

中微子质量与中微子振荡

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第二讲：质量与混合起源

- ◆ 超越标准模型的阳关道
- ◆ 味结构分块参数化方案
- ◆ 轻子味和轻子数不守恒
- ◆ 物质与反物质的不对称



江门中微子暑期学校，2023年6月22日

第二讲

- ◆ **超越标准模型的阳关道**
- ◆ **味结构分块参数化方案**
- ◆ **轻子味和轻子数不守恒**
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A MODEL OF LEPTONS*

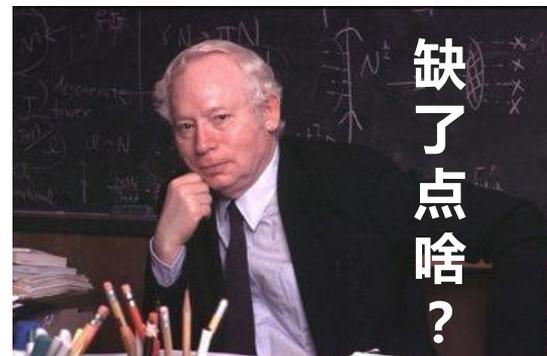
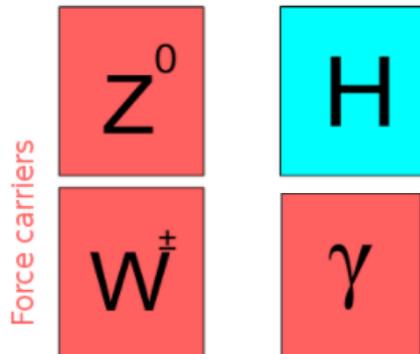
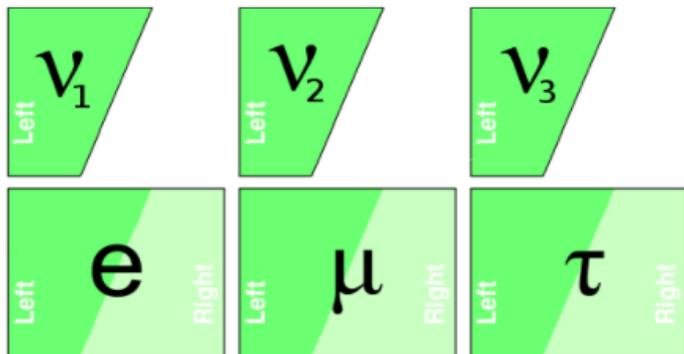
Steven Weinberg†

Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)

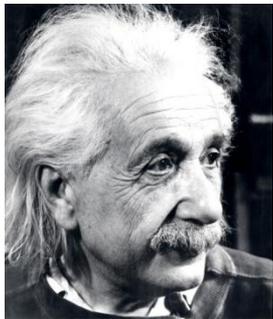
粒子内容：电子 (左、右手性)、电子型中微子 (左手性)；光子、W、Z、希格斯粒子

理论框架：洛伦兹不变性、定域规范对称性、 $SU(2)_L \times U(1)_Y$ 群结构、希格斯机制



左右手场的不对称

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} u_R \\ d_R \end{matrix}$$
$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} ? \\ e_R \end{matrix}$$



Albert Einstein:

Everything should be made as simple as possible, but not simpler!



简单性和经济性的
误导性

中微子质量 = 0

1957年

宇称最大破坏与中微子的二分量理论

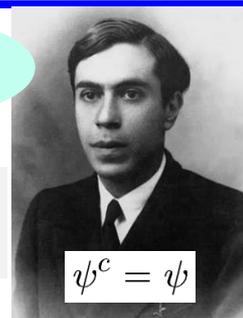
吴健雄
Leon Lederman
李政道、杨振宁
Lev Landau
Abdus Salam

代价最小的方案

引入右手中微子场及其自相互作用，以及中微子场的汤川相互作用：

$$-\mathcal{L}_{SS} = \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \overline{(N_R)^c} M_R N_R + \text{h.c.}$$

1937



Dirac (IQ>90)

Majorana (IQ>130)

