Reactor Neutrino Experiments

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提 纲

① 反应堆中微子实验
② **反应堆产生中微子**（为什么要近点实验厅）
③ **天然放射性**（为什么要泡在水里，要3层探测器）
④ **宇宙线**（为什么要到地下，要有反符合）
⑤ **液体闪烁体**（为什么能探测到中微子）
⑥ 探测器响应与刻度（看到的数字信号与物理量）
⑦ 事例的构成与挑选（从物理量到中微子事例）
⑧ **本底**（真的是中微子吗？）
⑨ 效率与误差（不能多挑也不能少挑）
⑩ χ^2分析（从中微子事例数到振荡）

1 – Reactor Neutrino Experiments

Hanford experiment

◆ **Hanford reactor**

◆ **$0.3m^3$ liquid scintillator**

◆ **90 2" PMTs**

◆ **Paraffin to shield neutrons**

◆ **Lead to shield gammas**

◆ **Expected events: 0.1~0.3 /min**

◆ **Observed: 5/min（bkg >>signals）**
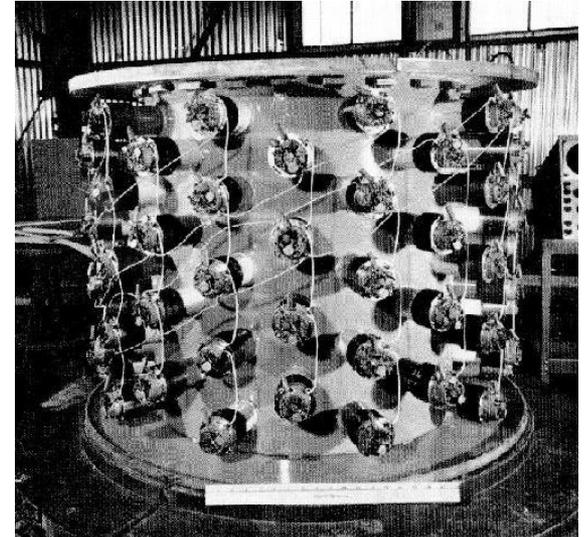
Cowan: The lesson of the work was clear: It is easy to shield out the noise men make, but impossible to shut out the cosmos.

Savannah River experiment：12 m rock overburden + veto → Discovery of Neutrino

Savannah River experiment

◆ **The first observation of neutrinos in 1956 by Reines & Cowan.**

  ⇨ **Inverse beta decay in CdCl$_3$ water solution → coincidence of prompt and delayed signal**

  ⇨ **Liquid scintillator + PMTs**

  ⇨ **Underground**

◆ **Modern experiments are still quite similar, except**

  ⇨ **Loading Gd into liquid scintillator**

  ⇨ **Larger, better detector**
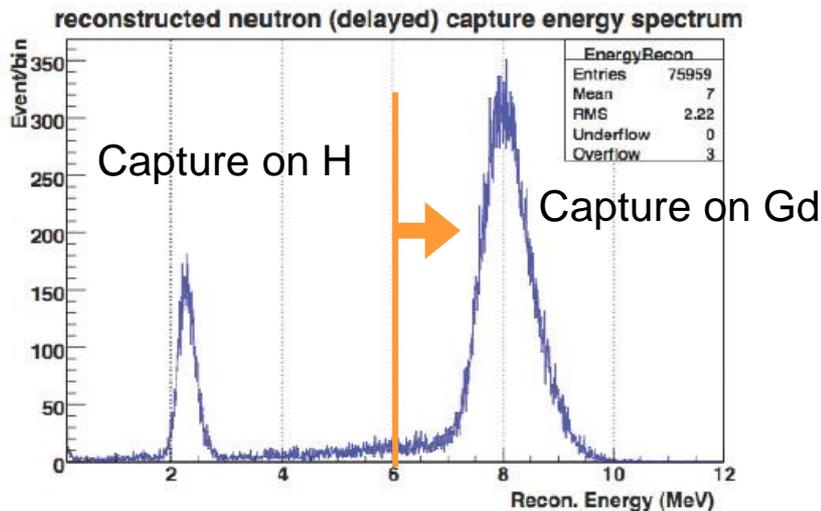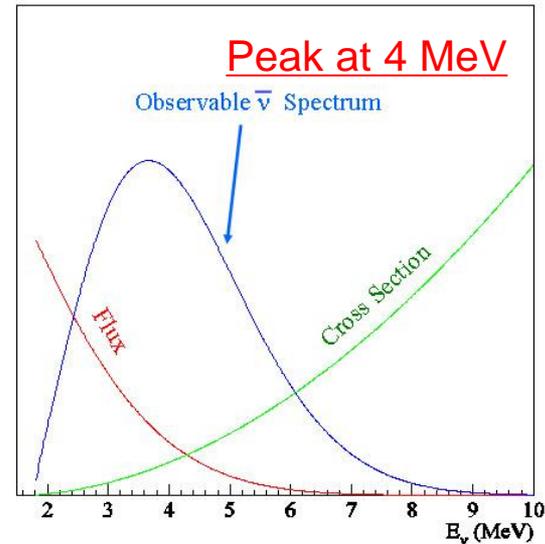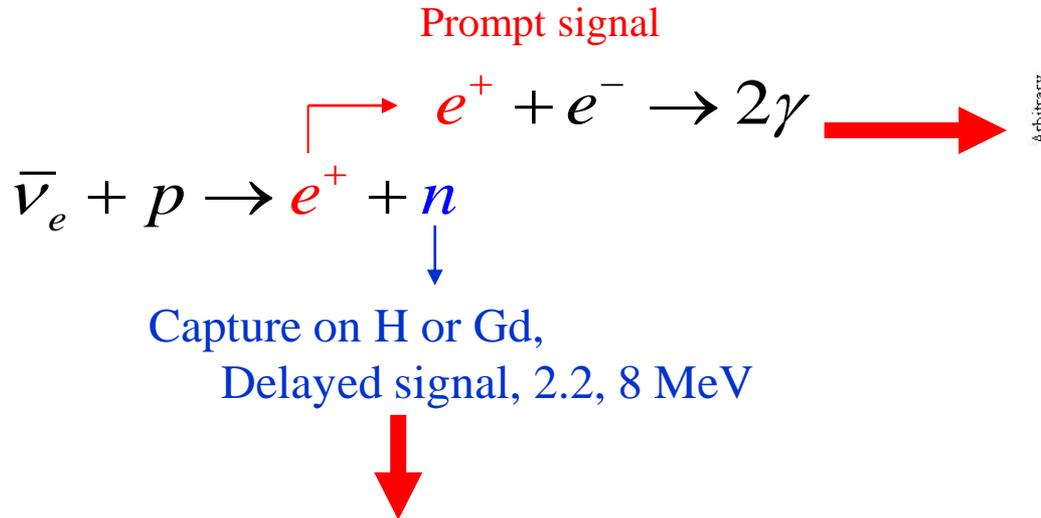
  ⇨ **Deeper underground, better shielding**

$$\overline{\nu}_e + p \rightarrow e^+ + n$$
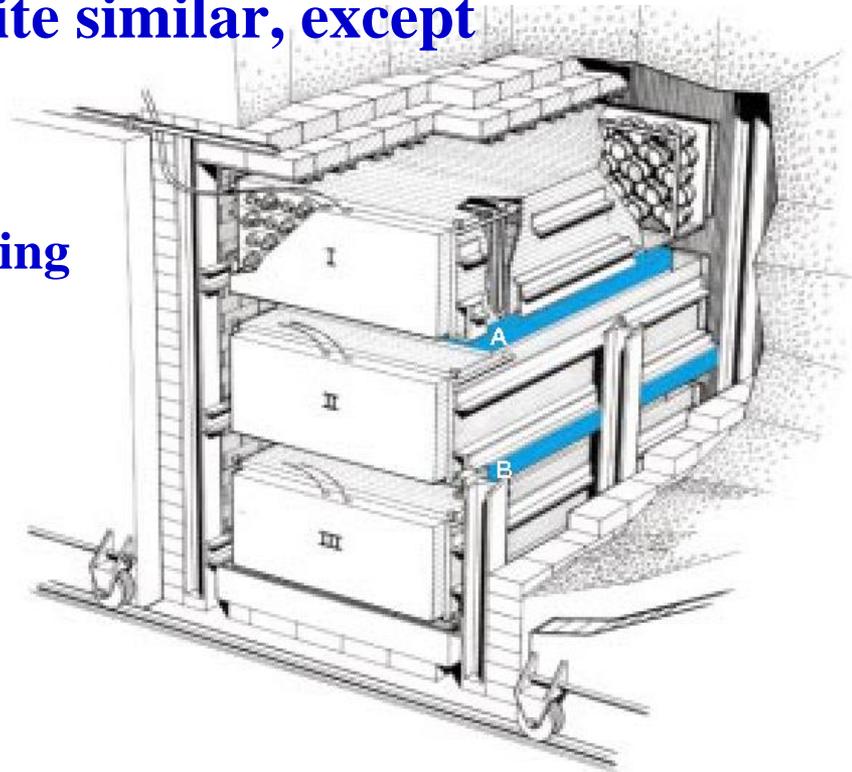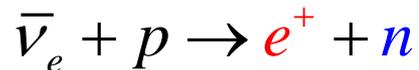
Reactor Antineutrinos
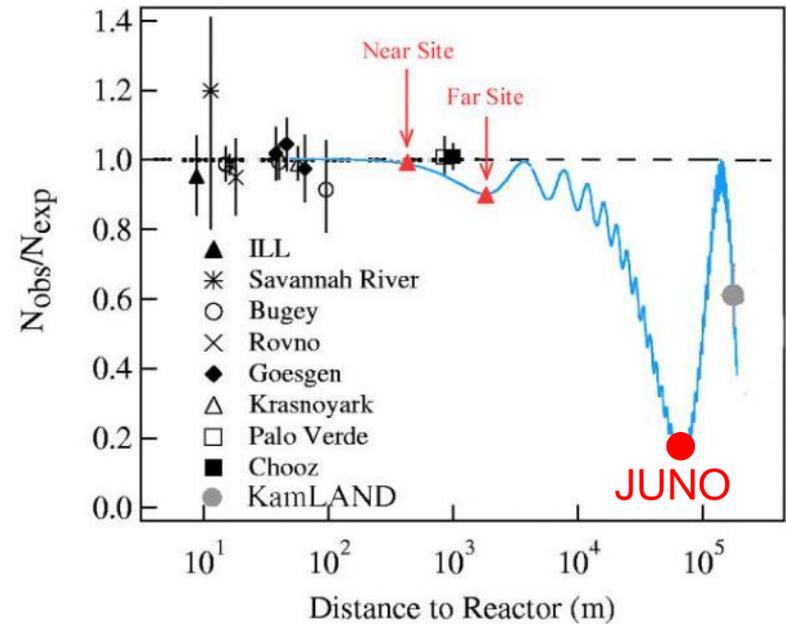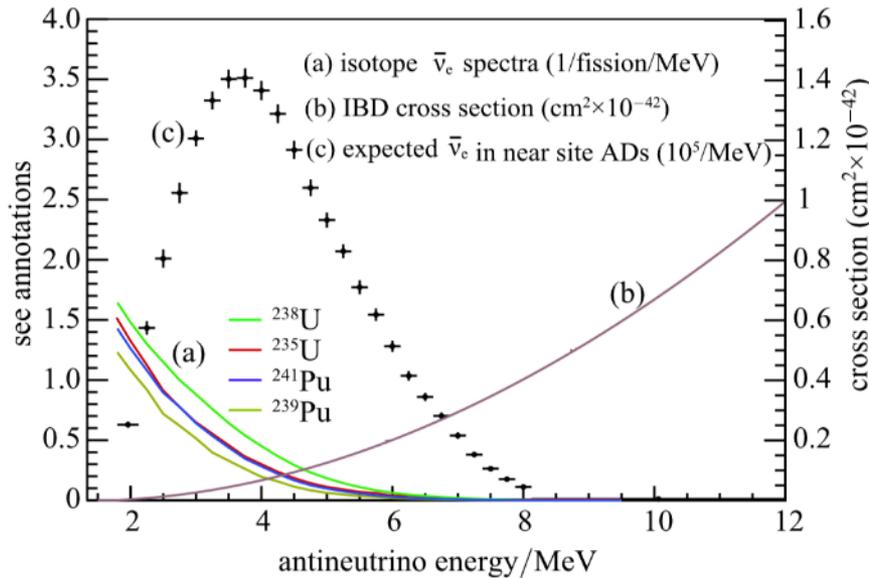
- ◆ **Reactor antineutrino: $\overline{\nu}_e$ emitted as fission products decay**
- ◆ **Commercial reactor (LEU) ^{235}U, ^{238}U, ^{239}Pu, ^{241}Pu; Research HEU (^{235}U)**
- ◆ **Usually detected via Inverse Beta Decay (IBD)**

~10 m	Very Short Baseline	Rate anomaly → sterile nu	
~100 m	θ_{13} exp. Near Detector	Spectrum anomaly	
~1 km	θ_{13} exp. Far Detector	θ_{13}	
~60 km	KamLAND, JUNO	θ_{12}, Mass Ordering	

Neutrino Signal (IBD)

Prompt signal

$$e^+ + e^- \to 2\gamma$$

$$\bar{\nu}_e + p \to e^+ + n$$

Capture on H or Gd,
Delayed signal, 2.2, 8 MeV

Peak at 4 MeV

Observable $\bar{\nu}$ Spectrum

Flux

Cross Section

E_ν (MeV)

reconstructed neutron (delayed) capture energy spectrum

EnergyRecon	
Entries	75959
Mean	7
RMS	2.22
Underflow	0
Overflow	3

Capture on H

Capture on Gd

Recon. Energy (MeV)

Neutron capture after thermalization

Time constant ~30 us（0.1% Gd）

◆ **Inverse beta decay reaction,** proposed by Pontecorvo, called Cowan-Reines reaction

◆ **Coincidence of**

⇨ Prompt: positron, energy correlated to neutrino energy

⇨ Delayed: neutron capture

◆ **10,000 times bkg reduction**

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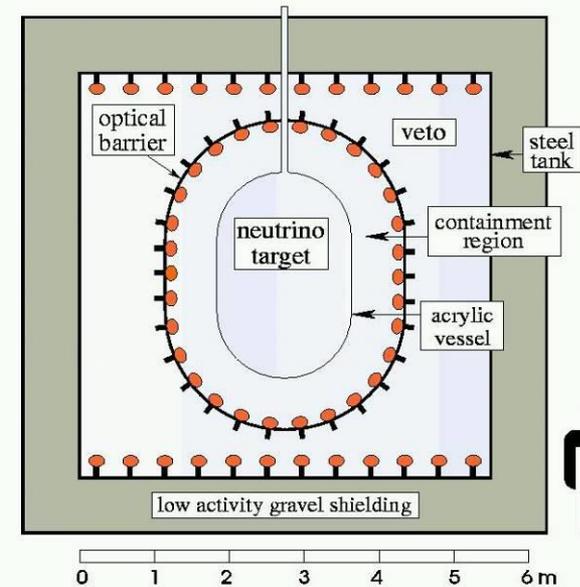# CHOOZ

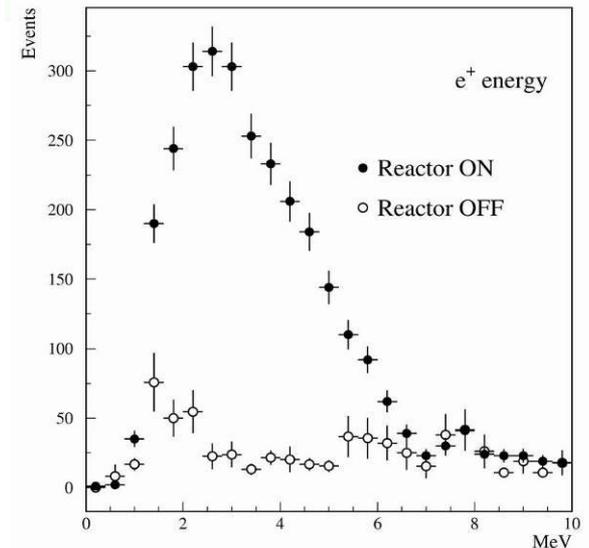**Baseline 1.05 km**

1997-1998, France

8.5 GWth

300 mwe

5 ton 0.1% Gd-LS

Bad Gd-LS

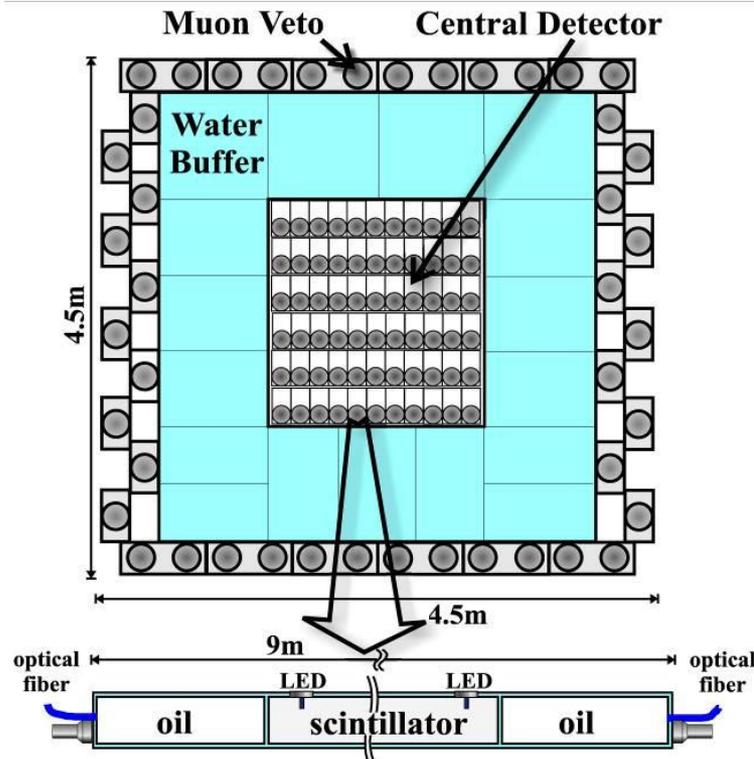$R=1.01\pm2.8\%(\text{stat}) \pm2.7\%(\text{syst}),\ \sin^2 2\theta_{13}<0.17$

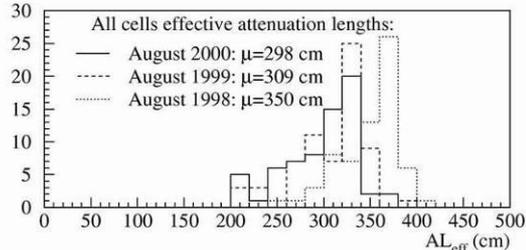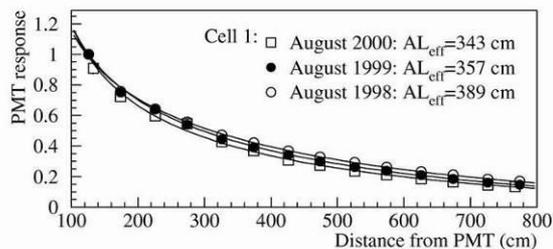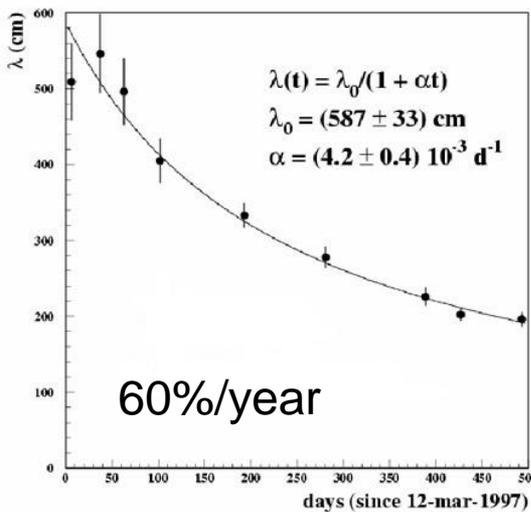| Parameter	Relative error
Reaction cross section	1.9 %
Number of protons	0.8 %
Detection efficiency	1.5 %
Reactor power	0.7 %
Energy released per fission	0.6 %
Combined	**2.7 %**

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Palo Verde

1998-1999, US

11.6 GWth

Segmented detector

12 ton 0.1% Gd-LS

Shallow overburden

32 mwe

Baseline 890m & 750m

$$\lambda(t) = \lambda_0/(1 + \alpha t)$$
$$\lambda_0 = (587 \pm 33) \text{ cm}$$
$$\alpha = (4.2 \pm 0.4) \, 10^{-3} \, d^{-1}$$

60%/year

Chooz Gd-LS

Cell 1:
- □ August 2000: AL_{eff}=343 cm
- ● August 1999: AL_{eff}=357 cm
- ○ August 1998: AL_{eff}=389 cm

All cells effective attenuation lengths:
- August 2000: μ=298 cm
- August 1999: μ=309 cm
- August 1998: μ=350 cm
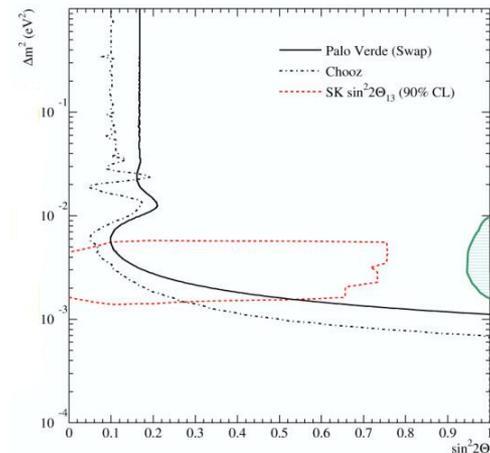
Palo Verde Gd-LS
1st year 12%, 2nd year 3%
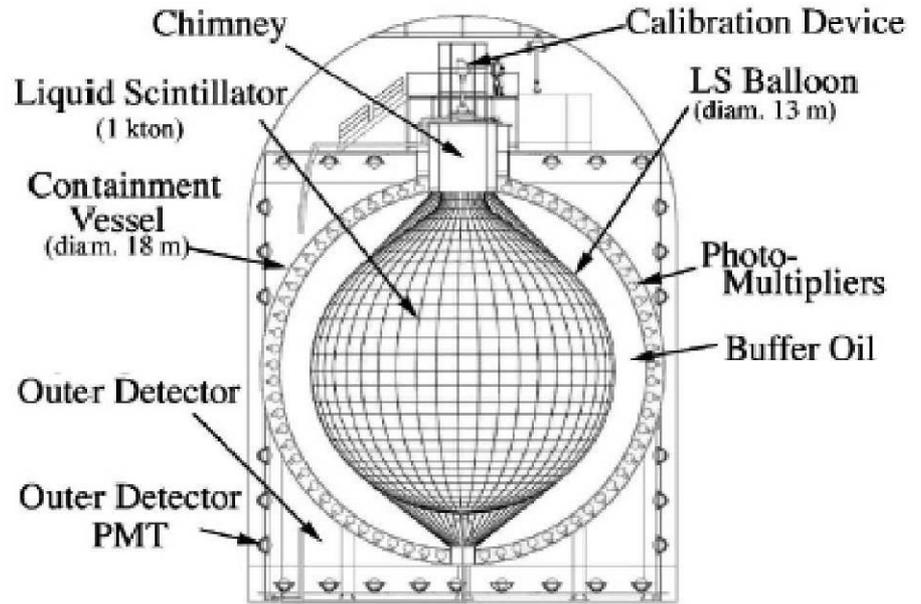
R=1.01±2.4%(stat) ±5.3%(syst)

- Palo Verde (Swap)
- Chooz
- SK $\sin^2 2\theta_{13}$ (90% CL)

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Baseline 180 km

2002-, Japan

53 reactors, 80 GWth

1000 ton LS

2700 mwe

**Radioactivity → fiducial cut,
Energy threshold**

TABLE I: Estimated systematic uncertainties (%).

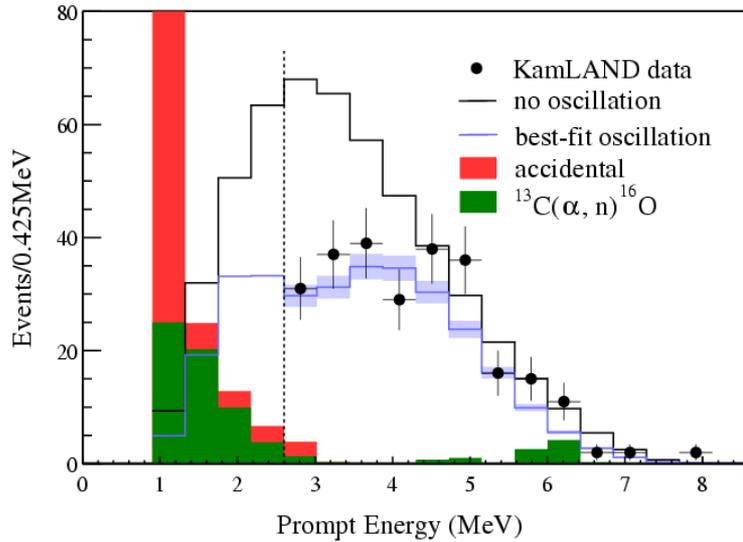| Fiducial Volume	4.7	Reactor power	2.1
Energy threshold	2.3	Fuel composition	1.0
Efficiency of cuts	1.6	$\overline{\nu}_e$ spectra [3]	2.5
Livetime	0.06	Cross section [5]	0.2
Total systematic uncertainty			6.5

KamLAND

R=0.658±0.044(stat) ±0.047(syst)

The first observation of reactor anti-neutrino disappearance

Confirmed antineutrino disappearance at 99.998% CL
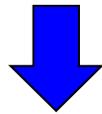
Excluded neutrino decay at 99.7% CL

Excluded decoherence at 94% CL
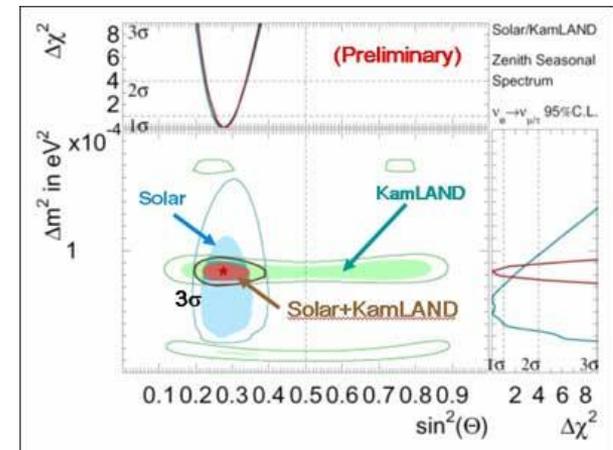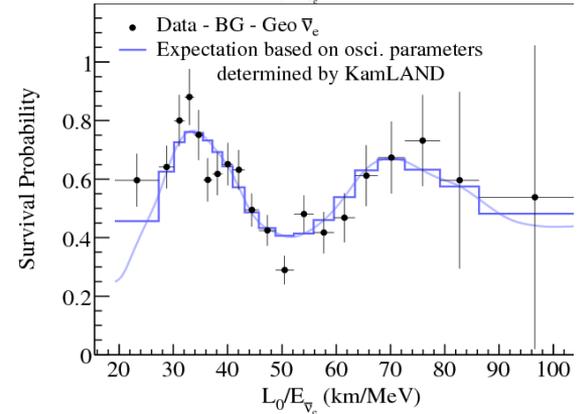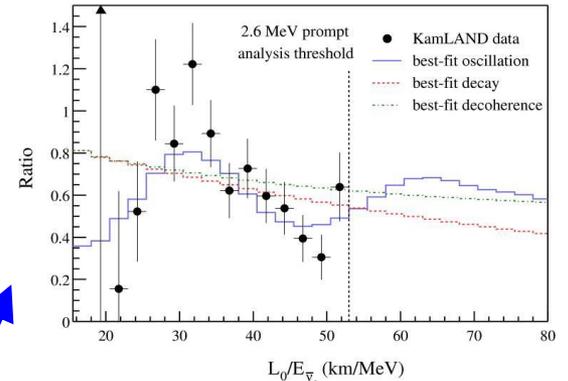
In a 3-ν framework

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$
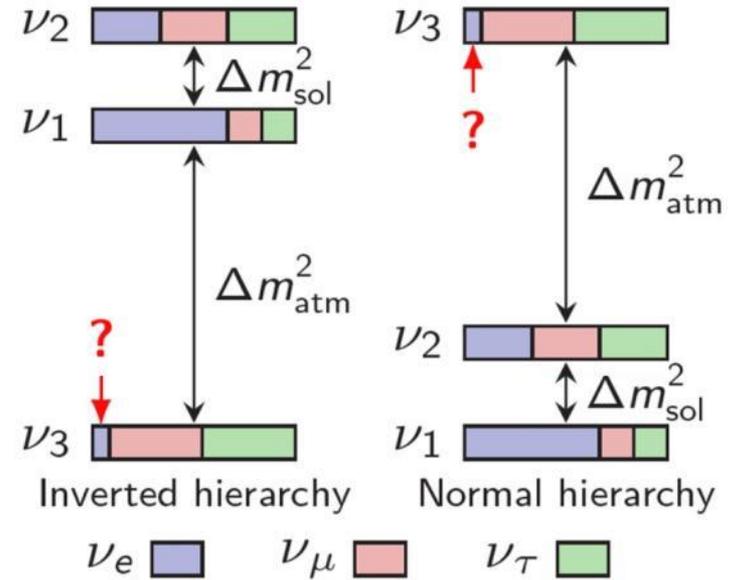

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta_{23} \sim 45°$	$\theta_{13} = ?$	$\theta_{12} \sim 34°$	
Atmospheric	Reactor	Solar	$0\nu\beta\beta$
Accelerator	Accelerator	Reactor	

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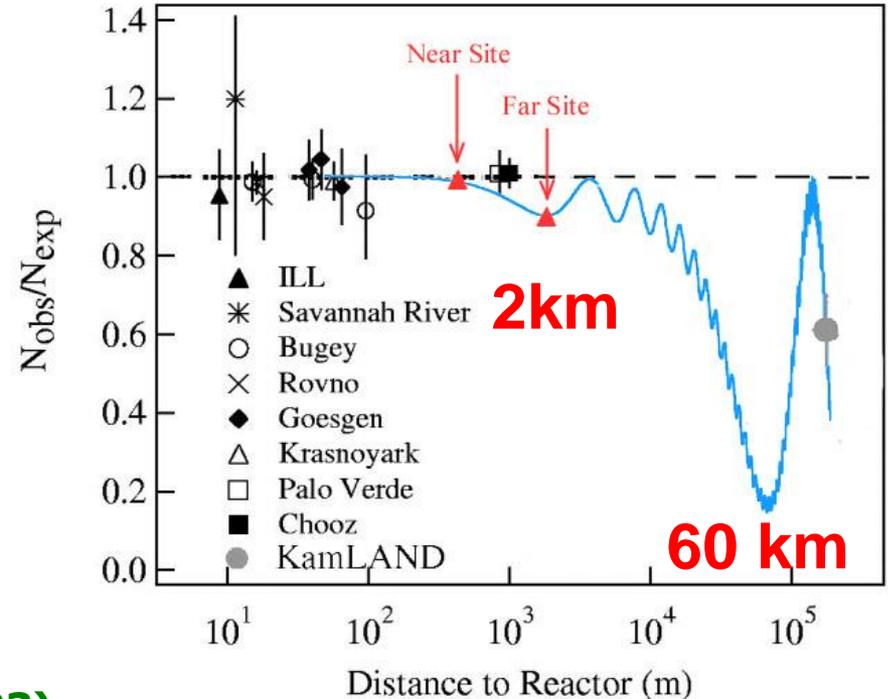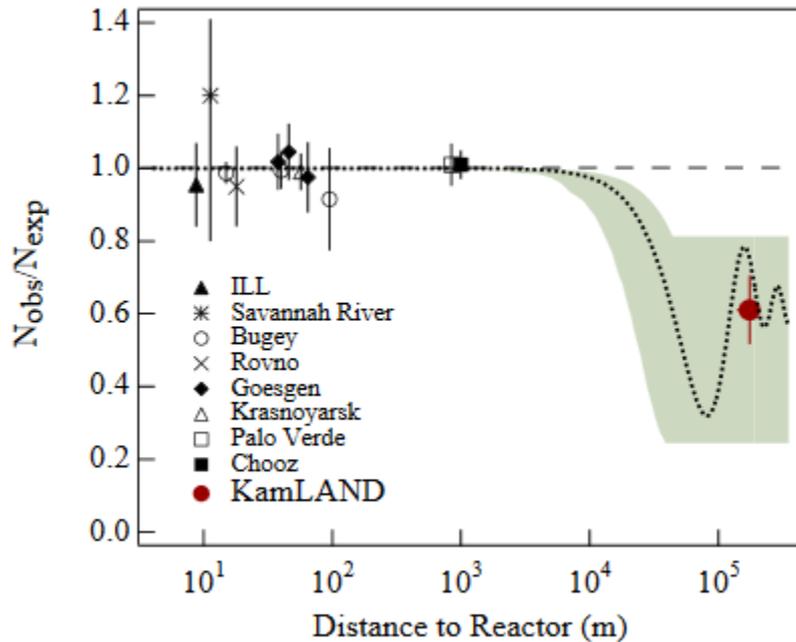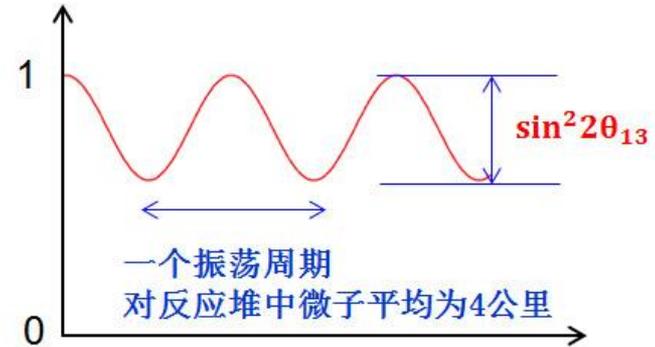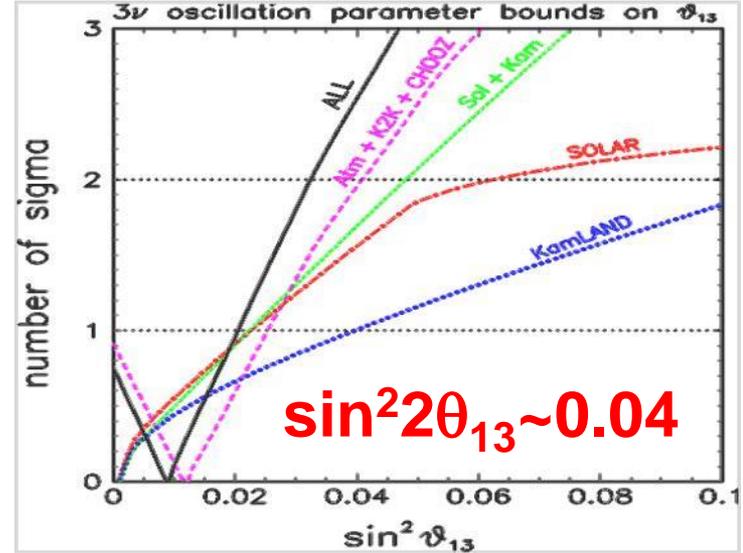# Reactor Neutrino Oscillation

飞行距离/中微子能量
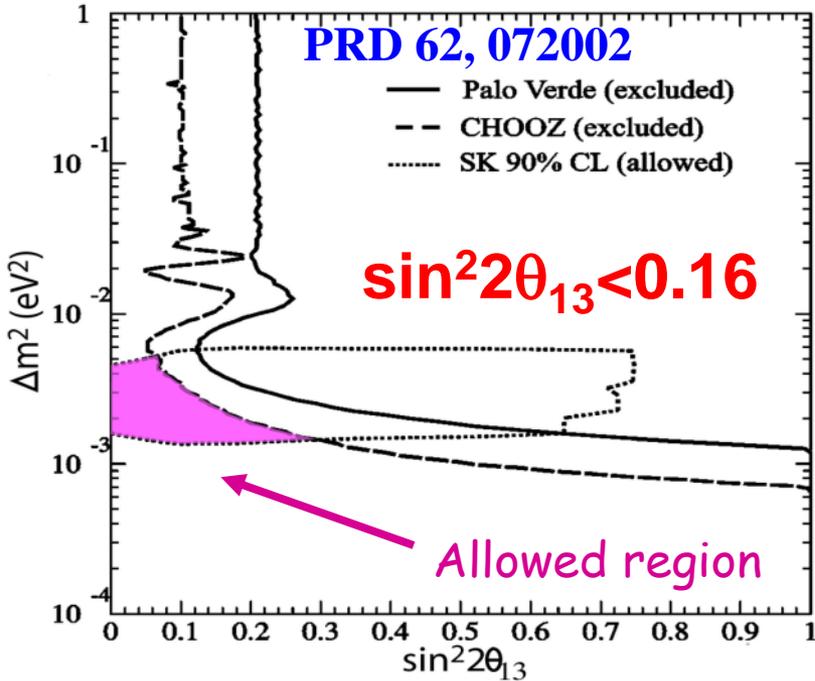
$$P_{sur} \approx 1 - \sin^2 2\theta_{13} \cdot \sin^2 \left(1.27 \cdot \Delta m^2_{31} \cdot \frac{L}{E}\right)$$

振幅大小   振荡频率

$\sin^2 2\theta_{13}$

一个振荡周期
对反应堆中微子平均为4公里

2km

60 km

KamLAND: Phys.Rev.Lett.90, 021802 (2003)

How large is θ_{13}?

PRD 62, 072002

— Palo Verde (excluded)
-- CHOOZ (excluded)
··· SK 90% CL (allowed)

$\sin^2 2\theta_{13} < 0.16$

Allowed region

3ν oscillation parameter bounds on ϑ_{13}

$\sin^2 2\theta_{13} \sim 0.04$

Fogli et al., hep-ph/0506307

Gonzalez-Garcia et al., JHEP1004:056, 2010

$\sin^2 2\theta_{13} \sim 0.04$

Atm & LBL & CHOOZ

Solar & KamLAND

ALL ν oscillation data 2008

ALL + MINOS 2009
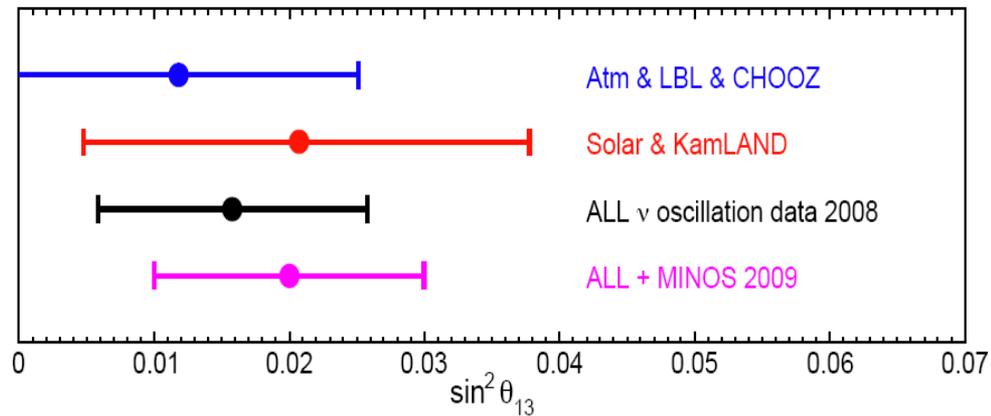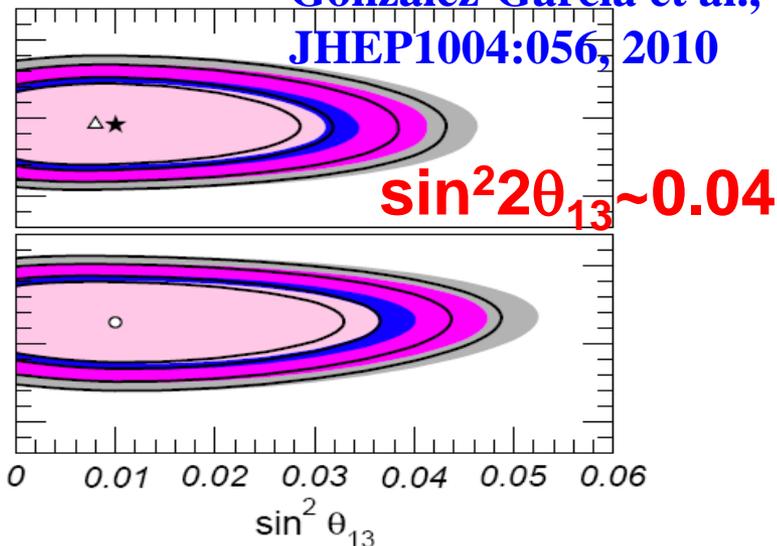
$\sin^2 2\theta_{13} \sim 0.08$, non-zero 2σ

Fogli et al., J.Phys.Conf.Ser.203:012103 (2010)

How to measure θ₁₃
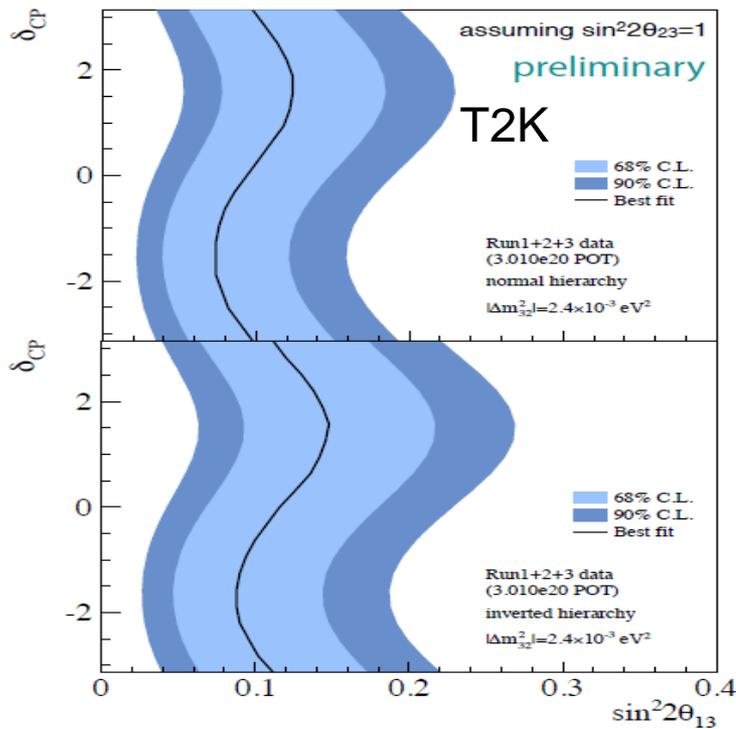
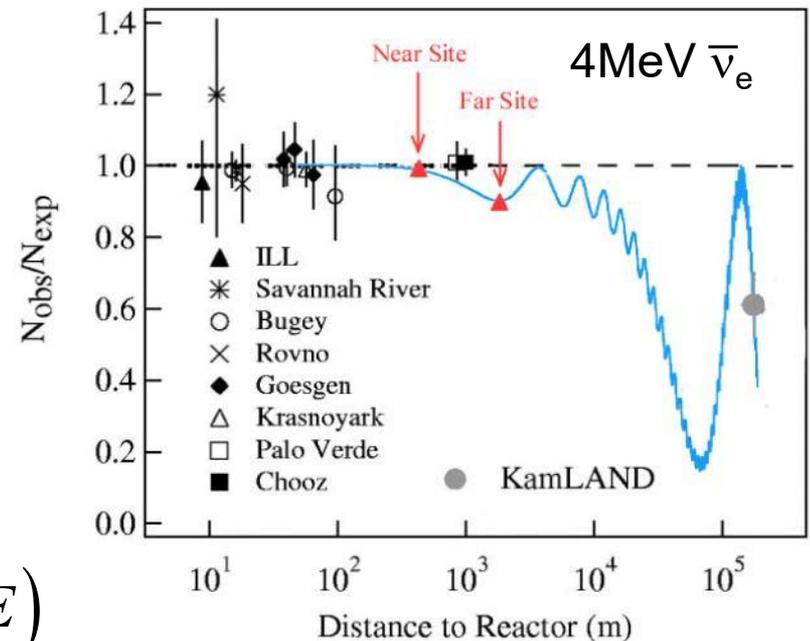**Accelerator** (appearance)
  Related with CPV and matter effect

$$P_{\nu_\mu \to \nu_e} = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2\left(\Delta m_{31}^2 L / 4E\right)$$

$$+ \text{(CPV term)} + \text{(matter term)}$$

$$+ \cdots\cdots$$

Reactor (disappearance)
  Clean in physics, only related to θ₁₃
  Precision measurement

$$P_{\overline{\nu}_e \to \overline{\nu}_e} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\Delta m_{31}^2 L / 4E\right)$$

$$- \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2\left(\Delta m_{21}^2 L / 4E\right)$$

Precision Measurement at Reactors

Major sources of uncertainties:

- ◆ **Reactor related   ~2%**
- ◆ **Detector related   ~2%**
- ◆ **Background   1~3%**

Lessons from past experience:

- ◆ **CHOOZ: Good Gd-LS**
- ◆ **Palo Verde: Better shielding**
- ◆ **KamLAND: No fiducial cut**

> **Near-far relative measurement**
> **Mikaelyan and Sinev, hep-ex/9908047**

| Parameter	Error	Near-far
Reaction cross section	**1.9 %**	**0**
Energy released per fission	**0.6 %**	**0**
Reactor power	**0.7 %**	**~0.1%**
Number of protons	**0.8 %**	**< 0.3%**
Detection efficiency	**1.5 %**	**0.2~0.6%**
CHOOZ Combined	**2.7 %**	**< 0.6%**

Proposed Reactor Experiments

Krasnoyarsk, Russia

Braidwood, USA

KASKA, Japan

RENO, Korea

Double Chooz, France

Diablo Canyon, USA

Daya Bay, China

Angra, Brazil

8 proposals, most in 2003 **(3 on-going)**
- **Fundmental parameter**
- **Gateway to ν-CPV and Mass Hierachy measurements**
- **Less expensive**

The Daya Bay Experiment

- **6 reactor cores, 17.4 GW$_{th}$**
- **Relative measurement**
  - **2 near sites, 1 far site**
- **Multiple detector modules**
- **Good cosmic shielding**
  - **250 m.w.e @ near sites**
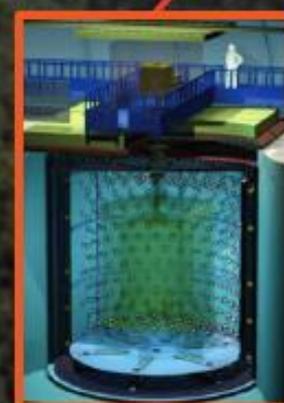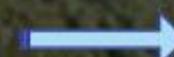 - **860 m.w.e @ far site**
- **Redundancy**

Double Chooz

P_{sur} vs L [km]

