Lattice calculation of the intrinsic soft function and the Collins-Soper kernel

M. Chu et, al JHEP 08 (2023) 172

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(Lattice Parton Collaboration)

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第二十届重味物理和CP破坏研讨会
(HFCPV-2023)

Lattice Parton Collaboration

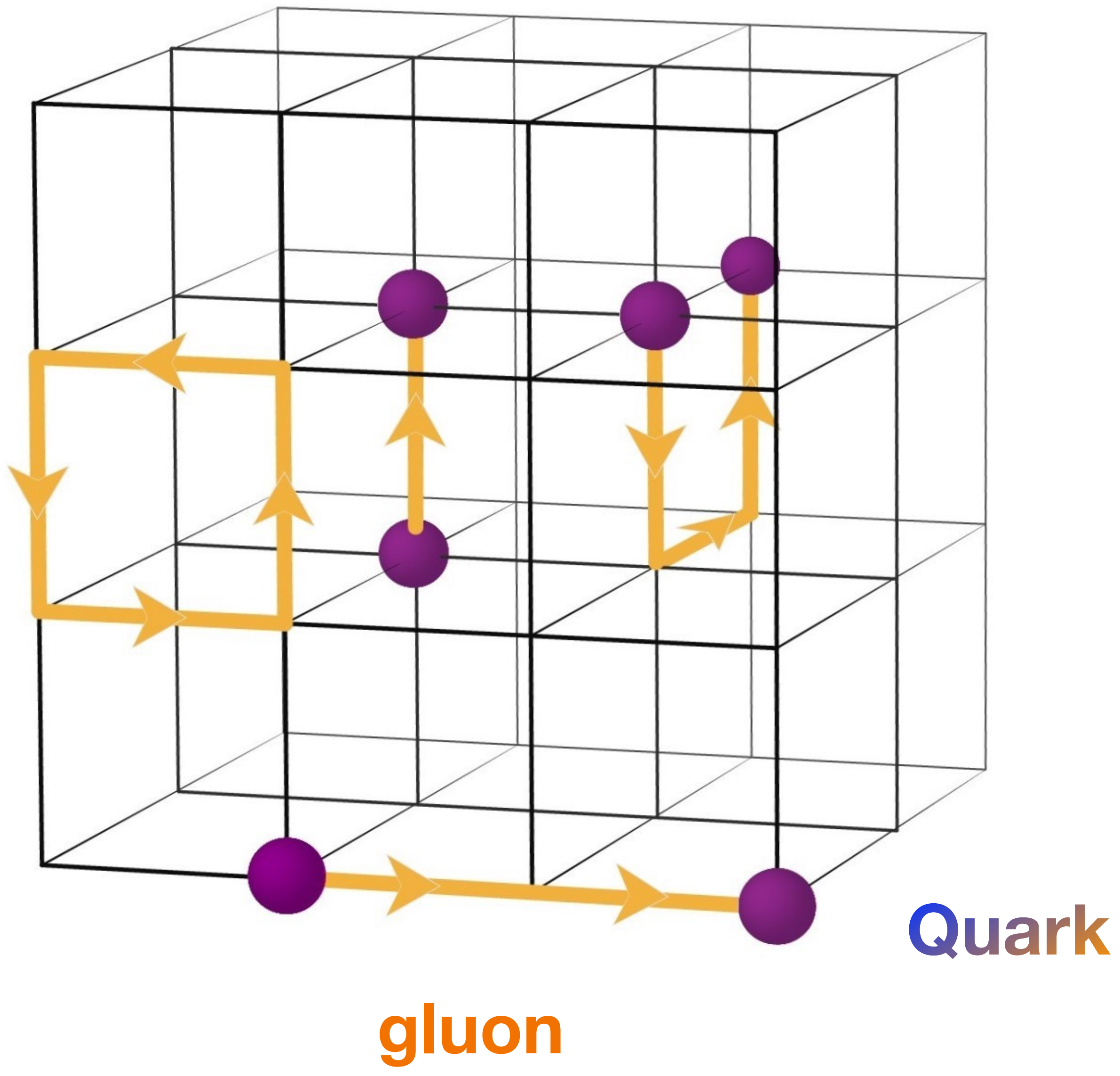
Outline

- Motivation:
  - Lattice QCD; LaMET; TMD physics; Soft function
- Theoretical framework:
  - CS kernel; Intrinsic soft function
- Numerical results:
  - quasi-TMDWF; Intrinsic soft function; CS kernel; TMDs
- Summary and outlook

Motivation: Lattice QCD

From Minkowski to Euclidean

Lattice QCD is a numerical method used to simulate Quantum Chromodynamics on a discrete space-time lattice. We can construct quark and gluon fields on Euclidean space.



Wick rotation: $t_M \to i\tau_E, \quad id^4x_M \to -d^4x_E$

Path integral

Minkowski space

$$\langle \hat{O}(\psi, \bar{\psi}, A) \rangle = \frac{\int [D\psi][D\bar{\psi}][DA]exp\{-\int d^4xL\}\hat{O}(\psi, A)}{\int [D\psi][D\bar{\psi}][DA]exp\{-\int d^4xL\}}$$

↓ Haar measure

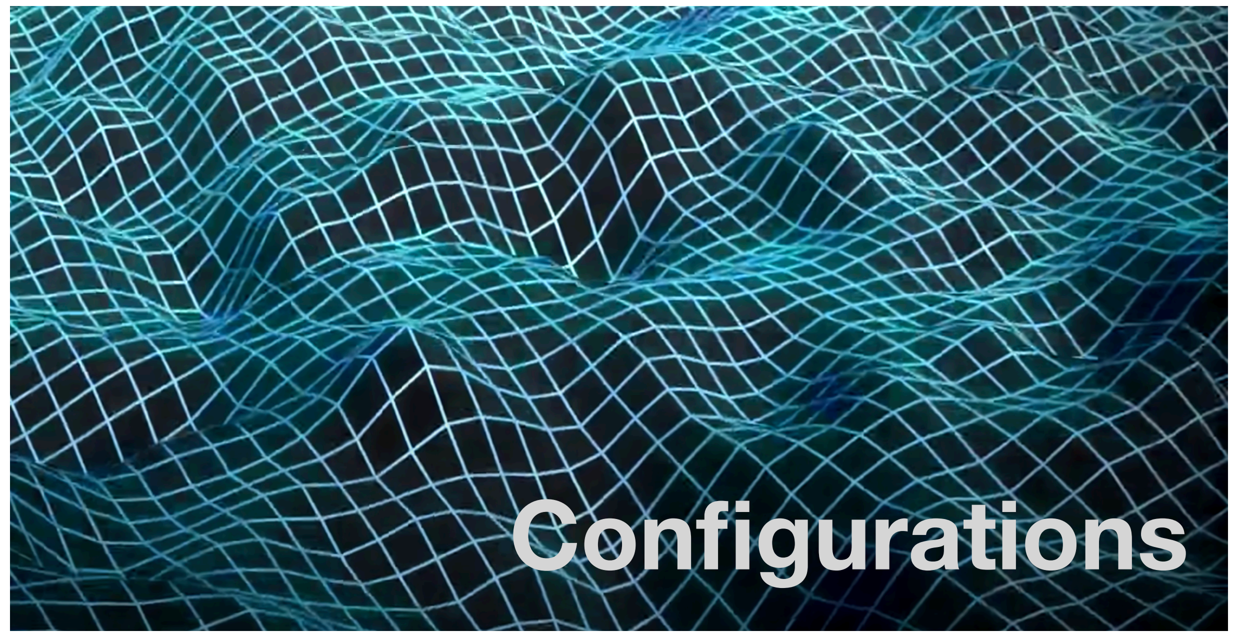
$$\langle \hat{O}(U) \rangle = \frac{\int [DU]exp\{-\int d^4xL_G\}\hat{O}(U)\det[D_{\text{QCD}}]}{\int [DU]exp\{-\int d^4xL_G\}\det[D_{\text{QCD}}]}$$

Euclidean space

discretization ↓

Probability density: $f(U) = e^{-\int [DU]L_G}\det[D_{\text{QCD}}]$

Monte Carlo with Markov chain



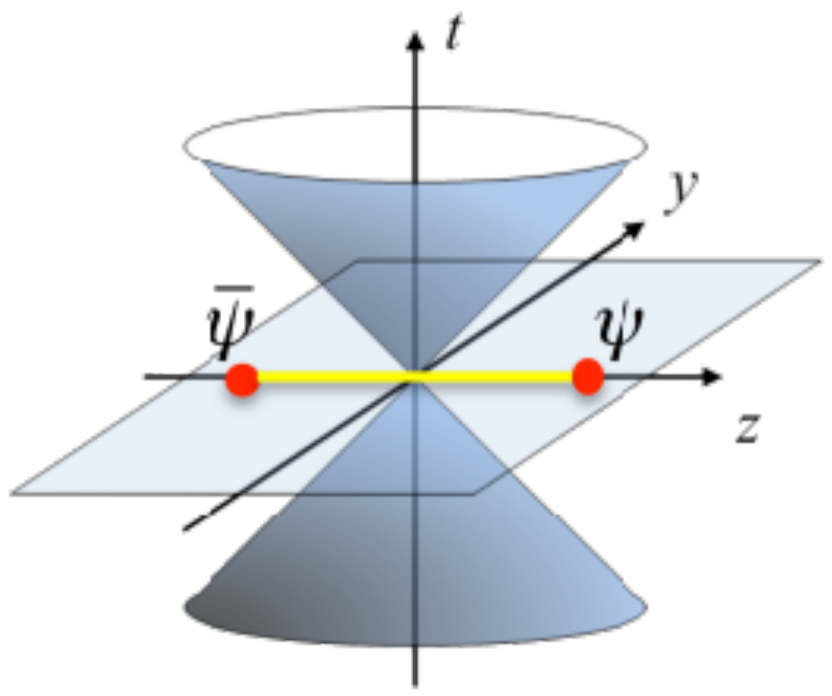
Motivation: Large Momentum Effective Theory

Time independent correlations

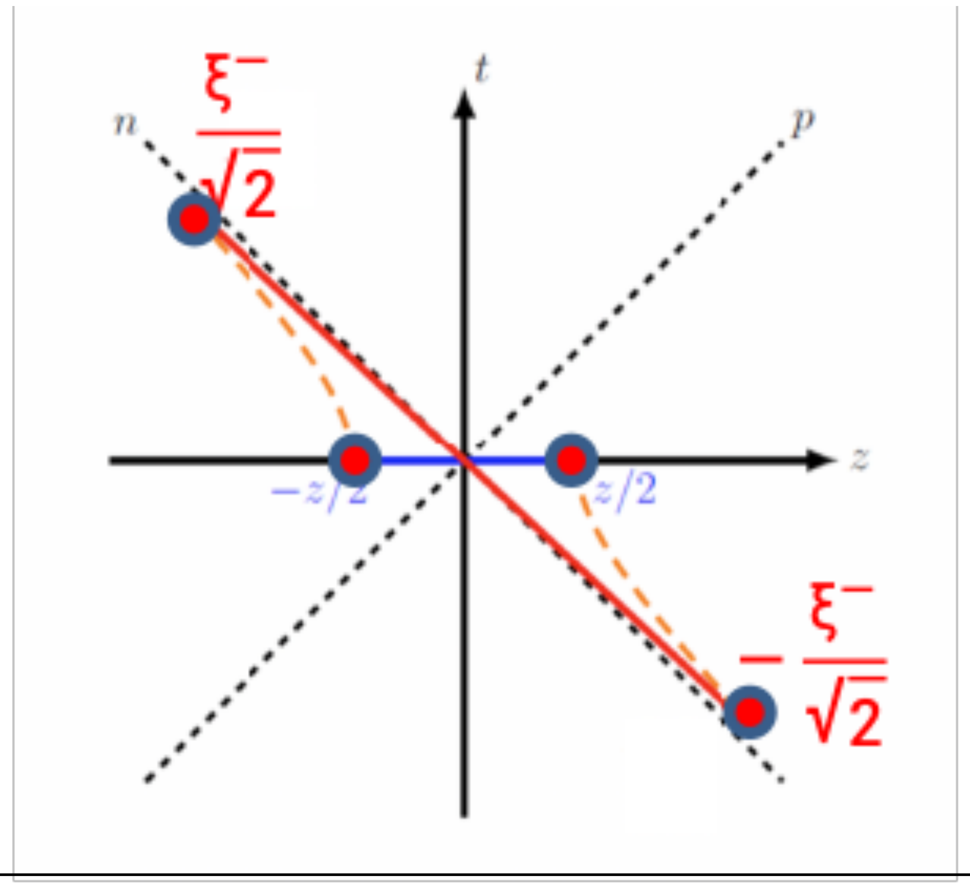
$$\tilde{\phi}(x) = \int P_z dz e^{-ixP_z z} \langle 0 | \bar{\psi}(\frac{z}{2}) \Gamma U(\frac{z}{2}, -\frac{z}{2}) \psi(-\frac{z}{2}) | \pi(P^z) \rangle$$

Time dependent correlations

$$\phi(x) = \int P^+ d\xi^+ e^{-ixP^+\xi^+} \langle 0 | \bar{\psi}(\frac{\xi^+}{\sqrt{2}}) \Gamma U(\frac{\xi^+}{\sqrt{2}}, -\frac{\xi^+}{\sqrt{2}}) \psi(-\frac{\xi^+}{\sqrt{2}}) | \pi(P^+) \rangle$$

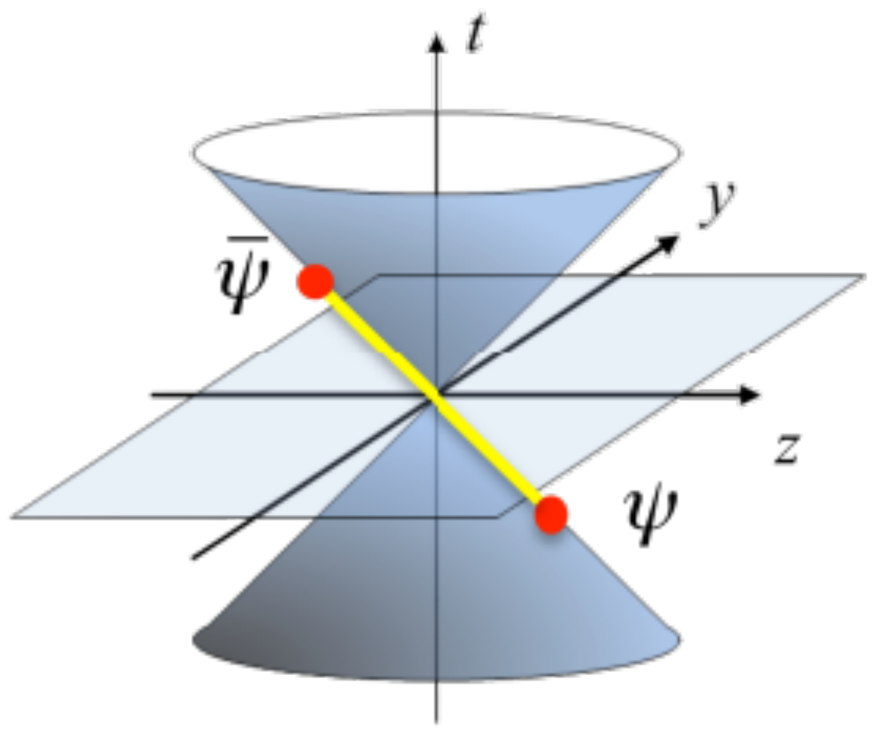


Lorentz boost →



X. Ji, Phys.Rev.Lett. 110 (2013) 262002

EFT →



There is same IR divergence for $\phi(x)$ and $\tilde{\phi}(x)$, and match them by perturbative kernel from free quark state.