相位变换依赖
= 破坏轻子数

问题的关键：电中性的基本费米子，在**独处**的时候，其反粒子可否是其自身？

代价与收益：基本理论结构不受影响；轻子数破坏；跷跷板压低中微子质量！

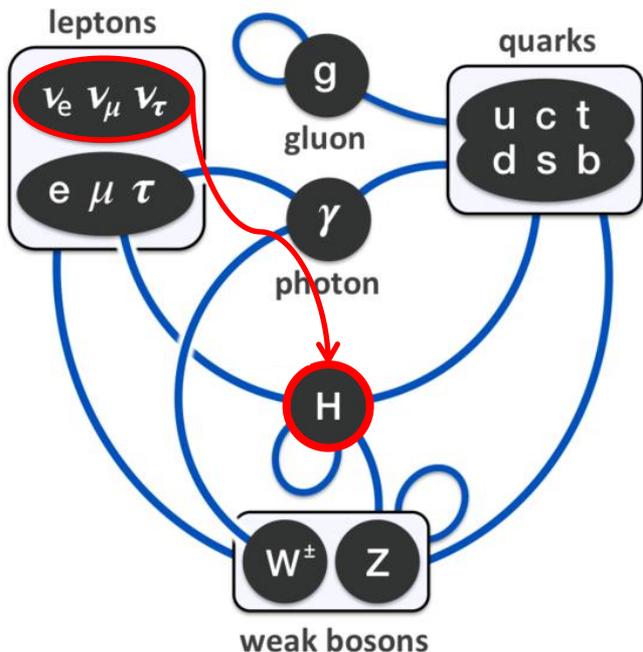


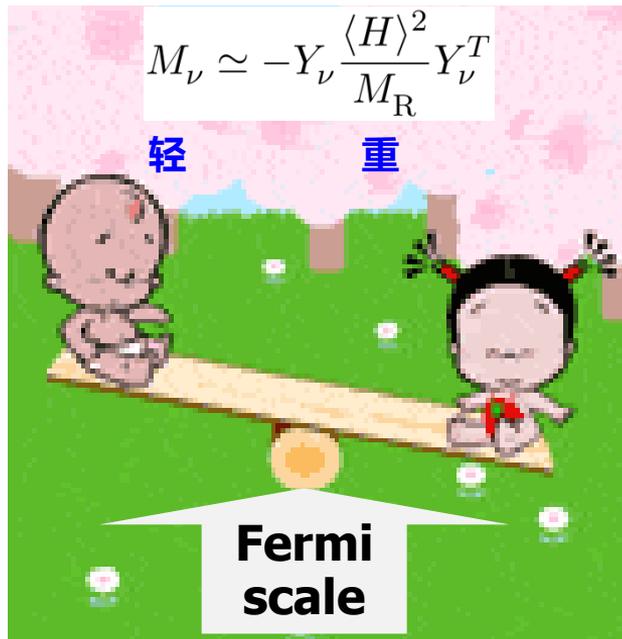
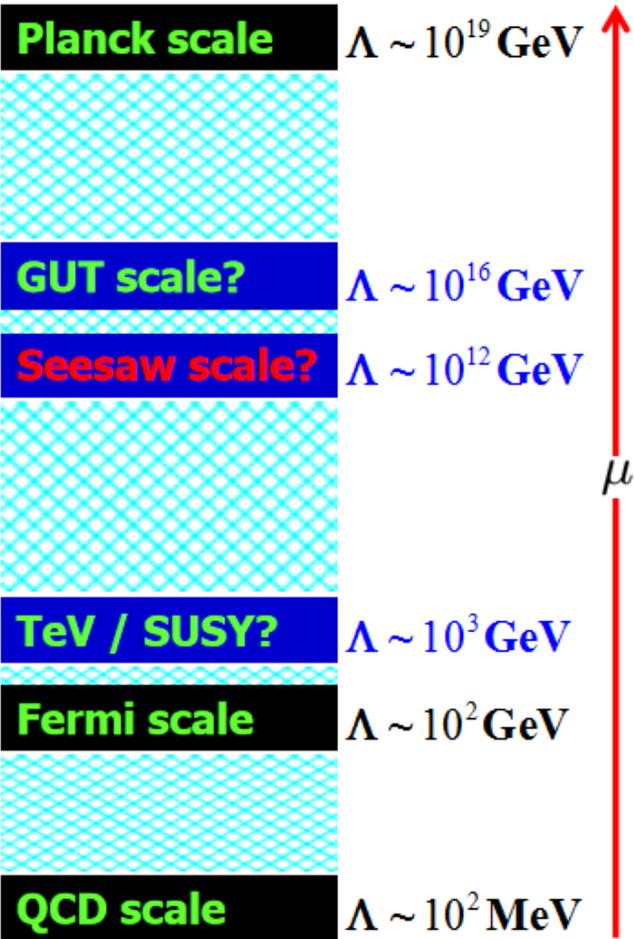
于是我陷入了沉思

有什么令人信服的指导原则吗？

Weinberg's 3rd law of progress in TH physics (1983): You may use any degree of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry.

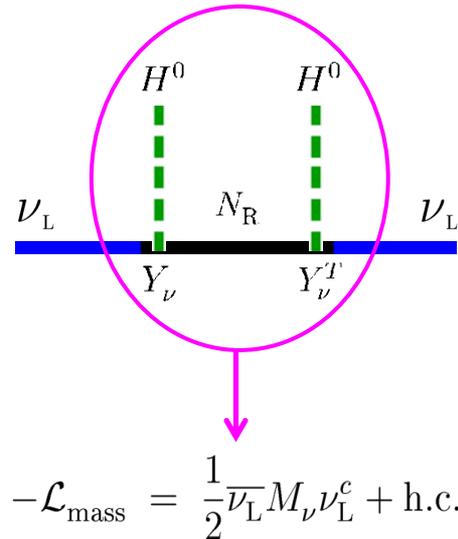
Gell-Mann's totalitarian principle (1956): Everything not forbidden is compulsory!





$$0.1 \text{ eV} \sim \frac{(100 \text{ GeV})^2}{10^{14} \text{ GeV}}$$

Seesaw — a footnote idea:
H. Fritzsch, M. Gell-Mann,
P. Minkowski (1975)

