Nuclear matrix elements for neutrinoless double-beta decay

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Outline

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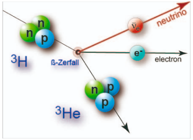
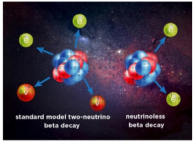
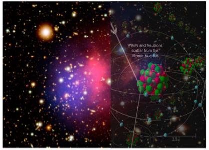
Low-energy nuclear probes



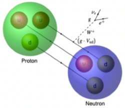
Single–beta decay



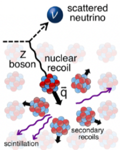
Neutrinoless double beta decay



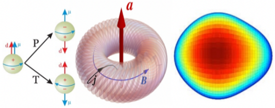
Dark matter direct detection



Superallowed Fermi transitions



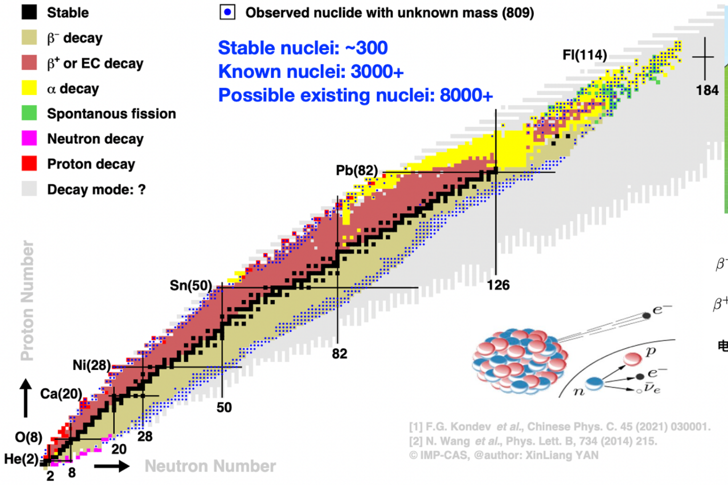
Neutrino scattering



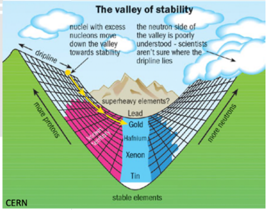
Symmetry-violating moments

- Probes to fundamental interactions and symmetries.
- Searches for new physics at nuclear level.
- All about Nuclear Matrix Elements (NME)

Nuclear Chart: decay mode of the ground state nuclide(NUBASE2020)



Stable nuclei: ~300
Known nuclei: 3000+
Possible existing nuclei: 8000+



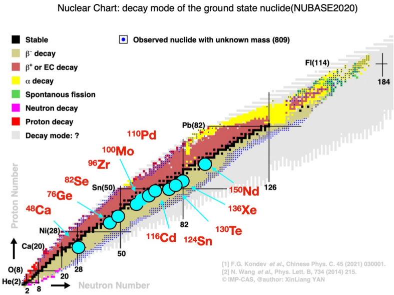
β^- 衰变 $\quad {}^A_Z X_N \to {}^{A}_{Z+1} Y_{N-1} + e^- + \bar{\nu}_e,$

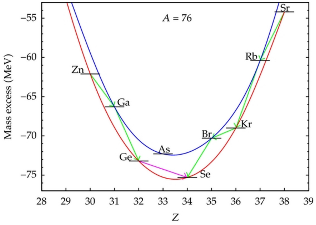
β^+ 衰变 $\quad {}^A_Z X_N \to {}^{A}_{Z-1} Y_{N+1} + e^+ + \nu_e,$

电子俘获 $\quad {}^A_Z X_N + e^- \to {}^{A}_{Z-1} Y_{N+1} + \nu_e,$

[1] F.G. Kondev *et al.*, Chinese Phys. C. 45 (2021) 030001.
[2] N. Wang *et al.*, Phys. Lett. B, 734 (2014) 215.

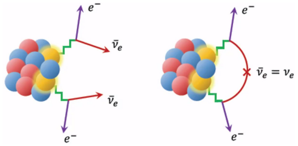
A special decay mode: $0\nu\beta\beta$ decay



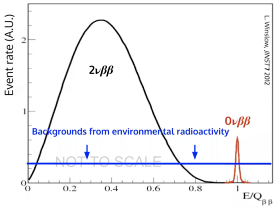


- The two modes of $\beta^-\beta^-$ decay:

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$



- Kinetic energy spectrum of electrons



$0\nu\beta\beta$ decay as a probe to neutrino properties

Neutrino oscillation

- From mass to flavor states

$$|\nu_\alpha\rangle = \sum_{j=1}^{N=3} U^*_{\alpha j} |\nu_j\rangle .$$

- $\Delta m^2_{ij}(\neq 0)$, and $\theta_{ij}(\neq 0)$.

Open questions

- The nature of neutrinos.
- Neutrino mass m_j and its origin.

The observation of $0\nu\beta\beta$ decay provide answers.

If $0\nu\beta\beta$ decay is driven by exchanging light massive Majorana neutrinos:

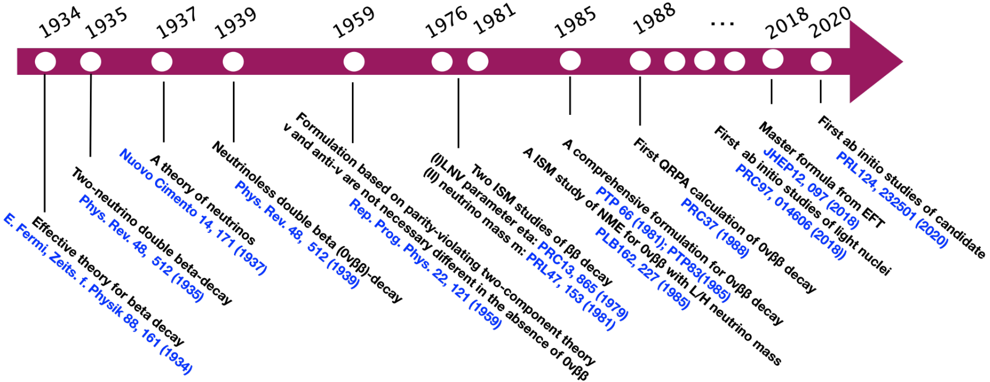
$$\langle m_{\beta\beta}\rangle \equiv |\sum_{j=1}^{3} U^2_{ej} m_j| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T^{0\nu}_{1/2} |M^{0\nu}|^2}\right]^{1/2}$$

- U_{ej}: elements of the PMNS matrix
- $M^{0\nu}$: the NME

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

- Transition operator: $\hat{O}^{0\nu}$
- Nuclear many-body wfs: $|\Psi_{I/F}\rangle$


Fermi
Goeppert-Mayer
Majorana
Furry
Primakoff
Vergados
Haxton
吴慧芳
Engel
...


1934
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1988
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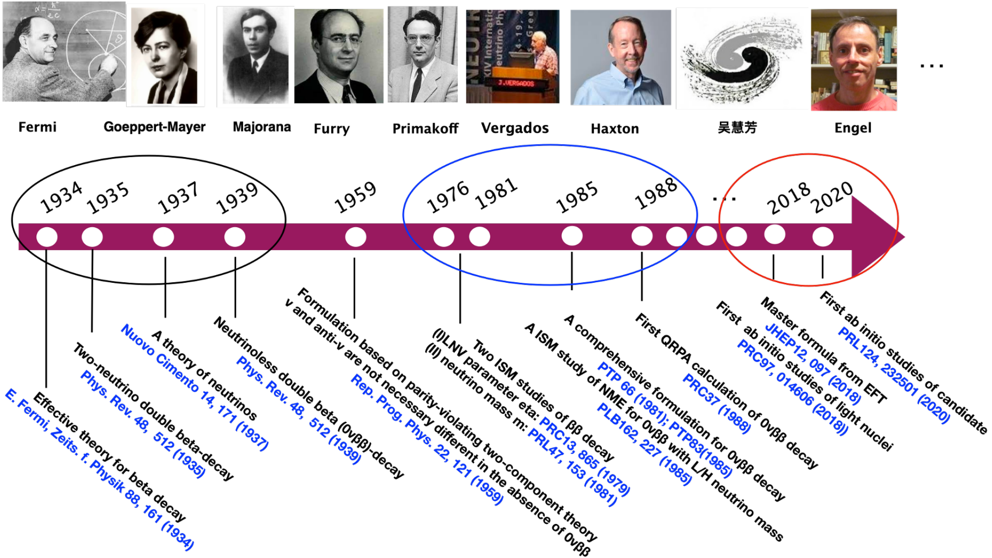
2018
2020
Effective theory for beta decay
E. Fermi, Zeits. f. Physik 88, 161 (1934)
Two-neutrino double beta-decay
Phys. Rev. 48, 512 (1935)
A theory of neutrinos
Nuovo Cimento 14, 171 (1937)
Neutrinoless double beta (0νββ)-decay
Phys. Rev. 48, 512 (1939)
Formulation based on parity-violating two-component theory
ν and anti-ν are not necessary different in the absence of 0νββ
Rep. Prog. Phys. 22, 121 (1959)
(I)LNV parameter eta: PRC13, 865 (1979)
(II) neutrino mass m: PRL47, 153 (1981)
Two ISM studies of ββ decay
A ISM study of NME for 0νββ with L/H neutrino mass
PLB162, 227 (1985)
A comprehensive formulation for 0νββ decay
PTP 66 (1981); PTP83(1985)
First QRPA calculation of 0νββ decay
PRC37 (1988)
First ab initio studies of light nuclei
PRC97, 014606 (2018))
Master formula from EFT
JHEP12, 097 (2018)
First ab initio studies of candidate
PRL124, 232501 (2020)

Brief history on modeling the decay rate



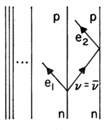
Earlier studies: **Estimation of NME $|M^{0\nu}|^2 \sim 10^{-2}/R_A^2$**

- Fermi theory (parity conserving), massless Majorana neutrino, $T_{1/2}^{0\nu} < 10^{17}$yr.