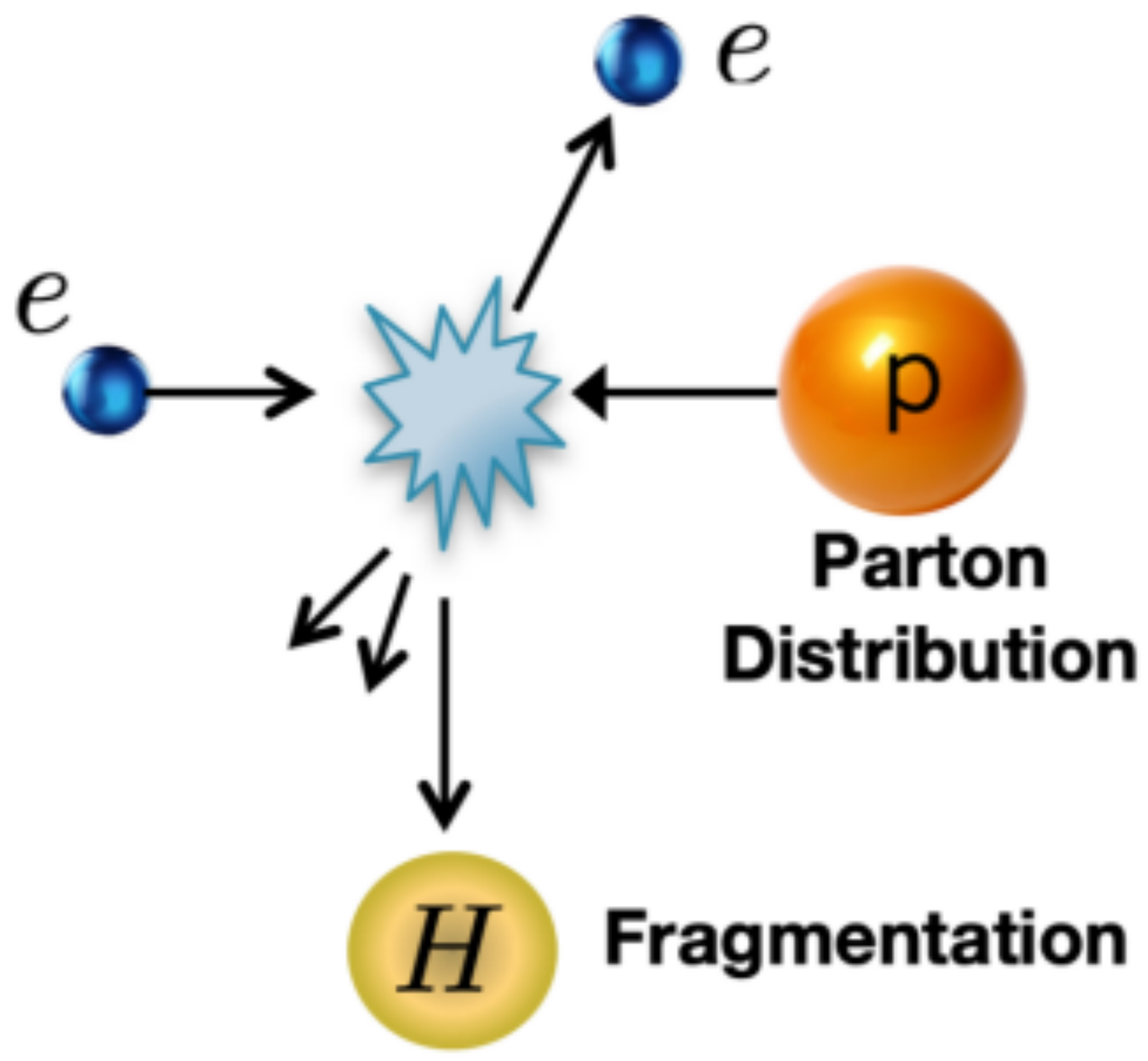
Not much clear in theory: factorization; renormalizations; power corrections……

Motivation: TMD physics

TMD process

TMDs are important inputs!