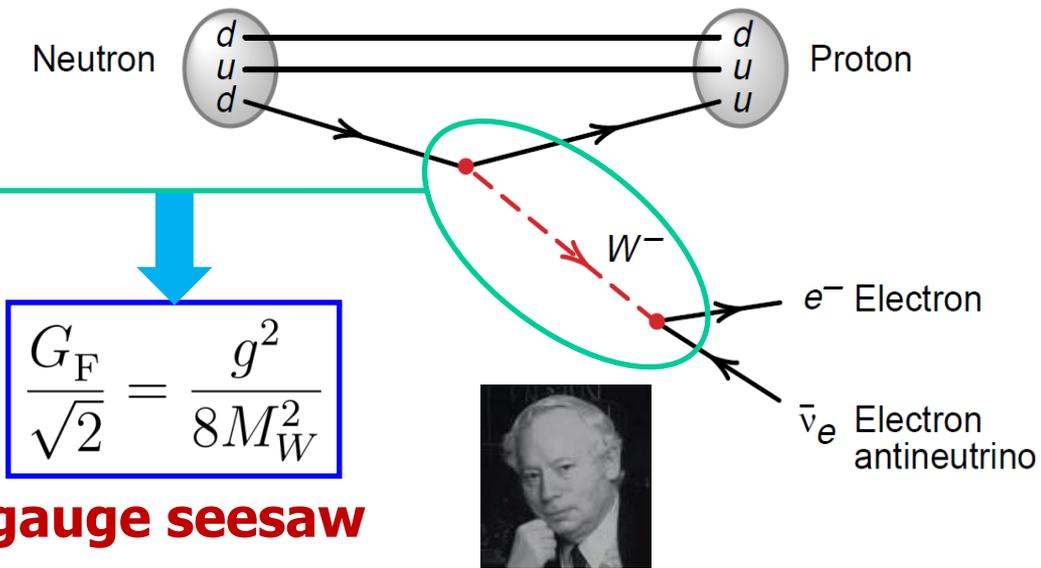
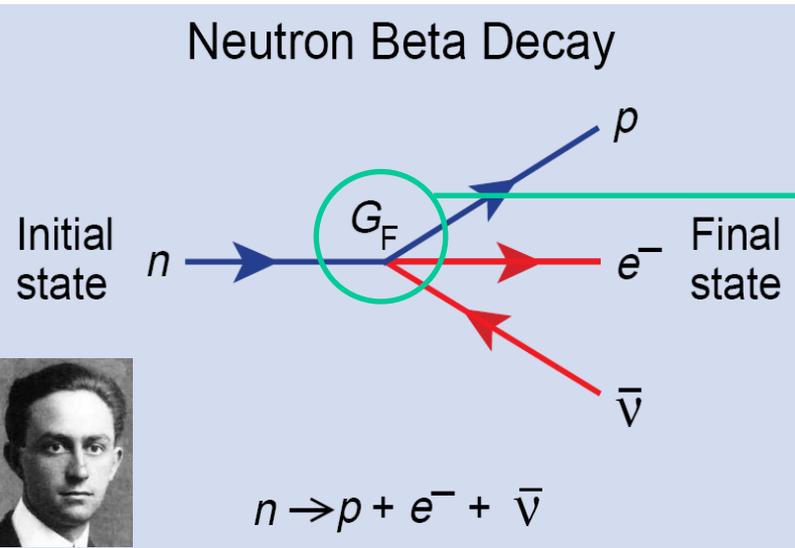


$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} \bar{\nu}_L M_\nu \nu_L^c + \text{h.c.}$$

从有效场论出发，积掉重自由度后，得轻中微子的有效质量项（破坏轻子数）。



从Enrico Fermi的贝塔衰变有效理论到Weinberg的标准模型，就是增加重自由度的游戏



Fermi耦合常数

$$G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

弱作用耦合常数

$$g \simeq 0.65 \quad \text{vs} \quad M_W \simeq 80.4 \text{ GeV}$$

中微子参与了这一弱带电流规范相互作用。但中微子的汤川 (Yukawa) 相互作用尚未知

该图像与Weinberg有效场论精神吻合，等价于标准模型唯一的 **5** 维有效算符（1979）：

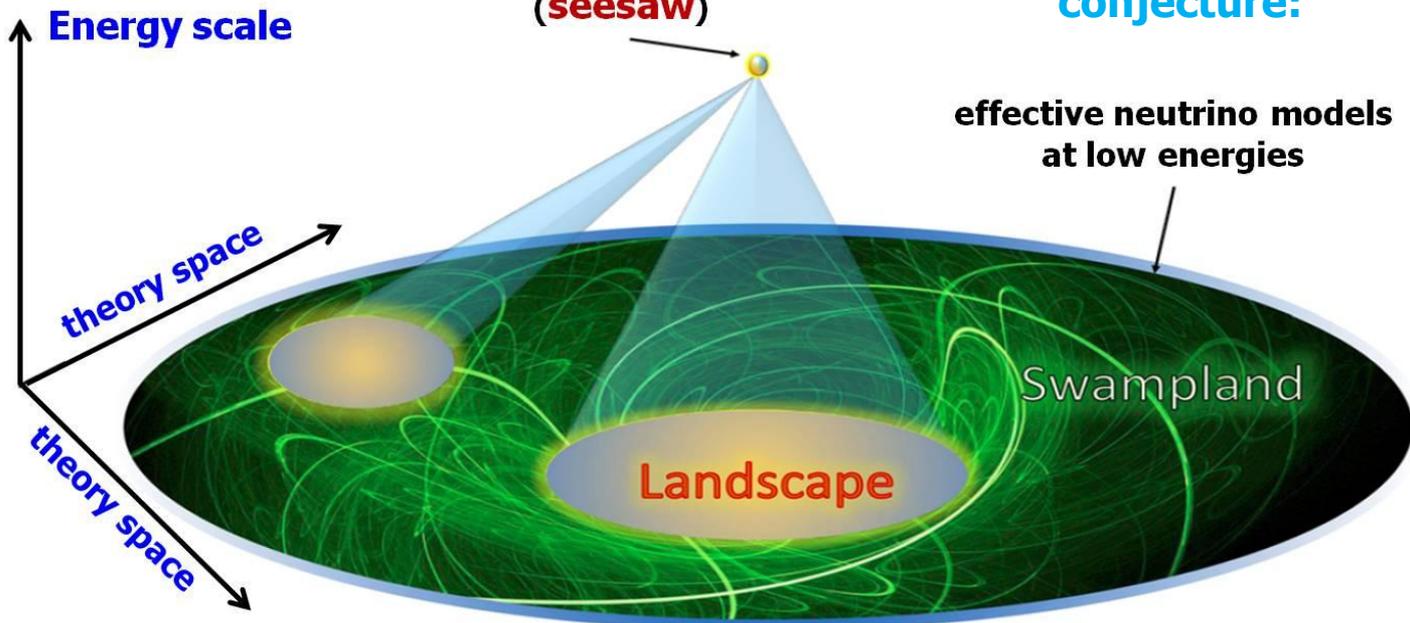
$$\mathcal{O}_w = \frac{\overline{\ell}_L \widetilde{H} H^T \ell_L^c}{\Lambda}$$