  Furry (1939); Primkoff (1952); Konopinski (1955)
- Parity non-conserving int, $T_{1/2}^{0\nu} \sim 10^{16}$yr $< T_{1/2}^{2\nu}$. Primkoff (1959)
- First considering LNV due to massive neutrinos m_ν, and admixture δ of RH current, $T_{1/2}^{0\nu}(m_\nu, \delta)$. Greuling (1960)

Modern studies: **Computing NMEs with nuclear models**

- "standard" mechanism ($\eta = \langle m_{\beta\beta}\rangle/m_e$), NME from ISM



Vergados (1976)

^{48}Ca: $|M^{0\nu}|^2 = 0.012$
^{130}Te: $|M^{0\nu}|^2 = 0.25$

Haxton (1981)

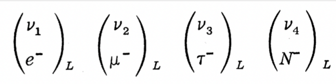
^{76}Ge: $|M^{0\nu}|^2 = 1.28^2$
^{82}Se: $|M^{0\nu}|^2 = 0.94^2$

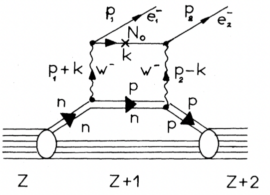
Constraint on the LNV parameter η

$$\eta^2 T_{1/2}^{0\nu}(0^+ \to 0^+) \sim 10^{12} \text{ yr}, \quad T_{1/2}^{0\nu} > 10^{22} \text{ yr}, \quad \eta < 10^{-4}$$

Modeling the transition operators

- Gauge theory (W^-), admixture of ν_L and a new Majorana neutrino N_0 with mass M_0: $\nu_{1,L} = \nu_{e,L} + \beta N_{0,L}$, NME from ISM. Vergados(1981)





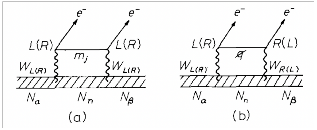
For $M_0 = 0.25, \cdots 10$ GeV,
$r_c = 0.4$ fm (SRC)

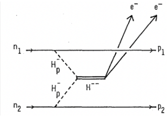
$$\beta^4 T_{1/2}^{0\nu} \sim 10^{10-15}\ \mathrm{yr}$$

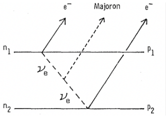
$$\beta^2 < 10^{-6} - 10^{-4}$$

- Reviews on different mechanisms: $\langle m_{\beta\beta}\rangle$, LRSM, Higgs boson, Majoron, N^*)

Doi (1981,1983,1985), Haxton (1984), Faessler (1998), · · ·







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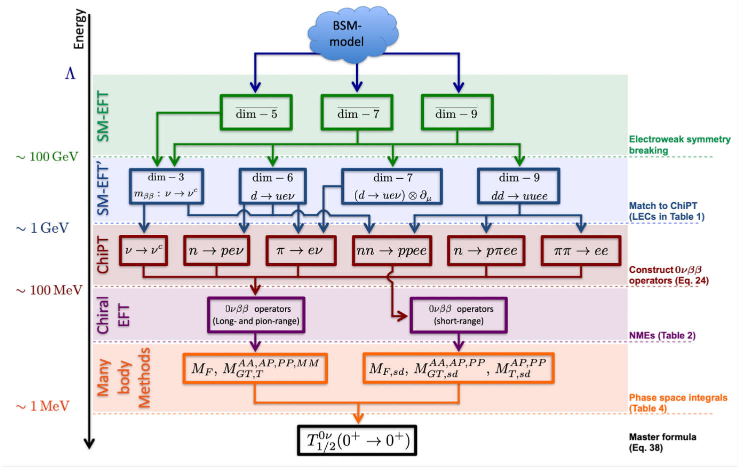
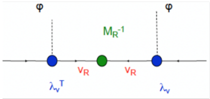
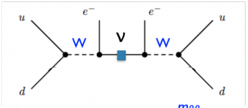
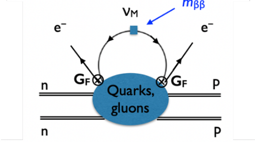
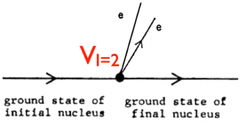
Recent studies: model-independent analysis of operators at different energy scales within EFT Cirigliano (2018)





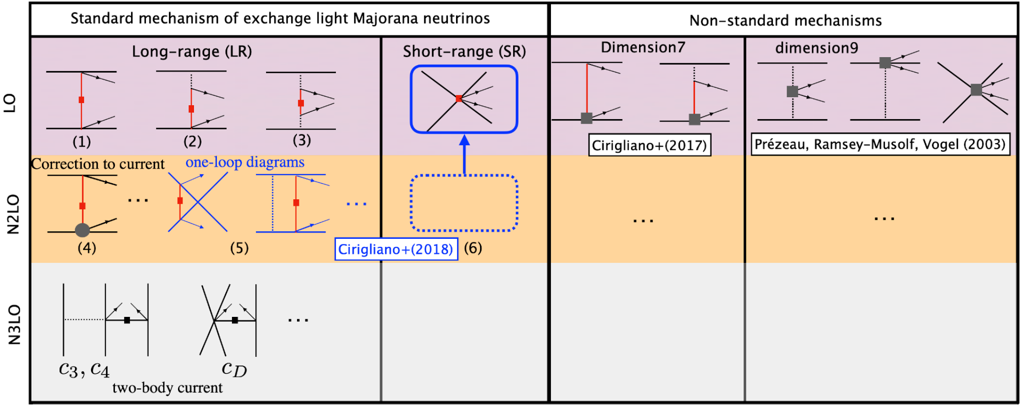






Modeling the transition operators

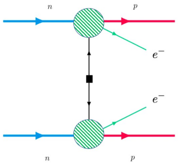
- At $E \sim$100 MeV: operators are expressed in terms of nucleons, pions, and leptons.
- Construction of transition operators within chiral EFT (or phenomenologically).



The half-life of $0\nu\beta\beta$ decay:

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{1}{\ln 2}\frac{1}{2}\int\frac{d^3\boldsymbol{k}_1}{(2\pi)^3 2\epsilon_1}\int\frac{d^3\boldsymbol{k}_2}{(2\pi)^3 2\epsilon_2}(2\pi)\delta(E_I - E_F - \epsilon_1 - \epsilon_2)|\mathcal{M}_{fi}|^2.$$

$$\simeq G_{0\nu} g_A^4(0)\left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2\left|M^{0\nu}\right|^2$$



- The phase space factor

$$G_{0\nu} \equiv \frac{1}{(\ln 2)(2\pi)^5}\frac{G_\beta^4 m_e^2}{R_0^2}\frac{1}{4}\int\int\sum_{\text{spins}}\left|\bar{u}(k_1, s_1)P_R C\bar{u}^T(k_2, s_2)\right|^2 k_1 k_2 d\epsilon_1 d\cos\theta_{12}$$

- The NME by an effective 1B current $\mathcal{J}_L$ with dipole form factors

$$M^{0\nu} \equiv \frac{4\pi R_0}{g_A^2(0)}\int d^3 r_1 d^3 r_2 \int\frac{d^3 q}{(2\pi)^3}e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}}\sum_N \frac{\langle\Psi_F|\,\mathcal{J}_{L,\mu}^\dagger(\boldsymbol{r}_1)\,|N\rangle\,\langle N|\,\mathcal{J}_L^{\mu\dagger}(\boldsymbol{r}_2)\,|\Psi_I\rangle}{q\,[q + E_N - (E_I + E_F)/2]}.$$

The effective one-body current operator takes the following form

$$\mathcal{J}_L^{\mu\dagger} = \bar{\psi}_N \gamma^\mu \left[g_V(\boldsymbol{q}^2) - g_A(\boldsymbol{q}^2)\gamma_5 + g_P(\boldsymbol{q}^2) q^\mu \gamma_5 - i g_W(\boldsymbol{q}^2)\sigma^{\mu\nu} q_\nu \right] \tau^+ \psi_N$$

The dipole form factors

$$\begin{aligned}
g_V(\boldsymbol{q}^2) &= g_V(0)\left(1 + \boldsymbol{q}^2/\Lambda_V^2\right)^{-2}, \\
g_A(\boldsymbol{q}^2) &= g_A(0)\left(1 + \boldsymbol{q}^2/\Lambda_A^2\right)^{-2}, \\
g_P(\boldsymbol{q}^2) &= g_A(\boldsymbol{q}^2)\left(\frac{2m_p}{\boldsymbol{q}^2 + m_\pi^2}\right), \\
g_W(\boldsymbol{q}^2) &= g_V(\boldsymbol{q}^2)\frac{\kappa_1}{2m_p},
\end{aligned}$$

where $g_V(0) = 1$, $g_A(0) = 1.27$, and the cutoff values are $\Lambda_V = 0.85$ GeV and $\Lambda_A = 1.09$ GeV. According to the conserved vector current (CVC) hypothesis, $g_W(0) = \kappa_1/2m_p$ with κ_1 being the anomalous nucleon isovector magnetic moment $\kappa_1 = \mu_n^{(a)} - \mu_p^{(a)} \simeq 3.7$.

- In the closure approximation, transition operator becomes a product of two current operators composed of five terms (VV, AA, PP, AP and MM),

$$g_V^2(\boldsymbol{q}^2)(\bar{\psi}\gamma_\mu\tau^+\psi)^{(1)}(\bar{\psi}\gamma^\mu\tau^+\psi)^{(2)}, \qquad \text{VV}$$

$$g_A^2(\boldsymbol{q}^2)(\bar{\psi}\gamma_\mu\gamma_5\tau^+\psi)^{(1)}(\bar{\psi}\gamma^\mu\gamma_5\tau^+\psi)^{(2)}, \qquad \text{AA}$$

$$g_P^2(\boldsymbol{q}^2)(q_\mu\gamma_5\tau^+\psi)^{(1)}(\bar{\psi}q^\mu\gamma_5\tau^+\psi)^{(2)}, \qquad \text{PP}$$

$$g_A(\boldsymbol{q}^2)g_P(\boldsymbol{q}^2)(\boldsymbol{\gamma}\gamma_5\tau^+\psi)^{(1)}(\bar{\psi}\boldsymbol{q}\gamma_5\tau^+\psi)^{(2)}, \qquad \text{AP}$$

$$g_W^2(\boldsymbol{q}^2)(\sigma_{\mu\nu}q_\mu\tau^+\psi)^{(1)}(\bar{\psi}\sigma^{\mu\nu}q^\mu\tau^+\psi)^{(2)}, \qquad \text{MM}$$

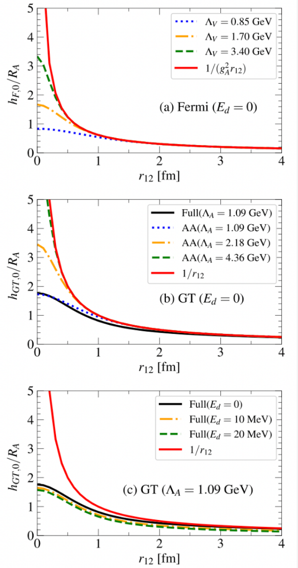
- In non-relativistic reduction (truncation in terms of $1/m_p$, no nucleon recoil terms), the transition operator in coordinate space

$$O^{0\nu} = \sum_{m\neq n=1}^{A} \tau_m^+ \tau_n^+ \Big[h^{0\nu}_{F,0}(r_{mn}, E_d) + h^{0\nu}_{GT,0}(r_{12}, E_d)\vec{\sigma}_m \cdot \vec{\sigma}_n + h^{0\nu}_{T,2}(r_{mn}, E_d) S^r_{mn} \Big],$$

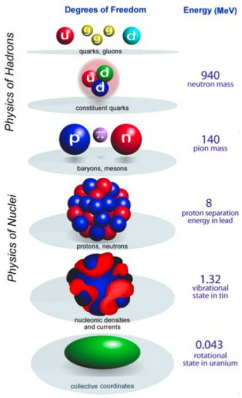
where the neutrino potentials in coordinate space are defined as

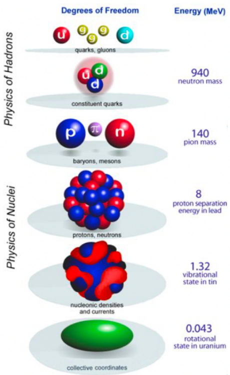
$$h^{0\nu}_{\alpha,L}(r_{12}, E_d) = \frac{2R_0}{\pi g_A^2(0)} \int_0^\infty \mathrm{d}q\, q^2 \frac{h_\alpha(\boldsymbol{q}^2)}{q(q+E_d)} j_L(qr_{12})$$

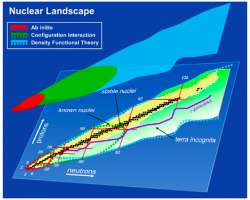
where $E_d = \langle E_N \rangle - (E_I + E_F)/2 \simeq 1.12 A^{1/2}$ MeV.