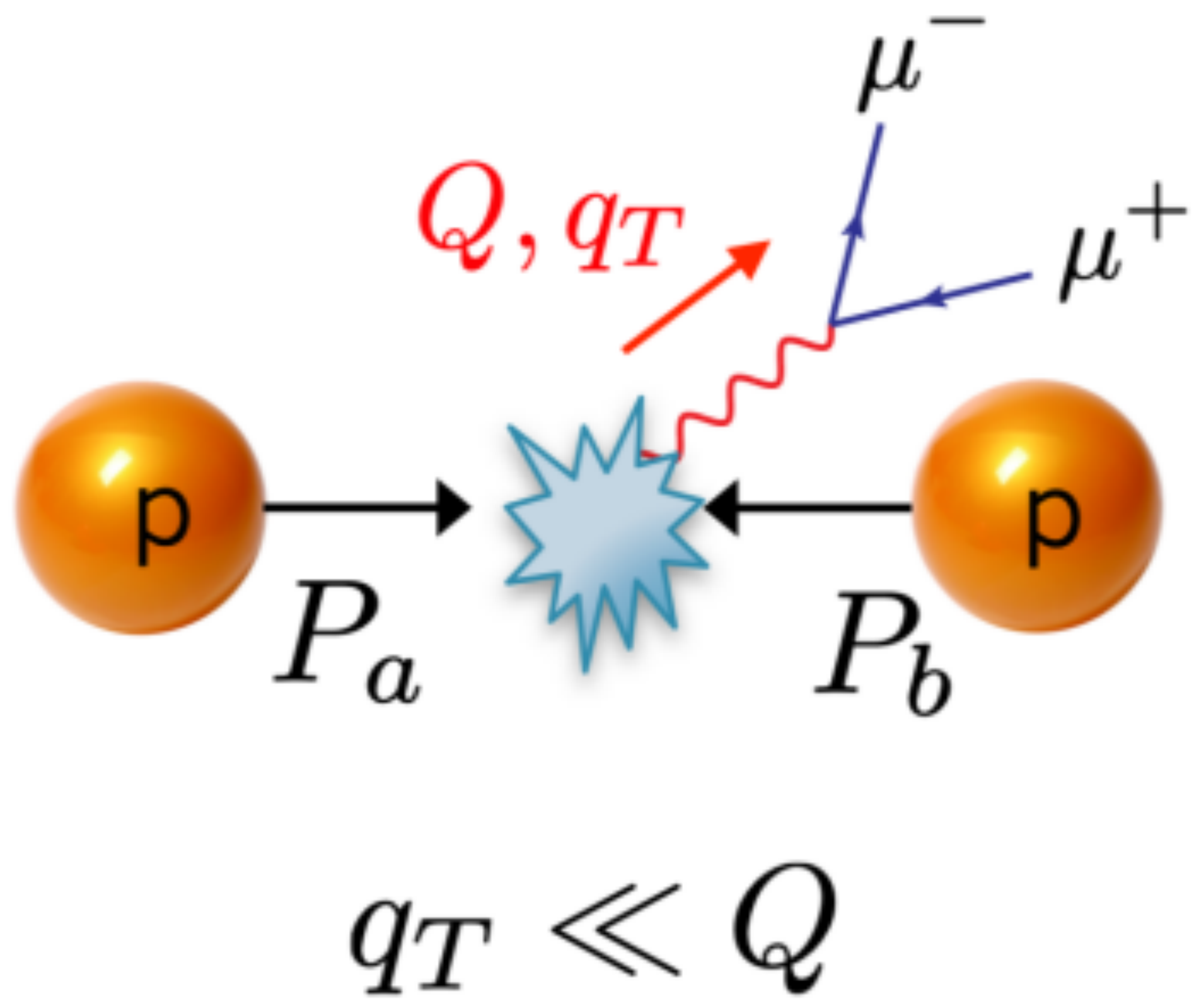
Semi-Inclusive DIS



LHC, FermiLab, RHIC, …

$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$

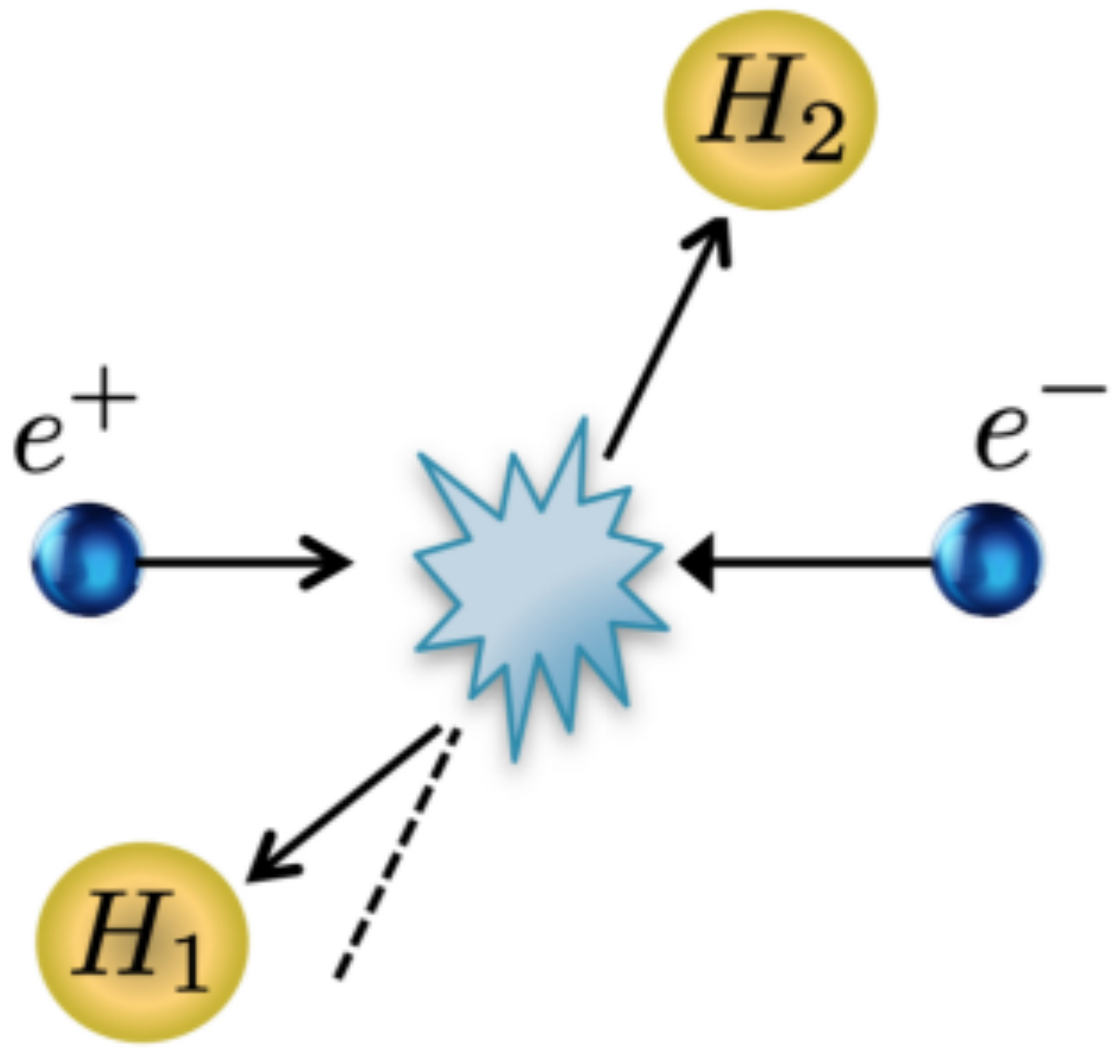
Drell-Yan



HERMES, COMPASS, JLab, EIC, …

$\sigma \sim f_{q/P}(x, k_T) D_{h/P}(x, k_T)$

Dihadron in e+e-

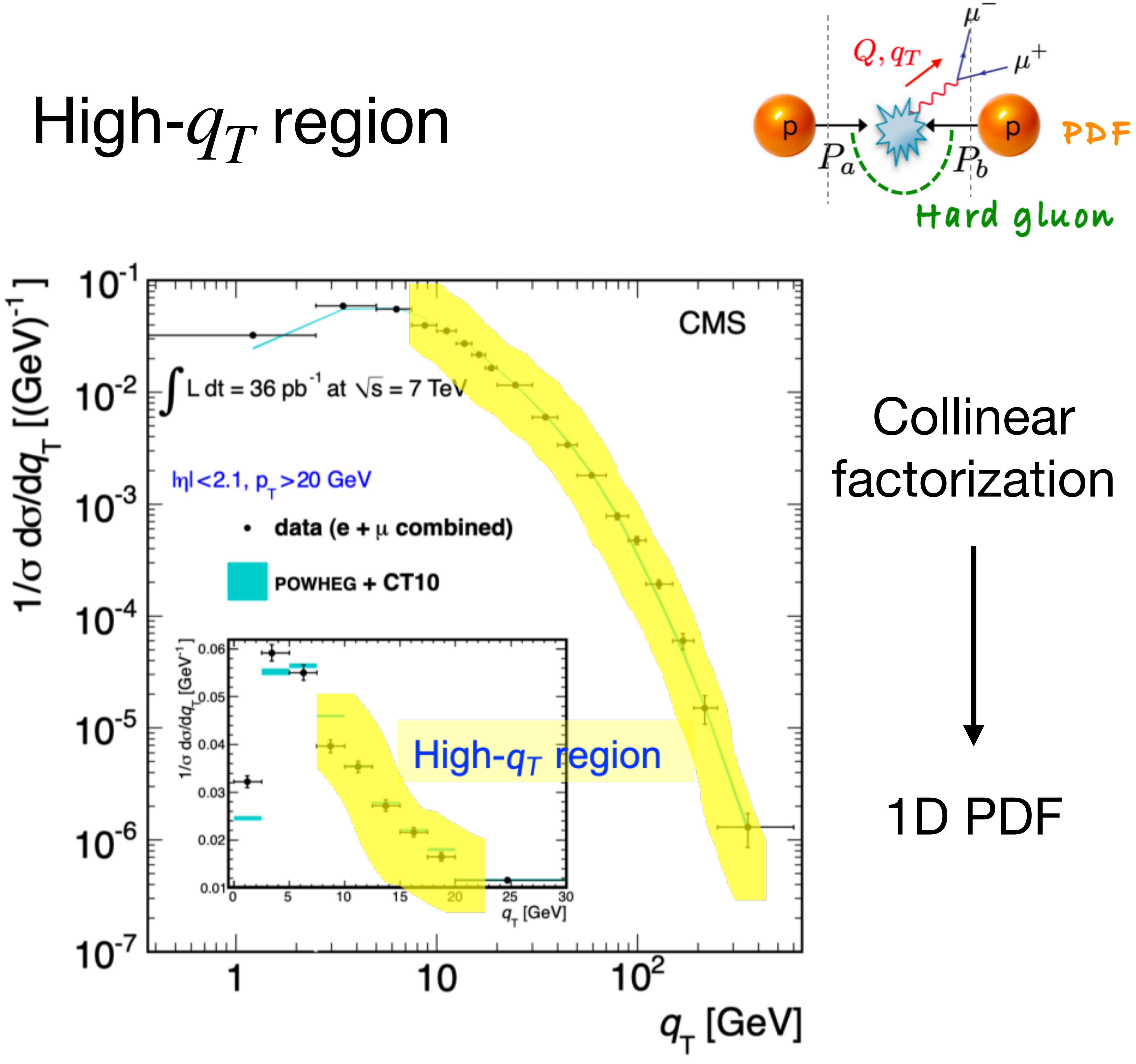


BESIII, Babar, Belle, …

$\sigma \sim D_{h_1/P}(x, k_T) D_{h_2/P}(x, k_T)$

Motivation: TMD physics

High-q_T region



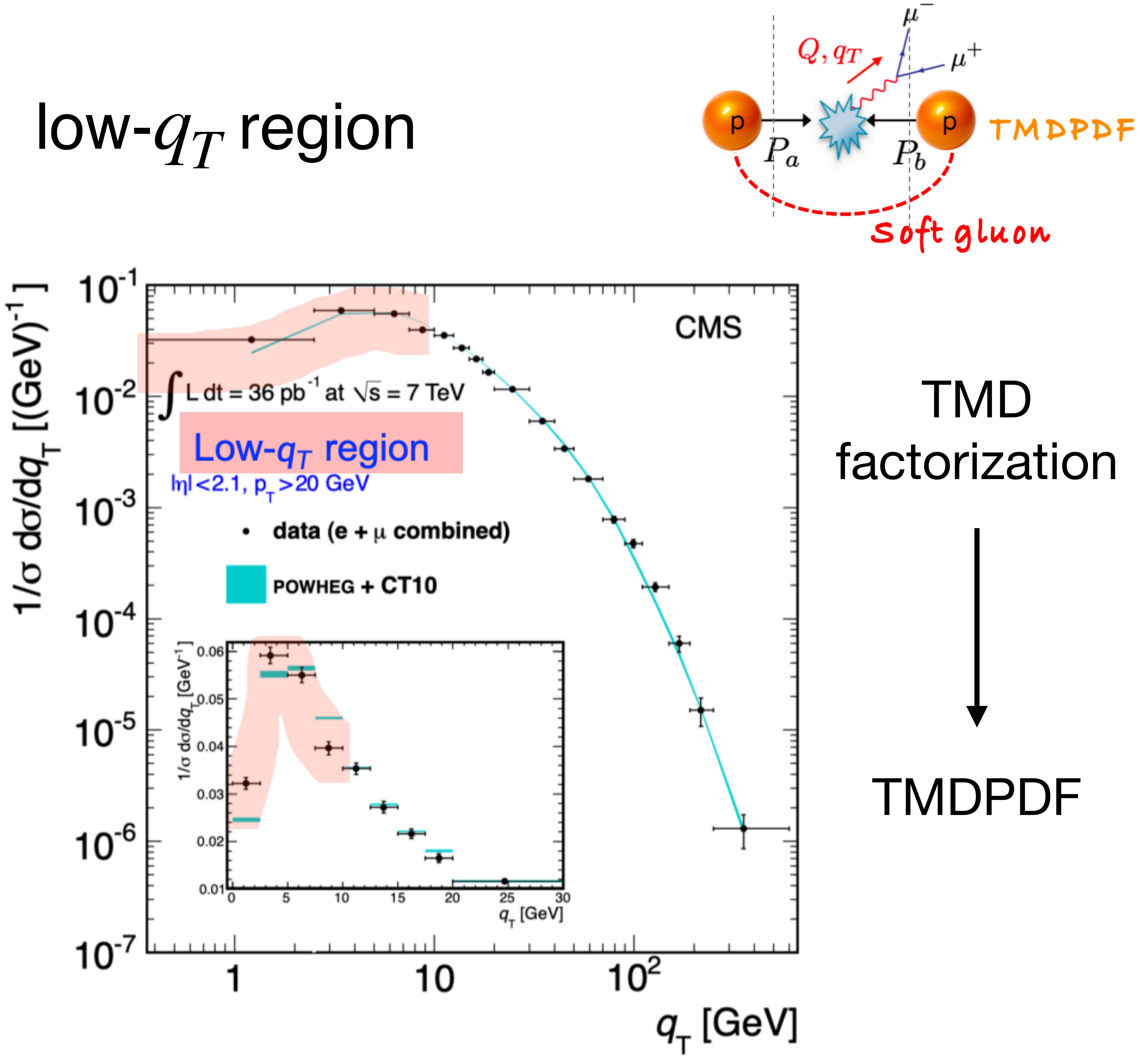
Z-production q_T spectrum at LHC

Collinear factorization

↓

1D PDF

low-q_T region



Z-production q_T spectrum at LHC

TMD factorization

↓

TMDPDF

- Theoretical analysis

TMD factorization, evolution and resummation

1. Boussarie et al. TMD handbook, arxiv: 2304.03302

2. Collins, Foundations of perturbative QCD, Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 32 (2011) 1-624

…

- Phenomenological results

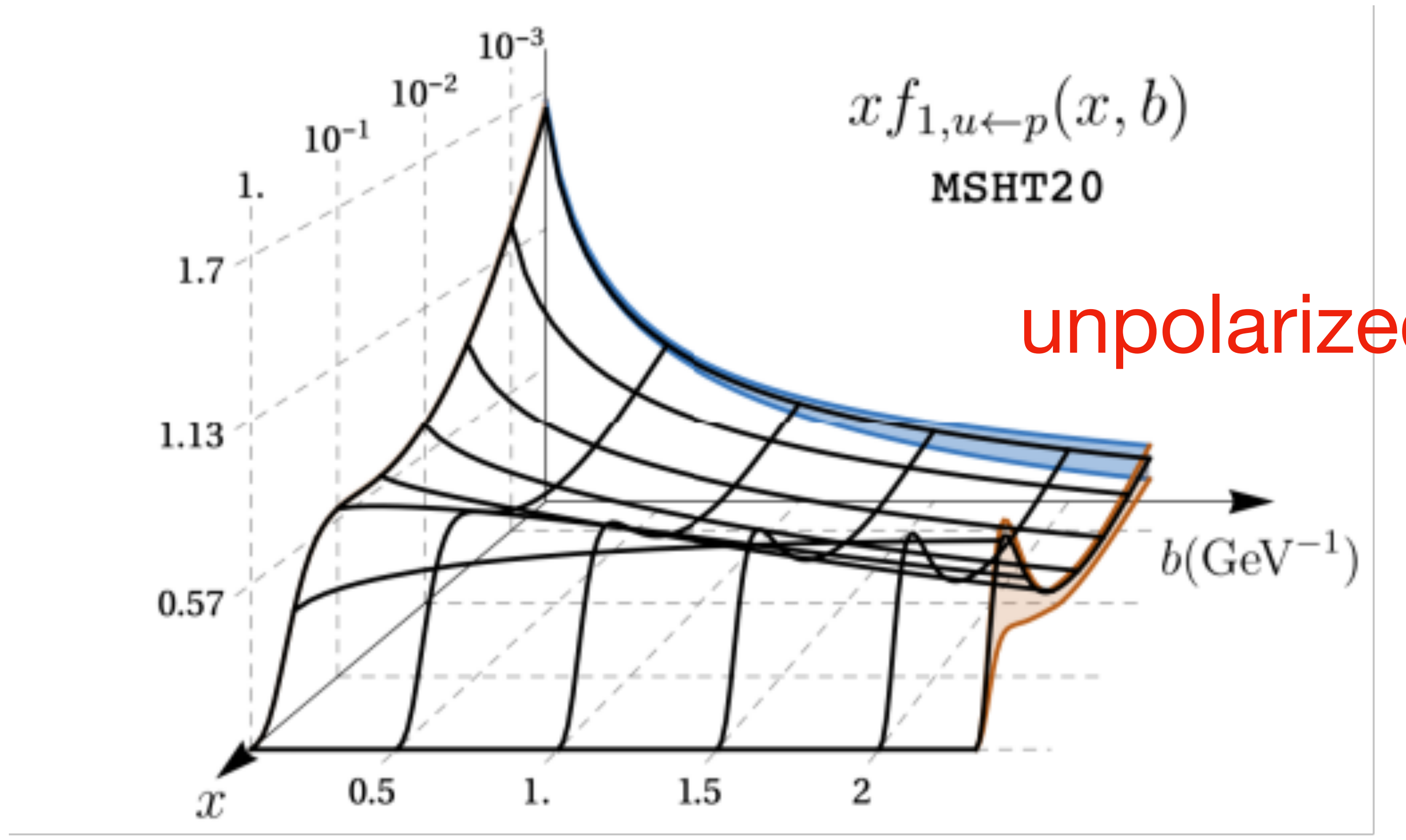
1. Bacchetta et al., JHEP 10 (2022) 127

2. Bury et al., JHEP 10 (2022) 118

3. Scimemi et al., JHEP 06 (2020) 137

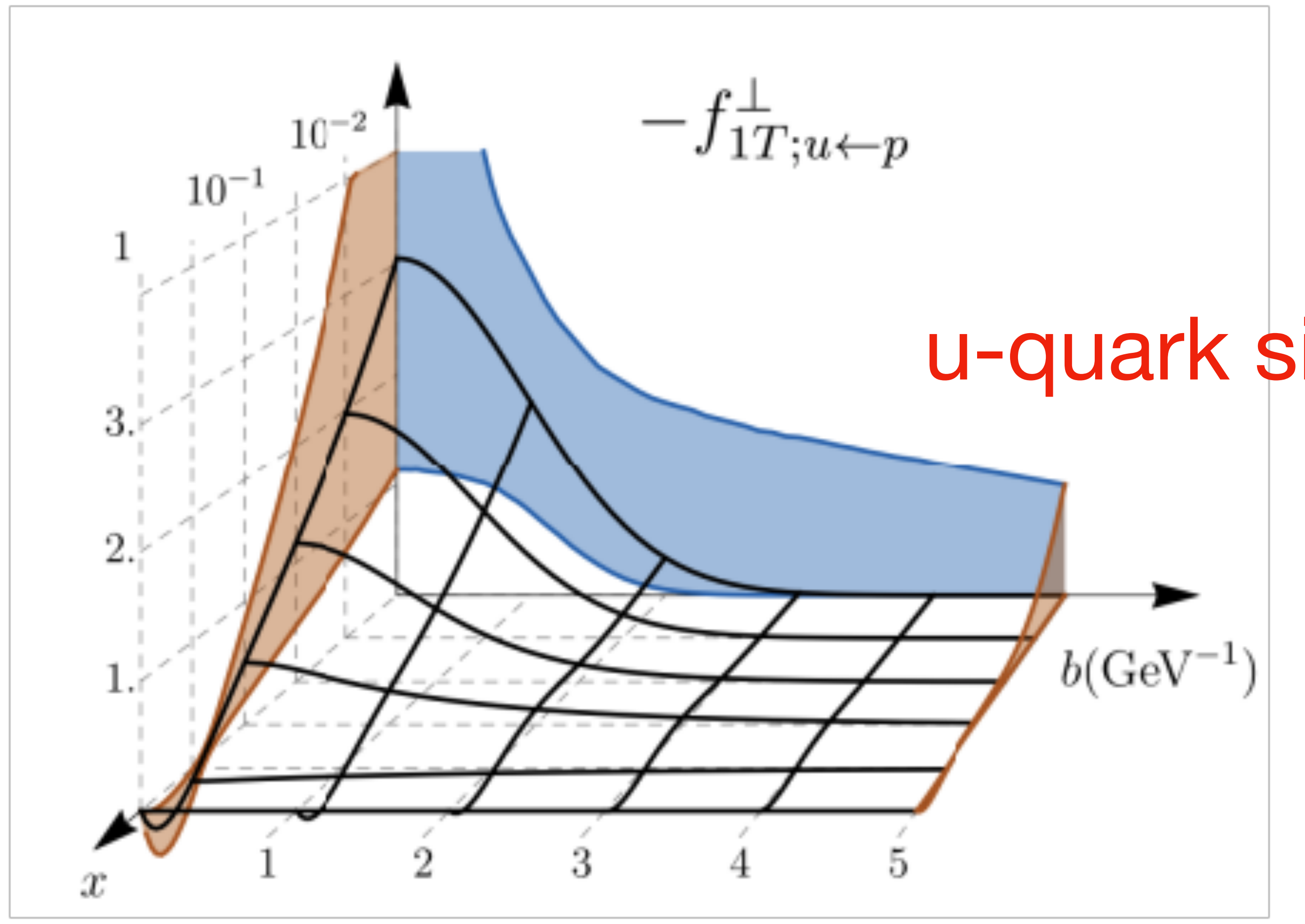
4. Bacchetta et al., JHEP 06 (2019) 051

…



unpolarized TMDPDF

M. Bury et al., JHEP 10 (2022) 118



u-quark sivers function

M. Bury et al., Phys.Rev.Lett. 126 (2021)

Motivation: Soft function

- TMD factorization(both for TMDPDF/WF)