(5 + 2n) 维度
Y. Liao, 2011

$$M_\nu \simeq -M_D M_R^{-1} M_D^T = -\langle H \rangle^2 Y_\nu M_R^{-1} Y_\nu^T$$

沼泽地猜想
swampland conjecture:



Cumrun Vafa 2005

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拉氏量

$$-\mathcal{L} = \frac{1}{2} \overline{\begin{bmatrix} \nu_L & (N_R)^c \end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_R \end{pmatrix} \begin{bmatrix} (\nu_L)^c \\ N_R \end{bmatrix} - \bar{l}_L Y_\nu N_R \phi^- + \text{h.c.}$$

新物理

标准模型

右手中微子场
+
自相互作用项

希格斯场

左手中微子场

汤川相互作用

味混合结构
分块参数化

$$U = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & U'_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix}$$

ZZX, 1110.0083

3 混合角 + 3 相位

$O_{56} O_{46} O_{45}$

interplay

$O_{23} O_{13} O_{12}$

3 混合角 + 3 相位

$O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14}$

9 混合角 + 9 相位

小白板：有没有耐心算出来？

$$O_{12} = \begin{pmatrix} c_{12} & \hat{s}_{12}^* & 0 & 0 & 0 & 0 \\ -\hat{s}_{12} & c_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{13} = \begin{pmatrix} c_{13} & 0 & \hat{s}_{13}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\hat{s}_{13} & 0 & c_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23}^* & 0 & 0 & 0 \\ 0 & -\hat{s}_{23} & c_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{14} = \begin{pmatrix} c_{14} & 0 & 0 & \hat{s}_{14}^* & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\hat{s}_{14} & 0 & 0 & c_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$O_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & \hat{s}_{24}^* & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\hat{s}_{24} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{34} & \hat{s}_{34}^* & 0 & 0 \\ 0 & 0 & -\hat{s}_{34} & c_{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{15} = \begin{pmatrix} c_{15} & 0 & 0 & 0 & \hat{s}_{15}^* & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\hat{s}_{15} & 0 & 0 & 0 & c_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{25} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{25} & 0 & 0 & \hat{s}_{25}^* & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\hat{s}_{25} & 0 & 0 & c_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$O_{35} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{35} & 0 & \hat{s}_{35}^* & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\hat{s}_{35} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{45} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{45} & \hat{s}_{45}^* & 0 \\ 0 & 0 & 0 & -\hat{s}_{45} & c_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{16} = \begin{pmatrix} c_{16} & 0 & 0 & 0 & 0 & \hat{s}_{16}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\hat{s}_{16} & 0 & 0 & 0 & 0 & c_{16} \end{pmatrix} \quad O_{26} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{26} & 0 & 0 & 0 & \hat{s}_{26}^* \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\hat{s}_{26} & 0 & 0 & 0 & c_{26} \end{pmatrix}$$

$$O_{36} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{36} & 0 & 0 & \hat{s}_{36}^* \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\hat{s}_{36} & 0 & 0 & c_{36} \end{pmatrix} \quad O_{46} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{46} & 0 & \hat{s}_{46}^* \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\hat{s}_{46} & 0 & c_{46} \end{pmatrix} \quad O_{56} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{56} & \hat{s}_{56}^* \\ 0 & 0 & 0 & 0 & -\hat{s}_{56} & c_{56} \end{pmatrix}$$

没有，我已经很满足了



在此基础上计算 9×9 活性与惰性中微子混合矩阵 (H.C. Han, ZZX, 2110.12705) 。

$U = AU_0$ 和 R 分别描述轻和重中微子的带电流相互作用强度：

$$UD_\nu U^T + RD_N R^T = 0$$

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^*c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix} \begin{matrix} \text{轻中微子} \\ \text{衍生参数} \end{matrix}$$

$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin \theta_{ij} \quad (\text{for } 1 \leq i < j \leq 6)$$

$$c_{ij} \equiv \cos \theta_{ij}$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ -c_{14}c_{15}\hat{s}_{16}\hat{s}_{26}^* - c_{14}\hat{s}_{15}\hat{s}_{25}^*c_{26} & c_{24}c_{25}c_{26} & 0 \\ -\hat{s}_{14}\hat{s}_{24}^*c_{25}c_{26} & & \\ -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}_{36}^* + c_{14}\hat{s}_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & -c_{24}c_{25}\hat{s}_{26}\hat{s}_{36}^* - c_{24}\hat{s}_{25}\hat{s}_{35}^*c_{36} & c_{34}c_{35}c_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}_{35}^*c_{36} + \hat{s}_{14}\hat{s}_{24}^*c_{25}\hat{s}_{26}\hat{s}_{36}^* & -\hat{s}_{24}\hat{s}_{34}^*c_{35}c_{36} & \\ +\hat{s}_{14}\hat{s}_{24}^*\hat{s}_{25}\hat{s}_{35}^*c_{36} - \hat{s}_{14}c_{24}\hat{s}_{34}^*c_{35}c_{36} & & \end{pmatrix}$$

轻与重中微子质量起源动力学味结构的原始参数

$$UU^\dagger + RR^\dagger = I$$

$$R = \begin{pmatrix} \hat{s}_{14}c_{15}c_{16} & \hat{s}_{15}c_{16} & \hat{s}_{16} \\ -\hat{s}_{14}^*c_{15}\hat{s}_{16}\hat{s}_{26}^* - \hat{s}_{14}^*\hat{s}_{15}\hat{s}_{25}^*c_{26} & -\hat{s}_{15}^*\hat{s}_{16}\hat{s}_{26}^* + c_{15}\hat{s}_{25}^*c_{26} & c_{16}\hat{s}_{26}^* \\ +c_{14}\hat{s}_{24}^*c_{25}c_{26} & & \\ -\hat{s}_{14}^*c_{15}\hat{s}_{16}c_{26}\hat{s}_{36}^* + \hat{s}_{14}^*\hat{s}_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & -\hat{s}_{15}^*\hat{s}_{16}c_{26}\hat{s}_{36}^* - c_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & c_{16}c_{26}\hat{s}_{36}^* \\ -\hat{s}_{14}^*\hat{s}_{15}c_{25}\hat{s}_{35}^*c_{36} - c_{14}\hat{s}_{24}^*c_{25}\hat{s}_{26}\hat{s}_{36}^* & +c_{15}c_{25}\hat{s}_{35}^*c_{36} & \\ -c_{14}\hat{s}_{24}^*\hat{s}_{25}\hat{s}_{35}^*c_{36} + c_{14}c_{24}\hat{s}_{34}^*c_{35}c_{36} & & \end{pmatrix}$$

