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ANNRI Data Analysis and PV Measurements at J-PARC

Clayton Auton NOPTREX China Meeting 2023

Enhancement of T-odd effects

$$\Delta \sigma_{PT} = \frac{4\pi}{k} \text{Im}(f_{\uparrow} - f_{\downarrow})$$
$$\Delta \sigma_{P} = \frac{4\pi}{k} \text{Im}(f_{-} - f_{+})$$

- Measure $\kappa(J)$ using (n, γ) resonance spectroscopy
- Almost no data on $\kappa(J)$ before NOPTREX
- Ongoing measurements using ANNRI Ge detector array at J-PARC





$$\langle \phi_s | V_P + W_{PT} | \phi_p \rangle = v + iw$$

$\kappa(J)$ Spin Factor



$$P: |\ell sI\rangle \to (-1)^{\ell} |\ell sI\rangle$$
$$\ell = 0,1$$
$$\bigcup$$

P-odd \Rightarrow s-wave and p-wave interference

 $T : |\ell sI\rangle \to (-1)^{i\pi S} K |\ell sI\rangle$ $S = I \pm 1/2$ \bigcup

T-odd ⇒ channel spin S interference



V. Gudkov and H. Shimizu, Phys. Rev. C 97, 065502 (2018)





$$\Delta \sigma_P = \frac{4\pi}{k} \operatorname{Im}(f_- - f_+) \qquad \Delta \sigma_{PT} = \frac{4\pi}{k} \operatorname{Im}(f_\uparrow - f_\downarrow)$$
ratio of differences in total neutron cross section
$$\frac{\Delta \sigma_{PT}}{\Delta \sigma_P} = \kappa (J) \frac{w}{v} \qquad \text{with} \\ \langle \phi_s | V_P + W_{PT} | \phi_p \rangle = v + iw$$

$$\int_{\kappa} \kappa (J = I + 1/2) = \left[\frac{\sqrt{I}}{2(I+1)} \right] \left(-2\sqrt{I} + \sqrt{2I + 3} \frac{y}{x} \right) \\ \kappa (J = I - 1/2) = \left[\frac{1}{2\sqrt{I+1}} \right] \left(2\sqrt{I+1} + \sqrt{2I - 1} \frac{y}{x} \right)$$

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$$\begin{cases} \kappa(J = I + 1/2) = \left[\frac{\sqrt{I}}{2(I+1)}\right] \left(-2\sqrt{I} + \sqrt{2I+3}\frac{y}{x}\right) \\ \kappa(J = I - 1/2) = \left[\frac{1}{2\sqrt{I+1}}\right] \left(2\sqrt{I+1} + \sqrt{2I-1}\frac{y}{x}\right) \end{cases}$$

$$J = I + 1/2$$
 corresponds
to the p-wave resonance

$$x \equiv \sqrt{\frac{\Gamma_{p,j=1/2}^n}{\Gamma_p^n}}$$
$$y \equiv \sqrt{\frac{\Gamma_{p,j=3/2}^n}{\Gamma_p^n}}$$

can be measured in (n, γ)

can reparametrize in terms of j=1/2, 3/2 mixing angle

$$x^2 + y^2 = 1 \implies \frac{x = \cos \phi}{y = \sin \phi}$$

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V. Gudkov and H. Shimizu, Phys. Rev. C 97, 065502 (2018)

ANNRI Data

ANNRI Detector at JPARC

- High-Purity Germanium Detectors (HPGE)
 - 14 vertical detectors (Type-A)
 - 8 horizontal detectors (Type-B)
- Bismuth Germinate (BGO)
 - 20 crystals surrounding HPGE detectors
 - Summed into 4 channels
- Increased JPARC power to 830 kW



Raw HPGE Data

- CAEN v1724 100-MHz 14-bit ADC
- Gamma ray events are converted from pulse shape to
 - Pulse height
 - Time-of-flight
- Overlapping event are flagged as pile-up events
- Coincidence, timestamp, and trigger number also recorded

1 Column 2 3 Flags 6 Dulocucient	Туре
5 Timestamp 6 Coin 7 detector 8 nTrigger 9 tof	UInt_t UShort_t ULong64_t UInt_t Int_t ULong64_t ULong64_t