Daya Bay
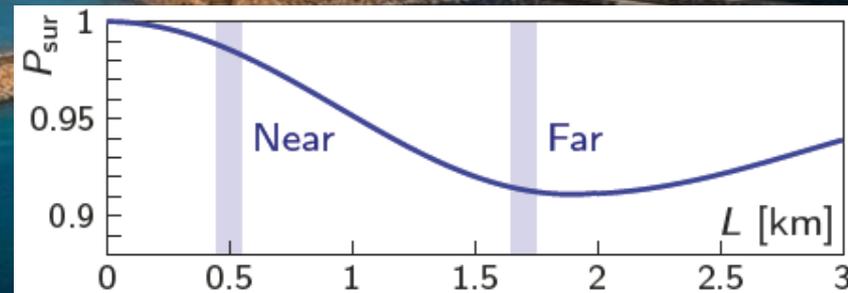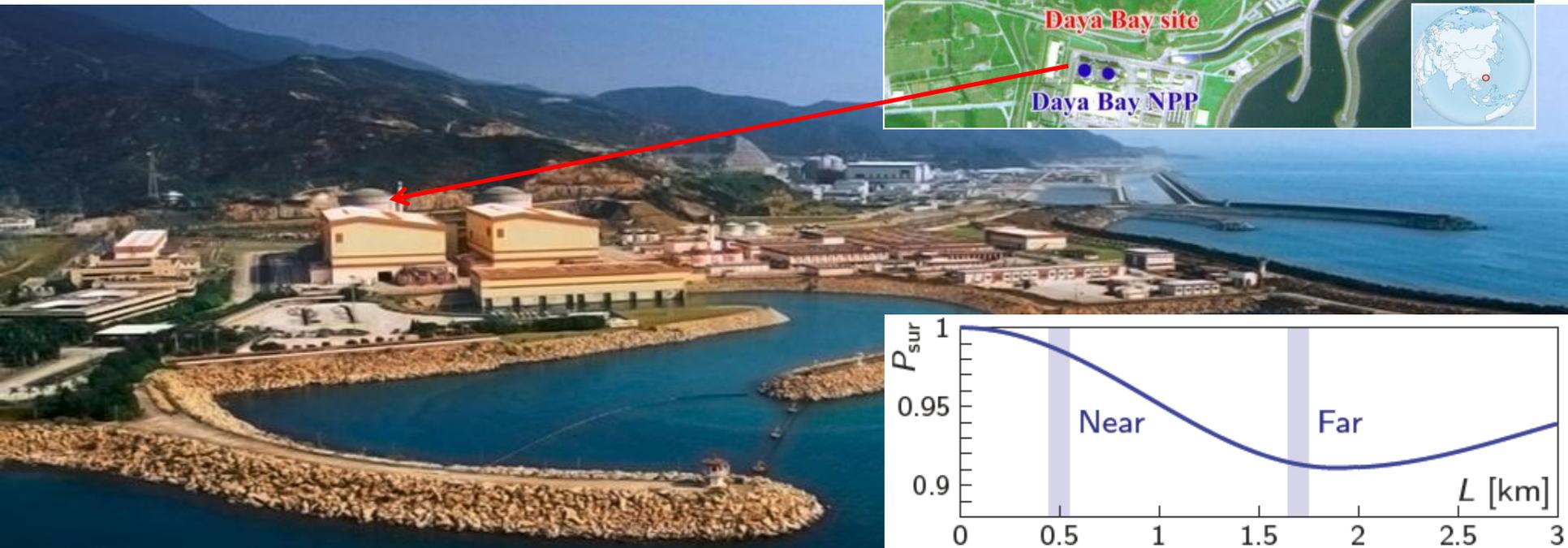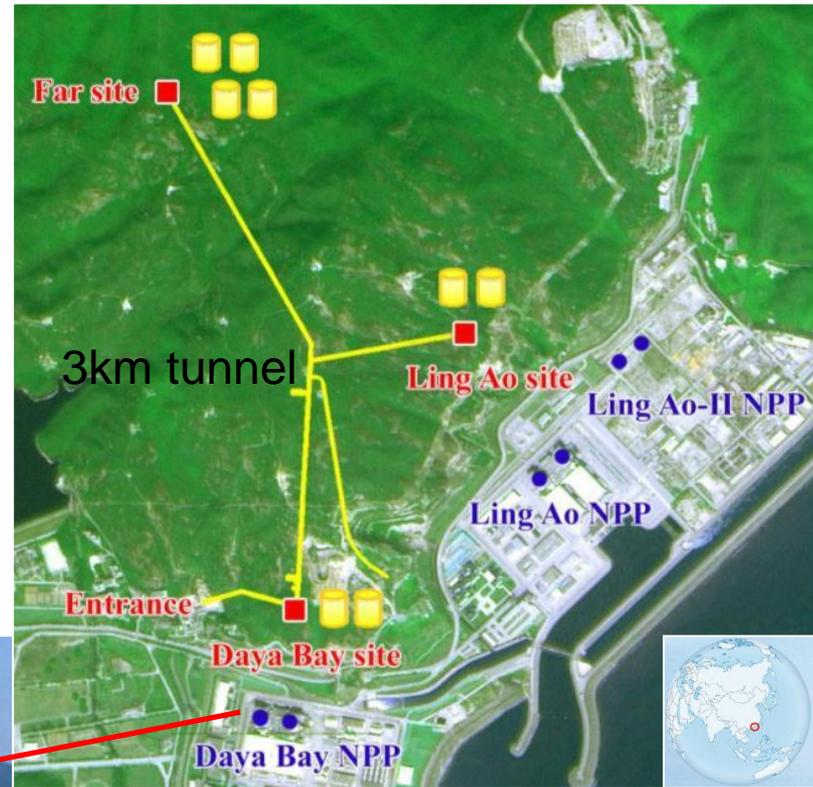
Double Chooz
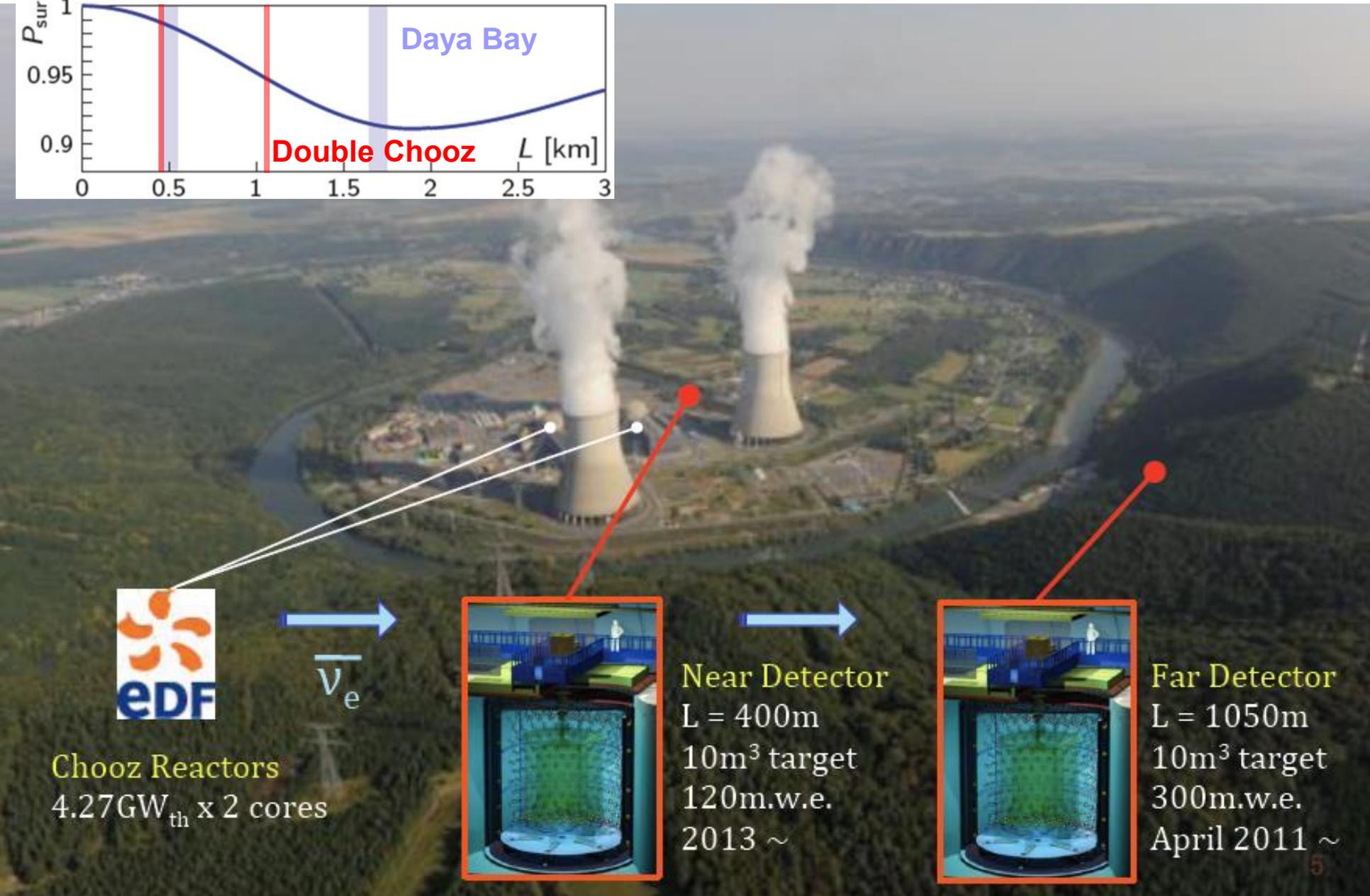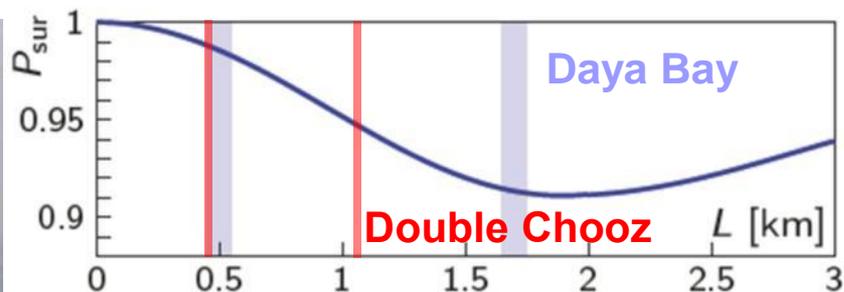
Chooz Reactors
$4.27GW_{th}$ x 2 cores

$\overline{\nu}_e$

Near Detector
L = 400m
$10m^3$ target
120m.w.e.
2013 ~
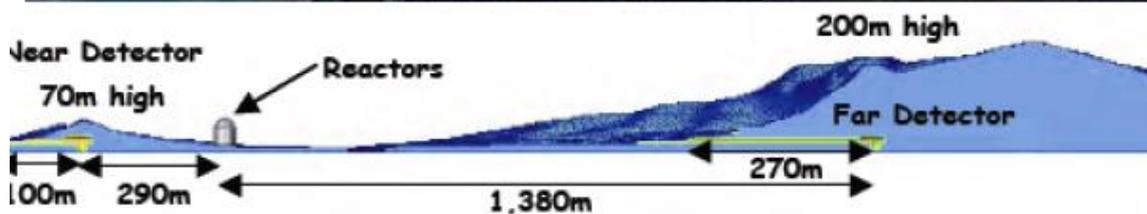
Far Detector
L = 1050m
$10m^3$ target
300m.w.e.
April 2011 ~

RENO

16t, 120 MWE

6 cores
16.5 GW

Near Detector
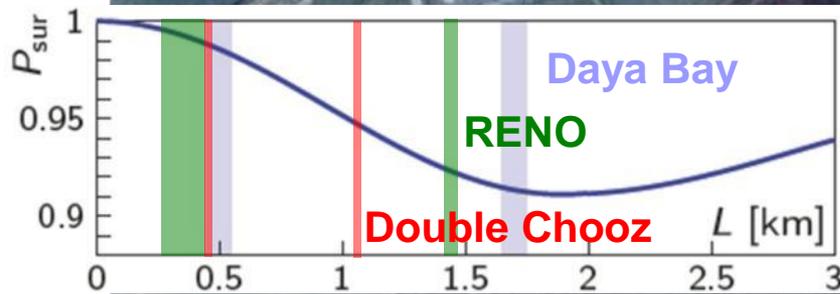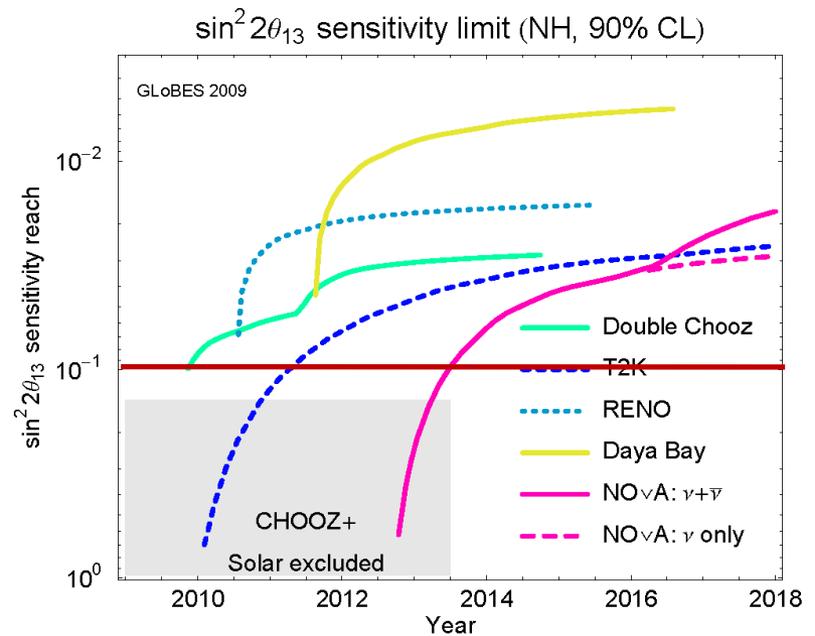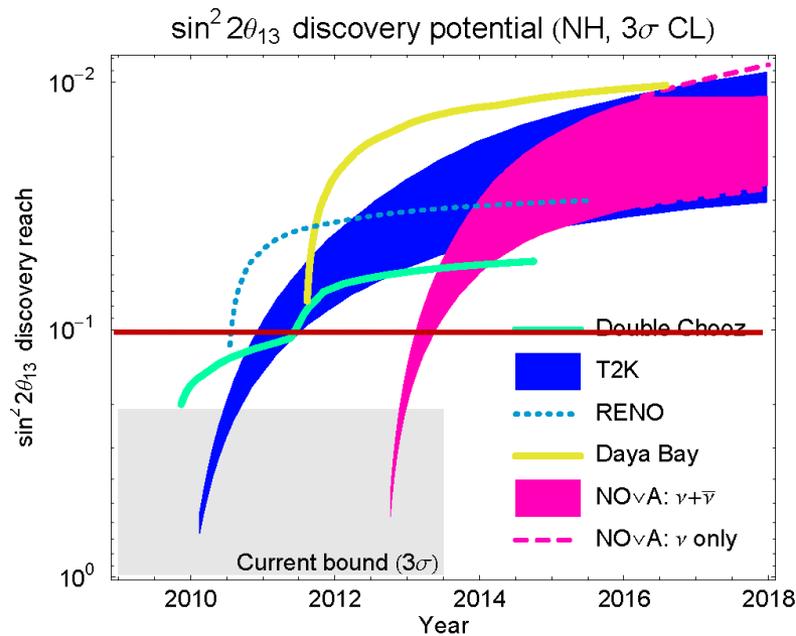
290m

1380m

Daya Bay

RENO

Double Chooz

Far Detector

16t, 450 MWE

200m high

Near Detector

70m high

Reactors

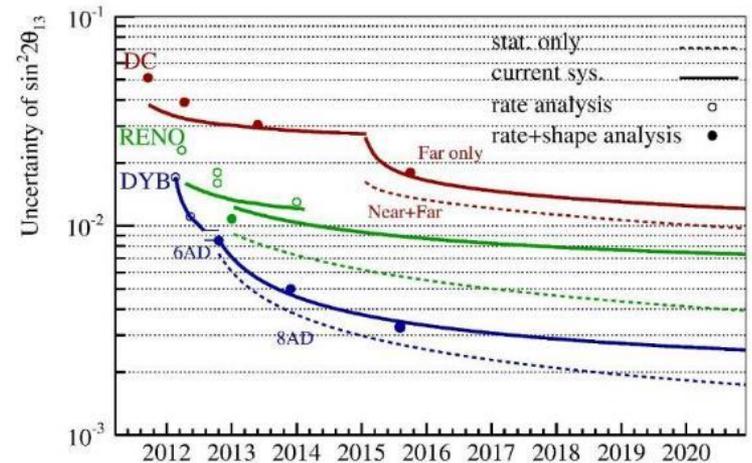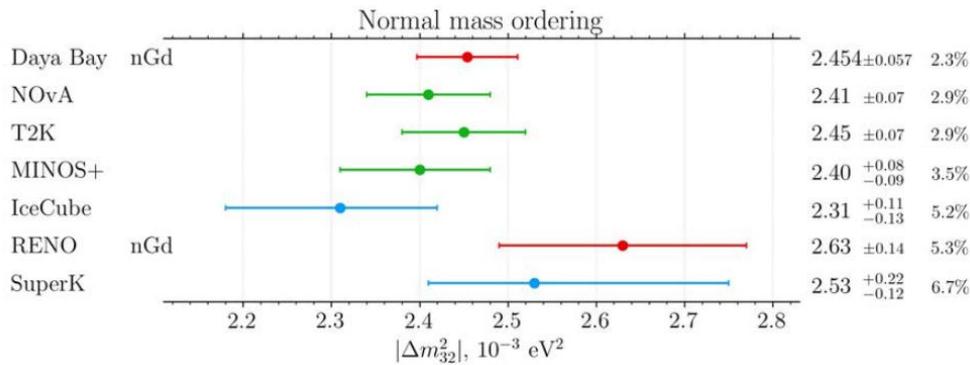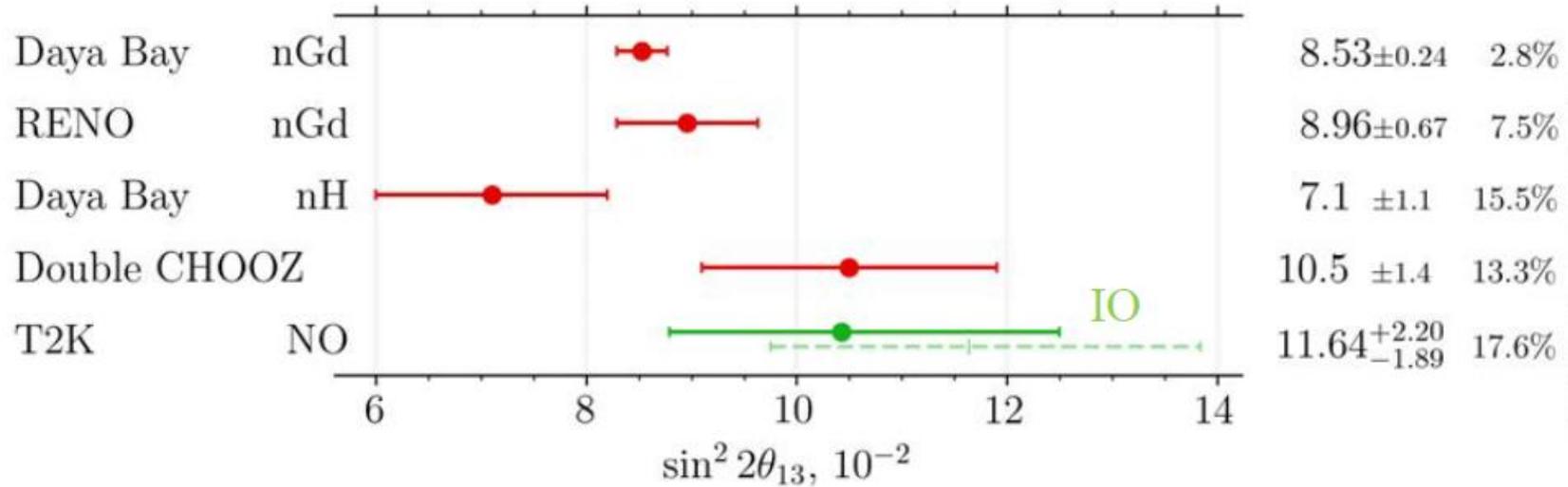Far Detector

100m   290m

1,380m

270m

Three on-going experiments

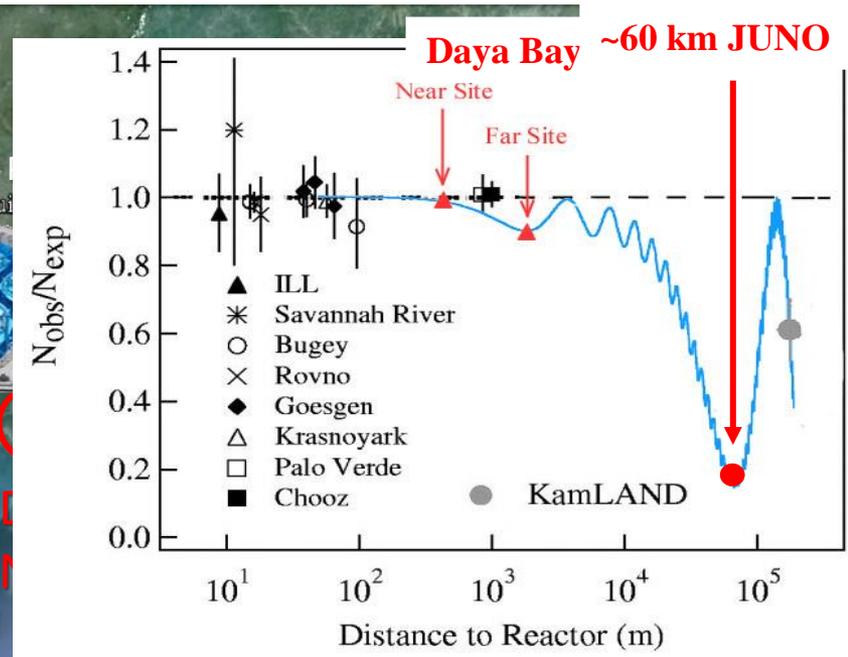| Experiment	Power (GW)	Detector(t) Near/Far	Overburden (m.w.e.) Near/Far	Sensitivity (3y,90%CL)
Daya Bay	17.4	40 / 80	250 / 860	~ 0.008
Double Chooz	8.5	8 / 8	120 / 300	~ 0.03
RENO	16.5	16 / 16	120 / 450	~ 0.02

Huber et al. JHEP 0911:044, 2009

θ₁₃ and Δm²ₑₑ

Daya Bay	nGd	8.53 ± 0.24	2.8%
RENO	nGd	8.96 ± 0.67	7.5%
Daya Bay	nH	7.1 ± 1.1	15.5%
Double CHOOZ		10.5 ± 1.4	13.3%
T2K	NO	$11.64^{+2.20}_{-1.89}$	17.6%

$\sin^2 2\theta_{13}, 10^{-2}$

Normal mass ordering

Daya Bay	nGd	2.454 ± 0.057	2.3%
NOvA		2.41 ± 0.07	2.9%
T2K		2.45 ± 0.07	2.9%
MINOS+		$2.40^{+0.08}_{-0.09}$	3.5%
IceCube		$2.31^{+0.11}_{-0.13}$	5.2%
RENO	nGd	2.63 ± 0.14	5.3%
SuperK		$2.53^{+0.22}_{-0.12}$	6.7%

$|\Delta m^2_{32}|, 10^{-3} \text{ eV}^2$

- DYB: shutdown in Dec. 2020
- RENO: 2020?
- Double Chooz: Dec. 2017

Neutrino Mass Ordering (in 2012)

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW

by 2020: 26.6 GW

Overburden ~ 700 m

Kaiping,
Jiangmen city,
Guangdong Province

Daya Bay

~60 km JUNO

| Cores	YJ-C1	YJ-C2	YJ-C3	YJ-C4	YJ-C5	YJ-C6
Power (GW)	2.9	2.9	2.9	2.9	2.9	2.9
Baseline (km)	52.75	52.84	52.42	52.51	52.12	52.21
Cores	TS-C1	TS-C2	TS-C3	TS-C4	DYB	HZ
Power (GW)	4.6	4.6	4.6	4.6	17.4	17.4
Baseline (km)	52.76	52.63	52.32	52.20	215	265

The JUNO Experiment

◆ **Jiangmen Underground Neutrino Observatory, a multiple-purpose neutrino experiment, approved in Feb. 2013. ~ 300 M$.**
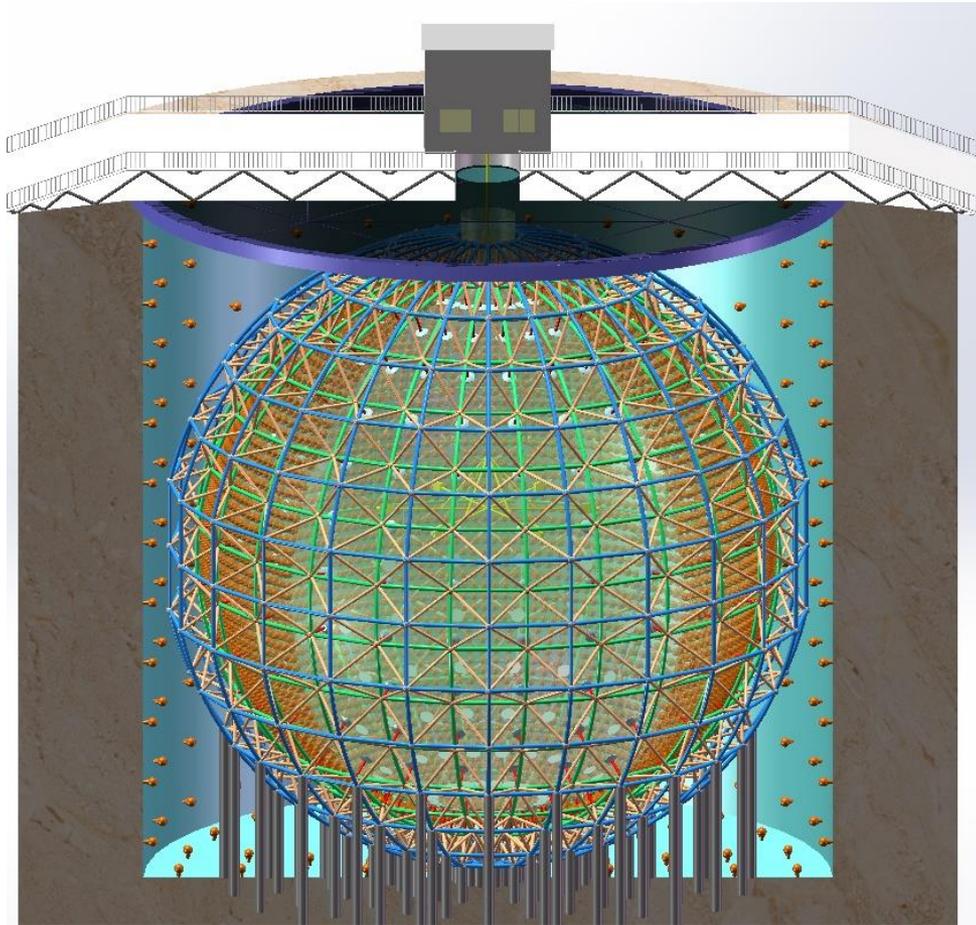
◆ **20 kton LS detector**

◆ **3% energy resolution**

◆ **700 m underground**

◆ **Rich physics possibilities**

⇨ **Reactor neutrino for Mass hierarchy and precision measurement of oscillation parameters**

⇨ **Supernovae neutrino**

⇨ **Geoneutrino**

⇨ **Solar neutrino**

⇨ **Atmospheric neutrino**

⇨ **Proton decay**

⇨ **Exotic searches**

L. Zhan, Y.F. Wang, J. Cao, L.J. Wen,
PRD78:111103, 2008; PRD79:073007,2009

Determine NMO with Reactors

4 MeV $\bar{\nu}_e$
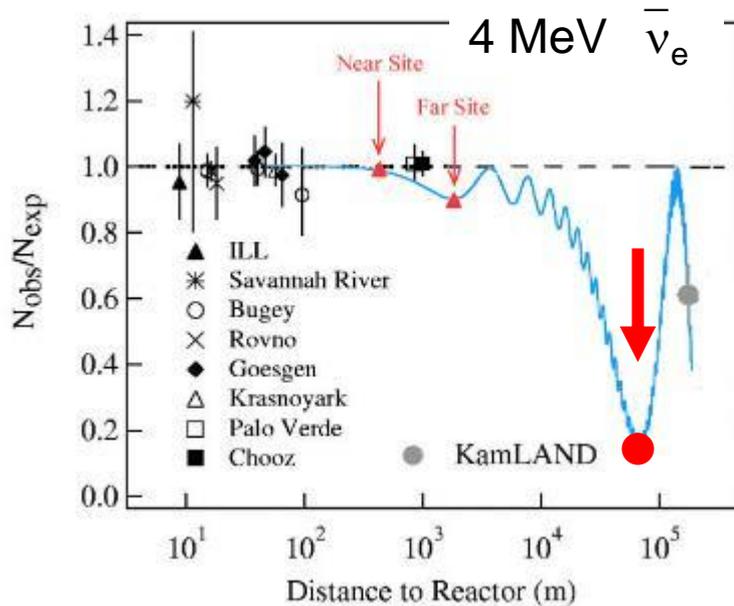
$$
\begin{aligned}
P_{ee}(L/E) &= 1 - P_{21} - P_{31} - P_{32} \\
P_{21} &= \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21}) \\
P_{31} &= \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31}) \\
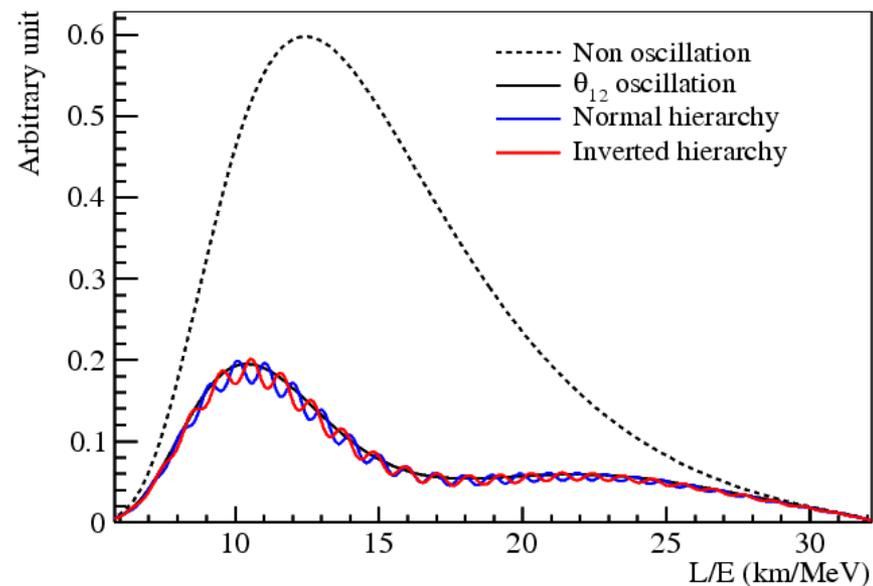P_{32} &= \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32})
\end{aligned}
$$

Precision energy spectrum measurement interference between P_{31} and P_{32}
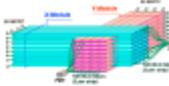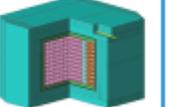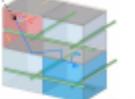→ ϕ: **Relative measurement**

Further improvement with $\Delta m^2_{\mu\mu}$ measurement from accelerator exp.
→ Δm^2_{ee}: **Absolute measurement**

Very Short Baseline Exps.

- Different technologies: (Gd, Li, B) (seg.)(movable)(2 det.)
- Most have sensitivity 0.02~0.03 @Δm~1eV^2 @90%CL

| Experiment		Reactor Power/Fuel	Overburden (mwe)	Detection Material	Segmentation	Optical Readout	Particle ID Capability
DANSS (Russia)		3000 MW LEU fuel	~50	Inhomogeneous PS & Gd sheets	2D, ~5mm	WLS fibers.	Topology only
NEOS (South Korea)		2800 MW LEU fuel	~20	Homogeneous Gd-doped LS	none	Direct double ended PMT	recoil PSD only
nuLat (USA)		40 MW ^{235}U fuel	few	Homogeneous ^{6}Li doped PS	Quasi-3D, 5cm, 3-axis Opt. Latt	Direct PMT	Topology, recoil & capture PSD
Neutrino4 (Russia)		100 MW ^{235}U fuel	~10	Homogeneous Gd-doped LS	2D, ~10cm	Direct single ended PMT	Topology only
PROSPECT (USA)		85 MW ^{235}U fuel	few	Homogeneous ^{6}Li-doped LS	2D, 15cm	Direct double ended PMT	Topology, recoil & capture PSD
SoLid (UK Fr Bel US)		72 MW ^{235}U fuel	~10	Inhomogeneous ^{6}LiZnS & PS	Quasi-3D, 5cm multiplex	WLS fibers	topology, capture PSD
Chandler (USA)		72 MW ^{235}U fuel	~10	Inhomogeneous ^{6}LiZnS & PS	Quasi-3D, 5cm, 2-axis Opt. Latt	Direct PMT/ WLS Scint.	topology, capture PSD
Stereo (France)		57 MW ^{235}U fuel	~15	Homogeneous Gd-doped LS	1D, 25cm	Direct single ended PMT	recoil PSD

Talk by Nathaniel Bowden @NEUTRINO2016

JUNO-TAO

◆ Taishan Antineutrino Observatory (TAO), a ton-level, high energy resolution LS detector at 40 m from the core, a satellite detector of JUNO.

◆ Measure reactor neutrino spectrum w/ sub-percent E resolution.

  ⇨ model-independent reference spectrum for JUNO

  ⇨ a benchmark for investigation of the nuclear database

◆ Taishan Nuclear Power Plant, 44.5 m from a 4.6 GW core, in a hall at -10 m underground.
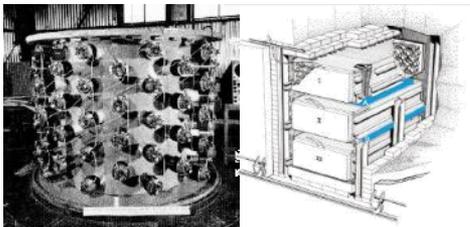
◆ First low-temperature LS experiment: SiPM + GdLS, 4500 p.e./MeV

TAO CDR: arXiv: 2005.08745
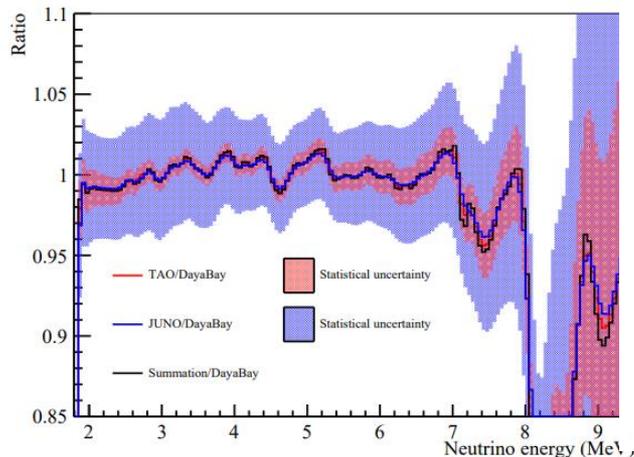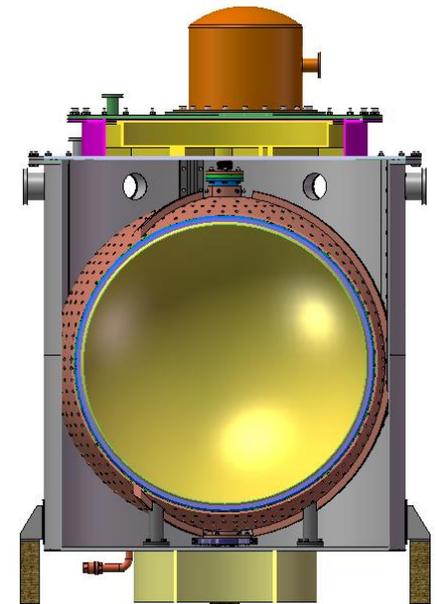
Constrain the fine structure in [2.5,6] MeV to < 1%

Core

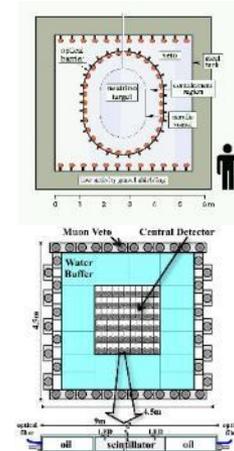# Reactor Neutrino Experiments

Discovery of ν

Early searches for oscillation

1980 Savannah, **YES**

1980 ILL, **NO**

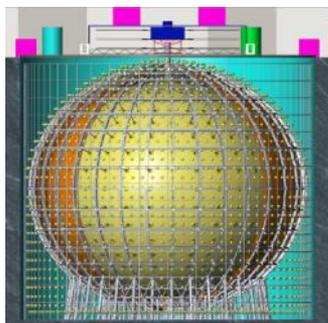1984 Bugey, **YES**

1986 Gosgen, **NO**

1995 Bugey-3, **NO**

Reactor ν spectra ~2%

1953, Hanford, 0.3 ton
1956, Savannah River, 4.2 ton

1997, CHOOZ, 8 ton
2000, Palo Verde, 12 ton

Mass Ordering,
Precision meas.
2024, JUNO, **20 000 ton**

2012,
Daya Bay, 160 ton
Double Chooz, 16 ton
RENO, 32 ton

$\sin^2 2\theta_{13} < 0.15$

Non-zero θ_{13}
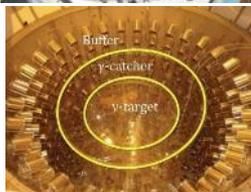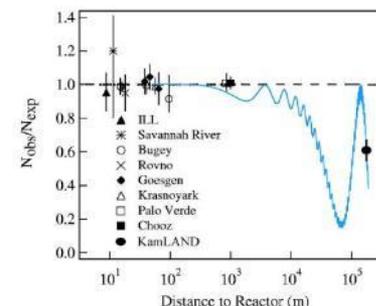
Reactor ν
oscillation (θ_{12})

VSBL for sterile ν

JUNO-TAO for spectrum

Moment, e scattering, Coherent Scattering

2002, KamLAND, **1000 ton**

28

2 - Neutrino from Reactor
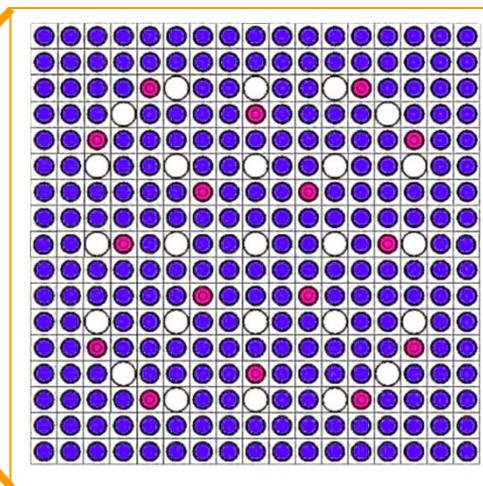
大亚湾核电站

绝大部分商用反应堆为压水堆或沸水堆，两者原理相同
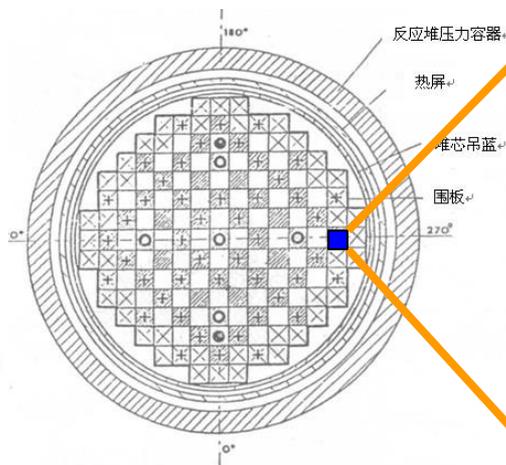电功率约为热功率的1/3。大亚湾：2.9 GW_{th}，900 MW_e→1080 MW_e
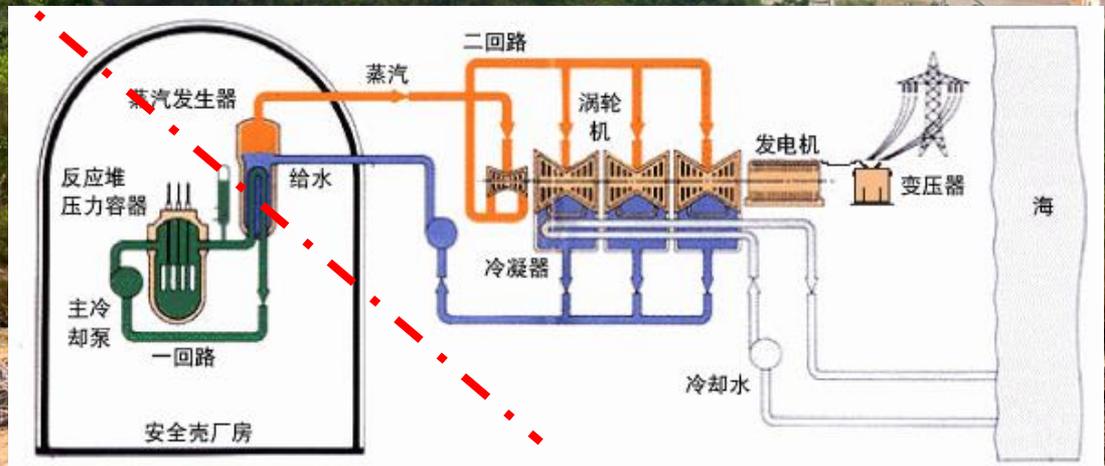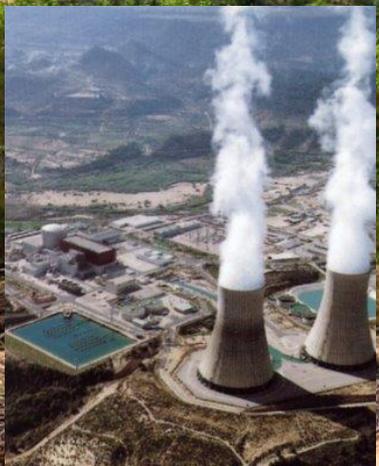
Structure of a Reactor Core

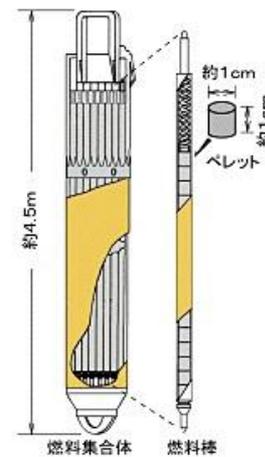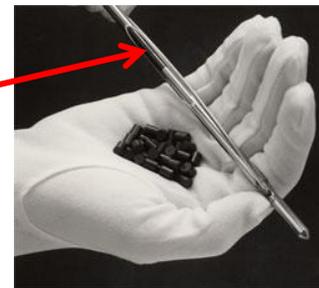- 压力容器、活性区、组件、燃料棒、元件
- 157个组件（assembly），3.7m高，3m直径
- 每组件17×17根棒（其中264根燃料棒，25根导管）
- 每燃料棒271个燃料元件。燃料元件二氧化铀烧结成1厘米大小的陶瓷状圆柱体，U-235 4.45%（1.8-4.45%)
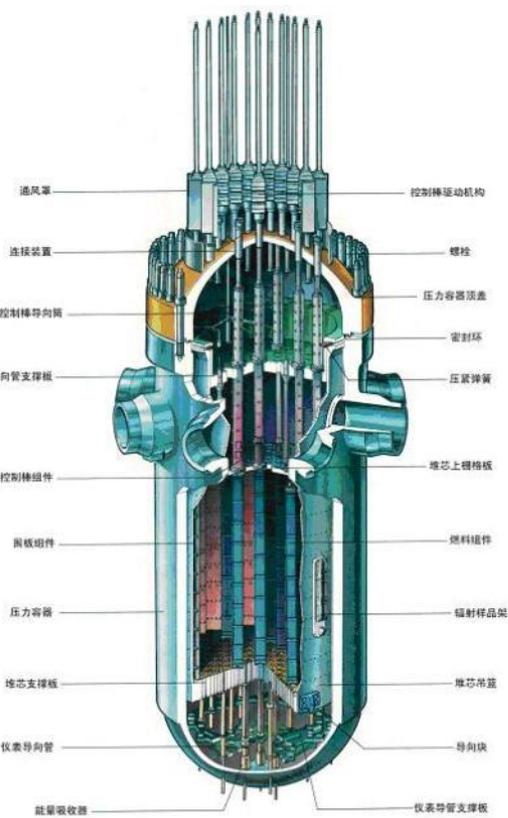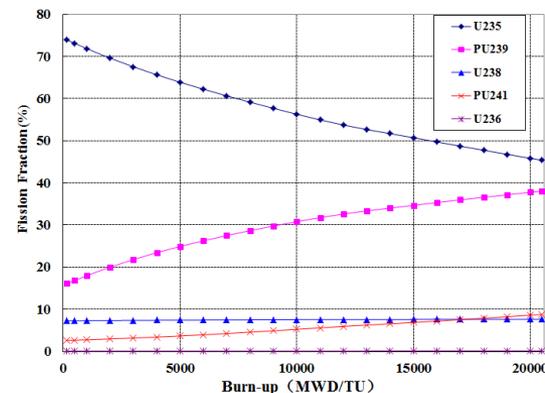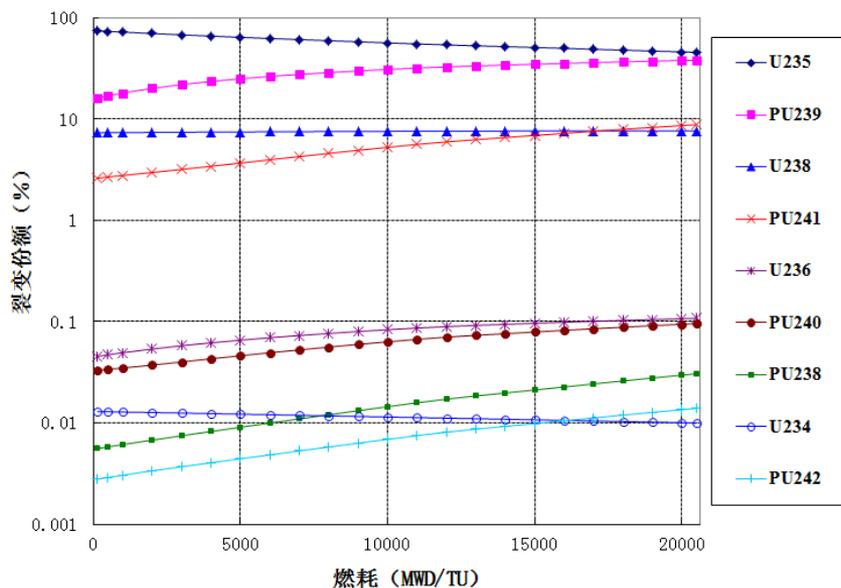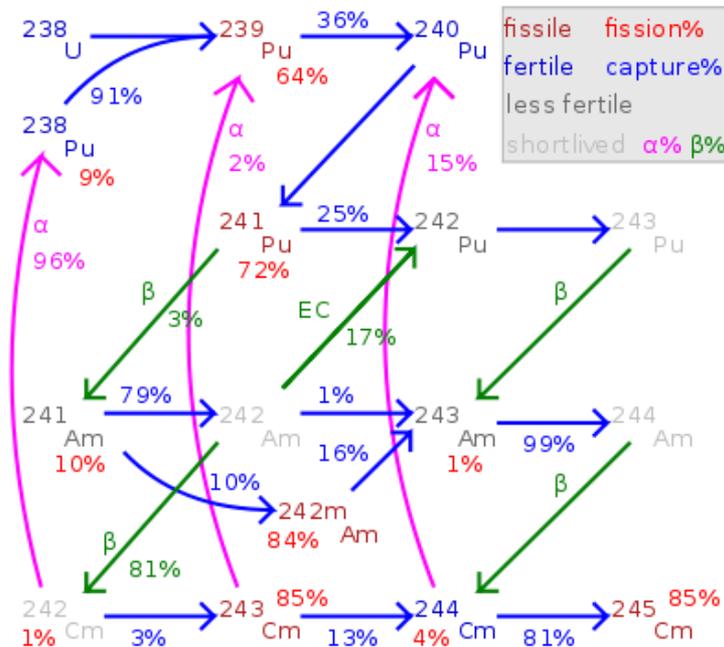

锆合金管

72吨核燃料
每天消耗3kg U235

红色：可燃毒物棒 8% Gd

Fuel Evolution

- ◆ **Initial 4.45% U235, Others:U238 and O**

- ◆ **U238 → n capture → Pu239 →2x n capture → Pu241**

- ◆ **Four major fission isotopes**
  - ⇨ **U235, Pu239, Pu241**
  - ⇨ **U238 fission w/ fast n**

- ◆ **燃耗(Burnup)：MW·day/ton U**

每裂变释放能量

◆ 每裂变释放能量是指核燃料裂变时放出的能量中，在反应堆内被吸收转化为热能的部分。

◆ 核燃料释放的能量

  ⇨ 一部分作为裂变产物的动能转化为热能；

  ⇨ 一部分被中微子带走；

  ⇨ 一部分来自子核的衰变能，长寿命子核能量释放延后（时间变化）

  ⇨ 还有一部分来自富余中子的俘获（时间变化）

◆ 采用整个寿期内的平均值做为近似，误差 **(0.30-0.47)%.**

Isotope	E_{fi}, MeV/fission
^{235}U	201.92 ± 0.46
^{238}U	205.52 ± 0.96
^{239}Pu	209.99 ± 0.60
^{241}Pu	213.60 ± 0.65

Isotopes	Energy (MeV)
U-235	201.7 ± 0.6
U-238	205.0 ± 0.9
Pu-239	210.0 ± 0.9
Pu-241	212.4 ± 1.0

