

# **Baryons in the light-front approach: The three-quark picture**

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In collaboration with Fu-Wei Zhang, Xiao-Hui Hu, Yu-Ji Shi

# Outline

- Introduction
- Framework and some applications
- Numerical results
- Summary and outlook

# Introduction

# Observation of $\Xi_{cc}^{++}$

PRL **119**, 112001 (2017)

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## Observation of the Doubly Charmed Baryon $\Xi_{cc}^{++}$

$$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$$

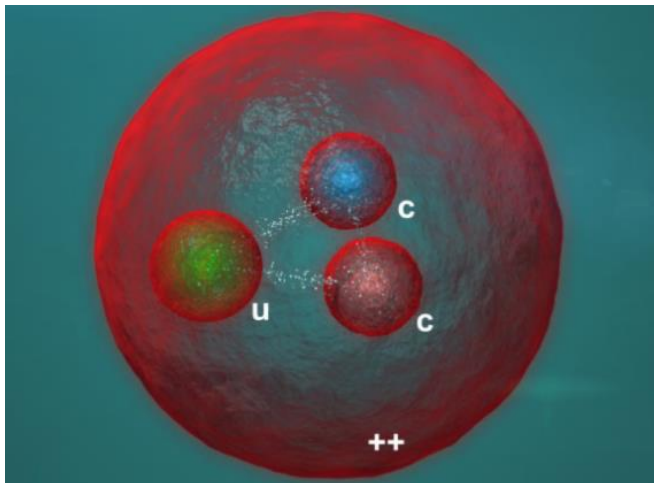
R. Aaij *et al.*\*

(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

A highly significant structure is observed in the  $\Lambda_c^+ K^- \pi^+ \pi^+$  mass spectrum, where the  $\Lambda_c^+$  baryon is reconstructed in the decay mode  $pK^- \pi^+$ . The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon  $\Xi_{cc}^{++}$ . The difference between the masses of the  $\Xi_{cc}^{++}$  and  $\Lambda_c^+$  states is measured to be  $1334.94 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \text{ MeV}/c^2$ , and the  $\Xi_{cc}^{++}$  mass is then determined to be  $3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+) \text{ MeV}/c^2$ , where the last uncertainty is due to the limited knowledge of the  $\Lambda_c^+$  mass. The state is observed in a sample of proton-proton collision data collected by the LHCb experiment at a center-of-mass energy of 13 TeV, corresponding to an integrated luminosity of  $1.7 \text{ fb}^{-1}$ , and confirmed in an additional sample of data collected at 8 TeV.

DOI: 10.1103/PhysRevLett.119.112001



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# Observation of $\Xi_{cc}^{++}$

## Weak decays of doubly heavy baryons: the $1/2 \rightarrow 1/2$ case


#21

Wei Wang (Shanghai Jiao Tong U. and Shanghai Jiaotong U.), Fu-Sheng Yu (Lanzhou U.), Zhen-Xing Zhao (Shanghai Jiaotong U. and Shanghai Jiao Tong U.) (Jul 10, 2017)

Published in: *Eur.Phys.J.C* 77 (2017) 11, 781 • e-Print: 1707.02834 [hep-ph]

 pdf

 DOI

 cite

 claim

 reference search

 134 citations

## Discovery Potentials of Doubly Charmed Baryons


#22

Fu-Sheng Yu (Lanzhou U. and Lanzhou, Inst. Modern Phys.), Hua-Yu Jiang (Lanzhou U.), Run-Hui Li (Neimunggu U.), Cai-Dian Lü (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Wei Wang (Shanghai Jiao Tong U. and Shanghai Jiaotong U.) et al. (Mar 27, 2017)

Published in: *Chin.Phys.C* 42 (2018) 5, 051001 • e-Print: 1703.09086 [hep-ph]