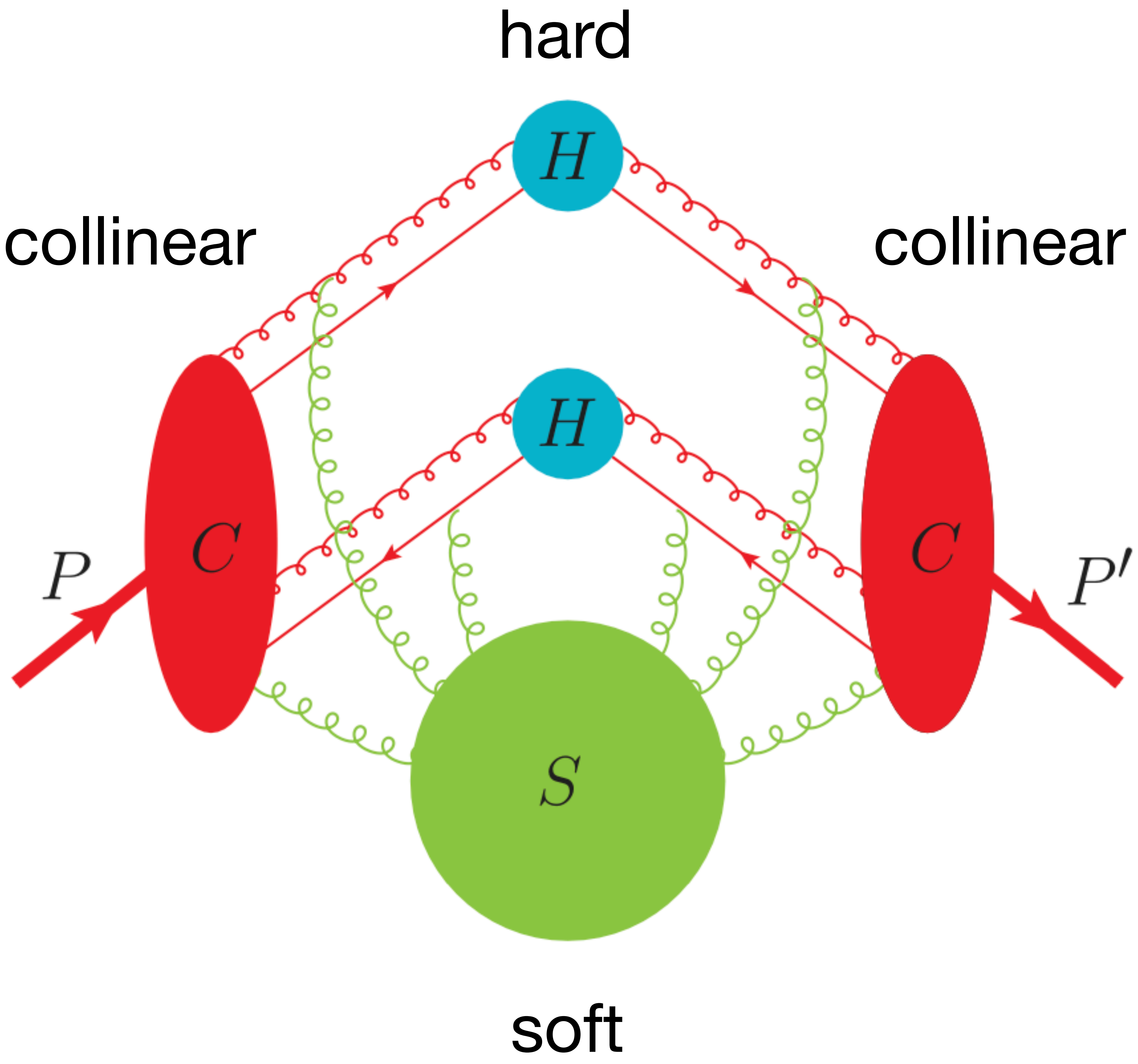
$$\tilde{f}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right) S_{I}^{\frac{1}{2}}\left(b_{\perp}, \mu\right)$$
$$= H^{\pm}\left(x, \zeta^{z}, \mu\right) e^{\left[\frac{1}{2}K\left(b_{\perp}, \mu\right) \ln \frac{\mp\zeta^{z} + i\epsilon}{\zeta}\right]} f^{\pm}\left(x, b_{\perp}, \mu, \zeta\right)$$

- Soft gluon effects

rapidity dependent part $e^{\frac{1}{2}K(b_{\perp},\mu)\frac{\mp\zeta^{z}+i\epsilon}{\zeta}}$

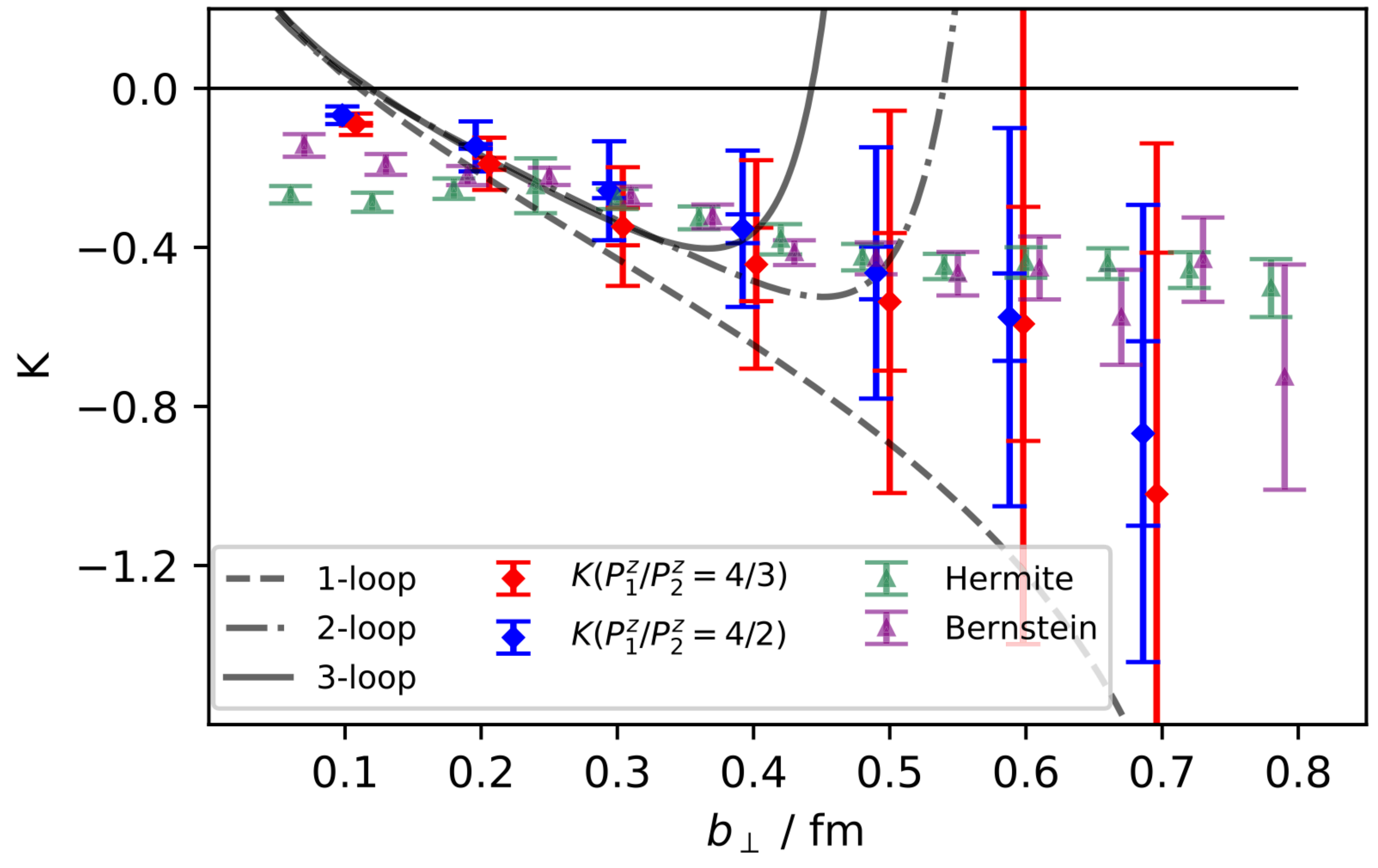
rapidity independent part $S_{\rm I}^{\frac{1}{2}}(b_{\perp}, \mu)$

Intrinsic soft function



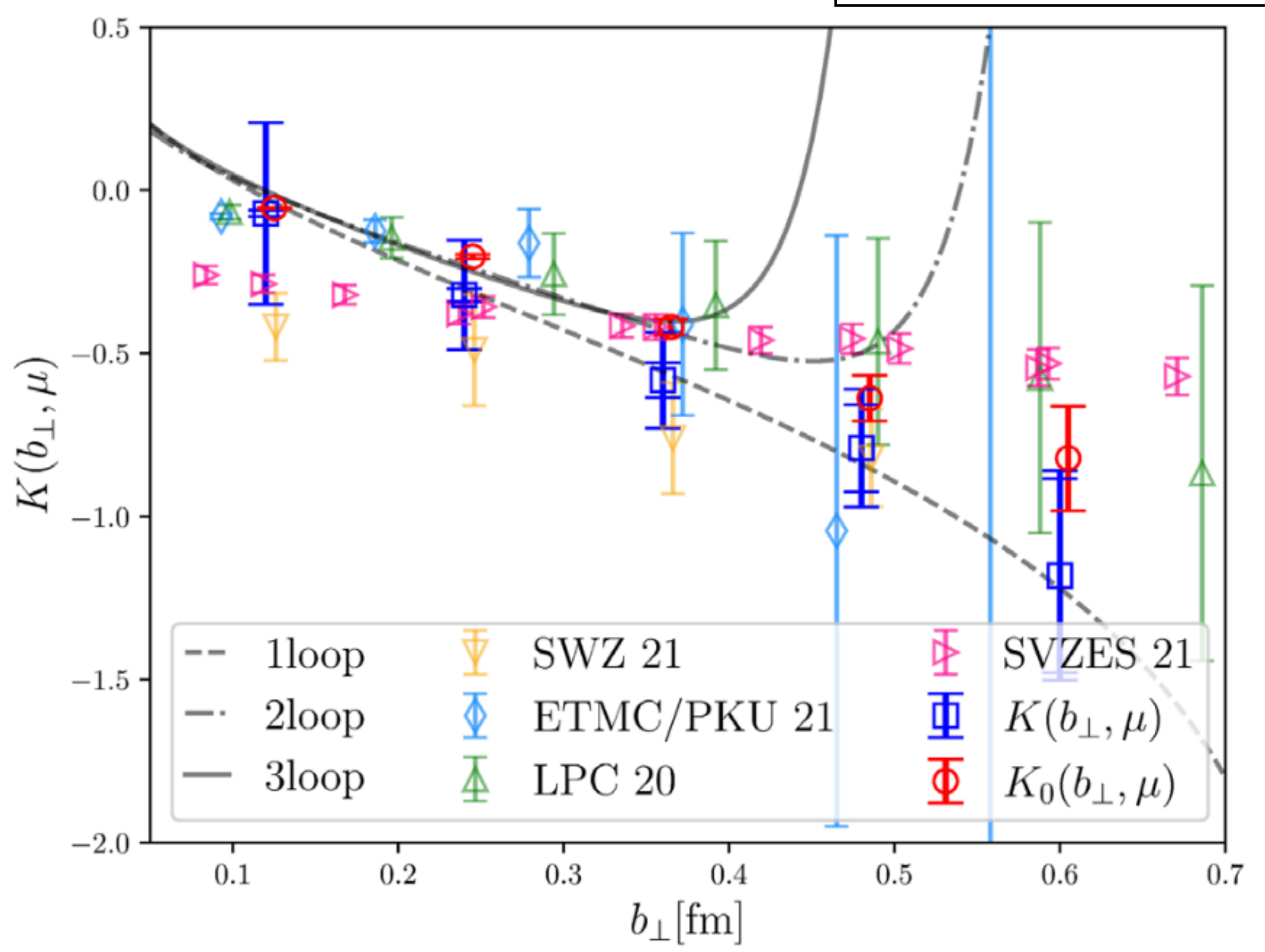
$K(b_\perp, \mu)$ with **tree level matching kernel**

Q. Zhang et al, Phys.Rev.Lett. 125 (2020)



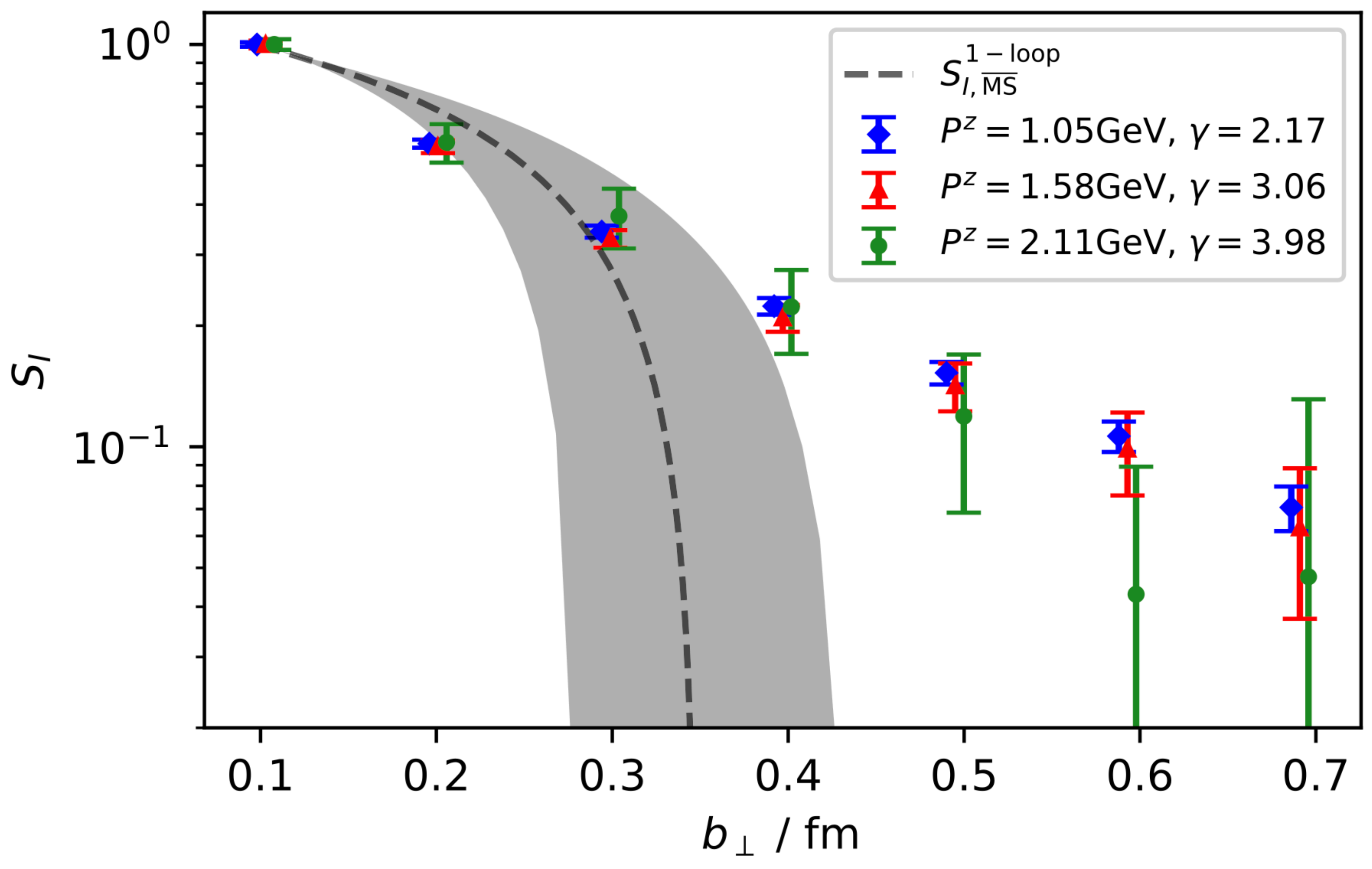
$K(b_\perp, \mu)$ with **one loop matching kernel**

