Standard Model Effective Field Theory at Future Lepton Colliders

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What is particle physics?

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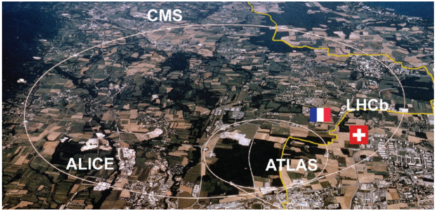
Quantum Mechanics

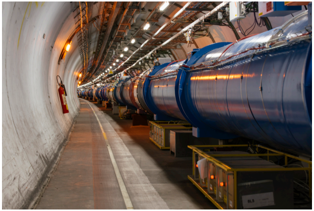
Special Relativity

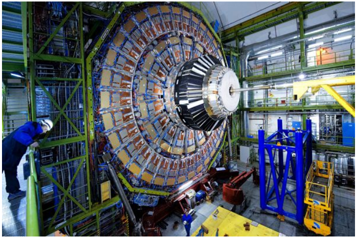
Quantum Field Theory

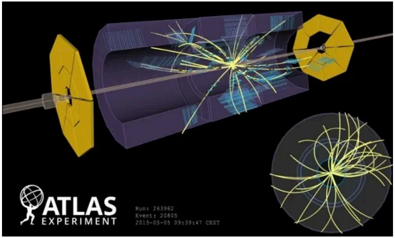
- **Quantum Field Theory** tells us:
  - Particles can be annihilated and created.
  - High energies $\Rightarrow$ heavy (new) particles.

particle physics ≈ collider physics





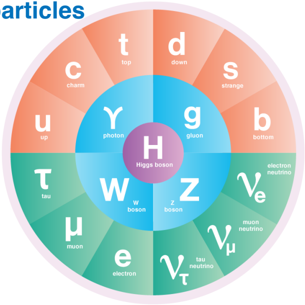




- **Build large colliders → go to high energy → discover new particles!**

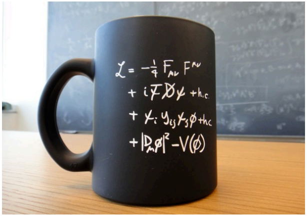
The Standard Model

particles

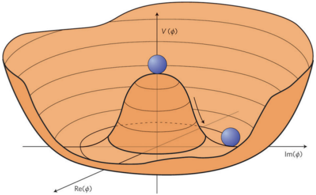


- **the Wheel**
  Only a few “elementary” particles.
- **the Mug**
  The Standard Model Lagrangian is simple!
- **the “Mexican Hat”**
  The Higgs Mechanism gives masses to the elementary particles.

interactions

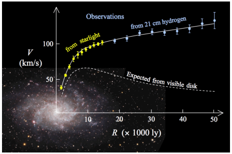


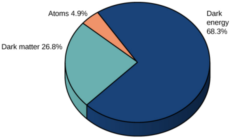
Higgs mechanism

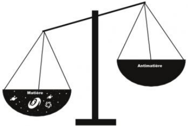


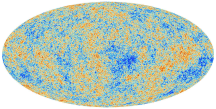
So many things we don’t know...

- What is dark matter?
- What is dark energy?
- Why are there more matter than anti-matter?
- What is the origin of neutrino masses?
- What caused the inflation (if it happened)?
- Why is the electroweak scale so much smaller than the Planck scale?
- Why is the strong CP phase θ so small?
- Why is the CKM matrix somewhat close to 1?
- What is the theory of quantum gravity?
- ......









We need experiments to find the answers!

- ~~LHC will find Supersymmetry (or something else), which has a dark matter candidate and solves the Hierarchy problem!~~

- **Higgs and nothing else?**

LHC will definitely find new physics!

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- **Higgs and nothing else?**
- **What's next?**
  - Build an even larger collider (~ 100 TeV)?
  - No guaranteed discovery!

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- **Higgs and nothing else?**
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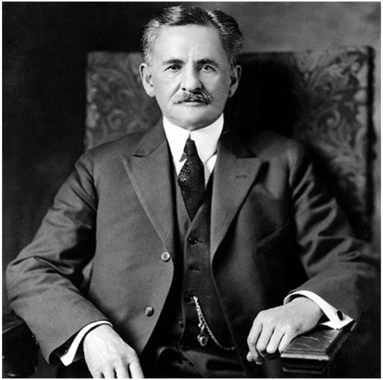
- **Build large colliders** $\rightarrow$ go to high energy $\rightarrow$ discover new particles!

  do precision measurements $\rightarrow$ **discover new physics indirectly!**

  - **Higgs factory!** (HL-LHC, or a future lepton collider)
  - **S**tandard **M**odel **E**ffective **F**ield **T**heory (model independent approach)

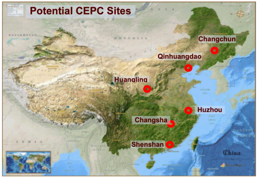
To summarize in one sentence...

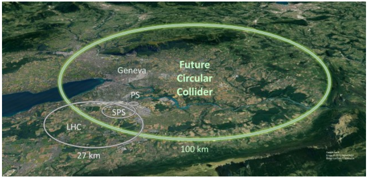


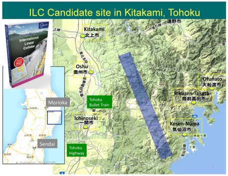
"Our future discoveries must be looked for in the sixth place of decimals."

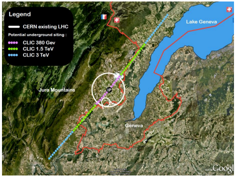
— Albert A. Michelson

Why lepton (e^+e^-) colliders?





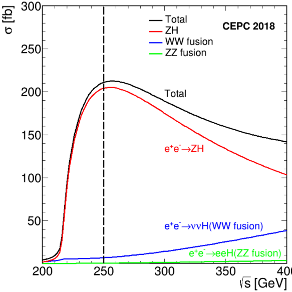


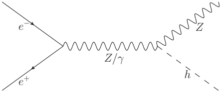


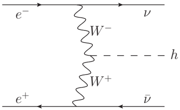
- **It's a Higgs (and Z, W, top) factory!**
  - Large statistics, clean environment
    $\Rightarrow$ **precision measurements!**
  - On the other hand, the LHC is designed to be a "discovery machine"...
- Circular vs. Linear
  - Circular: large luminosity, reuse the tunnel for a 100 TeV hadron collider.
  - Linear: high energy (up to a few TeVs), beam polarization.

