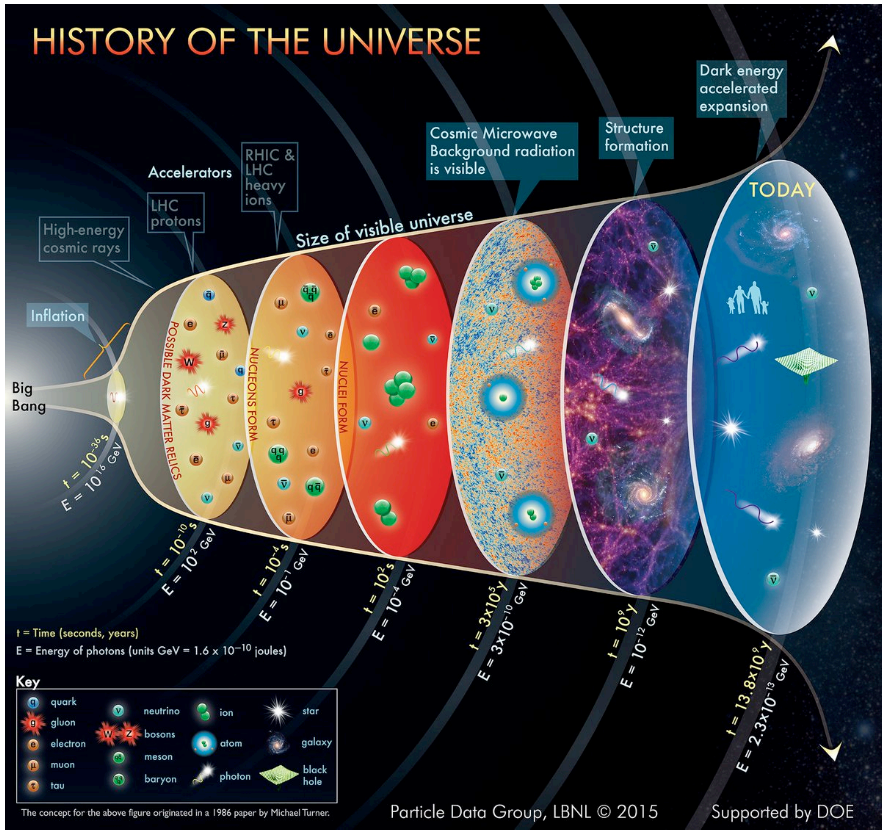
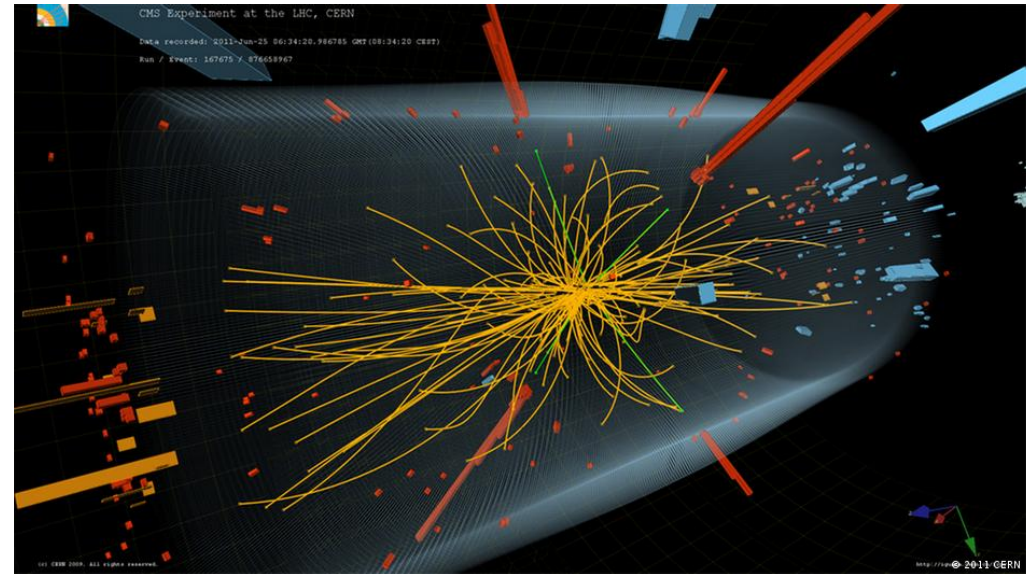
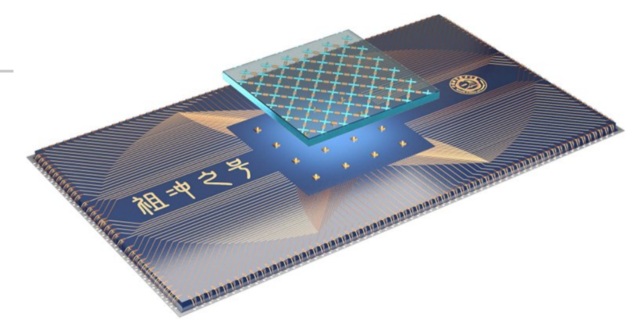
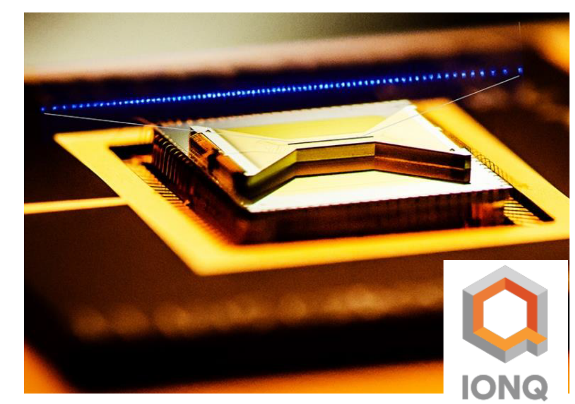








