Dispersive determination of 4th generation quark masses

Hsiang-nan Li

Presented at 20th HFCPV Workshop

Dec. 18, 2023

2309.15602

Motivation

2302.01761
2304.05921

- dispersive analyses of heavy meson decay widths, neutral meson mixing, etc. indicated that scalar sector of SM may not be free
- Higgs mass, fermion masses, mixing angles constrained dynamically through analyticity
- Bold conjecture: SM contains only three fundamental (gauge) parameters, and other parameters, governing interplay among various generations of fermions, are fixed by SM dynamics itself
- To maintain simplicity and beauty, natural extension of SM is to introduce sequential fourth generation of fermions, since associated parameters in scalar sector can be predicted

Merits of SM4 and experimental exclusion

- Condensates of 4th generation quarks and leptons as responsible mechanism of dynamical electroweak symmetry breaking

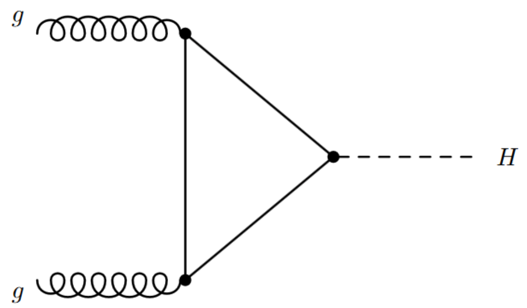
Holdom, 1986

- 1st-order phase transition for electroweak baryogenesis realized

Carena et al, 0410352

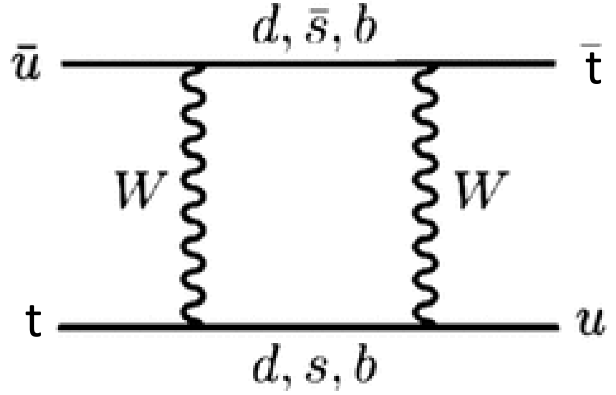
- Provide source of CP violation for baryon asymmetry of the Universe

Hou, 0803.1234



- But SM4 ruled out by data of Higgs production via gluon fusion and decay into photon pairs, (t + b’ + t’)^2 ~ 9 t^2
- Will show b’ mass 2.7 TeV and t’ mass 200 TeV, so heavy that bound states formed in Yukawa potential
- These bound states could bypass experimental constraints

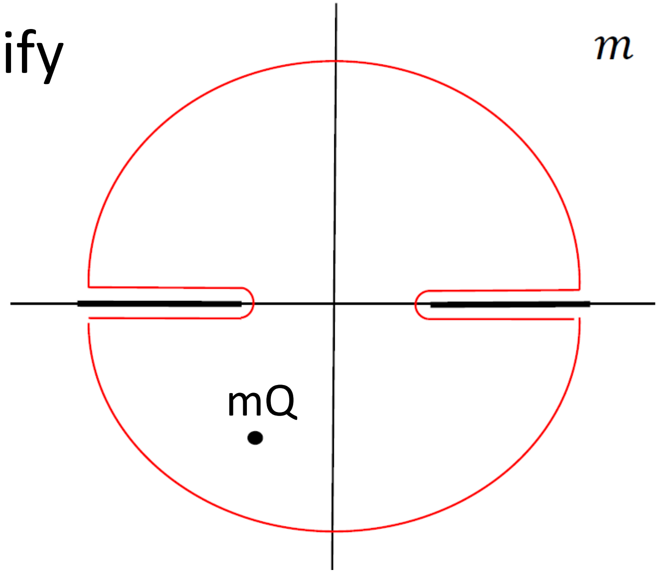
Strategy



- Reproduce top mass from dispersion relations for box diagrams
- Existence of common solution for 3 different channels $ij = db,\ sb$ and bb justifies our formalism and make predictions convincing
- Predict b' and t' masses

Framework

heavy quark mQ to Justify perturbative evaluation



- 2 contour integrations to reexpress box at mQ

unknowns to be solved

perturbative inputs from box diagrams

$$\int_{M_{ij}^2}^{R^2} \frac{\mathrm{Im}\Pi_{ij}(m)}{m_Q^2 - m^2} dm^2 = \int_{m_{ij}^2}^{R^2} \frac{\mathrm{Im}\Pi_{ij}^{\mathrm{box}}(m)}{m_Q^2 - m^2} dm^2$$

branch cuts along both m > 0, m < 0

big circle contributions cancel, because

$$\mathrm{Im}\Pi_{ij}(m) \rightarrow \mathrm{Im}\Pi_{ij}^{\mathrm{box}}(m)$$

due to analyticity

3 channels $ij = db,\ sb$ and bb

hadronic thresholds

$M_{db} = m_\pi + m_B,\ M_{sb} = m_K + m_B$ and $M_{bb} = 2m_B$

quark-level thresholds

$m_{db} = m_d + m_b,$
$m_{sb} = m_s + m_b$ and $m_{bb} = 2m_b.$

Box diagram inputs

- Box diagrams generate (V-A)(V-A), (S-P)(S-P) structures
- Focus on the former

Cheng 1982
Buras et al 1984

intermediate quark masses

$$\Gamma_{ij}^{\mathrm{box}}(m_Q) \propto \frac{C_2(m_Q)}{m_Q^4} \frac{\sqrt{[m_Q^2-(m_i+m_j)^2][m_Q^2-(m_i-m_j)^2]}}{(m_W^2-m_i^2)(m_W^2-m_j^2)}$$
$$\times\left\{2\left(m_W^4+\frac{m_i^2m_j^2}{4}\right)[m_Q^2-(m_i+m_j)^2][m_Q^2-(m_i-m_j)^2]\right.$$
$$\left.-3m_W^2m_Q^2(m_i^2+m_j^2)(m_Q^2-m_i^2-m_j^2)\right\},$$