Why lepton (e^+e^-) colliders?

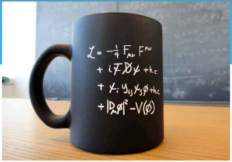
- Higgs
  - $e^+e^- \to hZ$
    cross section maximized at around 250 GeV
  - $e^+e^- \to \nu\bar{\nu}h$
    cross section increases with energy
  - $e^+e^- \to t\bar{t}h$,
    $e^+e^- \to Zhh$, $e^+e^- \to \nu\bar{\nu}hh$,
    ...
- and more
  - $e^+e^- \to Z \to f\bar{f}$
    Z-pole
  - $e^+e^- \to WW$
    WW threshold and above
  - $e^+e^- \to t\bar{t}$
    $t\bar{t}$ threshold and above







The Standard Model Effective Field Theory

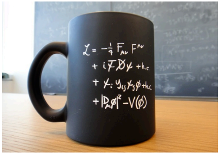


- $[\mathcal{L}_{\rm sm}] \leq 4$. Why?
  - ~~Bad things happen when we have non-renormalizable operators!~~
  - Everything is fine as long as we are happy with finite precision in perturbative calculation.
- **d=5:** $\frac{c}{\Lambda} LLHH \sim \frac{cv^2}{\Lambda}\nu\nu$, Majorana neutrino mass.
- Assuming Baryon and Lepton numbers are conserved,

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots .$$

- If $\Lambda \gg v, E$, then **SM + dimension-6 operators** are sufficient to parameterize the physics around the electroweak scale.

The Standard Model Effective Field Theory



+

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

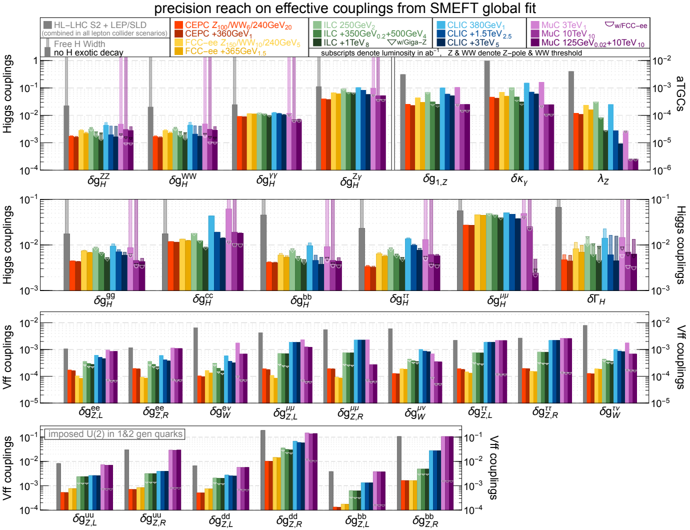
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

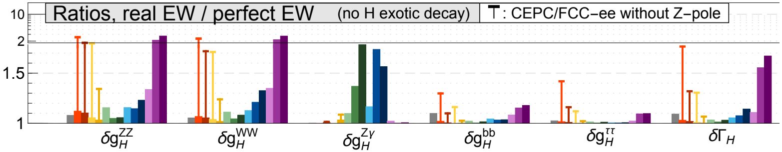
- Write down all possible (non-redundant) dimension-6 operators ...
- 59 operators (76 parameters) for 1 generation, or 2499 parameters for 3 generations. [arXiv:1008.4884] Grzadkowski, Iskrzyński, Misiak, Rosiek, [arXiv:1312.2014] Alonso, Jenkins, Manohar, Trott.
- A **full global fit** with all measurements to all operator coefficients?
  - We usually only need to deal with a subset of them, *e.g.* $\sim 20\text{-}30$ parameters for **Higgs and electroweak** measurements.
- Do a global fit and present the results with some fancy bar plots!

Higgs + EW, Results from the Snowmass 2021 (2022) study

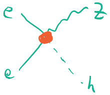
[2206.08326] de Blas, Du, Grojean, JG, Miralles, Peskin, Tian, Vos, Vryonidou



Impacts of (lack of) the Z-pole run

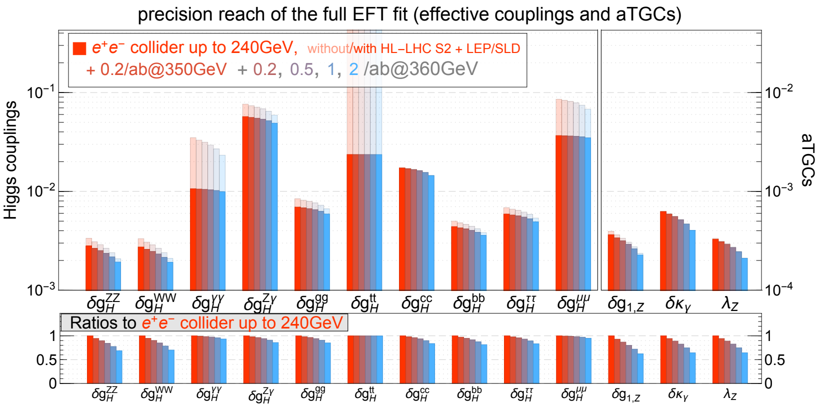


- Without good *Z*-pole measurements, the *eeZh* contact interaction may have a significant impact on the Higgs coupling determination.
- Current (LEP) *Z*-pole measurements are not good enough for CEPC/FCC-ee Higgs measurements!
  - **A future Z-pole run is important!**



- Linear colliders suffer less from the lack of a Z-pole run. **(Win Win!)**

Impact of a 350/360 GeV run



- $5.6\,\mathrm{ab}^{-1}$ at 240 GeV assumed.
- Measurements at 350/360 GeV provides additional handles on the anomalous couplings (*e.g.* $hZ^{\mu}Z_{\mu}$ *vs.* $hZ^{\mu\nu}Z_{\mu\nu}$).
- Also improves the measurements of $e^+e^- \to WW$ (aTGCs).