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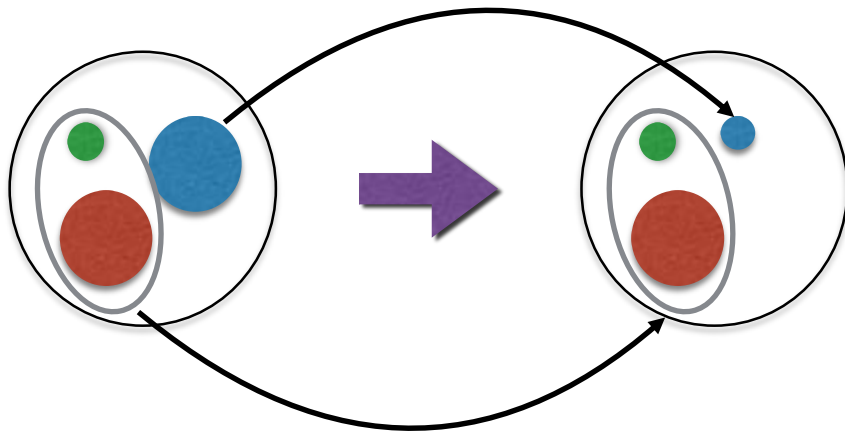
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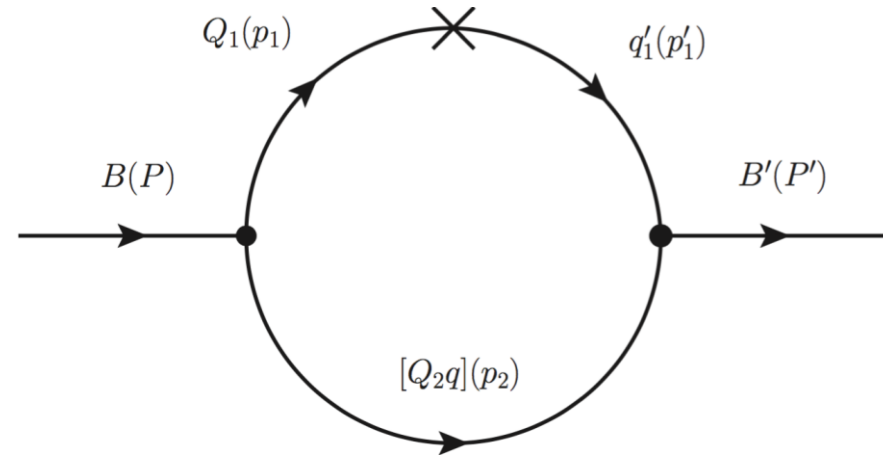
 125 citations

- Test the standard model
- Search for the origin of CP violation and new physics
- Understand the strong interactions

# Light-front quark model—the diquark picture



diquark  
 $0^+$  or  $1^+$



H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 77, 014020 (2008)

H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

Some defects:

--  $\Xi_{bc}(bcq)$ ,

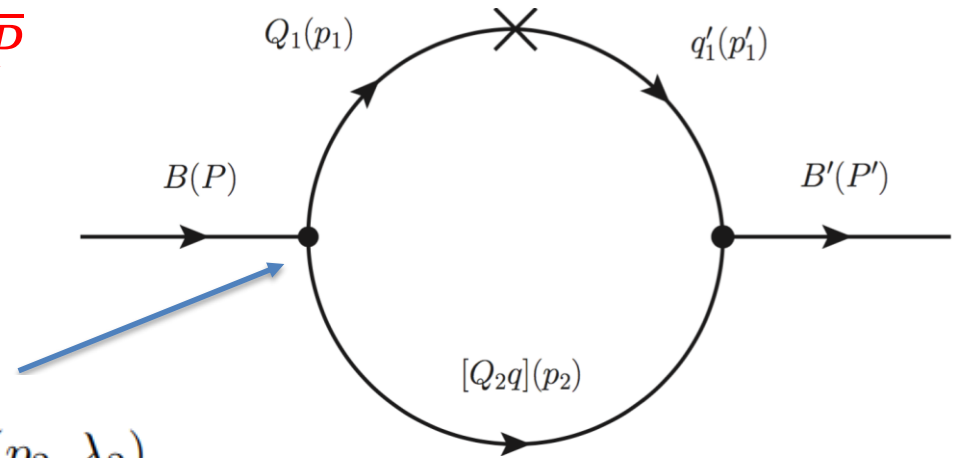
for c decay, bq -- diquark

for b decay, cq -- diquark

-- more parameters, such as  $m_{di}$  --  $m_{[ud]}$  and  $m_{\{ud\}}$

# Light-front quark model—the diquark picture

A Lorentz boost between  $p_2$  and  $\bar{P}$



$$\Gamma = -\frac{1}{\sqrt{3}}\gamma_5 \not{\epsilon}^*(p_2, \lambda_2),$$

H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

$$\begin{aligned} \Gamma &= \frac{1}{\sqrt{3}}\gamma_5 \not{\epsilon}^*(\bar{P}, \lambda_2) \\ &= \frac{1}{\sqrt{3}}\gamma_5 \left( \not{\epsilon}^*(p_2, \lambda_2) - \frac{M_0 + m_1 + m_2}{p_2 \cdot \bar{P} + m_2 M_0} \not{\epsilon}^*(p_2, \lambda_2) \cdot \bar{P} \right), \end{aligned}$$