W boson mass

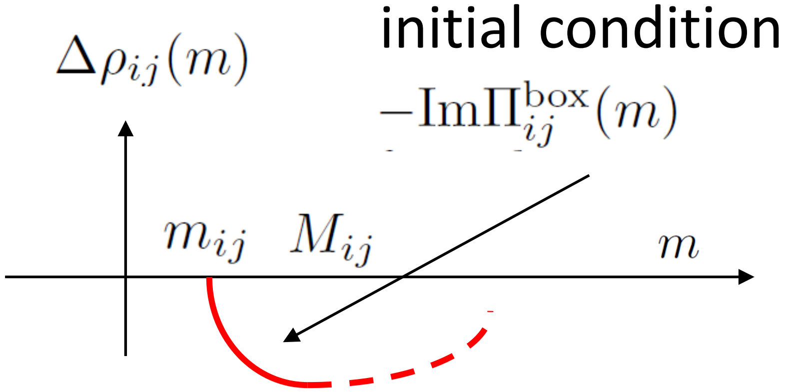
Solutions

- General form

originating from large circle radius R

$$\Delta\rho_{ij}(m_Q) \approx y_{ij}\left(\omega\sqrt{m_Q^2-(m_i+m_j)^2}\right)^{\alpha_{ij}} J_{\alpha_{ij}}\left(2\omega\sqrt{m_Q^2-(m_i+m_j)^2}\right)$$

arbitrary scale from scaling integration variable m^2



- Insensitivity to ω achieved by

vanishing to get roots of m_Q

Taylor expansion

$$\Delta\rho_{ij}(m_Q) = \Delta\rho_{ij}(m_Q)|_{\omega=\bar{\omega}_{ij}} + \frac{d\Delta\rho_{ij}(m_Q)}{d\omega}\Big|_{\omega=\bar{\omega}_{ij}}(\omega-\bar{\omega}_{ij}) + \frac{1}{2}\frac{d^2\Delta\rho_{ij}(m_Q)}{d\omega^2}\Big|_{\omega=\bar{\omega}_{ij}}(\omega-\bar{\omega}_{ij})^2+\cdots$$

fitted to initial conditions
to fix $\bar{\omega}_{ij}$, α_{ij}, y_{ij}

minimal to maximize stability window in ω

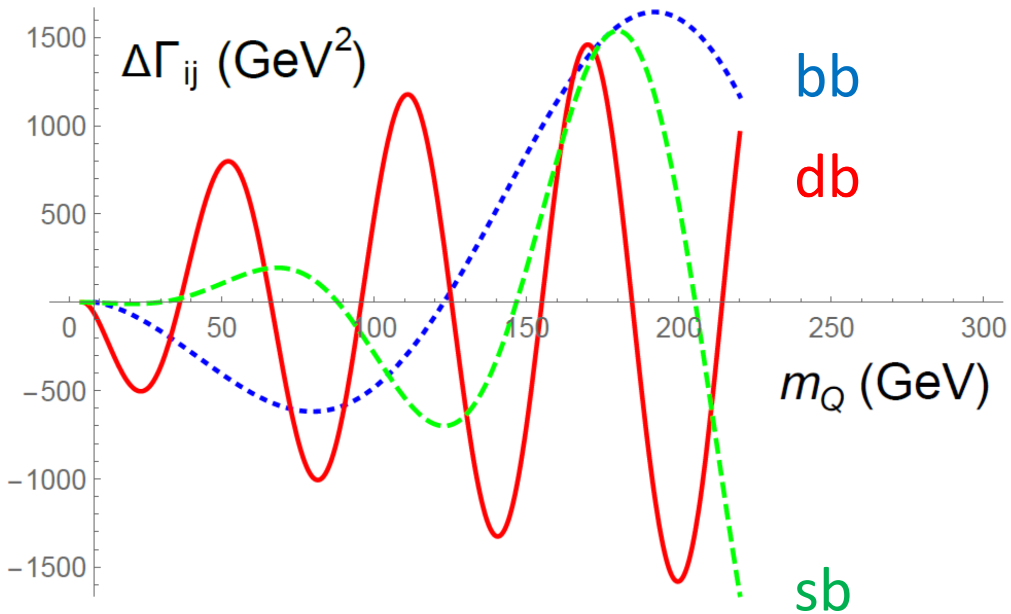
Roots

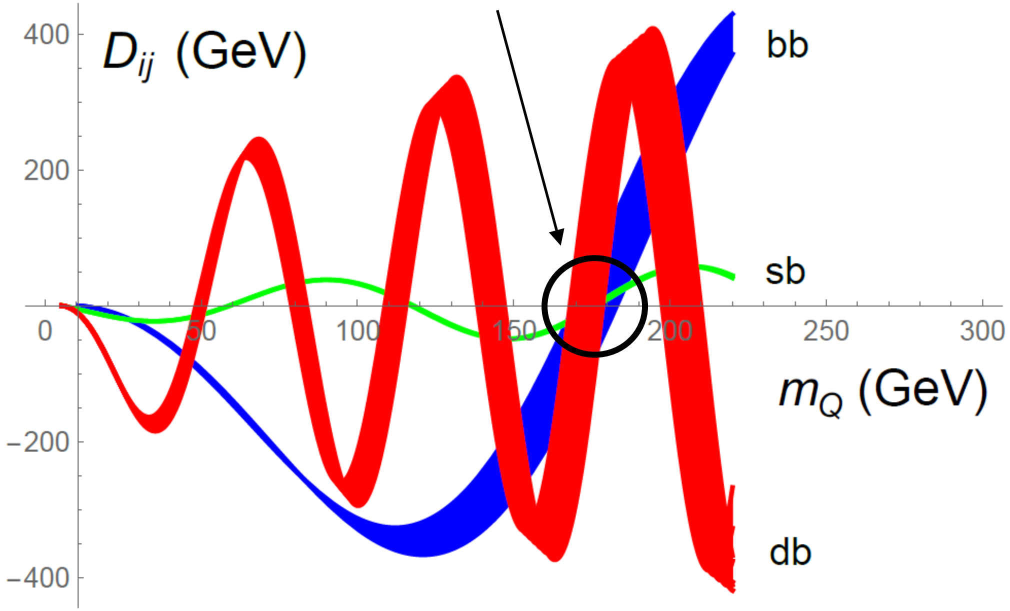
$$m_d = 0 \quad m_s = 0.1 \text{ GeV} \quad m_b = 4.16 \text{ GeV}$$
$$m_\pi = 0.14 \text{ GeV} \quad m_K = 0.49 \text{ GeV}, \; m_B = 5.28 \text{ GeV}$$