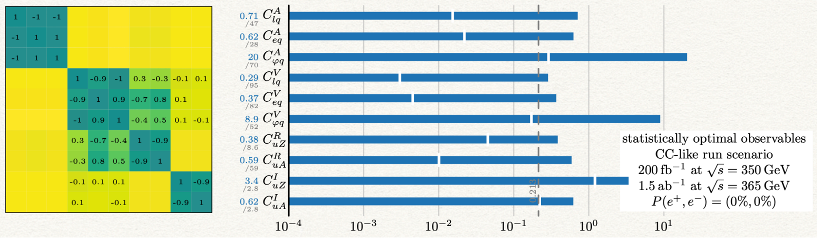
Probing Top operators with $e^- e^+ \to t\bar{t}$

[arXiv:1807.02121] Durieux, Perelló, Vos, Zhang

$$
\begin{aligned}
O^1_{\varphi q} &\equiv \tfrac{y_t^2}{2}\ \bar{q}\gamma^\mu q\ \ \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi, &
O_{uG} &\equiv y_t g_s\ \ \bar{q}T^A\sigma^{\mu\nu}u\ \ \epsilon\varphi^* G^A_{\mu\nu},\\
O^3_{\varphi q} &\equiv \tfrac{y_t^2}{2}\ \bar{q}\tau^I\gamma^\mu q\ \ \varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi, &
O_{uW} &\equiv y_t g_W\ \ \bar{q}\tau^I\sigma^{\mu\nu}u\ \ \epsilon\varphi^* W^I_{\mu\nu},\\
O_{\varphi u} &\equiv \tfrac{y_t^2}{2}\ \bar{u}\gamma^\mu u\ \ \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi, &
O_{dW} &\equiv y_t g_W\ \ \bar{q}\tau^I\sigma^{\mu\nu}d\ \ \epsilon\varphi^* W^I_{\mu\nu},\\
O_{\varphi ud} &\equiv \tfrac{y_t^2}{2}\ \bar{u}\gamma^\mu d\ \ \varphi^T\epsilon\, iD_\mu\varphi, &
O_{uB} &\equiv y_t g_Y\ \ \bar{q}\sigma^{\mu\nu}u\ \ \epsilon\varphi^* B_{\mu\nu},
\end{aligned}
$$

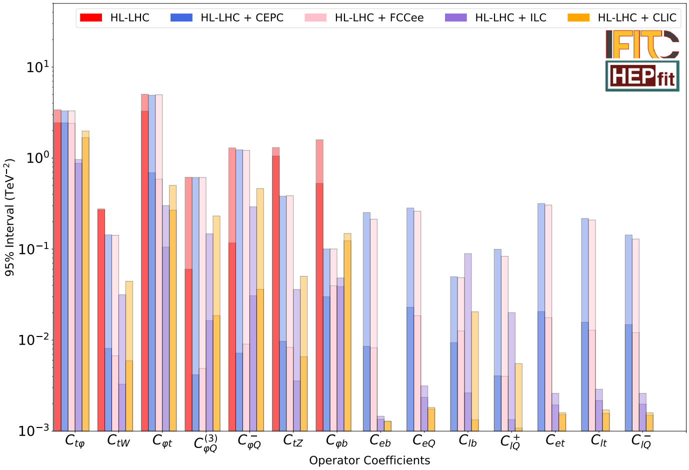
$$
\begin{aligned}
O^1_{lq} &\equiv \tfrac{1}{2}\ \bar{q}\gamma_\mu q\ \ \bar{l}\gamma^\mu l,\\
O^3_{lq} &\equiv \tfrac{1}{2}\ \bar{q}\tau^I\gamma_\mu q\ \ \bar{l}\tau^I\gamma^\mu l,\\
O_{lu} &\equiv \tfrac{1}{2}\ \bar{u}\gamma_\mu u\ \ \bar{l}\gamma^\mu l,\\
O_{eq} &\equiv \tfrac{1}{2}\ \bar{q}\gamma_\mu q\ \ \bar{e}\gamma^\mu e,\\
O_{eu} &\equiv \tfrac{1}{2}\ \bar{u}\gamma_\mu u\ \ \bar{e}\gamma^\mu e,
\end{aligned}
$$

- Also need to include **top dipole** interactions and *eett* contact interactions!
- Hard to resolve the top couplings from $4f$ interactions with just the 365 GeV run.
  - Can't really separate $e^+e^- \to Z/\gamma \to t\bar{t}$ from $e^+e^- \to Z' \to t\bar{t}$.
  - Is that a big deal?

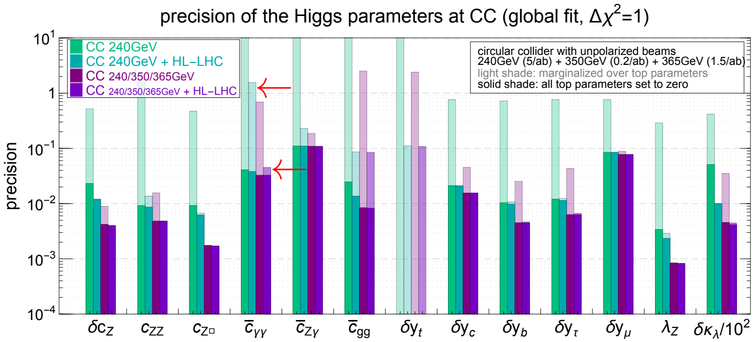


Results from the recent snowmass study

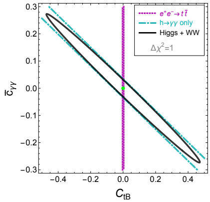
[2206.08326] de Blas, Du, Grojean, JG, Miralles, Peskin, Tian, Vos, Vryonidou



Top operators in loops (Higgs processes) [1809.03520] G. Durieux, JG, E. Vryonidou, C. Zhang



- $O_{tB} = (\bar{Q}\sigma^{\mu\nu}t)\,\tilde{\varphi}B_{\mu\nu} + h.c.$ is not very well constrained at the LHC, and it generates dipole interactions that contributes to the $h\gamma\gamma$ vertex.
- Deviations in $h\gamma\gamma$ coupling $\Rightarrow$ run at ~ 365 GeV to confirm?