中国科学技术大学
University of Science and Technology of China

Ying-Ying Li *yingyingli@ustc.edu.cn*

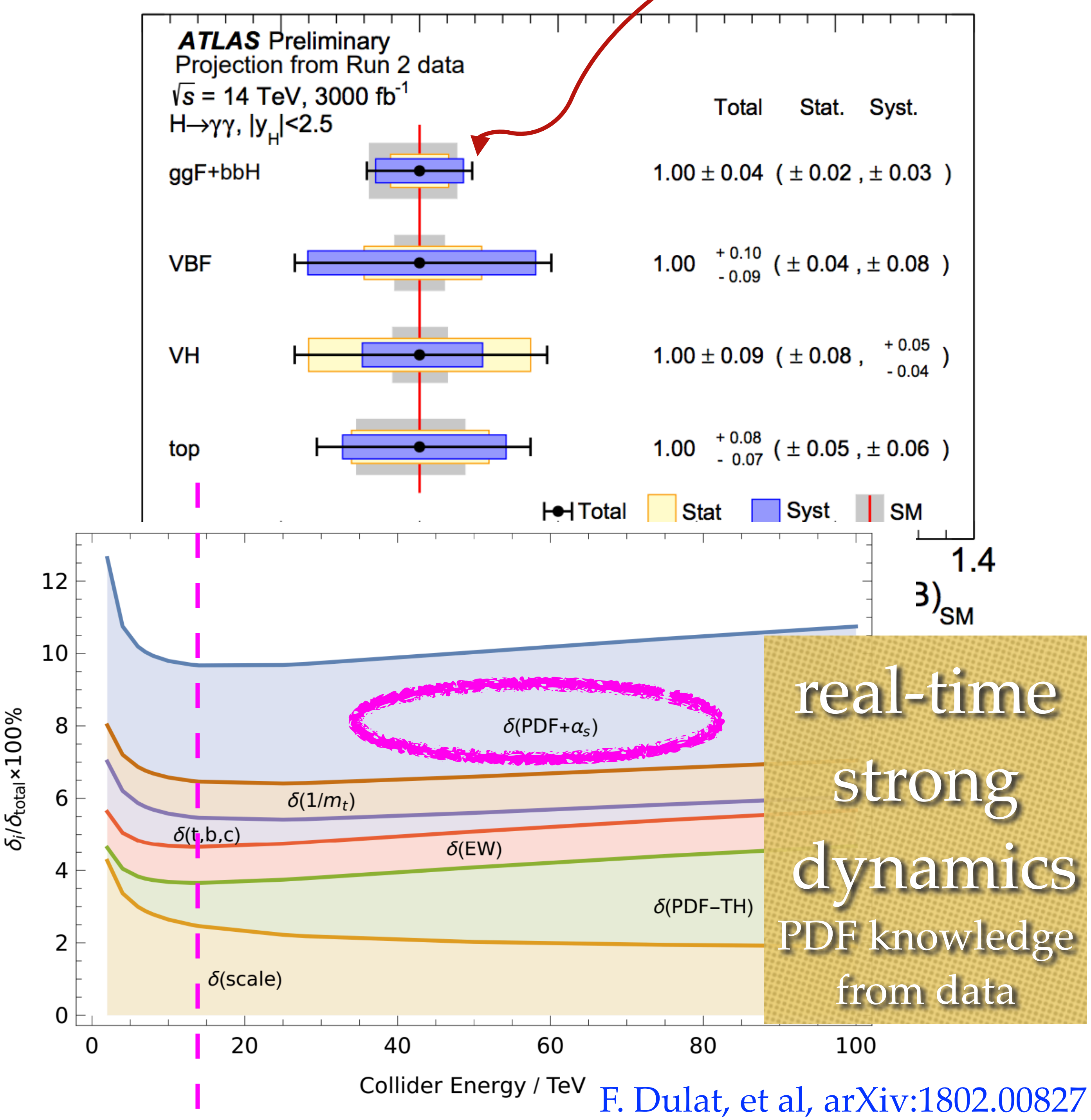
Quantum Simulation for High Energy Physics