M. Chu et al, Phys.Rev.D 106 (2022)

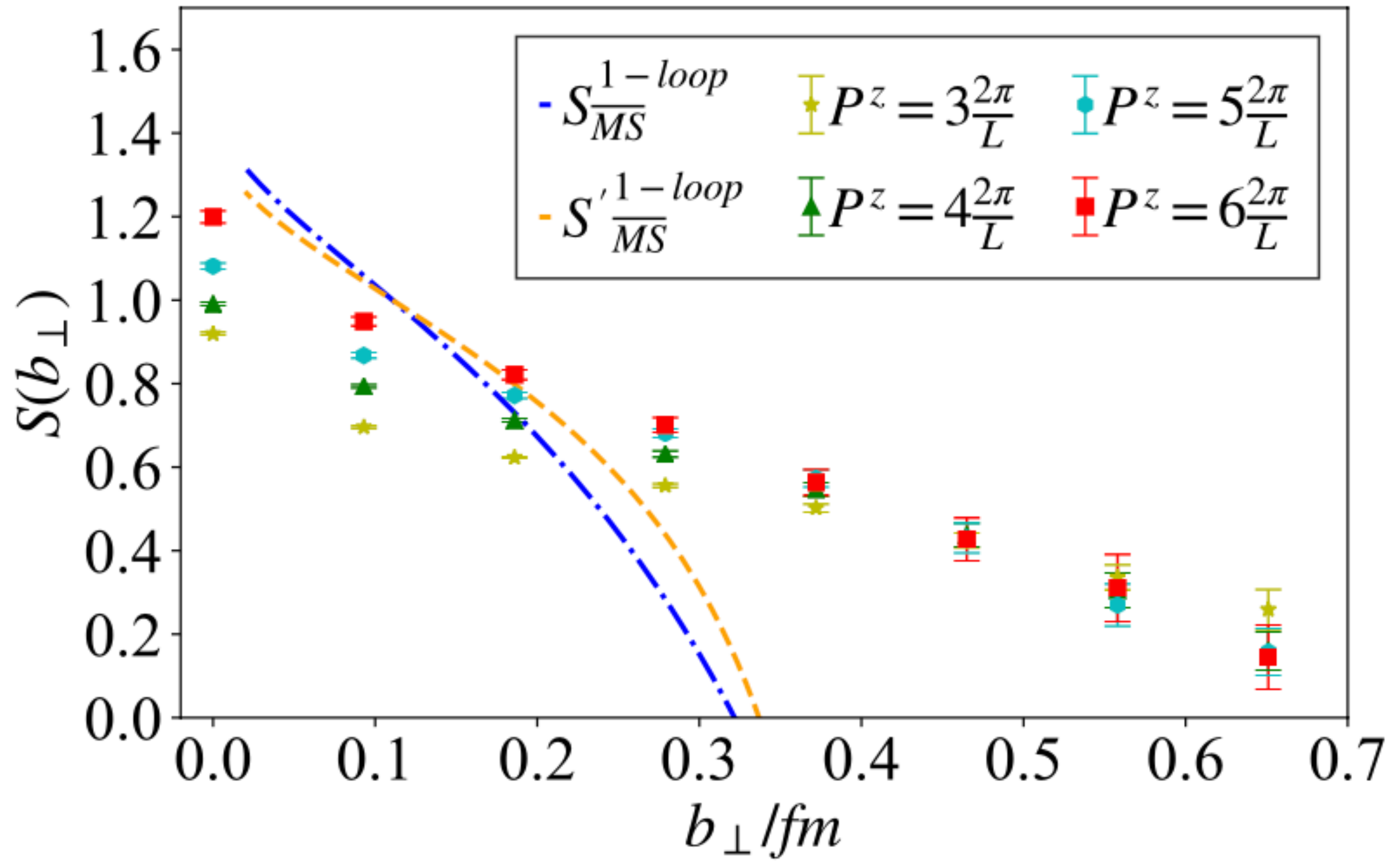


$S_I(b_\perp, \mu)$ with **tree level matching kernel**

Q. Zhang et al, Phys.Rev.Lett. 125 (2020)



Y. Li et al, Phys.Rev.Lett. 128 (2022)



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Physical meaning

- TMDs as important inputs

- RG evolution (μ)

$$\mu\frac{d}{d\mu}f_{q/P}^{TMD}(x,b_\perp,\mu,\zeta)=\gamma(\mu,\zeta)$$

- Rapidity evolution (ζ)

$$2\zeta\frac{d}{d\zeta}\ln f_{q/P}^{TMD}(x,b_\perp,\mu,\zeta)=K(b_\perp,\mu)$$

LaMET approach

- TMD factorization

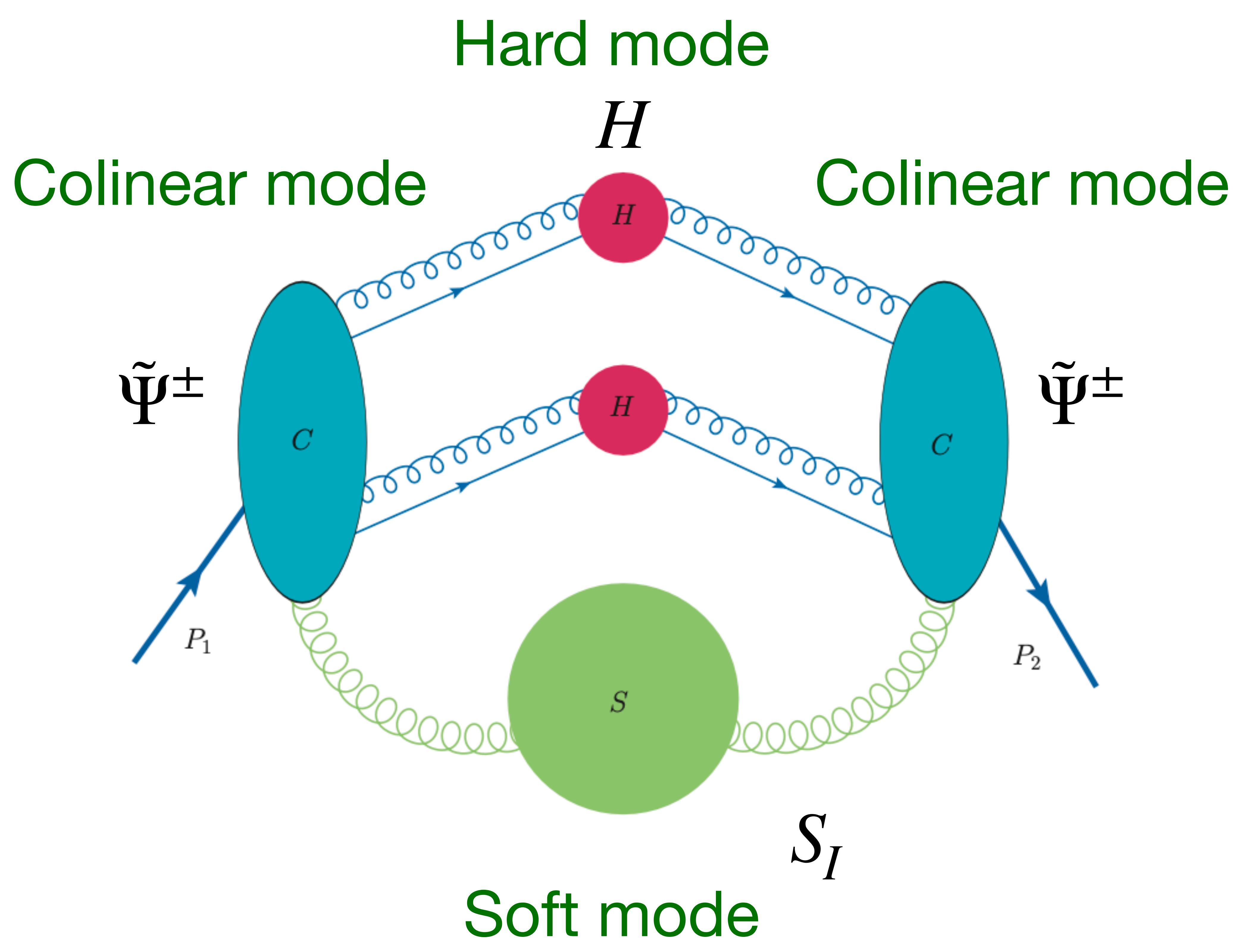
$$\tilde{\Psi}^{\pm}\left(x,b_\perp,\mu,\zeta^z\right)S_I^{\frac{1}{2}}\left(b_\perp,\mu\right)$$
$$=H^{\pm}\left(x,\zeta^z,\mu\right)e^{\left[\frac{1}{2}K\left(b_\perp,\mu\right)\ln\frac{\mp\zeta^z+i\epsilon}{\zeta}\right]}\Psi^{\pm}\left(x,b_\perp,\mu,\zeta\right)$$

Choose $\zeta^z=\zeta_1^z$, and ζ_2^z then divide $\tilde{f}^{\pm}(\zeta_1^z),\tilde{f}^{\pm}(\zeta_2^z)$

- CS kernel

$$K(b_\perp,\mu)=\frac{1}{\ln(P_2^z/P_1^z)}\ln\left[\frac{H^{\pm}(\zeta_1^z,\bar{\zeta}_1^z)\tilde{\Psi}^{\pm}(b_\perp,x,\zeta_2^z)}{H^{\pm}(\zeta_2^z,\bar{\zeta}_2^z)\tilde{\Psi}^{\pm}(b_\perp,x,\zeta_1^z)}\right]$$

Factorization



the leading order reduced diagram

Factorization of form factor

$$F\left(b_\perp, P_1, P_2, \Gamma, \mu\right) = \int dx_1 dx_2 H\left(x_1, x_2, \Gamma\right) S_I\left(b_\perp, \mu\right) \times \tilde{\Psi}^{\pm *}\left(x_2, b_\perp, \zeta^z\right) \tilde{\Psi}^{\pm}\left(x_1, b_\perp, \zeta^z\right)$$

Four quark form factor

$$F\left(b_\perp, P_1, P_2, \Gamma, \mu\right) = \left\langle P_2 \left| \bar{q}(b_\perp)\Gamma q(b_\perp)\bar{q}(0)\Gamma' q(0) \right| P_1 \right\rangle$$

Z.F. Deng et al., JHEP 09 (2022) 046

$F\left(b_\perp, P, \Gamma, \mu\right)$: four quark form factor, nonperturbative.

$H\left(x_1, x_2, \Gamma\right)$: Perturbative matching coefficient.

$\tilde{\Psi}^{\pm}(x, b_\perp, \zeta)$: quasi-TMDWF, nonperturbative.

Normalization

Z.F. Deng et al., JHEP 09 (2022) 046

$$F(b_\perp,\Gamma,P^z)=\frac{\langle\pi(P_2)|(\bar{q}\Gamma q)|_{b_\perp}(\bar{q}\Gamma q)|_0|\pi(P_1)\rangle}{f_\pi^2 P_1\cdot P_2}$$

$$\langle 0|(\bar{q}\gamma^\mu\gamma_5 q)|_0|P_1\rangle=-if_\pi P_1^\mu,\ \langle P_2|(\bar{q}\gamma_\mu\gamma_5 q)|_0|0\rangle=if_\pi P_{2\mu}$$

$$P_2=(P^z,0,0,-P^z),\ \ P_1=(P^z,0,0,P^z)$$

for the denominator

$$\begin{aligned}&\langle\pi(P_2)|(\bar{q}\gamma^\mu\gamma_5 q)|_0|0\rangle\langle 0|(\bar{q}\gamma_\mu\gamma_5 q)|_0|\pi(P_1)\rangle\\&=f_\pi^2(P_1\cdot P_2)=2f_\pi^2(P^z)^2\\&=2\langle\pi(P_1)|(\bar{q}\gamma^t\gamma_5 q)|_0|0\rangle\langle 0|(\bar{q}\gamma^t\gamma_5 q)|_0|\pi(P_1)\rangle\\&=2\langle 0|(\bar{q}\gamma^t\gamma_5 q)|_0|\pi(P_1)\rangle\langle\pi(P_1)|(\bar{q}\gamma^t\gamma_5 q)|_0|0\rangle\end{aligned}$$

Lattice simulation

3pt

$$F(b_\perp,P^z)=\frac{\langle\pi(P_2)|(\bar{q}\Gamma q)|_{b_\perp}(\bar{q}\Gamma q)|_0|\pi(P_1)\rangle}{2\langle 0|(\bar{q}\gamma^t\gamma_5 q)|_0|\pi(P_1)\rangle\langle\pi(P_2)|(\bar{q}\gamma^t\gamma_5 q)|_0|0\rangle}$$

local 2pt　　local 2pt

Which Γ or combinations of Γ should be used?