Chun-Khiang Chua, Phys. Rev. D 99, 014023 (2019)

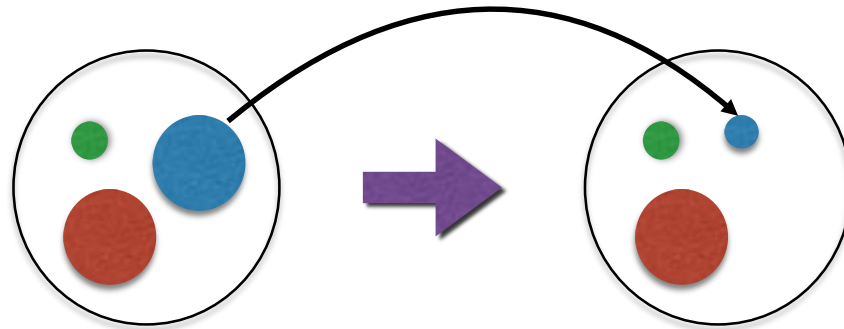
# Light-front quark model – the three-quark picture

S. Tawfiq, P. J. O'Donnell, and J. G. Körner, Phys. Rev. D 58, 054010 (1998)

H.-W. Ke, N. Hao, and X.-Q. Li, Eur. Phys. J. C 79, 540 (2019), ...

C.-Q. Geng, C.-W. Liu, and T.-H. Tsai, Phys.Lett.B 815, 136125 (2021), ...

Y.-S. Li, and X. Liu, Phys.Rev.D 107, 033005 (2023), ...




- lack a proof of spin wavefunctions
- shape parameters cannot be well determined
- relation between the diquark picture and the three-quark picture




# Framework and some applications

# The baryon state


$$\begin{aligned}
 |\mathcal{B}(P, S, S_z)\rangle &= \int \{d^3\tilde{p}_1\} \{d^3\tilde{p}_2\} \{d^3\tilde{p}_3\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \frac{1}{\sqrt{P^+}} \\
 &\times \sum_{\lambda_1, \lambda_2, \lambda_3} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3) C^{ijk} |q_1^i(p_1, \lambda_1) q_2^j(p_2, \lambda_2) q_3^k(p_3, \lambda_3)\rangle,
 \end{aligned}$$



spin and momentum



color



flavor

$$\Lambda_Q \quad A_0 \bar{u}(p_3, \lambda_3) (\bar{\not{P}} + M_0) (-\gamma_5) C \bar{u}^T(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) u(\bar{P}, S_z) \Phi(x_i, k_{i\perp}),$$

$$\Sigma_Q \quad A_1 \bar{u}(p_3, \lambda_3) (\bar{\not{P}} + M_0) (\gamma^\mu - v^\mu) C \bar{u}^T(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \left(\frac{1}{\sqrt{3}} \gamma_\mu \gamma_5\right) u(\bar{P}, S_z) \Phi(x_i, k_{i\perp}),$$

$$\Sigma_Q^* \quad A'_1 \bar{u}(p_3, \lambda_3) (\bar{\not{P}} + M_0) (\gamma^\mu - v^\mu) C \bar{u}^T(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) u_\mu(\bar{P}, S_z) \Phi(x_i, k_{i\perp}),$$

*(udQ)*

Three different flavors  $\left(\frac{1}{2} \otimes \frac{1}{2}\right) \otimes \frac{1}{2} = (0 \oplus 1) \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}.$

# Spin wavefunction

Take  $\Sigma_Q(udQ)$  as an example

where  $ud$  are considered as an axial-vector “diquark”