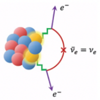


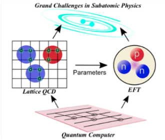
Modeling atomic nuclei







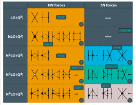




(Lattice) QCD for NP

2010s — **ab initio methods**

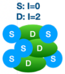
2000s — **High-precision NN interactions** Entem, Machleidt (2003)



1990s — **Chiral EFT for NN interaction** Weinberg (1991)

1970s — **Self-consistent mean-field/EDF approaches**

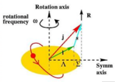
Interacting Boson Models Arima, Iachello (1974) IBM



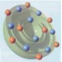
Hill, Griffin, Wheeler (1953,1957) GCM

1950s — **Collective models** Bohr and Mottelson (1953) PRM, BH

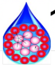
Shell models Bohm, Pines (1951-1953) RPA



Goeppert-Mayer (1949);
Haxel, Jensen, and Suess (1949) ISM



1930s — **Liquid drop model** von Weizsäcker (1935)



Modern studies: phenom. nuclear forces

- Interacting shell models (ISM) Vergados (1976), Haxton (1981), H.F.Wu (1985, 1993), Caurier (2008), Menéndez (2009), Horoi (2010), Coraggio (2020)
- Particle-number (and angular-momentum) projected BCS (HFB) with a schematic (PP+QQ) hamiltonian Grotz, Klapdor (1985), Chandra (2008), Rath (2010), Hinohara (2014)
- Quasi-particle random-phase approx. (QRPA) with a G-matrix residual interaction Vogel-2ν (1986), Engel (1988), Rodin (2003), Faessler (1998), Simkovic (1999), Fang (2010) or EDF Mustonen (2013), Terasaki (2015), Lv(2023), Bai (2023?)
- Interacting Boson Models (IBM) Barea (2009, 2012)
- GCM+EDFs Rodríguez (2010), Song (2014), Yao (2015)
- Angular momentum projected interacting shell model based on an effective interaction Iwata, Shimizu (2016) or EDF Wang (2021, 2023)
- Others: Generalized-seniority scheme Engel, Vogel, Ji, Pittel (1989)

Recent ab initio studies: chiral nuclear forces

- Quantum MC/NCSM for light nuclei: Pastore(2018); Yao(2021)
- Basis-expansion methods for candidates: Yao(2020), Belley(2021), Navorio(2021)

Early studies of $0\nu\beta\beta$ decay in China

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MAJORANA NEUTRINO AND LEPTON-NUMBER NON-CONSERVATION IN ^{48}Ca NUCLEAR DOUBLE BETA DECAY ☆

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We have made a nuclear shell-model calculation of the neutrinoless double beta decay of ^{48}Ca(0^+; g.s.) → ^{48}Ti(0^+; g.s.), using a diagramatic effective operator approach, together with the Paris and Reid NN potentials. Lepton-number non-conservation is due to the exchange of a Majorana neutrino between nucleon pairs. To reproduce the experimental value of $T_{1/2} \geq 2\times10^{21}$ yr, the limit on the Majorana neutrino mass is either $\leqslant 40$ eV or $\geqslant 3\times10^5$ GeV. Our calculation includes a microscopic treatment of the NN short-range correlation effects as well as finite-size nucleon effects.

NH ELSEVIER Physics Reports 242 (1994) 495–503 PHYSICS REPORTS

Neutrinoless double beta decays and nuclear matrix elements of ^{76}Ge and ^{82}Se†

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Abstract

We have made the nuclear shell-model calculations of the $0\nu\beta\beta$ decays in ^{76}Ge and ^{82}Se using a model-space approximation. Also we take into account the influences of both nuclear short-range correlation and finite nucleon size on the calculations of the transition matrix elements. The limits of Majorana neutrino mass and mixing parameter of right-handed current have been deduced by fitting the experimental values of $0\nu\beta\beta$ decay lifetimes of ^{76}Ge and ^{82}Se.

PHYSICAL REVIEW C VOLUME 40, NUMBER 1

Operator expansion method and the double beta decay of ^{48}Ca

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A new method, the operator expansion method, is derived and discussed. Using this method, the $\beta\beta$ decay of ^{48}Ca is recalculated. It is shown that by employing the Paris forces our method yields a matrix element suppressed by a factor of about 2–6 for the $2\nu\beta\beta$ mode, while for the $0\nu\beta\beta$ mode our calculation confirms the results obtained with the conventional closure approximation.

Physics Letters B 265 (1991) 53–56
North-Holland PHYSICS LETTERS B

中国首个本土无中微子双贝塔衰变

A search for neutrinoless double β decay of ^{48}Ca ☆

Ke You [a], Yucan Zhu [a], Junguang Lu [a], Hanseng Sun [a], Weihua Tian [a], Wenheng Zhao [a], Zhipeng Zheng [a,b], Minghan Ye [a,b], Chengrui Ching [b,c], Tsohsiu Ho [b,c], Fengzhu Cui [d], Changjiang Yu [d] and Guojing Jiang [d]
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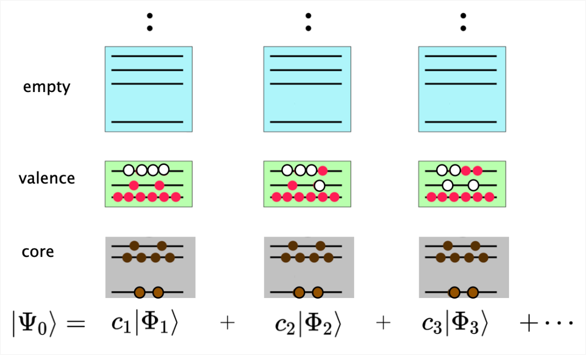
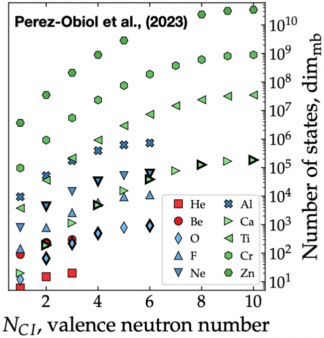
A search for the neutrinoless double β decay of ^{48}Ca is carried out in a coal mine near Beijing. Large scintillation crystals of natural CaF_2 were used as both detector and β source. Results obtained after a total of 7588.5 h of data taking give 9.5×10^{21} yr (76% confidence level) as the lower limit of the half-life of neutrinoless double β decay of ^{48}Ca.

The interacting shell model (ISM) in a nutshell





- Dimension of the model space: $d \sim C_M^{Z_v} C_M^{N_v}$, where M is the number of s.p. states in the valence space for neutrons and protons.
- Include all correlations, but in a limited model space determined by the combination of valence nucleons and valence orbits.

The basic building blocks of IBM are *s* and *d* bosons

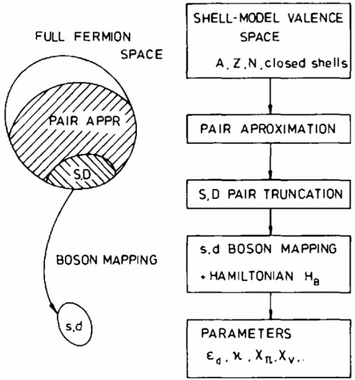
J. Barea and F. Iachello, PRC79, 044301 (2009)

$$|\Psi_0(N;\alpha_\mu)\rangle \propto \left(s^\dagger + \sum_\mu \alpha_\mu d^\dagger_\mu\right)^N |vac\rangle, \quad N = n_s + n_d$$

$$s^\dagger = \sum_j \alpha_j \sqrt{\frac{\Omega_j}{2}} \left(c^\dagger_j \times c^\dagger_j\right)^{(0)}$$

$$d^\dagger_\mu = \sum_{j\leqslant j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(c^\dagger_j \times c^\dagger_{j'}\right)^{(2)}_\mu$$

where s, d are Cooper pairs formed by two nucleons in the valence shell coupled to angular momenta $J = 0$ and $J = 2$, respectively. The structure coefficients α, β are to be determined.



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The pnQRPA in a nutshell

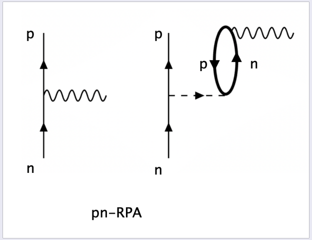
The proton-neutron quasiparticle random-phase approximation (pn-QRPA) as a method for small-amplitude (charge-changing) excitations

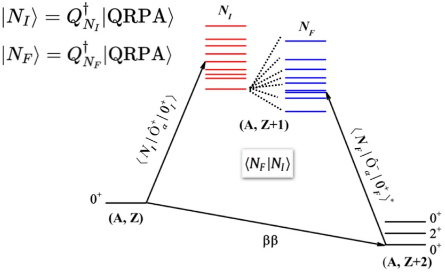
$$|\Psi_\nu(N_{I/F})\rangle = Q_\nu^\dagger|\mathrm{QRPA}\rangle, \quad Q_\nu|\mathrm{QRPA}\rangle = 0$$

where the excitation operator is usually constructed as

$$Q_\nu^\dagger = \sum_{pn} X_{pn}^\nu \beta_p^\dagger \beta_n^\dagger - Y_{pn}^\nu \beta_n \beta_p,$$

Basic diagrams (w/o pairing)





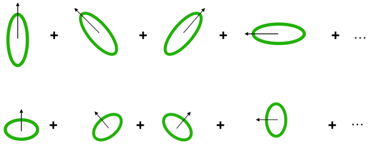
The GCM in a nutshell

The wave function in generator coordinate method (GCM) is constructed as a superposition of mean-field configurations with different intrinsically deformed shapes

$$|\Psi_{JM\pi}\rangle = \sum_{q_i,K} f_K^J(q_i)\hat{P}^J_{MK}\hat{P}^{N,Z,\pi}|\Phi(q_i)\rangle,$$

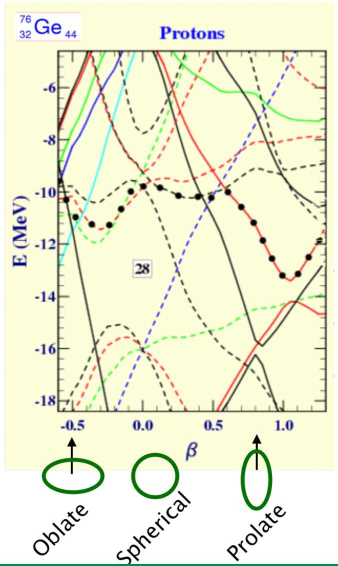
where the angular momentum projection operator

$$\hat{P}^J_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J*}_{MK}(\Omega)\hat{R}(\Omega).$$



$+ \cdots = \hat{R}(\Omega)|\Phi(q_i)\rangle$

$+ \cdots = \hat{R}(\Omega)|\Phi(q_j)\rangle$



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- Nuclear models produce many-body wave functions $\left|\Psi_{I/F}\right\rangle$
- By writing the transition operator in second quantization form, the NME becomes

$$M^{0\nu} = \sum_{pp'nn'} \langle pp'|O^{0\nu}|nn'\rangle \langle \Psi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Psi_I\rangle$$

- The two-body matrix element $\langle pp'|O^{0\nu}|nn'\rangle$ depends on transition operators.
- The two-body transition density $\langle \Psi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Psi_I\rangle$ depends on nuclear models.