Fierz transformation

$$F(\Gamma = I) - F(\Gamma = \gamma_5)$$
$$= (\bar{\psi}_a \psi_b)(\bar{\psi}_c \psi_d) - (\bar{\psi}_a \gamma_5 \psi_b)(\bar{\psi}_c \gamma_5 \psi_d)$$
$$= \frac{1}{2}\bar{\psi}_c \gamma^\mu \gamma_5 \psi_b \bar{\psi}_a \gamma_\mu \gamma_5 \psi_d - \frac{1}{2}\bar{\psi}_c \gamma^\mu \psi_b \bar{\psi}_a \gamma_\mu \psi_d$$

$$\sum F(\Gamma = \gamma^\mu) + F(\Gamma = \gamma^\mu \gamma_5)$$
$$= (\bar{\psi}_a \gamma^{x,y} \psi_b)(\bar{\psi}_c \gamma_{x,y} \psi_d) + (\bar{\psi}_a \gamma^{x,y} \gamma_5 \psi_b)(\bar{\psi}_c \gamma_{x,y} \gamma_5 \psi_d)$$
$$= \bar{\psi}_c \gamma^\mu \gamma_5 \psi_b \bar{\psi}_a \gamma_\mu \gamma_5 \psi_d + \bar{\psi}_c \gamma^\mu \psi_b \bar{\psi}_a \gamma_\mu \psi_d$$

Fierz rearrangement indicates $F(I) - F(\gamma_5)$ and $\sum F(\gamma^\mu) + F(\gamma^\mu \gamma_5)$ extract leading twist $\gamma^\mu \gamma_5$

UV divergence

Z.F. Deng et al., JHEP 09 (2022) 046

- The UV divergence in the I and γ_5 form factor. can be removed by the renormalization constant of scalar density operator

$$Z_S = 1 + \frac{\alpha_s C_F}{4\pi} \frac{3}{\epsilon_{\rm UV}}. \quad (59)$$

- There is no UV divergence in the $\gamma_\perp$ and $\gamma_\perp\gamma_5$ form factor. After some simplifications, Eq. (58) gives

$$F(b_\perp, P_1, P_2, \mu) = F^0\left[1 - \frac{\alpha_s C_F}{2\pi}\left(7 - \frac{3}{2}\ln\frac{Q^2\bar{Q}^2 b_\perp^4}{4e^{-4\gamma_E}} + \frac{1}{2}\ln^2\frac{Q^2 b_\perp^2}{2e^{-2\gamma_E}} + \frac{1}{2}\ln^2\frac{\bar{Q}^2 b_\perp^2}{2e^{-2\gamma_E}}\right)\right].$$

1. $F(\gamma^0)$, $F(\gamma^z)$, $F(\gamma^0\gamma_5)$ and $F(\gamma^z\gamma_5)$ have no contribution in leading order.
2. $F(I)$ and $F(\gamma^5)$ has UV divergence, while $F(\gamma^\perp)$ and $F(\gamma^\perp\gamma^5)$ have not.

- Motivation:
- Lattice QCD; LaMET; TMD physics; Soft function
- Theoretical framework:
- CS kernel; Intrinsic soft function
- Numerical results:
- quasi-TMDWF; Intrinsic soft function; CS kernel; TMDs
- Summary and outlook

Numerical results: quasi-TMDWF

Configuration sets

Ensemble	a(fm)	$N_\sigma^3 \times N_\tau$	$m_\pi^{\rm sea}$	$m_\pi^{\rm val}$	Measure
X650	0.098	$48^3\times$ 48	333 MeV	662 MeV	911×4
A654	0.098	$24^3\times$ 48	333 MeV	662 MeV	4923×20
a12m130	0.121	$48^3\times$ 64	132 MeV	310 MeV	1000×4
				220 MeV	1000×16
a12m310	0.121	$24^3\times$ 64	305 MeV	670 MeV	1053×8

Wilson fermion from CLS collaboration

Staggered fermion from MILC collaboration

Potential problem: large lattice spacing causes large discretization

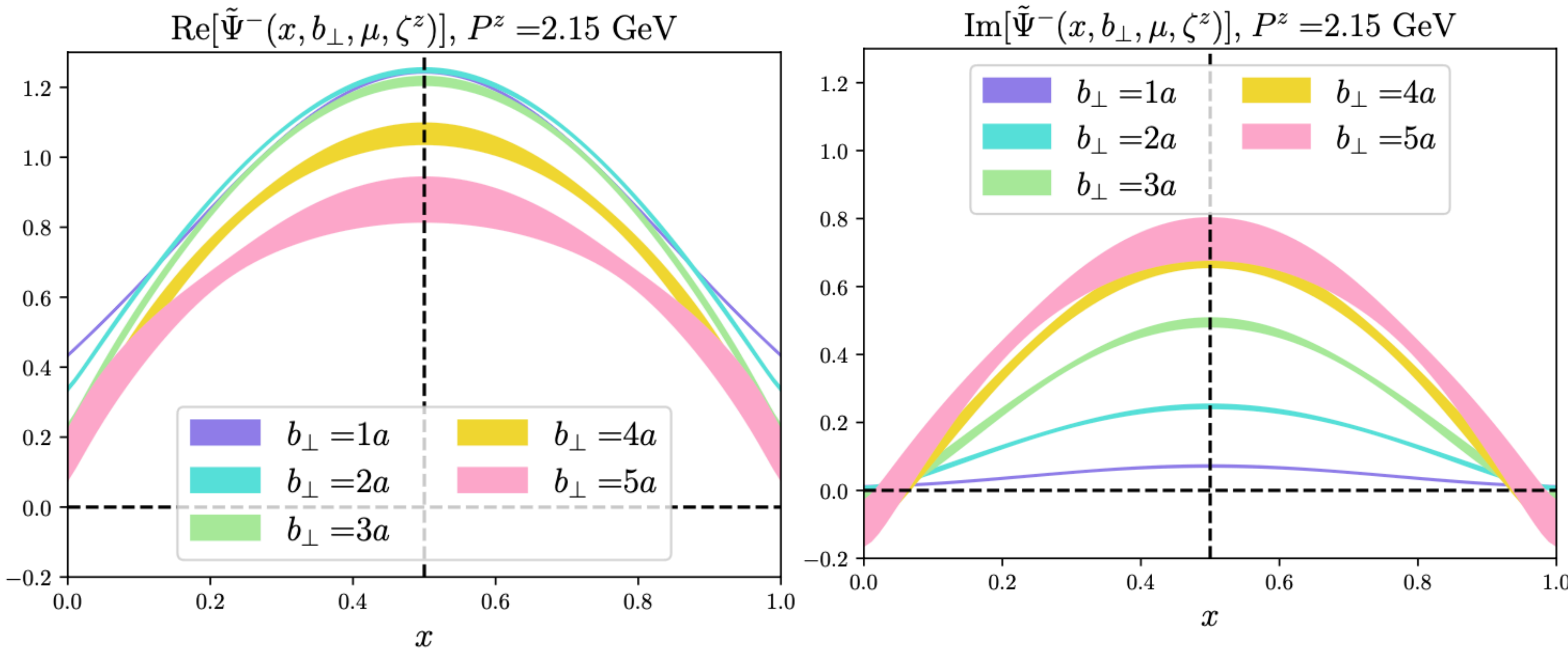
quasi-TMDWF matrix element

from 2pt

$$\tilde{\Psi}^{\pm}\left(z, b_\perp, \mu, \zeta^z\right) = \frac{\left\langle 0 \left| \bar{q}\left(z\hat{n}_z + b_\perp \hat{n}_\perp\right)\gamma^t\gamma_5 U_{c\pm} q(0)\right| \pi\left(P^z\right)\right\rangle}{\sqrt{Z_E\left(2L \pm z, b_\perp, \mu\right)}Z_O(1/a, \mu, \Gamma)}$$

Wilson loop: linear divergence, pinch pole singularity

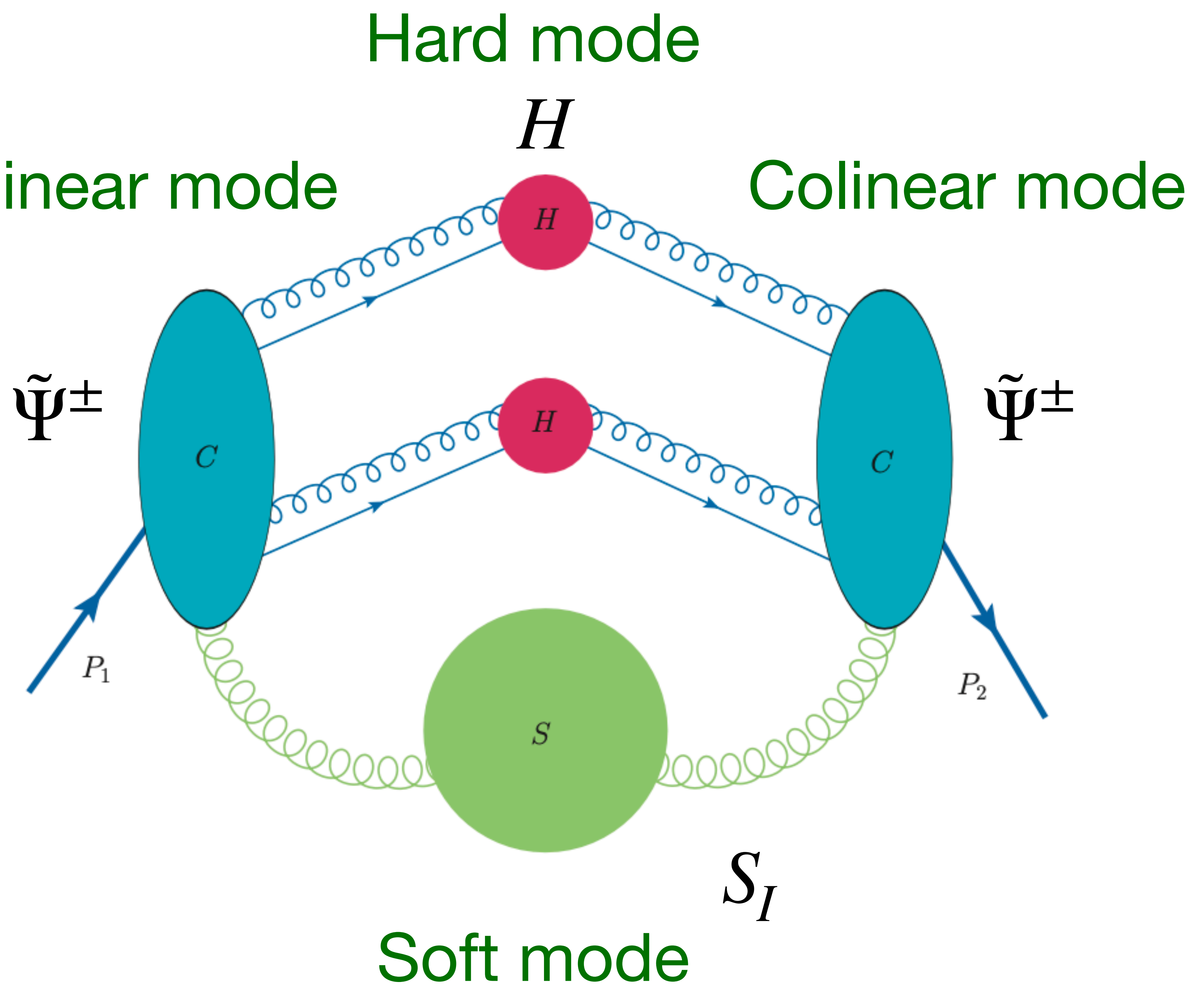
MSbar factor: logarithm divergence



Numerical results: Intrinsic soft function

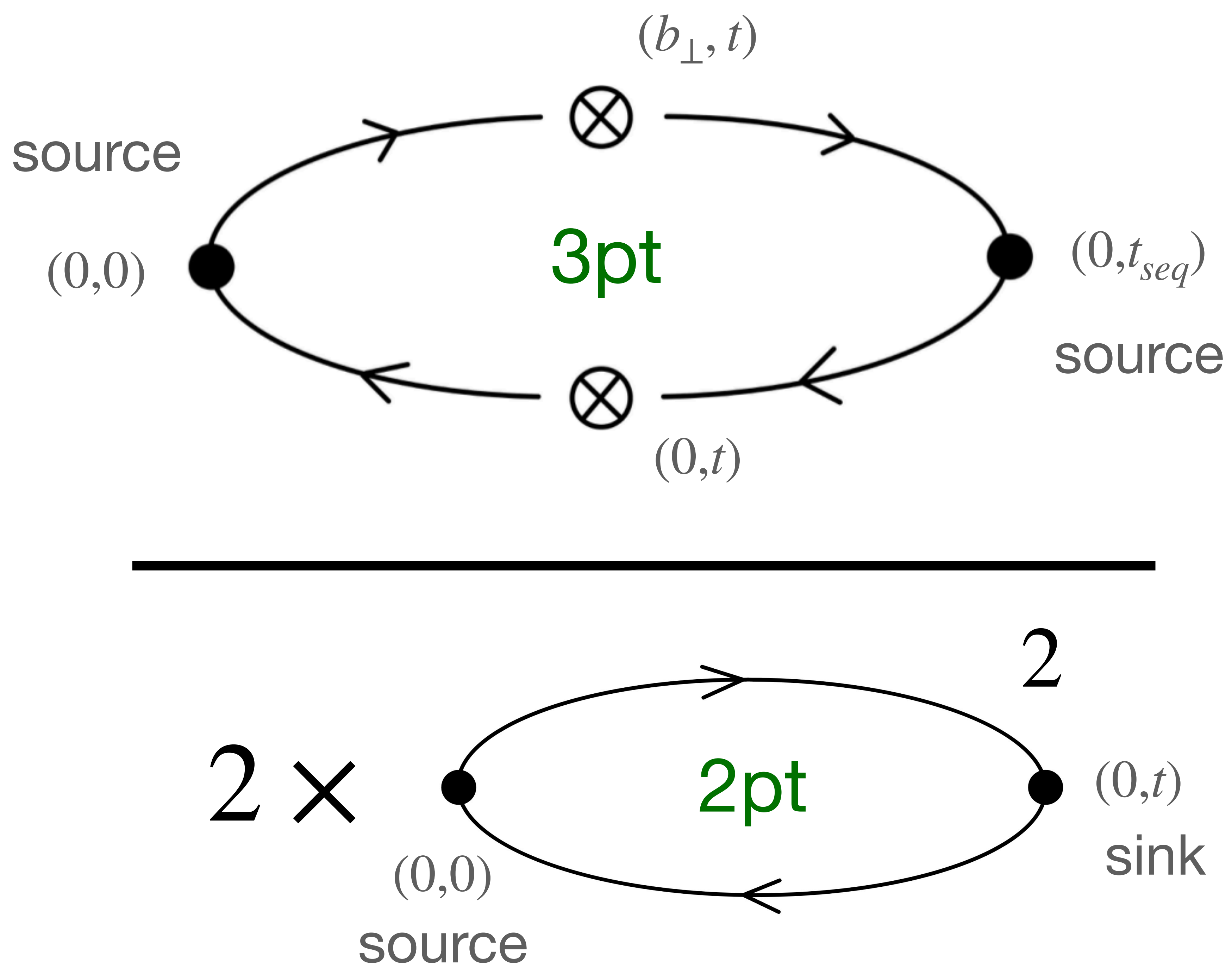
Form factor factorization

$$F\left(b_\perp, P_1, P_2, \Gamma, \mu\right) = \int dx_1 dx_2 H\left(x_1, x_2, \Gamma\right) S_I\left(b_\perp, \mu\right) \times \tilde{\Psi}^{\pm *}\left(x_2, b_\perp, \zeta^z\right) \tilde{\Psi}^{\pm}\left(x_1, b_\perp, \zeta^z\right)$$