Kopeikin et al, Physics of Atomic Nuclei, Vol. 67, No. 10, 1892 (2004)

M.F. James, J. Nucl. Energy 23, 517 (1969)

反应堆会不会发生核爆炸？

- 第一个反应堆：费米，芝加哥大学的体育场
- 链式反应，反应性 $\rho = (N_2 - N_1)/N_2$
- 燃料的温度效应（多普勒效应）：温度上升 → U238吸收共振峰覆盖的能谱加宽 → 吸收更多的中子 → 负反应性
  ⇨ 瞬发过程，对功率的变化响应很快：自稳机制
- 只有高浓度铀才能发生核爆炸（核弹）

- 高温下锆水反应生成氢气，化学爆炸抛撒放射性物质

铀同位素的微观截面
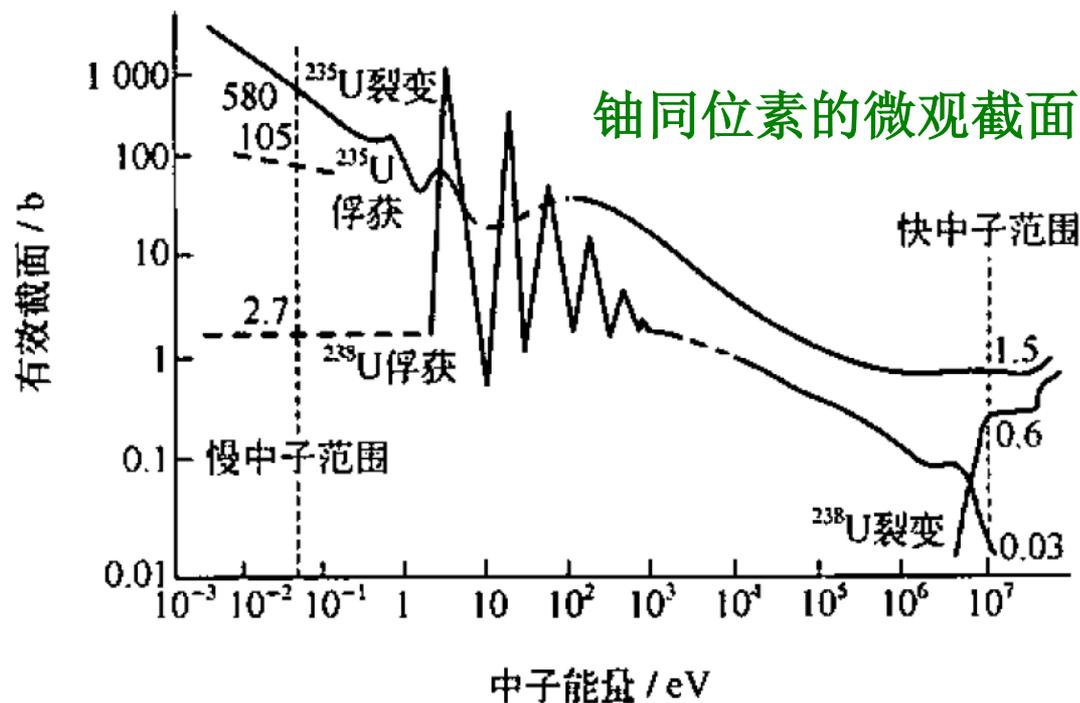
反应堆产生中微子

◆ **Neutrinos from subsequent β-decays of fission fragments.**

◆ 两裂变为主。钱三强、何泽慧发现存在三裂变

◆ 裂变产物是富中子核，平均每裂变释放6个中微子

β 衰变: $\quad {}^A_Z X \rightarrow {}^A_{Z+1} Y + e^- + \bar{\nu}_e$ $\qquad$ 忽略反冲核能量: $E_0 = E_e + E_\nu$

中微子能谱: $\quad S_f(E) = \sum_b \left(K_f^b \cdot F(Z_f, A_f, E) \cdot pE(E - E_{0f}^b)^2 \cdot C_f^b(E) \cdot (1 + \delta_f^b(Z_f, A_f, E)) \right)$

核素裂变的碎片有固定的质量分布

$${}^{235}_{92}U + n \rightarrow X_1 + X_2 + 2n$$
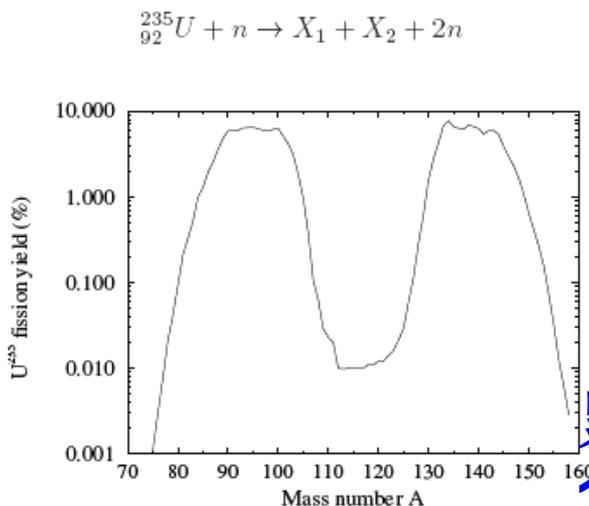

核素裂变有固定的中微子能谱

$$S_k = \sum_f A_f \cdot S_f$$

累加各核素得到的能谱误差较大：
1000多种核素，6000道，10%误差

Predicting the Flux and Spectrum

◆ **Summation (ab initio): Nuclear database, Σ fragments, Σ chains, Σ branches** (e.g. Vogel et al., PRC24, 1543 (1981), Estienne, et al, PRL 123, 022502 (2019))

◆ **Conversion: ILL measured the β-spectra → convert to neutrino spectra**

  ⇨ **ILL spectra: Use spectra of 30 virtual (allowed) decays, fit amplitude and endpoints** (ILL-Vogel spectra)

  ⇨ **Mueller: 90% ab initio + 10% fit → rate anomaly**

  ⇨ **Huber: fit w/ improved nuclear effects** (Huber-Mueller spectra)
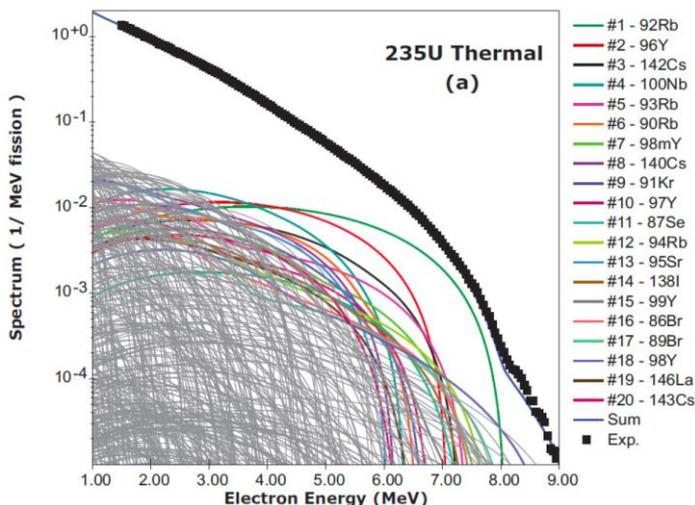
A. Sonzogni, AAP 2019
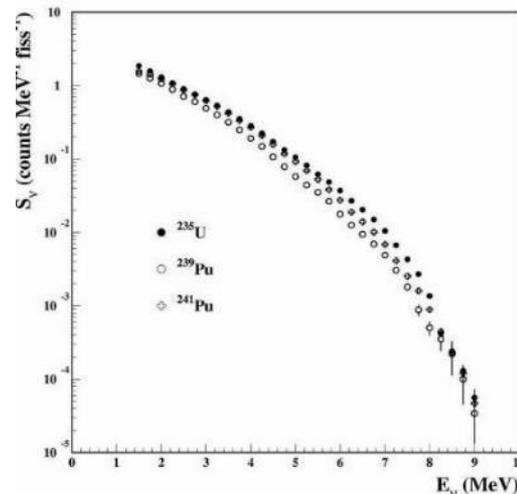
K. Schreckenbach et al. PLB118, 162 (1985)
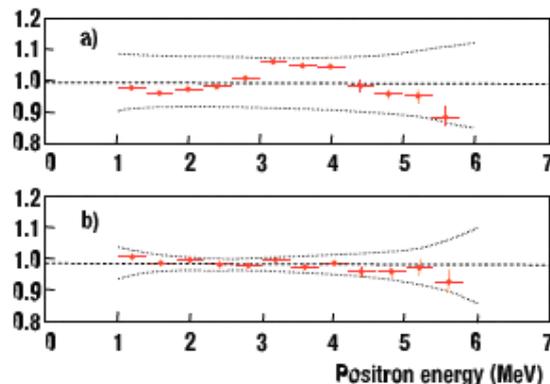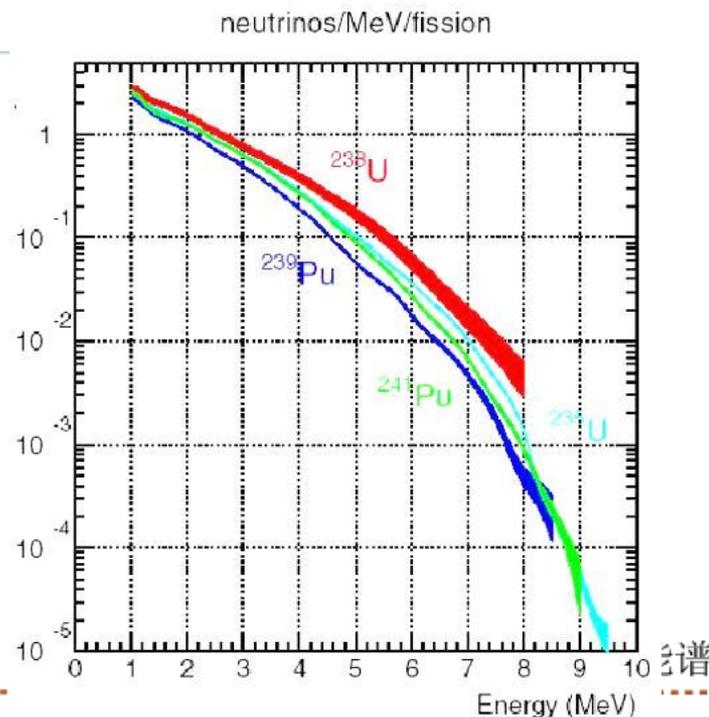A.A. Hahn et al. PLB160, 325 (1985)
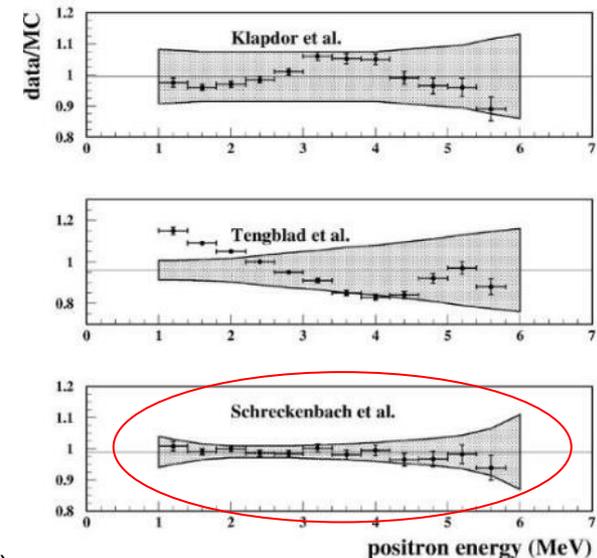
Shape verified by Bugey-3 data
Normalization by Bugey-4, 1.6%

ILL实验（劳厄-朗之万研究所）

neutrinos/MeV/fission

ℰ 测量原理:
- ℰ 将核素的薄片样品置入反应堆中接受中子照射(1~2天), 样品裂变发射的电子被高精度谱仪记录.
- ℰ 采用拟合的办法将测量的电子能谱转换成中微子能谱.(假设了20条虚拟的β分支)

ℰ ^{238}U采用Vogel的理论计算.
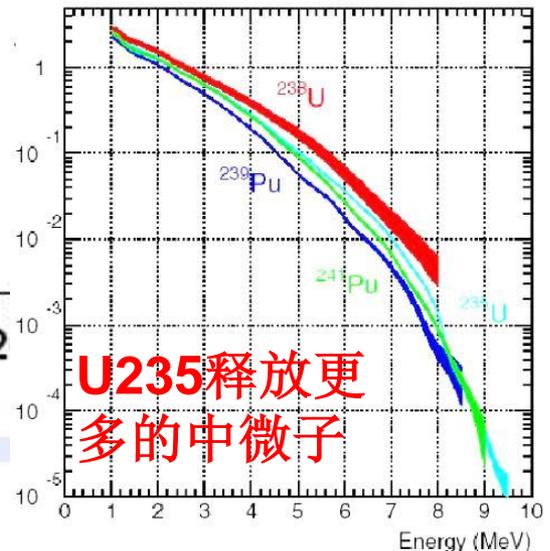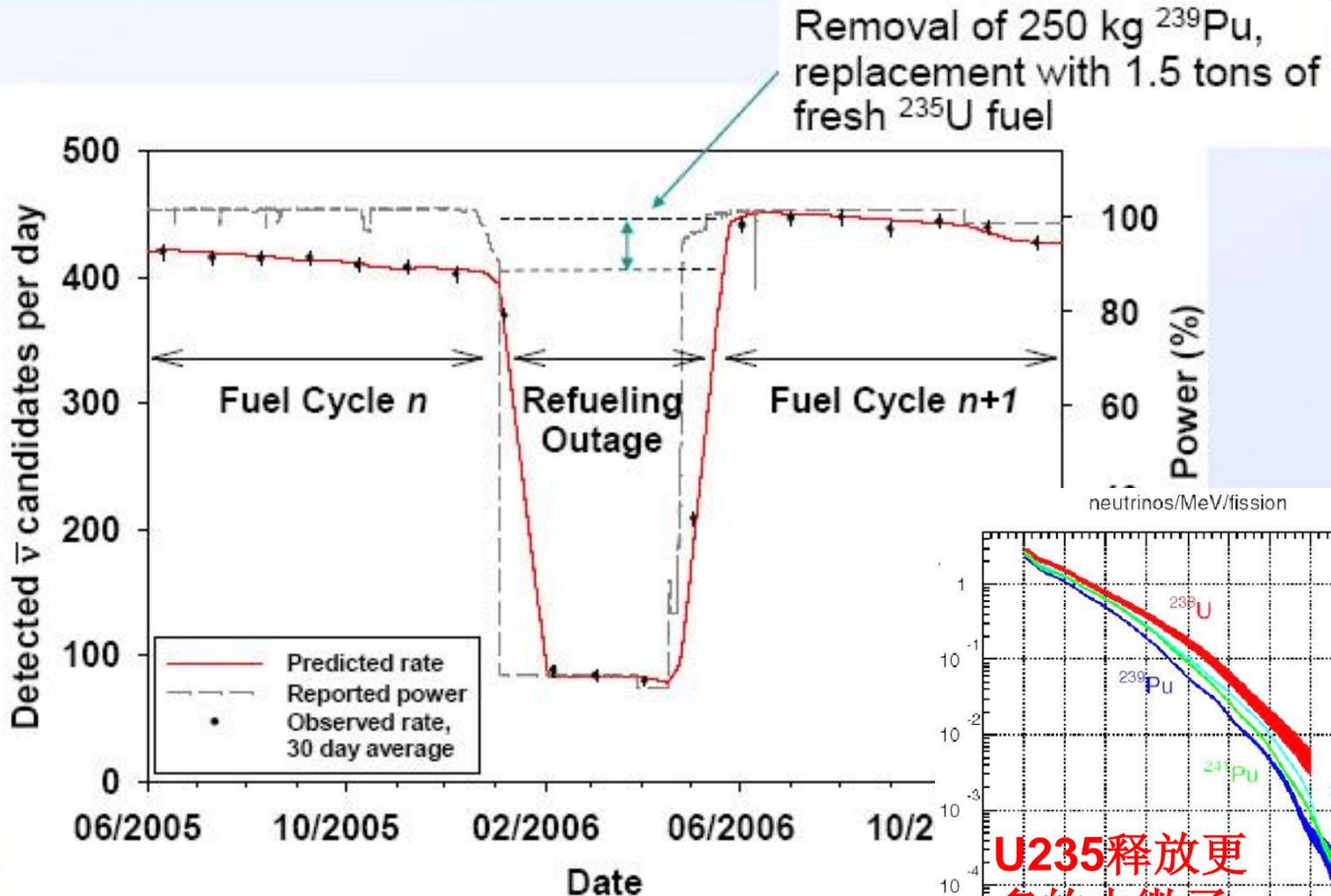- ℰ ^{238}U主要由快中子诱发裂变, 堆芯中快中子很少.
- ℰ ^{238}U在堆芯中裂变贡献较少(~10%), 误差影响有限.

Bugey实验在距离反应堆15米和40米的地方近距离测量反应堆中微子能谱, 利用ILL模型和Vogel计算的^{238}U能谱计算的总中微子能谱和实验结果符合的很好.

ILL的模型和Bugey实验测量符合最好

Long Term Monitoring – Fuel composition

Removal of 250 kg ^{239}Pu, replacement with 1.5 tons of fresh ^{235}U fuel

U235释放更多的中微子

Bowden, LLNL, 2008

Flux Calculation

Neutrino Flux $\quad S(E_\nu) = \sum_i^{isotopes} f_i S_i(E_\nu)$

$$S(E_\nu) = \frac{W_{th}}{\sum_i (f_i/F)\, e_i} \sum_i^{istopes} (f_i/F) S_i(E_\nu)$$

$$W_{th} = \sum_i f_i e_i \,, \qquad F = \sum_i f_i$$

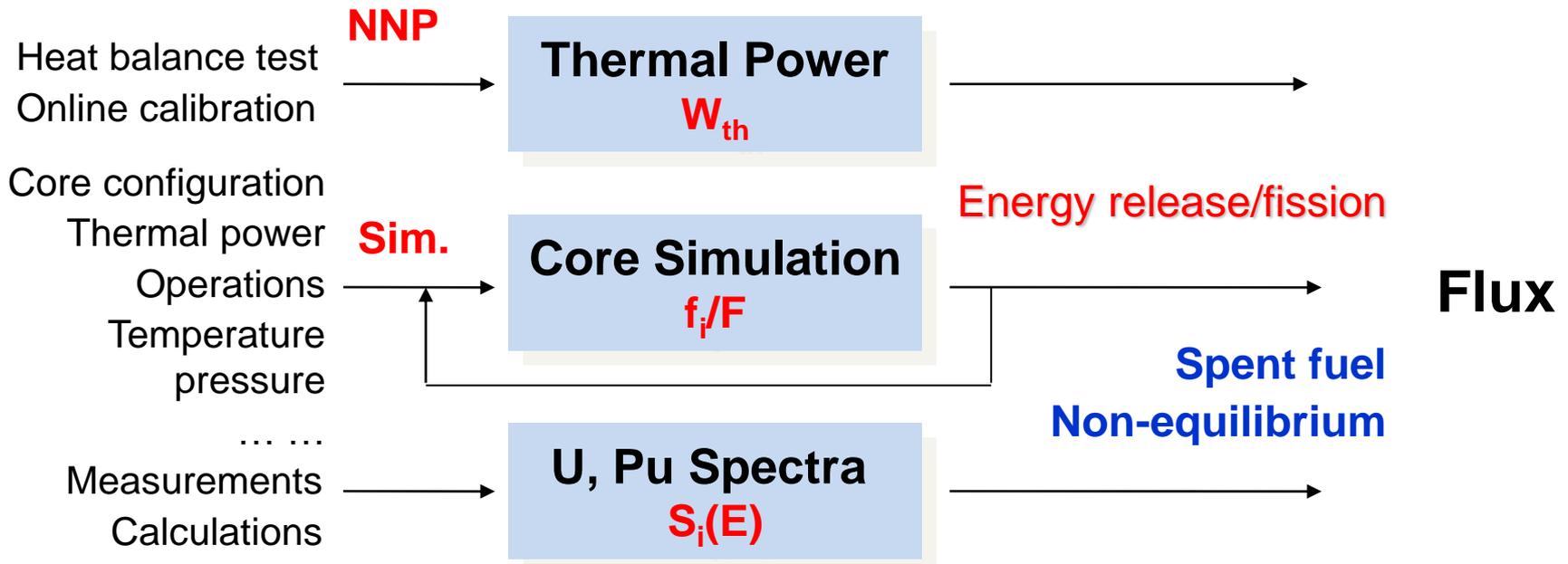E_ν : Neutrino energy

f_i : Fission rate of isotope i

$S_i(E_\nu)$: Neutrino energy spectra/f

(f_i /F): Fission fraction

W_{th} : Reactor thermal power
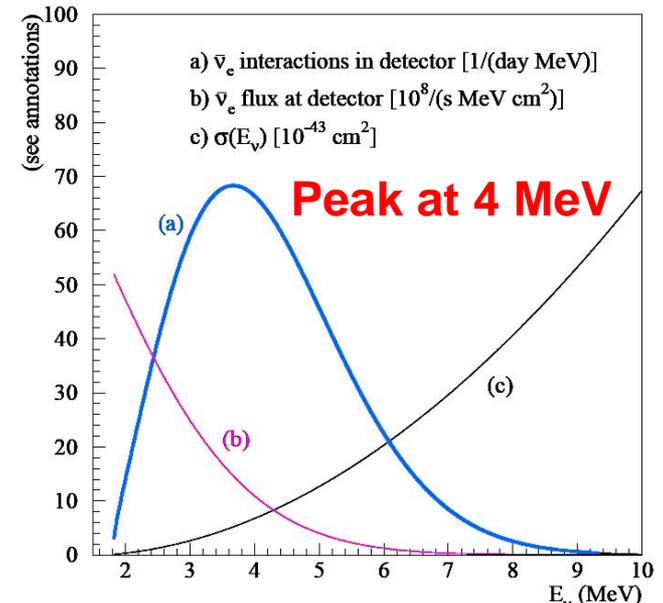
e_i : Energy release per fission

Heat balance test
Online calibration

NNP →

Thermal Power
W_{th}

→

Core configuration
Thermal power
Operations
Temperature
pressure

Sim. →

Core Simulation
f_i/F

→

Energy release/fission

Flux

Spent fuel
Non-equilibrium

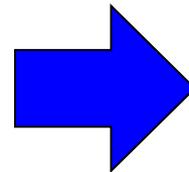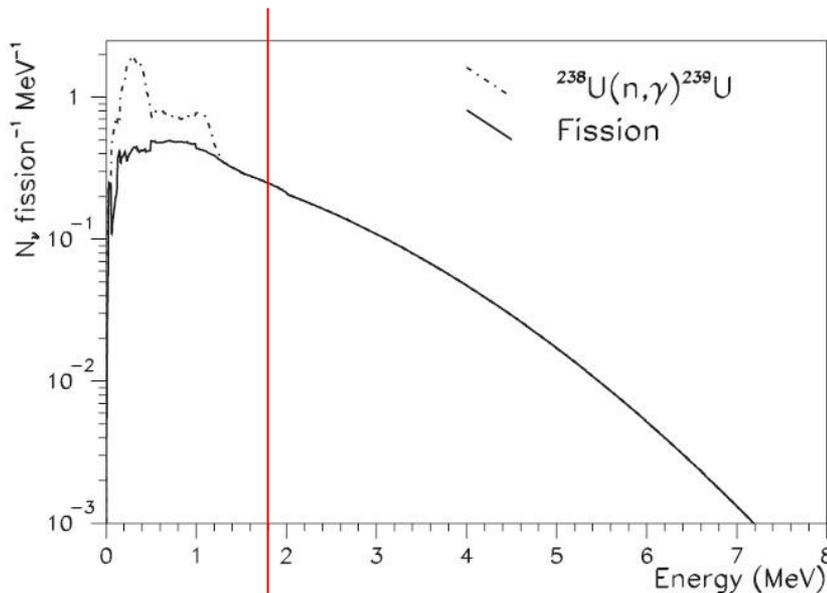… …
Measurements
Calculations

→

U, Pu Spectra
$S_i(E)$

→

Event Rate

◆ **Back-on-the-envelope: 2.9GW$_{th}$, 200 MeV/fission, 6 ν/fission**

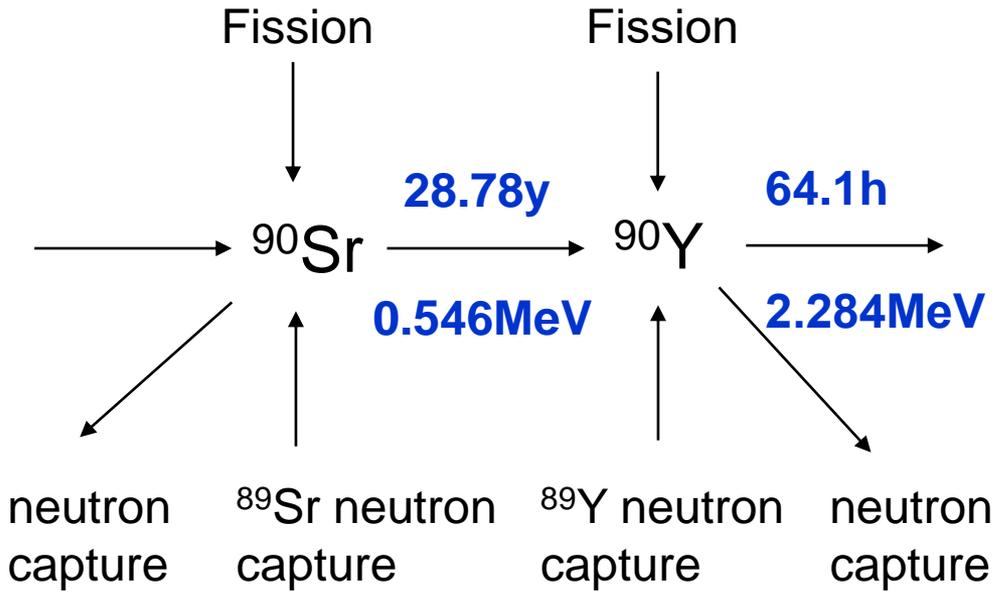⇨ **5.4×10^{20} $\overline{\nu}_e$/s (only 1/3 higher than 1.8 MeV threshold**

$$S(E_\nu) = \frac{W_{th}}{\sum_i \left(\frac{f_i}{F}\right) \cdot e_i} \sum_i \left(\frac{f_i}{F}\right) \cdot S_i (E_\nu)$$

◆ **IBD event rate at 1 km from reactor**

~ 1 IBD/(day·ton·GW) → 20 ton × 2.9GW×2/(0.36×0.36km)

a) $\overline{\nu}_e$ interactions in detector [1/(day MeV)]
b) $\overline{\nu}_e$ flux at detector [10^8/(s MeV cm^2)]
c) $\sigma(E_\nu)$ [10^{-43} cm^2]

Peak at 4 MeV

Small Corrections

1) Non-equilibrium isotope (~0.3%)

Fission   Fission

$\longrightarrow$ ^{90}Sr $\xrightarrow{\quad 28.78y \quad}$ ^{90}Y $\longrightarrow$

0.546MeV   2.284MeV

neutron ^{89}Sr neutron ^{89}Y neutron   neutron
capture   capture   capture   capture

Weighted by inverse-β decay Xsec.

Ratio to neutrinos in 2-4 MeV

Ratio to all neutrinos
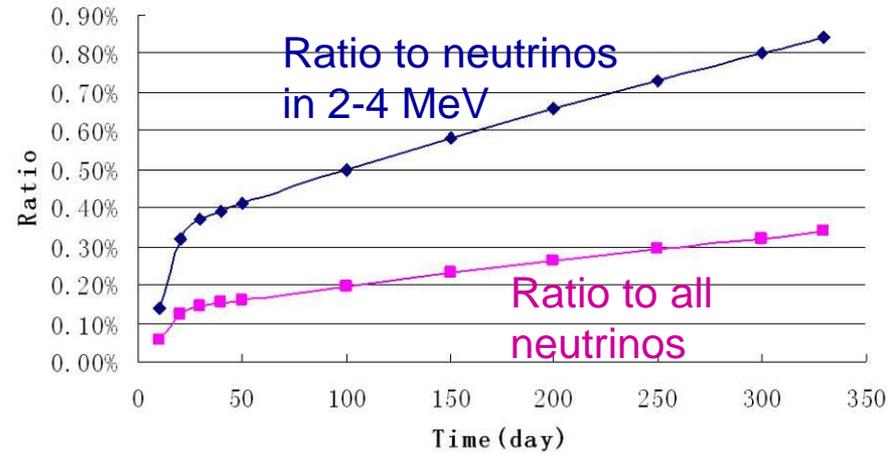
2）Spent fuel （~0.3%)

四个要素

$$S(E_\nu) = \frac{W_{th}}{\sum_i \left(\frac{f_i}{F}\right) \cdot e_i} \sum_i \left(\frac{f_i}{F}\right) \cdot S_i(E_\nu)$$

◆ **Isotope neutrino spectra (ILL+Vogel, Huber+Mueller, Vogel, Fallot, etc.)**

◆ **Thermal Power W_{th} (Provided by NPP)**

◆ **Fission Fraction (f_i/F)   (Provided by NPP or independent core simulation)**

◆ **Energy release per fission e_i (database）**

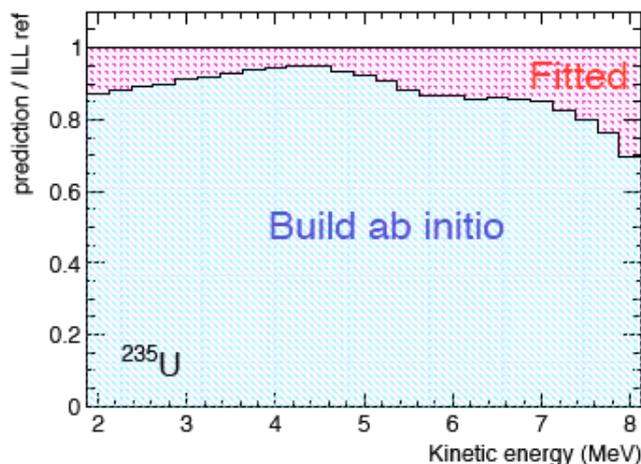◆ **Correlation among uncertainties.**

新的中微子能谱

- 2011年Mueller等人对ILL测量的电子能谱重新进行了先转换.
  - 利用了真实的核数据库计算出绝大部分能谱
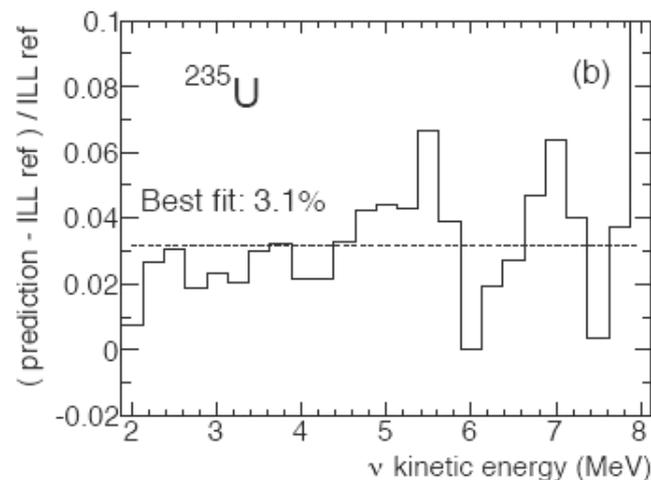 - 剩余的部分按照ILL的办法假设5条虚拟的β分支拟合.
  - 与 P.Huber 的独立计算相符合
- 影响: 所得到的核素裂变中微子能谱比原来ILL的能谱整体抬高~3%, 引起惰性中微子的讨论.

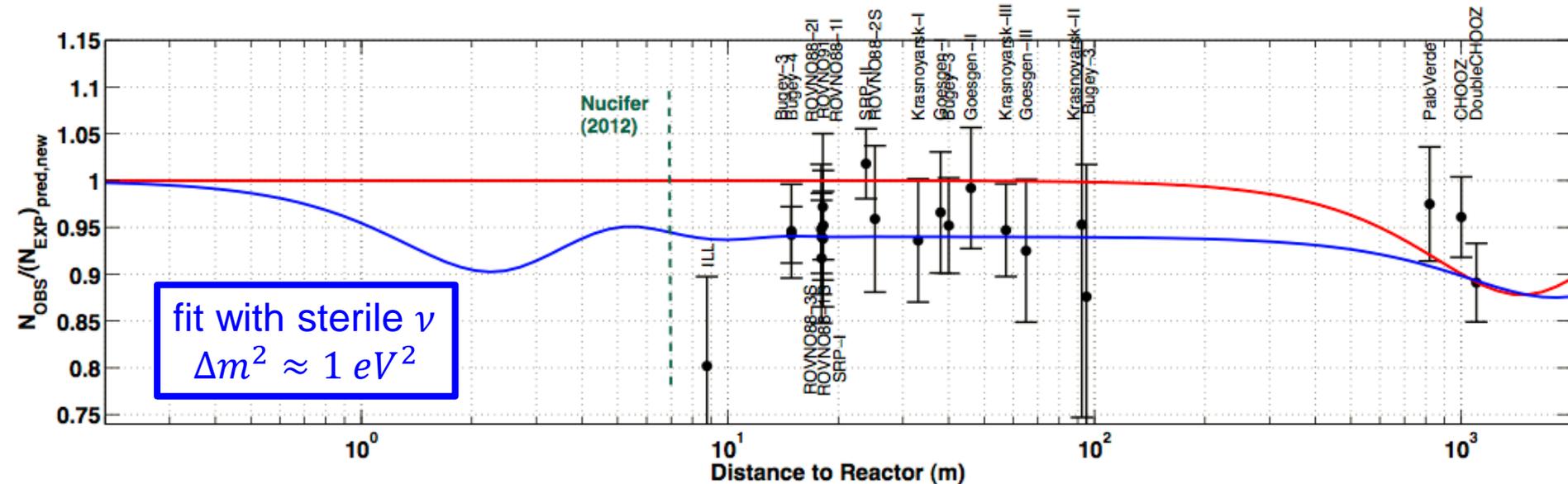
新方法的能谱同ILL能谱的比较

- 对大亚湾实验的影响
  - Rate Analysis: 同时用新能谱和ILL能谱进行能谱预测, 不对灵敏度分析产生影响
  - Shape Analysis: 可能会产生影响, 将同时使用两种能谱进行比较.

新的能谱比ILL能谱整体抬高

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Reactor Anomaly (Rate)

◆ **ILL spectra agree w/ data**

◆ **2011, Huber-Mueller spectra higher than data by 6%**

◆ **Sterile neutrino?**

fit with sterile ν
$\Delta m^2 \approx 1\ eV^2$

G. Mention et al.
Phys.Rev. D83 (2011) 073006

热功率

- **KME, thermal power, Secondary Heat Balance Method.**
  - ⇨ **The most accurate measurement.**
  - ⇨ **Offline measurement, weekly or monthly**
  - ⇨ **Generally cited with (0.6-0.7)% uncertainties in literature.**
- **KIT/KDO, thermal power. Good for analysis.**
  - ⇨ **Primary Heat Balance**
  - ⇨ **Online**
  - ⇨ **Weekly calibrated to KME power.**

$$\left| P_{KIT} - P_{KME} \right| < 0.1\% \ \text{FP}$$

- **RPN, nuclear power**
  - ⇨ **Ex-core neutron flux monitoring**
  - ⇨ **Online**
  - ⇨ **Safety and reactor operation control**
  - ⇨ **Daily calibrated to KIT/KDO power**

$$\left| P_{RPN} - P_{KME} \right| < 1.5\% \ \text{FP}$$
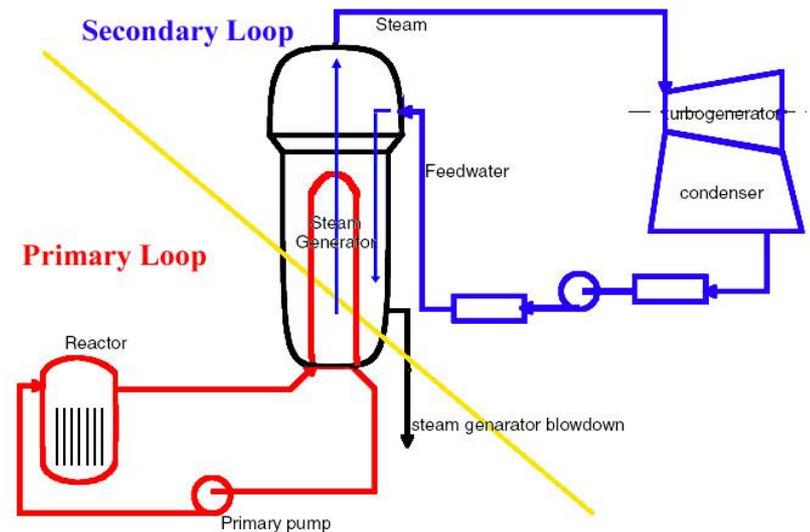
热功率误差

◆ **Chooz 0.6%, Palo Verde 0.7%.**

◆ **Motivation of power uprates by the power plants → Study the power uncertainties and improve the instrumentation.**

◆ **Uncertainties of secondary heat balance is dominated by the flow rate.**

⇨ **Venturi flow meter. Most US reactors. Uncertainty is often 1.4%. It can be as low as 0.7% if properly calibrated and maintained, but suffering from fouling effects, which could grow as high as 3% in a few years.**

⇨ **Orifice plate. France EDF reactors. Typically 0.72%. No fouling effects. Could be improved to 0.4% with lab tests.**

Note: Above flow meter uncertainties are at 95% C.L. as defined in ISO 5167. Unless specified, the thermal power uncertainty given by the power plant is also at 95% C.L.

⇨ **Ultrasonic. Start to use in some US and Japan reactors. Type I TT 0.45%, Type II TT 0.2% (Djurcic et al.)**

An example

- **EPRI document prepared by EDF,** *Improving Pressurized Water Reactor Performance Through Instrumentation:…… (2006)*
- **For N4 reactor (Chooz type) with 4 steam generators:**

Table 3-1
Summary of main components.

Empirical formula and uncertainty specified in ISO 5167-1-2003.

Correlated or Uncorrelated for the 4 flow meters?

Orifice Plate

Origin of uncertainty	Contribution [MWth]	Relative fraction [%] of the 17.2 MWth
Discharge coefficient	15.33	79.57
Differential pressure	6.33	13.57
Steam generator inlet temperature	2.81	2.68
Primary input	2.00	1.35
Others uncertainties accumulated	2.98	3.00

Final uncertainty at 95% confidence.

$$W_{th,reactor} = 4250 \pm 17.2 \text{ MW} (0.40 \%)$$

- If not assuming the discharge coefficients of the 4 orifice plates are independent, the power uncertainty at 68.3% C.L. will be 0.37%.

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Another Example

◆ **Daya Bay and Ling Ao reactors (EDF, 2.9GW$_{th}$) are all calibrated with SAPEC system, an EDF *portable* high precision secondary heat balance test system with its own sensors, databases, and data processing, of uncertainty 0.45%. Ling Ao KME is predicted to have an uncertainty of 0.48% (95% C.L.)**

◆ **4 tests on Ling Ao KME show differences from 0.031% to 0.065%. Why?**

  ⇨ **Used the same orifice plates but different pressure transmitters.**

  ⇨ **It proves that the uncertainty is dominated by discharge coefficient.**

  ⇨ **Ling Ao KME is in very good agreement with SAPEC.**

Table 1   Comparison of the core power calculation results between KME system and SAPEC system

		Test 1	Test 2	Test 3	Test 4
Thermal power	KME (MW)	2897.1	2904.4	2908.9	2906.9
	SAPEC (MW)	2896	2903	2907	2906
	Difference (MW)	1.1	1.4	1.9	0.9
	Difference	0.038%	0.048%	0.065%	0.031%
Uncertainty Analysis	KME 系统	0.4806%	0.4806%	0.4806%	0.4806%
	SAPEC 系统	0.45%	0.45%	0.45%	0.45%

Uncertainties of fission fraction

◆ **Depends on the simulation code. Only slightly on the inputs (has not been checked on other simulation code.)**

◆ **Compare measured and calculated concentration of fuel isotopes, sampled at different burn-up. Part of the qualification of the code.**

◆ **Apollo/Science模拟：裂变份额误差5%**
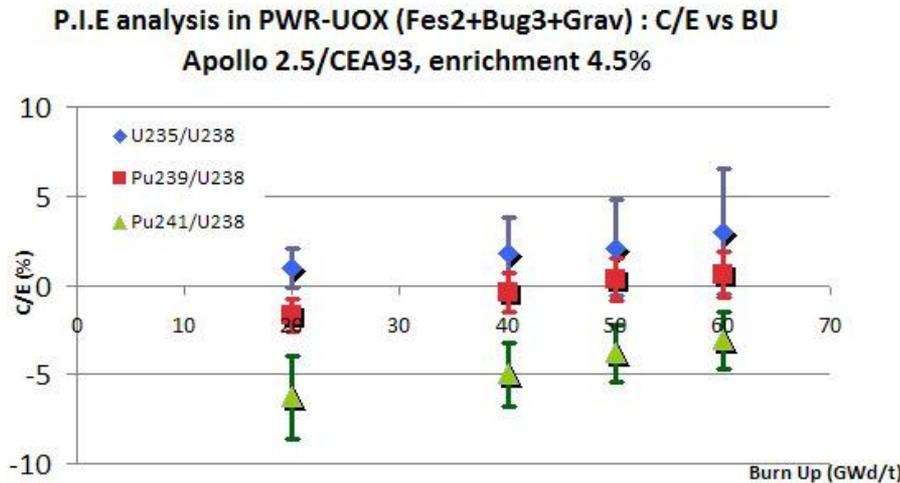
One analysis of Apollo 2.5

Lester Miller thesis, **ROCS**
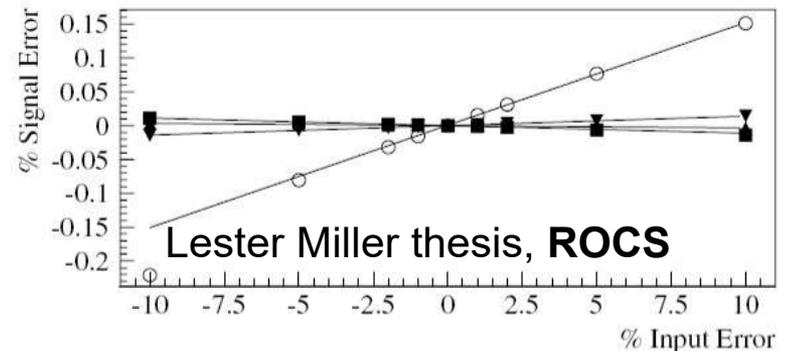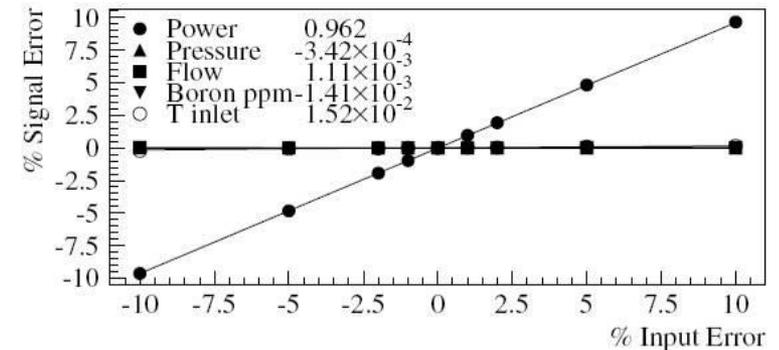
Correlation of fission fractions
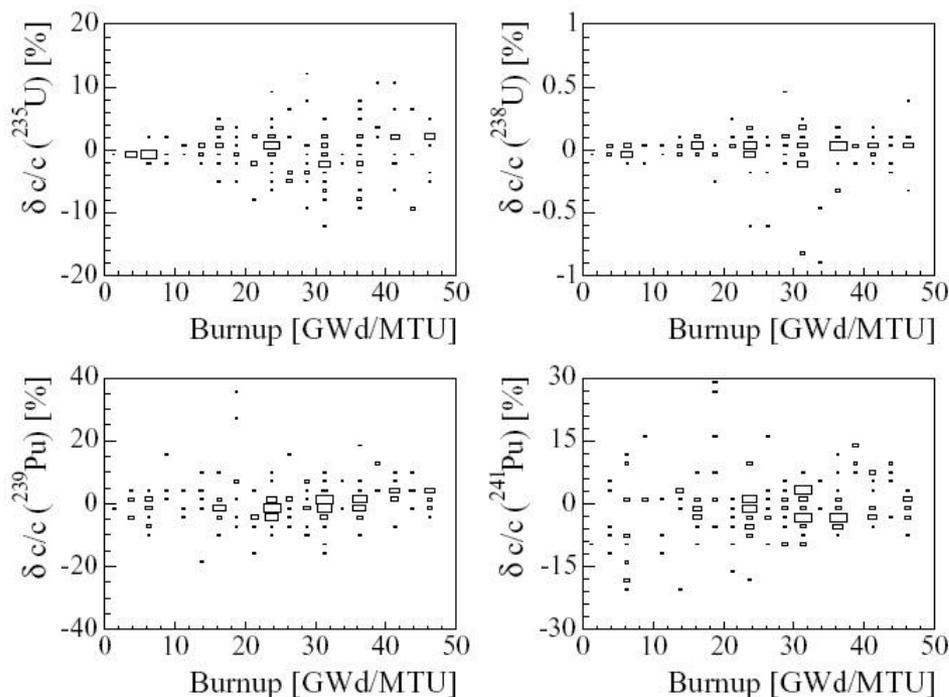
$$S(E_\nu) = \frac{W_{th}}{\sum_i \left(\frac{f_i}{F}\right) \cdot e_i} \sum_i \left(\frac{f_i}{F}\right) \cdot S_i(E_\nu)$$

◆ 由于总功率的限制，**5%** 的裂变份额误差对应 **0.5%** 的中微子数误差

◆ 考虑到各同位素之间的 关联，**0.5%→ 0.6%**

Djurcic et al. J. Phys. G: Nucl. Part. Phys. 36 (2009) 045002

Dragon模拟

	^{235}U	^{238}U	^{239}Pu	^{241}Pu
^{235}U	1.00	-0.22	-0.53	-0.18
^{238}U	-0.22	1.00	0.18	0.26
^{239}Pu	-0.53	0.18	1.00	0.49
^{241}Pu	-0.18	0.26	0.49	1.00

Uncertainties from Past Experiments

CHOOZ, Eur. Phys. J. C27, 331 (2003)

$$R = 1.01 \pm 2.8\% (\text{stat}) \pm 2.7\% (\text{syst})$$

Parameter	Relative error
Reaction cross section	1.9 %
Number of protons	0.8 %
Detection efficiency	1.5 %
Reactor power	0.7 %
Energy released per fission	0.6 %
Combined	2.7 %

- **Neutrino spectra** (1.9% → 1.6% with Bugey data)

- Inverse β-decay cross section (0.2%)

- **Fission fraction f_k** (~5%)

- Non-equilibrium fragments (0%)

Palo Verde, PRD62, 072002

Parameter	Relative error
Neutrinos/fission	1.4 %
Power, target, distance	1.5%
Combined	2.1 %

Power contributes ~0.7%

KamLAND,PRL94:081801, 2005.

TABLE I: Estimated systematic uncertainties (%).