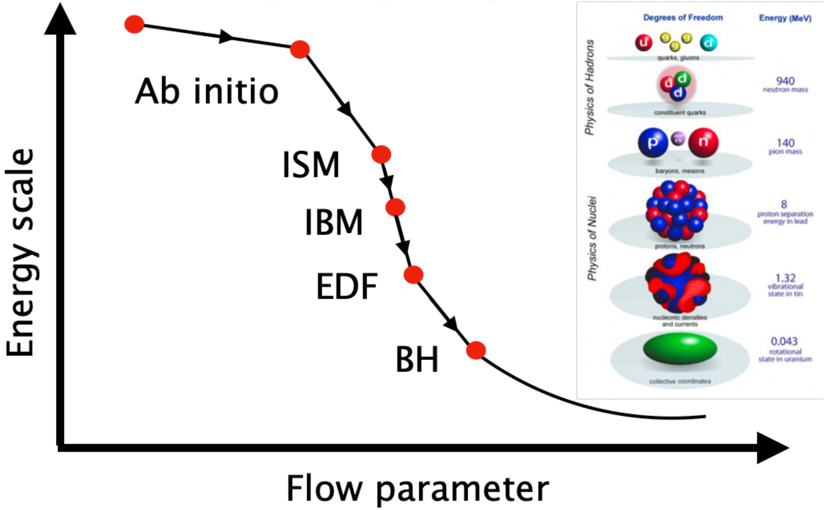
A simple case: HFB approximation for w.f.s. (Generalized Wick Theorem)

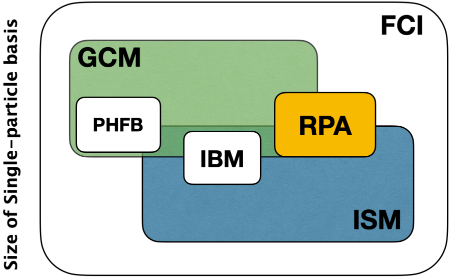
$$\langle \Phi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Phi_I\rangle = \langle \Phi_F|c_p^\dagger c_{p'}^\dagger|\Phi_I\rangle \langle \Phi_F|c_{n'} c_n|\Phi_I\rangle + \cdots .$$

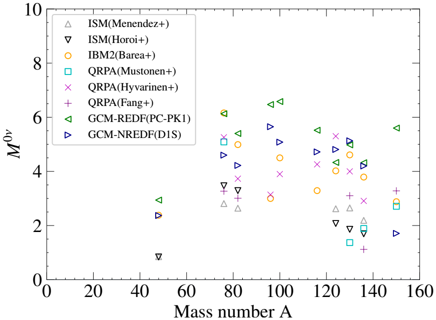
Caveat: both transition operators and wave functions are energy-scale and scheme-dependent!

Modeling atomic nuclei

If nuclear models are connected via a similarity renormalization group, they should give the same observables.





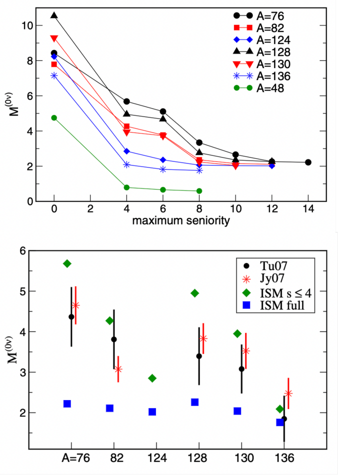


- A general argument: nuclear-structure properties reasonably reproduced (to be checked quantitatively).
- Nuclear models are not RG equivalent! Different schemes (model spaces and interactions): apples v.s. oranges
- ISM predicts small NMEs, while IBM and EDF predict large NMEs. Efforts in resolving the discrepancy: very difficult or even impossible?

Contents

Comparison of ISM with QRPA

- Seniority number: number of particles that are not in pairs.
- Enlarging the model space of ISM (with the nonzero seniority numbers) reduces significantly the NMEs.
- The NME by the spherical QRPA is comparable to that of ISM with $s \leq 4$.
  Caveat: different interactions are employed!

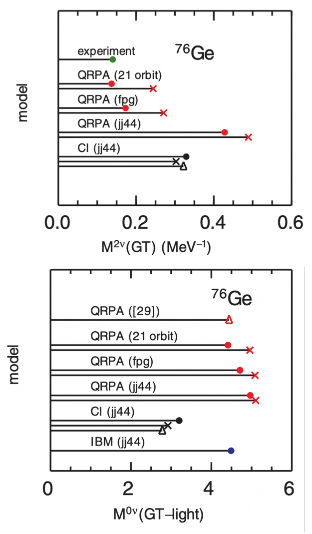


E. Caurier et al., PRL 100, 052503 (2008)

To understand the discrepancy between ISM (IBM) and QRPA, the same operators should be employed.

- With the same interaction *jj*44 (pf5g9), the QRPA and IBM produce systematically larger values for the NMEs than the ISM (CI).

Brown, Fang, Horoi, PRC 92, 041301(R) (2015)



Conclusion

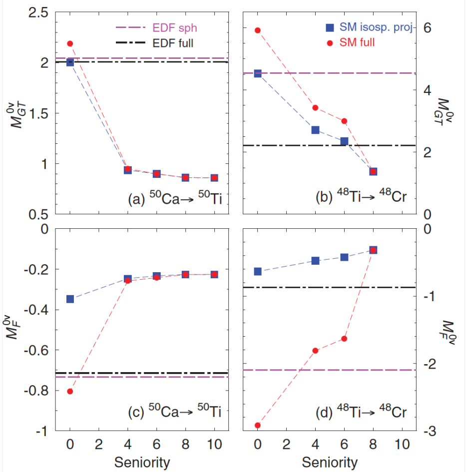
One of the main sources of discrepancy between ISM and QRPA is the different numbers of pairing-broken configurations which decreases the NME significantly.

Comparison of ISM and EDF studies

- The NMEs by the spherical EDF calculation approximately equal to the ISM with the seniority number $s = 0$.
- The full EDF (with deformation and other effects) produces a much smaller NME, much still larger than that by the ISM.
- Caveat: different interactions (Gogny D1S in EDF and KB3G in ISM) are used.

Conclusion

In the EDF studies, it is necessary to include pairing-broken configurations with higher-seniority numbers, which together with deformation effects quench the NME.



J. Menéndez et al., PRC90, 024311 (2014)

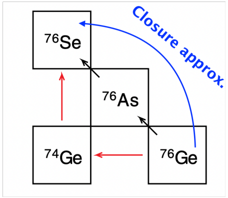
Uncertainties from different treatments of pairing correlations

- Re-writing the transition operator by inserting intermediate states $|N\rangle$

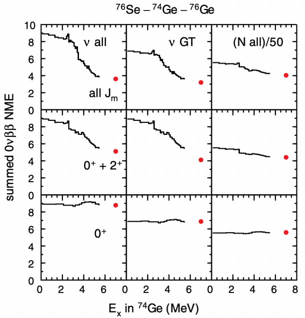
$$M^{0\nu} = \sum_{pp'nn'} \langle pp'|O^{0\nu}|nn'\rangle \sum_N \langle \Psi_F|c_p^\dagger c_{p'}^\dagger|N\rangle \langle N|c_{n'}c_n|\Psi_I\rangle$$

where the intermediate state is chosen differently.

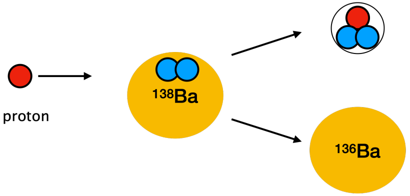
- The NME is decomposed into sums of products over the intermediate nucleus with two less nucleons.
- Pairing interaction enhances the two-nucleon transfer cross sections and thus the NME. G. Potel et al., PRC87, 054321 (2013)
- The QRPA and IBM treat the pairing correlation differently.



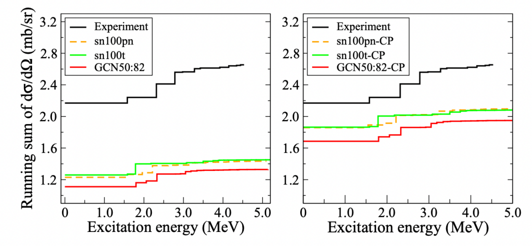
Two-nucleon transfer



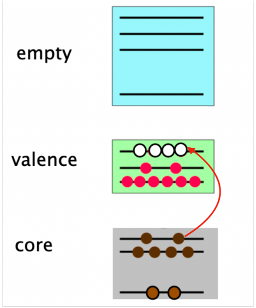
Model-space effect on the NME in ISM



ISM (w/o core polarization) ISM (w/ core polarization)



B.M. Rebeiro et al., Phys.Lett.B 809 (2020) 135702



Core polarization

- Core polarization effect is necessary for the ISM to reproduce the $^{138}\mathrm{Ba}(p,t)^{136}\mathrm{Ba}$ reaction cross section.
- With this effect, the $J=0$ component of $M^{0\nu}_{\mathrm{GT}}$ increases from 5.67 to 8.96.

Model-space effect on the NME in ISM

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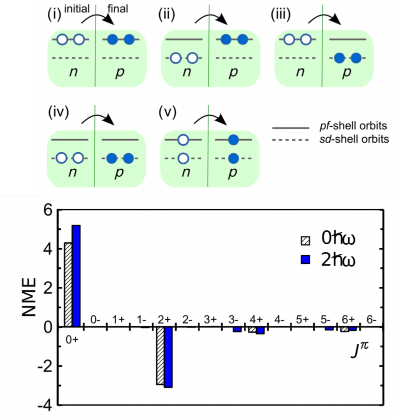
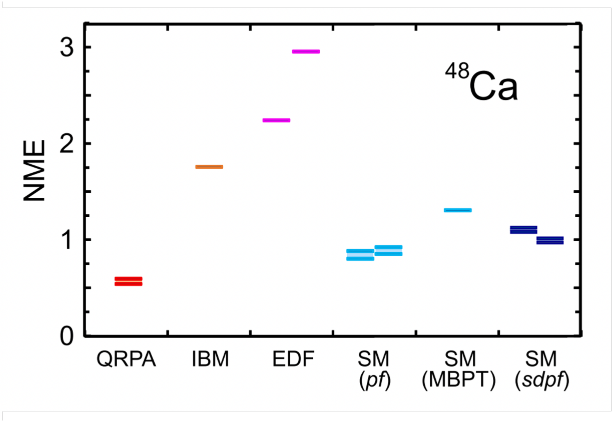




- The NME increases by about 30% ($M^{0\nu} = 1.1$), which is due to cross-shell $sd - pf$ pairing correlations. Y. Iwata et al., Phys Rev Lett 116, 112502 (2016)

Conclusion: enlarge the model space of ISM generally increases the NME.

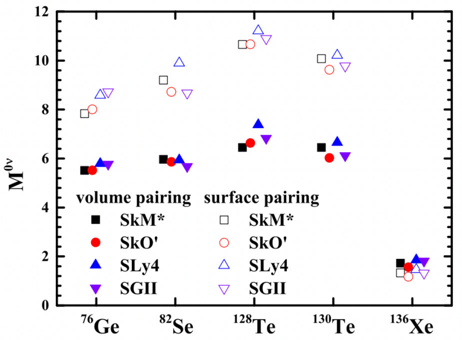
The isovector pairing interaction,

$$V^{pp}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \left[t_0' + \frac{t_3'}{6}\rho\left(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}\right)\right]\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

Parameters t_i' are fitted to pairing gaps determined from odd-even mass difference and

- Volume type: $t_3' = 0$
- Surface type: $t_3' \neq 0$, reducing pairing in nuclear interior.

Two different choices of isovector pairing interactions leads to quite different NMEs.

