
$0(1^+)$	[3.8 2.9]	[8.6 −8.6]	[−2.5 −6.8]

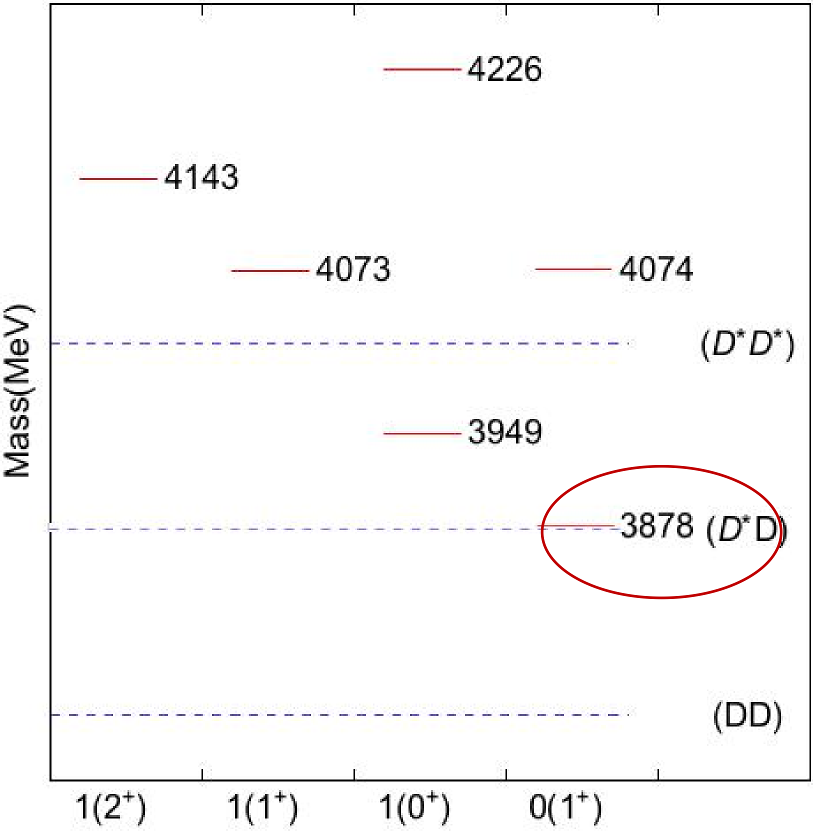
C_{ij}	n	s	c	b	$C_{i\bar{j}}$	$\bar{n}$	$\bar{s}$	$\bar{c}$	$\bar{b}$
n	18.3	12.1	4.0	1.3	n	29.8	18.7	6.6	2.1
s		6.5	4.3	1.3	s		9.8	6.7	2.3
c			3.5	2.0	c			5.3	3.3
b				1.9	b				2.9

		$cc\bar{n}\bar{n}$		
$I(J^P)$	Mass	Channels		Γ
$1(2^+)$	[4143.2]	D^*D^* [(33.3, 20.8)]		[20.8]
$1(1^+)$	[4072.8]	D^*D [(16.7, 53.0)]		[53.0]
$1(0^+)$	[4225.9 3948.8]	D^*D^* [(55.7, 43.2) (2.6, −)]	DD [(0.3, 0.3) (41.4, 35.9)]	[43.5 35.9]
$0(1^+)$	[4074.0 3878.2]	D^*D^* [(48.4, 20.9) (1.6, −)]	D^*D [(6.2, 19.8) (18.8, 7.2)]	[40.7 7.2]

If M → 3876 MeV, Γ=3.0 MeV;
If M → 3880 MeV, Γ=9.7 MeV.

Width sensitive to mass for near-threshold states.

Lowest $I(J^P) = 0(1^+)\, cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$



(a) $cc\bar{n}\bar{n}$

Capstick, Roberts, PRD 49, 4570 (1994);
Roberts, Silvestre-Brac, PRD 57, 1694 (1998);
Segovia et al, PRD 80, 054017 (2009);
Ferretti et al, PRD 90, 054010 (2014);
Gui et al, PRD 98, 016010 (2018).