Fiducial Volume	4.7	Reactor power	2.1
Energy threshold	2.3	Fuel composition	1.0
Efficiency of cuts	1.6	$\overline{\nu}_e$ spectra [3]	2.5
Livetime	0.06	Cross section [5]	0.2
Total systematic uncertainty			6.5

Correlated		Uncorrelated	
Energy/fission	0.2%	Power	0.5%
IBD reaction/fission	3%	Fission fraction	0.6%
		Spent fuel	0.3%
Combined	3%	Combined	0.8%

Daya Bay Absolute Rate Measurement
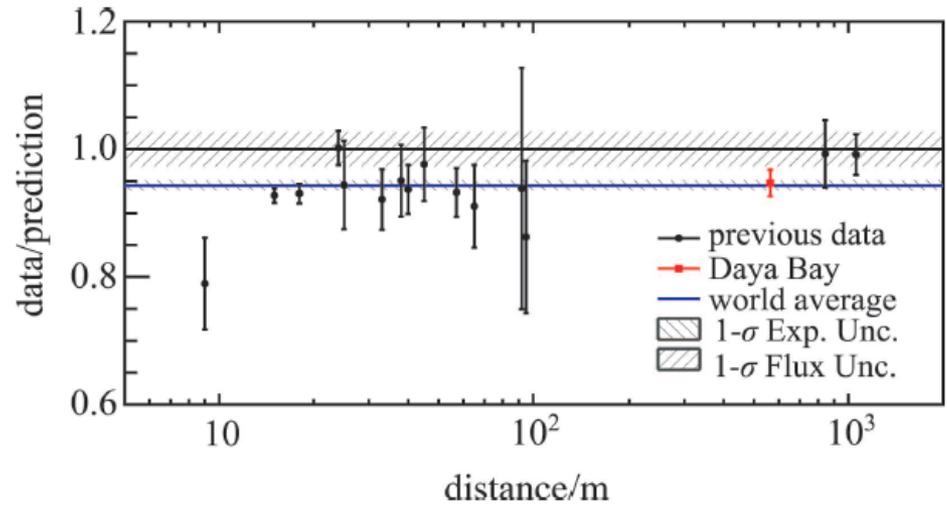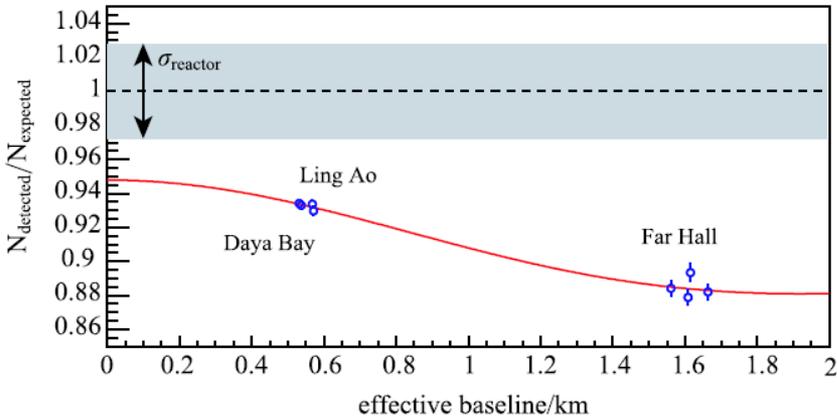
Chin. Phys. C41, 013002 (2017)

⇨ **Data/(Huber+Mueller)：** **0.946±0.020**

⇨ **Past global average:** **0.942±0.009**

⇨ **Data/(ILL+Vogel)：** **0.992±0.021**

contribution	uncertainty
statistics	0.1%
oscillation	0.1%
reactor	0.9%
detection efficiency	1.93%
total	2.1%

Daya Bay Fuel Evolution

- Combined fit for major fission isotopes ^{235}U and ^{239}Pu

- $\sigma235$ is 7.8% lower than Huber-Mueller model (2.7% meas. uncertainty)

- $\sigma239$ is consistent with the prediction (6% meas. uncertainty)

- 2.8σ disfavor equal deficit (H-M model & sterile hypothesis)
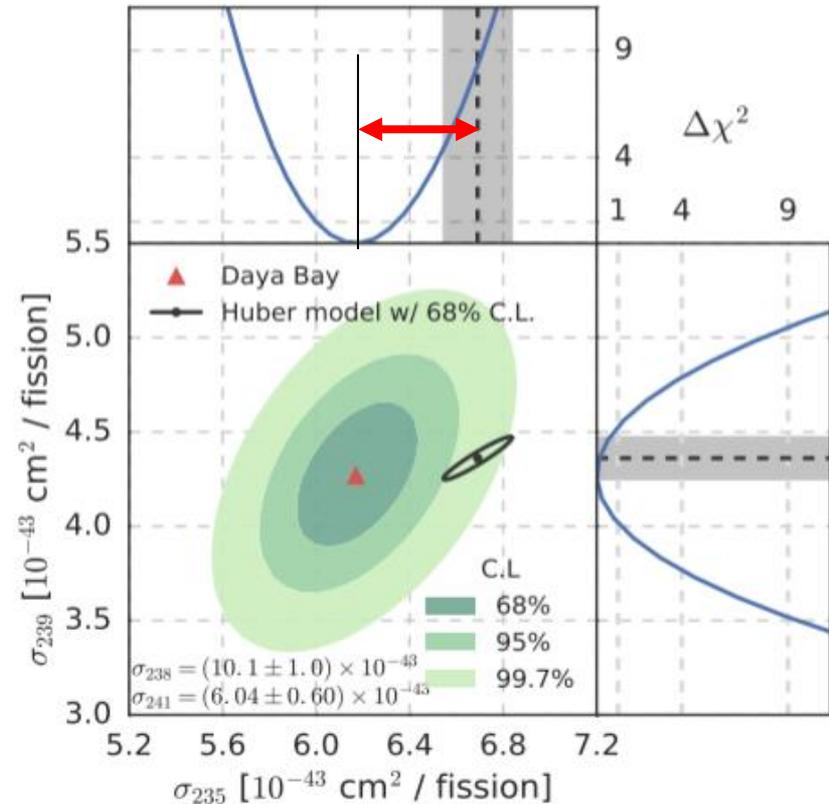
PRL118, 251801 (2017)

Reactor Anomaly (Spectrum)

- ◆ 5 MeV Bump
- ◆ Not due to energy non-linearity
- ◆ Not due to sterile ν
- ◆ Possibly due to forbidden decays (PRL112: 2021501; PRL114:012502)

DC, JHEP 1410 (2014) 086   **RENO:arXiv:1610.04326**   **Chin. Phys. C41, 013002 (2017)**
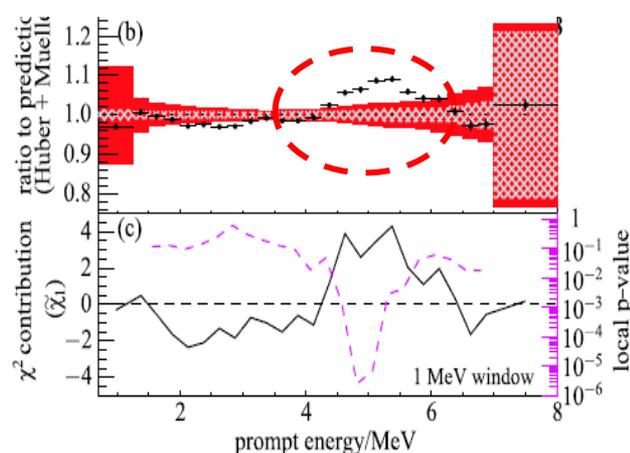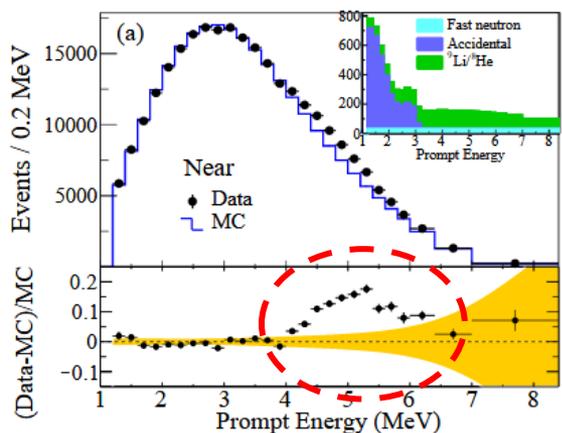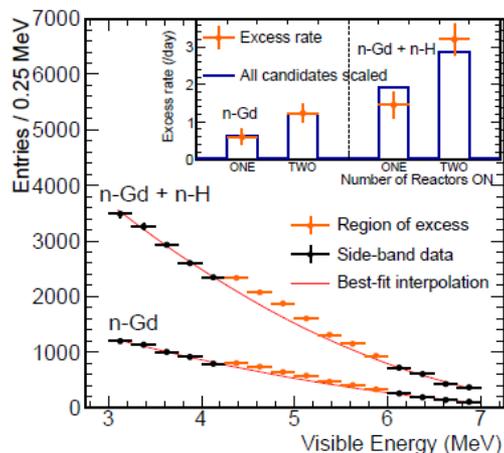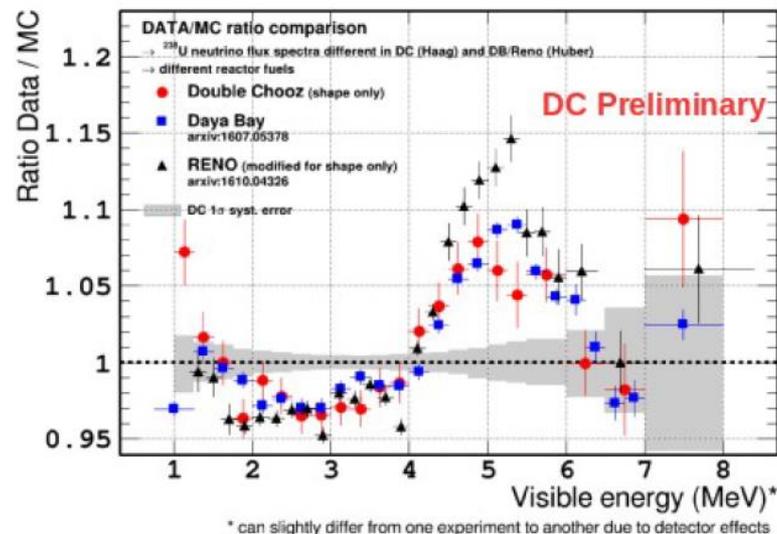
Measuring the spectrum

◆ **Unfolding the reactor neutrino spectrum**

⇨ Between 1.5 and 7MeV: 1.0% at 3.5 MeV, 6.7% at 7 MeV

⇨ Above 7 MeV it is larger than 10%.
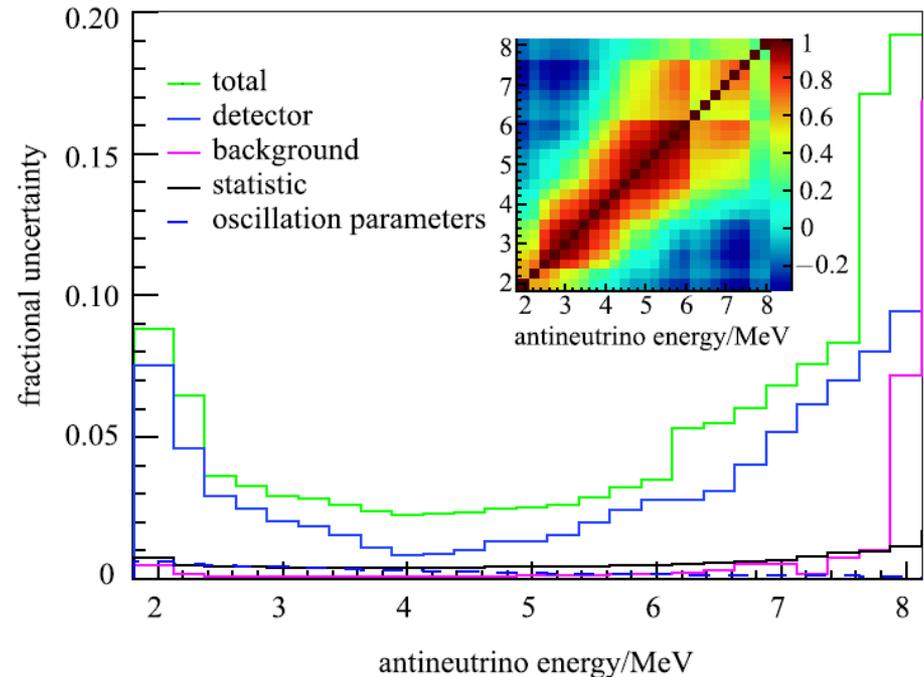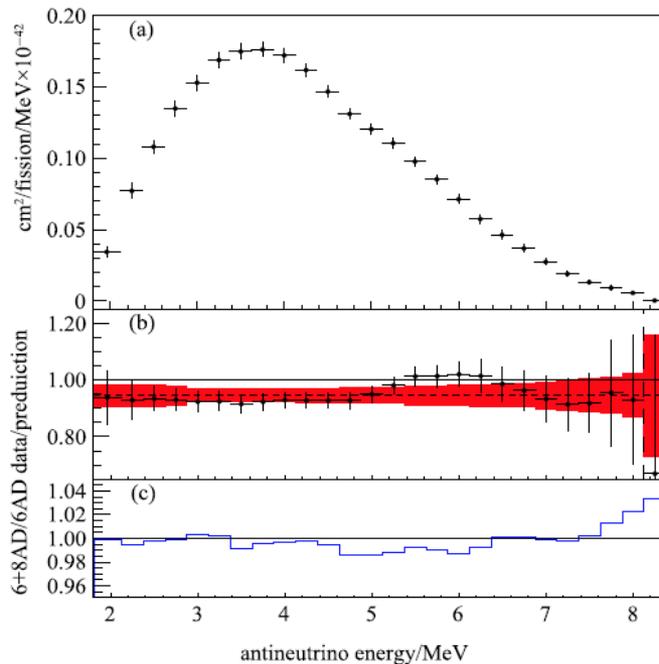
◆ **New prediction besides *ab initio* method and conversion method**

◆ **W/ the direct measurement, spectra uncertainty comes mainly from energy non-linearity uncertainty (do not double counting): 1% energy scale → 10% uncertainty in spectrum.**

◆ **Aim at 1% for JUNO**

Reactor Neutrino Experiments

- **Elastic scattering with electron**
  - ⇨ **Abnormal magnetic moment (TEXONO, GEMMA, NUMU)**
  - ⇨ **Weak mixing angle θw**
- **Inverse Beta Decay**
  - ⇨ **Most experiments (LS detector)**
- **1.8-10 MeV means**
  - ⇨ **Natural radioactivity**
  - ⇨ **Long-lived isotopes produced by cosmic muons**

Peak at 4 MeV

a) $\bar{\nu}_e$ interactions in detector [1/(day MeV)]
b) $\bar{\nu}_e$ flux at detector [10^8/(s MeV cm^2)]
c) $\sigma(E_\nu)$ [10^{-43} cm^2]

E_ν (MeV)
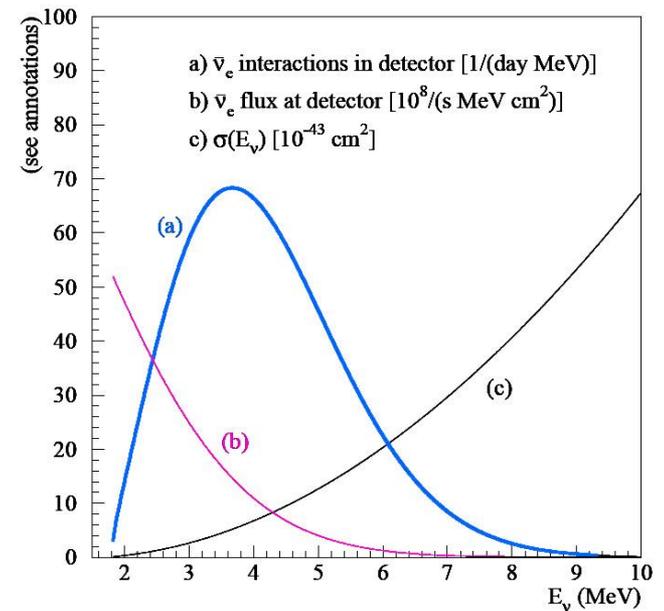
3 – Natural Radioactivity

Natural Radiation

- **Natural radiation is everywhere, born w/ the Earth**
  - ^{238}U $\rightarrow$ ^{222}Rn **In rock, mud, air, sea water**
  - ^{232}Th
  - ^{40}K

$$-dN = \lambda N dt,$$
$$N = N_0 e^{-\lambda t}$$

- **1 Bq = 1 decay per second**
  - **Human body has ~ 5000 Bq K-40**
  - **1 kg rock (granite) ~ 150 Bq**
  - **1 kg Uranium ~ 12,000,000 Bq**
  - **1 kg Co-60 = ? Bq**

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\lambda \tau = 1, \qquad \tau = \frac{1}{\lambda}$$

- **Cosmogenic radiation: ^{14}C, neutron, and many**
- **Artificial radiation, e.g.**
  - ^{60}Co **(In stainless steel, for industry and research)**
  - ^{137}Cs **(N bomb test, for research)**

Decay of ^{232}Th to ^{208}Pb

Alpha Lines

	KeV	%	Level
^{232}Th	4.011	77	0
	3.957	23	59
^{228}Th	5.423	72.7	0
	5.340	26.7	85
^{224}Ra	5.686	95.0	0
	5.449	5.0	241
^{220}Rn	6.288	99.9	0
^{216}Po	6.778	99+	0
^{212}Bi	6.090	27.2	0
	6.051	69.9	40
^{212}Po	8.785	??	0

Gamma Lines

	KeV	%
^{228}Ac	911	29?
	969	17.4
	1588	3.6
^{224}Ra	241	3.9
^{212}Pb	239	43.6
^{212}Bi	727	6.6
^{208}Tl	511	7.8
	583	30.9
	860	4.3
	2615	35.9
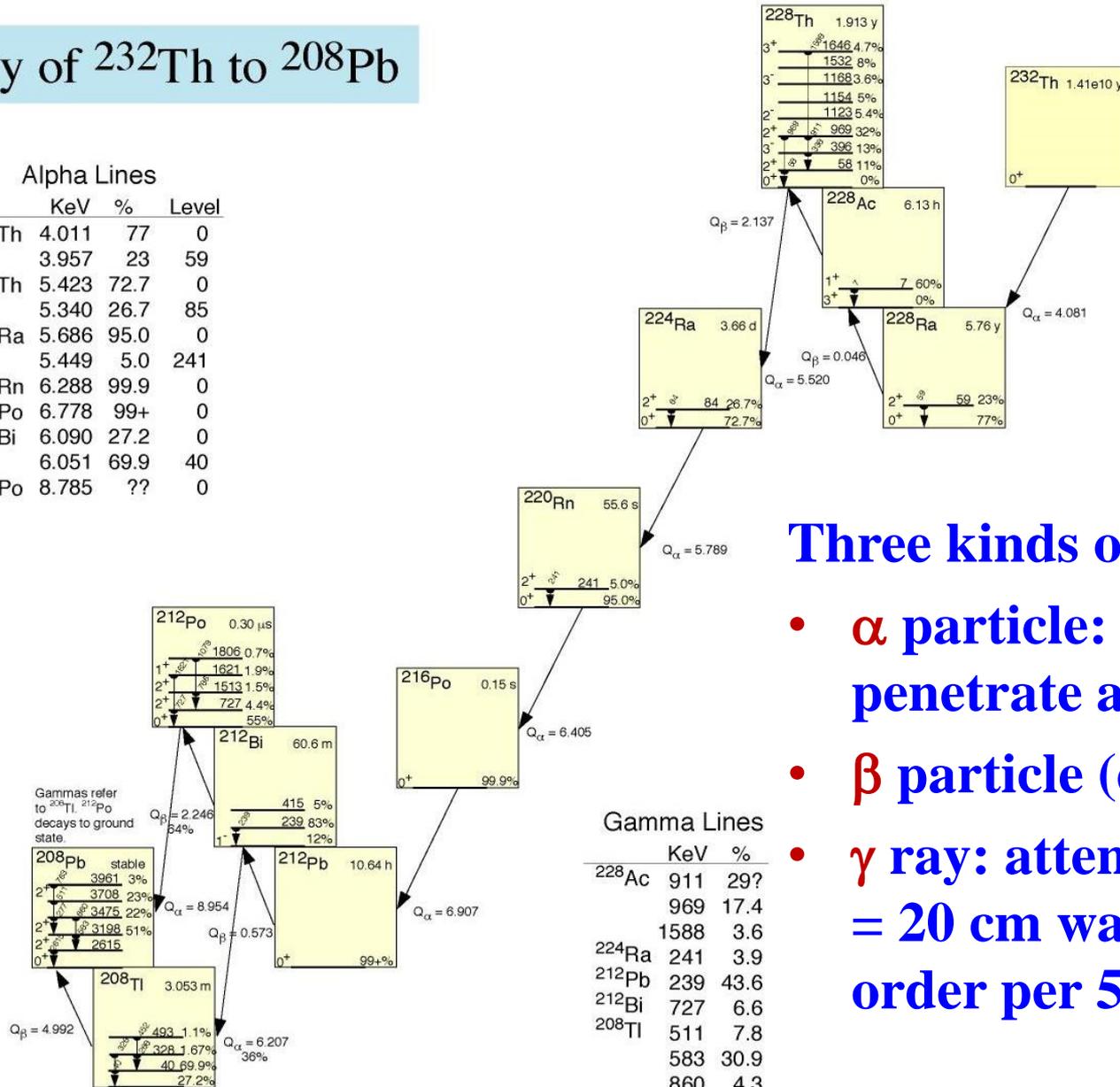
Three kinds of radiation:

- **α particle: can not penetrate a paper**
- **β particle (electron): cm**
- **γ ray: attenuation length = 20 cm water, or one order per 50 cm water**

Gamma spectrum of DYB granite
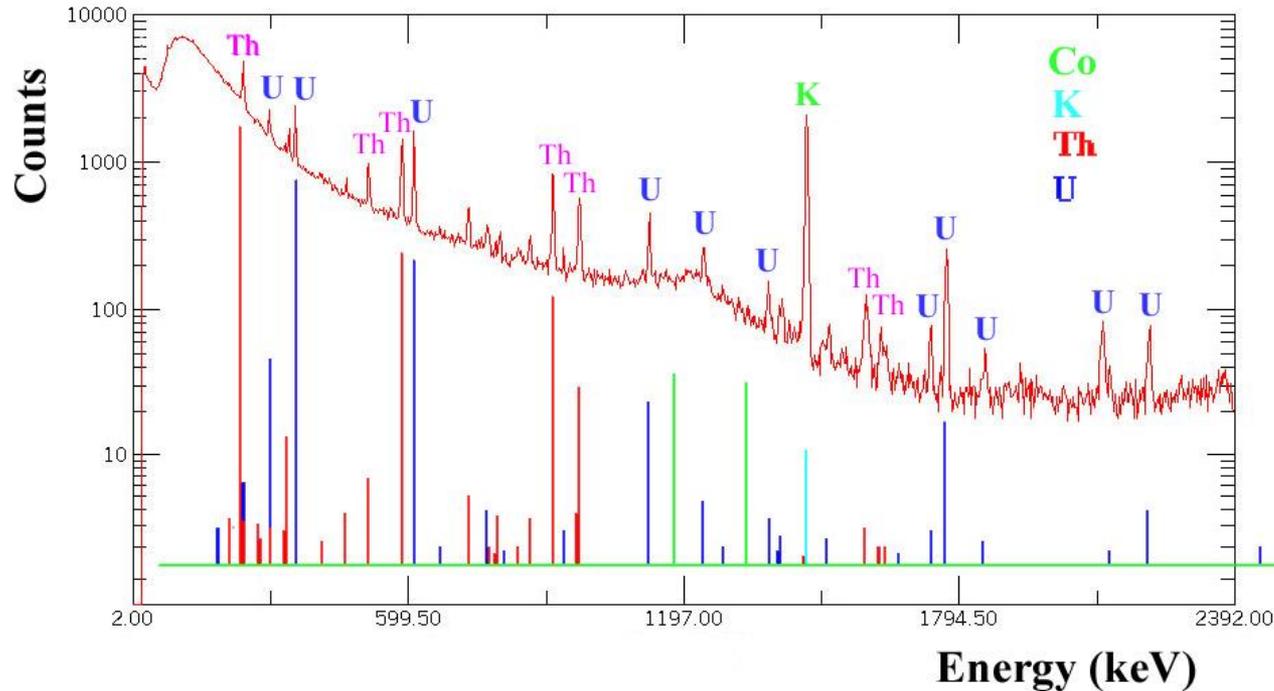
γ Energy < 2.6 MeV (TI-208)

◆ **Granite has high radiation in all kinds of rock. Radiation of Daya Bay granite is 3 times higher than world average**

◆ **10 ppm U**

◆ **30 ppm Th**

◆ **5 ppm K40**

ppb to Bq conversion
    1 ppb 40K = 258.4 mBq/kg
    1 ppb 238U = 12.4 mBq/kg
    1 ppb 232Th =   4.0 mBq/kg

Where the radioactivities come from?

- **Rock**
  - 8.8ppm U, 28.7ppm Th, 4.5ppm K
  - Almost 0 after 2.5m water shielding
- **Liquid scintillator**
  - **Radon** (product of U238 decay, emitted from rock)
  - Contamination, additives
  - Liquid scintillator need cover with N2
- **PMT glass,** can not contact with LS directly (oil shielding); use low background glass
- **Stainless steel tank:** need low bkg steel.
- Avoid aluminum, glass components in detector
- **All detector material need radio-assay**
- **Assembly of detector in clean room (10,000 class)**

Use water to shield gamma from rock

AD Reconstructed Position

in air

in water

Boundary of 4-m acrylic vessel

Entries 65328
Mean x 0.08425
Mean y 1.15
RMS x 1.033
RMS y 0.5944

Detector 1

AD PMT gain: 1.5×10^7
Trigger: NPMT > 55
(~0.5-0.7 MeV)
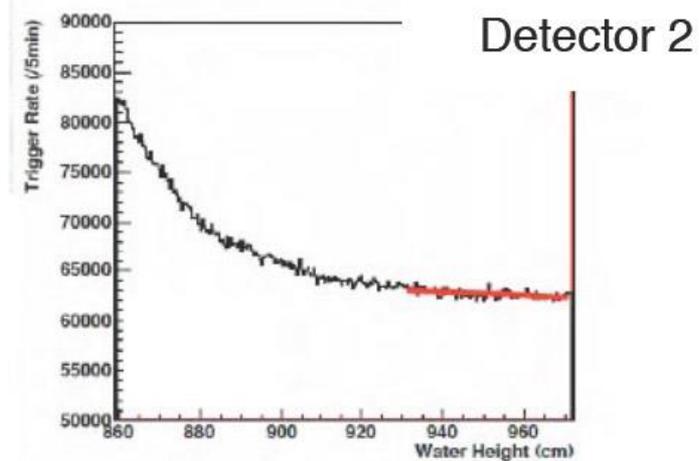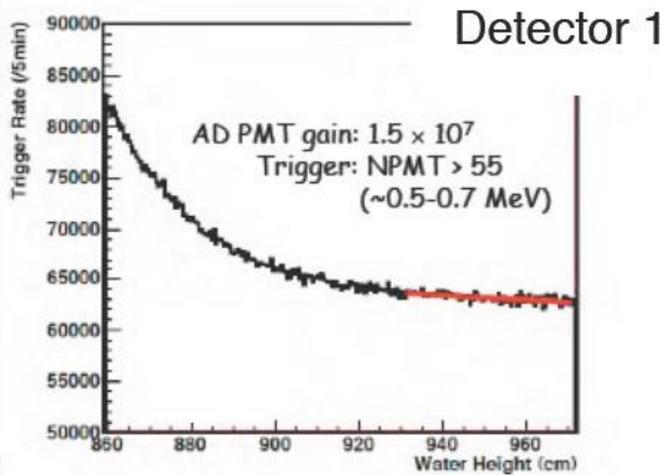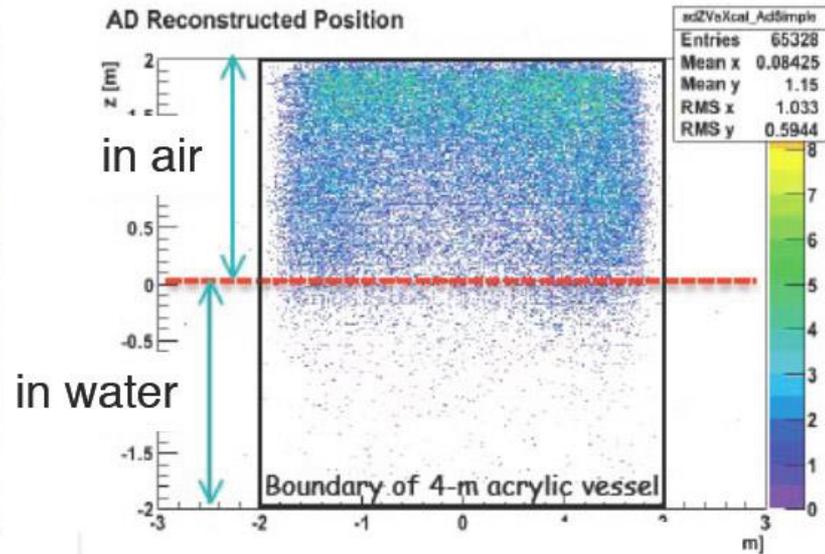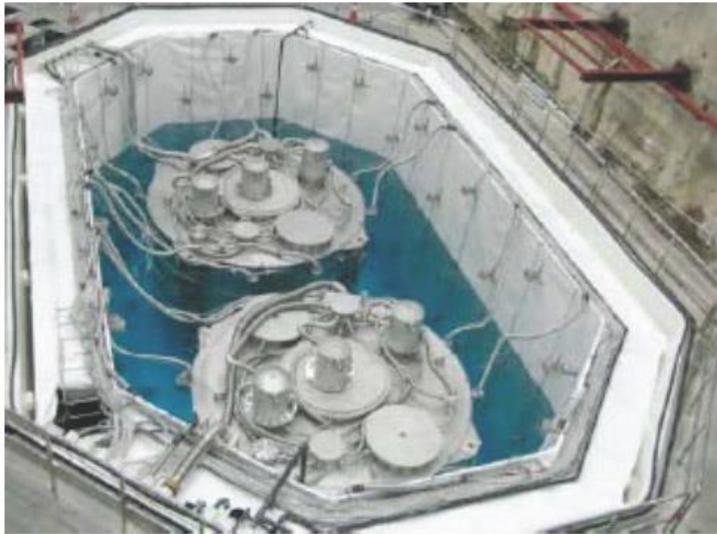
Detector 2

w/o water, DYB detector will see 1,000,000 Hz from rock
w/ 2.5m water, rate lowers to 200 Hz (almost 0 from rock)

Estimate the Shielding Power

◆ **MeV γ deposit energy in matter via Compton scattering on electron. Therefore, attenuation is proportional to electron density, thus matter density (almost independent of material)**

◆ **"mass attenuation coefficient"**

◆ **attenuation length ~ 20 cm water. Reduce 1 order/50 cm.**

◆ **What's the shielding power of DYB water pool?**

	γs (>1 MeV)	E (MeV)	mass attenuation coefficient in oil(cm^2/g)	mass attenuation coefficient in steel(cm^2/g)
1 Bq U	0.82	1.5	0.0575	0.0486
1 Bq Th	0.88	2	0.0493	0.0425
1 Bq K	0.105	1.5	0.0575	0.0486
1 Bq Co	2	1.17+1.33	0.0493	0.0425

W/o enough shielding, we can't see any neutrino:

Imaging how to find 0.002 neutrinos in 1,000,000 backgrounds

Why DYB use Gd-LS? Why JUNO won't dope Gd in LS?
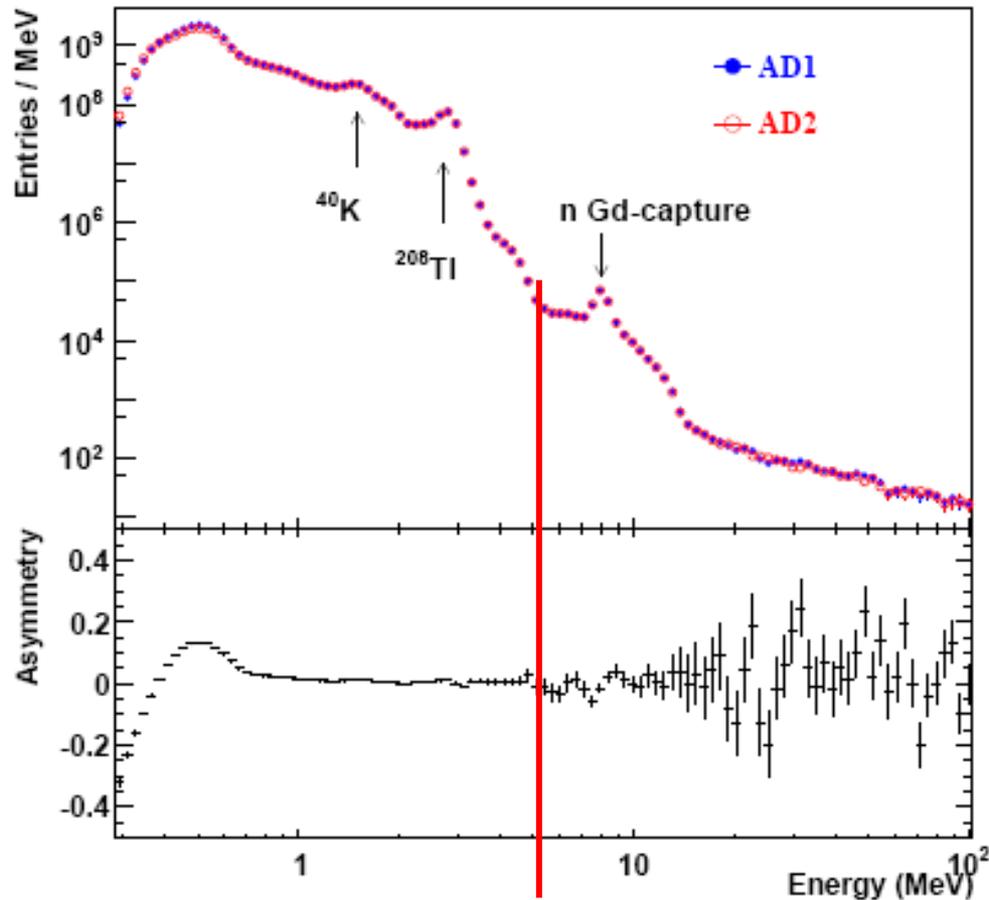
4 – Cosmic Muon

Cosmic Muon

p — proton
n — neutron
π^+, π^-, π^0 — pions
μ^+, μ^- — muons
e^- — electron
e^+ — positron
ν — neutrino
γ — gamma-ray

μ **Flux on surface ~ 200Hz/m^2**
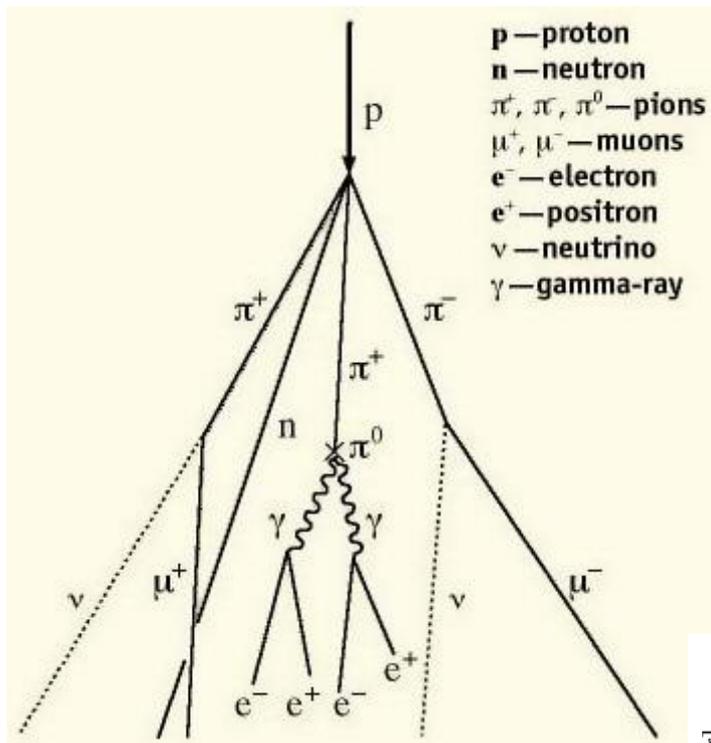
**Minimum ionization muon:
~200MeV/m.w.e.**
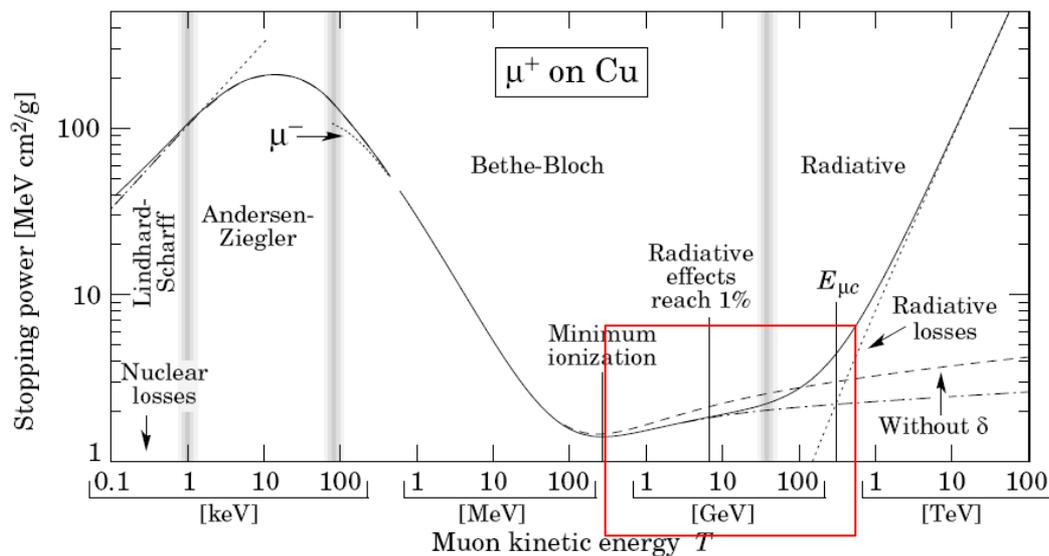
Muon Tomography

Discovery of a big void in Khufu's Pyramid by observation of cosmic-ray muons
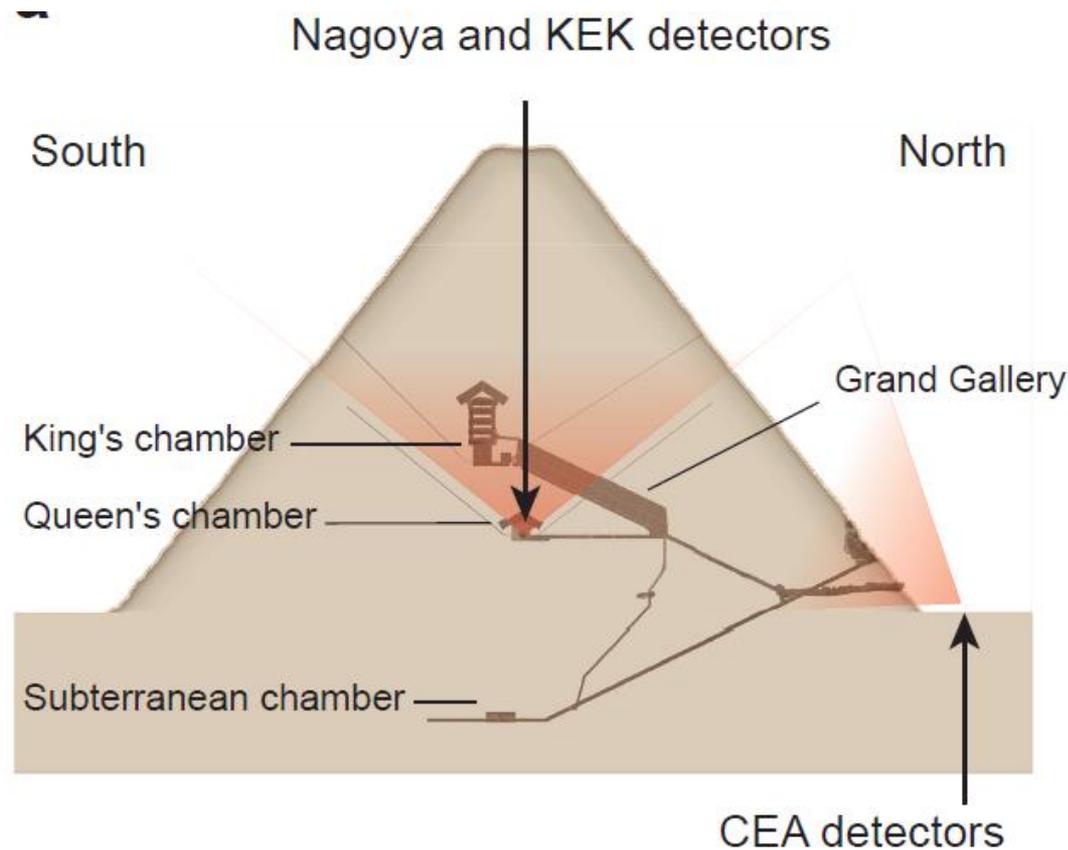
Muon Flux

◆ Muon on surface: Gaisser formula

$$\frac{dI_\mu}{dE_\mu d\cos(\theta)} = 0.14 \left(\frac{E_\mu}{GeV}\right)^{-2.7} \left[\frac{1}{1 + \dfrac{1.1 E_\mu \cos(\theta)}{115 GeV}} + \frac{0.054}{1 + \dfrac{1.1 E_\mu \cos(\theta)}{850 GeV}}\right]$$

$$\frac{dI_\mu}{dE_\mu d\cos(\theta)} = 0.14 \left(\frac{E_\mu}{GeV}\left(1 + \frac{3.64 GeV}{E_\mu [\cos(\theta^*)]^{1.29}}\right)\right)^{-2.7} \left[\frac{1}{1 + \dfrac{1.1 E_\mu \cos(\theta^*)}{115 GeV}} + \frac{0.054}{1 + \dfrac{1.1 E_\mu \cos(\theta^*)}{850 GeV}}\right]$$

point line: the Gaisser's formula
solid line: the modified formula

✛ L3+C00; θ = 0°	◇ Mars75; θ = 0°
□ Rastin84; θ = 0°	▼ L3+C00; θ = 56°
☆ CAPRICE97; θ = 0°	● Kiel-Desy data; θ = 75°
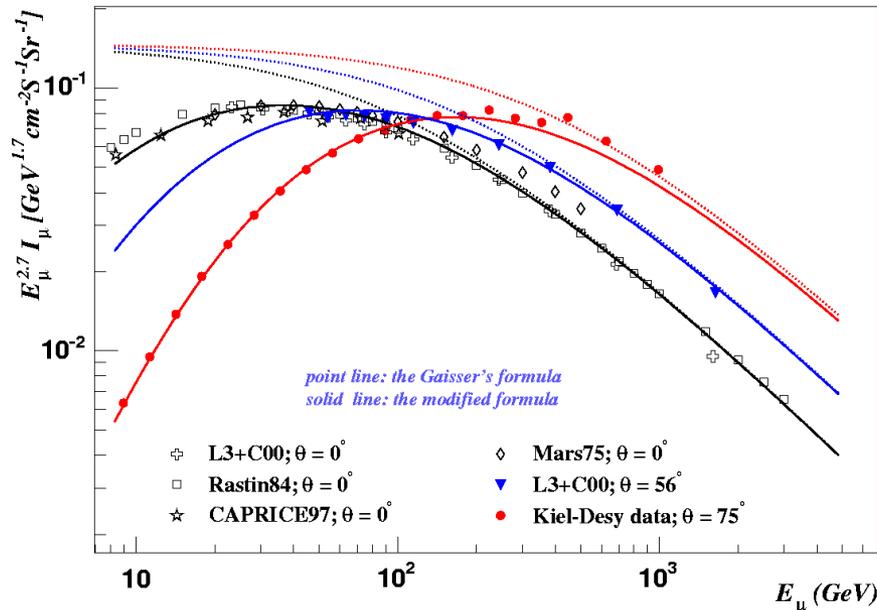
Modified Gaisser formula

More precise flux need simulation (Earth Magnetic field), See Honda's lecture

Muon Energy Loss

◆ **Continuous process —— Ionization energy loss**

$$\frac{dE}{dx} \approx -[1.9 + 0.08 ln(\frac{E_\mu}{m_\mu})]$$

◆ **Discrete process ——Bremsstrahlung, pair production, hadron process, important for high energy muon**

$$\frac{dE}{dx} = -\frac{E_\mu}{\xi} \qquad 在岩石中，\ \xi \approx 2.5 \times 10^5 g \cdot cm^{-2}.$$

◆ **Minimum muon energy to pass x_{min} thick rock**

$$E_0^{min} = \epsilon \left(e^{\frac{x_{min}}{\xi}} - 1 \right)$$

ε=500GeV, the energy at which the continuous process has equal contribution to discrete process

Propagate Muon to Underground Lab
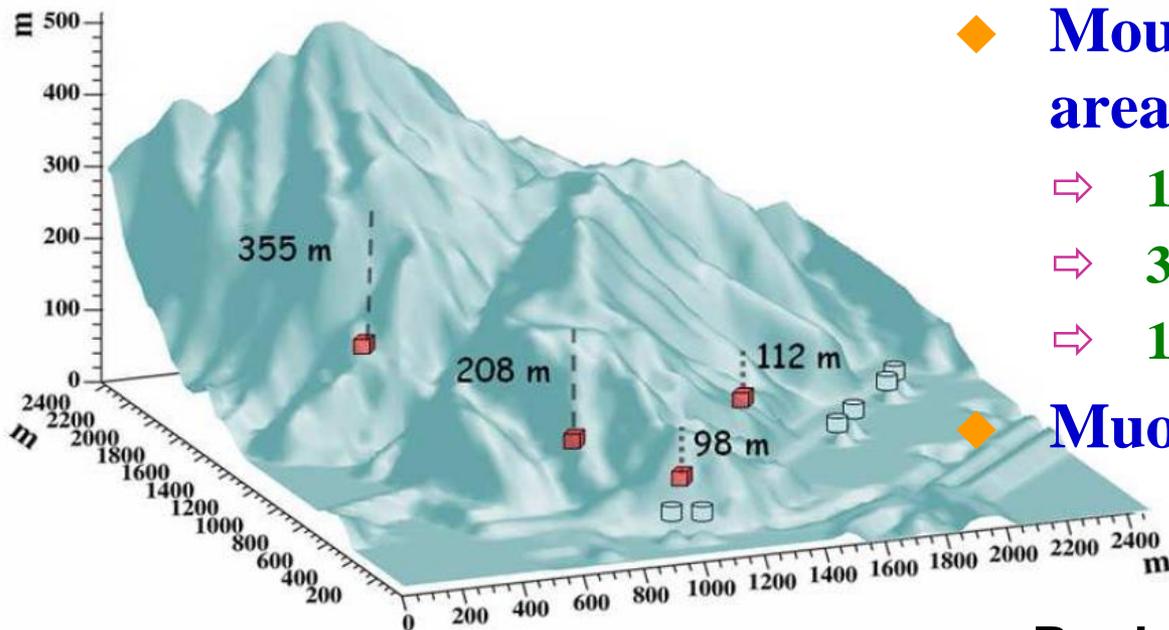
- ◆ **Mount map of Daya Bay area**
  - ⇨ **1×2 km high precision (1m)**
  - ⇨ **3×4 km 1:5000 map (5m)**
  - ⇨ **10×10 km SRTM map**
- ◆ **Muon propagation software: MUSIC, FLUKA**

Rock density：~2.6 g/cm^3
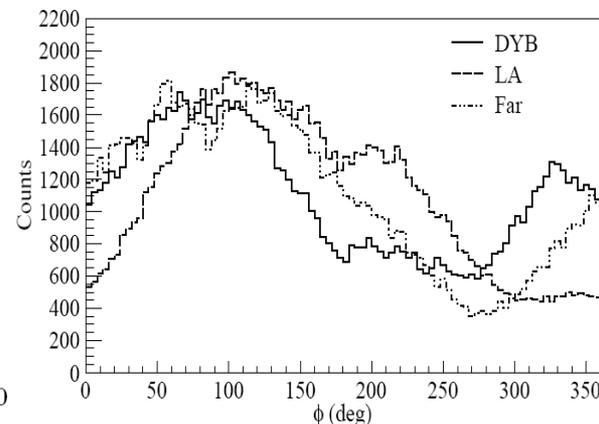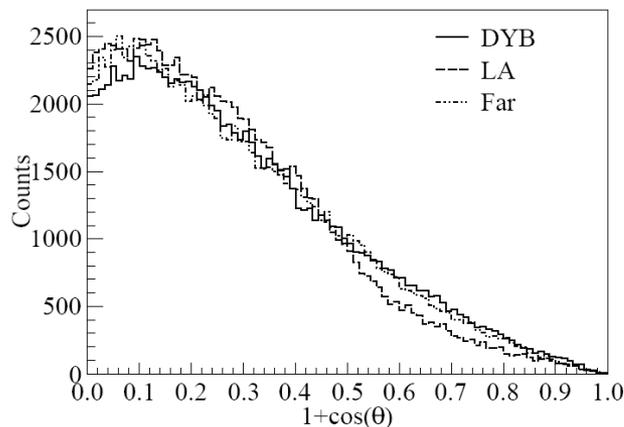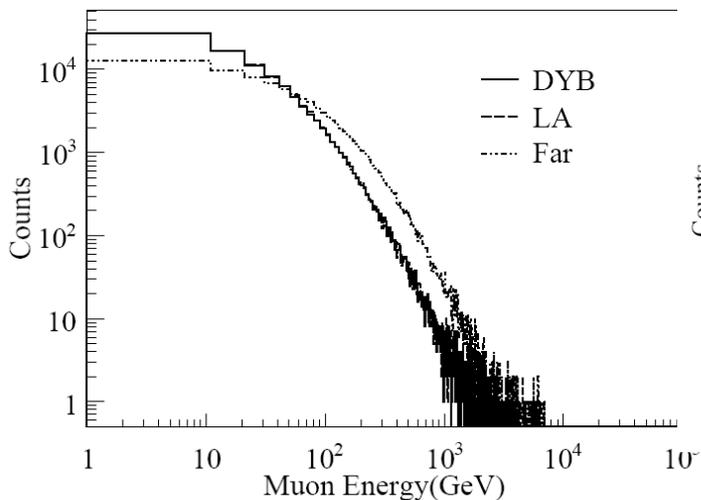
Muons at Daya Bay Lab