- Solutions of unknowns

higher roots, larger 2nd derivative

3 derivatives first vanish simultaneously at $m_t = (173 \pm 3)$ GeV





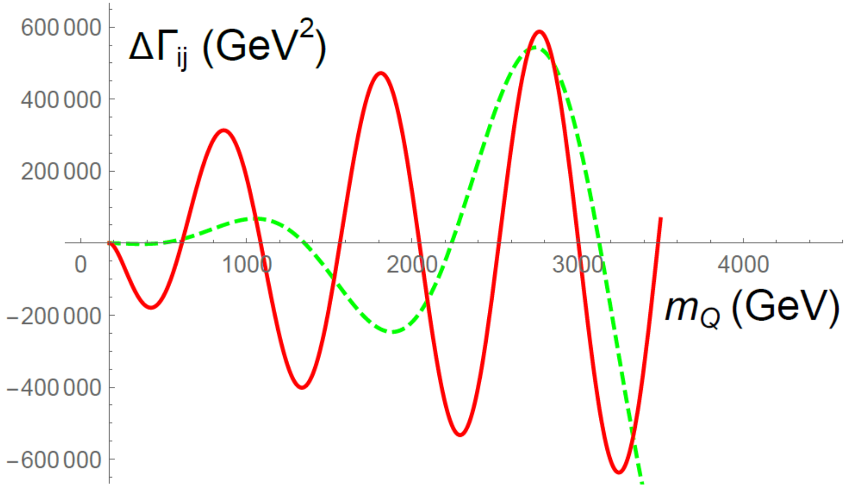
1st peak of bb, 2nd peak of sb, 3rd peak of db overlap around mQ ~ 180 GeV!

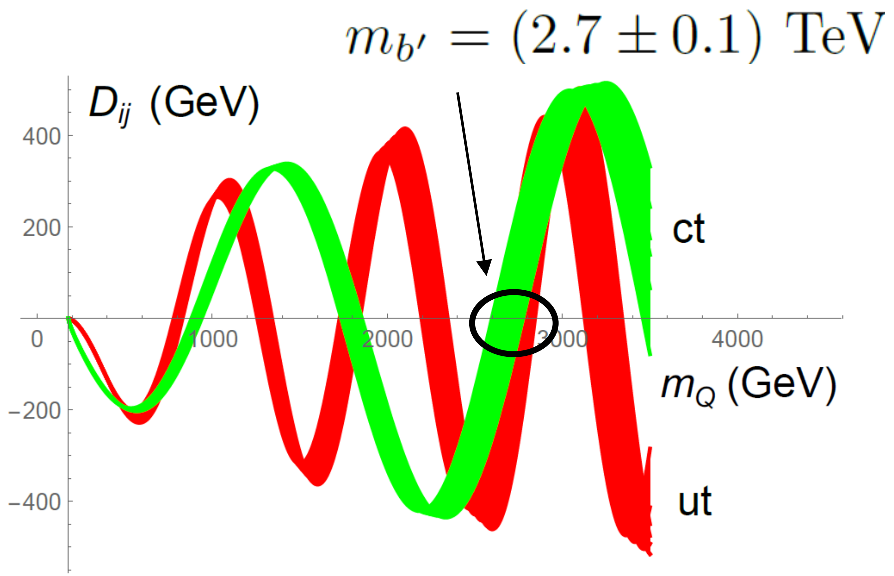
$$D_{ij}(m_Q) \equiv \frac{d}{d\omega} \frac{J_{\alpha_{ij}}\left(2\omega\sqrt{m_Q^2 - (m_i + m_j)^2}\right)}{J_{\alpha_{ij}}\left(2\omega\sqrt{M_{ij}^2 - (m_i + m_j)^2}\right)}\Bigg|_{\omega=\bar{\omega}_{ij}}$$

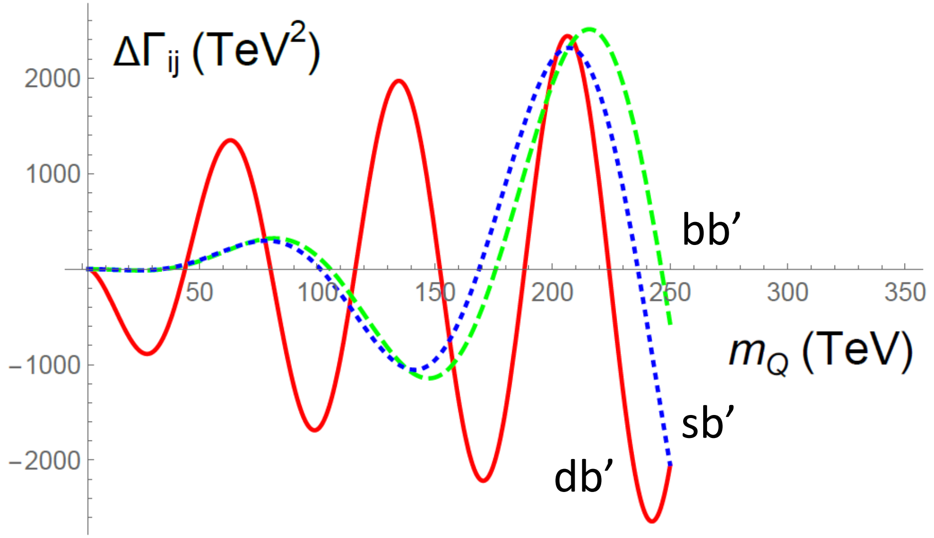
uncertainties from $m_b = (4.16 \pm 0.01)$ GeV and different ways of fixing $\bar{\omega}_{ij}$