Step 1: In the **rest frame** of quark 2 and 3 – “diquark”

$$I^\mu \equiv \bar{u}(p_3, s_3) \frac{(\bar{\not{P}} + M_0)}{2M_0} \gamma_\perp^\mu(p_{23}) (-C) \bar{u}^T(p_2, s_2)$$

$$\gamma_\perp^\mu(p_{23}) = \gamma_\perp^\mu(\bar{P}) - \frac{M_0 p_{23}^\mu + m_{23} \bar{P}^\mu}{m_{23} M_0} \frac{\gamma_\perp(\bar{P}) \cdot p_{23}}{e_{23} + m_{23}},$$

$$p_{23} = p_2 + p_3, \quad m_{23}^2 = p_{23}^2,$$

$$\gamma_\perp^\mu(\bar{P}) = \gamma^\mu - \psi v^\mu, \quad v^\mu = \bar{P}^\mu / M_0.$$

$$I^\mu \sim \left\langle \frac{1}{2} \frac{1}{2}; s_3 s_2 \left| \frac{1}{2} \frac{1}{2}; 1, s_{23} \right. \right\rangle \epsilon^{*\mu}(p_{23}, s_{23}).$$

# Spin wavefunction

Step 2: Couple the “diquark” to quark 1

$$T \equiv I^\mu \cdot \bar{u}(p_1, s_1) \Gamma_{1,23\mu} u(\bar{P}, S_z)$$

$$\Gamma_{1,23\mu} = \frac{\gamma_5}{\sqrt{3}} \left( \gamma_\mu - \frac{M_0 + m_1 + m_{23}}{M_0(e_{23} + m_{23})} \bar{P}_\mu \right).$$

$$T \sim \left\langle \frac{1}{2} \frac{1}{2}; s_3 s_2 \middle| \frac{1}{2} \frac{1}{2}; 1 s_{23} \right\rangle \left\langle \frac{1}{2} 1; s_1 s_{23} \middle| \frac{1}{2} 1; \frac{1}{2} S_z \right\rangle.$$

Step 3: Tensor simplification

$$\begin{aligned} & \bar{u}(p_3, \lambda_3) (\bar{\not{P}} + M_0) \underline{\gamma_\perp^\mu} (p_{23}) (-C) \bar{u}^T(p_2, \lambda_2) \underline{\Gamma_{1,23\mu}} u(\bar{P}, S_z) \\ = & \dots \\ = & \bar{u}(p_3, \lambda_3) (\bar{\not{P}} + M_0) (\gamma^\mu - v^\mu) C \bar{u}^T(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \left( \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 \right) u(\bar{P}, S_z) \end{aligned}$$

- Lorentz boost plays a crucial role
- Same method can be applied to multi-quark states!

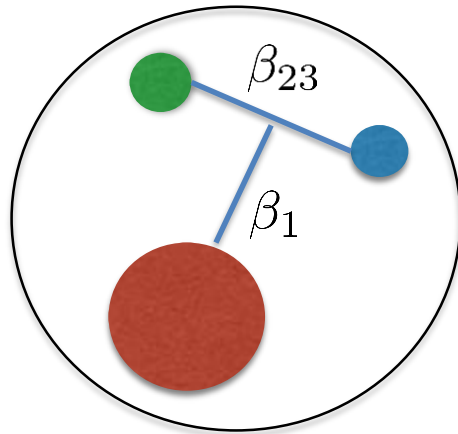
# Momentum wavefunction

$$\Phi(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}) = \sqrt{\frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}} \varphi(\vec{k}_1, \beta_1) \varphi\left(\frac{\vec{k}_2 - \vec{k}_3}{2}, \beta_{23}\right)$$

$$\varphi(\vec{k}, \beta) \equiv 4 \left(\frac{\pi}{\beta^2}\right)^{3/4} \exp\left(\frac{-k_{\perp}^2 - k_z^2}{2\beta^2}\right)$$

shape parameters

$$\int \left(\prod_{i=1}^3 \frac{dx_i d^2 k_{i\perp}}{2(2\pi)^3}\right) 2(2\pi)^3 \delta(1 - \sum x_i) \delta^2(\sum k_{i\perp}) |\Phi(x_i, k_{i\perp})|^2 = 1$$



Other choices may be possible!