A 和 R 中实际包含6个独立CP破坏相位或其组合

- ◆ 轻和重中微子的标准带电流相互作用：

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_L} \gamma^\mu \left[\underset{\text{振荡}}{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + \underset{\text{衰变}}{R} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L \right] W_\mu^- + \text{h.c.}$$



Pontecorvo-Maki-Nakagawa-Sakata (PMNS) 轻子混合矩阵 $U = AU_0$ 的么正性完好度已被检验到 **0.1%** 的精度。

- ◆ 利用原始的跷跷板味参数计算所有中微子可观测量，有助于限制该机制的参数空间：

$$M_\nu \equiv U_0 D_\nu U_0^T = (iA^{-1}R) D_N (iA^{-1}R)^T$$

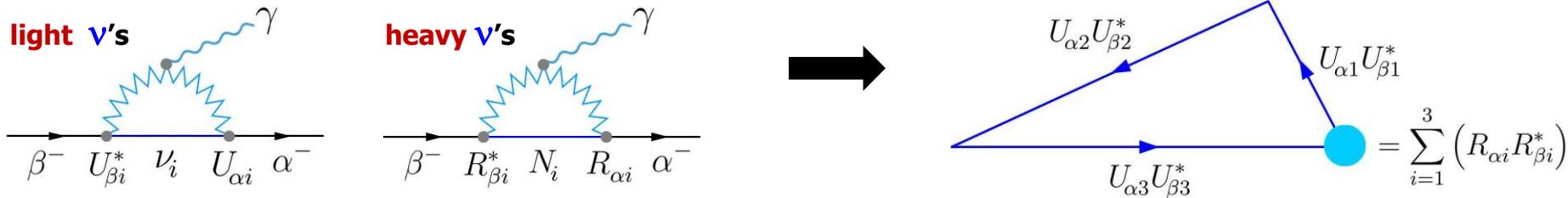
$$\begin{cases} D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \text{Diag}\{M_1, M_2, M_3\} \\ \Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \end{cases} \quad \begin{matrix} \uparrow \\ A^{-1}R \simeq \end{matrix} \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix}$$

- 中微子振荡：味破坏
- 带电轻子味破坏过程
- 轻子数破坏过程

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在跷跷板机制中，轻和重中微子可引发带电轻子的辐射衰变（**轻子味破坏**），但尚未被实验观测到。



可在完整跷跷板机制 (ZZX, D. Zhang, 2009.09717) 或 seesaw EFT 框架 (D. Zhang, S. Zhou, 2107.12133) 中计算:

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \rightarrow \alpha^- + \gamma)}{\Gamma(\beta^- \rightarrow \alpha^- + \bar{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\text{em}}}{2\pi} \left| \sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \left(-\frac{5}{6} + \frac{1}{4} \cdot \frac{m_i^2}{M_W^2} \right) - \frac{1}{3} \sum_{i=1}^3 R_{\alpha i} R_{\beta i}^* \right|^2 \simeq \frac{3\alpha_{\text{em}}}{8\pi} \left| \sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \right|^2$$

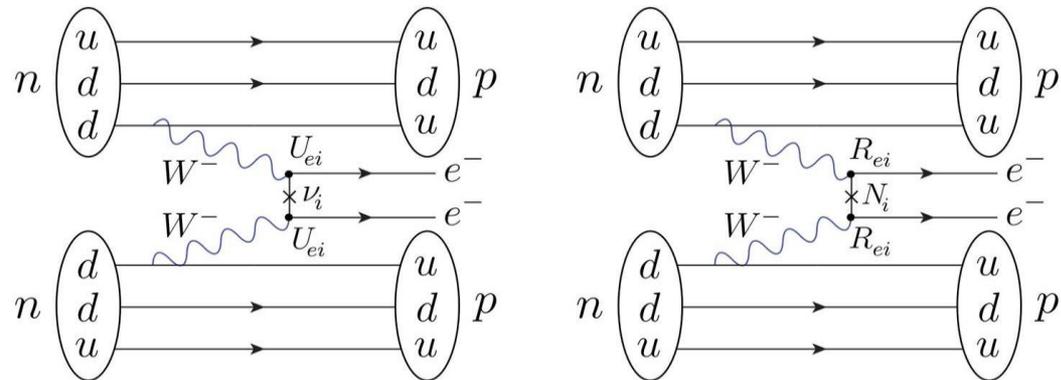
从而利用目前的实验数据（上限）限制**轻子么正多边形**简化为**有效三角形的有效顶点的尺寸**:

$$\left| \sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \right| = \left| \sum_{i=1}^3 R_{\alpha i} R_{\beta i}^* \right| \simeq \sqrt{\frac{8\pi \xi_{\alpha\beta}}{3\alpha_{\text{em}}}} \simeq 33.88 \sqrt{\xi_{\alpha\beta}} \longrightarrow \left\{ \begin{array}{l} \left| \sum_{i=1}^3 U_{ei} U_{\mu i}^* \right| = \left| \sum_{i=1}^3 R_{ei} R_{\mu i}^* \right| < \underline{2.20 \times 10^{-5}} \\ \left| \sum_{i=1}^3 U_{ei} U_{\tau i}^* \right| = \left| \sum_{i=1}^3 R_{ei} R_{\tau i}^* \right| < \underline{1.46 \times 10^{-2}} \\ \left| \sum_{i=1}^3 U_{\mu i} U_{\tau i}^* \right| = \left| \sum_{i=1}^3 R_{\mu i} R_{\tau i}^* \right| < 1.70 \times 10^{-2} \end{array} \right.$$