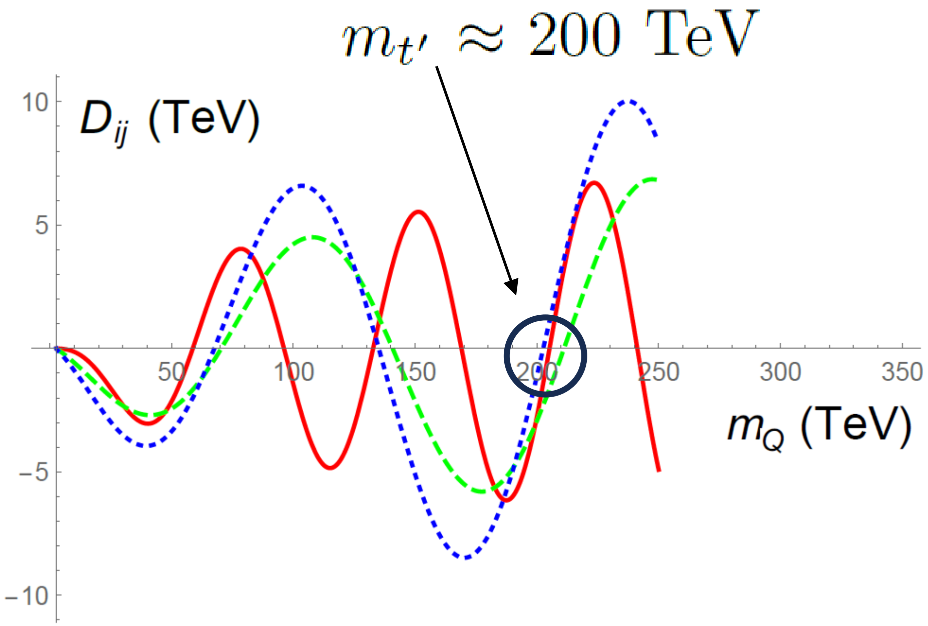
b′ and t′ masses

2nd peak of ct,
3rd peak of ut
overlap at
mQ ~ 2.7 TeV









$\bar{b}'b'$ bound states

$v = 246$ GeV

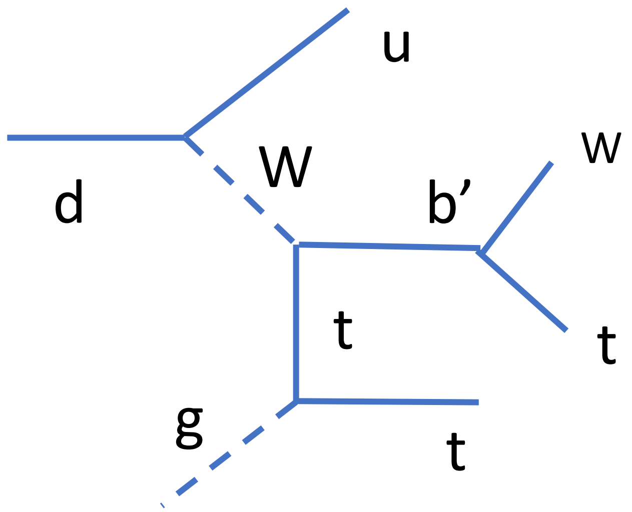
- As 4th generation quark mass meets criterion $K_Q = m_Q^3/(4\pi v^2 m_H) > 1.68$ bound states formed

Hung, Xiong 2011

- With b' mass 2.7 TeV, $\bar{b}'b'$ bound states formed definitely
- Should analyze gluon fusion involving internal b' in effective theory
- Gluon fusion into S via effective operator $A^\mu A^\nu S$, coupling $\sqrt{s} g_{ggS}$
- Scalar S propagates according to BW factor $1/(s - m_S^2 - i\sqrt{s}\Gamma_S)$
- S transforms into H with magnitude $s g_{SH}$
- Total amplitude $\mathcal{M} \sim \dfrac{\sqrt{s}^3 g_{ggS} g_{SH}}{s - m_S^2 - i\sqrt{s}\Gamma_S}$
- Matching to fundamental theory ➡ $g_{ggS} g_{SH} = (2/3)\Gamma_S/v$
- New scalar contribution of O(10E-3) to Higgs production negligible

Search modes

- Impossible to detect t' in near future
- Gluon fusion into $\bar{b}'b'$ ground state of mass 3.2 TeV not efficient owing to small gluon PDFs
- Weak boson fusion $qq \to WW, ZZ \to S$ more promising
- For single b' production, consider associated production $dg \to u\bar{t}b'$, power enhanced by one fewer virtual weak boson, but down by gluon PDFs. Similar to vector-like quark search
- Another single b' production $ug \to W^{+}b'$ down by diminishing 4X4 CKM matrix element $V_{ub'}$



Conclusion

- Dispersion relations which physical observables must obey impose stringent constraints on dynamics at various scales
- Analyticity dictates scalar sector that couples generations
- Tested formalism by finding common solution for top mass from 3 channels, highly nontrivial and convincing
- Predicted b’ mass 2.7 TeV and t’ mass 200 TeV
- Bound states formed with huge Yukawa couplings, whose contributions to Higgs production via gluon fusion tiny
- Worthwhile to continue search of b’ quarks and ground state of mass 3.2 TeV

Back-up slides

Polynomial expansion

arbitrary scale ↓

- Introduce dimensionless variables, $m_S^2 - 4m_b^2 = u\Lambda$, $m^2 - 4m_b^2 = v\Lambda$

$$\int_0^\infty dv \frac{\Delta\rho(v)}{u-v} = 0$$

$\Delta\rho(v) \to 0$ at large v, because $\mathrm{Im}\Pi(m) \to \mathrm{Im}\Pi^{\mathrm{p}}(m)$

power series in $1/u$ using $1/(u-v) = \sum_{i=1}^{\infty} v^{i-1}/u^i$

- Start with case of N vanishing coefficients, N large

contained in $L_0^{(\alpha)}(v), L_1^{(\alpha)}(v), \cdots, L_{N-1}^{(\alpha)}(v)$

$$\int_0^\infty dv v^{i-1} \Delta\rho(v) = 0, \quad i = 1, 2, 3 \cdots, N$$

- Imply expansion in generalized Laguerre polynomials because of orthogonality

weight

$$\Delta\rho(v) = \sum_{j=N}^{N'} a_j \underline{v^\alpha e^{-v}} L_j^{(\alpha)}(v), \quad N' > N \qquad \int_0^\infty \underline{y^\alpha e^{-y}} L_m^{(\alpha)}(y) L_n^{(\alpha)}(y) dy = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{mn}$$

↑ fixed by initial condition in principle, needs not be infinite

Solution

- Large j approximation, subject to correction of $1/\sqrt{j}$

$$L_j^{(\alpha)}(v) \approx j^{\alpha/2} v^{-\alpha/2} e^{v/2} J_\alpha(2\sqrt{jv})$$

- Solution

arbitrary degree and scale appear in ratio

$$\Delta\rho(m) \approx \sum_{j=N}^{N'} a_j \sqrt{\frac{j(m^2-4m_b^2)}{\Lambda}}^{\alpha} e^{-(m^2-4m_b^2)/(2\Lambda)} J_\alpha\left(2\sqrt{\frac{j(m^2-4m_b^2)}{\Lambda}}\right)$$

- Scaling variable $\omega \equiv \sqrt{N/\Lambda}$, large N limit $N'/\Lambda = \omega^2 + (N'-N)/N \approx \omega^2$

$$J_\alpha(2\sqrt{j(m^2-4m_b^2)/\Lambda}) \approx J_\alpha(2\omega\sqrt{m^2-4m_b^2}) \qquad e^{-(m^2-4m_b^2)/(2\Lambda)} = e^{-\omega^2(m^2-4m_b^2)/(2N)}$$

≈ 1

$$\Delta\rho(m) \approx y\left(\omega\sqrt{m^2-4m_b^2}\right)^{\alpha} J_\alpha\left(2\omega\sqrt{m^2-4m_b^2}\right)$$