# Color and flavor wavefunctions

Color wf:

$$\epsilon^{ijk} / \sqrt{6}$$

Flavor wf:

$$\begin{aligned}\Lambda_Q &= \frac{1}{\sqrt{2}} (ud - du)Q && \text{-- scalar } (0^+) \\ \Sigma_Q &= \frac{1}{\sqrt{2}} (ud + du)Q = \Sigma_Q^* && \text{-- axial-vector } (1^+)\end{aligned}$$

# To determine the shape parameters

Take  $\Lambda_Q$  as an example

$$\langle 0 | J_{\Lambda_Q} | \Lambda_Q(P, S_z) \rangle$$

Step 1: Calculate it in LFQM

Step 2: Use the definition of

$$\langle 0 | J_{\Lambda_Q} | \Lambda_Q(P, S_z) \rangle = \lambda_{\Lambda_Q} u(P, S_z).$$

Step 3: Extract the pole residue

$$\lambda_{\Lambda_Q} = \int \frac{dx_2 d^2 k_{2\perp}}{2(2\pi)^3} \frac{dx_3 d^2 k_{3\perp}}{2(2\pi)^3} \frac{1}{\sqrt{x_1 x_2 x_3}} \Phi(x_i, k_{i\perp}) \sqrt{6} A_0$$

$$\times \frac{\text{Tr}[\dots] \text{Tr}[\gamma^+(\not{p}_1 + m_1)(\bar{\not{P}} + M_0)]}{\text{Tr}[\gamma^+(\not{P} + M)]},$$

**M can be extracted!**

$$\lambda_{\Lambda_Q} = \text{the above equation with } \gamma^+ \rightarrow \gamma^+ \gamma^-$$

Here contains  $\beta_1$  and  $\beta_{23}$

$$\text{Tr}[\dots] = \text{Tr}[C \gamma_5 (\not{p}_3 + m_3) (\bar{\not{P}} + M_0) (-\gamma_5) C (\not{p}_2 + m_2)^T]$$

# Form factors

$\Lambda_b \rightarrow \Lambda_c$  Form factors

Step 1: Calculate the matrix elements in LFQM

Step 2: Write the matrix elements in terms of form factors

$$\langle \Lambda_c(P', S'_z) | \bar{c} \gamma^\mu b | \Lambda_b(P, S_z) \rangle = \bar{u}(P', S'_z) \left[ \gamma^\mu f_1(q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{M} f_2(q^2) + \frac{q^\mu}{M} f_3(q^2) \right] u(P, S_z),$$

$$\langle \Lambda_c(P', S'_z) | \bar{c} \gamma^\mu \gamma_5 b | \Lambda_b(P, S_z) \rangle = \bar{u}(P', S'_z) \left[ \gamma^\mu g_1(q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{M} g_2(q^2) + \frac{q^\mu}{M} g_3(q^2) \right] \gamma_5 u(P, S_z),$$

Step 3: Extract the form factors

$$f_1 = \frac{1}{8P^+ P'^+} \int \{d^3 \tilde{p}_2\} \{d^3 \tilde{p}_3\} \frac{\Phi'^* \Phi}{\sqrt{P^+ P'^+ p_1^+ p_1'^+}} A'_0 A_0 \text{Tr}[\dots]$$

$$\times \text{Tr}[(\bar{\not{P}} + M_0) \gamma^+ (\bar{\not{P}}' + M'_0) (\not{p}'_1 + m'_1) \gamma^+ (\not{p}_1 + m_1)],$$

$$\text{Tr}[\dots] = \text{Tr}[(\bar{\not{P}} + M_0) (-\gamma_5) C (\not{p}_2 + m_2)^T C \gamma_5 (\bar{\not{P}}' + M'_0) (\not{p}_3 + m_3)]$$

f2, g1, g2 can also be obtained in a similar way

$\Sigma_b \rightarrow \Sigma_c, \Xi_{cc} \rightarrow \Lambda_c$  Form factors



# The relation between the two pictures

$$\Lambda_Q \quad \psi_0(321) \equiv \bar{u}(p_3, \lambda_3)(\bar{\not{P}} + M_0)(-\gamma_5)C\bar{u}^T(p_2, \lambda_2)\bar{u}(p_1, \lambda_1)u(\bar{P}, S_z),$$

$$\Sigma_Q \quad \psi_1(321) \equiv \bar{u}(p_3, \lambda_3)(\bar{\not{P}} + M_0)(\gamma^\mu - v^\mu)C\bar{u}^T(p_2, \lambda_2)\bar{u}(p_1, \lambda_1)\left(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5\right)u(\bar{P}, S_z)$$

Quark 3 and 2 form a diquark

$$\left\{ \begin{array}{l} \psi_0(321) = -\psi_0(231), \\ \psi_1(321) = \psi_1(231) \end{array} \right.$$