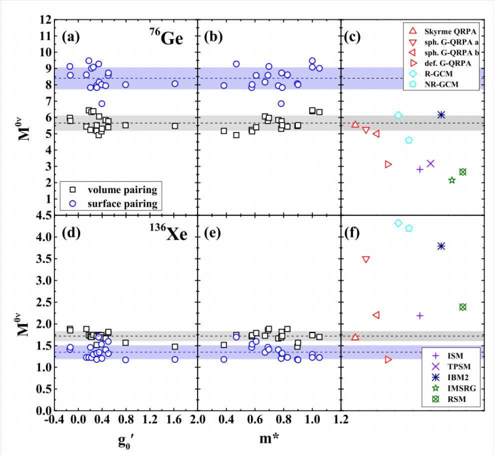


W. L. Lv, Y.-F. Niu, D.-L. Fang, JMY, C.-L. Bai, J. Meng, arXiv:2302.04423v1 [nucl-th]

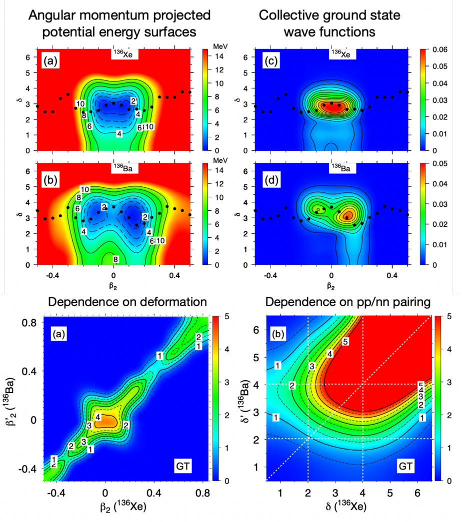
18 Skyrme EDFs characterized with

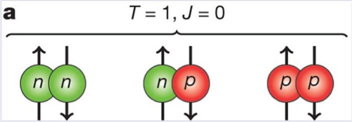
- different nucleon effective mass m^* (single-particle properties),
- different Landau parameter g_0' (spin-isospin excitation properties)

and two types of isovector pairing interactions are employed in the spherical QRPA calculation.



Lv, Niu, Fang, JMY, Bai, Meng, arXiv:2302.04423v1 [nucl-th]

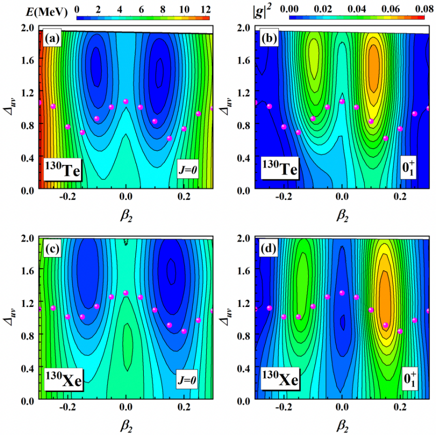


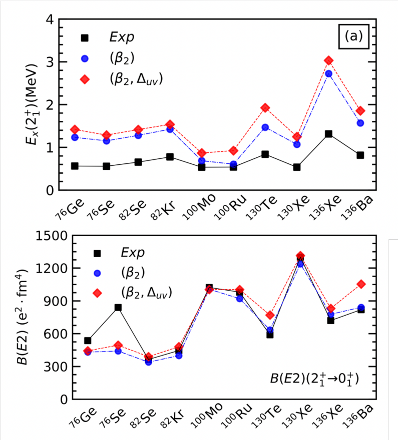


Isovector pairing

- The inclusion of isovector pairing fluctuation increases the NMEs of candidate nuclei by 10%–40%

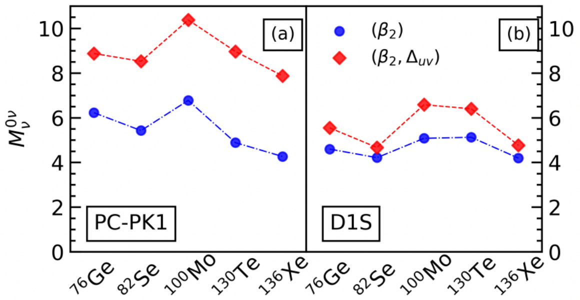
N. López-Vaquero et al., PRL111, 142501 (2013)





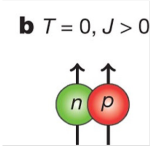
- The inclusion of isovector pairing fluctuation enhances the pairing correlation, increasing the excitation energies of low-lying states.

C.R. Ding, X. Zhang, JMY, P. Ring, J. Meng, arXiv:2305.00742 (2023)



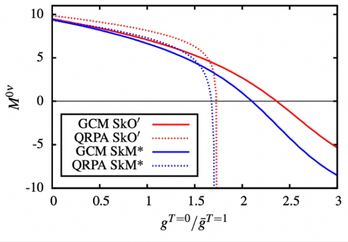
C.R. Ding, X. Zhang, JMY, P. Ring, J. Meng, arXiv:2305.00742 (2023)

- Confirm the conclusion that the inclusion of isovector pairing fluctuation increases the NME.
- Next: readjustment of parameters of the EDFs to fit energy spectra (like pairing strengths) and inclusion of isosclar pairing effect.

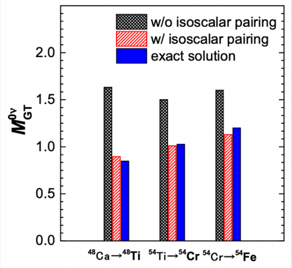


Isoscalar pairing

N. Hinohara & J. Engel, PRC90, 031301(R) (2014)



C. F. Jiao, J. Engel, and J. D. Holt, PRC96, 054310 (2017)



- The NME decreases with the increase of the strength of isoscalar pairing.
- The isoscalar pairing fluctuation decreases the NMEs of candidate nuclei.

Conclusion: The strength parameters of pairing correlations between nucleons in QRPA and MR-EDF need to be further constrained by other data than odd-even mass difference.

Contents

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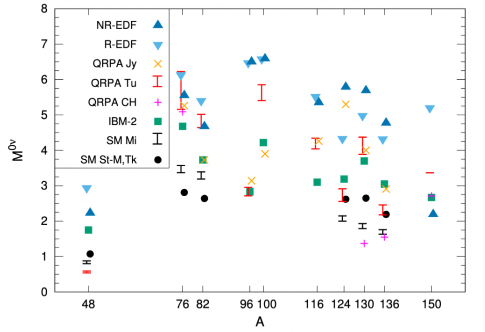
- Statistical uncertainty quantification remains to be done for most calculations (QRPA: g_A, g_{pp})
- Average value of the NMEs by different nuclear models:

$$\bar{M}^{0\nu} = \sum_{k=1}^{K} M_k^{0\nu} \omega_k,$$

where ω_k is the weight of the k-th model.

– No bias: same ω_k for each model
– Bayesian Model Averaging (BMA)



Engel, Menéndez, Rep. Prog. Phys. 80, 046301 (2017)

Bayesian Model Averaging for the NMEs of $0\nu\beta\beta$

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- The posterior distribution of $M^{0\nu}$ for a given data y,

$$p(M^{0\nu}|y) = \sum_{k=1}^{p} p(M^{0\nu}|M_k, y)\omega_k(y),$$

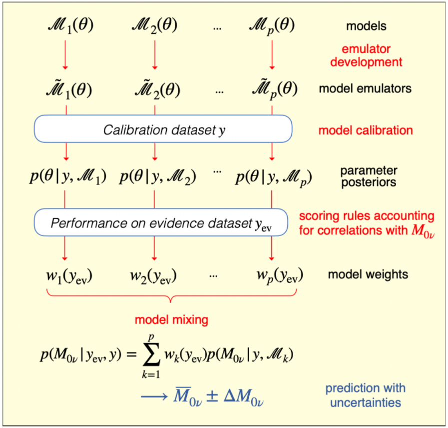
- The weight of each model M_k

$$\omega_k(y) = \frac{p(y|M_k)\pi(M_k)}{\sum_{k=1}^{K} p(y|M_k)\pi(M_k)}$$

- The marginal density of the data

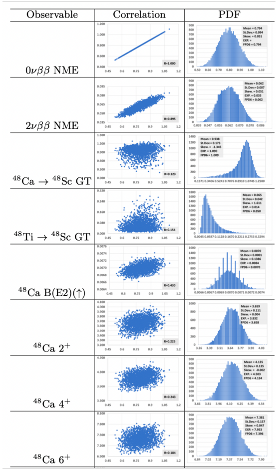
$$p(y|M_k) = \int d\theta p(y|\theta, M_k)\pi(\theta|M_k)d\theta$$

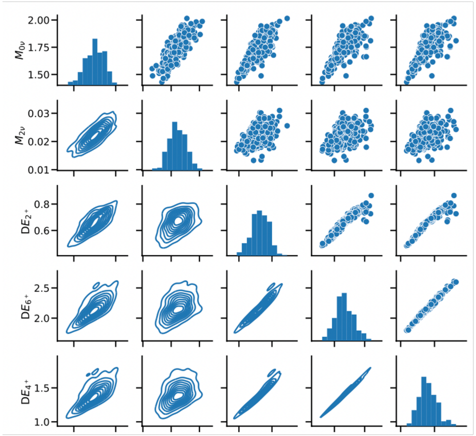
where θ is the parameter of the model M_k.



- Repeat for all nuclei of 0νββ interest
- Repeat with calibration dataset refined for 0νββ

V. Cirigliano et al., J. Phys. G: Nucl. Part. Phys. 49, 120502



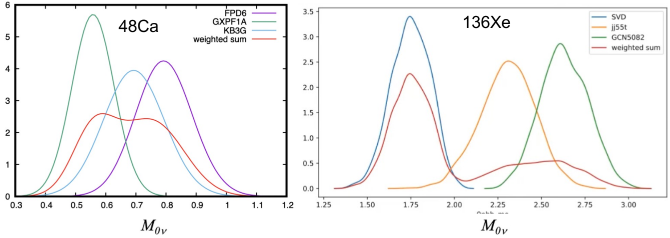


Starting from three different shell-model Hamiltonians, 1k samples of SM Hamiltonians were generated around the optimal values (10%) for each case. M. M. Horoi, A. Neacsu, S. Stoica, PRC106, 054302 (2022); Phys. Rev. C 107, 045501 (2023)

Bayesian Model Averaging of the probability distribution functions (PDF)

$$P(x = M^{0\nu}) = \sum_{k=1}^{3} P_k(x)\omega_k,$$

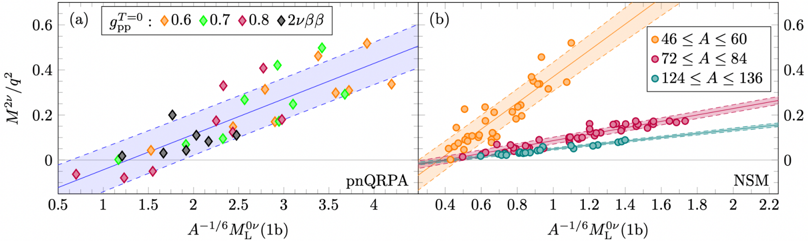
- The weighted PDF is predominated by the model that reproduces the $M^{2\nu}$, which unfortunately depends on the employed quenching factor q.

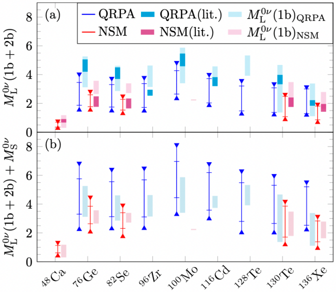


M. Horoi, A. Neacsu, S. Stoica, arXiv:2203.10577 [nucl-th]

With the quenching factor $q = 0.7$, the SVD model becomes predominant and the NME $M^{0\nu} \in [1.55, 2.65]$ for ^{136}Xe with 90%C.L.

Correlation relations with $2\nu\beta\beta$: pnQRPA and SM





- Good correlations between $M^{0\nu}$ and $M^{2\nu}$ (nucleus-dependent!)
- Using the correlation relation and the data of $M^{2\nu}$ to determine $M^{0\nu}$
- Uncertainty in the NMEs is still large.