施加 μ - τ reflection symmetry, 可得更强的限制:

$$\left| \sum_{i=1}^3 U_{ei} U_{\tau i}^* \right| = \left| \sum_{i=1}^3 R_{ei} R_{\tau i}^* \right| < 2.20 \times 10^{-5}$$

在跷跷板机制中，轻和重中微子可引发无中微子双贝塔衰变（**轻子数破坏**），但尚未被实验发现。

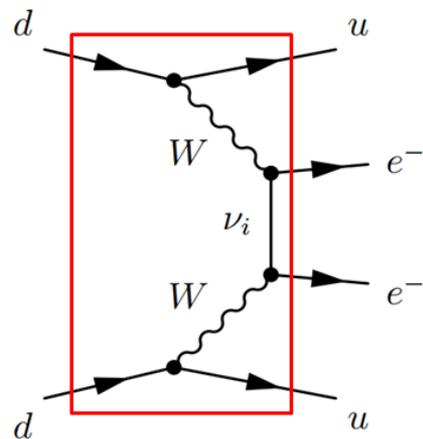


Seesaw + Unitarity:

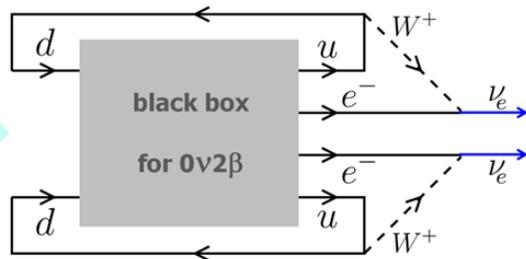
$$\sum_{i=1}^3 m_i U_{ei}^2 + \sum_{j=1}^3 M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^3 |U_{ei}|^2 + \sum_{j=1}^3 |R_{ej}|^2 = 1$$

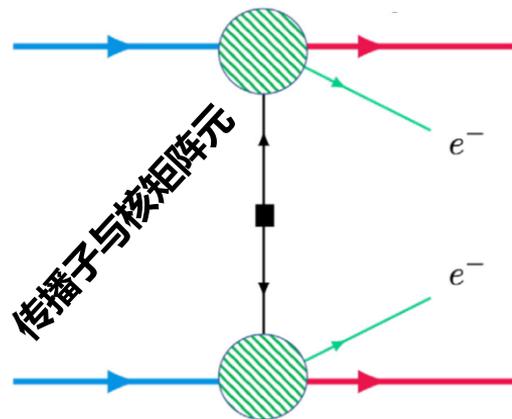
Schechter-Valle定理 (1982)：若发现 **0ν2β** 信号，则中微子具有**马约拉纳**属性



将中微子导出来

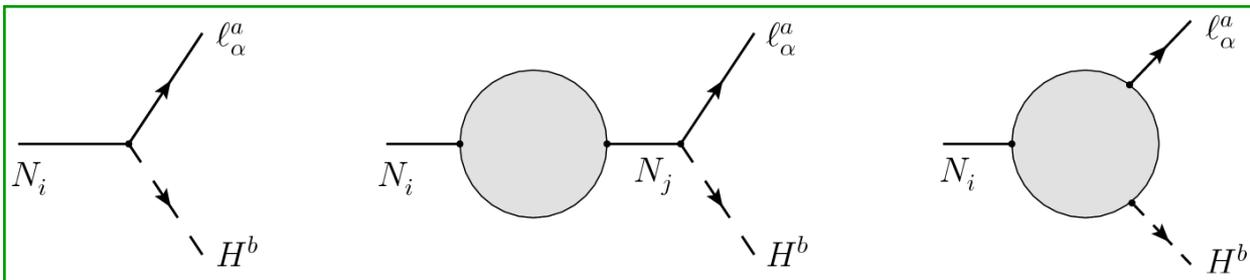


重要：马约拉纳费米子，既是新物理，也是新物态



J.M. Yao et al, PNP 2022

在跷跷板机制中，重马约拉纳中微子的衰变过程既破坏轻子数，又破坏CP对称性，可导致轻子生成



M. Fukugita, T. Yanagida 1986

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma(N_i \rightarrow \ell_\alpha + H) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \bar{H})}{\sum_\alpha [\Gamma(N_i \rightarrow \ell_\alpha + H) + \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \bar{H})]}$$

$$= \frac{1}{8\pi(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ii}} \sum_{j \neq i} \left\{ \text{Im} \left[(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij} \xi(x_{ji}) + (\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij}^* \zeta(x_{ji}) \right] \right\}$$

$$\mathcal{Y}_\nu \equiv Y_\nu U_0^*$$

$$x_{ji} \equiv M_j^2 / M_i^2$$

其中 $\xi(x_{ji}) = \sqrt{x_{ji}} \{ 1 + 1/(1 - x_{ji}) + (1 + x_{ji}) \ln [x_{ji}/(1 + x_{ji})] \}$, $\zeta(x_{ji}) = 1/(1 - x_{ji})$