A diquark bases

$$\left\{ \begin{array}{l} \text{normalization factor} \\ \text{orthogonal} \end{array} \right. \quad \frac{1}{4\sqrt{M_0^3(e_1 + m_1)(e_2 + m_2)(e_3 + m_3)}} \sum_{\lambda_1\lambda_2\lambda_3} \psi_0^\dagger(321')\psi_1(321) = 0$$

T matrix

$$\begin{pmatrix} \psi_0(312) \\ \psi_1(312) \end{pmatrix} = T \begin{pmatrix} \psi_0(321) \\ \psi_1(321) \end{pmatrix} \quad T = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$T^{-1} = T$$

# The relation between the two pictures

1. Calculate the overlap factors of  $\Xi_{bc}^+(cbu) \rightarrow \Lambda_b(dbu)$

$$\psi_1(bcu) = -\frac{\sqrt{3}}{2}\psi_0(buc) - \frac{1}{2}\psi_1(buc),$$

$$\begin{aligned}\psi_0(udb) &= \frac{1}{2}\psi_0(ubd) - \frac{\sqrt{3}}{2}\psi_1(ubd) \\ &= -\frac{1}{2}\psi_0(bud) - \frac{\sqrt{3}}{2}\psi_1(bud)\end{aligned}$$

$$\langle \psi_0(udb) | \psi_1(bcu) \rangle = \frac{\sqrt{3}}{4} \langle \psi_0(bud) | \psi_0(buc) \rangle + \frac{\sqrt{3}}{4} \langle \psi_1(bud) | \psi_1(buc) \rangle,$$

2. Calculate the overlap factors of  $\Xi_{cc}^{++}(ccu) \rightarrow \Lambda_c(dc u)$

$$\frac{2}{\sqrt{2}} \left\{ \frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4} \right\} = \left\{ \frac{\sqrt{6}}{4}, \frac{\sqrt{6}}{4} \right\}$$

W. Wang, F.-S. Yu, and Z.-X. Zhao, Eur. Phys. J. C 77, 781 (2017)

# Possibly better definitions of interpolating currents

Hermite conjugate  $\psi_{0,1}$

$$J_{\Lambda_Q}^{\text{new}} = \epsilon_{abc} [u_a^T C \gamma_5 (1 + \psi) d_b] Q_c,$$

$$J_{\Sigma_Q}^{\text{new}} = \epsilon_{abc} [u_a^T C (\gamma^\mu - v^\mu) (1 + \psi) d_b] \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 Q_c$$

$$v^\mu \equiv p^\mu / \sqrt{p^2}$$

Traditional definitions

$$J_{\Lambda_Q} = \epsilon_{abc} [u_a^T C \gamma_5 d_b] Q_c,$$

$$J_{\Sigma_Q} = \epsilon_{abc} [u_a^T C \gamma^\mu d_b] \gamma_\mu \gamma_5 Q_c$$

Comments:

1. The factor  $1/\sqrt{3}$

$$\lambda_{\Sigma_Q} \approx \lambda_{\Lambda_Q}$$

2. Let  $v \rightarrow 0$  in  $J^{\text{new}} \rightarrow J$  -- in fact, we cannot do that.

# Numerical results

# Inputs and shape parameters

Inputs  $m_u = m_d = 0.25$  GeV,  $m_c = 1.4$  GeV,  $m_b = 4.8$  GeV.

$$\lambda_{\Lambda_b} = 0.030 \pm 0.009, \quad \lambda_{\Lambda_c} = 0.022 \pm 0.008,$$

$$\lambda_{\Sigma_b} = 0.062 \pm 0.018, \quad \lambda_{\Sigma_c} = 0.045 \pm 0.015,$$

$$\lambda_{\Xi_{cc}} = 0.115 \pm 0.027.$$

Z.-G. Wang, Eur. Phys. J. C 68, 479 (2010)

Z.-G. Wang, Phys. Lett. B 685, 59 (2010)