With experiment $M_{Tcc} = M_{D^{*+}} + M_D - 273$ keV,
quasi-two-body decay width:

$$\Gamma = \int_0^{k_{max}} dk \frac{\Gamma_{D^{*+}\to D^0\pi^+}}{(M_{T_{cc}^+} - E_{D^{*+}}(k) - E_{D^0}(k))^2 + \frac{1}{4}\Gamma_{D^{*+}}} \frac{k^2|\mathcal{M}|^2}{(2\pi)^2 M_{T_{cc}^+} E_{D^{*+}}(k) E_{D^0}(k)}$$

~105 keV

$$k_{max} = \frac{\sqrt{M_{T_{cc}^+}^2 - (2M_{D^0} + M_\pi)^2}\sqrt{M_{T_{cc}^+}^2 - M_\pi^2}}{2M_{T_{cc}^+}}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$

Unlike Pc case where ratios between different partial widths of rearrangement channels can be adopted, one cannot get further information here.

PHYSICAL REVIEW D **104**, 114009 (2021)

Color and baryon number fluctuation of preconfinement system in production process and T_{cc} structure

Yi Jin,[1] Shi-Yuan Li,[2] Yan-Rui Liu,[2] Qin Qin,[3] Zong-Guo Si,[2] and Fu-Sheng Yu[4,5,6]

IV. CONCLUSION

The consistency between the theoretical analysis on the T_{cc} production by Qin, Shen and Yu [37] and the data [8,9] strongly favors that the newly discovered resonance T_{cc} is produced as a real four-quark state. We in this paper clarify

$QQ\bar{q}\bar{q}$ tetraquark states: rearrangement decay

$I(J^P)$	Mass	Channels			Γ
$cc\bar{n}\bar{n}$					
		D^*D^*			
$1(2^+)$	4143.2	(33.3, 20.8)			20.8
		D^*D			
$1(1^+)$	4072.8	(16.7, 53.0)			53.0
		D^*D^*	DD		
$1(0^+)$	4225.9	(55.7, 43.2)	(0.3, 0.3)		43.5
	3948.8	(2.6, −)	(41.4, 35.9)		35.9
		D^*D^*	D^*D		
$0(1^+)$	4074.0	(48.4, 20.9)	(6.2, 19.8)		40.7
	3878.2	(1.6, −)	(18.8, 7.2)		7.2
$cc\bar{n}\bar{s}$					
		$D^*D_s^*$			
2^+	4217.5	(33.3, 35.5)			35.5
		$D^*D_s^*$	D^*D_s	DD_s^*	
1^+	4182.2	(49.6, 42.7)	(4.1, 6.4)	(4.5, 7.0)	56.1
	4146.6	(0.0, 0.0)	(16.7, 24.1)	(16.6, 23.7)	47.8
	4009.2	(0.4, −)	(20.9, 13.9)	(20.5, 13.1)	27.0
		$D^*D_s^*$	DD_s		
0^+	4295.1	(55.3, 76.7)	(0.2, 0.4)		77.1
	4018.8	(3.0, −)	(41.5, 65.0)		65.0
$cc\bar{s}\bar{s}$					
		$D_s^*D_s^*$			
2^+	4293.5	(33.3, 14.6)			14.6
		$D_s^*D_s$			
1^+	4222.0	(16.7, 42.7)			42.7
		$D_s^*D_s^*$	D_sD_s		
0^+	4366.6	(55.0, 33.6)	(0.1, 0.1)		33.8
	4090.7	(3.3, −)	(41.5, 29.1)		29.1