◆ CP破坏可导致轻子数不对称，后者通过电弱反常效应导致重子数不对称，可解释宇宙的原初反物质消失之谜——轻子生成机制 (leptogenesis)。

◆ 但跷跷板机制只是一个定性有效的机制，其味结构完全未知，一般情况下没有任何定量预言能力。

第二讲

- ◆ 超越标准模型的阳关道
- ◆ 味结构分块参数化方案
- ◆ 轻子味和轻子数不守恒
- ◆ 物质与反物质的不对称

利用跷跷板机制中的原始味参数，计算轻马约拉纳中微子振荡过程中的Jarlskog CP破坏不变量：

$$UD_\nu U^T + RD_N R^T = 0 \longrightarrow M_\nu \equiv U_0 D_\nu U_0^T = (iA^{-1}R) D_N (iA^{-1}R)^T$$

自由度统计 (mass + mixing angle + CPV phase): **3 + 3 + 3 (derivational)** ← **3 + 9 + 6 (original)**

◆ 不计微小的么正性破坏效应，Jarlskog 不变量定义如下：

$$\mathcal{J}_\nu \sum_\gamma \varepsilon_{\alpha\beta\gamma} \sum_k \varepsilon_{ijk} = \text{Im} \left[(U_0)_{\alpha i} (U_0)_{\beta j} (U_0)_{\alpha j}^* (U_0)_{\beta i}^* \right]$$

$$\begin{cases} D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \text{Diag}\{M_1, M_2, M_3\} \\ \Delta_{ij} \equiv m_i^2 - m_j^2 \end{cases}$$

◆ 一方面，可以通过下式给出 Jarlskog 不变量 (ZZX, arXiv:2306.02362)

$$\mathcal{J}_\nu = \prod_{i>j} \Delta_{ij}^{-1} \text{Im} \left[(M_\nu M_\nu^\dagger)_{e\mu} (M_\nu M_\nu^\dagger)_{\mu\tau} (M_\nu M_\nu^\dagger)_{\tau e} \right]$$

◆ 另一方面，可以通过跷跷板关系，使用原始的跷跷板味参数计算上式中的质量矩阵元和质量平方差，进而表达出 Jarlskog 不变量。

但表达式太冗长，这里退而求其次，选取最小跷跷板机制举例说明。

$$A^{-1}R \simeq \begin{pmatrix} \hat{S}_{14}^* & \hat{S}_{15}^* & \hat{S}_{16}^* \\ \hat{S}_{24}^* & \hat{S}_{25}^* & \hat{S}_{26}^* \\ \hat{S}_{34}^* & \hat{S}_{35}^* & \hat{S}_{36}^* \end{pmatrix}$$

最小跷跷板机制只包含两个右手中微子场，因此在树图阶导致一个轻中微子质量为零，相应的马约拉纳相位消失。该benchmark机制的**原始参数11个**：两个重中微子质量、6个混合角、3个CP相位；与轻中微子相关的**衍生参数7个**：两个轻中微子质量、3个混合角、2个CP相位。

$$\alpha \equiv \delta_{14} - \delta_{15}, \beta \equiv \delta_{24} - \delta_{25}, \gamma \equiv \delta_{34} - \delta_{35}$$

$$\begin{aligned} \text{Im} \left[(M_\nu M_\nu^\dagger)_{e\mu} (M_\nu M_\nu^\dagger)_{\mu\tau} (M_\nu M_\nu^\dagger)_{\tau e} \right] &= C_0 [C_\alpha \sin 2\alpha + C_\beta \sin 2\beta + C_\gamma \sin 2\gamma \\ &+ C_{\alpha+\beta} \sin(\alpha + \beta) + C_{\beta+\gamma} \sin(\beta + \gamma) + C_{\gamma+\alpha} \sin(\gamma + \alpha) \\ &+ C_{\alpha-\beta} \sin(\alpha - \beta) + C_{\beta-\gamma} \sin(\beta - \gamma) + C_{\gamma-\alpha} \sin(\gamma - \alpha)] = \mathcal{J}_\nu \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 \end{aligned}$$

◆ 系数

$$\begin{aligned} C_0 &= M_1^2 M_2^2 [s_{14}^2 (s_{25}^2 + s_{35}^2) + s_{24}^2 (s_{15}^2 + s_{35}^2) + s_{34}^2 (s_{15}^2 + s_{25}^2) \\ &- 2s_{14}s_{15}s_{24}s_{25} \cos(\alpha - \beta) - 2s_{24}s_{25}s_{34}s_{35} \cos(\beta - \gamma) - 2s_{14}s_{15}s_{34}s_{35} \cos(\gamma - \alpha)] \end{aligned}$$

$$C_\alpha = M_1 M_2 s_{14}^2 s_{15}^2 (s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2)$$

$$C_{\alpha+\beta} = M_1 M_2 s_{14} s_{15} s_{24} s_{25} (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2)$$

$$C_\beta = M_1 M_2 s_{24}^2 s_{25}^2 (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2)$$

$$C_{\beta+\gamma} = M_1 M_2 s_{24} s_{25} s_{34} s_{35} (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2)$$

$$C_\gamma = M_1 M_2 s_{34}^2 s_{35}^2 (s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2)$$

$$C_{\gamma+\alpha} = M_1 M_2 s_{14} s_{15} s_{34} s_{35} (s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2)$$

◆ 结论：

low seesaw scale

$\mathcal{J}_\nu \leftarrow \text{9 ways} \rightarrow \alpha, \beta, \gamma$

$$C_{\alpha-\beta} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{34}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{35}^2] s_{14} s_{15} s_{24} s_{25}$$

$$C_{\beta-\gamma} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{14}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{15}^2] s_{24} s_{25} s_{34} s_{35}$$

$$C_{\gamma-\alpha} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{24}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{25}^2] s_{14} s_{15} s_{34} s_{35}$$