Muon Simulation

Far

Mid
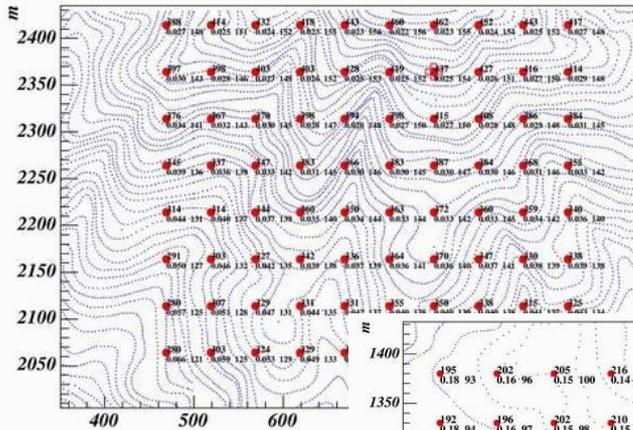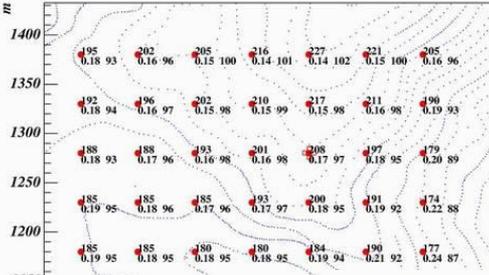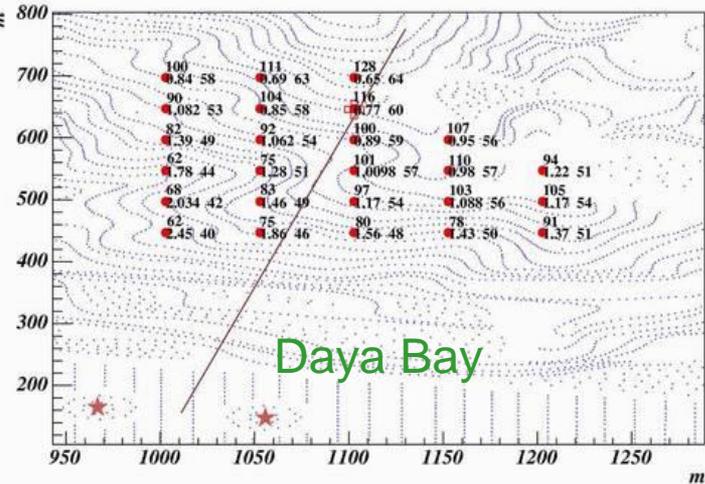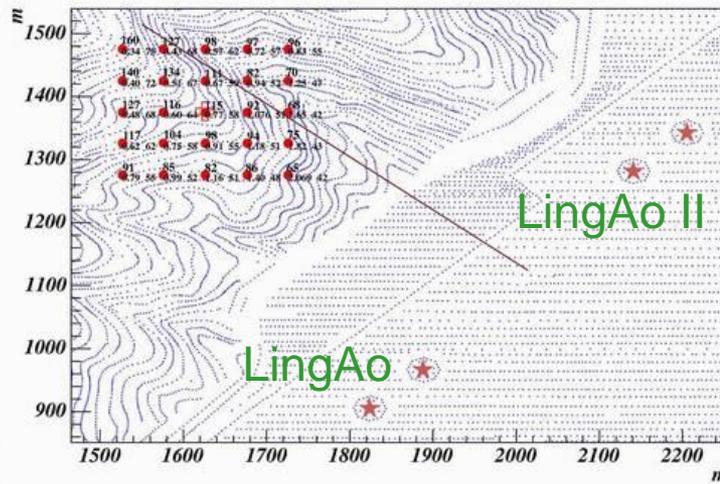
	DYB	LA	Mid	Far
Elevation (m)	93	100	208	324
Flux (Hz/m²)	**0.88**	**0.69**	0.17	**0.039**
Mean Energy (GeV)	57	58	97	142

Muon rate at surface ~200Hz/m²

Daya Bay

LingAo II

LingAo

Spallation Neutron
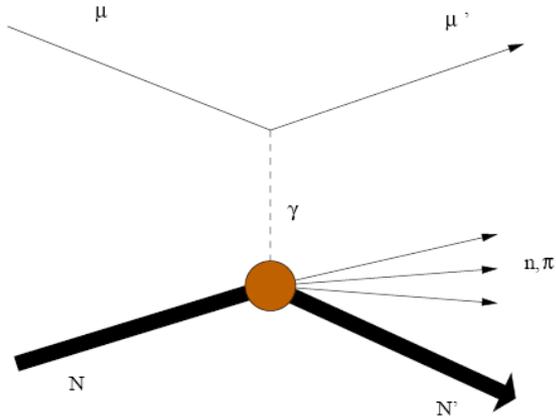
muon Spallation   Gamma-N
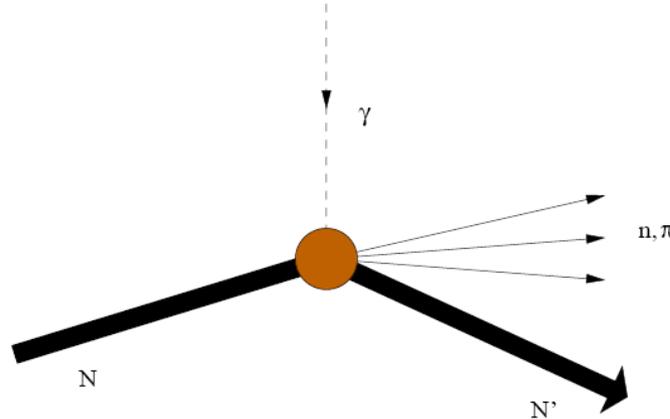
- 中子可打出次级中子
- **muon**在核上俘获产生中子（低能）

◆ **Cosmogenic neutron is one of the most important bkg**

◆ **Neutron yield**

$$N_n = 4.14 E_\mu^{0.74} \times 10^{-6} \, neutron /(muon \bullet g / cm^2)$$ Y.F. Wang et al

（当物质密度为**1 g/cm³时**）每个**muon**经过**1cm**产生的中子数（中子的海洋）

◆ **Neutron density at Daya Bay near (far) site**
0.03（0.001）neutron/m³/sec

Muon capture on nuclei

- **Stopping muon: $\mu \to e + \nu$, Michel e, lifetime 2.19703 μs**
- **Form a μ^- molecule, then decay $\mu^- \to e^- + \bar{\nu}$, or capture on nuclei.**

$$\mu^- + p \to \nu_\mu + n$$

$$\mu^- + {}^{12}C \to \nu_\mu + {}^{12}B^*$$
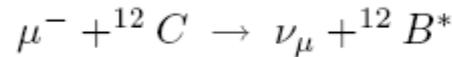
元素	μ^- 寿命 (ns)	核俘获率 (s^{-1})	核俘获过程几率 (%)	平均中子个数 (/反应)
C	2026.3	0.388×10^5	7.85	1
H	2194.9	0.420×10^3	0.11	1
O	1795.4	1.026×10^5	18.43	0.98
Fe	201	45.30×10^5	91.08	1.12
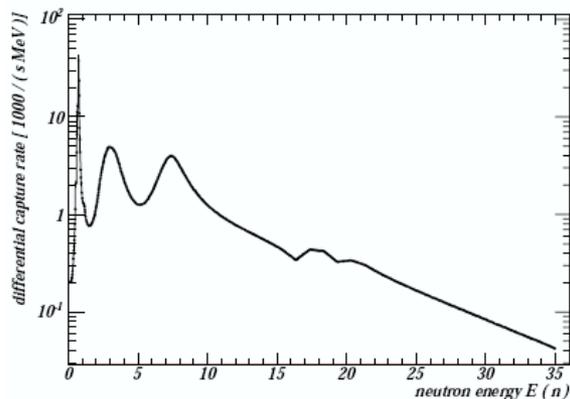
图 1.33: 碳核俘获带负电荷的 μ 子后放出的中子能谱 [64]
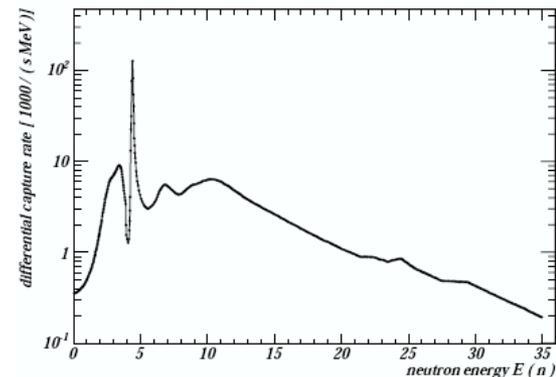
图 1.35: 氧核俘获带负电荷的 μ 子后放出的中子能谱 [64]

Cosmogenic Longlived Isotopes

◆ **Hard to remove by muon veto**

◆ **The most important bkg in DYB: He8/Li9 with a delayed neutron**

$$\sigma_{\text{tot}}(E_\mu) \propto E_\mu^{0.73}$$
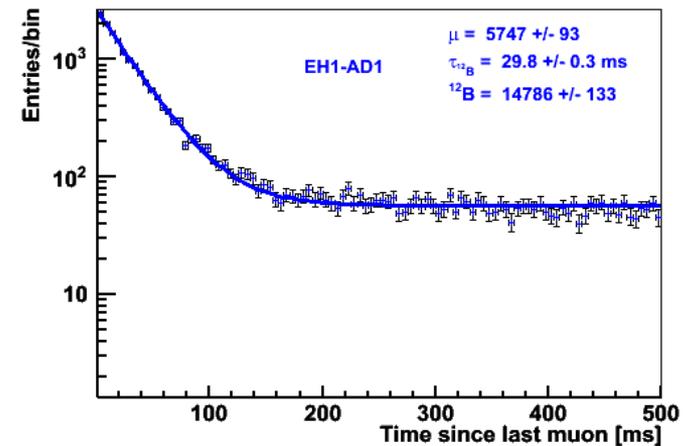
TABLE V: Summary of the neutron and isotope production yields from muon-initiated spallation in KamLAND. The results of the FLUKA calculation shown in this table include corrections for the muon spectrum and the μ^+/μ^- composition of the cosmic-ray muon flux.

	Lifetime in KamLAND LS	Radiation Energy	Spallation Production Yield ($\times 10^{-7}$ $(\mu \cdot (\text{g/cm}^2))^{-1}$)			Fraction from showering μ this measurement
			Hagner, et al. [10]	FLUKA calc.	this measurement	
n	207.5 μs	2.225 MeV (capt. γ)	—	2097 ± 13	2787 ± 311	64 ± 5%
^{12}B	29.1 ms	13.4 MeV (β^-)	—	27.8 ± 1.9	42.9 ± 3.3	68 ± 2%
^{12}N	15.9 ms	17.3 MeV (β^+)	—	0.77 ± 0.08	1.8 ± 0.4	77 ± 14%
^{8}Li	1.21 s	16.0 MeV ($\beta^-\alpha$)	1.9 ± 0.8	21.1 ± 1.4	12.2 ± 2.6	65 ± 17%
^{8}B	1.11 s	18.0 MeV ($\beta^+\alpha$)	3.3 ± 1.0	5.77 ± 0.42	8.4 ± 2.4	78 ± 23%
^{9}C	182.5 ms	16.5 MeV (β^+)	2.3 ± 0.9	1.35 ± 0.12	3.0 ± 1.2	91 ± 32%
^{8}He	171.7 ms	10.7 MeV ($\beta^-\gamma n$)	$\Big\} 1.0 \pm 0.3$	0.32 ± 0.05	0.7 ± 0.4	76 ± 45%
^{9}Li	257.2 ms	13.6 MeV ($\beta^-\gamma n$)		3.16 ± 0.25	2.2 ± 0.2	77 ± 6%
^{11}C	29.4 min	1.98 MeV (β^+)	421 ± 68	416 ± 27	866 ± 153	62 ± 10%
^{10}C	27.8 s	3.65 MeV ($\beta^+\gamma$)	54 ± 12	19.1 ± 1.3	16.5 ± 1.9	76 ± 6%
^{11}Be	19.9 s	11.5 MeV (β^-)	< 1.1	0.84 ± 0.09	1.1 ± 0.2	74 ± 12%
^{6}He	1.16 s	3.51 MeV (β^-)	7.5 ± 1.5	12.08 ± 0.83	—	—
^{7}Be	76.9 day	0.478 MeV (EC γ)	107 ± 21	105.3 ± 6.9	—	—

Pickup Strip

Insulator

Electrode

HV

Gas gap

bakelite resistivity 10^{10}--10^{13} Ωcm

~2000 ton pure water

μ

高10米

长16米　宽10米（近点）或16米（远点）

RPC

Pool Cover

Concrete

Tyvek

PMTs

OWS

IWS

AD

AD Support

μ

1m

$$\cos\theta = \frac{c}{nv},$$
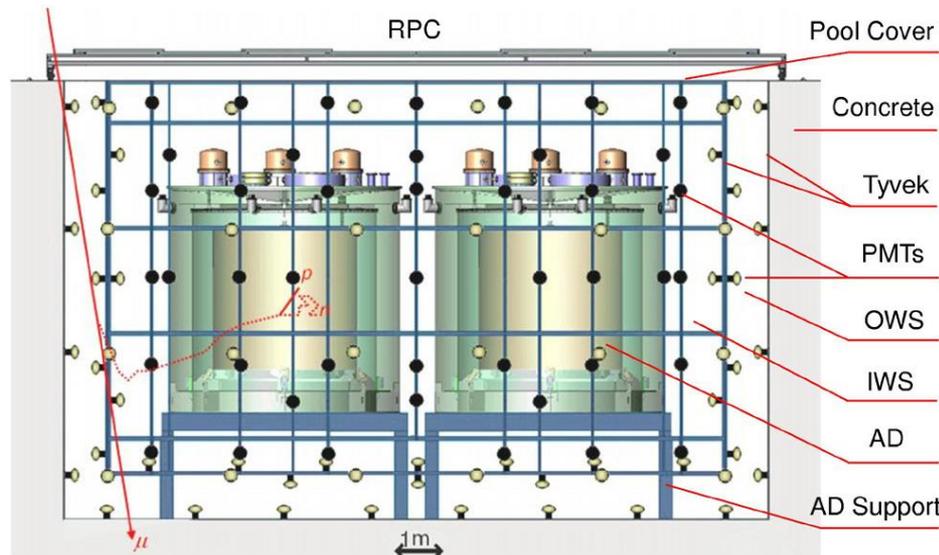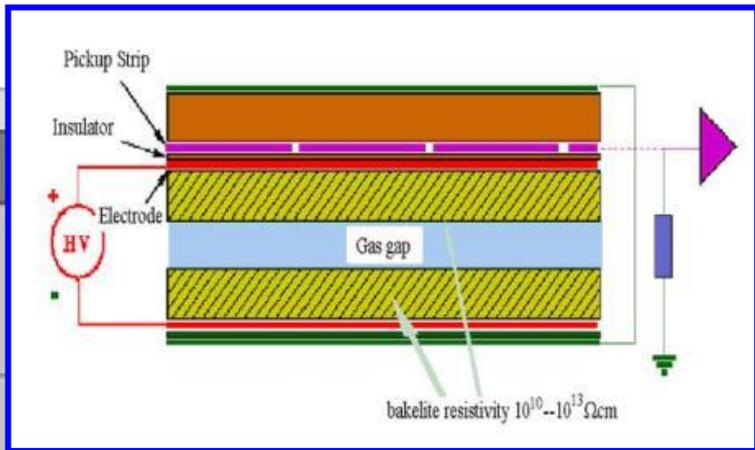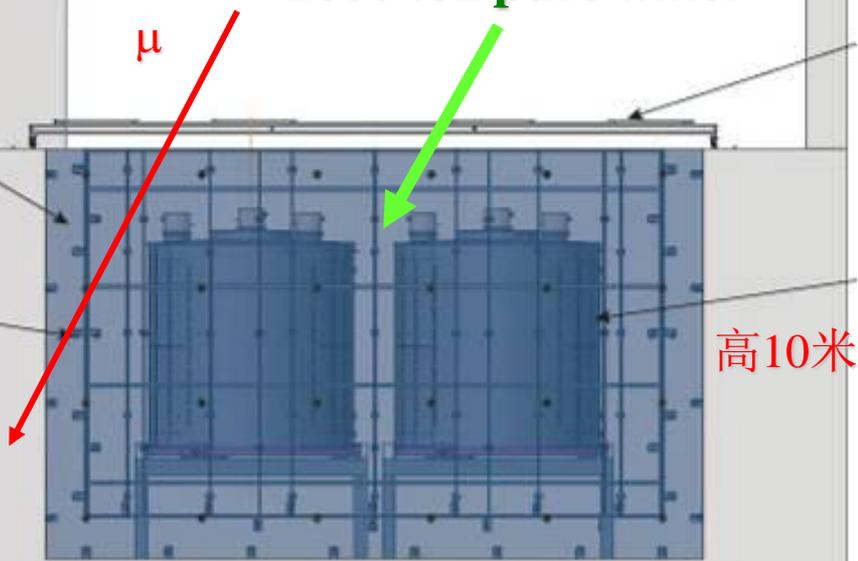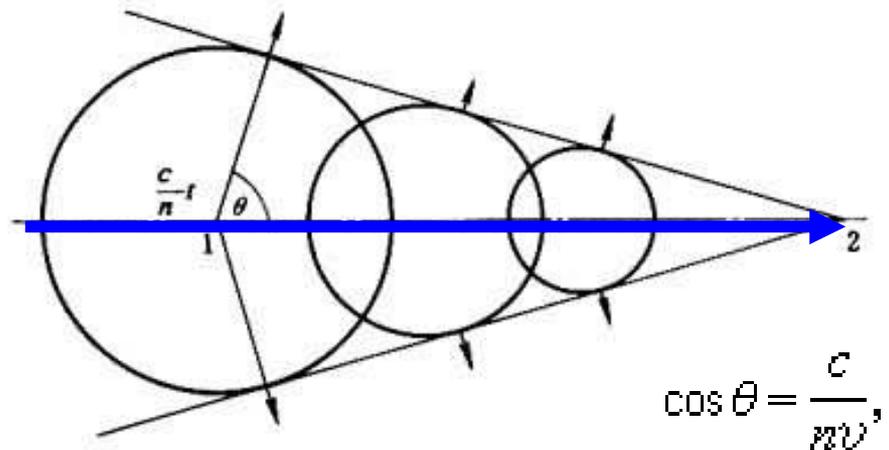
运动带电粒子所激发的电磁场

5 - Liquid Scintillator

Daya Bay Liquid Scintillator

◆ **LS: ~ 10 k photons/MeV (NaI 40 k photons/MeV), isotropic**

◆ **Recipe: LAB + 3g/L PPO + 15mg/L bis-MSB   (+0.1% Gd)**

◆ **Daya Bay photoelectron yield: 163 p.e./MeV**
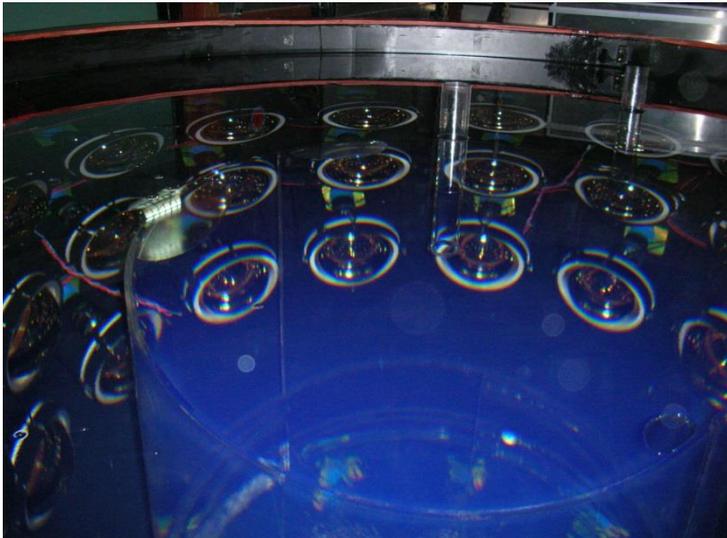
10 k photon/MeV (?)
Attenuated: 70%
PMT photocathode coverage: 12%
PMT quantum efficiency: ~ 20%

1.68% ➔ 168 p.e./ MeV

Entries	741249
C	6.507e+05 ± 8.825e+02
sigma	2.269 ± 0.008
T	43.78 ± 0.01
t0	3.564 ± 0.023
t1	17.76 ± 0.24
t2	122.6 ± 2.9
w1	0.7441 ± 0.0024
w2	0.1887 ± 0.0021
offset	45.06 ± 0.53

TPMHB0580EA

CATHODE RADIANT SENSITIVITY

QUANTUM EFFICIENCY

CATHODE RADIANT SENSITIVITY (mA/W)
QUANTUM EFFICIENCY (%)
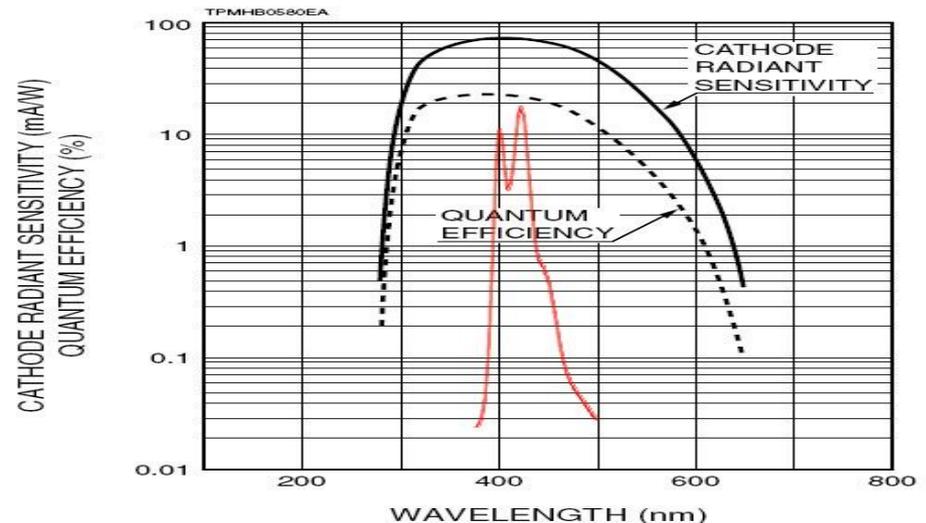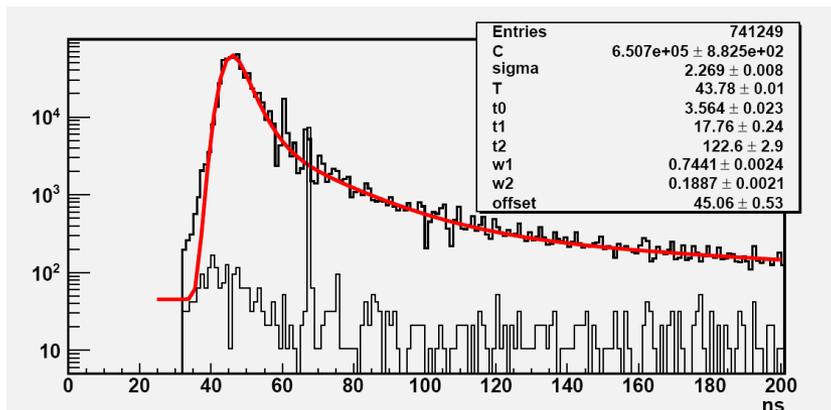
WAVELENGTH (nm)

Make a Liquid Scintillator

◆ **Solvent**

⇨ 含苯环的芳香族化合物

⇨ 三甲苯、二甲苯**(PX)**、甲苯、苯、**PXE、DIN、LAB**

⇨ **Solvent itself has very low light yield**
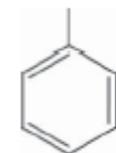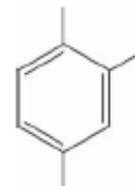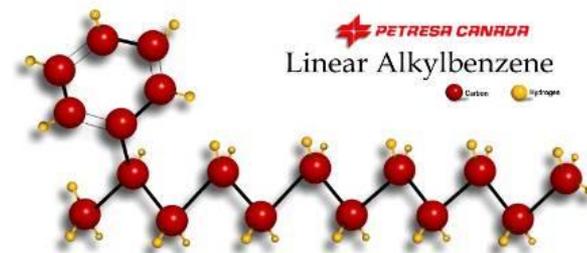
◆ **Fluor (1-5 g/L)**

⇨ **PPO、PBD、TP、PMP**
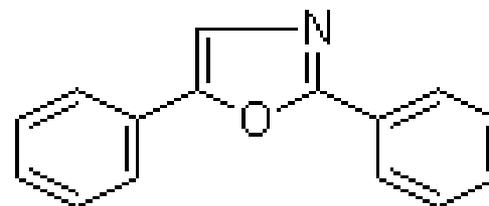
◆ **(Optional) Wavelength Shifter (~10 mg/L）**

⇨ **bis-MSB、POPOP、BBQ**

⇨ **Match PMT response, shifting photon wavelength to transparent reagion**

Toluene

Pseudocumene

PETRESA CANADA
Linear Alkylbenzene
Carbon Hydrogen

H₃C

CH₃
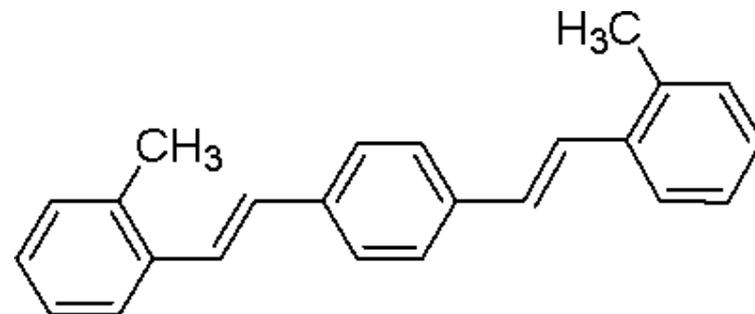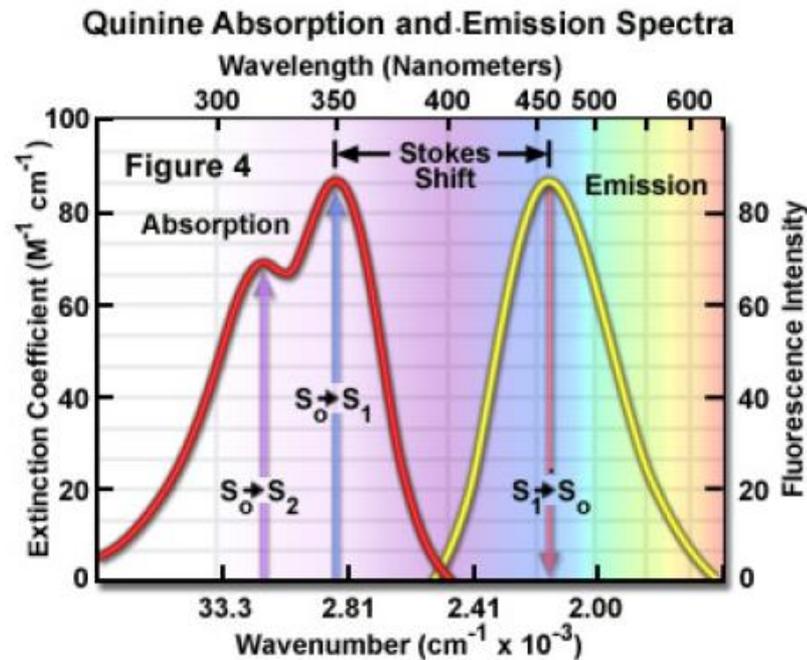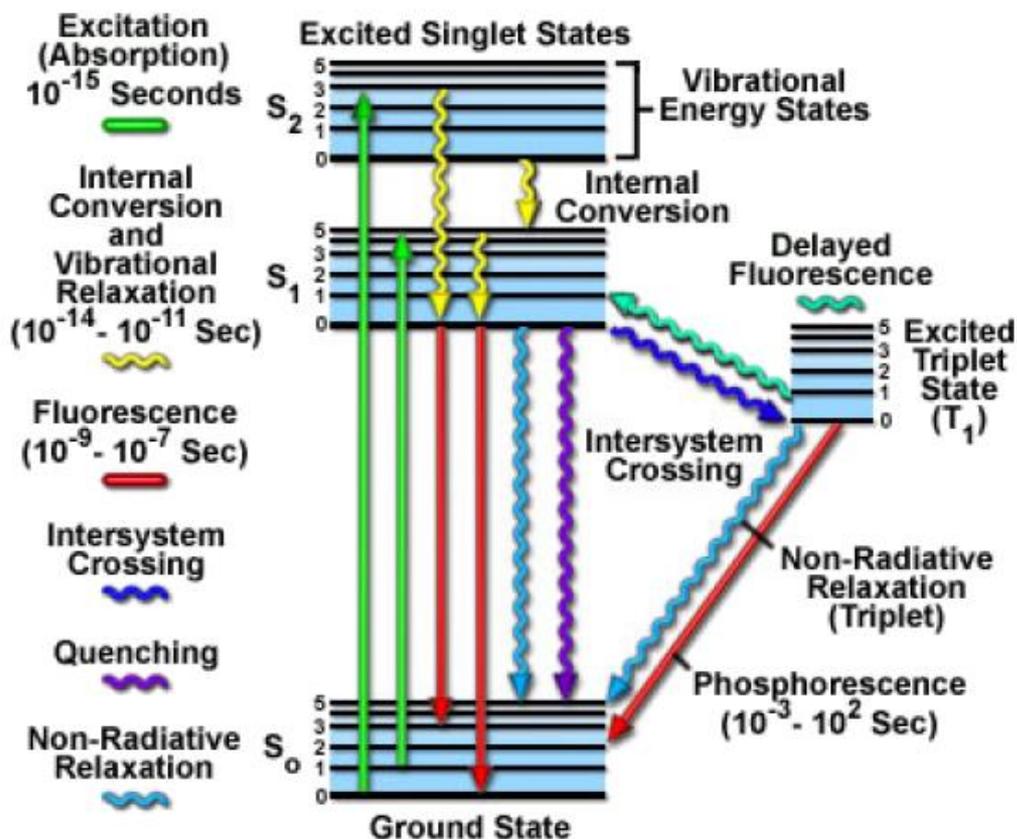
分子的能级结构

图 2.2: 斯托克斯位移。 [52]

◆ π 电子的能级结构。$S0$ 为基态，$S1, S2, S3$ 为激发单重态，$T1$ 为激发三重态。

◆ 正是有了斯托克斯位移（1852年发现），闪烁体产生的荧光才可以在自身材料中传播

Linear Alkylbenzene

Resonance

Ionization excitation of **solvent** — solvent

10^{-8} m | Forster energy transfer

primary fluor (~1% wt/wt)

emit UV. ~340 nm

10^{-4} m γ

absorb UV photon — secondary fluor (~0.05% wt/wt)

emit blue. ~400 nm

1 m γ

absorb blue photon — photodetector
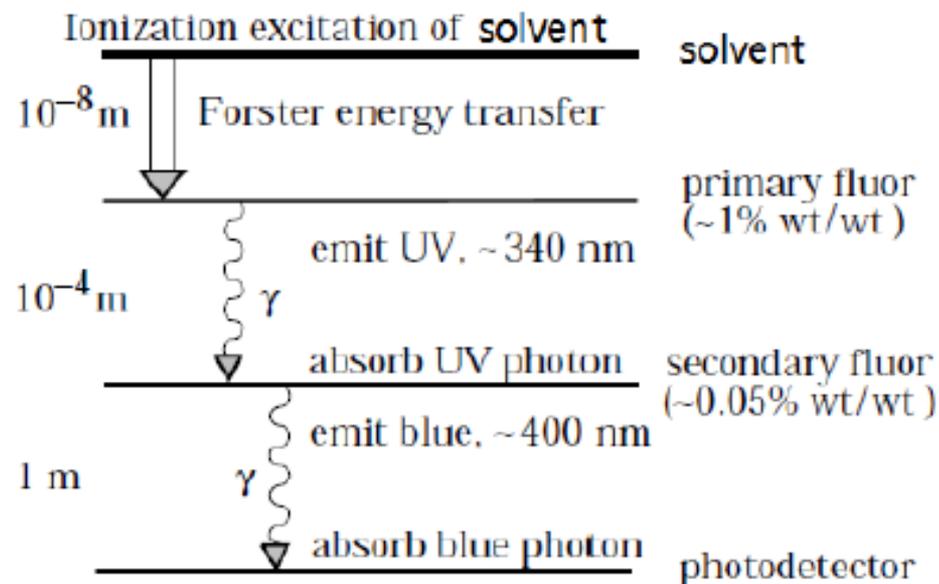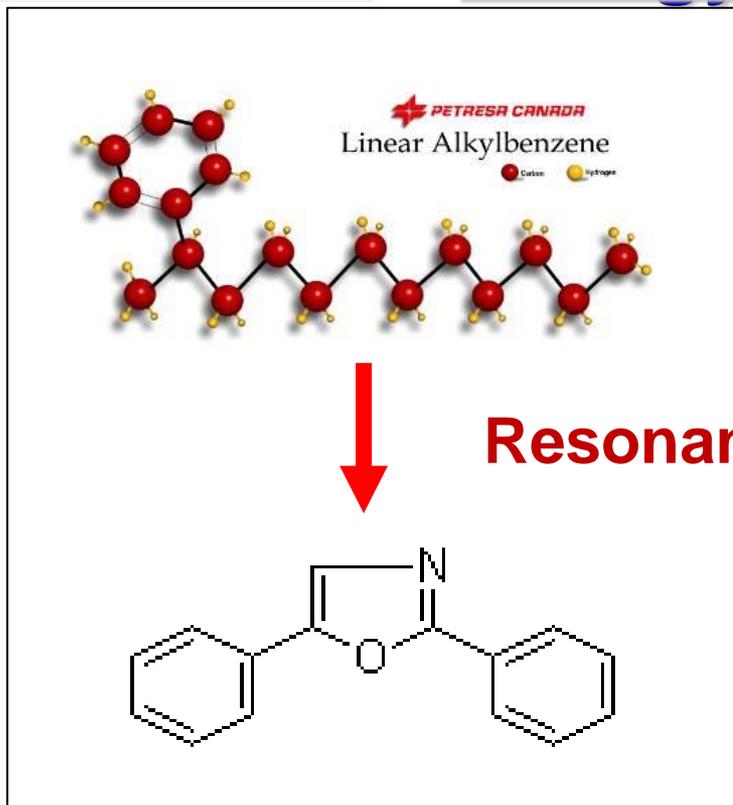
Absorption and reemisson of UV light
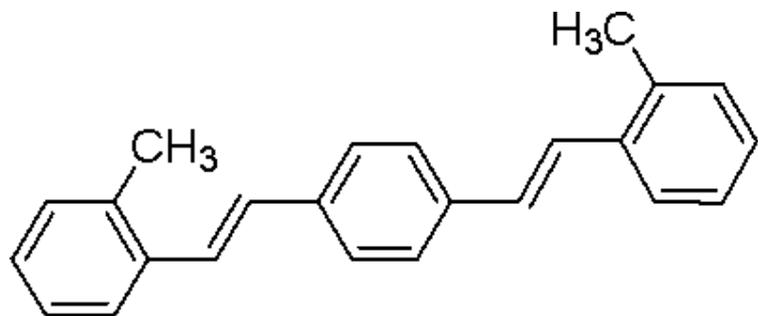
- 二元液闪（不带波长位移）
- 三元液闪

Light Yield vs PPO (Bis-MSB) Concentration

Concentration Quenching
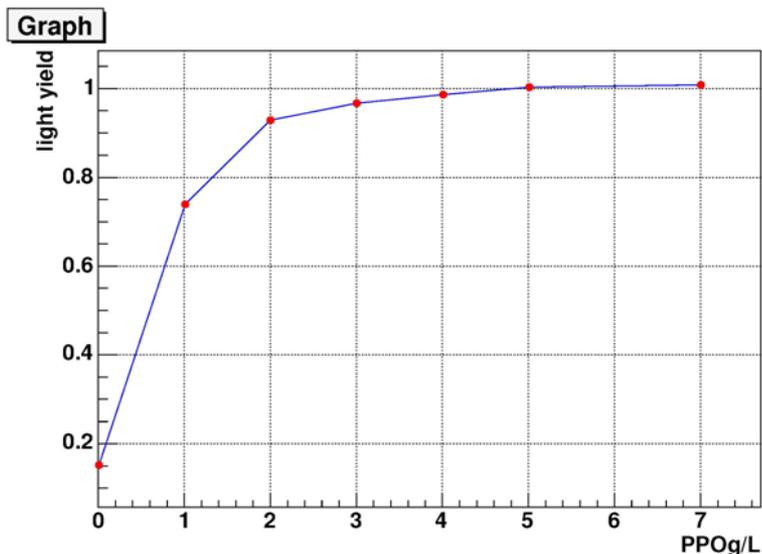
LAB为溶剂时PPO
的浓度与发光效率
的关系
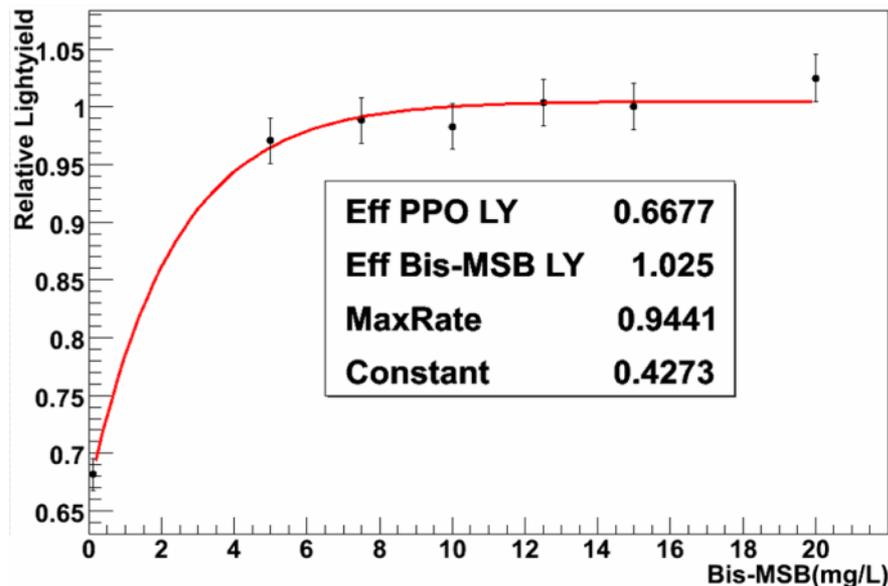
bis-MSB的浓度与
发光效率的关系

???

Attenuation, absorption, scattering

Bouguer's law: $\quad I = I_0 e^{-\frac{x}{l}}$

$$\frac{1}{l_{atten}} = \frac{1}{l_{absorb}} + \frac{1}{l_{ray}} + ...$$

Mie, Raman, absorb&re-emit…

Absorption: the molecular are heated by incident light.

Rayleigh scattering: scatter off the bound electrons of molecular.

Attenuation: also noted as extinction length.

Attenuation

$$I = I_0 \exp(-x/L)$$

⇨ **L=2~3 m (Tap water)，50% light absorbed in DYB detector**

⇨ **L=10m ： 20% light absorbed in DYB**

39.6 m

Result of water sample

5米
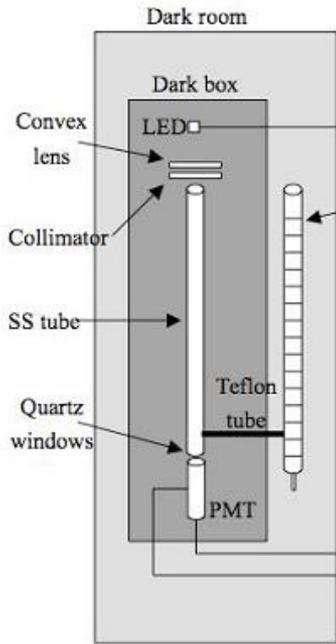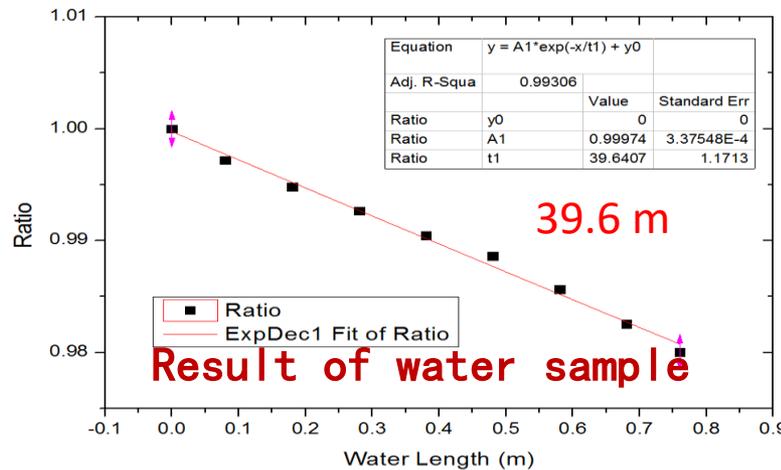
分光光度计 UV-vis

吸光度的定义 $Abs = -\log_{10}(I/I_0)$

衰减长度的定义 $I = I_0 \exp(-x/\lambda)$

$10cm$的比色池 $\lambda = 0.04343/Abs$

1-m tube
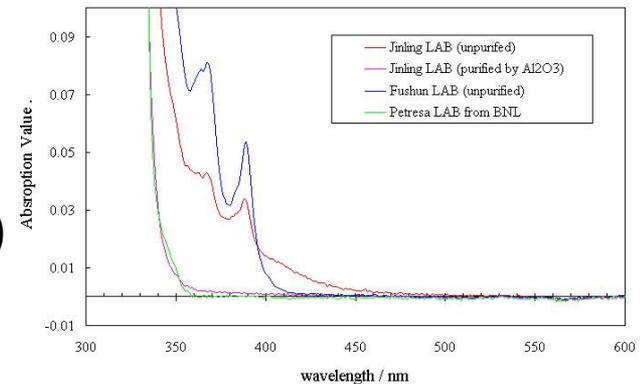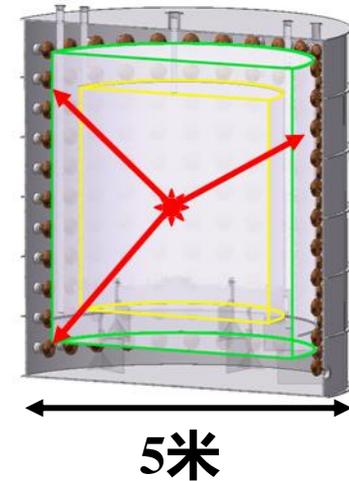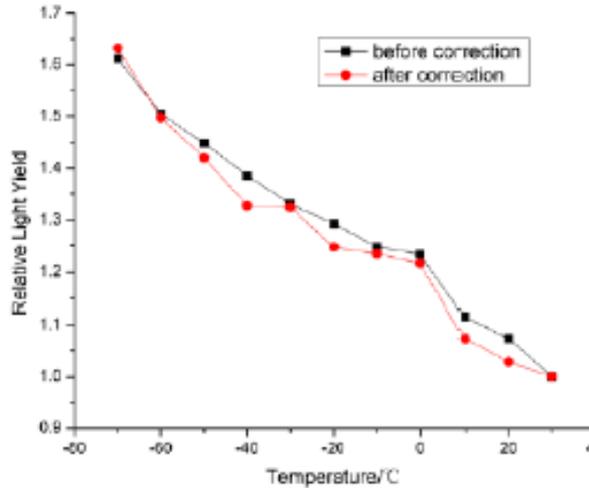
Quenching

Thermal Quenching
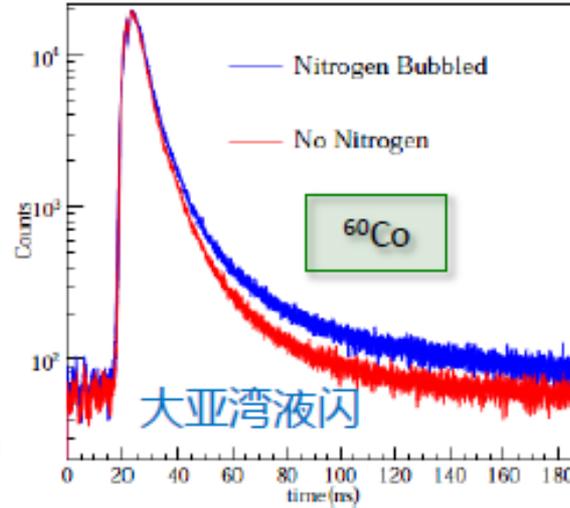
Oxygen quenching ~11%

NIM 158,1(1979), NIM 400,1(1997), …
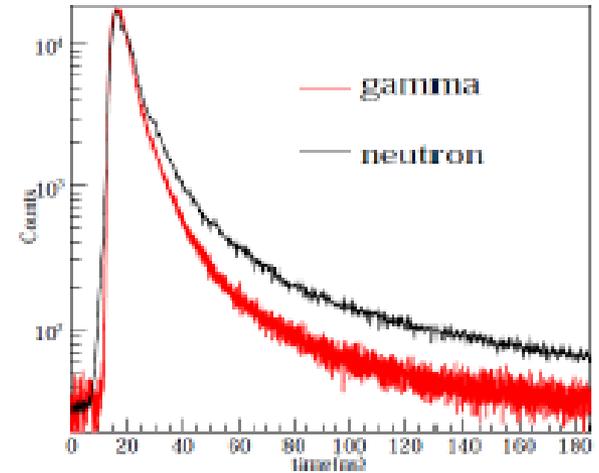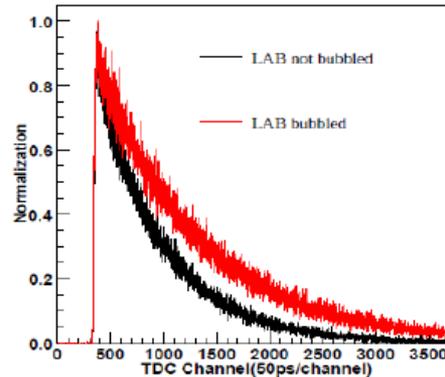
Energy quenching Pulse shape
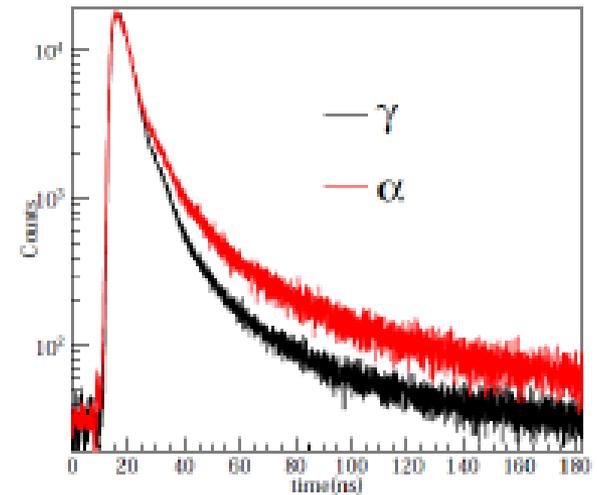
Non-linearity due to energy quenching
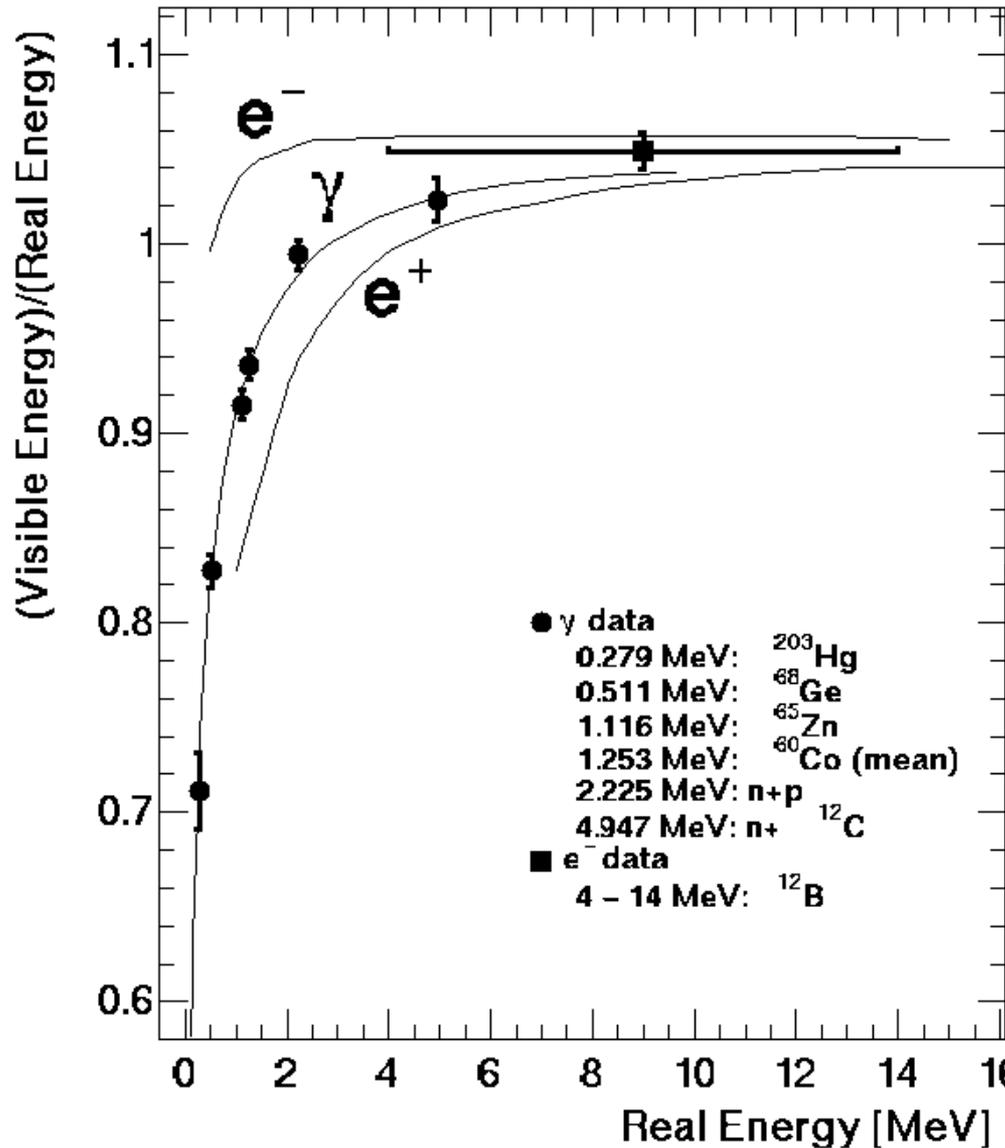
$$\frac{dS}{dr} = \frac{A \, dE/dr}{1 + kB \, dE/dr}$$
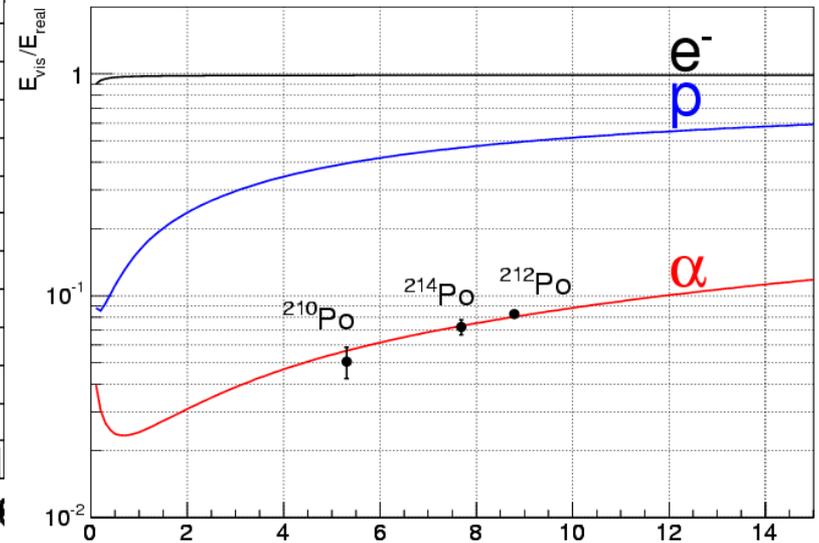
大亚湾液体闪烁体的主要性能

◆ 光产额：~1万光子/MeV

◆ 发射光谱：380-500 nm

◆ 衰减长度：15米（未扣除散射）@430nm

◆ 吸收谱

◆ 吸收重发射几率?