Raw Data Example

Rows are single detector events

	•							
+	Row	Coin	Flags	PulseHeight	Timestamp	detector	nTrigger	+ tof
+	0	0	0	481	3941640000	20	0	 394164
l	1	1	0	435	30796670000	21	0	3079667
ļ	2	1	0	435	30796670000	22	0	3079667
ļ	3	1	0	435	30796670000	23	0	3079667
I	4	0	0	1343	4094890000	19	 0	 409489



Analysis Using RDataFrame

- Similar to popular frameworks like Pandas/Dask
- Can work with data sets larger than memory
- Allows multi-threading
- Lazy actions reduce number of event loops in computation graph
- JIT compiled C++ in PyROOT





RDataFrame

Analysis Using RDataFrame

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RDataFrame Example



RDataFrame Example



RDataFrame Example



Why RDataFrame?

- Enables interactive and idiomatic largescale distributed data analysis
- Python API (PyROOT) with C++ event loop
- Short learning-curve:
 - No TTreeReader + explicit event loop
 - No TTRee::Draw()
- Native multi-threading
- Scalable with cluster backends such as Spark, Dask, ...



κ(J) Measurement & Analysis Method

Angular distribution of (n, y) reactions

V. V. Flambaum et al, Nuclear Physics A, vol. 435, no. 2, pp. 352 - 380, 1985.

 $\frac{\mathrm{d}\sigma_{\mathbf{n}\gamma_{f}}}{\mathrm{d}\Omega_{\gamma}} = \frac{1}{2} \left(a_{0} + \frac{a_{1}\hat{k}_{\mathbf{n}} \cdot \hat{k}_{\gamma}}{a_{2}\sigma_{\mathbf{n}}} \cdot (\hat{k}_{\mathbf{n}} \times \hat{k}_{\gamma}) + \frac{a_{3}}{a_{3}} \left((\hat{k}_{\mathbf{n}} \cdot \hat{k}_{\gamma})^{2} - \frac{1}{3} \right) \right)$ + $a_4(\hat{k}_p \cdot k_\gamma)(\sigma_n \cdot (\hat{k}_n \times \hat{k}_\gamma)) + a_5(\sigma_\gamma \cdot \hat{k}_\gamma)(\sigma_n \cdot \hat{k}_\gamma)$ $a_{6}(\boldsymbol{\sigma}_{\gamma}\cdot\hat{\boldsymbol{k}}_{\gamma})(\boldsymbol{\sigma}_{n}\cdot\hat{\boldsymbol{k}}_{n}) + a_{7}(\boldsymbol{\sigma}_{\gamma}\cdot\hat{\boldsymbol{k}}_{\gamma})\left((\boldsymbol{\sigma}_{n}\cdot\hat{\boldsymbol{k}}_{\gamma})(\hat{\boldsymbol{k}}_{\gamma}\cdot\hat{\boldsymbol{k}}_{n}) - \frac{1}{3}\boldsymbol{\sigma}_{n}\cdot\hat{\boldsymbol{k}}_{n}\right)$ $+ \frac{a_8}{\sigma_{\gamma} \cdot \hat{k}_{\gamma}} \left((\sigma_n \cdot \hat{k}_n) (\hat{k}_n \cdot \hat{k}_{\gamma}) - \frac{1}{3} \sigma_n \cdot \hat{k}_{\gamma} \right)$ + $a_9\sigma_{\mathbf{n}}\cdot\hat{k}_{\gamma} + a_{10}\sigma_{\mathbf{n}}\cdot\hat{k}_{\mathbf{n}} + a_{11}\left((\sigma_{\mathbf{n}}\cdot\hat{k}_{\gamma})(\hat{k}_{\gamma}\cdot\hat{k}_{\mathbf{n}}) - \frac{1}{3}(\sigma_{\mathbf{n}}\cdot\hat{k}_{\mathbf{n}})\right)$ + $a_{12}(\sigma_{n}\cdot\hat{k}_{n})\left((\hat{k}_{n}\cdot\hat{k}_{\gamma})-\frac{1}{3}(\sigma_{n}\cdot\hat{k}_{\gamma})\right)$ + $a_{13}\sigma_{\gamma}\cdot\hat{k}_{\gamma}+a_{14}(\sigma_{\gamma}\cdot\hat{k}_{\gamma})(\hat{k}_{n}\cdot\hat{k}_{\gamma})$ $+ a_{15}(\boldsymbol{\sigma}_{\boldsymbol{\gamma}} \cdot \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}})\boldsymbol{\sigma}_{\mathrm{n}} \cdot (\hat{\boldsymbol{k}}_{\mathrm{n}} \times \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}}) + a_{16}(\boldsymbol{\sigma}_{\boldsymbol{\gamma}} \cdot \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}}) \left((\hat{\boldsymbol{k}}_{\mathrm{n}} \cdot \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}})^2 - \frac{1}{3} \right)$ $+ \ \ \, \left| \frac{a_{17}(\boldsymbol{\sigma}_{\boldsymbol{\gamma}} \cdot \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}})(\hat{\boldsymbol{k}}_{\mathbf{n}} \cdot \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}})(\boldsymbol{\sigma}_{\mathbf{n}} \cdot (\hat{\boldsymbol{k}}_{\mathbf{n}} \times \hat{\boldsymbol{k}}_{\boldsymbol{\gamma}}))}{\right|,$