Z.-G. Wang, Eur. Phys. J. A 45, 267 (2010)

$$\Lambda_b \rightarrow \Lambda_c \quad \beta_{b,[ud]} = 0.63 \pm 0.05 \text{ GeV}, \quad \beta_{[ud]} = 0.27 \pm 0.03 \text{ GeV},$$
$$\beta_{c,[ud]} = 0.45 \pm 0.05 \text{ GeV};$$

$$\Sigma_b \rightarrow \Sigma_c \quad \beta_{b,\{ud\}} = 0.66 \pm 0.04 \text{ GeV}, \quad \beta_{\{ud\}} = 0.28 \pm 0.03 \text{ GeV},$$
$$\beta_{c,\{ud\}} = 0.49 \pm 0.04 \text{ GeV};$$

$$\Xi_{cc} \rightarrow \Lambda_c \quad \beta_{u,\{cc\}} = 0.490 \pm 0.040 \text{ GeV}, \quad \beta_{\{cc\}} = 0.400 \pm 0.025 \text{ GeV}.$$

$$\lambda_1 \approx \lambda_2 \approx \lambda_{\text{QCDSR}}$$

# Form factors and comparison

TABLE II: Our form factors are compared with other results in the literature. The asterisk on Ref. [Shi19a] indicates that, in this literature, we made a mistake in the calculation of the axial-vector form factors, which led us to get the wrong symbol, and here we have corrected it.

$\Lambda_b \rightarrow \Lambda_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	$0.469 \pm 0.029$	$-0.105 \pm 0.011$	–	$0.461 \pm 0.027$	$0.006 \pm 0.005$	–
Three-quark picture [Ke19]	0.488	–0.180	–	0.470	–0.048	–
Diquark picture [Zhao18]	0.670	–0.132	–	0.656	–0.012	–
Diquark picture [Ke07]	0.506	–0.099	–	0.501	–0.009	–
QCD sum rules [Zhao20]	0.431	–0.123	0.022	0.434	0.036	–0.160
Lattice QCD [Detmold15]	0.418	–0.099	–0.075	0.390	–0.004	–0.206
$\Sigma_b \rightarrow \Sigma_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	$0.490 \pm 0.018$	$0.467 \pm 0.006$	–	$-0.163 \pm 0.005$	$0.007 \pm 0.001$	–
Three-quark picture [Ke19]	0.494	0.407	–	–0.156	–0.0529	–
Diquark picture [Ke12]	0.466	0.736	–	–0.130	–0.0898	–
$\Xi_{cc} \rightarrow \Lambda_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	$0.517 \pm 0.071$	$-0.036 \pm 0.007$	–	$0.155 \pm 0.019$	$-0.072 \pm 0.012$	–
Diquark picture [Wang17]	0.790	–0.008	–	0.224	–0.050	–
QCD sum rules [Shi19a] *	0.63	–0.05	–0.81	0.24	–0.11	–0.84
Light-cone sum rules [Shi19b]	$0.81 \pm 0.01$	$0.32 \pm 0.01$	$-0.90 \pm 0.07$	$1.09 \pm 0.02$	$-0.86 \pm 0.02$	$0.76 \pm 0.01$
NRQM [Perez-Marcial89]	0.36	0.14	0.08	0.20	0.01	–0.03
MBM [Perez-Marcial89]	0.45	0.01	–0.28	0.15	0.01	–0.70

$\beta_{u[cq]}$	$\beta_{d[cq]}$	$\beta_{s[cq]}$	$\beta_{c[cq]}$	$\beta_{b[cq]}$	$\beta_{u[bq]}$	$\beta_{d[bq]}$	$\beta_{s[bq]}$	$\beta_{c[bq]}$	$\beta_{b[bq]}$
0.470	0.470	0.535	0.753	0.886	0.562	0.562	0.623	0.886	1.472

Diquark picture

# Decay widths and comparison

TABLE III: Our decay widths (in units of  $10^{-14}$  GeV) are compared with other results in the literature.

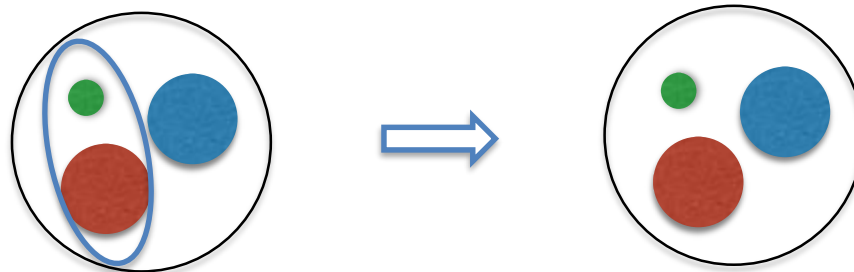
$\Lambda_b \rightarrow \Lambda_c e^- \bar{\nu}_e$	Decay width	$\Sigma_b \rightarrow \Sigma_c e^- \bar{\nu}_e$	Decay width	$\Xi_{cc} \rightarrow \Lambda_c e^+ \nu_e$	Decay width
This work	2.54	This work	0.870	This work	0.755
Three-quark picture [Ke19]	2.78	Three-quark picture [Ke19]	1.03	Diquark picture [Wang17]	1.05
Diquark picture [Zhao18]	3.96	Diquark picture [Ke07]	0.908	QCD sum rules [Shi19a] *	$0.76 \pm 0.37$
Diquark picture [Ke07]	3.39			Light-cone sum rules [Shi19b]	$3.95 \pm 0.21$
QCD sum rules [Zhao20]	$2.96 \pm 0.48$				
Lattice QCD [Detmold15]	$2.35 \pm 0.15$				