◆ 衰减时间：3ns+10ns+150ns

◆ 氧气淬灭：通氮可提高~10%的光产额

◆ 温度效应：从20度到-75度，光产额提高70%

◆ 波形分辨：有些液闪对不同粒子的时间谱相差较大

Liquid Scintillator Hall

Mineral Oil

Liquid Scintillator

185 ton 0.1% Gd-LS

Filling Equipment

LS mixing equipment

ISO tank equiped with load cell. Target mass errorr ~0.1%
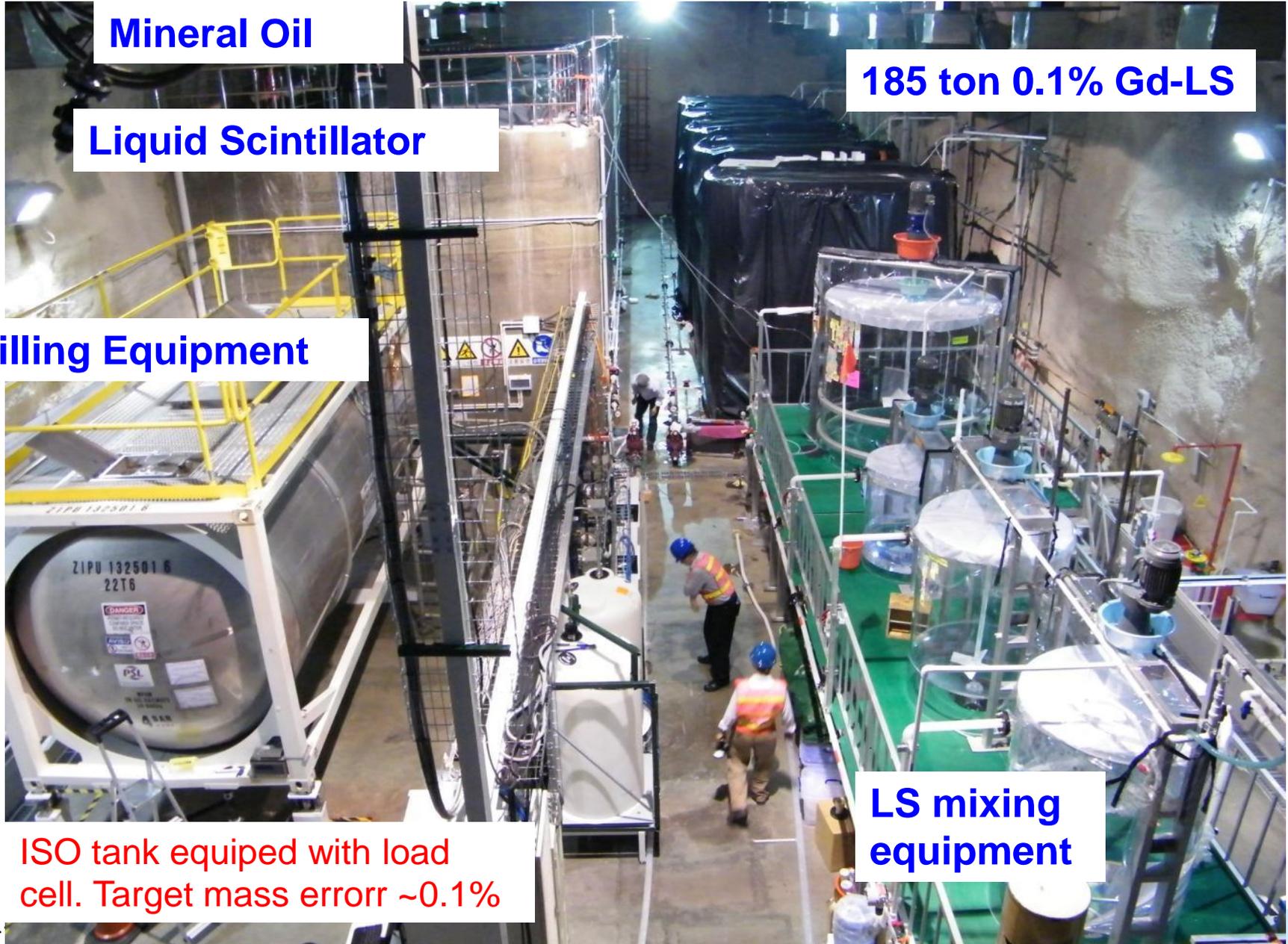
6 – Detector Response and Calibration

Detectors

3 layers separated by Acrylic Vessels
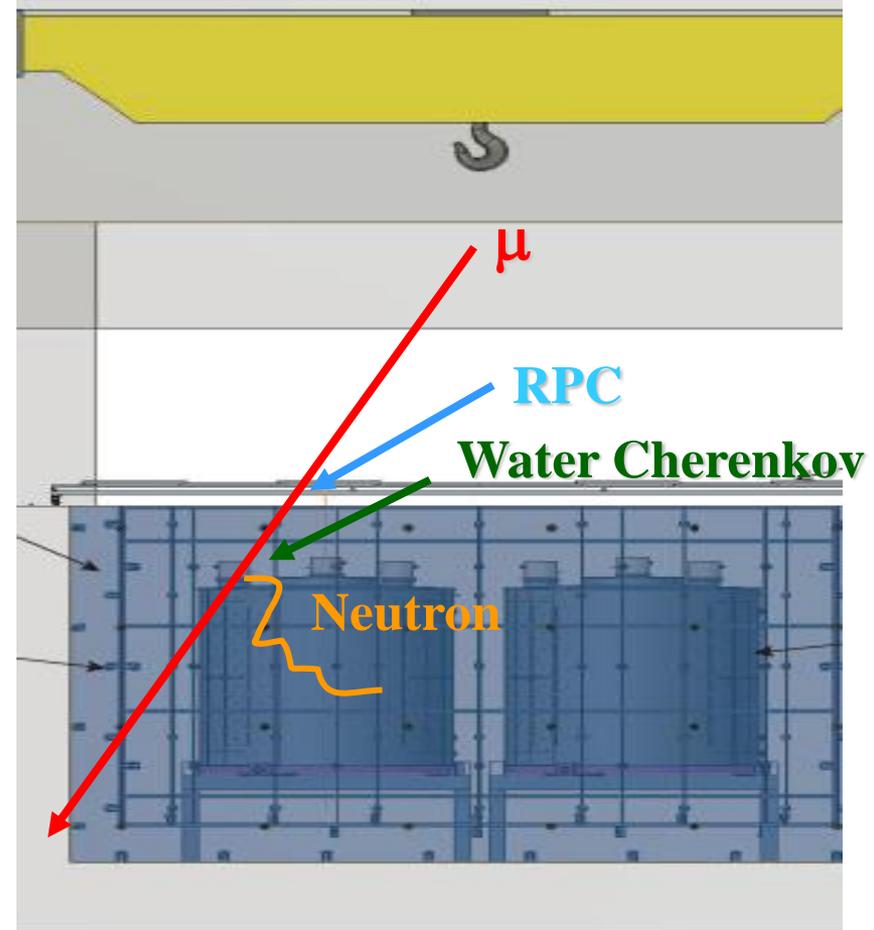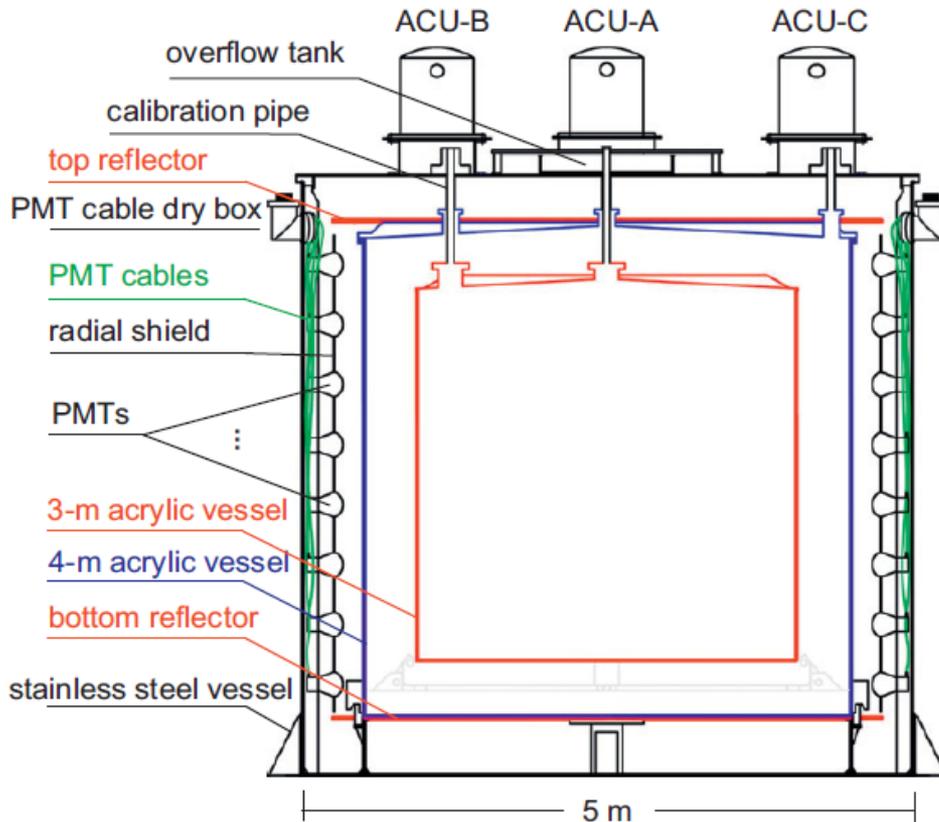
Water shields radioactivity and neutron

- **Two layer water Cherenkov detector**
- **RPC**
- **Combined eff. 99.5%+-0.25%**

◆ 中微子与氢核反应，生成正电子与中子

◆ 正电子直接电离激发液闪发光；然后与电子湮灭生成一对伽马；伽马与电子发生康普顿散射，将能量传递给电子，电子再电离激发液闪发光。

◆ 中子在液闪中慢化（平均~30微秒），在钆和氢上俘获，释放总能量为8MeV或2.2MeV的伽马光子，同上，激发液闪发光。

◆ 液闪发的光由PMT探测。

可见光子

中子

中微子

正电子

掺钆液闪

PMT

液闪

白油

Photoelectrons

- **LS ~ 10k photons/MeV, isotropic, point-like**
- **Photocathode coverage (with reflector): 12%**
- **Light attenucation ~exp(-3/20)=0.86**
- **PMT quantum eff. ~20%, PMT collection eff. 0.8**
- **10k*0.12*0.86*0.8 ~ 165 p.e./MeV**
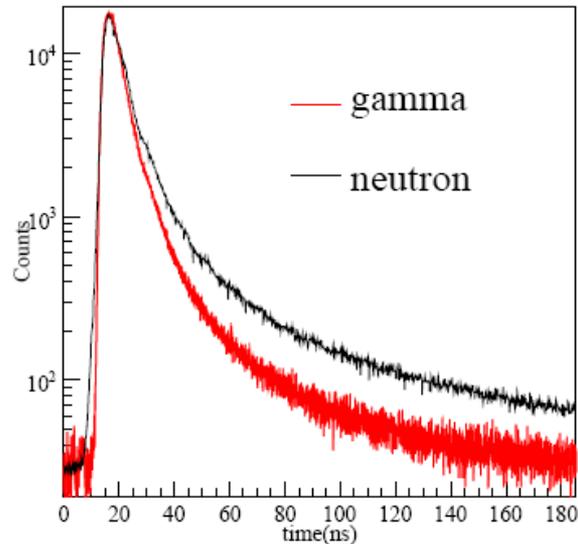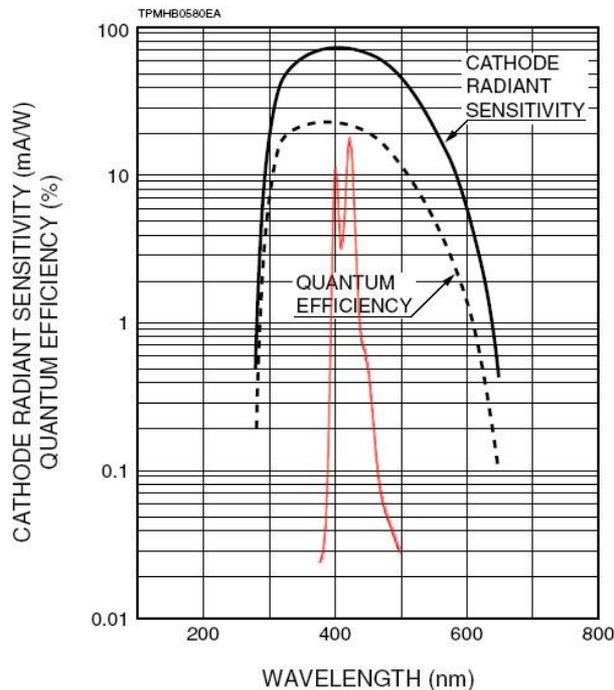
Fig. 15. Experimental light pulse measured for LAB+3 g/L PPO +15 mg/L bis-MSB under γ and fast neutron radiation
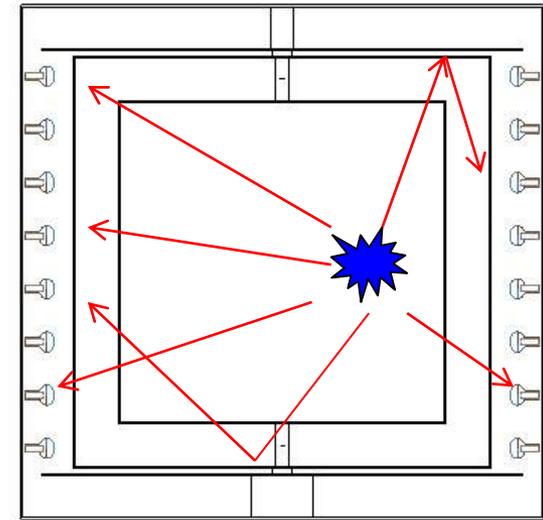
触发

- ◆ 去掉噪声和低能本底事例
- ◆ **NHIT（100ns内PMT击中数）**
- ◆ **ESUM（总光电子数）**
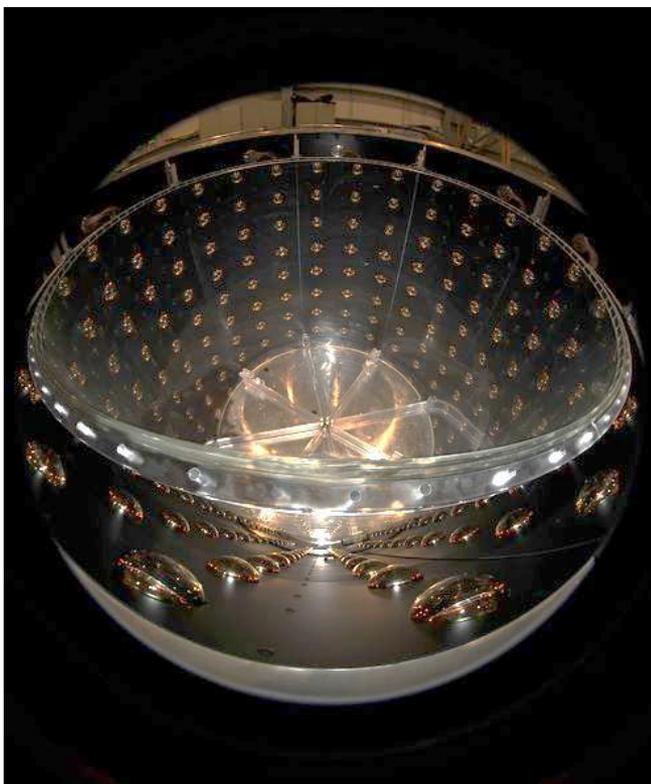
PMT暗噪声，常温下 kHz – 10 kHz
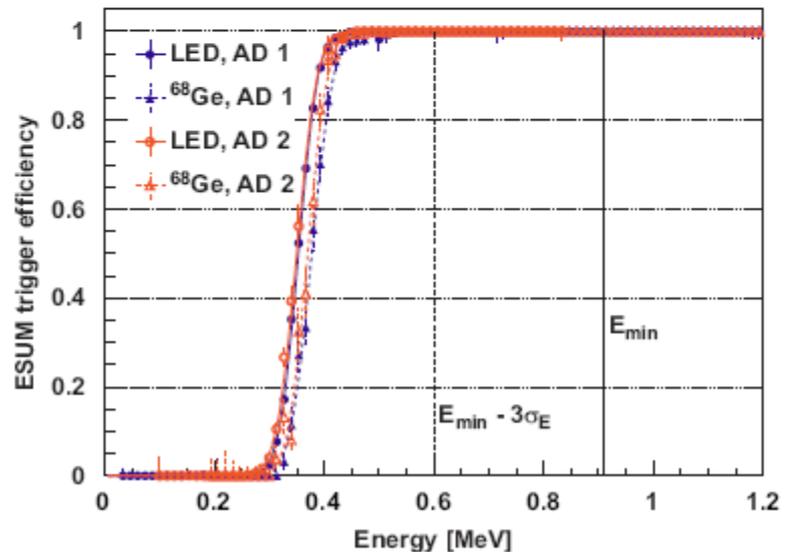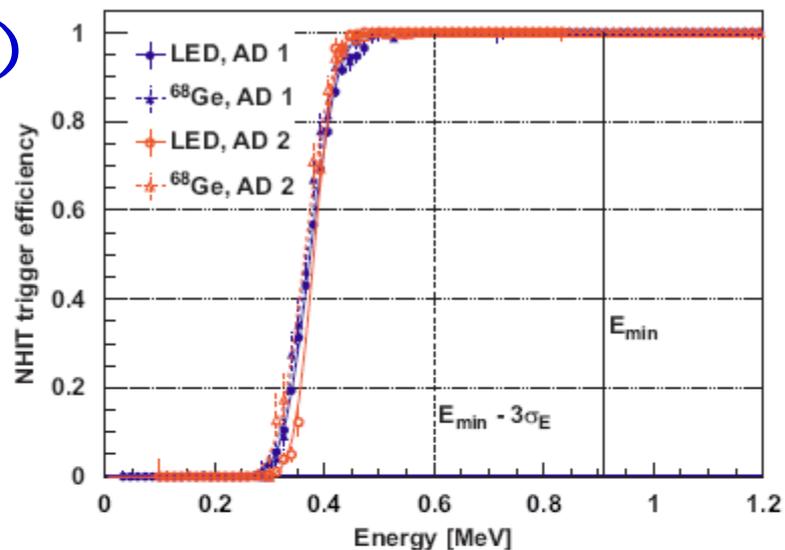
Energy Calibration

- **LED (PMT gain，timing)**
- **Ge68 (positron threshold 1.022 MeV)**
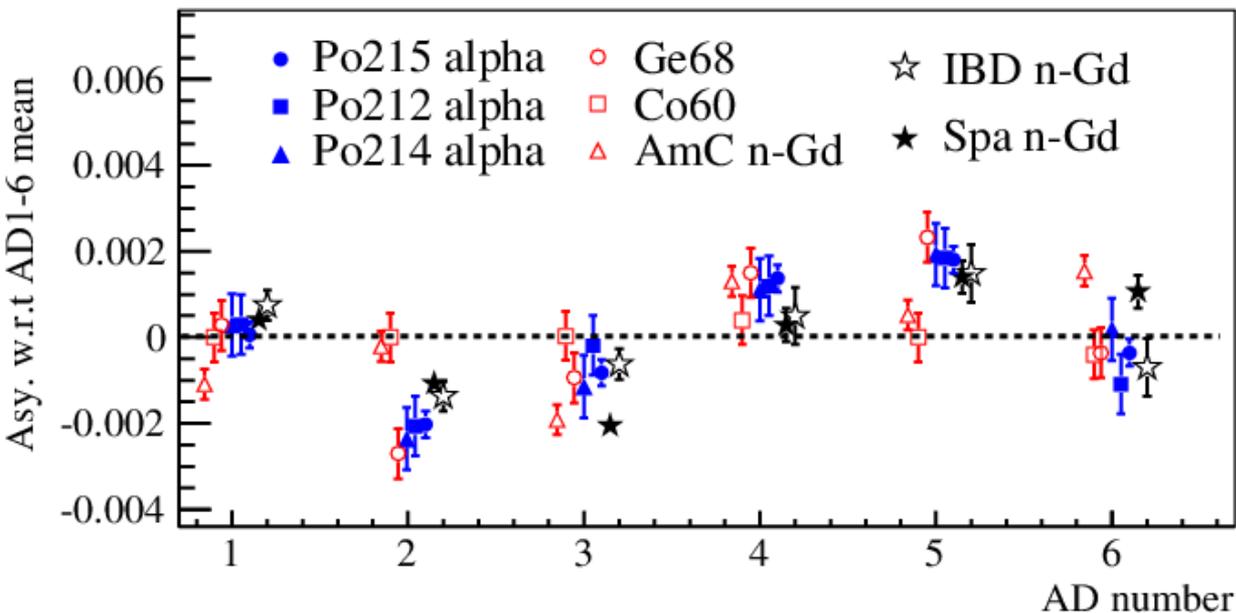- **Co60 (2.506 MeV) + Am-C (neutron)**
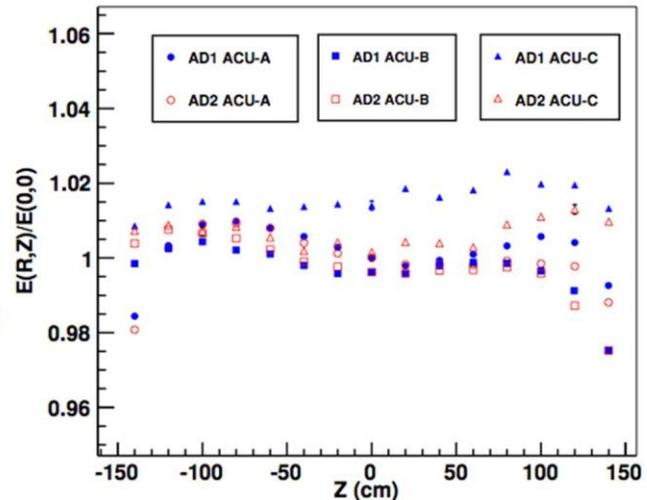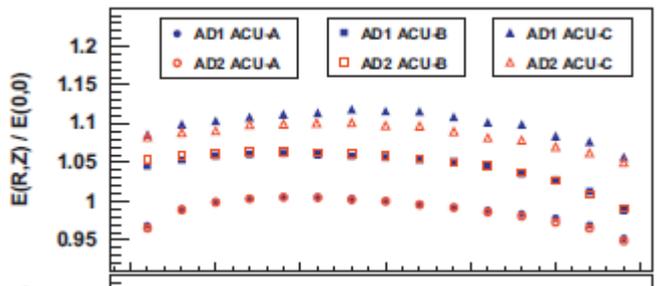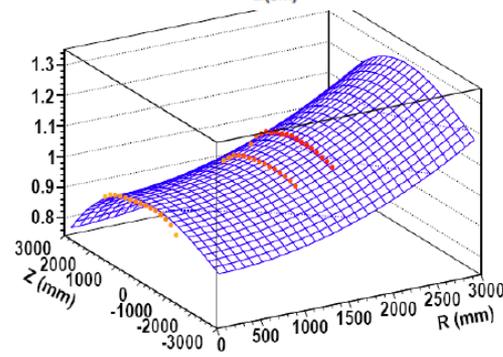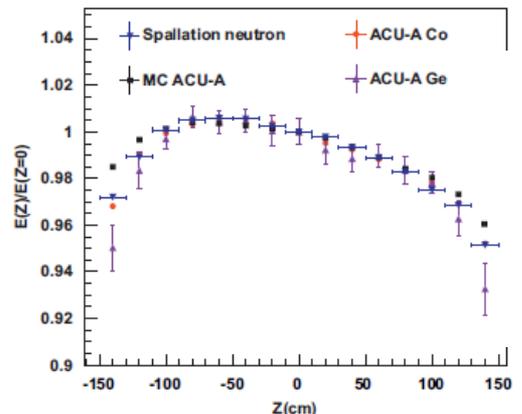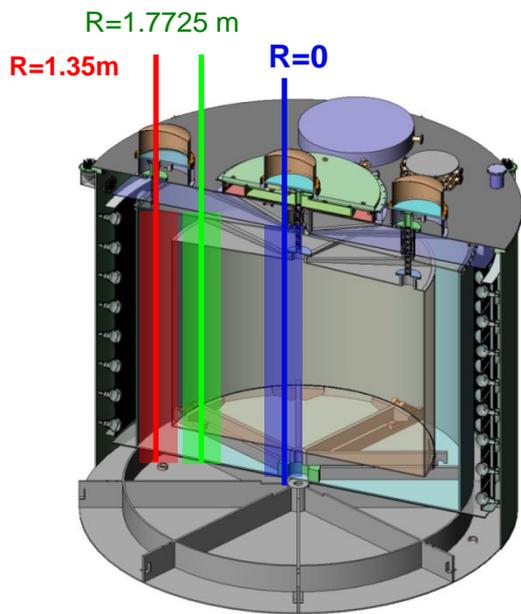
Ge68: 15 Hz
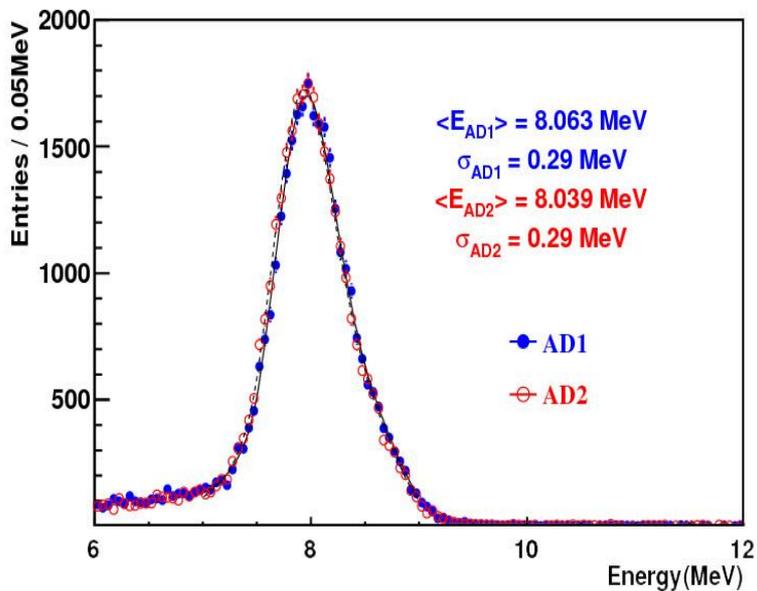**0.511x2 MeV**

Am-C：
0.5 Hz

Co60: 100 Hz
1.173 + 1.332 MeV

Energy Scale Uncertainty

能标误差 **0.2%**

通过Monte Carlo方法传递能标误差到效率误差➜不同**AD**有 **0.12%** 的相对效率差别

7- Event in Detector and Event Selection

数据的构成

◆ **Neutrino event rate（0.01 Hz@near site)**

◆ **Trigger ~ 0.4 MeV**

◆ **AD event rate 280 Hz**

  ⇨ **PMT dark noise**
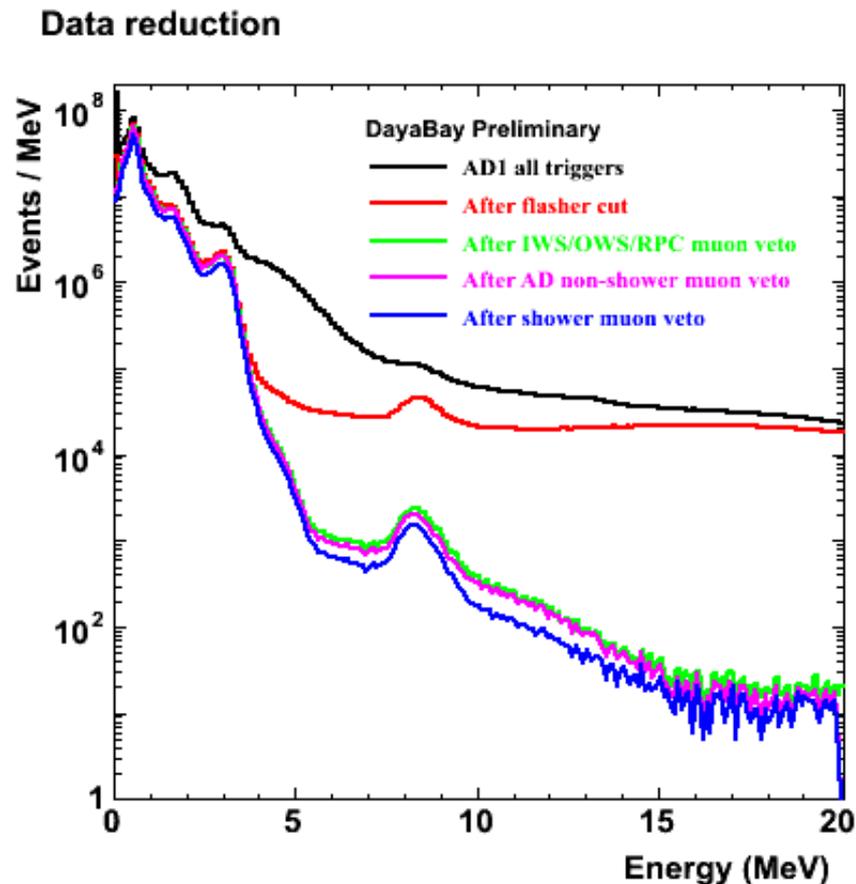
  ⇨ **PMT flasher**

  ⇨ **Radioactivity (low E）**

  ⇨ **Muon(high E)**

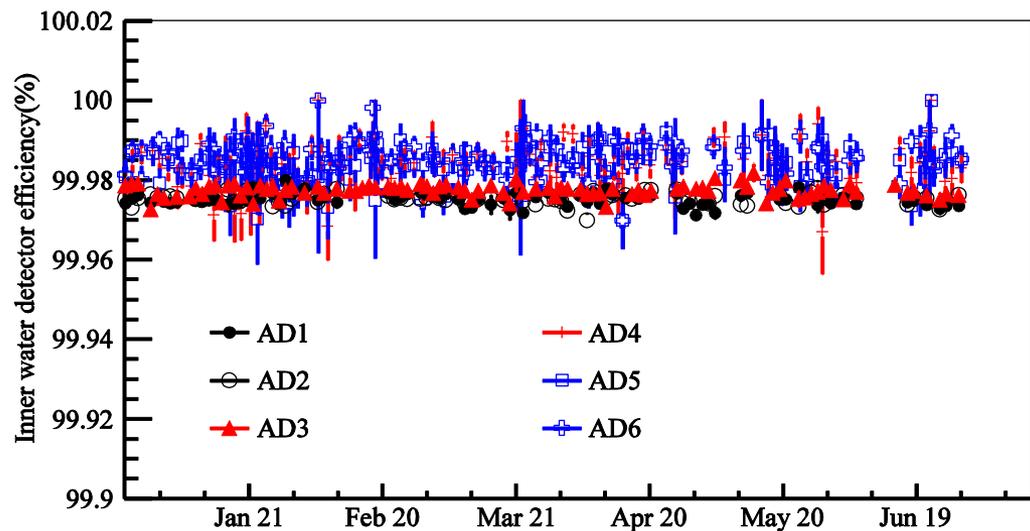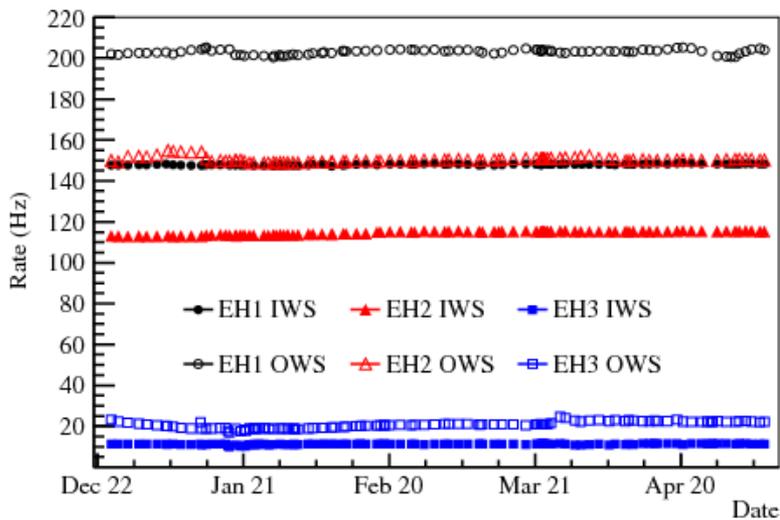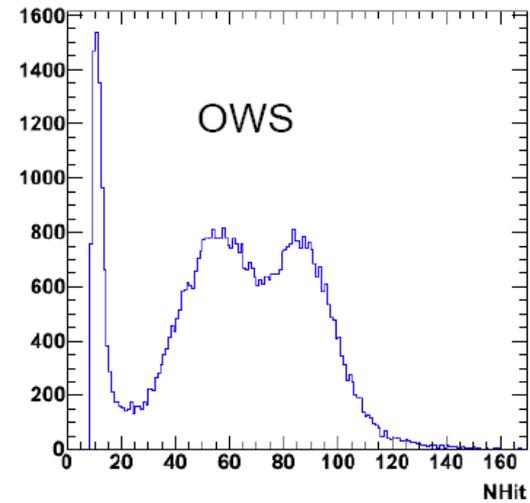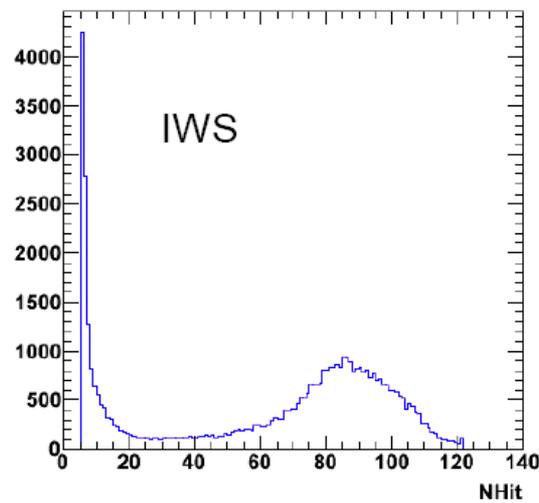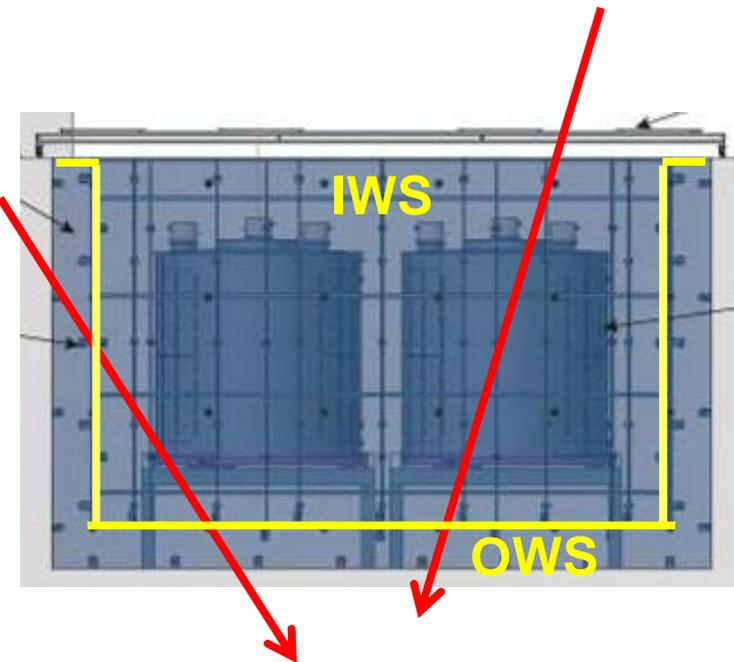 ⇨ **Cosmogenic n, isotope**

  ⇨ **Neutron source bkg**

  ⇨ **Neutrino**

◆ **Muons in Inner Water Pool (IWS), outer (OWS)**

◆ **RPC、FADC readout**

Data reduction

DayaBay Preliminary
AD1 all triggers
After flasher cut
After IWS/OWS/RPC muon veto
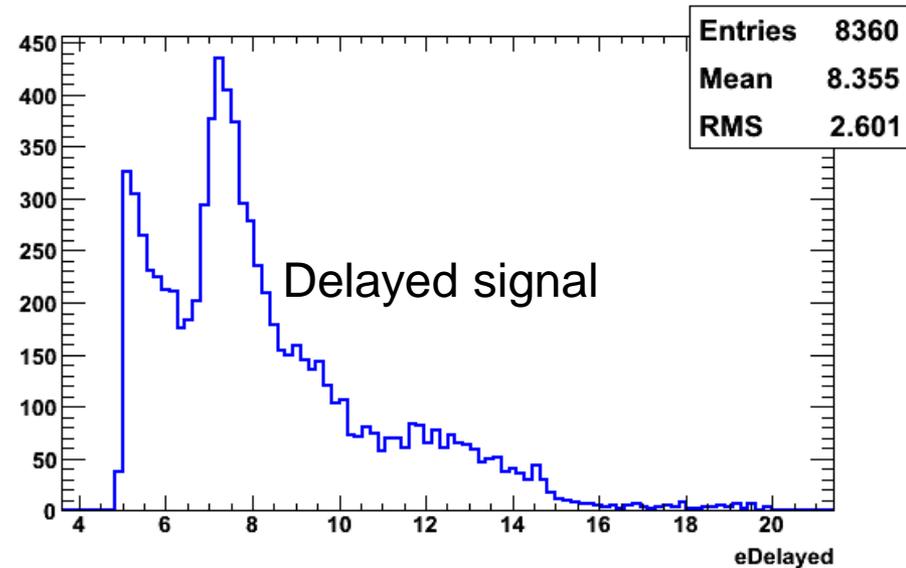After AD non-shower muon veto
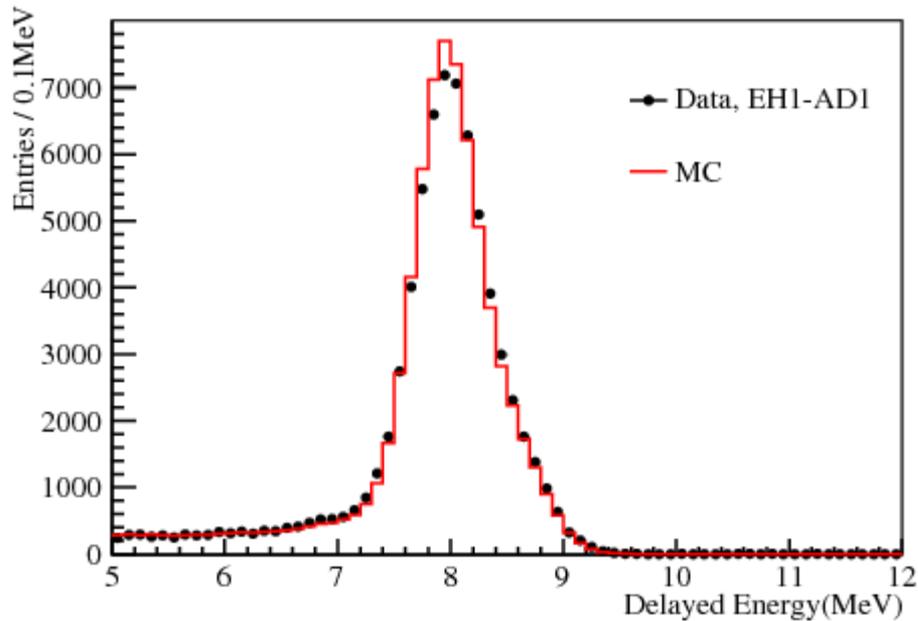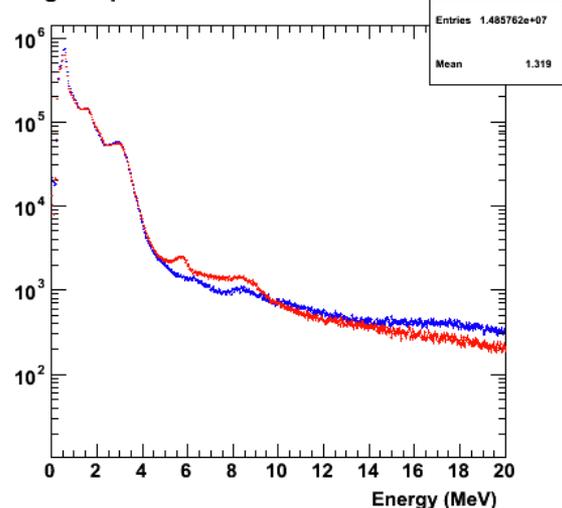After shower muon veto

Energy (MeV)

水池宇宙线事例

Nhit > 12

First look at the delayed (n) signal
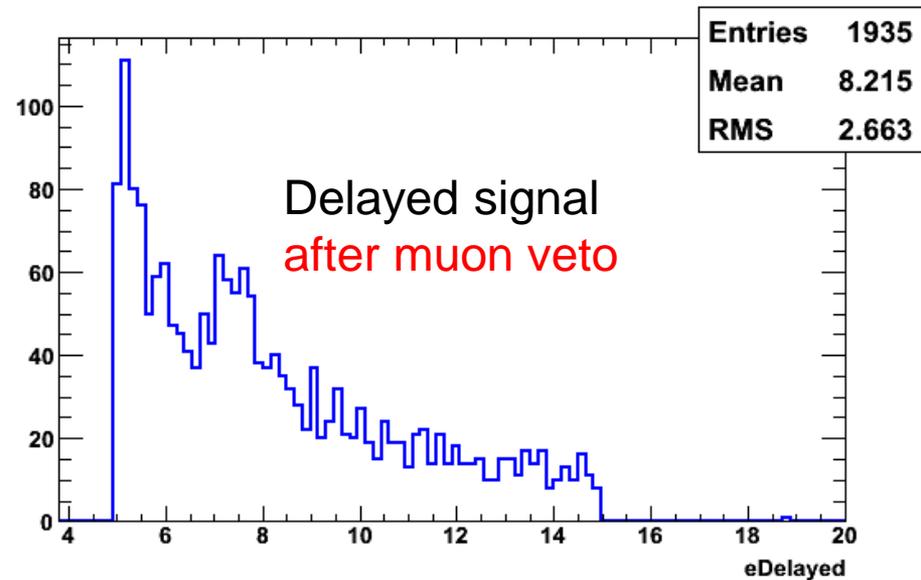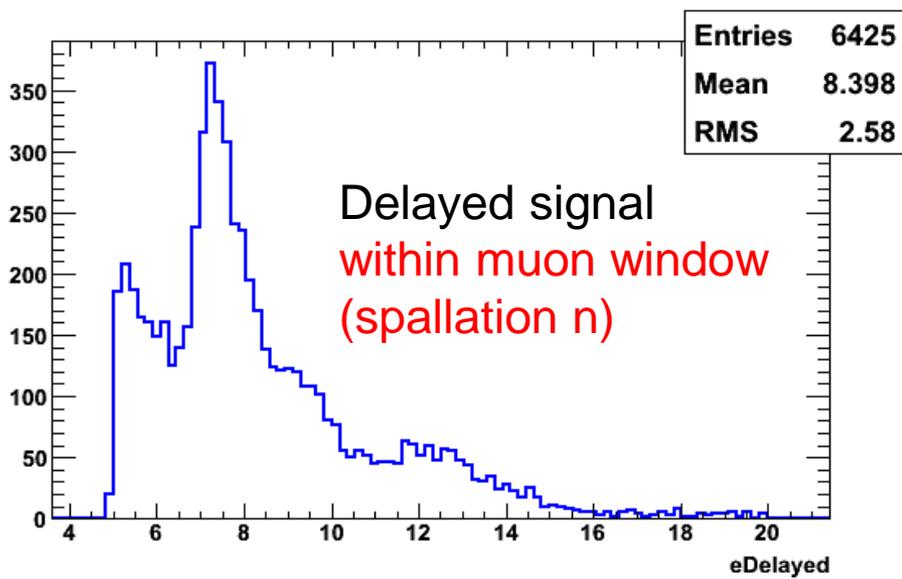
PMT Flasher

singlesSpec

Y/1.e3:X/1.e3 {det==2}

AD1/2的能谱应该相同
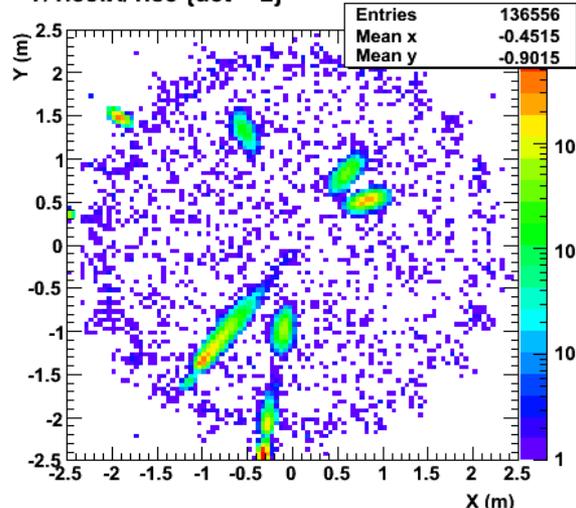
顶点分布，说明存在未知本
底，本底有固定**hit pattern**