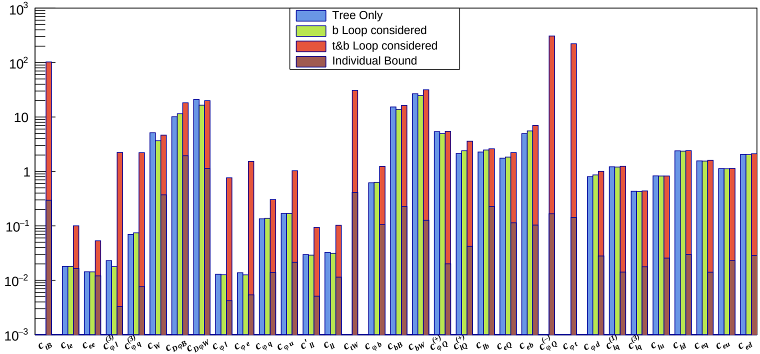
Top operators in loops (current EW processes)

[2205.05655] Y. Liu, Y. Wang, C. Zhang, L. Zhang, JG

	Experiment	Observables
Low Energy	CHARM/CDHS/ CCFR/NuTeV/ APV/QWEAK/ PVDIS	Effective Couplings
Z-pole	LEP/SLC	Total decay width Γ_Z Hadronic cross-section σ_{had} Ratio of decay width R_f Forward-Backward Asymmetry A^f_{FB} Polarized Asymmetry A_f
W-pole	LHC/Tevatron/ LEP/SLC	Total decay width Γ_W W branching ratios $Br(W \to l\nu_l)$ Mass of W Boson M_W
$ee \to qq$	LEP/TRISTAN	Hadronic cross-section σ_{had} Ratio of cross-section R_f Forward-Backward Asymmetry for b/c A^f_{FB}
$ee \to ll$	LEP	cross-section σ_f Forward-Backward Asymmetry A^f_{FB} Differential cross-section $\frac{d\sigma_f}{dcos\theta}$
$ee \to WW$	LEP	cross-section σ_{WW} Differential cross-section $\frac{d\sigma_{WW}}{dcos\theta}$

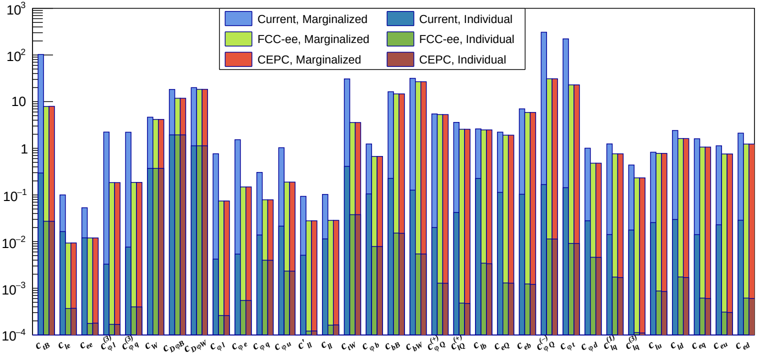
- Top operators (1-loop) + EW operators (tree, including bottom dipole operators)
- $e^+e^- \to f\bar{f}$ at different energies, $e^+e^- \to WW$.

Top operators in loops (current EW processes)



- Good sensitivities, but too many parameters for a global fit...

Top operators in loops (future EW processes)



- Good sensitivities, but too many parameters for a global fit...
- It shows the importance of directly measuring $e^+e^- \to t\bar{t}$.

Machine learning in SMEFT analyses

Machine learning is not physics!

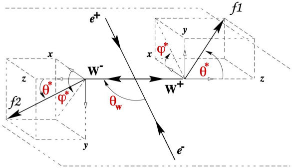
past

now

- Current work with Shengdu Chai (柴声都), Lingfeng Li (李凌风) on $e^+e^- \to WW$.
- Future work with many other students on more processes...

Why Machine learning?

- In many cases, the new physics contributions are sensitive to the differential distributions.
  - *e.g.* $e^+e^- \to WW$
  - How to extract information from the differential distribution?
  - If we have the full knowledge of $\frac{d\sigma}{d\Omega} \Rightarrow$ matrix-element method, optimal observables...



- The ideal $\frac{d\sigma}{d\Omega}$ we can calculate is not the $\frac{d\sigma}{d\Omega}$ that we actually measure!
  - detector acceptance, measurement uncertainties, ISR/beamstrahlung ...
  - In practice we only have **MC samples**, not analytic expressions, for $\frac{d\sigma}{d\Omega}$.
  - With **Neural Network** we can (in principle) reconstruct $\frac{d\sigma}{d\Omega}$ from MC samples.

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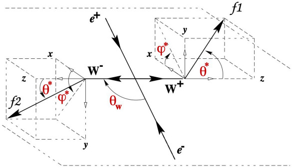
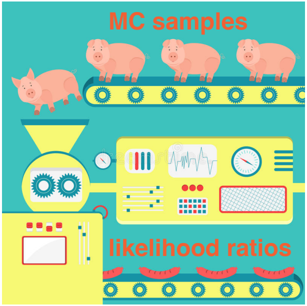




A rough sketch

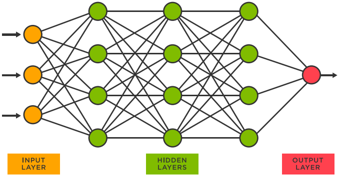
- We have a theory (SMEFT) that gives a differential cross section $\frac{d\sigma}{d\Omega}$ which is a function of the parameters of interest $\mathbf{c}$ (Wilson coefficients).
  - For simplicity, let's ignore the total rate and focus on $\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \equiv p(\mathbf{x}|\mathbf{c})$, *i.e.* it's a probability density function of the observables $\mathbf{x}$.
  - Define the likelihood function $\mathcal{L}(\mathbf{c}|\mathbf{x}) \equiv p(\mathbf{x}|\mathbf{c})$. For a sample of N events, maximizing the joint likelihood $\prod_{i=1}^{N} \mathcal{L}(\mathbf{c}|\mathbf{x}_i)$ (or the log likelihood) gives the best estimator for $\mathbf{c}$. (matrix-element method)
- Suppose we have two equal-size samples $\{\mathbf{x}_{i,\mathbf{c}_0}\} \sim p(\mathbf{x}|\mathbf{c}_0)$ and $\{\mathbf{x}_{i,\mathbf{c}_1}\} \sim p(\mathbf{x}|\mathbf{c}_1)$, one could define the cross-entropy loss function(al)

$$L(\hat{s}) = -\sum_{i=1}^{N} \log \hat{s}(\mathbf{x}_{i,\mathbf{c}_1}) - \sum_{i=1}^{N} \log\left(1 - \hat{s}(\mathbf{x}_{i,\mathbf{c}_0})\right) ,$$

which is minimized by the optimal decision function

$$s(\mathbf{x}|\mathbf{c}_0, \mathbf{c}_1) = \frac{p(\mathbf{x}|\mathbf{c}_1)}{p(\mathbf{x}|\mathbf{c}_0) + p(\mathbf{x}|\mathbf{c}_1)} .$$