$I(J^P)$	Mass	Channels			Γ
$bb\bar{n}\bar{n}$					
		B^*B^*			
$1(2^+)$	10795.3	(33.3, 5.3)			5.3
		$\bar{B}^*\bar{B}$			
$1(1^+)$	10772.9	(16.7, 11.5)			11.5
		$\bar{B}^*\bar{B}^*$	$\bar{B}\bar{B}$		
$1(0^+)$	10834.4	(57.4, 10.3)	(1.2, 0.3)		10.5
	10738.4	(0.9, 0.1)	(40.5, 7.2)		7.4
		$\bar{B}^*\bar{B}^*$	$\bar{B}^*\bar{B}$		
$0(1^+)$	10717.8	(41.2, 4.6)	(12.2, 7.0)		11.6
	10584.5	(8.8, −)	(12.8, −)		0
$bb\bar{n}\bar{s}$					
		$B^*B_s^*$			
2^+	10869.9	(33.3, 10.0)			10.0
		$\bar{B}^*\bar{B}_s^*$	$\bar{B}^*\bar{B}_s$	$\bar{B}\bar{B}_s^*$	
1^+	10846.5	(0.1, 0.0)	(15.1, 4.9)	(18.2, 5.9)	10.9
	10819.1	(44.2, 10.4)	(11.3, 3.4)	(8.8, 2.6)	16.3
	10722.2	(5.7, −)	(15.3, 2.3)	(14.7, 2.1)	4.3
		$\bar{B}^*\bar{B}_s^*$	$\bar{B}\bar{B}_s$		
0^+	10903.8	(56.7, 18.9)	(0.7, 0.3)		19.2
	10807.8	(1.7, 0.4)	(41.0, 13.8)		14.2
$bb\bar{s}\bar{s}$					
		$B_s^*B_s^*$			
2^+	10946.1	(33.3, 4.7)			4.7
		$\bar{B}_s^*\bar{B}_s$			
1^+	10921.6	(16.7, 10.3)			10.3
		$\bar{B}_s^*\bar{B}_s^*$	$\bar{B}_s\bar{B}_s$		
0^+	10975.7	(55.8, 8.7)	(0.3, 0.1)		8.8
	10878.7	(2.5, 0.2)	(41.3, 6.5)		6.8

J^P	Mass	$bc\bar{s}\bar{s}$ Channels			Γ
		$\bar{B}_s^*D_s^*$			
2^+	7617.9	(33.3, 12.8)			12.8
		$\bar{B}_s^*D_s^*$	$\bar{B}_s^*D_s$	$\bar{B}_sD_s^*$	
1^+	7640.5	(46.2, 19.7)	(0.4, 0.2)	(8.1, 4.1)	24.1
	7586.4	(3.0, 0.9)	(1.4, 0.8)	(29.3, 12.3)	14.1
	7510.2	(0.8, −)	(39.8, 18.2)	(4.3, 1.0)	19.2
		$\bar{B}_s^*D_s^*$	$\bar{B}_sD_s$		
0^+	7668.6	(55.2, 26.1)	(0.2, 0.1)		26.3
	7482.6	(3.2, −)	(41.5, 20.6)		20.6

Ratios between partial widths as predictions for tetraquark states having two or three rearrangement decay channels.

Summary

- It seems that the lowest $\mathbf{0(1^+)}$ $\boldsymbol{cc\bar{u}\bar{d}}$ tetraquark state can also be used to understand the LHCb $\boldsymbol{T_{cc}}$ state. [mass, width, production]

- In the S-wave $\boldsymbol{QQ\bar{q}\bar{q}}$ states, only the $\mathbf{0(1^+)}$ $\boldsymbol{bb\bar{u}\bar{d}}$ tetraquark is stable. It is located ~20 MeV below the $\boldsymbol{\bar{B}\bar{B}}$ threshold.

- The lowest $\mathbf{0(0^+)}$ $\boldsymbol{bc\bar{u}\bar{d}}$ state is very close to the $\boldsymbol{\bar{B}D}$ threshold. We can not exclude it as a stable tetraquark. The lowest $\mathbf{0(1^+)}$ $\boldsymbol{bc\bar{u}\bar{d}}$ is also a near-threshold state.

Thanks for your attention!