Lattice simulation

$$\frac{VC_3(b_\perp, t_{seq}, t, P)}{2C_2^2(t = \frac{t_{seq}}{2})} = \frac{\langle \pi(P_2) | (\bar{q}\Gamma q)|_{b_\perp} (\bar{q}\Gamma q)|_0 | \pi(P_1) \rangle}{2\langle 0 | (\bar{q}\gamma^t\gamma_5 q)|_0 | \pi(P_1)\rangle\langle \pi(P_1) | (\bar{q}\gamma^t\gamma_5 q)|_0 | 0 \rangle} = F(b_\perp, \Gamma, P^z)$$



Numerical results: Intrinsic soft function

Lattice results

$$S_I(b_\perp,\mu) = \frac{F(b_\perp, P^z, \Gamma, \mu)}{\int dx_1 dx_2 H(x_1,x_2)\,\tilde{\Psi}^{\pm *}(x_2, b_\perp, \zeta^z)\,\tilde{\Psi}^{\pm}(x_1, b_\perp, \zeta^z)}$$

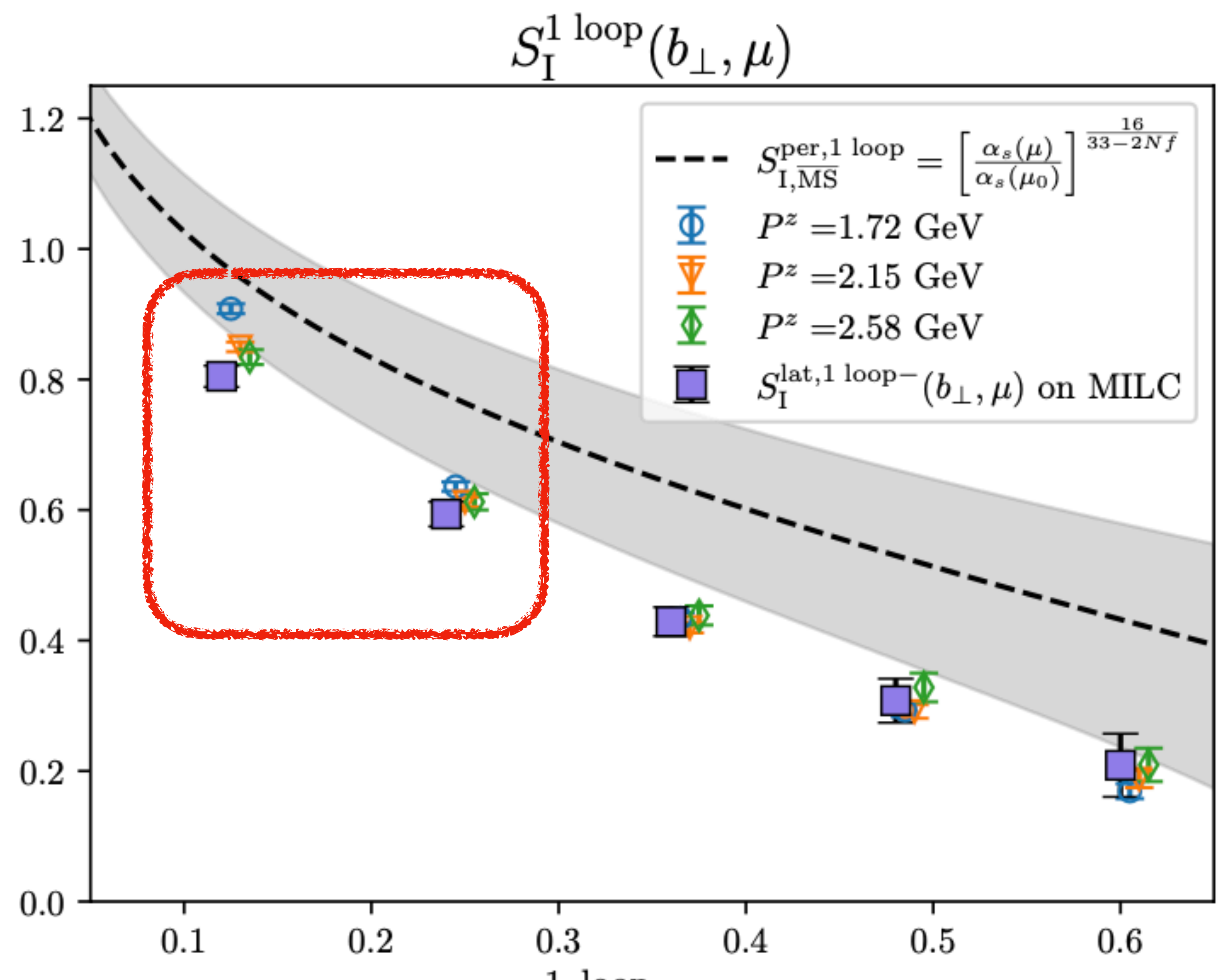
Infinite P^z limit $S_I(P^z) = S_I(P^z = limit) + \frac{C}{(P^z)^2}$

Perturbative calculation

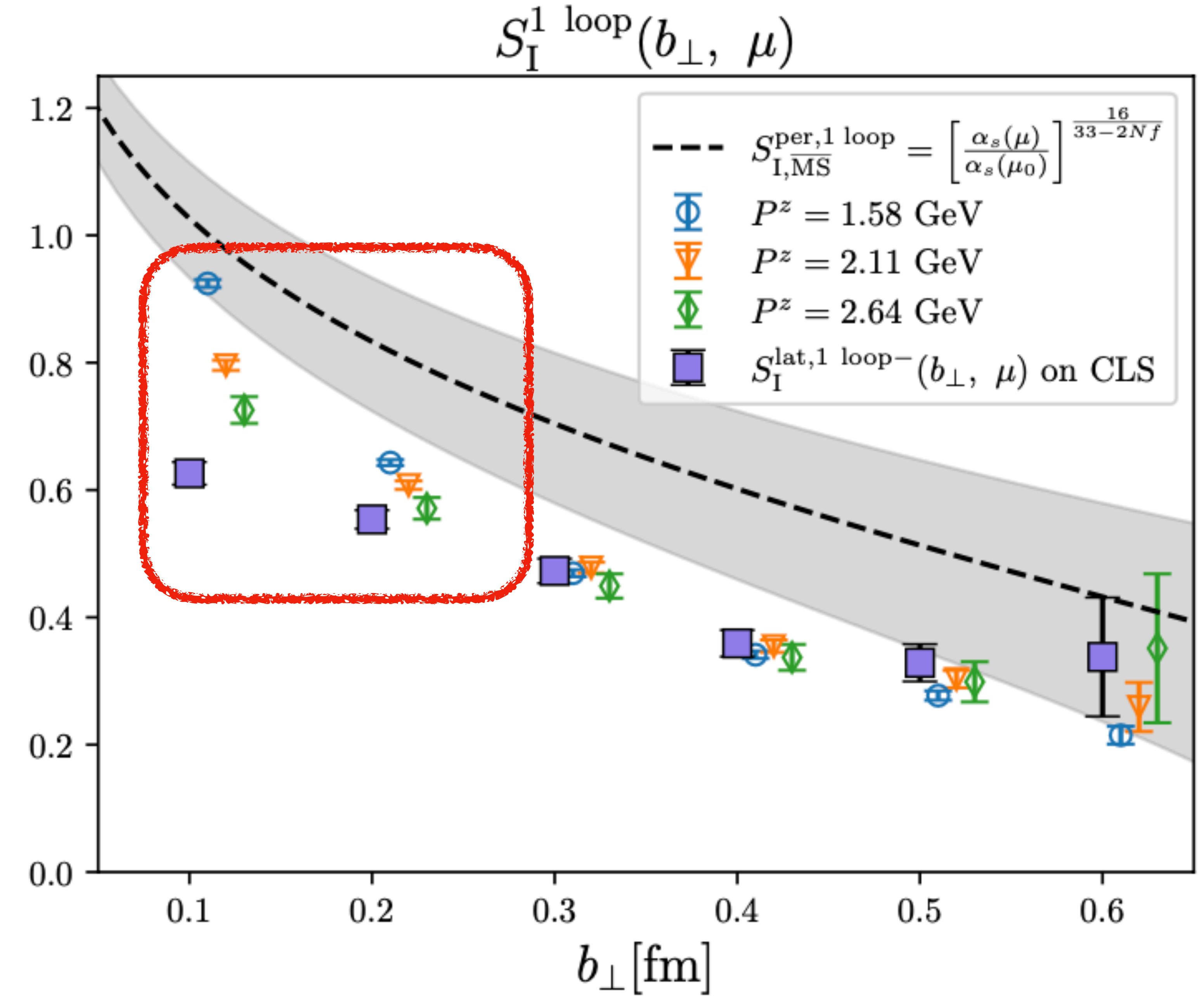
$$S^{\text{per,1 loop}}_{\text{I},\overline{\text{MS}}} = \left[\frac{\alpha_s(\mu = 2\text{GeV})}{\alpha_s(\mu_0 = 1/b^*_\perp)}\right]^{\frac{16}{33-2N_f}}$$

scale $b^*_\perp \in \left[1/\sqrt{2}, \sqrt{2}\right] b_\perp$

MILC



CLS



Numerical results: CS kernel

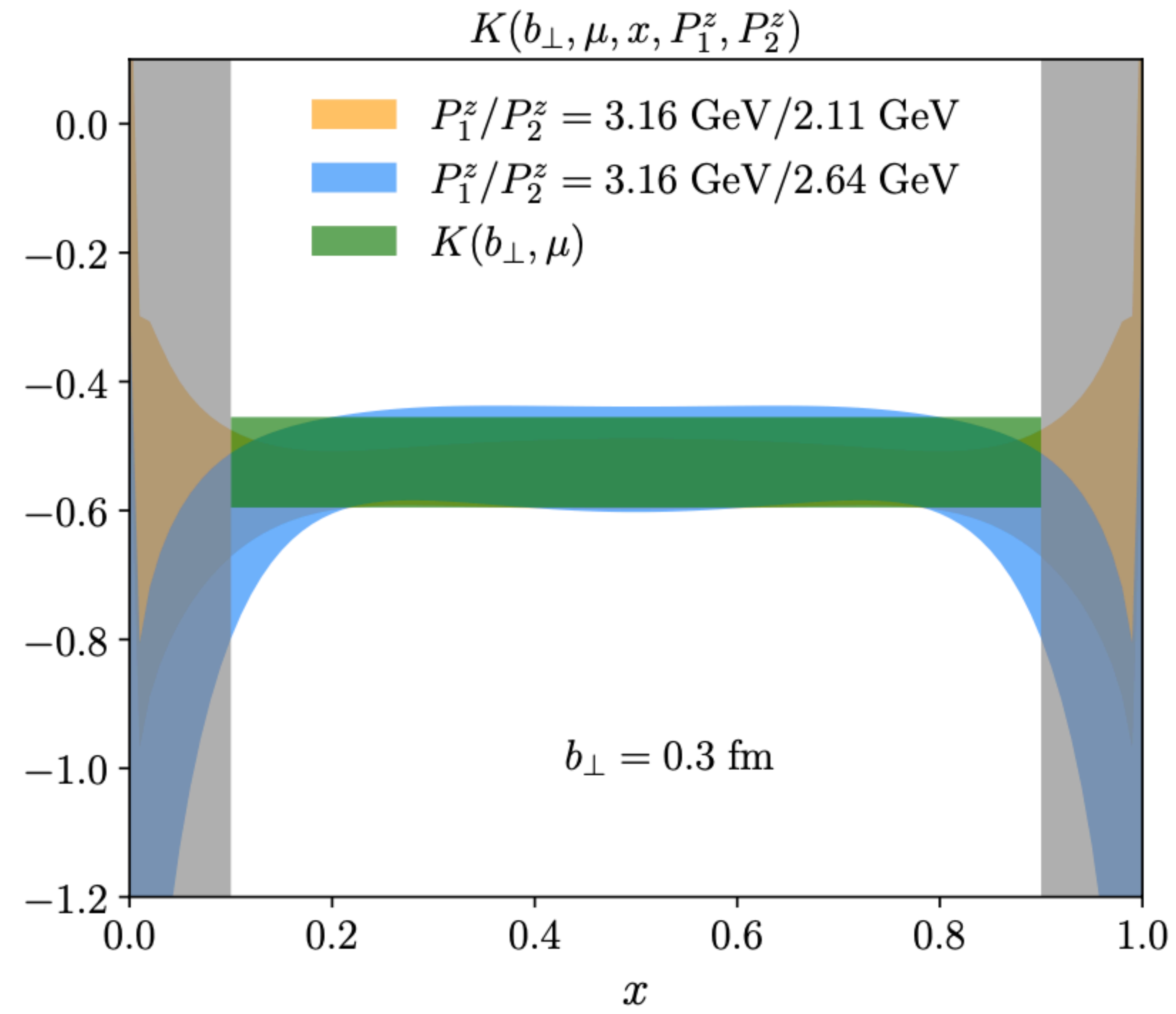
Lattice results

$$K(b_\perp,\mu)=\frac{1}{\ln(P_2^z/P_1^z)}\ln\left[\frac{H^{\pm}(\zeta_1^z,\bar{\zeta}_1^z)\tilde{\Psi}^{\pm}(b_\perp,x,\zeta_2^z)}{H^{\pm}(\zeta_2^z,\bar{\zeta}_2^z)\tilde{\Psi}^{\pm}(b_\perp,x,\zeta_1^z)}\right]$$