A rough sketch



- From neural network we can construct a function $\hat{s}(\mathbf{x})$. By minimizing $L(\hat{s})$ with respect to $\hat{s}(\mathbf{x})$ we can obtain an estimator for the likelihood ratio

$$\hat{r}(\mathbf{x}|\mathbf{c}_0,\mathbf{c}_1) = \frac{1-\hat{s}(\mathbf{x}|\mathbf{c}_0,\mathbf{c}_1)}{\hat{s}(\mathbf{x}|\mathbf{c}_0,\mathbf{c}_1)} = \frac{\hat{p}(\mathbf{x}|\mathbf{c}_0)}{\hat{p}(\mathbf{x}|\mathbf{c}_1)},$$

  which is the same as the true likelihood ratio in the ideal limit (large sample, perfect training).

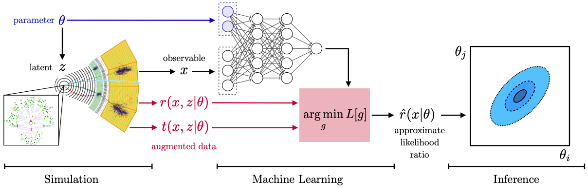
- There are many other ways to construct a loss function(al)....

- With additional assumptions on how $\frac{d\sigma}{d\Omega}$ depends on $\mathbf{c}$ (*i.e.*, a quadratic relation), we only need to train a finite number of times to know how the likelihood ratio depend on $\mathbf{c}$.

Machine Learning

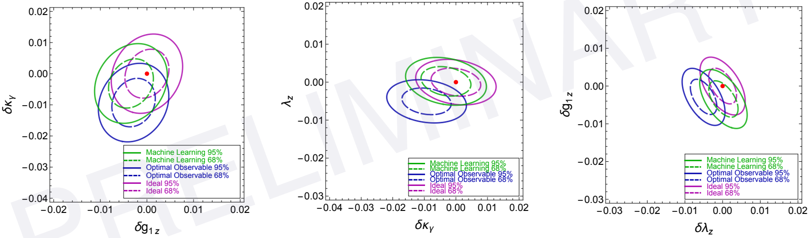
- Several ML SMEFT studies already exist (mostly for LHC)
  - $pp \to ZW$ [2007.10356] Chen, Glioti, Panico, Wulzer
  - $pp \to tt$, $pp \to hZ$ [2211.02058] Ambrosio, Hoeve, Madigan, Rojo, Sanz
  - ...
- One could make use of latent variable "z" (the parton level analytic result for $\frac{d\sigma}{d\Omega}$) to increase the performance of ML.

[1805.00013, 1805.00020] Brehmer, Cranmer, Louppe, Pavez



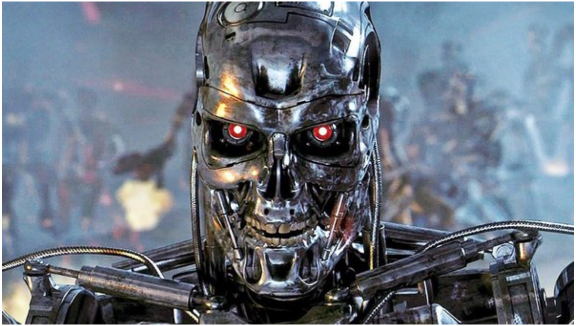
- Assuming linear dependences $\frac{d\sigma}{d\Omega} = S_0 + \sum_i S_{1,i}\, c_i$, there is a method called SALLY (Score approximates likelihood locally) that is basically the ML version of Optimal Observables.

Machine Learning in $e^+e^- \to WW$ (preliminary results, Shengdu Chai, JG, Lingfeng Li)



- Scale (size of the ellipses) is arbitrary.
- Semileptonic channel, MadGraph/Pythia/Delphes (CEPC detector card), 3-aTGC fit
  - Naively applying truth-level optimal observables could lead to a large bias!
  - It's easier for machine learning to take care of systematics! (Residue bias possibly due to the narrow width approximation made with the analytic result.)

Machine learning



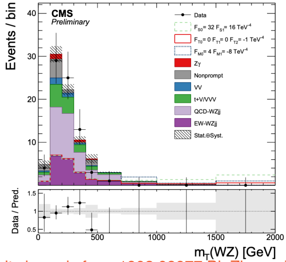
- **When will Machine take over?**
  - Before or after a future lepton collider is built?

Probing dimension-8 operators?

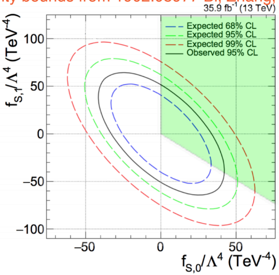
- The dimension-8 contribution has a large energy enhancement ($\sim E^4/\Lambda^4$)!
- It is difficult for LHC to probe these bounds.
  - Low statistics in the high energy bins.
  - Example: Vector boson scattering.
  - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- Can we separate the dim-8 and dim-6 effects?
  - Precision measurements at several different $\sqrt{s}$?

    (A **very** high energy lepton collider?)
  - Or find some special process where dim-8 gives the leading new physics contribution?

CMS-PAS-SMP-18-001



positivity bounds from 1902.08977 Bi, Zhang, Zhou

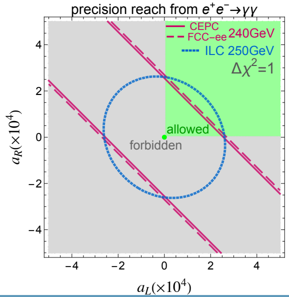


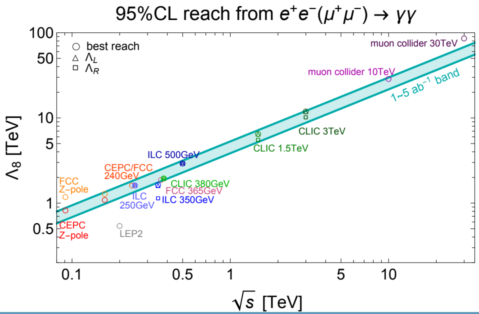
The diphoton channel [arXiv:2011.03055] Phys.Rev.Lett. 129, 011805, JG, Lian-Tao Wang, Cen Zhang

- $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$), SM, non-resonant.
- Leading order contribution: **dimension-8 contact interaction**.
  ($f^+f^- \to \bar{e}_L e_L$ or $e_R \bar{e}_R$)

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\mathrm{SM+d8}} = 2e^2 \frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 \,.$$

- Can probe dim-8 operators (and their positivity bounds) at a **Higgs factory** (~ 240 GeV)!