$\sum_{j=N}^{N'} a_j$

solution in terms of single Bessel function

Scale invariance

Xiong, Wei, Yu 2022

- Solution to this type of integral (Fredholm) equation, if existing, is unique, given boundary condition. Will construct a solution
- It must be insensitive to arbitrary Λ , i.e., to ω from variable change
- To realize this insensitivity, consider

minimal to maximize stability window

$$\Delta\rho(m_S) = \Delta\rho(m_S)|_{\omega=\bar{\omega}} + \frac{d\Delta\rho(m_S)}{d\omega}\Big|_{\omega=\bar{\omega}}(\omega-\bar{\omega}) + \frac{1}{2}\frac{d^2\Delta\rho(m_S)}{d\omega^2}\Big|_{\omega=\bar{\omega}}(\omega-\bar{\omega})^2 + \cdots$$

fit to initial condition to determine $\bar{\omega}$, α y

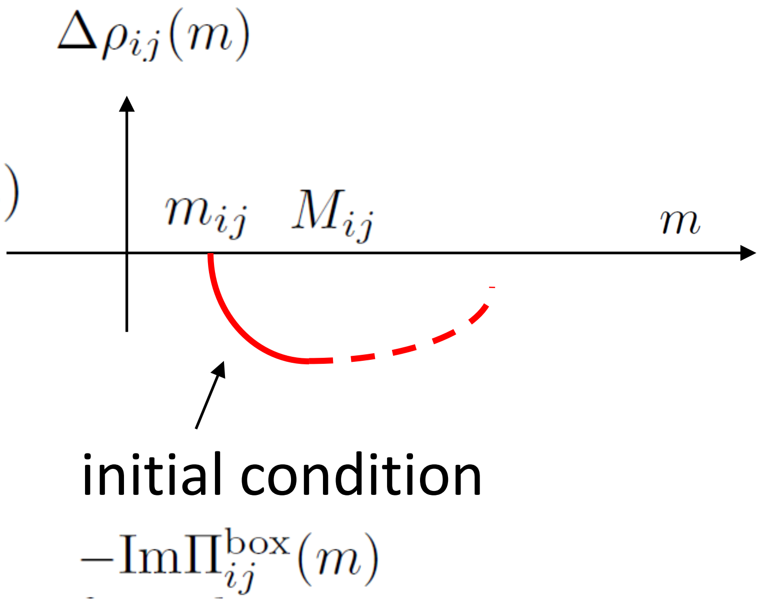
$d\Delta\rho(m_S)/d\omega|_{\omega=\bar{\omega}} = 0$ discrete roots! stability window exists

- Single root of m_S is allowed ➡ Higgs mass
- Both N and Λ can be arbitrarily large, large N approximation justified

Initial conditions

- Move RHS to LHS,

$$\Delta\rho_{ij}(m) \equiv \mathrm{Im}\Pi_{ij}(m) - \mathrm{Im}\Pi_{ij}^{\mathrm{box}}(m)$$



extended to infinity

$$\int_{m_{ij}^2}^{\infty} \frac{\Delta\rho_{ij}(m)}{m_Q^2 - m^2} dm^2 = 0$$

- Threshold behaviors around $m_Q \sim m_{ij}$

$$\Gamma_{db}^{\mathrm{box}}(m_Q) \sim \frac{(m_Q^2 - m_b^2)^3}{m_Q^4},$$

$$\Gamma_{sb}^{\mathrm{box}}(m_Q) \sim \frac{\sqrt{[m_Q^2 - (m_b + m_s)^2][m_Q^2 - (m_b - m_s)^2]}^3}{m_Q^4}$$

$$\Gamma_{bb}^{\mathrm{box}}(m_Q) \sim \frac{\sqrt{m_Q^2 - 4m_b^2}^3}{m_Q}.$$

governed by 1st term
in curly brackets
2nd term down by $(m_i^2 + m_j^2)/m_W^2$

Integrands

- Motivated by threshold behaviors, choose integrands (to simplify initial conditions)

suppress low-m residues like D meson mass or $m = \pm(m_i + m_j)$ relative to $m = \pm m_Q$

$$\mathrm{Im}\Pi_{db}(m) = \frac{m^4\Gamma_{db}(m)}{(m^2 - m_b^2)^2},$$

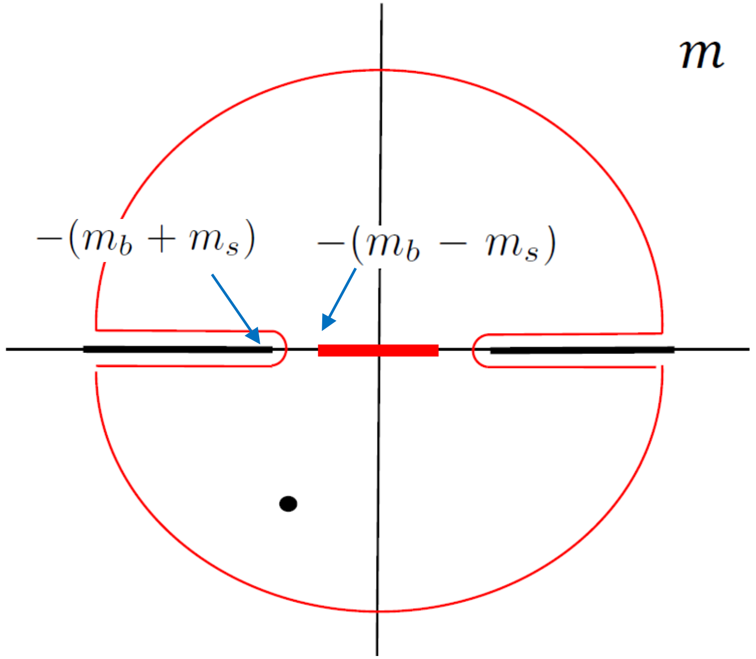
alleviate divergent behaviors in numerators

$$\mathrm{Im}\Pi_{sb}(m) = \frac{m^4\Gamma_{sb}(m)}{[m^2 - (m_b + m_s)^2]^2\sqrt{m^2 - (m_b - m_s)^2}^3}$$

$$\mathrm{Im}\Pi_{bb}(m) = \frac{m\Gamma_{bb}(m)}{m^2 - 4m_b^2},$$

additional branch cut does not contribute

odd power of m due to odd function $\Gamma_{bb}^{\mathrm{box}}(m)$ in m



- Definitions of $\mathrm{Im}\Pi_{ij}^{\mathrm{box}}(m)$ are self-evident

Parameter fixing

$m_d = 0 \quad m_s = 0.1\ \mathrm{GeV} \quad m_b = 4.16\ \mathrm{GeV}$

$m_\pi = 0.14\ \mathrm{GeV} \quad m_K = 0.49\ \mathrm{GeV},\ m_B = 5.28\ \mathrm{GeV}$