a term can be written by Φ parameter

- $\kappa(J)$ related to a_1 via ϕ
- Measured using unpolarized neutrons and target



 139 La (n, γ) data example

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T. Okudaira et al., Phys, Rev. C 97, 034622 (2018).

Measure a lower/higher asymmetry on pwave resonance

$$A_{LH} \equiv \frac{N_L - N_H}{N_L + N_H}$$

 ${\rm A_{LH}}$ has correlation with $\cos\theta_{\gamma}$

 $A_{LH}(\theta_{\gamma}) = A\cos\theta_{\gamma} + B$

Can extract values for A, B from fit





can solve for x and y giving two solutions!



- Measured in ¹³⁹La 0.73 eV p-wave
 - κ(J) ~0.53
 - The first measurement of $\kappa(J)$ ever



 $x = cos\phi$, $y = sin\phi$

¹²⁷I and ¹¹¹Cd a₁ measurements

Targets



- Nal
- 10 mmt, 24 mm¢
- 20 mmt self-filter



- ^{nat}Cd
- 2 mmt
- 4 mmt self-filter

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¹²⁷I resonances

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G.E. Mitchell et al. | Physics Reports 354 (2001) 157–241

Table 16 Resonance parameters and PNC asymmetries p for ¹²⁷I. Parameters A_J are in units eV⁻¹ $A_3^{\rm b}$ E (eV) A_2^{b} $|p|/\Delta p$ l J^{a} $g\Gamma_n$ (meV) p (%) -57.7^{a} 0 2 3 -52.3^{a} 0 largest PV 7.51° 0.00012 ± 0.0001 37.6 0.13 ± 0.14 0.9 22.5 asymmetry 5.2 10.34^c 0.0028 ± 0.0003 9.0 -0.005 ± 0.03 0.2 13.93° 0.0014 ± 0.0001 15.3 9.0 0.01 ± 0.04 0.3 20.43^a 0 3 0.68 ± 0.05 24.63° 51.7 10.3 1 0.00064 ± 0.0006 19.3 1.65 ± 0.16 31.24^a 0 2 13.0 ± 1.5 37.74^a 0 2 26.0 ± 2.5 45.39^a 2 11.5 ± 2.0 0 52.20^c 0.5 0.00085 ± 0.0008 47.8 13.1 0.10 ± 0.18 53.82 0.019 ± 0.002 8.8 2.9 0.24 ± 0.02 12.0 64.04 13.5 6.8 3.0 0.008 ± 0.001 0.06 ± 0.02 65.93^a 0 2 0.80 ± 0.15 78.53^a 0 15.5 ± 2.0 13.6 85.84 0.0174 ± 0.002 4.7 0.24 ± 0.02 11.0 90.38^a 0 3 10.4 ± 1.5

Self-Filter

used upstream self-filters to reduce pileup from nearby s-waves



¹²⁷I Beam Intensity with Self-Filter

beam intensity Intensity 0 Entries 14283.91 Mean Std Dev 10 10 10 50 E_n[eV] 10 15 20 25 30 35 40 45 5

hEn all gated Bo

14.66

13.30

- Measure beam intensity using ^{nat}Bo target
- Can clearly see drop in intensity from self-filter

Nal Interesting Features



Discovery still possible!