L. Jokiniemi et al., PRC 107 044305 (2023)

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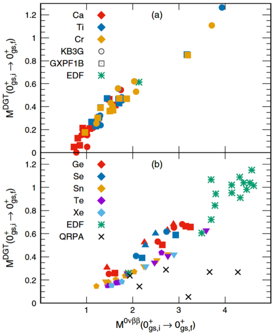
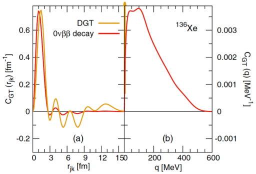
$$M^{\mathrm{DGT}} = \langle 0_f^+ | \sum_{1,2} [\sigma_1 \otimes \sigma_2]^0 \tau^{(1)+} \tau^{(2)+} | 0_i^+ \rangle .$$

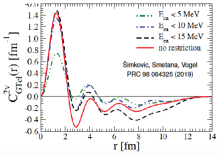
$$M^{0\nu\beta\beta} = \sum_{\alpha=F,GT,T} \langle 0_f^+ | \sum_{1,2} h_{\alpha,K}(r_{12})\, C_\alpha^K \cdot S_\alpha^K \tau^{(1)+} \tau^{(2)+} | 0_i^+ \rangle$$



N. Shimizu et al., PRL120, 142502 (2018)

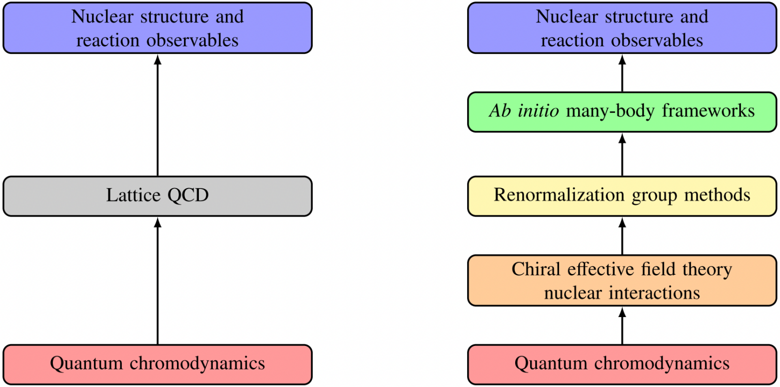


From QRPA calculations



- Linear correlation between DGT and $M^{0\nu\beta\beta}$ found in all phenom. models except for QRPA
- Both transitions are dominated by short-range contribution
- The above conclusion is model-dependent and nucleus-dependent!

Contents



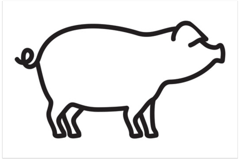
K. Hebeler, Phys. Rep. 890, 1 (2021)

Basic ideas of constructing an EFT

S. Weinberg, Physica 96A, 327 (1979); S. Weinberg, Phys. Lett. B 251, 288 (1990); Nucl. Phys. B363, 3 (1991)

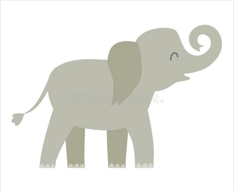
- Symmetry consideration (chiral symmetry of QCD)
- Identification of important energy scales (active and break down), and the effective degrees of freedom (pions and nucleons)
- Writing down a most general Lagrangian (order by order convergence)

EFT for an elephant:



LO

NLO



N2LO

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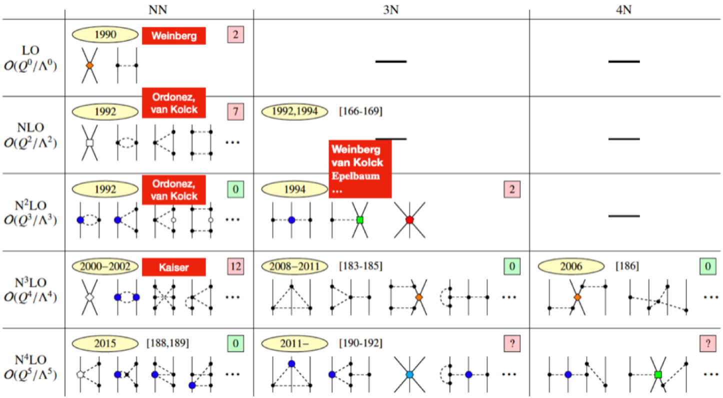
Strategy for developing ab initio methods for $0\nu\beta\beta$ decay

- Operator forms: (Chiral) effective field theory (EFT) to specify the forms of nuclear forces and transition operators at different orders.
- Parametrization: Scattering data or Lattice QCD calculations to determine the low-energy constants (LECs) of the operators.
- Many-body solvers: A systematically improvable nuclear-structure theory to solve the nuclear many-body problem and compute observables.

- Non-relativistic chiral 2N+3N interactions (Weinberg power counting and others)

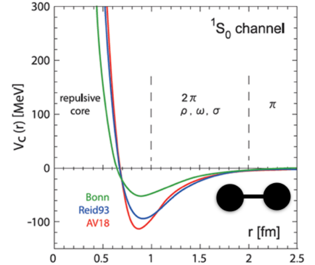


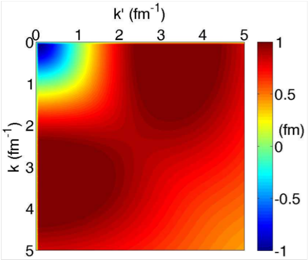
K. Hebeler, Phys. Rep. 890, 1 (2020)

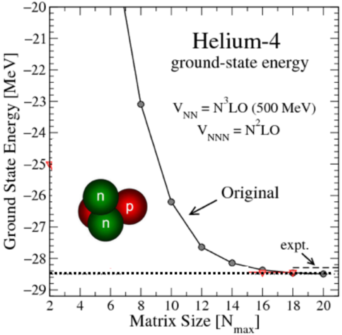
- Relativistic chiral 2N interaction (up to N^{2}LO)

J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, P. Ring, PRL128, 142002 (2022)

Features of realistic nuclear forces

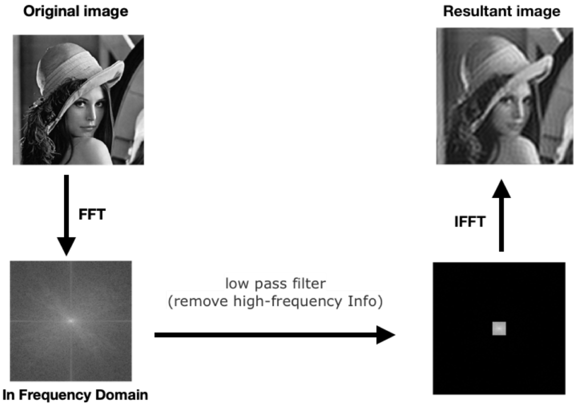




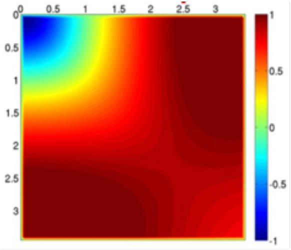


- Repulsive core & strong tensor force: low and high k modes strongly coupled.
- non-perturbative, poorly convergence in basis expansion methods.

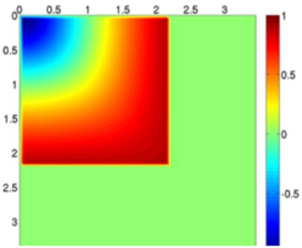
Low-pass filter

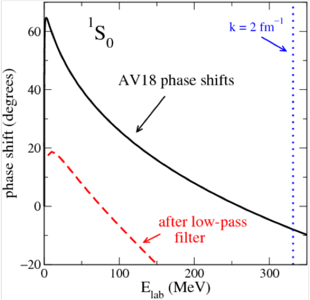


- Low-pass filter: passes signals with a frequency lower than a selected cutoff frequency and attenuates signals with frequencies higher than the cutoff frequency.
- Long-wavelength (low-E) information is preserved.



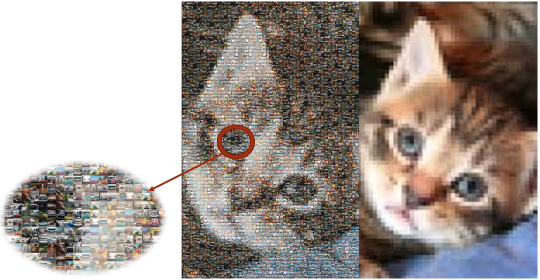
low pass filter
(remove high-frequency Info)





- Cut at $\Lambda = 2.2\ \mathrm{fm}^{-1}$
- Fails to reproduce the phase shift
- because low and high k are coupled

R. J. Furnstahl, K. Hebeler, RPP(2013)



- Renormalization group (RG) is an iterative coarse-graining procedure designed to tackle difficult physics problems involving many length scales.
- To extract relevant features of a physical system for describing phenomena at large length scales by integrating out short distance degrees of freedom.
- The effects of high-E physics can be absorbed into LECs with the RG.

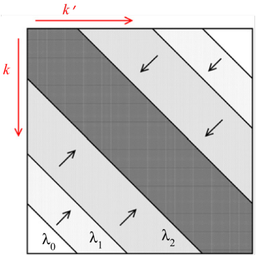
K. G. Wilson (1983); S. D. Glazek & K. G. Wilson (1993); F. Wegner (1994)

- Apply unitary transformations to Hamiltonian

$$H_s = U_s H U_s^\dagger \equiv T_{\mathrm{rel}} + V_s$$

from which one finds the flow equation

$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\mathrm{rel}}, H_s]$$



The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1}. S. K. Bogner etal. (2007)

Evolution of the potential

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

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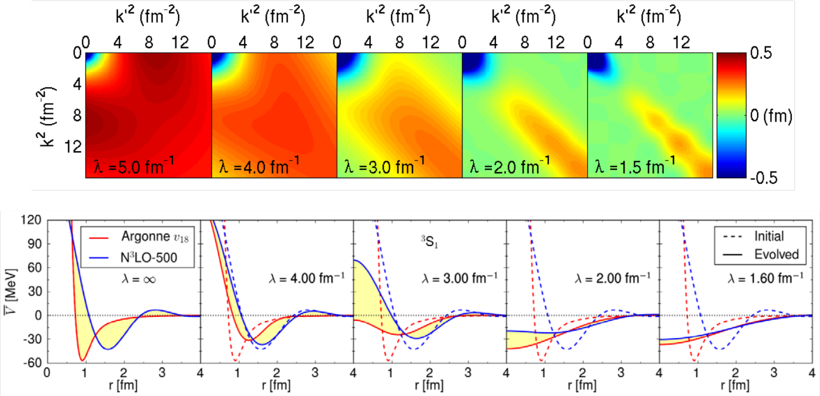


Local projection of AV18 and N^{3}LO(500 MeV) potentials $V(r)$.

- The hard core "disappears" in the softened interactions

S. K. Bogner et al. (2010); Wendt et al. (2012)

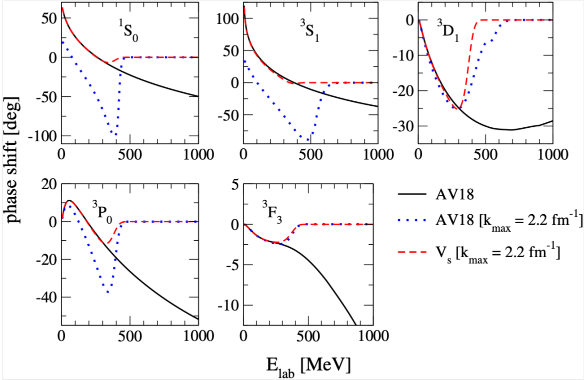
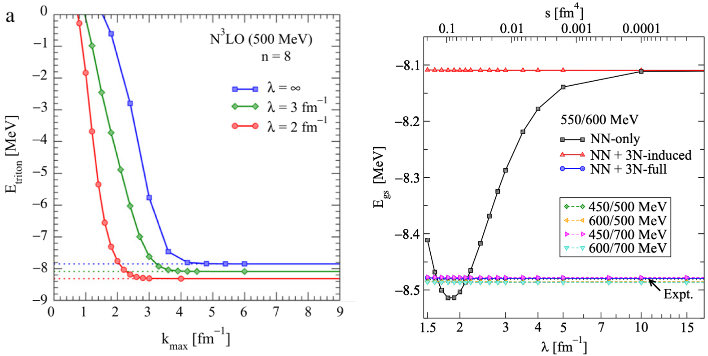


Figure: Blue dotted lines for the low-pass filter of AV18; red dashed lines for the low-pass filter of the SRG-softened AV18; black solid for the full AV18, on top of that for the SRG-softened AV18.