- Initial conditions around $m_Q \sim m_{ij}$

$$\Delta\rho_{db}(m_Q) \sim m_Q^2 - m_b^2,$$
$$\Delta\rho_{sb}(m_Q) \sim [m_Q^2 - (m_b + m_s)^2]^{-1/2}$$
$$\Delta\rho_{bb}(m_Q) \sim (m_Q^2 - 4m_b^2)^{1/2}.$$

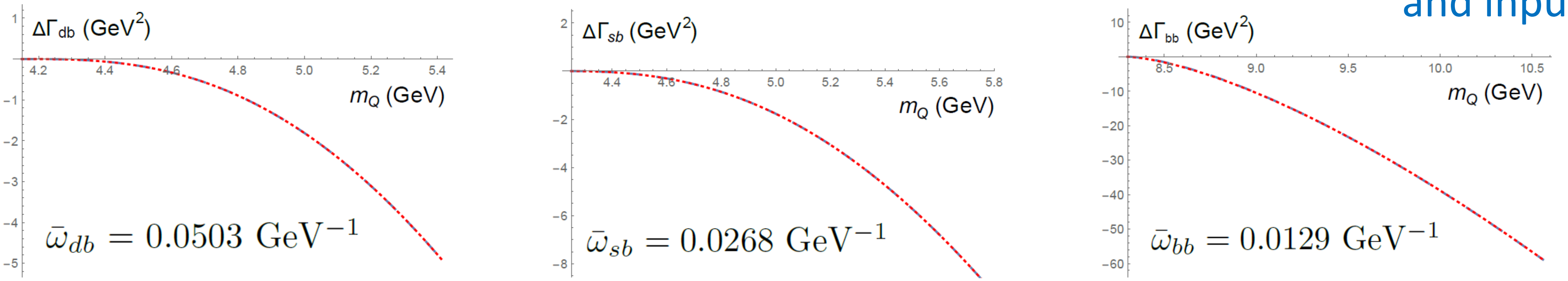
clear why considering complicated integrands: to have simple power of $m_Q^2 - (m_i + m_j)^2$

➡ $\alpha_{db} = 1, \quad \alpha_{sb} = -1/2, \quad \alpha_{bb} = 1/2$

- Boundary conditions $\Delta\rho_{ij}(m_Q)$ set coefficients

$$y_{ij} = -\mathrm{Im}\Pi_{ij}^{\mathrm{box}}(M_{ij}) \left[\left(\omega\sqrt{M_{ij}^2 - (m_i + m_j)^2} \right)^{\alpha_{ij}} J_{\alpha_{ij}} \left(2\omega\sqrt{M_{ij}^2 - (m_i + m_j)^2} \right) \right]^{-1}$$

comparison of fitted results and inputs



b' mass

- Similar box diagrams with ut, ct channels (t does not hadronize)

$$m_{ut} = m_t \ (m_u = 0) \qquad M_{ut} = m_\pi + m_t \qquad m_{ct} = m_c + m_t \qquad M_{ct} = m_D + m_t$$

- Threshold behaviors

governed by 2nd term in curly Brackets, enhanced by $(m_i^2 + m_j^2)/m_W^2 \approx m_t^2/m_W^2$

$$\Gamma_{ut}^{\rm box}(m_Q) \sim \frac{(m_Q^2 - m_t^2)^2}{m_Q^2},$$

$$\Gamma_{ct}^{\rm box}(m_Q) \sim \frac{m_Q^2 - m_t^2 - m_c^2}{m_Q^2} \sqrt{[m_Q^2 - (m_t + m_c)^2][m_Q^2 - (m_t - m_c)^2]}.$$

- Integrands

$$\mathrm{Im}\Pi_{ut}(m) = \frac{m^2 \Gamma_{ut}(m)}{m^2 - m_t^2},$$

$$\mathrm{Im}\Pi_{ct}(m) = \frac{m^2 \Gamma_{ct}(m)}{[m^2 - (m_t + m_c)^2](m_Q^2 - m_t^2 - m_c^2)\sqrt{m^2 - (m_t - m_c)^2}}$$

$\bar{b}'b'$ bound states

- As 4th generation quark mass meets criterion $K_Q = m_Q^3/(4\pi v^2 m_H) > 1.68$, $v = 246$ GeV, bound states formed

  Hung, Xiong 2011

- Binding energy for $m_Q^* \approx 1.26$ TeV and $m_H^* \approx 1.45$ TeV at fixed point of RG evolution in SM4 estimated to be -4.9 GeV
- With b' mass 2.7 TeV, $\bar{b}'b'$ bound states formed definitely
- Should analyze gluon fusion involving internal b' in effective theory
- Gluon fusion into S via effective operator $A^\mu A^\nu S$, coupling $\sqrt{s} g_{ggS}$
- Scalar S propagates according to BW factor $1/(s - m_S^2 - i\sqrt{s}\Gamma_S)$
- S transforms into H with magnitude $s g_{SH}$
- Total amplitude

$$\mathcal{M} \sim \frac{\sqrt{s}^3 g_{ggS} g_{SH}}{s - m_S^2 - i\sqrt{s}\Gamma_S}$$

Parameter fixing and roots

$m_D = 1.87$ GeV $m_t = (173 \pm 3)$ GeV

- Initial conditions around $m_Q \sim m_{ij}$

$$m_c(m_t) = m_c(m_c)\left[\frac{\alpha_s(m_t)}{\alpha_s(m_c)}\right]^{4/\beta_0} \approx 0.7 \text{ GeV}$$

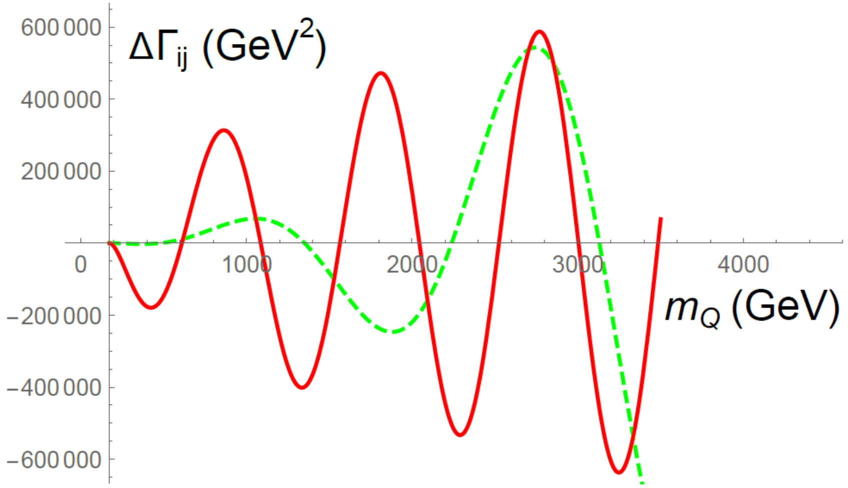
$$\Delta\rho_{ut}(m_Q) \sim m_Q^2 - m_t^2,$$
$$\Delta\rho_{ct}(m_Q) \sim [m_Q^2 - (m_t + m_c)^2]^{-1/2}$$