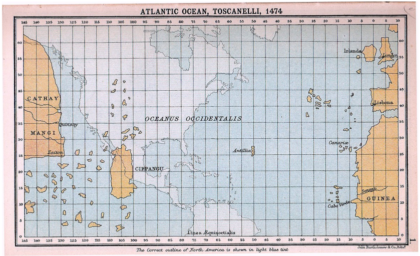


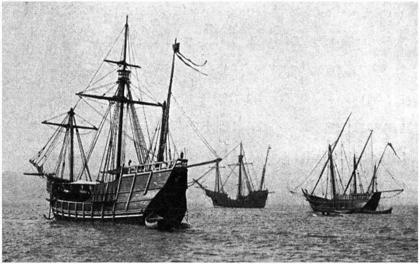
Conclusion

- **We have no idea what is the new physics beyond the Standard Model.**
- **One important direction to move forward is to do precision measurements of the Standard Model processes.**
  - A future lepton collider is an ideal machine for that.
  - SMEFT is a good theory framework (but is not everything).
  - Expanding the theory framework?
    - Loop contributions, dimension-8 operators, HEFT ...
- **Machine learning is** (likely to be) **the future!**

A lesson from Christopher Columbus (哥伦布发现美洲大陆)

- **You need to have a theory.**
  - The earth is round, India is in the east...
- **Your theory can be wrong!**
  - Columbus did not find India, but found America instead...
- **You need to ask money from the government!**
  - Columbus convinced the monarchs of Spain to sponsor him.
- **Will we discover the new world?**

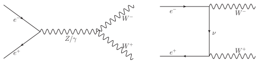


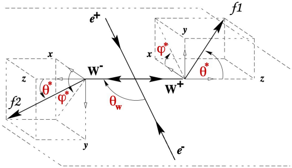


backup slides

$e^+e^- \to WW$ with Optimal Observables

- TGCs (and additional EFT parameters) are sensitive to the differential distributions!
  - One could do a fit to the binned distributions of all angles.
  - Not the most efficient way of extracting information.
  - Correlations among angles are sometimes ignored.



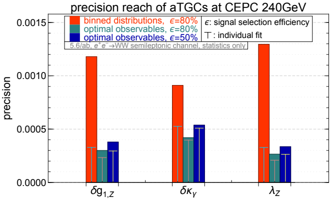


- What are optimal observables?

  (See *e.g.* Z.Phys. C62 (1994) 397-412 Diehl & Nachtmann)
  - In the limit of large statistics (everything is Gaussian) and small parameters (linear contribution dominates), the best possible reaches can be derived analytically!

$$\frac{d\sigma}{d\Omega} = S_0 + \sum_i S_{1,i}\, g_i \,, \qquad c_{ij}^{-1} = \int d\Omega \frac{S_{1,i} S_{1,j}}{S_0} \cdot \mathcal{L} \,,$$

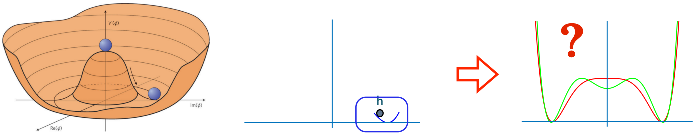
- The optimal observables are given by $\mathcal{O}_i = \frac{S_{1,i}}{S_0}$, and are functions of the 5 angles.



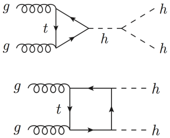
[arXiv:1907.04311] de Blas, Durieux, Grojean, JG, Paul

Higgs self-coupling

- We know very little about the Higgs potential!

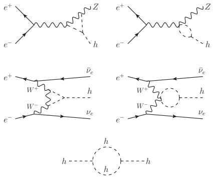


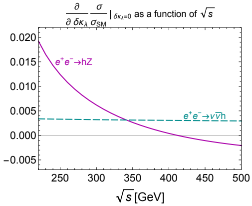
- To know more about the Higgs potential, we need to measure the Higgs self-couplings (**hhh** and **hhhh** couplings).
- The $(H^\dagger H)^3$ operator can modify the Higgs self-couplings.
- Probing the *hhh* coupling at Hadron colliders.
  - $gg \to hh$
  - $\lesssim 50\%$ at HL-LHC.
  - $\lesssim 5\%$ at a 100 TeV collider.