S(n) = 6826.13 keV 5

highest energy decays with known final state spins F

Difficult to isolate decays from single p-wave resonance

Eγ [†]	Ι _γ ‡#	E_i (level)	\mathbf{J}_i^{π}	E_f	J_f^{π}
6645.61 2	185 8	(6826.20)	$2^+, 3^+$	180.40	$(3)^{+}$
6665.24 5	53 <i>3</i>	(6826.20)	$2^+, 3^+$	160.77	1+,2+
6674.53 9	25 1	(6826.20)	$2^+, 3^+$	151.48	$(3)^{+}$
6681.98 2	227 9	(6826.20)	$2^+, 3^+$	144.03	(3)-
6688.05 4	107 5	(6826.20)	$2^+, 3^+$	137.96	4-
6692.36 2	507 21	(6826.20)	2+,3+	133.65	2-
6697.73 4	126 5	(6826.20)	2+,3+	128.28	$(4)^+$
6740.46 5	72 8	(6826.20)	2+,3+	85.54	3+
6798.68 6	38 2	(6826.20)	2+,3+	27.32	2+
6826.00 9	21 2	(6826.20)	2+,3+	0.0	1+



Gate on suspected full absorption and single escape peaks for A_{LH} analysis

Gate is wide to capture γ -rays responsible



¹²⁷I 13.6 eV p-wave



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A_{LH} Summary

nuclei	Resonance energy (eV)	A _{LH} slope	A _L (%)
¹¹¹ Cd	4.5	0.04627 ± 0.07446	1.3 ± 0.4
117	7.4	0.02724 ± 0.05476	0.13 ± 0.14
117	10.35	0.02845 ± 0.02671	-0.005 ± 0.0.03
117	13.6	0.08749 ± 0.04396	0.01 ± 0.04
117	22.2	0.11350 ± 0.05768	unmeasured
117	24.2	0.07808 ± 0.05999	1.65 ± 0.16

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PV-Search at JPARC

Search for Parity Violation in Unmeasured Heavy Nuclei PV resonances found by TRIPLE

- Unmeasured range 140<A<180 with nonzero spin
 - ¹⁶⁹Tm, I=1/2 (100%), DNP possible in a diamagnetic salt
 - ¹⁷¹Yb, I=1/2 (14.1%), has been hyperpolarized
 - ¹⁴⁹Sm, ¹⁵¹Eu, ¹⁶⁷Er, and ¹⁶⁵Ho polarizable
- Many unmeasured nuclei → just one discovery is meaningful!



Search for Parity Violation in Unmeasured Heavy Nuclei

PV resonances found by TRIPLE



- ¹⁶⁹Tm, I=1/2 (100%), DNP possible in a diamagnetic salt
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Original Plan

- Original plan was to use Nal array or pure Csl in current mode on BL04
- Could not use to to space constrains and necessity for large spin-transport coils
- Decided to use existing HPGe array instead





PV Search Experiential Setup

- ³He spin-filter gives net longitudinal polarization of neutrons
- Measure gammas and transmission through target
- Can also easily measure a₁





³He spin-filter

spin-transport

upper cluster





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Thank you!

BL04 Long-Term Plans



BL04 Experimental Area 2 (downstream)