Mar. 29, 2023 @IHEP

Theoretical inputs to colliders

theoretical uncertainties

PDF



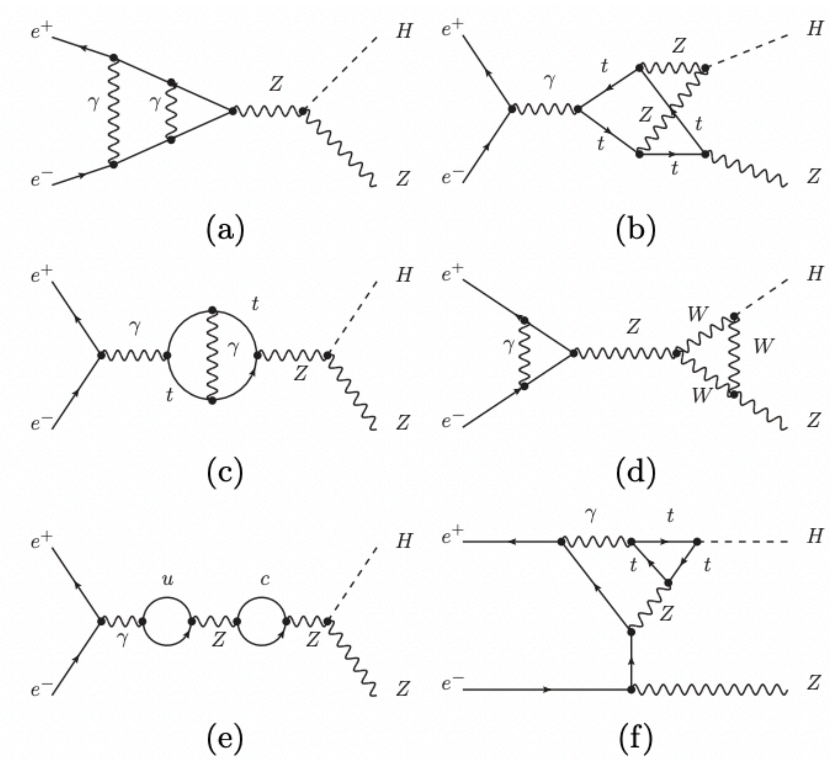
F. Dulat, et al, arXiv:1802.00827
M. Cepeda, et al, arXiv:1902.00134