# Summary and outlook



# Summary

- A three-quark picture in LFQM is built up using the quark spinors and Dirac matrices
- The shape parameters are determined with the help of pole residue
- The relation between the diquark picture and the three-quark picture is clarified



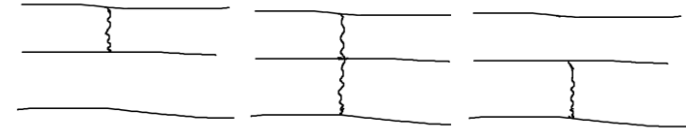
A small flaw?

$$\lambda_1 \approx \lambda_2 \approx \lambda_{\text{QCDSR}}$$

# Outlook

- Various applications

- ✓ arXiv: 2309.16386 – QED correction to  $\Xi_c - \Xi'_c$  mixing
- ✓ W-exchange diagram



- Lorentz boost plays a crucial role when building the model-- Multiquark states? – the most famous X(3872)

- ✓ H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D 70, 034007 (2004)
- ✓ H. Y. Cheng and C. K. Chua, J. High Energy Phys. 11 (2004) 072

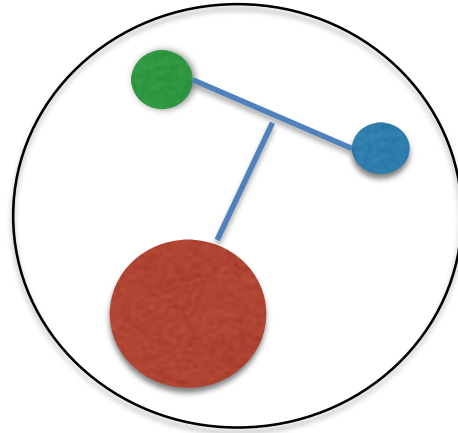
- Interpolating currents of baryons, even multi-quark states

# To make a tetraquark state

Spin wf: S-wave

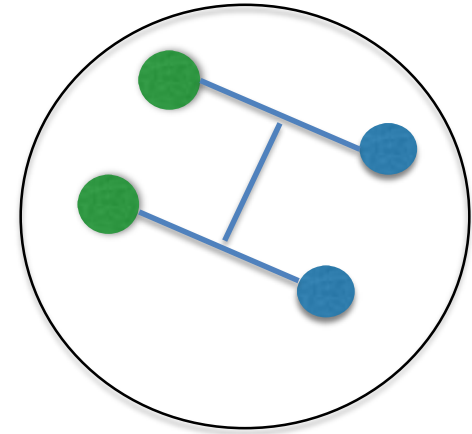
Building blocks:

- Quark
- Diquark



baryon

- RF of 23
- RF of 123



tetraquark state

- RF of 12
- RF of 34
- RF of 1234

Rest Frame

- $0^+$  Diquark 
$$I \equiv \bar{u}(p_3, s_3) \frac{(\bar{\not{P}} + M_0)}{2M_0} \gamma_5 (-C) \bar{u}^T(p_2, s_2)$$
- $1^+$  Diquark 
$$I^\mu \equiv \bar{u}(p_3, s_3) \frac{(\bar{\not{P}} + M_0)}{2M_0} \gamma_\perp^\mu(p_{23}) (-C) \bar{u}^T(p_2, s_2)$$

## To make a tetraquark state

- **Momentum wf:**  $\beta_{12}, \beta_{34}, \beta_{12,34}$
- **Color wf:**  $3 \times \bar{3} \times 3 \times \bar{3}$
- **Flavor wf:** SU(3)

**Matching**  
between diquark picture and full-quark picture



# 重味物理暑期学校

国家自然科学基金委理论物理专款项目

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Thank you for your attention!