Large sized Nal detectors for higher energy cross-section measurements

Target location

Neutron shielding



Backup Slides

¹¹¹Cd 4.5 eV p-wave



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¹²⁷I 7.4 eV p-wave



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¹²⁷I 10.35 eV p-wave



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¹²⁷I 22.2 eV p-wave



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¹²⁷I 24.2 eV p-wave



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E_{γ}^{\dagger} $I_{\gamma}^{\ddagger \#}$ $E_i(\text{level})$ J_i^{π} E_f J^{γ}	τ f
$5962.15^{\textcircled{0}}$ 10 31 2 (6826.20) 2 ⁺ ,3 ⁺ 863.90?	
5976.71 5 73 4 (6826.20) $2^+, 3^+$ 849.34 (\leq 4)	
5982.53 8 59 <i>3</i> (6826.20) 2 ⁺ ,3 ⁺ 843.52	
$5986.00 \ 4 \qquad 128 \ 5 \qquad (6826.20) \ 2^+, 3^+ \qquad 840.04 \qquad (1^-, 2, 3^+) \ (1^-, 3^+) \ (1^$	3+)
5997.70 <i>4</i> 105 5 (6826.20) 2 ⁺ ,3 ⁺ 828.34	
$6004.60^{(@)} 6$ 58 3 (6826.20) $2^+, 3^+$ 821.44?	
6030.42 <i>19</i> 13 <i>1</i> (6826.20) 2 ⁺ ,3 ⁺ 795.62	
$6034.1^{\textcircled{0}}$ 5 4 1 (6826.20) $2^+, 3^+$ 791.9?	
6038.05 <i>1</i> 7 13 2 (6826.20) 2 ⁺ ,3 ⁺ 787.99	
$6055.3^{\textcircled{0}}4$ 8 1 (6826.20) $2^+.3^+$ 770.7?	
$6076.03 \ 13$ 22 1 (6826.20) 2 ⁺ ,3 ⁺ 750.01 (\leq 4)	
6121.14 <i>17</i> 15 <i>1</i> (6826.20) 2 ⁺ ,3 ⁺ 704.90	
6139.01 8 43 <i>3</i> (6826.20) 2 ⁺ ,3 ⁺ 687.03 (3,4) ⁻	
6149.31 9 37 2 (6826.20) 2 ⁺ ,3 ⁺ 676.73	
$6164.64 \ 6 \qquad 61 \ 3 \qquad (6826.20) \qquad 2^+, 3^+ \qquad 661.40$	
$6169.6 \ 4 \qquad 6 \ 1 \qquad (6826.20) 2^+, 3^+ 656.4$	
$6212.06 \ 6 \qquad 52 \ 3 \qquad (6826.20) \ 2^+, 3^+ 613.97$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	4-)
$6276.32 \ 19 \qquad 19 \ 2 \qquad (6826.20) \ 2^+, 3^+ \qquad 549.71 (3^+, 4^-)$	⁻)
$6289.64^{\textcircled{0}}8$ 44 2 (6826.20) 2 ⁺ ,3 ⁺ 536.39?	
$6307.53 2 576 23 (6826.20) 2^+, 3^+ 518.50 (3,4)^-$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	\+
6434.050 472 (0820.20) $2,5$ 391.97 $(1,2,5)6440.72.6 64.3 (6826.20) 2^+3^+ 385.30 2^+3^+$)
$6440.72.0$ 04.5 (0620.20) $2^{+},5^{+}$ 365.50 $2^{+},5^{+}$	
6453666 59 3 (6826.20) 2 ,5 570.75 (4)	
(470) $\otimes @$ (6826.20) $2^+, 3^+, 346.00$	
6420.20 2,5 540.2: 6481.76.10 51 4 (6826.20) 2+3+ 344.26 (2.3.4))+
6530 44 14 19 1 (6826 20) 2 ⁺ 3 ⁺ 295 58 (2 3 4))+
6593.8.4 7 1 (6826.20) 2 ⁺ 3 ⁺ 232.2 4 ⁺)
6605.44 91 (6826.20) $2^+,3^+$ 220.6 (1.2.3)+
$6645.61 2 185 8 (6826.20) 2^+, 3^+ 180.40 (3)^+$,
6665.24 5 53 <i>3</i> (6826.20) 2 ⁺ ,3 ⁺ 160.77 1 ⁺ ,2 ⁺	
$6674.53 9 25 1 (6826.20) 2^+, 3^+ 151.48 (3)^+$	
$6681.98\ 2 \qquad 227\ 9 \qquad (6826.20) \qquad 2^+, 3^+ \qquad 144.03 \qquad (3)^-$	
6688.05 4 107 5 (6826.20) 2 ⁺ ,3 ⁺ 137.96 4 ⁻	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
669/.734 126 5 (6826.20) 2 ⁺ ,3 ⁺ 128.28 (4) ⁺	
6740.46.5 72.8 (6826.20) 2 ⁺ ,3 ⁺ 85.54 3 ⁺	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	



³He Spin-Filter

- Polarized ³He has a large spin dependent cross-section
- Effective for meV to eV neutron energies
- Polarize ³He with Spin Exchange Optical Pumping



Spin Exchange Optical Pumping

- Use SEOP to polarize ³He gas cell
- Optically pump Rb vapor with 795 nm laser
 - Uniform holding field \overline{B}_0
 - Photons with circular polarization transfer spin angular momentum to Rb electrons
- Rb electrons exchange spin with ³He atoms upon collision



(T.R. Gentile et al. 2017)



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$\kappa(J)$ Measurement & Analysis cont.

rotationally invariant and independent terms in (n, γ) differential cross section for unpolarized neutrons

$$\frac{d\sigma_{n\gamma_f}}{d\Omega_{\gamma}} = \frac{1}{2} \left(a_0 + a_1 \cos \theta_{\gamma} + a_3 \left(\cos^2 \theta_{\gamma} - \frac{1}{3} \right) \right)$$

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Okudaira Takuyu. PhD. thesis, 2018. T. Okudaira *et al.*, Phys, Rev. C **97**, 034622 (2018).