The *NN* phase shifts are preserved in the SRG. S. K. Bogner et al. (2007); D. Jurgenson et al. (2008)



- Convergence becomes faster as the decreases of the λ (resolution/degree of decoupling).
- importance of (induced) three-body forces, NO2B approximation

Bogner et al. (2008); Furnstahl et al. (2013)

Two variants of IMSRG: MR and VS

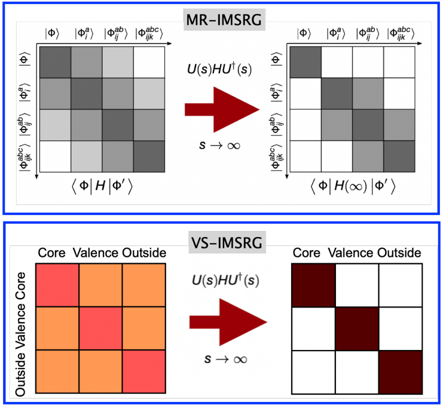
- Unitary transformations

$$H(s) = U(s)H_0U^\dagger(s)$$

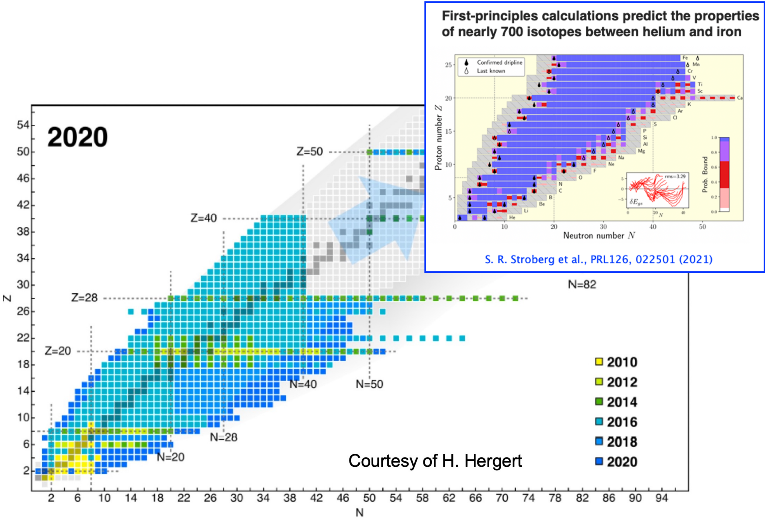
Flow equation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

- Generator $\eta(s)$: chosen either to decouple a given reference state from its excitations or to decouple the valence space from the excluded spaces.
- Not necessary to construct the whole H matrix, computation complexity scales polynomially with nuclear size.



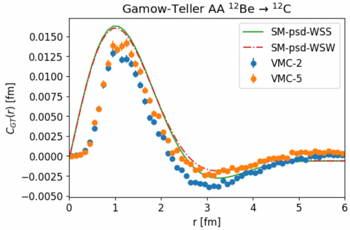
H. Hergert et al., Phys. Rep. 621, 165 (2016); S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)


First-principles calculations predict the properties of nearly 700 isotopes between helium and iron
Confirmed dripline
Last known
Proton number Z
Neutron number N
Prob. Bound
S. R. Stroberg et al., PRL126, 022501 (2021)
2020
Z=50
Z=40
Z=28
Z=20
N=20
N=28
N=40
N=50
N=82
Z
N
2010
2012
2014
2016
2018
2020
Courtesy of H. Hergert

For light nuclei: benchmark studies

- Quantum Monte-Carlo (QMC) S. Pastore et al., PRC97, 014606 (2018)
- No-core shell model (NCSM) R. A. M. Basili et al., PRC102, 014302 (2020); S. Novario et al., PRL126, 182502 (2021); JMY et al., PRC103, 014315 (2021)

Comparison of ISM and QMC calculations



Wang et al., PLB798, 134974 (2019); Weiss et al., PRC106 065501 (2022)

- Good agreement at long distances
- Missing short-range correlations in shell model which may quench the NME $\simeq 30\%$.

Ab initio methods for the lightest candidate ^{48}Ca

- Multi-reference in-medium generator coordinate method (IM-GCM)

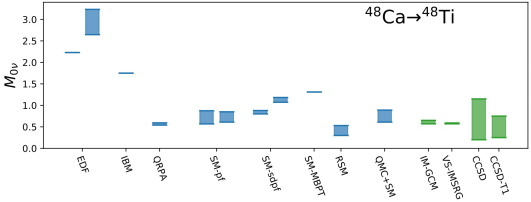
  JMY et al., PRL124, 232501 (2020)

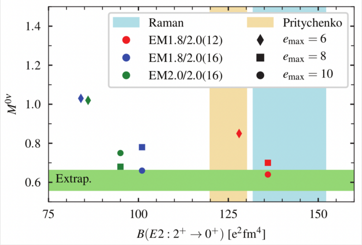
- IMSRG+ISM (VS-IMSRG)

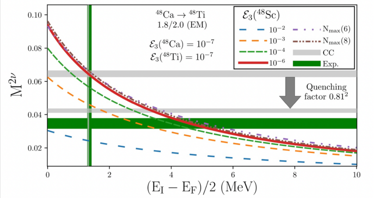
  A. Belley et al., PRL126, 042502 (2021)

- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

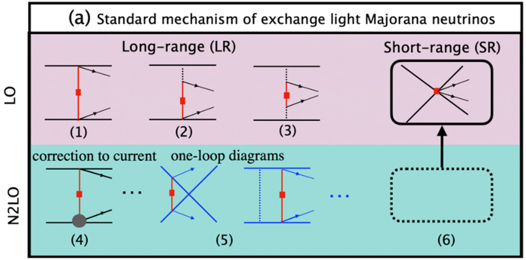
  S. Novario et al., PRL126, 182502 (2021)

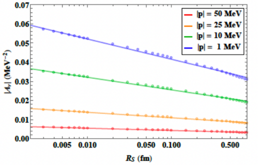




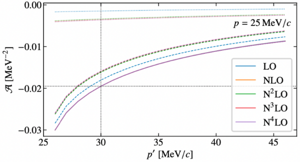


Promotion of the contact operator to LO



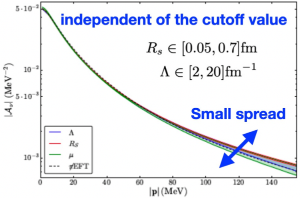


Lines fitted to $\mathcal{A}_\nu = a + b \ln R_S$



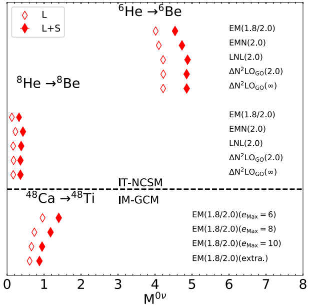
- The $nn \to ppe^- e^-$ transition amplitude $\mathcal{A}_\nu$ by the long-range transition operator at the LO is regulator-dependent.
- With the following contact term at LO, the $\mathcal{A}_\nu$ becomes regulator independent

$$V_{\nu,S} = -2 g_\nu^{NN} \tau^{(1)+} \tau^{(2)+}$$

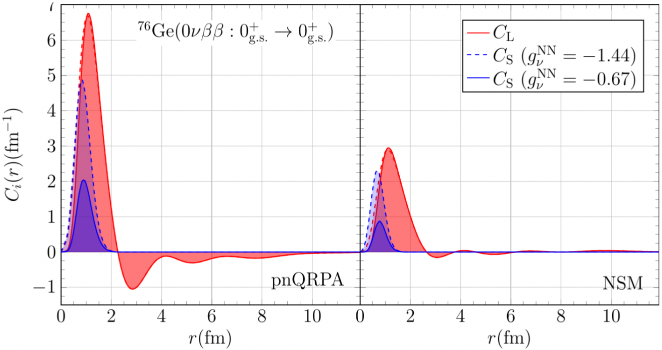


- The contact term (S) enhances the NME for ^{48}Ca by 43(7)%, the uncertainty is propagated only from the synthetic datum.
- The half-life $T_{1/2}^{0\nu\beta\beta}$ is only half of the previously expected value.
- More accurate predictions require the LEC (g_ν^{NN}) from LQCD.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)



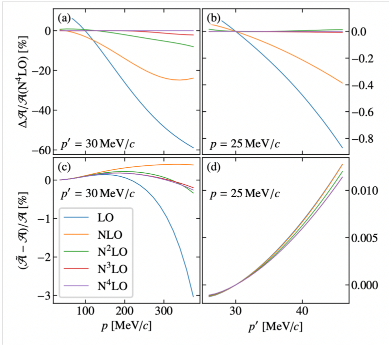
The contact transition operator for $0\nu\beta\beta$ decay



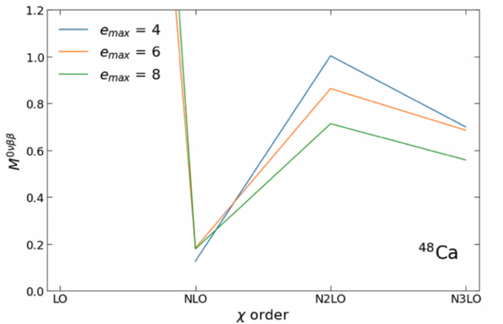
Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

- Caveat: inconsistent consideration of the short-range term, but the conclusion is consistent with ab initio calculations.
- ISM: increase NME by 20% – 50%
- QRPA: increase NME by 30% – 70%

$2n \to 2p + 2e^-$



$^{48}\mathrm{Ca} \to ^{48}\mathrm{Ti} + 2e^-$

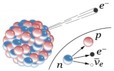


- The $\mathcal{A}_\nu(2n \to 2p + 2e^-)$ converges quickly w.r.t. the chiral expansion order of nuclear interactions. Negligible contribution beyond NLO, particular true for low momentum cases. R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)
- Convergence is slower in candidate nuclei ^{48}Ca.