X dependence fit   MILC   *M. Chu et al., Phys.Rev.D 106 (2022)*

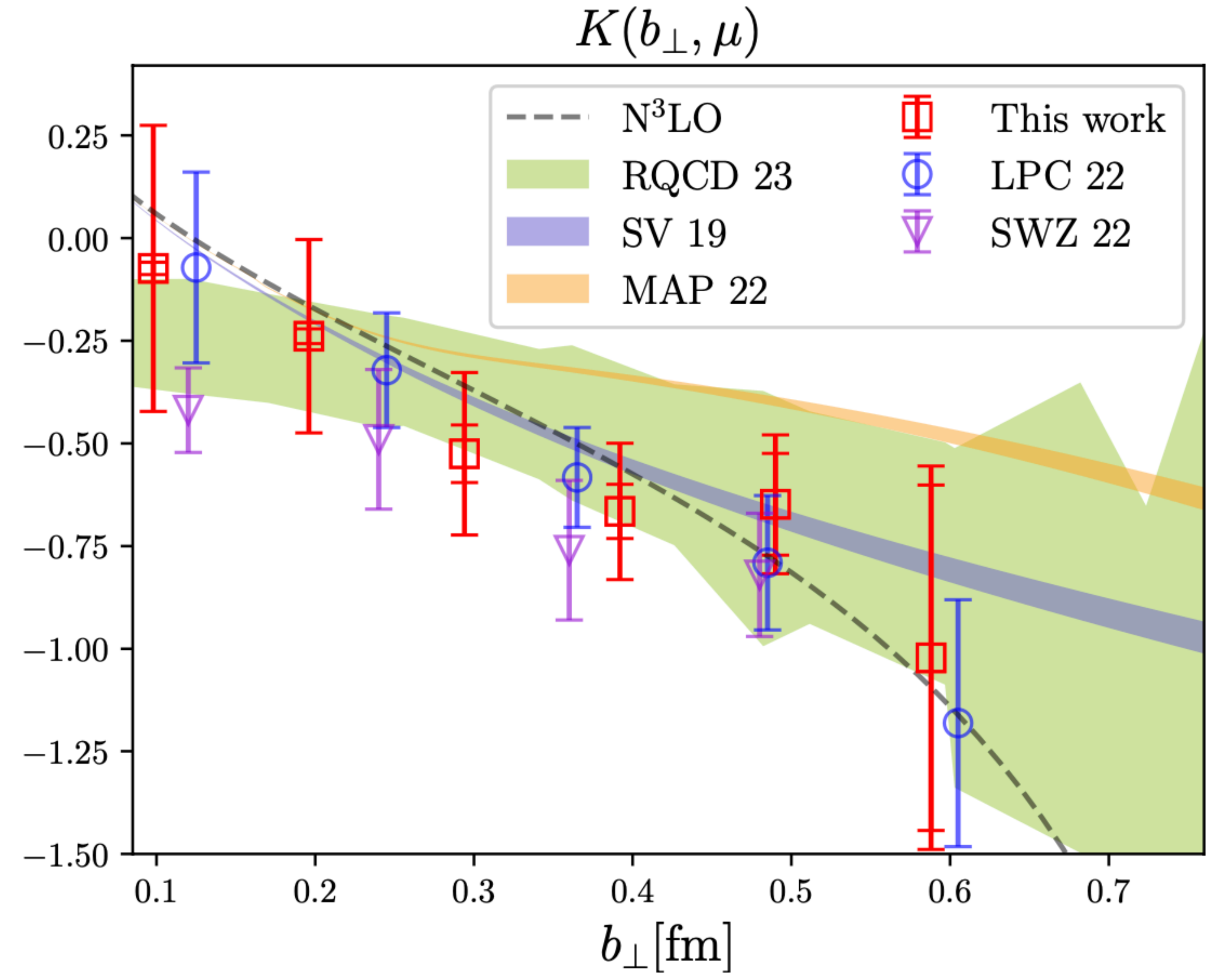
$$K(b_\perp,\mu,x,P_1^z,P_2^z)=K(b_\perp,\mu)+A\left[\frac{1}{x^2(1-x)^2(P_1^z)^2}-\frac{1}{x^2(1-x)^2(P_2^z)^2}\right]$$

CLS

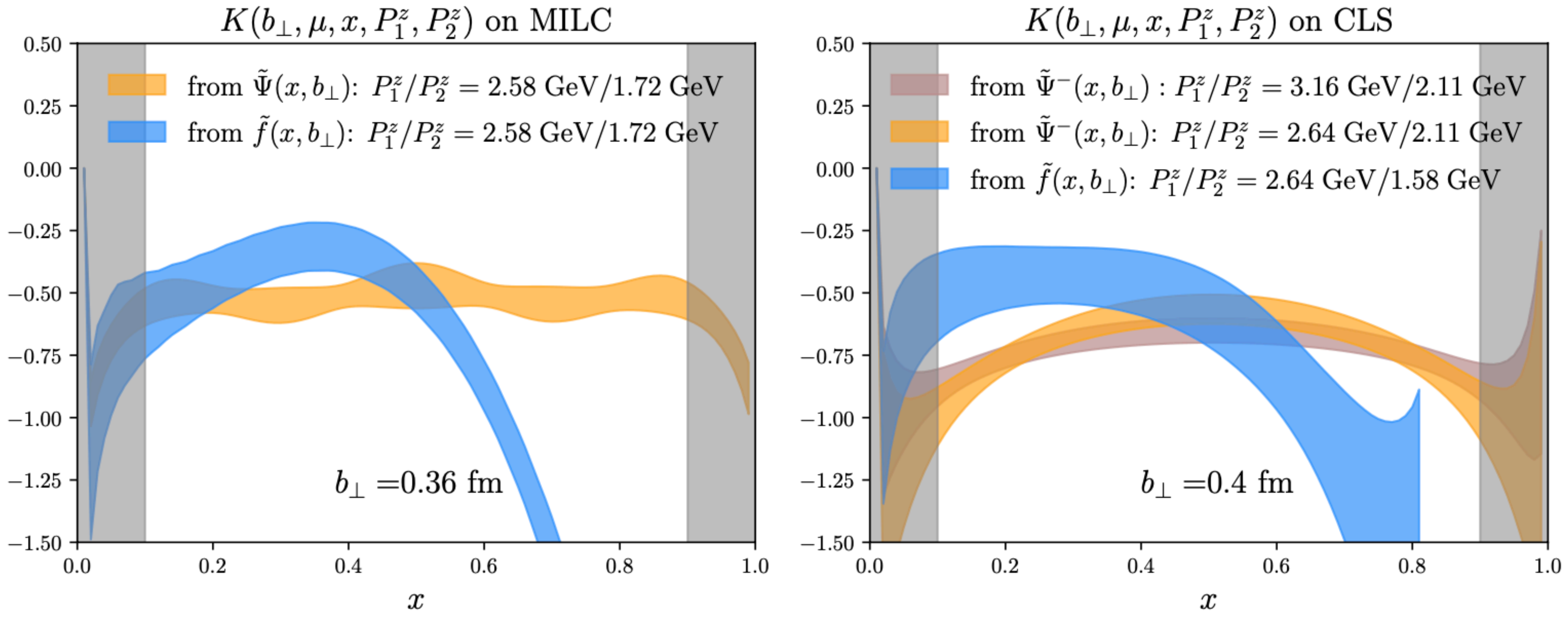


CS kernel result

CLS



Comparison CS kernel from TMDWF/PDF



Numerical results: TMDs

Lattice results from two ensembles

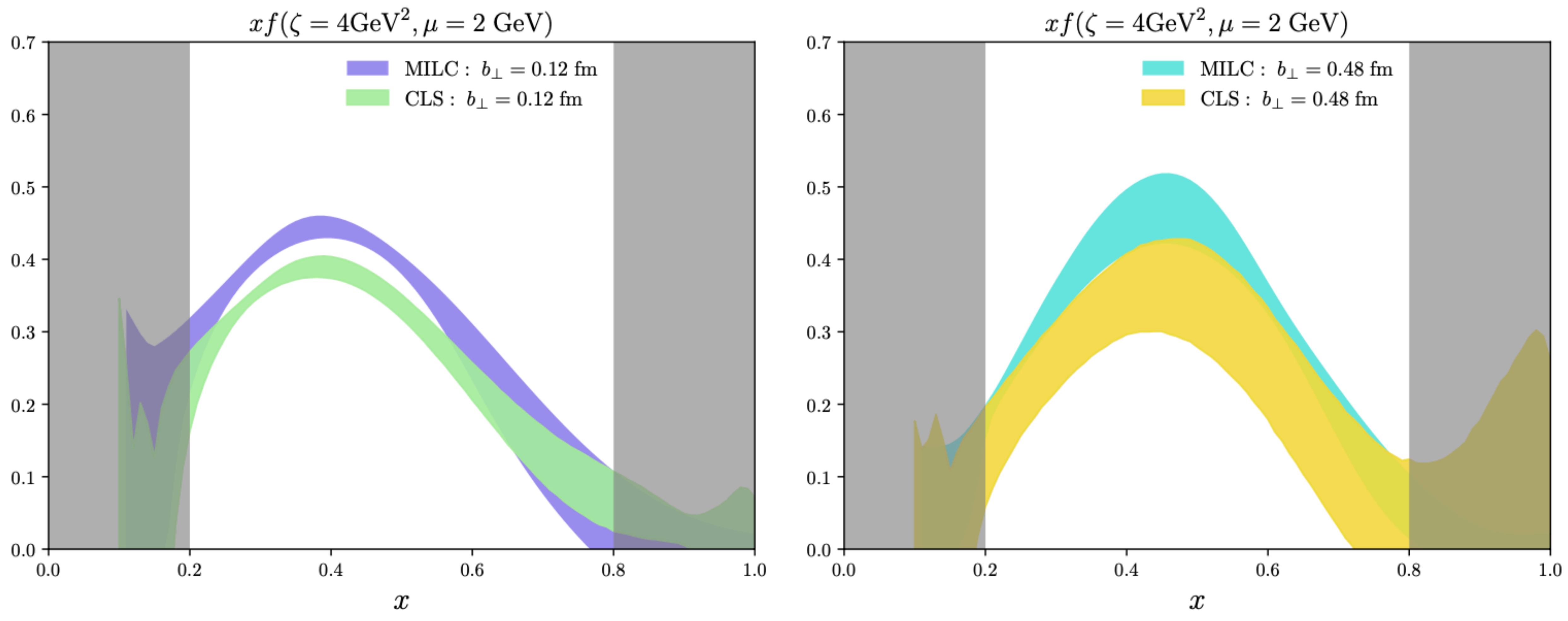
TMDPDF and TMDWF

$$\tilde{f}^{\pm}\left(x, b_\perp, \mu, \zeta^z\right) S_I^{\frac{1}{2}}\left(b_\perp, \mu\right) = H^{\pm}\left(x, \zeta^z, \mu\right) e^{\left[\frac{1}{2} K\left(b_\perp, \mu\right) \ln \frac{\mp \zeta^z + i\epsilon}{\zeta}\right]} f^{\pm}\left(x, b_\perp, \mu, \zeta\right)$$

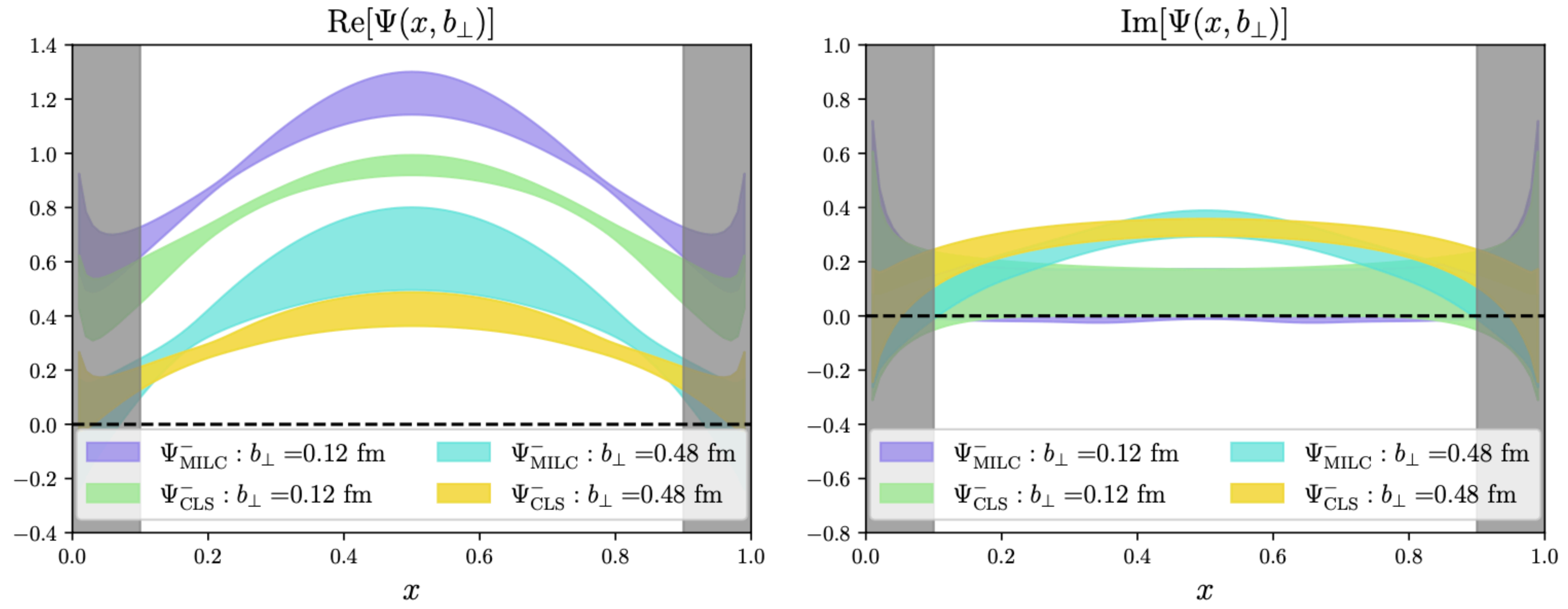
Intrinsic soft function

CS kernel

TMDPDF from MILC and CLS



TMDWF from MILC and CLS



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Summary and outlook

- TMDs are crucial for analyzing scattering processes, with the soft function detailing soft gluon radiation.
- This research marks the first attempt into extracting the intrinsic soft function using a one-loop matching kernel, alongside accurate normalization and Dirac structure considerations.
- Further study on smaller lattices is needed to explore discrete effects.
- Deeper examination of TMDPDFs and TMDWFs is necessary for a more comprehensive understanding.

Thanks for your attention!