第一个重马约拉纳中微子的轻子数破坏过程的CP不对称度：

$$\varepsilon_{1e} = \frac{M_1^2 s_{14} s_{15}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) [s_{14} s_{15} \sin 2\alpha + s_{24} s_{25} \sin(\alpha + \beta) + s_{34} s_{35} \sin(\alpha + \gamma)] \right. \\ \left. + x_{21} \zeta(x_{21}) [s_{24} s_{25} \sin(\alpha - \beta) + s_{34} s_{35} \sin(\alpha - \gamma)] \right]$$

$$\varepsilon_{1\mu} = \frac{M_1^2 s_{24} s_{25}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) [s_{14} s_{15} \sin(\alpha + \beta) + s_{24} s_{25} \sin 2\beta + s_{34} s_{35} \sin(\beta + \gamma)] \right. \\ \left. + x_{21} \zeta(x_{21}) [s_{14} s_{15} \sin(\beta - \alpha) + s_{34} s_{35} \sin(\beta - \gamma)] \right]$$

$$\varepsilon_{1\tau} = \frac{M_1^2 s_{34} s_{35}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) [s_{14} s_{15} \sin(\alpha + \gamma) + s_{24} s_{25} \sin(\beta + \gamma) + s_{34} s_{35} \sin 2\gamma] \right. \\ \left. + x_{21} \zeta(x_{21}) [s_{14} s_{15} \sin(\gamma - \alpha) + s_{24} s_{25} \sin(\gamma - \beta)] \right]$$

$$\alpha \equiv \delta_{14} - \delta_{15}, \quad \beta \equiv \delta_{24} - \delta_{25}, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\varepsilon_1 = + \frac{M_1^2 x_{21} \xi(x_{21})}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma \right. \\ \left. + 2s_{14} s_{15} s_{24} s_{25} \sin(\alpha + \beta) + 2s_{14} s_{15} s_{34} s_{35} \sin(\alpha + \gamma) + 2s_{24} s_{25} s_{34} s_{35} \sin(\beta + \gamma) \right]$$

第二个重马约拉纳中微子的轻子数破坏过程的CP不对称度：

$$\varepsilon_{2e} = -\frac{M_2^2 s_{14} s_{15}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) [s_{14} s_{15} \sin 2\alpha + s_{24} s_{25} \sin(\alpha + \beta) + s_{34} s_{35} \sin(\alpha + \gamma)] \right. \\ \left. + x_{12} \zeta(x_{12}) [s_{24} s_{25} \sin(\alpha - \beta) + s_{34} s_{35} \sin(\alpha - \gamma)] \right]$$

$$\varepsilon_{2\mu} = -\frac{M_2^2 s_{24} s_{25}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) [s_{14} s_{15} \sin(\alpha + \beta) + s_{24} s_{25} \sin 2\beta + s_{34} s_{35} \sin(\beta + \gamma)] \right. \\ \left. + x_{12} \zeta(x_{12}) [s_{14} s_{15} \sin(\beta - \alpha) + s_{34} s_{35} \sin(\beta - \gamma)] \right]$$

$$\varepsilon_{2\tau} = -\frac{M_2^2 s_{34} s_{35}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) [s_{14} s_{15} \sin(\alpha + \gamma) + s_{24} s_{25} \sin(\beta + \gamma) + s_{34} s_{35} \sin 2\gamma] \right. \\ \left. + x_{12} \zeta(x_{12}) [s_{14} s_{15} \sin(\gamma - \alpha) + s_{24} s_{25} \sin(\gamma - \beta)] \right]$$

$$\alpha \equiv \delta_{14} - \delta_{15}, \quad \beta \equiv \delta_{24} - \delta_{25}, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\varepsilon_2 = -\frac{M_2^2 x_{12} \xi(x_{12})}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma \right. \\ \left. + 2s_{14} s_{15} s_{24} s_{25} \sin(\alpha + \beta) + 2s_{14} s_{15} s_{34} s_{35} \sin(\alpha + \gamma) + 2s_{24} s_{25} s_{34} s_{35} \sin(\beta + \gamma) \right]$$

轻中微子振荡与重马约拉纳中微子衰变的CP破坏效应对原始CP相位的依赖 (ZZX, 2306.02362)

CP不守恒量之间一般不存在正比关系

	2α	2β	2γ	$\alpha + \beta$	$\beta + \gamma$	$\alpha + \gamma$	$\alpha - \beta$	$\beta - \gamma$	$\gamma - \alpha$
\mathcal{J}_ν	✓	✓	✓	✓	✓	✓	✓	✓	✓
ε_{1e}	✓			✓		✓	✓		✓
$\varepsilon_{1\mu}$		✓		✓	✓		✓	✓	
$\varepsilon_{1\tau}$			✓		✓	✓		✓	✓
ε_1	✓	✓	✓	✓	✓	✓			
ε_{2e}	✓			✓		✓	✓		✓
$\varepsilon_{2\mu}$		✓		✓	✓		✓	✓	
$\varepsilon_{2\tau}$			✓		✓	✓		✓	✓
ε_2	✓	✓	✓	✓	✓	✓			

◆ 我们早已有了中微子与物质相互作用的标准理论：**标准模型**；但还没有中微子质量起源的标准理论。

◆ 对标准模型理论框架的最自然、最小尺度扩充，导致中微子质量起源的**正统跷跷板机制**。后果：中微子具有**马约拉纳属性**（新物理和新物态）；**轻子生成机制**原则上起作用，理解宇宙的原初反物质消失之谜（一石二鸟）

◆ 但是包括**跷跷板机制**在内的所有中微子质量起源模型，都是**定性**有效的，没有**定量**预言能力，原因在于它们与标准模型一样，都无法确定**费米子的味结构**。这是所有理论家面临的困境：标准模型的自由参数已经够多了，新物理带来更多的新参数。这与**费米的大象**原则背道而驰，不好，非常不好！

◆ 敢问路在何方：1）探索**味对称性**（聪明人）；2）发展**跷跷板有效场论**（聪明人）；3）给出跷跷板结构的一般且明确的**分块欧拉型参数化**（我老人家自娱自乐）；4）**新想法**和**新手段**（天才 + 年轻人）....



LÉONARD DE VINCI

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason