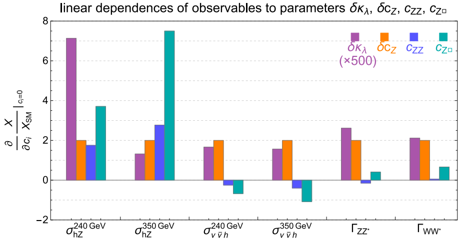


Triple Higgs coupling at one-loop order

[arXiv:1711.03978] Di Vita, Durieux, Grojean, JG, Liu, Panico, Riembau, Vantalon

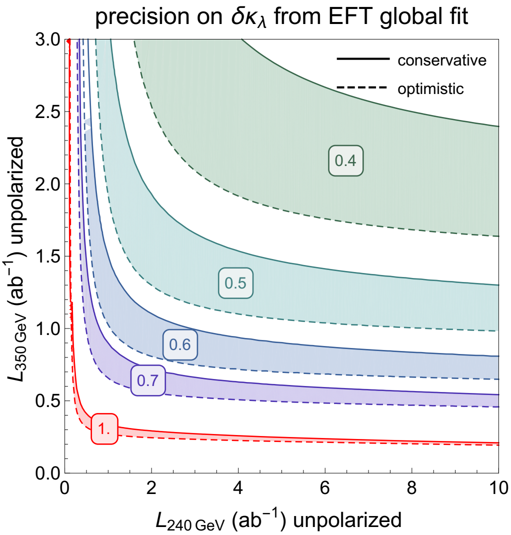






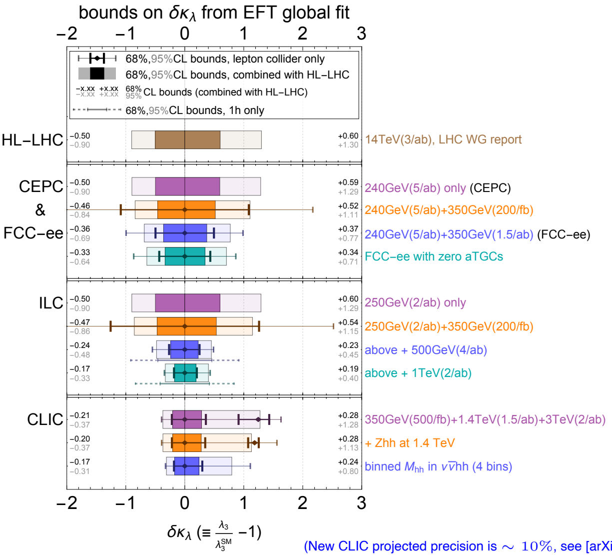
- $\kappa_\lambda \equiv \frac{\lambda_{hhh}}{\lambda_{hhh}^{SM}}$,
  $\delta\kappa_\lambda \equiv \kappa_\lambda - 1 = c_6 - \frac{3}{2}c_H$,
  with $\mathcal{L} \supset -\frac{c_6\lambda}{v^2}(H^\dagger H)^3$.
- One loop corrections to all Higgs couplings (production and decay).
- 240 GeV: hZ near threshold (more sensitive to $\delta\kappa_\lambda$)
- at 350-365 GeV:
  - WW fusion
  - hZ at a different energy
- $h \to WW^* / ZZ^*$ also have some discriminating power (but turned out to be not enough).

Triple Higgs coupling from EFT global fits



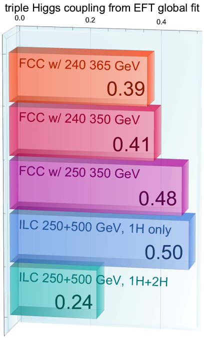
- Runs at two different energies (240 GeV and 350/365 GeV) are needed to obtain good constraints on the triple Higgs coupling in a global fit!

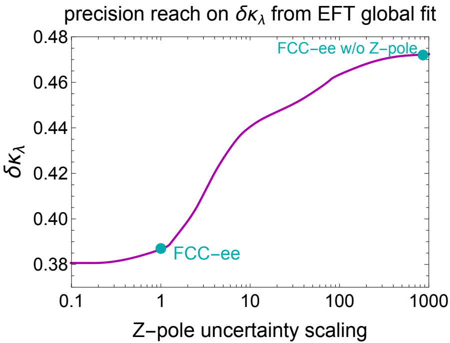
Triple Higgs coupling from global fits [arXiv:1711.03978]



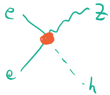
(New CLIC projected precision is $\sim 10\%$, see [arXiv:1901.05897] Roloff *et al.*)

Updates on the triple Higgs coupling determination from EFT global fits



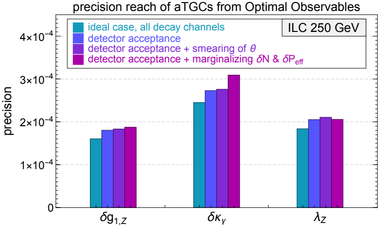


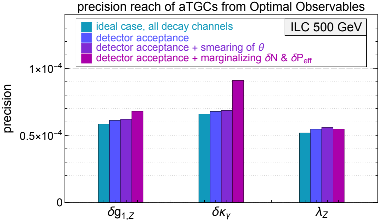
- 240, 365 GeV are better than 250, 350 GeV.
- Impacts of Z-pole measurements are not negligible.
  ($eeZ(h)$ contact interaction enters $e^+e^- \to hZ$.)



Updates on the *WW* analysis with Optimal Observables

- How well can we do it in practice?
  - detector acceptance, measurement uncertainties, ...
- What we have done (current work for the snowmass study)
  - detector acceptance ($|\cos\theta| < 0.9$ for jets, < 0.95 for leptons)
  - some smearing (production polar angle only, $\Delta = 0.1$)
  - ILC: marginalizing over total rate (δN) and effective beam polarization (δP_{eff})
- Constructing full EFT likelihood and feed it to the global fit. (For illustration, only showing the 3-aTGC fit results here.)
- Further verifications (by experimentalists) are needed.



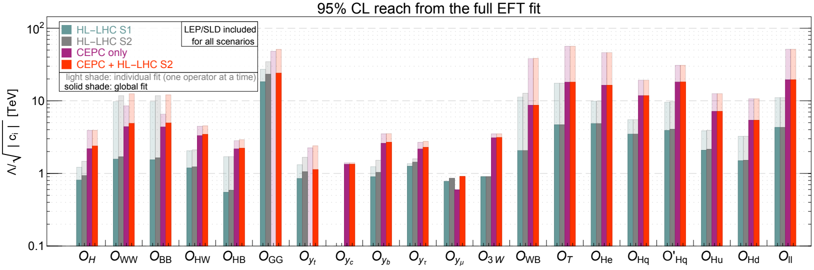


D6 operators

$\mathcal{O}_H = \frac{1}{2}(\partial_\mu \lvert H^2 \rvert)^2$ $\mathcal{O}_{WW} = g^2 \lvert H \rvert^2 W^a_{\mu\nu} W^{a,\mu\nu}$ $\mathcal{O}_{BB} = g'^2 \lvert H \rvert^2 B_{\mu\nu} B^{\mu\nu}$ $\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$ $\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\mathcal{O}_{GG} = g_s^2 \lvert H \rvert^2 G^A_{\mu\nu} G^{A,\mu\nu}$ $\mathcal{O}_{y_u} = y_u \lvert H \rvert^2 \bar{q}_L \tilde{H} u_R + \text{h.c.} \quad (u \to t, c)$ $\mathcal{O}_{y_d} = y_d \lvert H \rvert^2 \bar{q}_L H d_R + \text{h.c.} \quad (d \to b)$ $\mathcal{O}_{y_e} = y_e \lvert H \rvert^2 \bar{l}_L H e_R + \text{h.c.} \quad (e \to \tau, \mu)$ $\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_{\mu} W^b_{\nu\rho} W^{c\,\rho\mu}$
$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}$	$\mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{WB} = gg' H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}$ $\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$ $\mathcal{O}_{\ell\ell} = (\bar{\ell}_L \gamma^\mu \ell_L)(\bar{\ell}_L \gamma_\mu \ell_L)$	$\mathcal{O}_{H\ell} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{\ell}_L \gamma^\mu \ell_L$ $\mathcal{O}'_{H\ell} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{\ell}_L \sigma^a \gamma^\mu \ell_L$ $\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R \gamma^\mu e_R$
$\mathcal{O}_{Hq} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{q}_L \gamma^\mu q_L$ $\mathcal{O}'_{Hq} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}_L \sigma^a \gamma^\mu q_L$	$\mathcal{O}_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{u}_R \gamma^\mu u_R$ $\mathcal{O}_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{d}_R \gamma^\mu d_R$