正常事例

AD1R5C20a
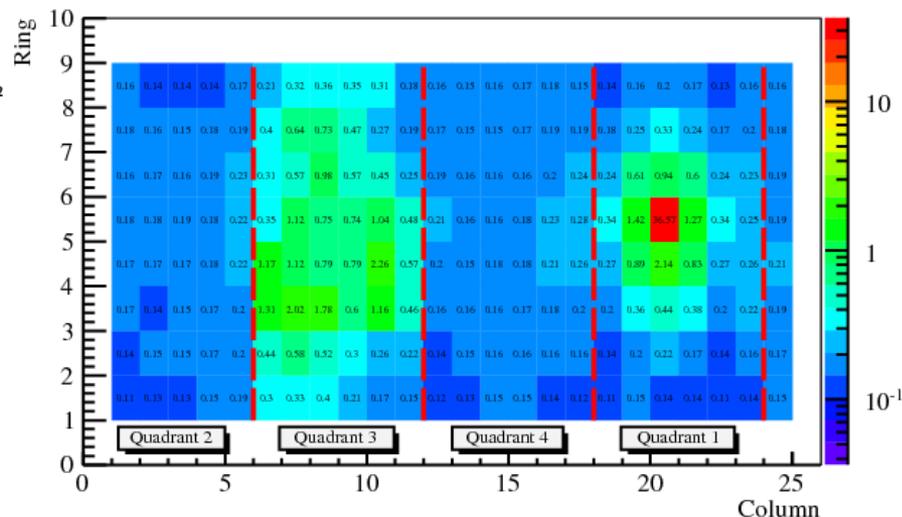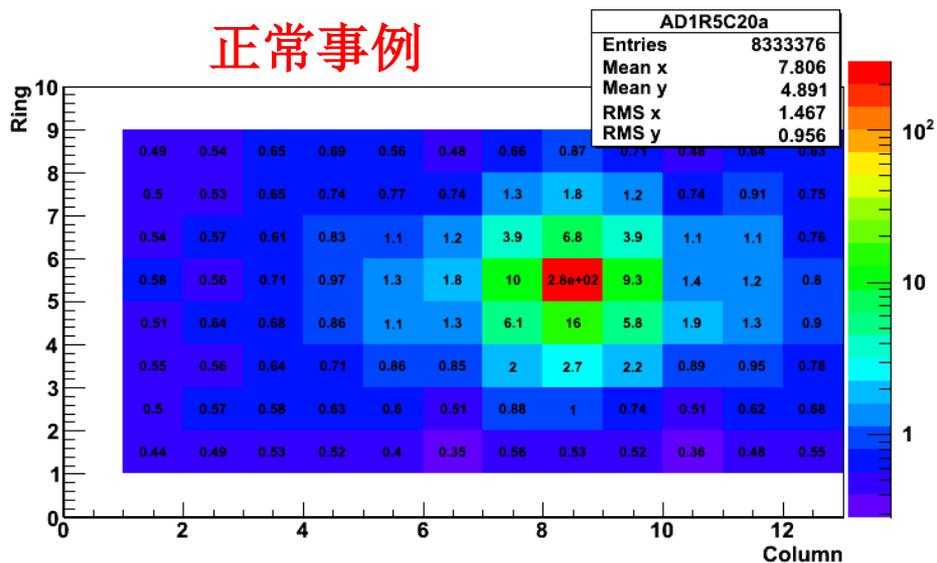
Flasher本底

PMT Flasher

◆ 光电倍增管底座打火。

◆ 打火PMT电荷最大

◆ 对面一些PMT也被照亮

(5MeV, 12MeV) region

$$\left(\frac{Quadrant}{1.0}\right)^2 + \left(\frac{MaxQ}{0.45}\right)^2 < 1$$
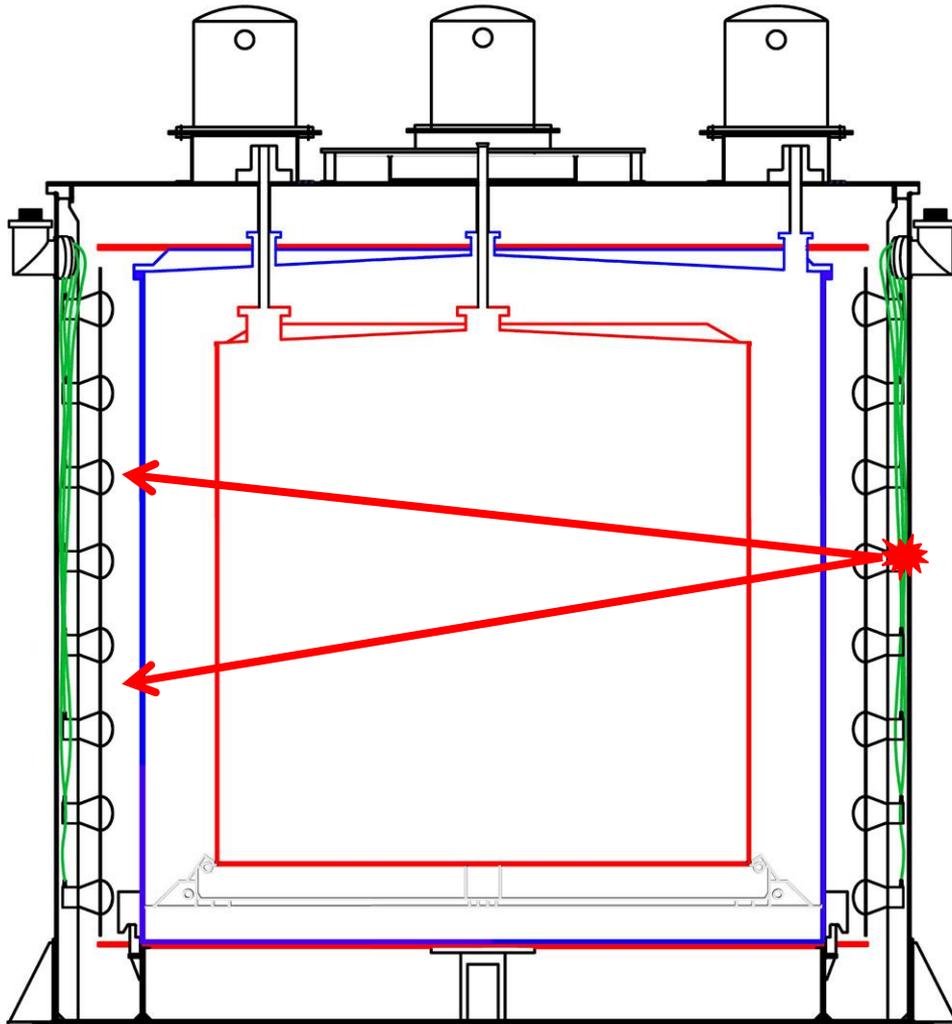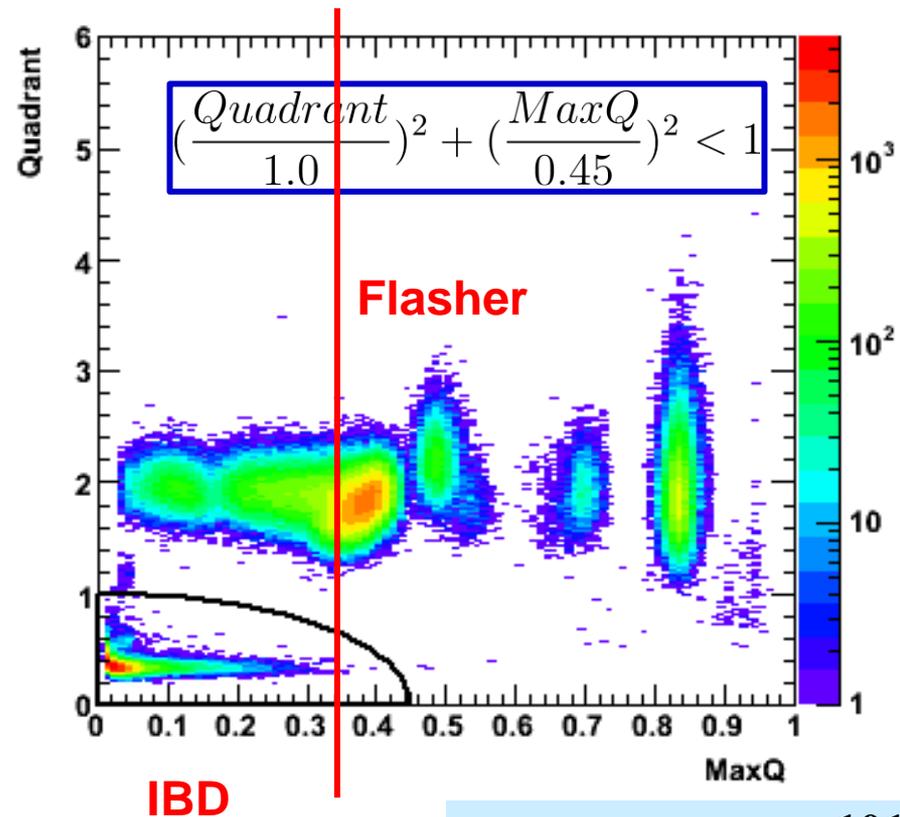
Flasher

IBD

Reject PMT Flasher

◆ **5%的PMT会打火，形成的本底事例占总事例5%**
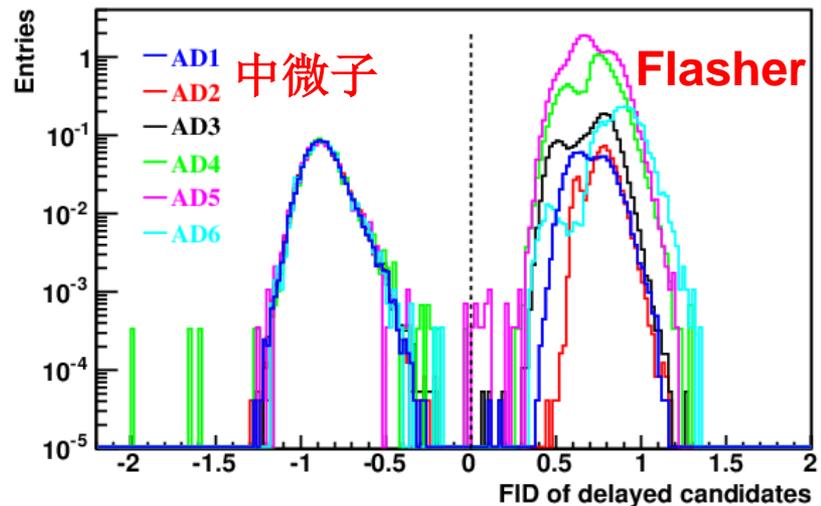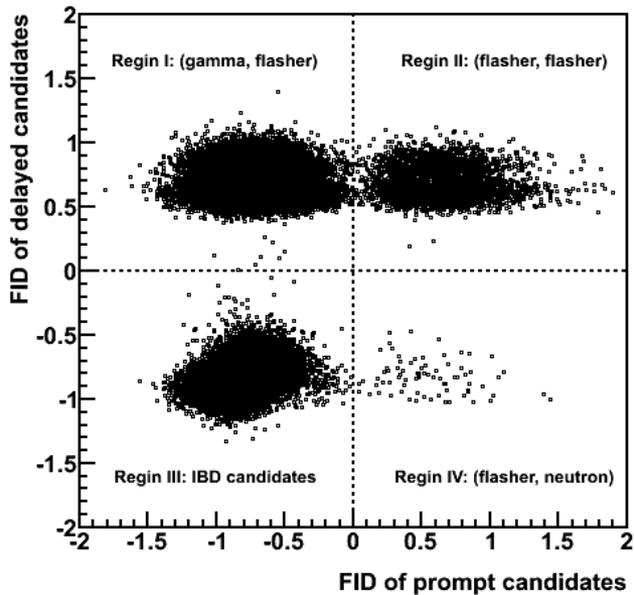
会带来0.01%的相对误差 →

Data Reduction

Data reduction
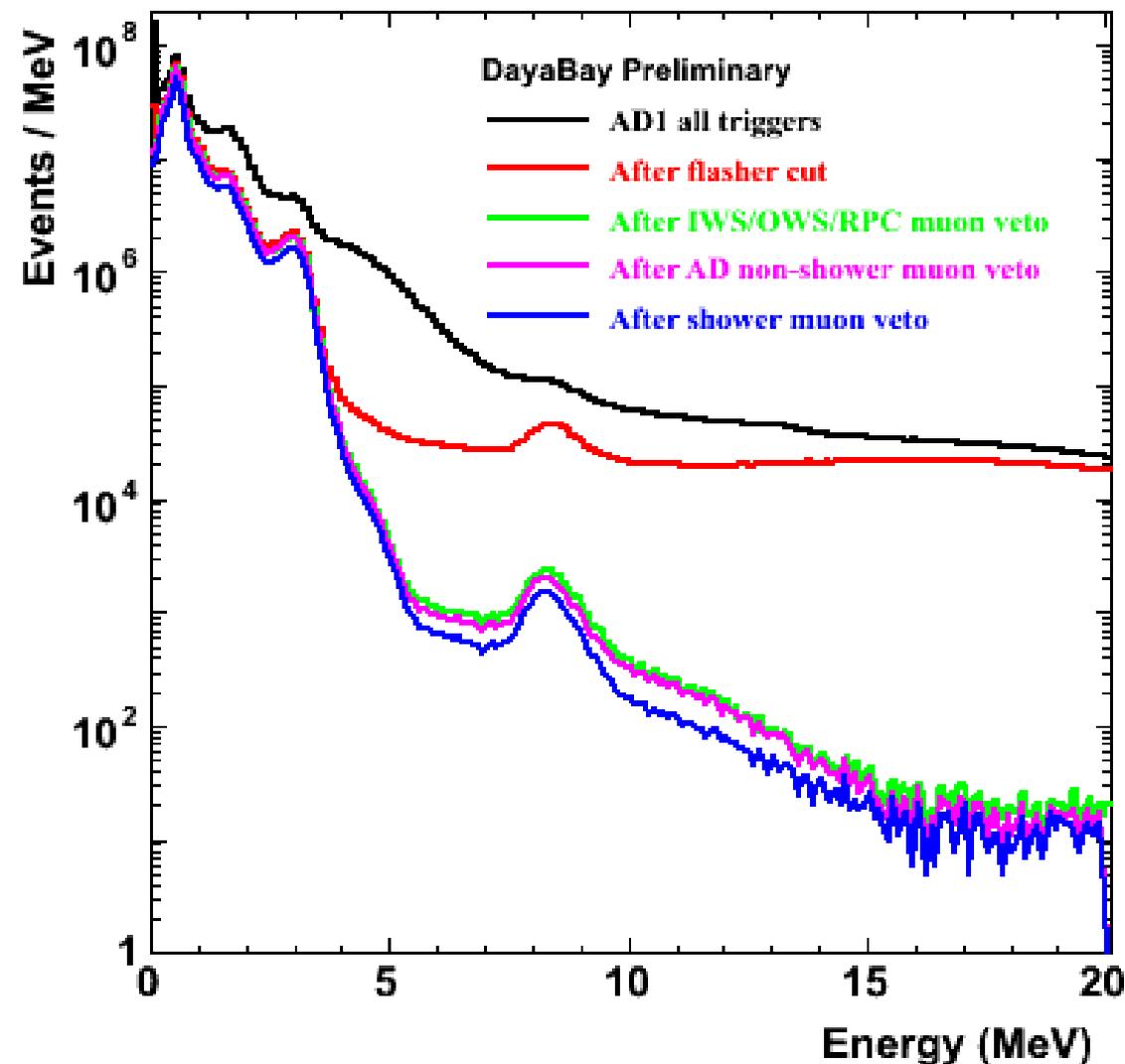
- ◆ 首先去掉非物理事例——**Flasher**
- ◆ 再去掉宇宙线、以及宇宙线引起的本底
  - ⇨ 大信号后**PMT**过冲形成的电子学噪声
  - ⇨ 米歇尔电子
  - ⇨ **spallation**中子
  - ⇨ 其它长寿命同位素

全能谱

◆ **去掉Flasher，排除宇宙muon及其后200微秒的事例后，得到的能谱**
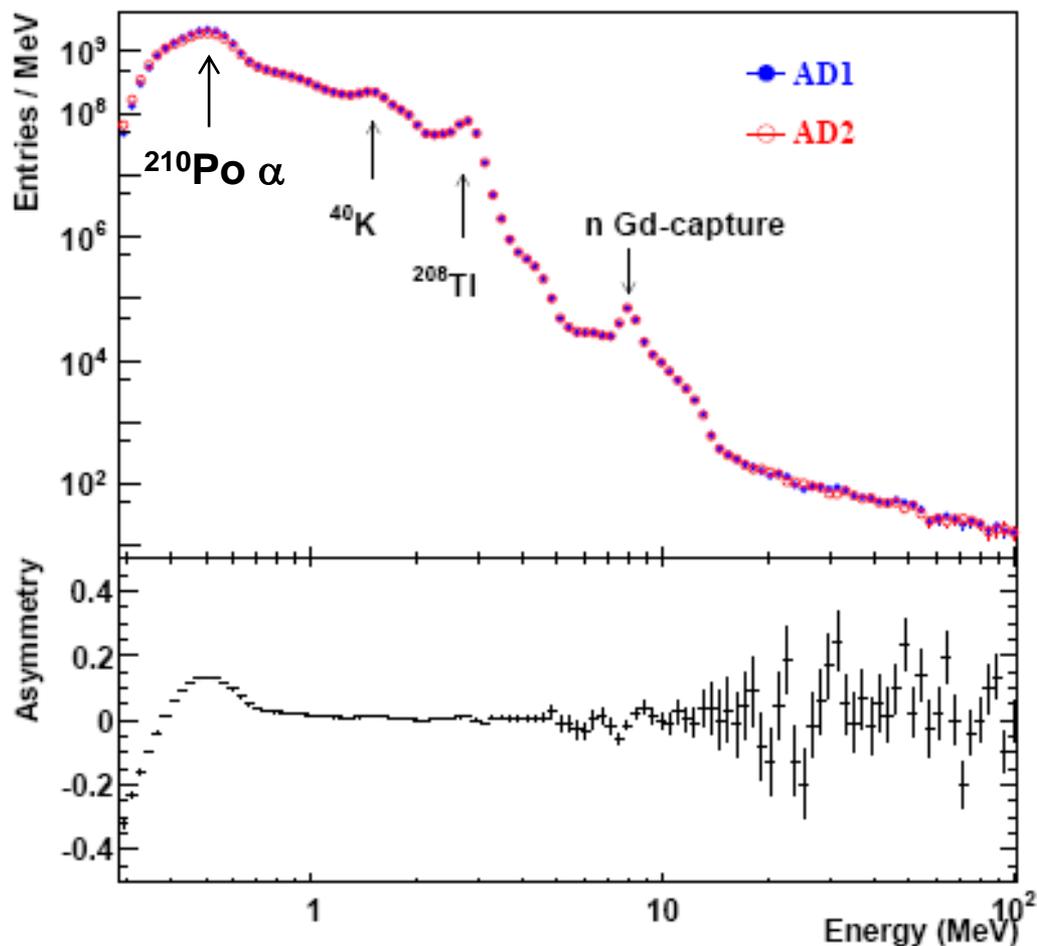
Neutrino Selections

0.7-12 MeV

Prompt candidate

6-12 MeV
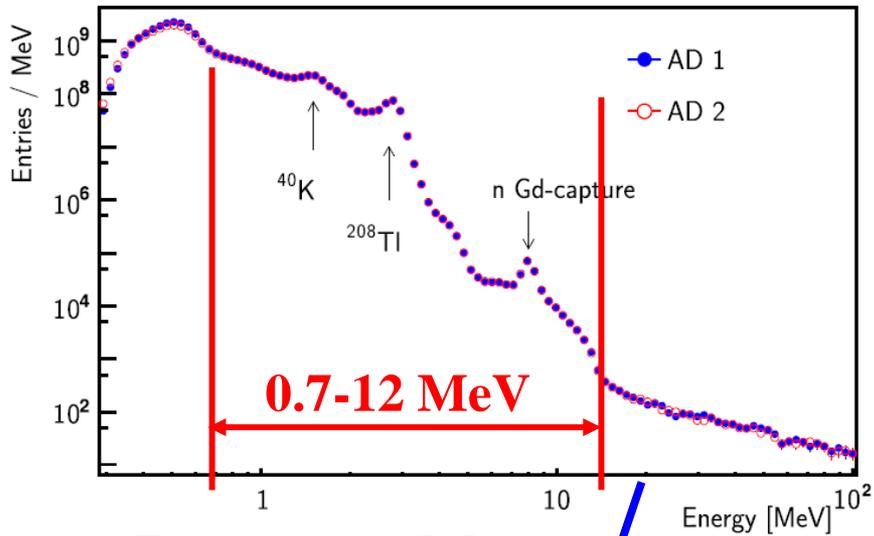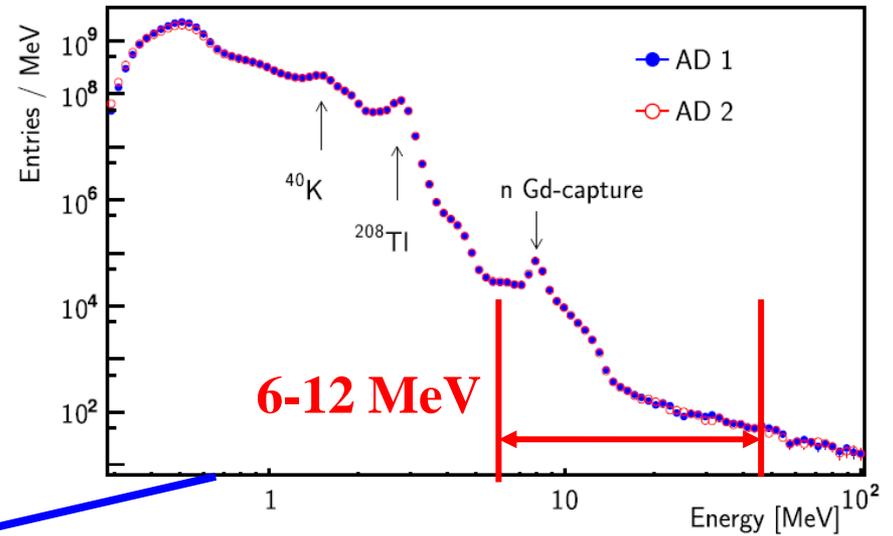
Delayed candidate
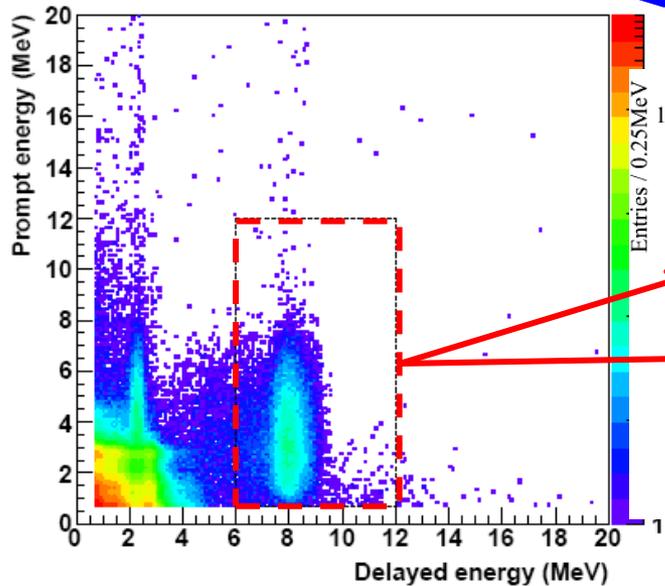
Correlated Events in 1-200 μs
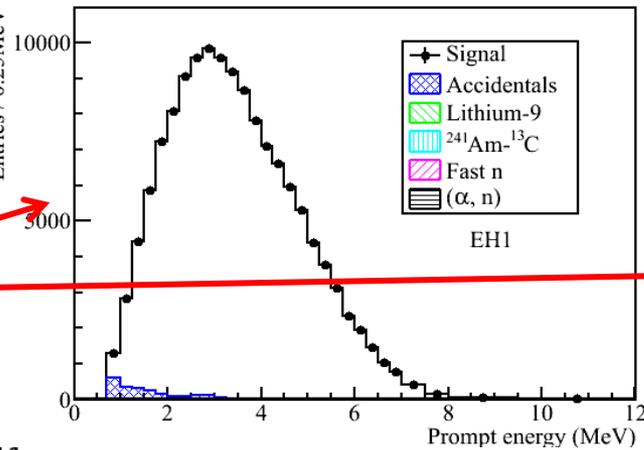
Reactor Neutrinos (Prompt)
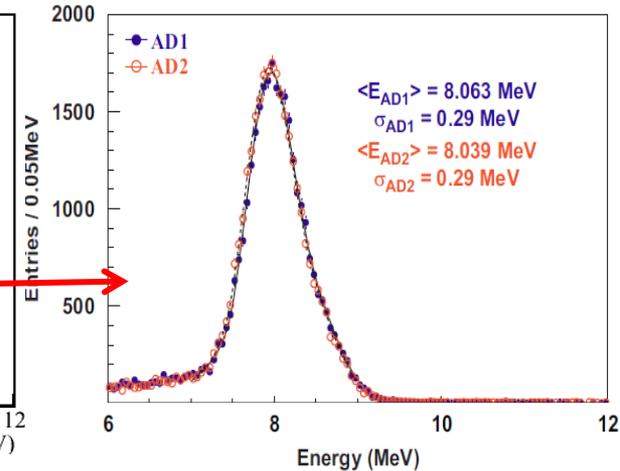
Neutrons (Delayed)

Neutrino Event Selection

◆ **Pre-selection**

⇨ **Reject Flashers**

⇨ **Reject Triggers within (-2 μs, 200 μs) to a tagged water pool muon**

◆ **Neutrino event selection**

⇨ **Multiplicity cut**

● Prompt-delayed pairs within a time interval of 200 μs

● No triggers(E > 0.7MeV) before the prompt signal and after the delayed signal by 200 μs

⇨ **Muon veto**

● *1s* after an AD shower muon

● *1ms* after an AD muon

● *0.6ms* after an WP muon

⇨ **0.7MeV < E_{prompt} < 12.0MeV**

⇨ **6.0MeV < $E_{delayed}$ < 12.0MeV**

⇨ **1μs < Δt_{e^+-n} < 200μs**

Selected Signal Events

Time between prompt-delayed

Distance between prompt-delayed
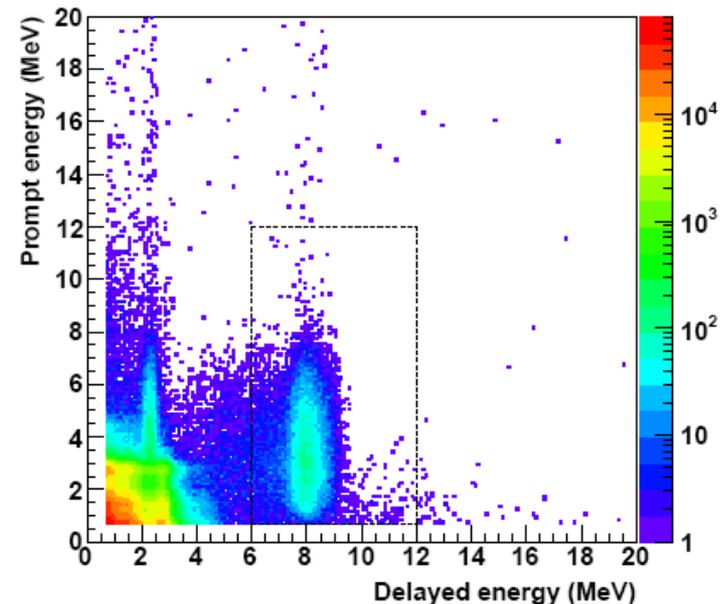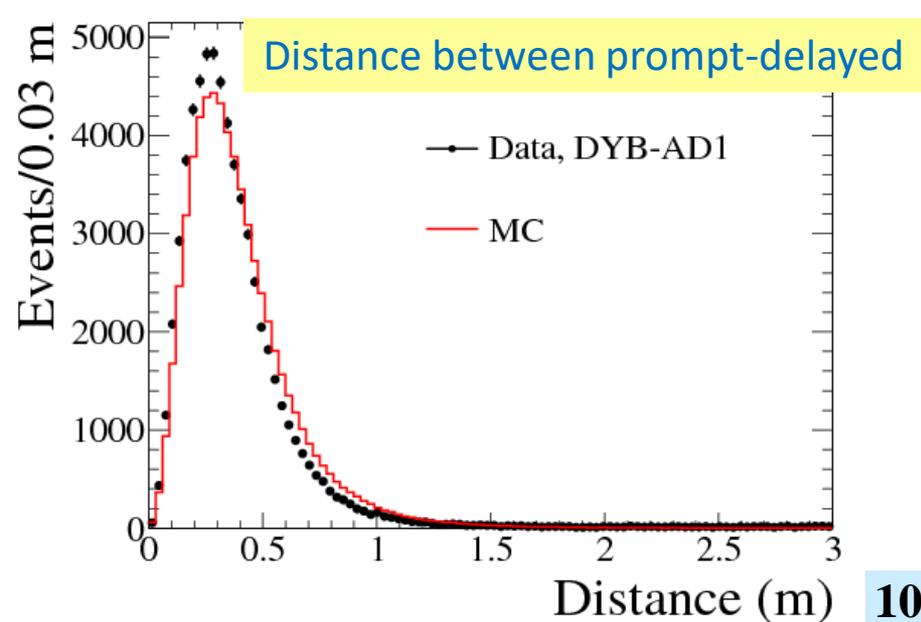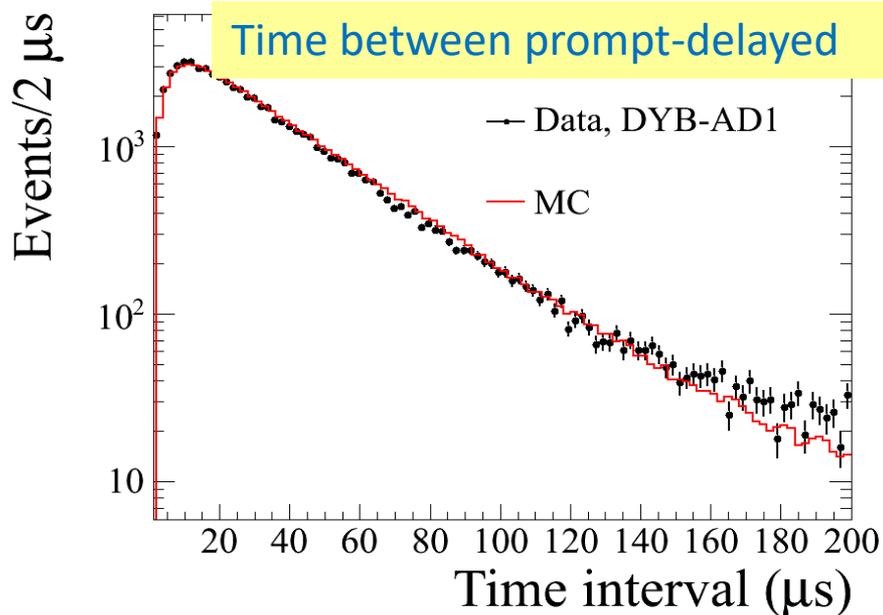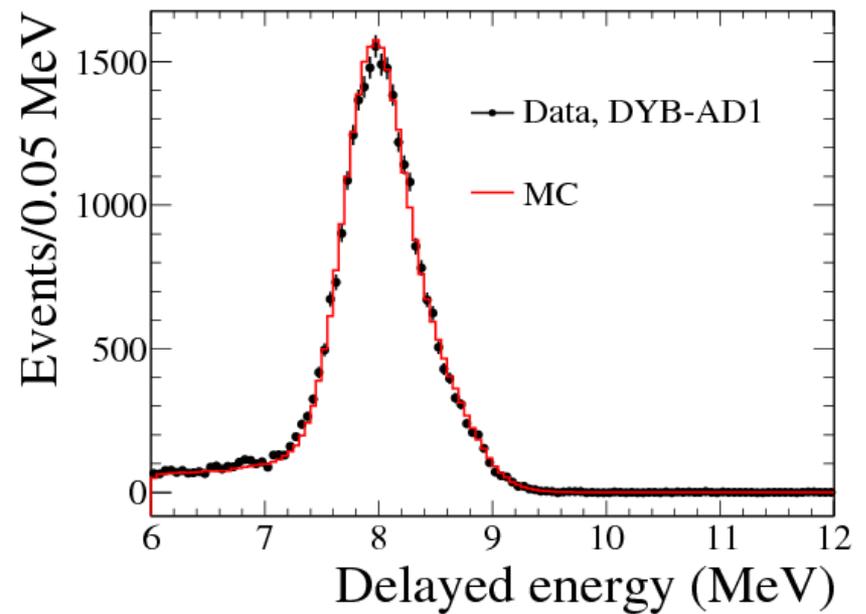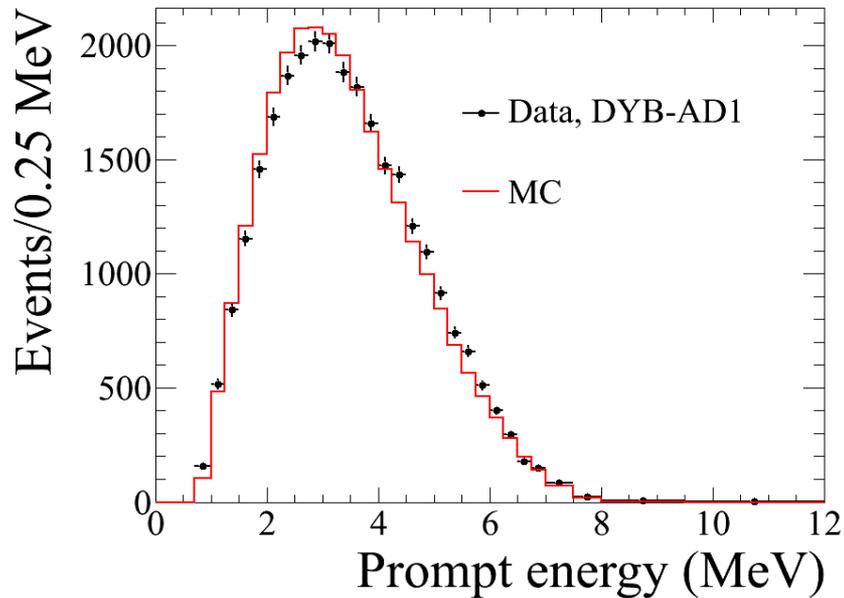
Efficiency and Uncertainty

每做一次挑选（**cut**条件）就会带来一个误差！

◆ 不同探测器之间的差别当成误差（非关联）

	Efficiency	Correlated	Uncorrelated
Target Protons		0.47%	0.03%
Flasher cut	99.98%	0.01%	0.01%
Delayed energy cut	90.9%	0.6%	0.12%
Prompt energy cut	99.88%	0.10%	0.01%
Multiplicity cut		0.02%	<0.01%
Capture time cut	98.6%	0.12%	0.01%
Gd capture ratio	83.8%	0.8%	<0.1%
Spill-in	105.0%	1.5%	0.02%
Livetime	100.0%	0.002%	<0.01%
Total	78.8%	1.9%	0.2%

Functional Identical Detectors

◆ **Why systematics is so small?** c.f. An et al. NIM. A 685 (2012) 78

⇨ **Idea of "identical detectors" throughout the procedures of design / fabrication / assembly / filling.**

⇨ **For example: Inner Acrylic Vessel, designed D=3120±5 mm**

- Variation of D by geometry survey=**1.7mm**, Var. of volume: 0.17%

- Target mass var. by load cell measurement during filling: 0.19%

Diameter	IAV1	IAV2	IAV3	IAV4	IAV5	IAV6
Surveyed(mm)	3123.12	3121.71	3121.77	3119.65	3125.11	3121.56
Variation (mm)	1.3	2.0	2.3	1.8	1.5	2.3

⇨ **"Same batch" of liquid scintillator**

4-m AV in pairs

Assembly in pairs

5x40 t Gd-LS, circulated

20 t filling tank

200 t LS, circulated
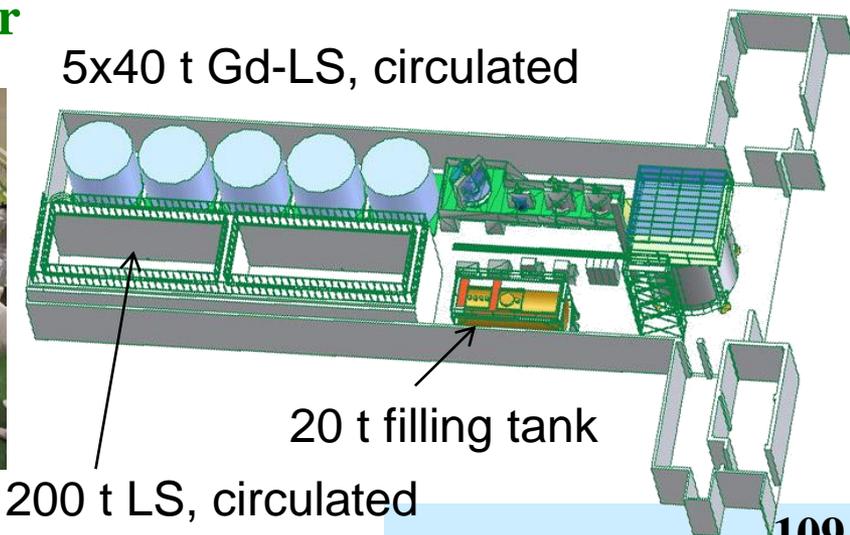
Side-by-side Comparison

◆ **Expected ratio of neutrino events: R(AD1/AD2) = 0.982**

◆ **Measured ratio: 0.987 ± 0.004(stat) ± 0.003(syst)**
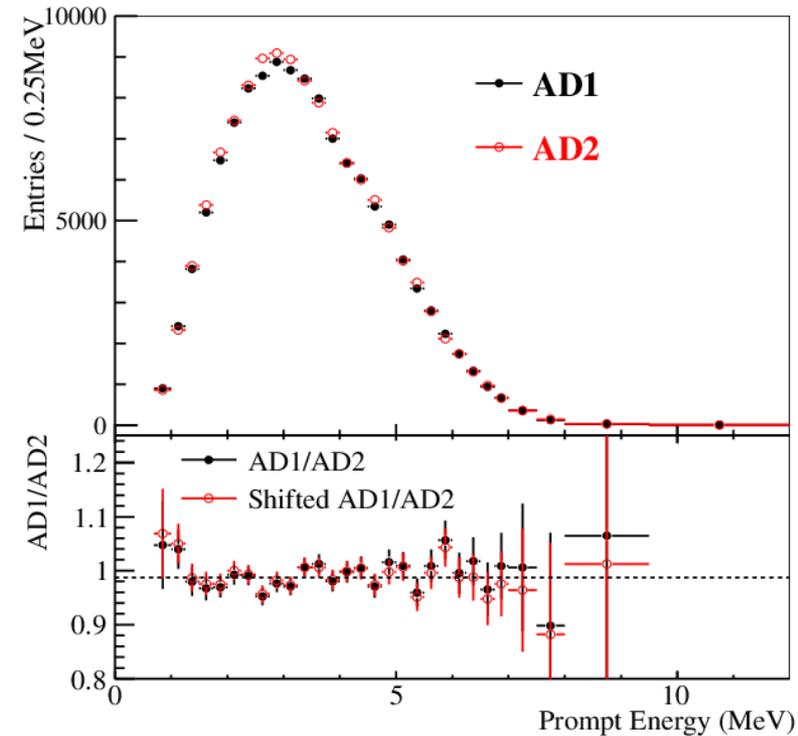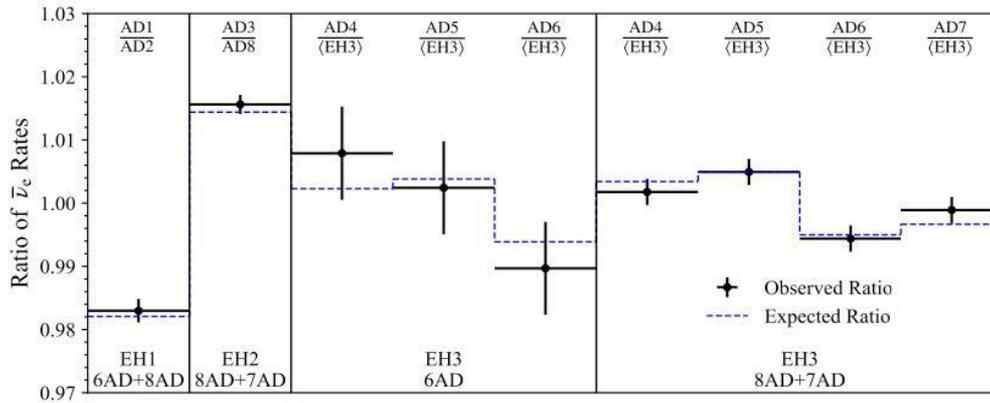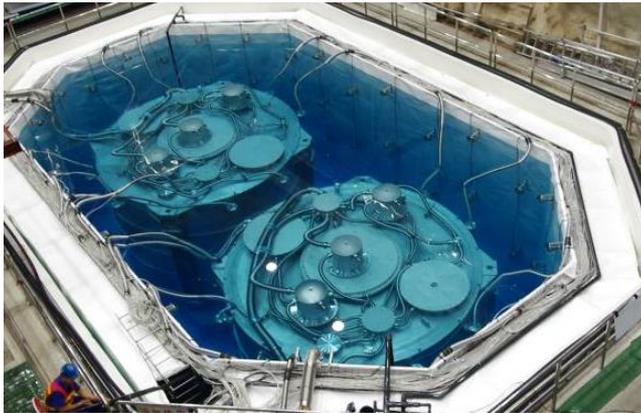
This check shows that systematic errors are under control

Data set: Dec 24 to May 11

8 - Background

本底（设计值）

来源：宇宙线，天然放射性，从以前实验的经验：

- ◆ **$^9Li/^8He$ （关联本底）（估计B/S 0.3%）**
  - ⇨ 宇宙线产生的长寿命同位素，β-n级联衰变，与中微子信号类似。
- ◆ **"快中子" 本底 （关联本底）（估计B/S 0.1-0.2%)**
  - ⇨ 宇宙线产生的高能中子，中子与质子碰撞形成快信号，慢化中子形成慢信号。
  - ⇨ 中子在碳、氧核中俘获，发射高能中子。
- ◆ **偶然符合本底（估计B/S ~1%，减除误差 ~0.1%）**
  - ⇨ 类正电子信号 (单事例率 <100 Hz)
    - ● 天然放射性 (PMT, 岩石, 钢罐, 液闪, ... <50 Hz)
    - ● 宇宙线、宇宙线产生的同位素，等等
  - ⇨ 类中子信号 (<200/day)
    - ● 单中子：宇宙线产生的、未留下质子反冲信号的中子。
    - ● 宇宙线产生的长寿命同位素 (例如 $^{12}B/^{12}N$)
    - ● 其它在 6-10 MeV 范围内的事例 (例如 Michel's electron, 宇宙线等)

1 – ^{8}He/^{9}Li

♦ **Cosmic μ produced ^{9}Li/^{8}He in LS**

⇨ **β-decay + neutron emitter**

⇨ **τ(^{8}He/^{9}Li) = 171.7ms/257.2ms**

⇨ **^{8}He/^{9}Li, Br(n) = 12%/48%, ^{9}Li dominant**

⇨ **Production rate follow $E_\mu^{0.74}$ power law**

♦ **Measurement:**

⇨ **Time-since-last-muon fit**

$$f(t) = B/\lambda \cdot e^{-t/\lambda} + S/T \cdot e^{-t/T}$$

⇨ **Improve the precision by reducing the muon rate:**

● **Select only muons with an energy deposit >1.8MeV within a [10us, 200us] window**

● **Issue: possible inefficiency of ^{9}Li**

⇨ **Results w/ and w/o the reduction is studied**

$\alpha \times E^{0.74}$

muon energy (GeV)

^{9}Li/^{8}He Fit

^{9}Li yield

time since last muon (s)
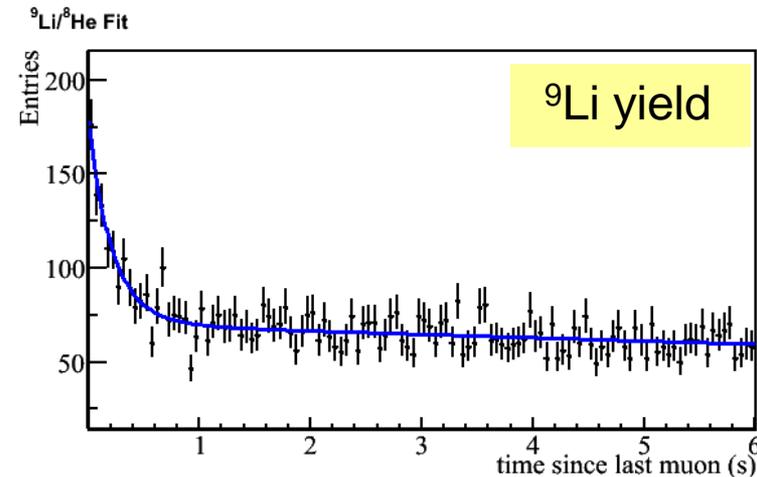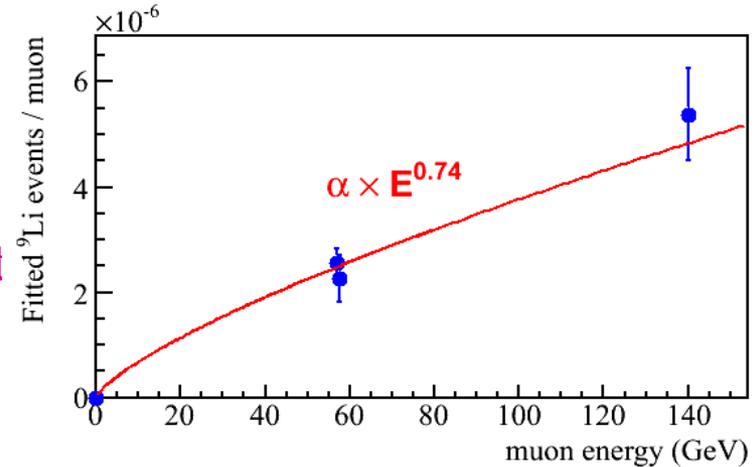
Error follows

$$\sigma_b = \frac{1}{N} \cdot \sqrt{(1 + \tau R_\mu)^2 - 1}$$

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2 - Fast Neutron

宇宙线muon产生高能中子
- 质子反冲（快信号）
- 慢化中子俘获（慢信号）
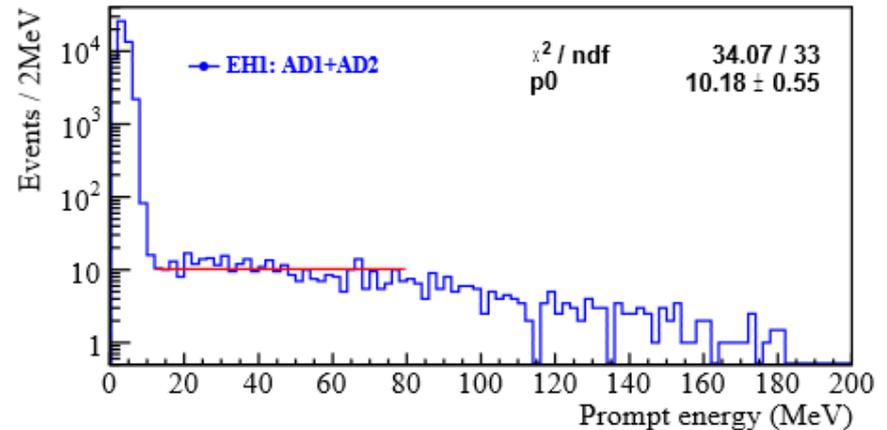
方法1 ：外推

方法2： 按来源累加
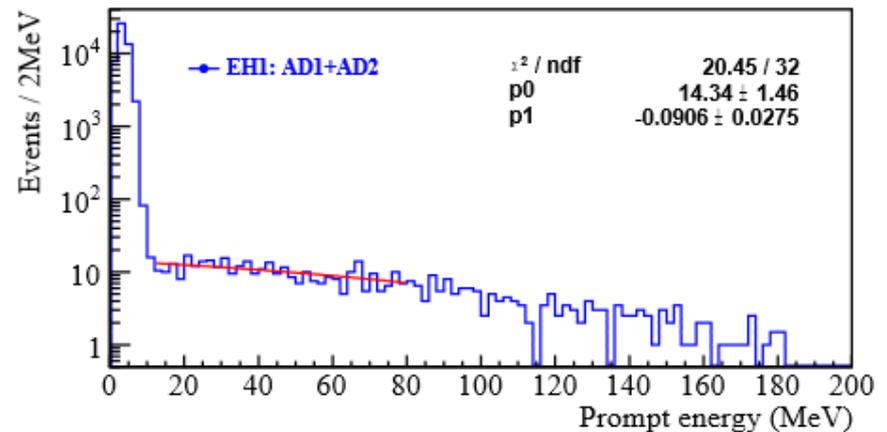
$$n_f = n_f^{iws} \cdot \left(1 - \epsilon_{iws}\right) + n_f^{ows} \cdot \left(1 - \epsilon_{ows}\right) + n_f^{rock}$$

0.7-12 MeV

Prompt candidate

6-12 MeV
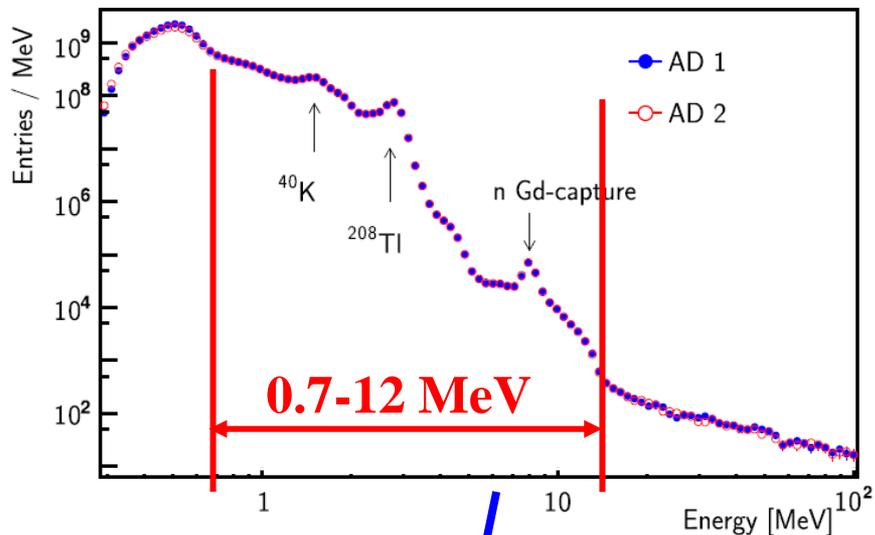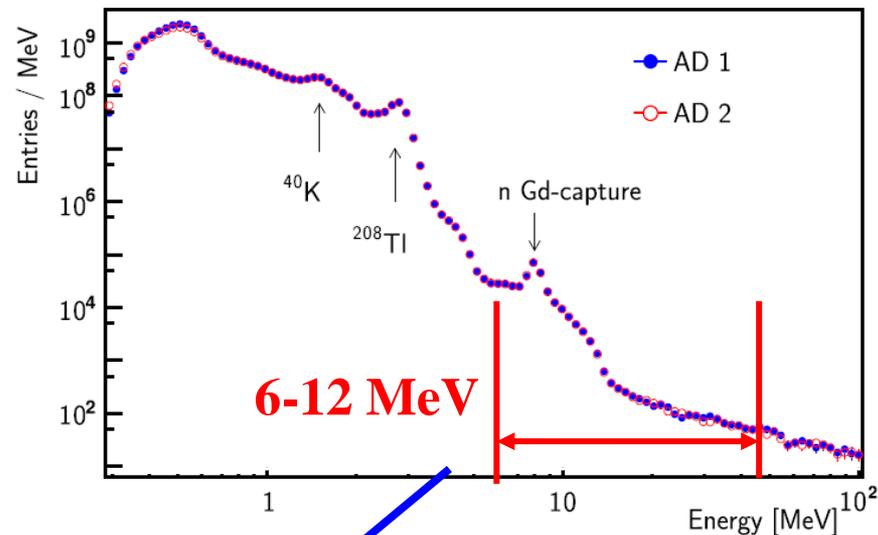
Delayed candidate
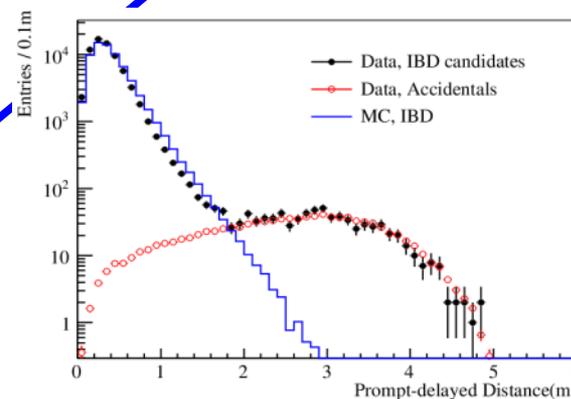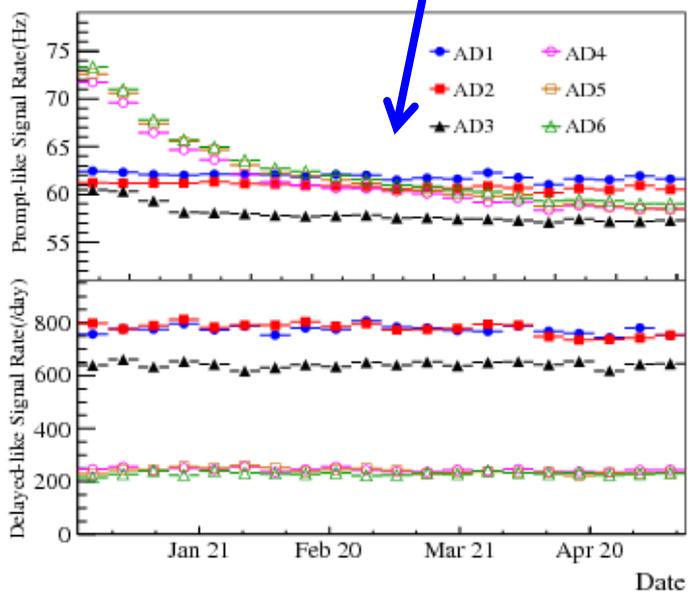
方法1：**Evaluated by coincidence probability**

方法2：**off-windows coincidence**

方法3： **vertex distance distribution**

4,5 – neutron souce and $^{13}C(\alpha,n)^{16}O$

- 中子源本底：大亚湾独有
- Alpha-n本底，KamLAND实验

B/S @ EH1/2 ~ 0.01%,   B/S @ EH3 ~ 0.05%,   ΔB/B ~ 50%

Time correlations of the cascade decays
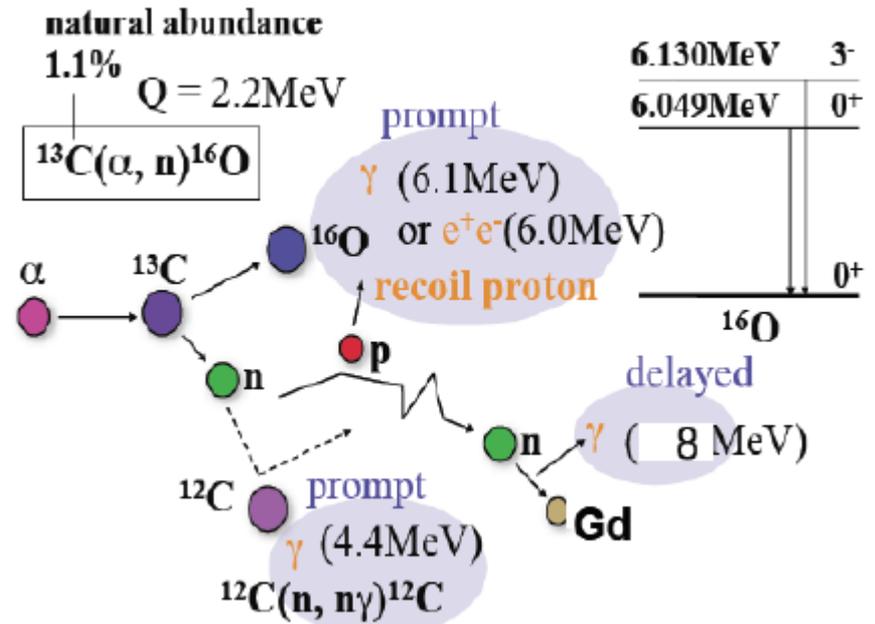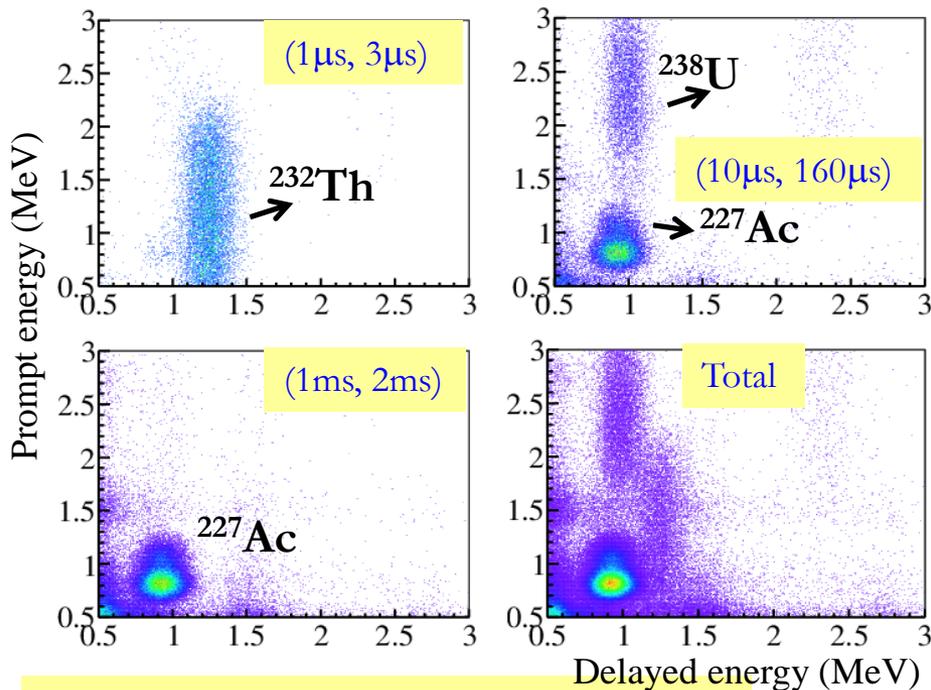
排它性分析

◆ **为什么中子能区有这么多事例？(6.0~12.0MeV)?**

◆ **Possible sources**

Spectrum of single triggers in (0.7MeV, 12.0MeV)
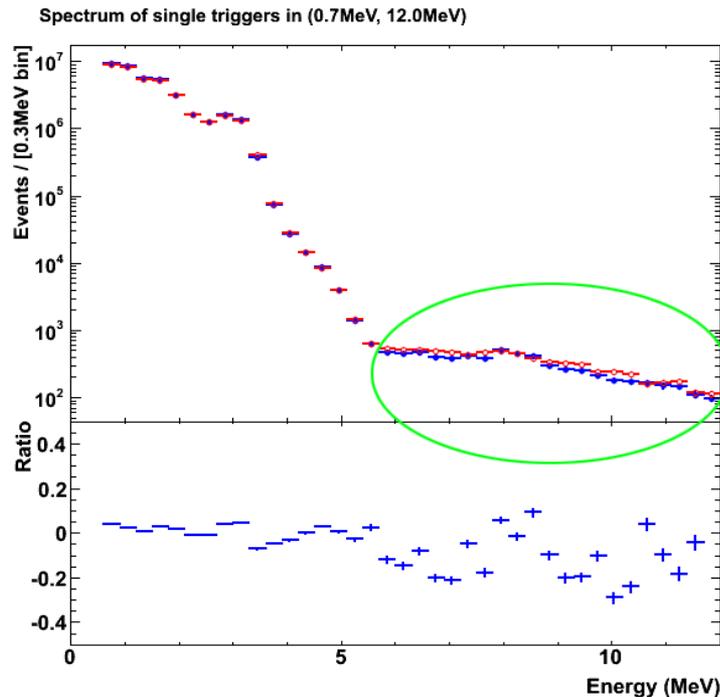
◆ 排它性分析（**Exclusive**）：假如我们找到了全部事例的来源，就不存在未知本底

中子能区事例

Sources	EH1		EH2		EH3	
	Rate (/day/AD)	Fraction	Rate (/day/AD)	Fraction	Rate (/day/AD)	Fraction
AmC neutron	271+-10	26.3+-1.0	277+-7	33.0+-0.8	205+-11	74.3+-5.5%
$^{12}B/^{12}N$	478+-13	46.4+-1.3%	354+-4	42.1+-0.5%	35+-2	12.7+-1.0%
$^{8}Li/^{8}B$	216+-18	21.0+-1.8%	155+-16	18.5+-1.9%	16+-5	5.8+-1.8%
^{9}C	40+-16	3.8+-1.6%	24+-9	2.9+-1.1%	4+-4	1.4+-1.4%
$^{9}Li/^{8}He$	4+-2	0.4+-0.2%	3+-2	0.4+-0.2%	<1	<0.4%
^{11}Be	7+-4	0.7+-0.4%	5+-3	0.6+-0.4%	<1	<0.4%
IBD e+ (n captured on H)	14+-1	1.4+-0.1%	12+-1	1.4+-0.1%	2+-1	0.7+-0.4%
Sum	1030+-29	100.0+-2.9%	830+-20	98.8+-2.4%	262+-13	94.9+-6.7%
All singles	1030+-7	-----	840+-3	-----	276+-14	-----
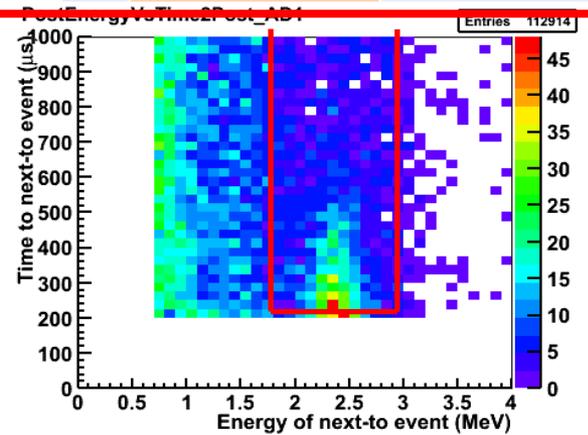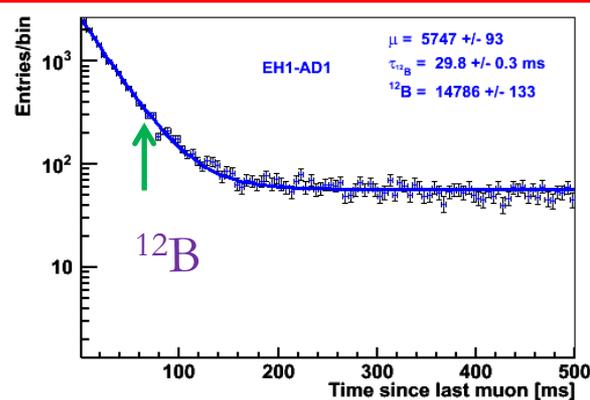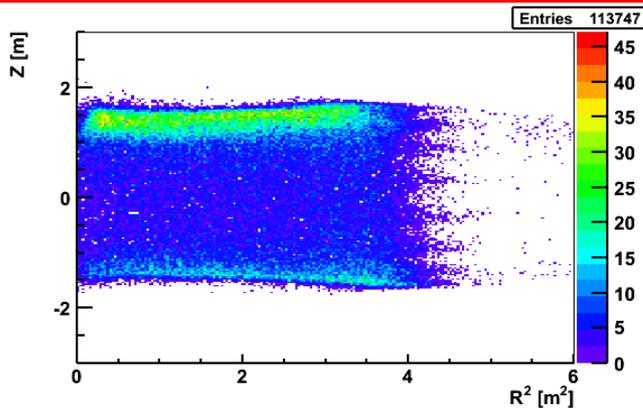
中子能区事例能谱的比较

◆ 各分量求和得到的能谱与总能谱比较

◆ 理解了探测器事例，没有大的未知本底（排它性）
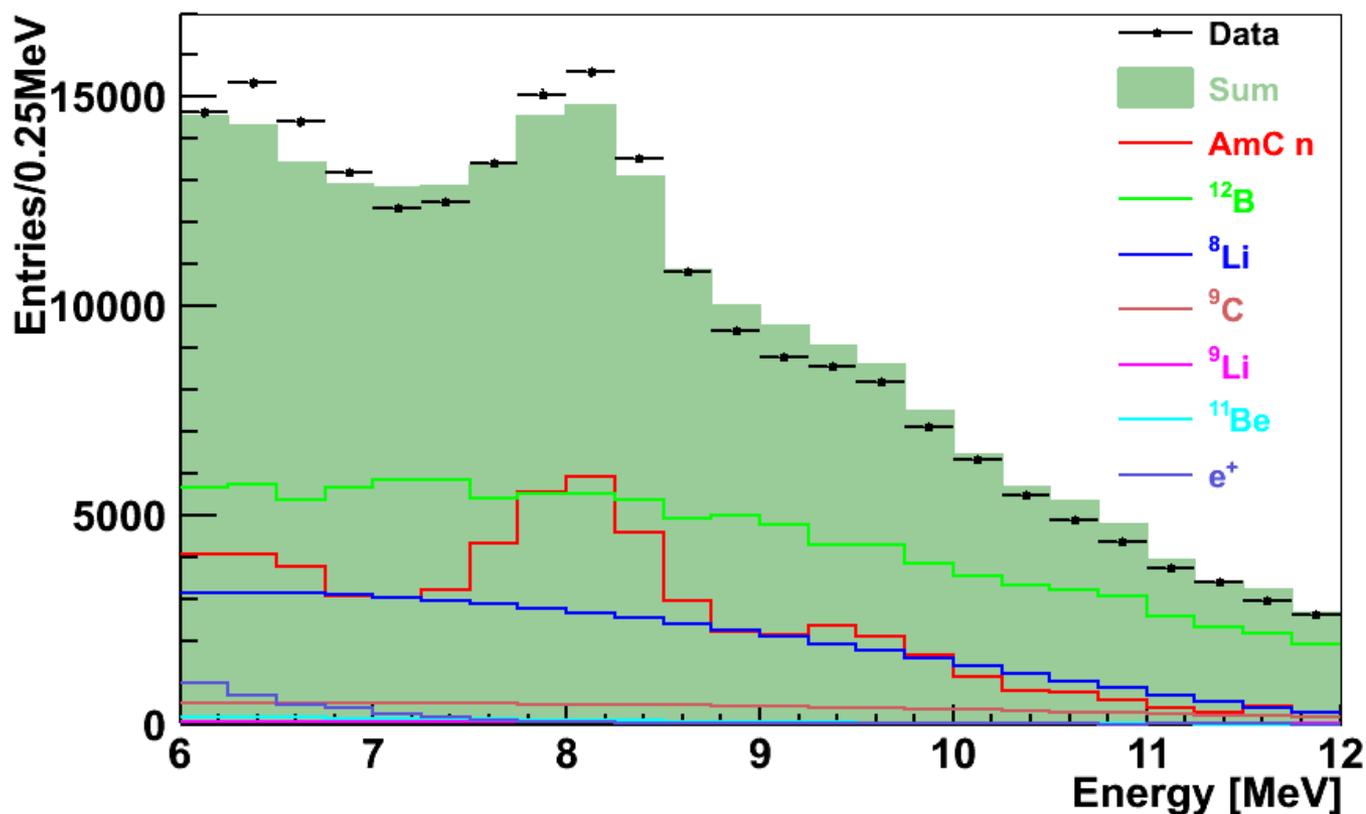
信号与本底
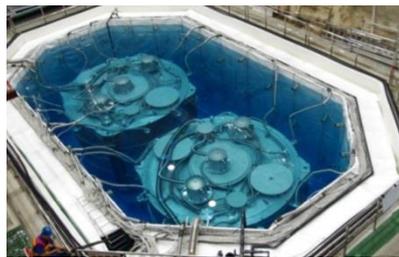
	AD1	AD2	AD3	AD4	AD5	AD6
Antineutrino candidates	69121	69714	66473	9788	9669	9452
DAQ live time (day)	127.5470		127.3763	126.2646		
Efficiency $\varepsilon_\mu * \varepsilon_m$	0.8015	0.7986	0.8364	0.9555	0.9552	0.9547
Accidentals (/day)	9.73 ± 0.10	9.61 ± 0.10	7.55 ± 0.08	3.05 ± 0.04	3.04 ± 0.04	2.93 ± 0.03
Fast neutron (/day)	0.77 ± 0.24	0.77 ± 0.24	0.58 ± 0.33	0.05 ± 0.02	0.05 ± 0.02	0.05 ± 0.02
^{8}He/^{9}Li (/day)	2.9 ± 1.5		2.0 ± 1.1	0.22 ± 0.12		
Am-C corr. (/day)	0.2 ± 0.2					
^{13}C(α, n)^{16}O (/day)	0.08 ± 0.04	0.07 ± 0.04	0.05 ± 0.03	0.04 ± 0.02	0.04 ± 0.02	0.04 ± 0.02
Antineutrino rate (/day)	**662.47 ±3.00**	**670.87 ±3.01**	**613.53 ±2.69**	**77.57 ±0.85**	**76.62 ±0.85**	**74.97 ±0.84**

10 - χ² Analysis
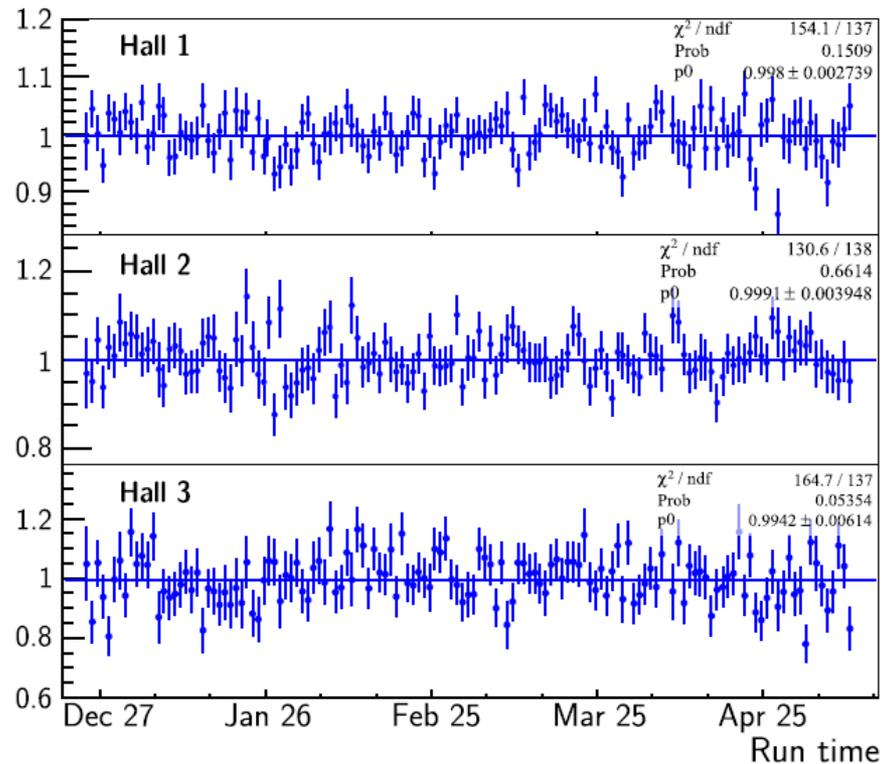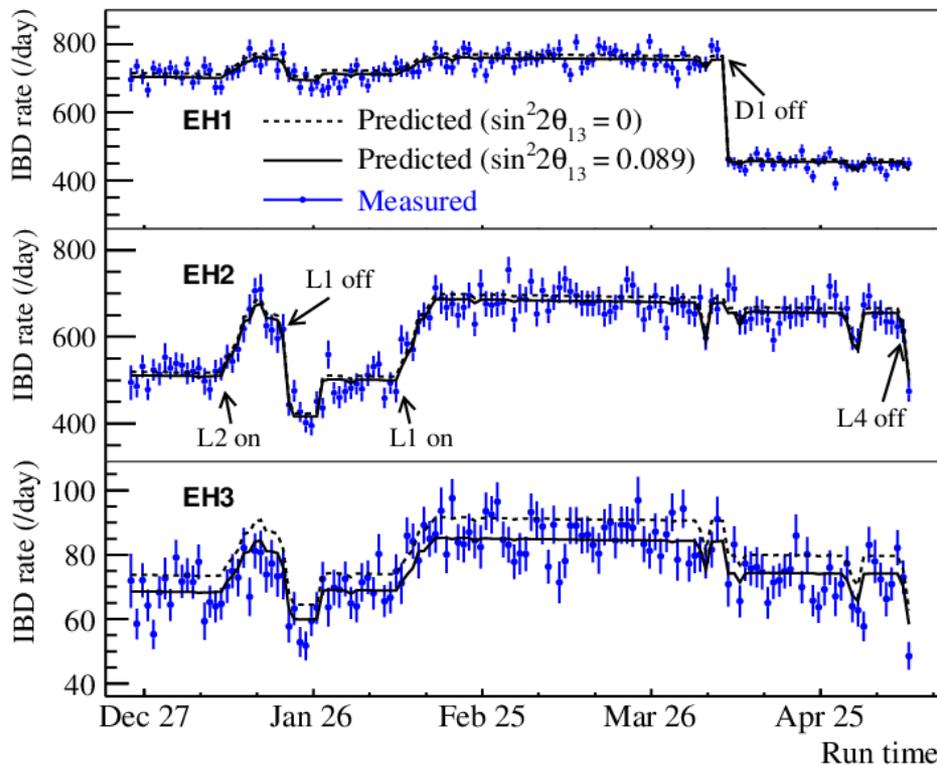
$$\chi^2 = \sum_{d=1}^{6} \frac{\left[M_d - T_d \left(1 + \varepsilon + \sum_r \omega_r^d \alpha_r + \varepsilon_d \right) + \eta_d \right]^2}{M_d + B_d} + \sum_r \frac{\alpha_r^2}{\sigma_r^2} + \sum_{d=1}^{6} \left(\frac{\varepsilon_d^2}{\sigma_d^2} + \frac{\eta_d^2}{\sigma_B^2} \right),$$

6个数据点（探测器），三大误差来源：反应堆、探测器、本底

$$R = \frac{M_f}{\overline{N_f}} = \frac{M_f}{\alpha M_a + \beta M_b}$$

◆ **归一化条件：无振荡时R=1**

$$\sum_i f_i \overline{\nu}_i = \alpha \sum_i a_i \overline{\nu}_i + \beta \sum_i b_i \overline{\nu}_i \,.$$

denote $F = \sum_i f_i \overline{\nu}_i,\ A = \sum_i a_i \overline{\nu}_i,\ \text{and}\ B = \sum_i b_i \overline{\nu}_i.$

$$\beta = (F - \alpha A)/B \,.$$

◆ **误差最小化条件**

$$\Delta N_f = N_f - \overline{N}_f = \sum_i f_i \nu_i - (\alpha M_a + \beta M_b)$$

$$\sigma^2_{\Delta N} = \sum_i (f_i - \alpha a_i - \beta b_i)^2 \sigma^2$$

$$\frac{\partial \sigma^2_{\Delta N}}{\partial \alpha} = 0 \,,$$
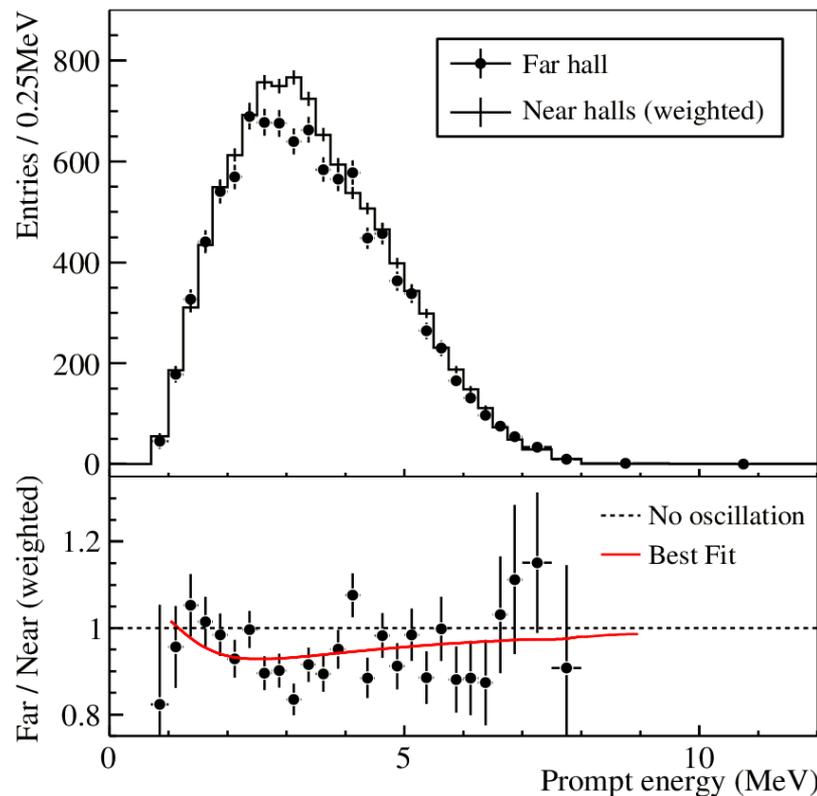
$$R = 0.944 \pm 0.007(\text{stat}) \pm 0.003(\text{syst})$$

假如无振荡，则远点中微子丢失**5.6%**。总误差为**0.0076**。由统计涨落造成缺失的概率为**7.4 sigma**

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χ² function

How to take in account systematic errors correctly?

Simplest one:

$$\chi_1{}^2 = \frac{(T-O)^2}{T + T^2 \sigma_{sys}^2}$$

Suppose divide the data into two sub-sets:

$$\chi_2{}^2 = \frac{(T_1 - O_1)^2}{T_1 + T_1{}^2 \sigma_{sys}^2} + \frac{(T_2 - O_2)^2}{T_2 + T_2{}^2 \sigma_{sys}^2} \qquad \textbf{?}$$

χ² function

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Let each sub-set to be one half, $T_1 = T_2 = T/2$.

$$\chi_2^2 = \frac{(T-O)^2}{T + T^2 \sigma_{sys}^2 / 2} \neq \chi_1^2$$

Same data, different binning, results should be the same.

Correlation

Two sub-sets share the same systematic error. They are correlated.

$$\chi_3{}^2 = \left(T_1 - O_1, T_2 - O_2\right) \begin{pmatrix} T_1 + T_1{}^2 \sigma_{sys}^2 & T_1 T_2 \sigma_{sys}^2 \\ T_1 T_2 \sigma_{sys}^2 & T_2 + T_2{}^2 \sigma_{sys}^2 \end{pmatrix}^{-1} \begin{pmatrix} T_1 - O_1 \\ T_2 - O_2 \end{pmatrix}$$

Now $\chi_3{}^2 = \chi_1{}^2$

Another form:

$$\chi_1{}^2 = \frac{(T-O)^2}{T + T^2 \sigma_{sys}^2} = \min_\alpha \left\{ \frac{\left[T(1+\alpha) - O\right]^2}{T} + \frac{\alpha^2}{\sigma_{sys}^2} \right\}$$

It is straight forward to get for two sub-sets.

$$\chi_4{}^2 = \min_\alpha \left\{ \frac{\left[T_1(1+\alpha) - O_1\right]^2}{T_1} + \frac{\left[T_2(1+\alpha) - O_2\right]^2}{T_2} + \frac{\alpha^2}{\sigma_{sys}^2} \right\} = \chi_3{}^2 = \chi_1{}^2$$

We know there is only one σ_{sys}. Correlations are included automatically.

General Form

We have N measurements m_i

t_i **is the true value, σ_i is uncorrelated errors,**

β_{ji} **is the j-th kind correlated error to measurement i. (K parameters)**

r_i **and r'_j are supposed to have standard gaussian distribution.**

$$m_i = t_i + \sigma_i r_i + \sum_{j=1,K} \beta_{ji} r'_j \qquad (1)$$

The probability distribution of the measurements is

$$dP = \int \prod_{i=1,N} p(r_i) dr_i \prod_{j=1,K} p(r'_j) dr'_j \prod_{i=1,N} \delta\left(m_i - t_i - \sigma_i r_i - \sum_{j=1,K} \beta_{ji} r'_j \right) d^N m \qquad (2)$$

Integrate over r_i

$$dP = \int \prod_{j=1,K} dr'_j \, C \exp(-\chi^2 / 2) d^N m \qquad (3)$$

$$\chi^2 = \sum_{i=1,N} \left(\frac{m_i - t_i - \sum_{j=1,K} \beta_{ji} r'_j}{\sigma_i} \right)^2 + \sum_{j=1,K} r'^2_j \qquad (4)$$

- **We are finding the "most probable" solution in the probability space by varying the fitting physical parameters like θ_{13} (not r'$_j$, but implicit in t_i here). Minimizing with respect to r'$_j$ will lead to the most probably solution for the fitting parameters.**

- **A minimization with K parameters.**

- **We may need many bins in spectrum analysis.**

- **Fitting on a quadratic polynomial is easy and stable.**

χ^2 method 2

Equation (3) can be integrated over r'$_j$ analytically.

$$\chi^2 = \sum_{i=1,N} \left(\frac{m_i - t_i}{\sigma_i} \right)^2 - \sum_{j=1,K} \sum_{j'=1,K} B_j (A^{-1})_{jj'} B_{j'}$$

(5)

$$B_j = \sum_{i=1,N} \beta_{ji} (m_i - t_i) / \sigma_i^2$$

$$A_{jj'} = \delta_{jj'} + \sum_{i=1,N} \beta_{ji} \beta_{j'i} / \sigma_i^2$$

Evaluation of χ^2 function need inverse a K$\times$K matrix.

Covariance Matrix Estimator

$$\chi^2 = \sum_{i=1,N} \sum_{i'=1,N} (m_i - t_i)(V^{-1})_{ii'}(m_{i'} - t_{i'})$$

(6)

$$V_{ii'} = \sigma_i^2 \delta_{ii'} + \sum_{j=1,K} \beta_{ji} \beta_{ji'}$$

Evaluation χ^2 function need inverse a N×N matrix.

Inverse V analytically, we can find that method 3 is equivalent to method 2.

===

Now we have 3 equivalent χ^2 function.

- **Method 1: K parameter fitting. K is number of correlated errors.**
- **Method 2: K×K matrix inversion.**
- **Method 3: N×N matrix inversion. N is number of data points.**

χ² 与 Δχ²

◆ 统计分析的两大功能：假设检验与参数拟合

◆ 假设检验：先提出一个假设，用数据检验这个假设与数据符合得好不好

◆ 例如，大亚湾55天的数据如上图。"是否存在振荡？"或者等价地，"θ_{13}是否为0？"是一个待检验的假设。我们令振荡为0，然后看数据与这个预期的符合程度。下图小图中有θ_{13}=0时的χ^2值。我们得到$\chi^2 \sim 30$，对应5.2σ。因此对θ_{13}=0这个假设的排除度是5.2σ
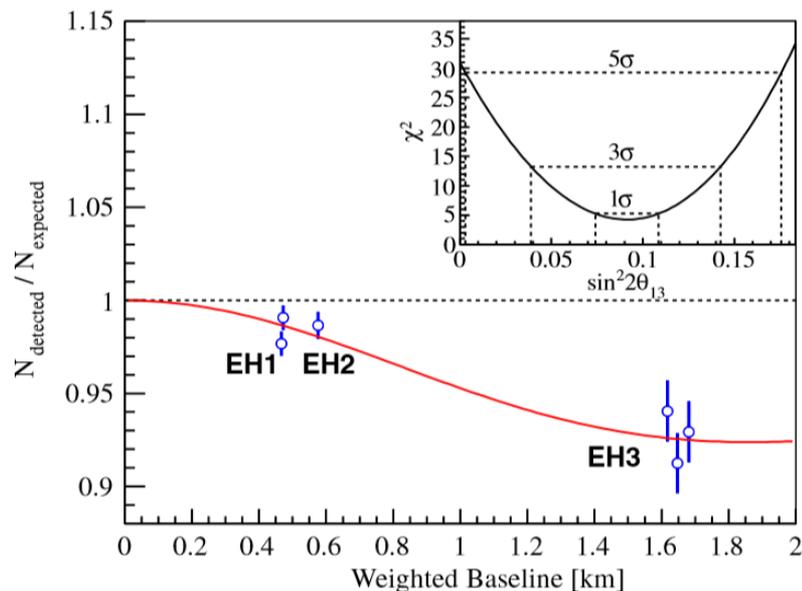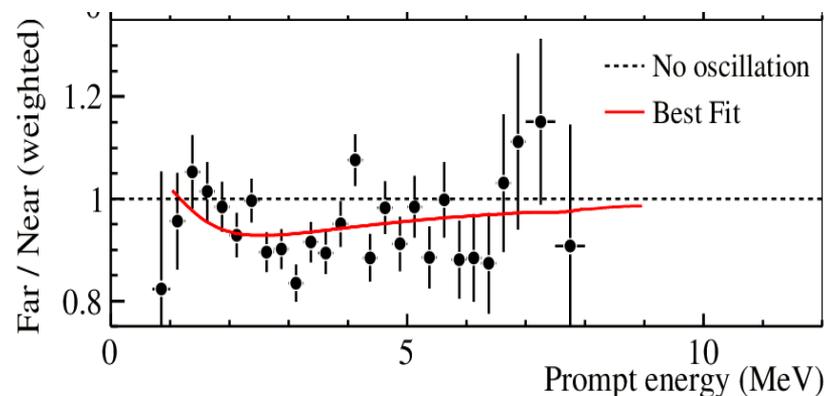
χ² 与 Δχ²

◆ **参数拟合**：假定理论是对的，确定参数。只有理论是对的，参数才有意义。

◆ 先拟合出最小的χ²，对应最佳的参数值。我们得到 $\sin^2 2\theta_{13} = 0.092$:

◆ χ^2/ndf（拟合的优度）表征这个拟合好不好。一般应在1左右，太大或太小都可能有问题，但没有绝对的好坏。

◆ 得到最佳参数后，求参数的误差，通过人为使θ_{13}偏离最佳拟合值，求相应的Δχ²。对参数自由度为1的情况，Δχ²=1,4,9,16,25对应1-5σ（理想高斯分布情况）， 原因见：J. Pumplin et. al. PRD65,014011(2001)

◆ 从右图中：χ^2_{min}=4.26，此时 $\sin^2 2\theta_{13}$=0.092。拟合优度为4.26/4。扫描不同的θ_{13}值，得到χ²曲线。当 χ²=5.26时，即Δχ²=1，对应的θ_{13}范围即1倍标准偏差的范围。
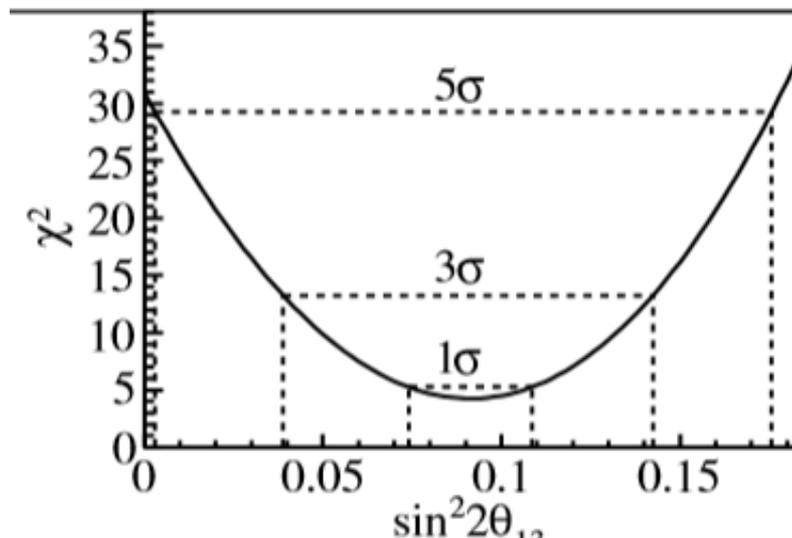
data. The best-fit value is

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat.}) \pm 0.005(\text{syst.}),$$

with a χ^2/NDF of 4.26/4 (where NDF is the number of degrees of freedom). All best estimates of pull parameters

在假设检验和参数拟合中，每变化一次拟合参数，都要对pull参数做最小化。即不同的θ_{13}值，对应的pull参数是不同的。只有这样才算考虑了误差。

$$\chi^2 = \sum_{d=1}^{6} \frac{\left[M_d - T_d\left(1+\varepsilon+\sum_r \omega_r^d \alpha_r + \varepsilon_d\right) + \eta_d\right]^2}{M_d + B_d}$$

$$+ \sum_r \frac{\alpha_r^2}{\sigma_r^2} + \sum_{d=1}^{6}\left(\frac{\varepsilon_d^2}{\sigma_d^2} + \frac{\eta_d^2}{\sigma_B^2}\right),$$

大亚湾分析中**ndf**应该等于几？

M_d： 第**d**个**AD**的测量中微子数
T_d： 第**d**个**AD**的预测中微子数
B_d： 第**d**个**AD**的本底数

ε： 整体归一化系数，对应反应关联误差与探测器关联误差**3.6%**。分析时未加约束，即完全相对测量

ω_r^d： 第**r**个反应堆对第**d**个探测器的中微子贡献比例

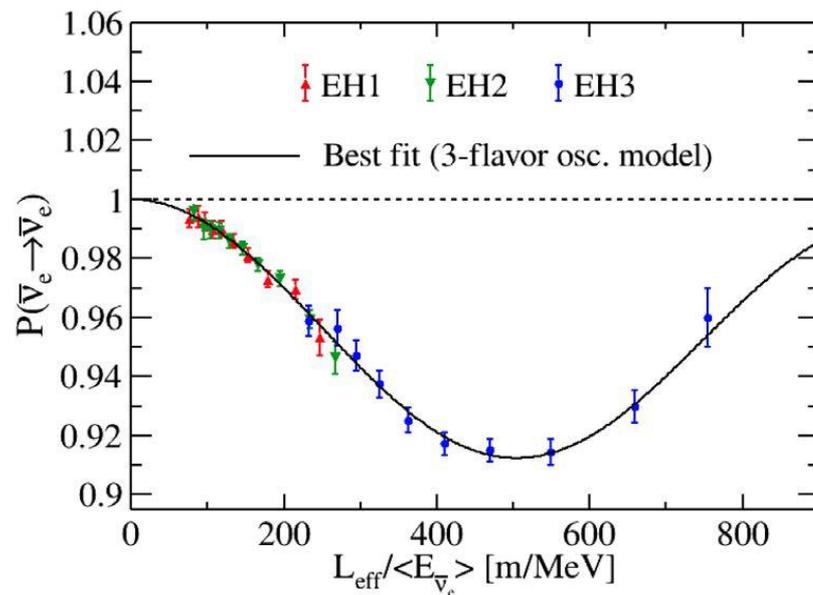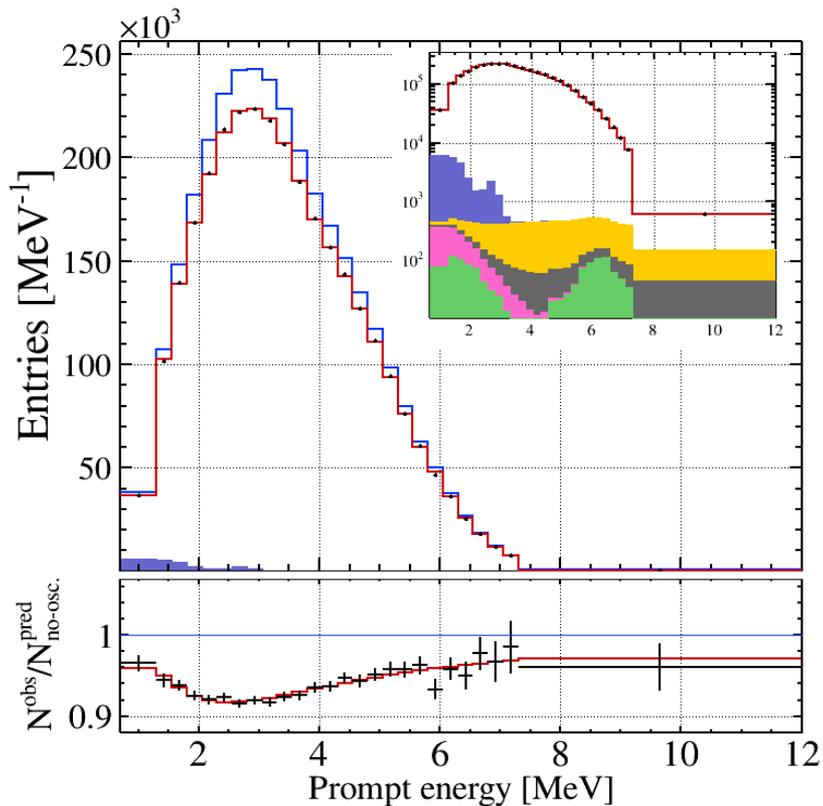**α_r**： 反应非关联误差，对应σ_r=**0.8%**

ε_d： 探测器非关联误差，对应σ_d=**0.2%**

η_d： 本底误差，对应σ_d=**0.2-0.37%**（对应的信号与本底表）
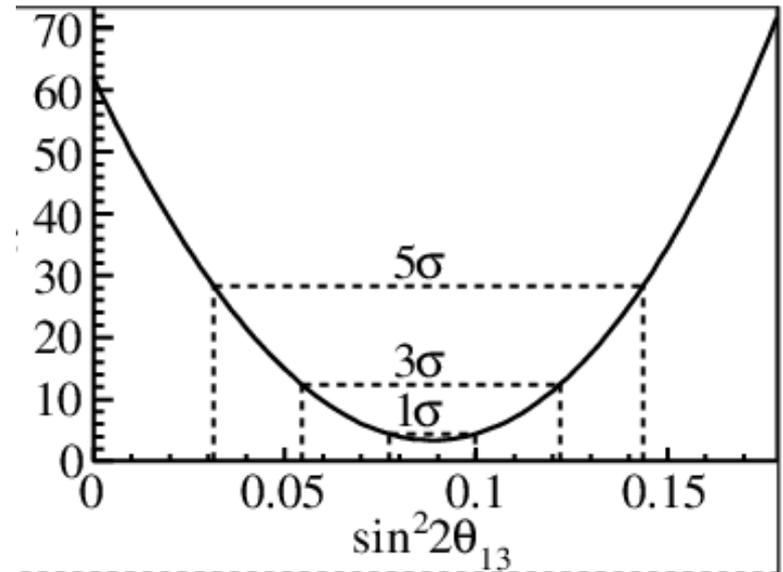
$$\sin^2 2\theta_{13} = 0.089 \pm 0.010(\text{stat}) \pm 0.005(\text{syst})$$

5.55 M neutrinos (nGd) selected from 3158 days operation.

$$\sin^2 2\theta_{13} = 0.0851 \pm 0.0024$$

$$\Delta m_{32}^2 = 2.466 \pm 0.060 \text{ e-3 eV}^2 \text{ (NO)}$$

◆ 最精确的反应堆中微子能谱

◆ 首次测量U235和Pu239中微子能谱

◆ 首次测量>10 MeV的反应堆中微子

◆ 在很大参数空间上基本排除轻质量惰性中微子