 $\frac{\Gamma_{p,j=1/2}}{\Gamma_p^n}$

 $y \equiv \sqrt{\frac{\Gamma_{p,j=3/2}^n}{\Gamma_p^n}}$

 $x \equiv 1$

$$a_1 = a_{1x}x + a_{1y}y$$
$$a_3 = a_{3xy}xy + a_{3yy}y^2$$

dispersive shape as a function of neutron energy near the p-wave resonance



define $(\bar{a}_{0,1,3})_{L,H}$ the weighted average for lower and higher sides of the resonance

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$$(\bar{a}_{0,1,3})_L = \int_{E_p - 2\Gamma_p}^{E_p} dE' \int d^3 p_A a_{0,1,3}(E') \Phi(t^m, E', p_A)$$
$$(\bar{a}_{0,1,3})_H = \int_{E_p}^{E_p + 2\Gamma_p} dE' \int d^3 p_A a_{0,1,3}(E') \Phi(t^m, E', p_A)$$

Okudaira Takuyu. PhD. thesis, 2018. T. Okudaira *et al.*, Phys, Rev. C **97**, 034622 (2018).

$$(I_{\gamma,d})_{L,H} = \frac{I_0}{2} [(\bar{a}_0)_{L,H} \bar{P}_{d0} + (\bar{a}_1)_{L,H} \bar{P}_{d1}]$$

Lower and higher γ -ray count in d^{th} detector

$$(A_{LH})_{d} = \frac{(I_{\gamma,d})_{L} - (I_{\gamma,d})_{H}}{(I_{\gamma,d})_{L} + (I_{\gamma,d})_{H}}$$
$$= \frac{\underline{[(\bar{a}_{0})_{L} - (\bar{a}_{0})_{H}] + [(\bar{a}_{1})_{L} - (\bar{a}_{1})_{H}]\frac{\bar{P}_{d1}}{\bar{P}_{d0}}}{[(\bar{a}_{0})_{L} + (\bar{a}_{0})_{H}] + [(\bar{a}_{1})_{L} + (\bar{a}_{1})_{H}]\frac{\bar{P}_{d1}}{\bar{P}_{d0}}}$$
$$= \frac{[(\bar{a}_{1})_{L} - (\bar{a}_{1})_{H}]}{[(\bar{a}_{0})_{L} + (\bar{a}_{0})_{H}]} \cos \bar{\theta}_{\gamma} + \frac{[(\bar{a}_{1})_{L} - (\bar{a}_{1})_{H}]}{[(\bar{a}_{1})_{L} + (\bar{a}_{1})_{H}]}$$

 $P_n(\cos\bar{\theta}_\gamma) = \bar{P}_{dn}/\bar{P}_{d0}$

ignoring
$$a_3$$
, lower higher
asymmetry for the d^{th}
detector can be written as
$$(A_{LH})_d = \frac{[(\bar{a}_1)_L - (\bar{a}_1)_H]}{[(\bar{a}_0)_L + (\bar{a}_0)_H]} \cos \bar{\theta}_{\gamma} + \frac{[(\bar{a}_1)_L - (\bar{a}_1)_H]}{[(\bar{a}_1)_L + (\bar{a}_1)_H]}$$

comparing to
$$A_{LH}$$
 \rightarrow $A_{LH}(\theta_{\gamma}) = \underline{A}\cos\theta_{\gamma} + B$

from
$$A_{LH}$$
 vs. $\cos(\theta_{\gamma})$ fit

$$A = \frac{\left[(\bar{a}_{1})_{L} - (\bar{a}_{1})_{H} \right]}{\left[(\bar{a}_{0})_{L} + (\bar{a}_{0})_{H} \right]} \stackrel{(\bar{a}_{1})_{L,H}}{=} Cx + Dy$$

$$from resonance parameters
Can solve for x and y (or ϕ)
giving two solutions!
Can solve for x and y (or ϕ)$$

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T. Okudaira et al., Phys, Rev. C 97, 034622 (2018).