- SILH' basis (eliminate $\mathcal{O}_{WW}$, $\mathcal{O}_{WB}$, $\mathcal{O}_{H\ell}$ and $\mathcal{O}'_{H\ell}$)
- Modified-SILH' basis (eliminate $\mathcal{O}_W$, $\mathcal{O}_B$, $\mathcal{O}_{H\ell}$ and $\mathcal{O}'_{H\ell}$)
- Warsaw basis (eliminate $\mathcal{O}_W$, $\mathcal{O}_B$, $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$)

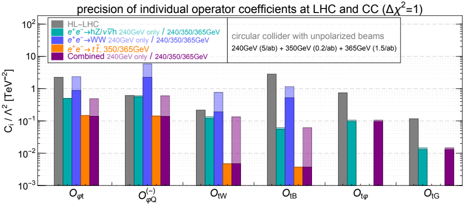
Reach on the scale of new physics

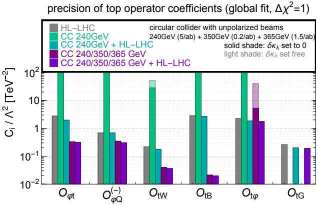


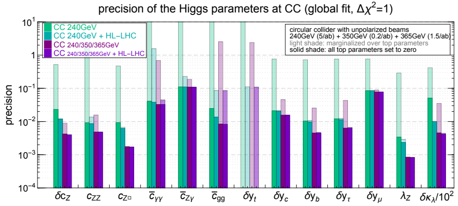
- Reach on the scale of new physics Λ.
- Note: reach depends on the couplings c_i!

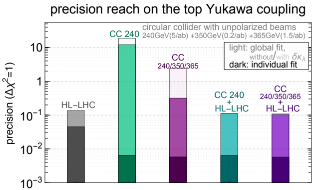
Top operators in loops

[arXiv:1809.03520] G. Durieux, JG, E. Vryonidou, C. Zhang







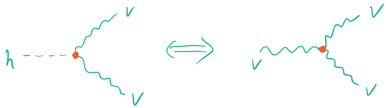


- Higgs precision measurements have sensitivity to the top operators in the loops.
  - But it is challenging to discriminate many parameters in a global fit!
- HL-LHC helps, but a 360 or 365 GeV run is better.
- Indirect bounds on the top Yukawa coupling.

You can't really separate Higgs from the EW gauge bosons!

- $\mathcal{O}_{H\ell} = iH^\dagger \overleftrightarrow{D_\mu} H \bar{\ell}_L \gamma^\mu \ell_L,$
  $\mathcal{O}'_{H\ell} = iH^\dagger \sigma^a \overleftrightarrow{D_\mu} H \bar{\ell}_L \sigma^a \gamma^\mu \ell_L,$
  $\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D_\mu} H \bar{e}_R \gamma^\mu e_R$
  (or the ones with quarks)
  - modifies gauge couplings of fermions,
  - also generates $hVff$ type contact interaction.

- $\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu},$
  $\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
  - generate **aTGCs** $\delta g_{1,Z}$ and $\delta \kappa_\gamma$,
  - also generates HVV **anomalous couplings** such as $hZ_\mu \partial_\nu Z^{\mu\nu}$.



You also have to measure the Higgs!

- Some operators can only be probed with the **Higgs particle**.
- $|H|^2 W_{\mu\nu} W^{\mu\nu}$ and $|H|^2 B_{\mu\nu} B^{\mu\nu}$
  - $H \to v/\sqrt{2}$, corrections to gauge couplings?
  - Can be absorbed by field redefinition! This applies to any operators in the form $|H|^2 \mathcal{O}_{\rm SM}$.

$$
\begin{aligned}
c_{\rm SM} \mathcal{O}_{\rm SM} \quad \textit{vs.} \quad & c_{\rm SM} \mathcal{O}_{\rm SM} + \frac{c}{\Lambda^2} |H|^2 \mathcal{O}_{\rm SM} \\
&= (c_{\rm SM} + \frac{c\, v^2}{2\, \Lambda^2}) \mathcal{O}_{\rm SM} + \text{terms with } h \\
&= c'_{\rm SM} \mathcal{O}_{\rm SM} + \text{terms with } h
\end{aligned}
$$

  - probed by measurements of the $h\gamma\gamma$ and $hZ\gamma$ couplings, or the hWW and hZZ anomalous couplings.
  - or Higgs in the loop (different story...)
- Yukawa couplings, Higgs self couplings, ...

Why lepton colliders?

- EFT is good for lepton colliders.
  - A systematic parameterization of Higgs (and other) couplings.
- Lepton colliders are also good for EFT!
  - High precision $\Rightarrow$ $E \ll \Lambda$
    Ideal for EFT studies!
  - LHC is built for discovery, but ....

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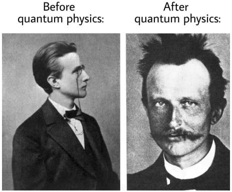
- **Energy** *vs.* **Precision**
  - Poor measurements at the high energy tails lead to problems in the interpretation of EFT...

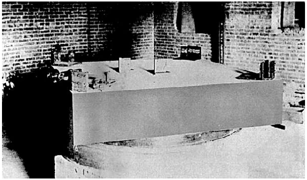
A lesson from history

- In 1875, a young Max Planck was told by his advisor Philipp von Jolly not to study physics, since there was nothing left to be discovered.
    - **Planck did not listen.**

Max Planck:



- In 1887, Michelson and Morley tried to find ether, the postulated medium for the propagation of light that was widely believed to exist.
    - **They didn't find it.**



- **"Our future discoveries must be looked for in the sixth place of decimals." — Albert A. Michelson**

Conclusion

Waiting for a future lepton collider to be built...