Contents

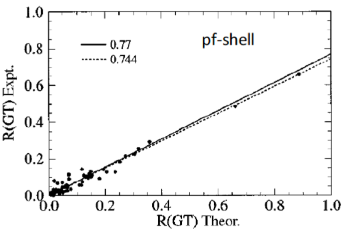
- The half-life of single-beta decay

$$t_{1/2} = \frac{\kappa}{f_0(B_F + B_{GT})},$$

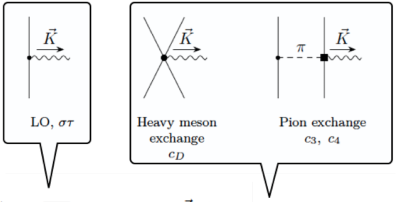


$$B_F = \frac{g_V^2}{2J_i+1}|M_F|^2, \quad B_{GT} = \frac{g_A^2}{2J_i+1}|M_{GT}|^2$$

G. Martinez-Pinedo et al, PRC **53**, R2602 (1996)



- The charge-changing axial-vector current



$$\vec{J}^A(\vec{K}) = \sum_j \imath g_A \boldsymbol{\sigma}_j \boldsymbol{\tau}_j^{\pm} e^{\imath \vec{K}\cdot\vec{r}_j}.$$

2B currents

Park, T.-S. et al. PRC67, 055206 (2003);
M. Hoferichter et al., PRD102, 074018 (2020)

$$g_A^{\text{eff}}(q,0,\rho) \equiv g_A \left\{ 1 - \frac{\rho}{f_\pi^2}\left[-\frac{c_D}{4}\frac{1}{g_A \Lambda_\chi} + \frac{2c_3}{3}\frac{q^2}{4m_\pi^2+q^2} + \frac{I(\rho,0)}{3}\left(2c_4 - c_3 + \frac{1}{2m_p}\right)\right]\right\}$$

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The quenching factor:

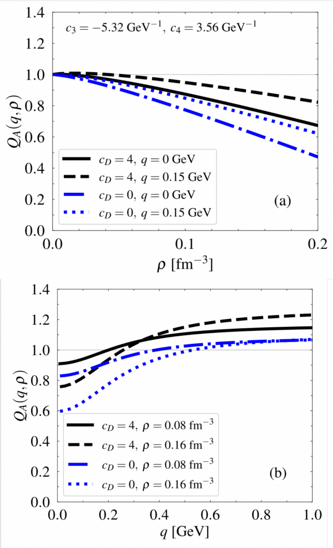
$$\mathcal{Q}_A(q,\rho) \equiv g_A^{\mathrm{eff}}(q,\rho)/g_A = 1 + A[q]\rho + B\rho^{1/3} + C$$

where the coefficients A, B and C are defined as

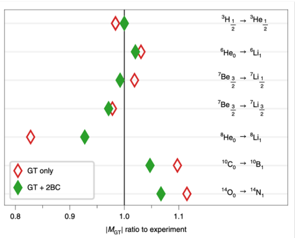
$$A[q] = \frac{c_D}{4f_\pi^2}\frac{1}{g_A\Lambda_\chi} - \frac{1}{3f_\pi^2}\left[\left(2c_4 - c_3 + \frac{1}{2m_p}\right) + 2c_3\frac{q^2}{4m_\pi^2+q^2}\right]$$

$$B = \frac{m_\pi^2}{f_\pi^2}\left(\frac{2}{3\pi^2}\right)^{2/3}\left(2c_4 - c_3 + \frac{1}{2m_p}\right),$$

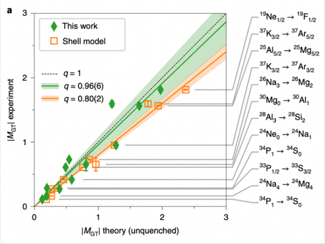
$$C = -\frac{2m_\pi^3}{3\pi^2 f_\pi^2}\left(2c_4 - c_3 + \frac{1}{2m_p}\right)\tan^{-1}\left(\frac{k_F}{m_\pi}\right).$$



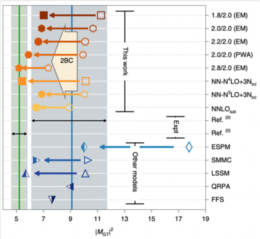
Ab initio methods for nuclear single-β decay



NCSM



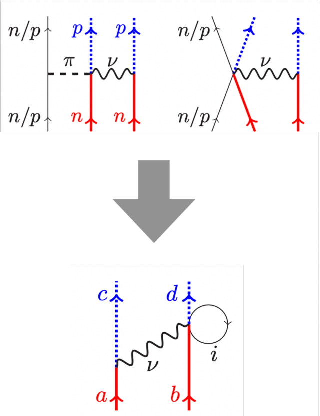
VS–IMSRG

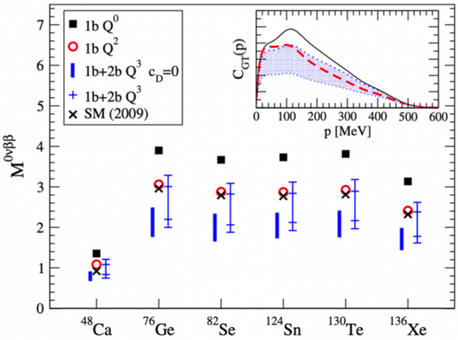


CC

- **VS-IMSRG**: a unitary transformation is constructed to decouple a valence-space Hamiltonian H_{vs}. The eigenstates are obtained by a subsequent diagonalization of the H_{vs}.
- A proper treatment of strong nuclear correlations and the consistency between 2BCs and three-nucleon forces explain the g_A-quenching puzzle in conventional valence-space shell-model calculations.

P. Gysbers et al., Nature Physics 15, 428 (2019)

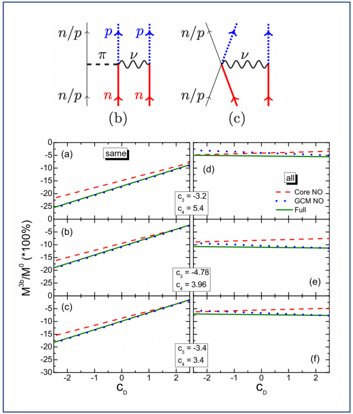


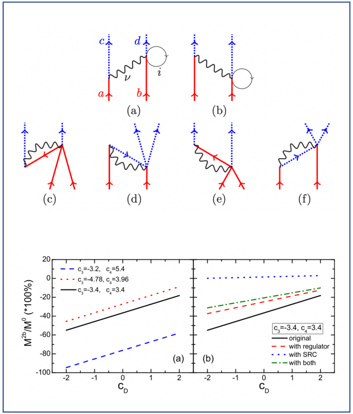


J. Menendez et al., PRL107, 062501 (2011)

- The 2B current changes NMEs ranging from -35% to 10%.

Two-body current effect





- The 3B operators quench matrix elements by about 10%,
- The 2B operators can produce somewhat larger quenching.

L.J. Wang, J. Engel, JMY, PRC 98, 031301(R) (2018)

Contents

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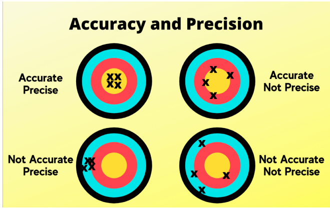
- $0\nu\beta\beta$ decay: lepton-number-violation process, a complementary way to determine the absolute mass scale of neutrinos.
- Next-generation experiments: tonne-scale detectors with a half-life sensitivity of up to 10^{28} years.
- NME: large model uncertainty, impacting extracted neutrino mass, attracting a lot of efforts from nuclear community.
- Ab initio studies of NMEs: remarkable progress, disclosing non-trivial contributions from high-energy light neutrinos. The NMEs for heavier candidate nuclei (^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe) are within reach. Stay tuned!

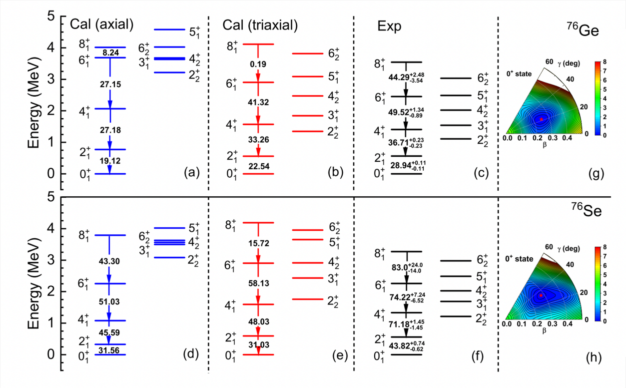


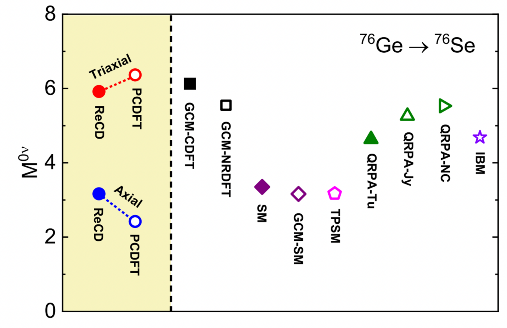
Uncertainty Quant. in ab initio studies

- Accuracy and precision of (ab initio) nuclear many-body calculations need to be improved.
- More constraints to shrink the uncertainty.
- Disentangle contributions from other mechanisms.

- How does the pairing correlation between nucleons affect the (neutrinoless) double-beta decay? What would happen if there was no pairing correlation between nucleons in atomic nuclei?
- About 35 isotopes can decay via double-beta decay. Why only a few of them with large $Q_{\beta\beta}$ (normally than 2 MeV) are selected as candidates for measurements?
- What can we learn if the $0\beta\beta$-decay is not observed in the next-generation ton-scale experiments?

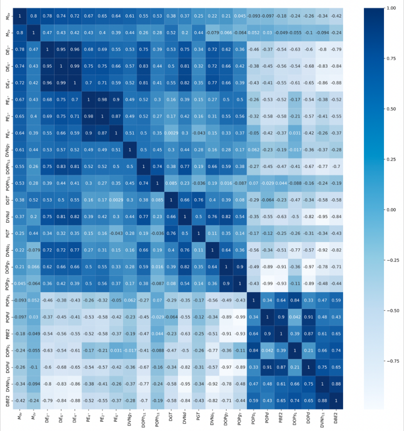
Relativistic Configuration-interaction EDF for ^{76}Ge





Y. K. Wang, P. W. Zhao, and J. Meng, arXiv:2304.12009 [nucl-th]

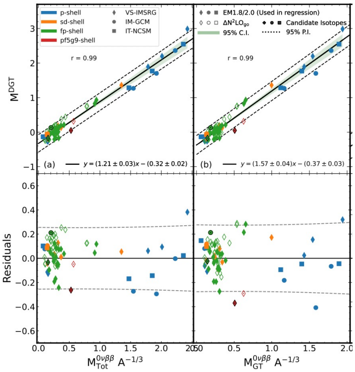
Uncertainty quantification within ISM

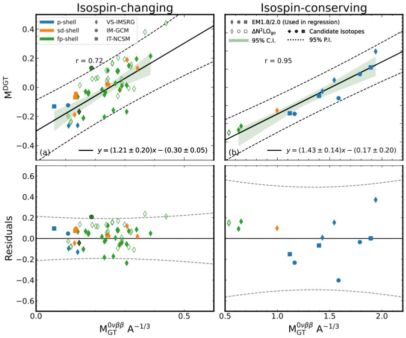


M. M. Horoi, A. Neacsu, S. Stoica, arXiv:2302.03664 (2023)

- $M^{0\nu}$ is strongly correlated with $M^{2\nu}$, and with the excitation energies of $2_1^+, 4_1^+, 6_1^+$ states in both initial and final nuclei.

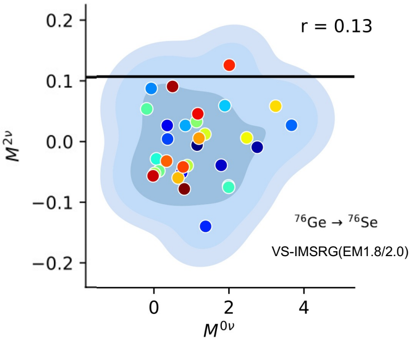
Correlation relation between NLDBD and DGT





- Weak correlation between $M^{0\nu}$ and M^{DGT}.
- Other observables: $2\nu\beta\beta$ decay, excitation energies?

JMY et al., PRC106, 014315 (2022)


r = 0.13
0.2
0.1
0.0
−0.1
−0.2
$M^{2\nu}$
0
2
4
$M^{0\nu}$
^{76}Ge → ^{76}Se
VS-IMSRG(EM1.8/2.0)