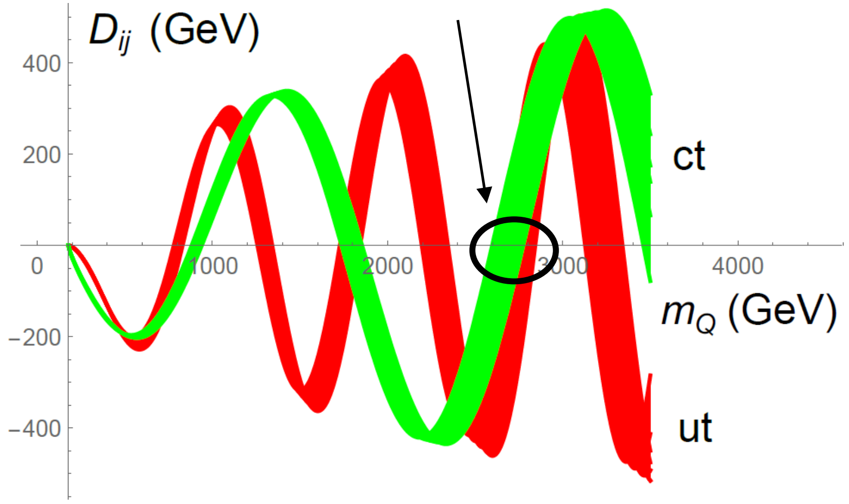
➡ $\alpha_{ut} = 1, \quad \alpha_{ct} = -1/2.$

- Same forms of solutions and coefficients
- Fits to initial conditions give $\bar{\omega}_{ut} = 0.00326 \text{ GeV}^{-1}$ and $\bar{\omega}_{ct} = 0.00176 \text{ GeV}^{-1}$

$m_{b'} = (2.7 \pm 0.1)$ TeV

2nd peak of ct, 3rd peak of ut overlap at mQ ~ 2.7 TeV



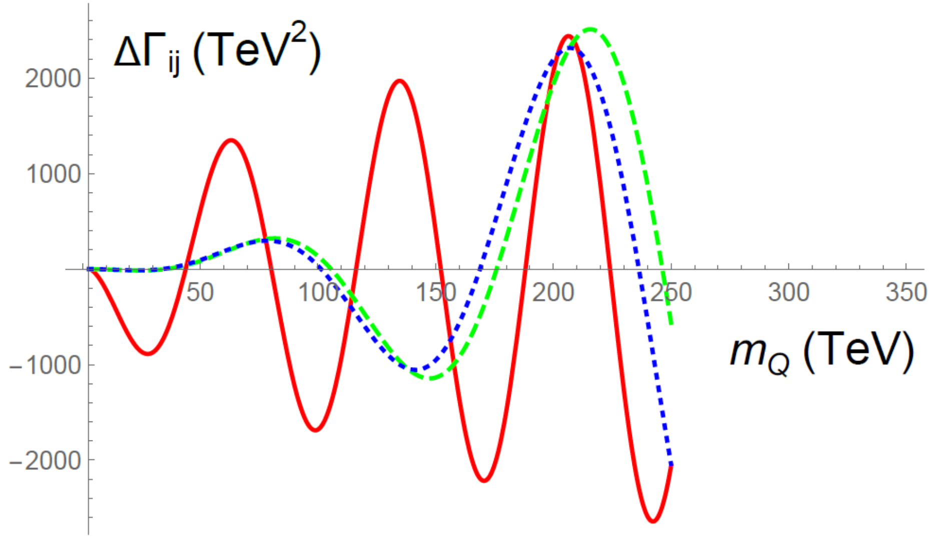


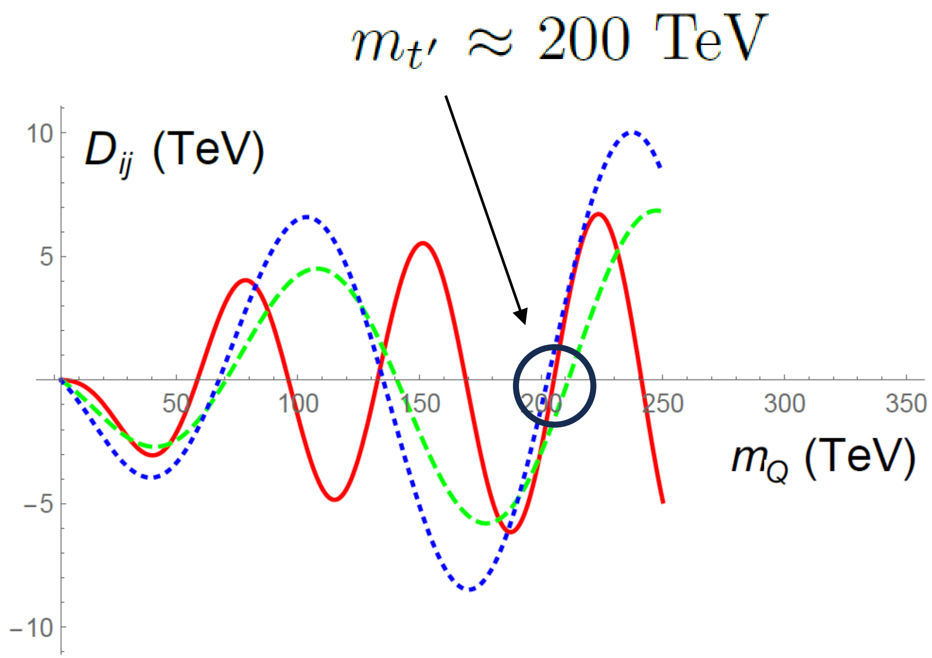
t' mass

- Similar box diagrams with db', sb', bb' channels
- Same analysis

$$m_{db'} = m_{b'} \ (m_d = 0),\ m_{sb'} = m_s + m_{b'} \text{ and } m_{bb'} = m_b + m_{b'}$$
$$M_{db'} = m_\pi + m_{b'},\ M_{sb'} = m_K + m_{b'} \text{ and } M_{bb'} = m_B + m_{b'}$$

- sb', bb' curves close in shape





Heavy quarkonia in Yukawa potential

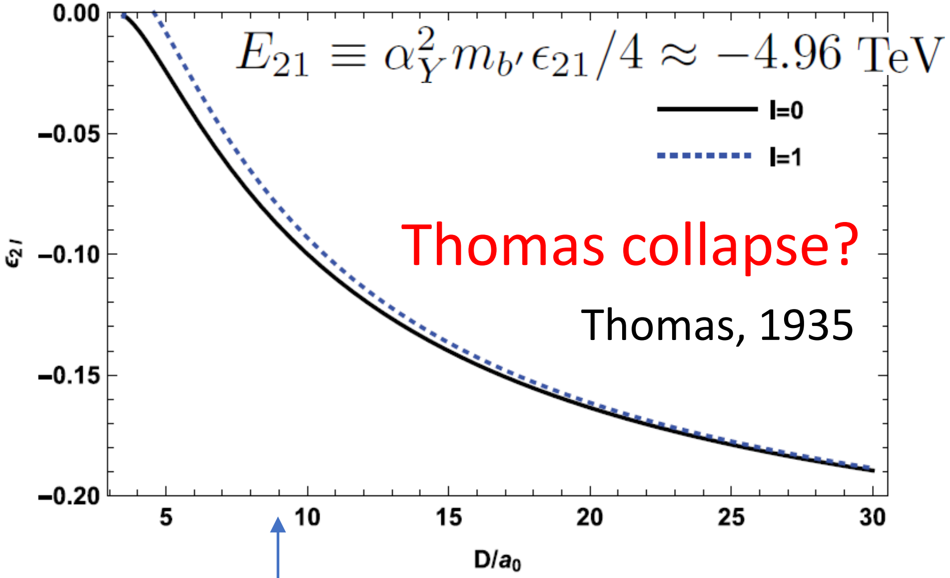
- Yukawa potential $$V(r) = -\alpha_Y \frac{e^{-m_H^* r}}{r}$$ ← b' mass higher than fixed point

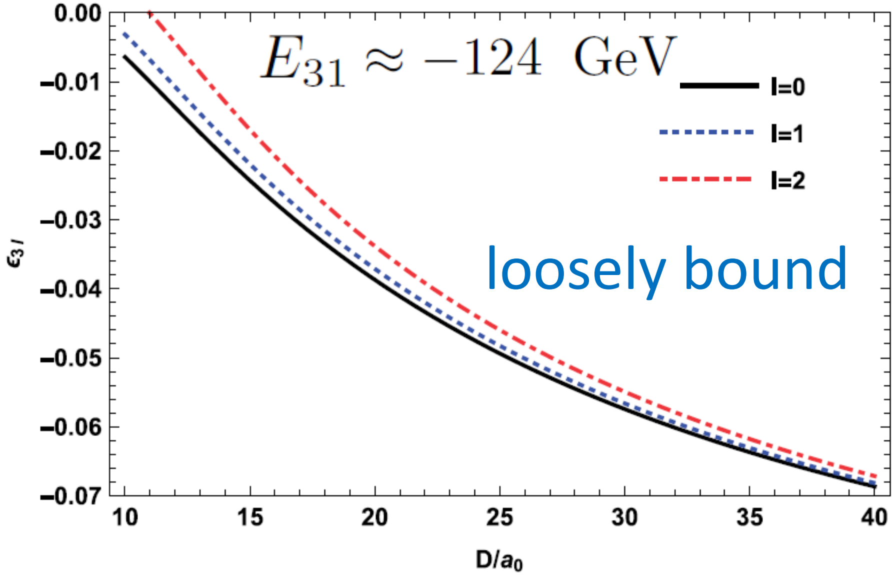
$$\alpha_Y = m_{b'}^2/(4\pi v^2)$$

Napsuciale, Rodriguez 2021

pseudoscalar or vector — P-wave scalars

- Only 5 bound states exist $(n, l) = (1, 0),\ (2, 0),\ (2, 1),\ (3, 0)$ and $(3, 1)$





$1/(m_H^* a_0) = 8.9,\ a_0 \equiv 2/(\alpha_Y m_{b'})$ being Bohr radius

Contribution to Higgs production

Lansberg, Pham 2009

- Width approximated by $S \to gg$ decay width

1st derivative of radial wave function at origin

$$\Gamma_S = 48\alpha_S^2(2m_{b'})\frac{|R'_{21}(0)|}{m_S} \approx \underline{570\ \text{GeV}} \quad > \quad m_S = 2m_{b'} + E_{21} \approx 440\ \text{GeV}$$

call for relativistic calculation

- Imagine fictitious Higgs with $s \approx m_S^2$, matched to fundamental theory

Georgi et al. 1978; Spira et al. 1995

$$\left|\frac{v}{s}\frac{\sqrt{s}^3 g_{ggS}g_{SH}}{s - m_S^2 - i\sqrt{s}\Gamma_S}\right|^2 \approx \left(\frac{v g_{ggS}g_{SH}}{\Gamma_S}\right)^2 \approx \left(\frac{3}{2}\right)^2 \quad \Rightarrow \quad g_{ggS}g_{SH} = (2/3)\Gamma_S/v$$

- Extrapolated to $s = m_H^2$, relative to top-loop contribution in SM

$$\left|\frac{v}{s}\frac{\sqrt{s}^3 g_{ggS}g_{SH}}{s - m_S^2 - i\sqrt{s}\Gamma_S}\right|^2 \approx \left(\frac{2}{3}\frac{m_H\Gamma_S}{m_S^2}\right)^2 \approx 6.2\%$$

down by m_S^{-4}

Contribution to Higgs production

- Contribution of $(n, l) = (3, 1)$ $\left(\frac{2}{3}\frac{m_H \Gamma_S}{m_S^2}\right)^2 \approx 4.3 \times 10^{-6}$

$$\Gamma_S = 48\alpha_S^2(2m_{b'})\frac{|R'_{31}(0)|}{m_S} \approx \underline{694 \ \text{GeV}} \qquad m_S = 2m_{b'} + E_{31} \approx 5.28 \ \text{TeV}$$

- Relativistic calculation---solving Dirac (not Schrodinger) equation
- Crude approximation, spectrum degenerate in l

Ikhdair, 2012

- Ground state mass 3.23 TeV, n=2 mass 4.45 TeV, n=3 mass < 5.4 TeV
- n=3 state indeed loosely bound
- n=2 state contributes at 10E-3 level, assuming width insensitive to bound state masses
- Conclusion: new scalar contribution to Higgs production negligible