higher order corrections

$$\text{CEPC}: \sigma(e^+e^- \to ZH), 0.51\%$$

NLO EW
NNLO EW-QCD 1%

complete two loop

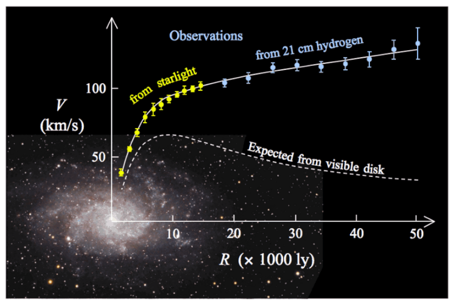


F. An, et al, arXiv:1810.09037
Y. Gong et al., Q. -F. Sun et al.,
X. Chen et al., arXiv: 2209.14953

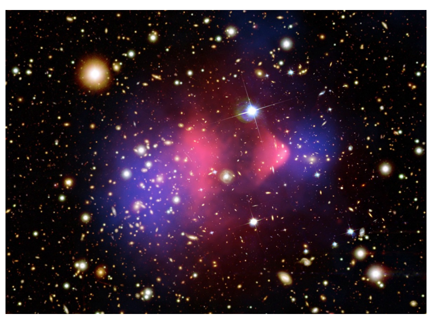
Puzzles We Encountered

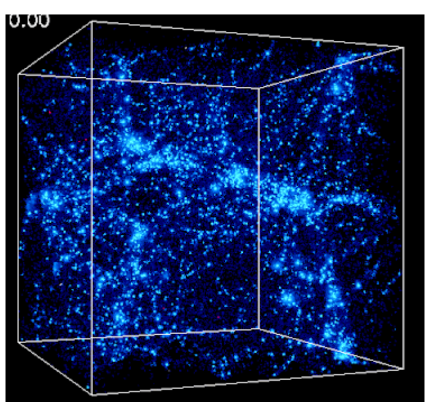
1

1933-Now
dark matter



rotation curves


bullet cluster


structure formation

axion dark matter-misalignment

$$m_a^2(T) f_a^2 \propto \frac{\partial^2 F(\theta, T)}{\partial \theta^2}\Big|_{\theta=0}$$

$F(\theta, T)$

QCD free energy

lattice non-perturbative calculations

$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$

Imaginary time problem : $W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$

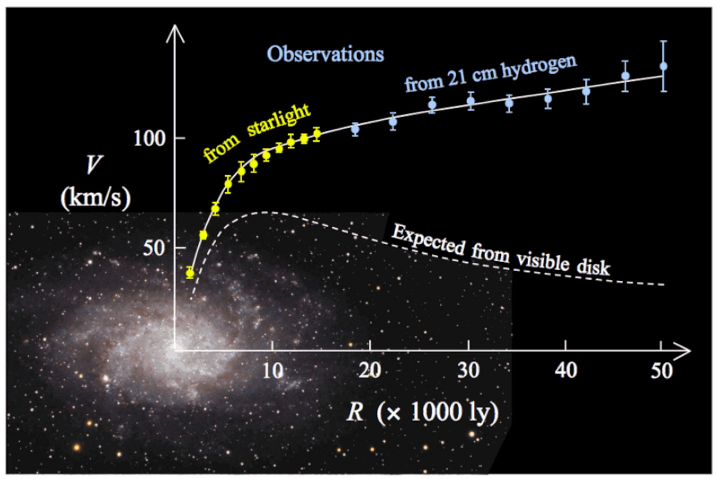
complex $S(\mathcal{C})$ for non vanishing θ

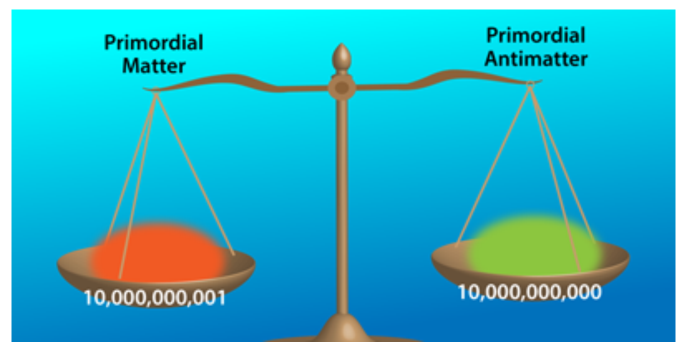
Sign Problem!!!

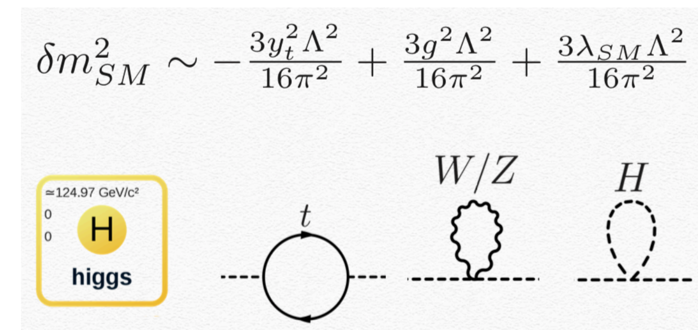
configuration space $\mathcal{C}$ is exponentially large in system size

Puzzles encountered

1933-Now dark matter	1940s-Now baryon asymmetry	1967-Now naturalness problem







- WIMP dark matter freeze-out,
- axion dark matter (relic abundance depends on the QCD topological term—complex action)

Sakharov

Out of equilibrium

C- and CP-violation

B-violation

- Composite Higgs (strong dynamics),
- Higgs Precision measurement (higher-order rare processes—rare events)

out-of-equilibrium,
non-perturbative
higher order processes,
quantum interference
cannot be solved classically
due to theoretical or computational limitations:

sign problem or rare events that has exponential-scaling of the complexity

$$\langle x | e^{-iHt} | y \rangle = \int \mathcal{D}\phi \, e^{iS}$$

[arXiv:2204.03381]

Quantum Simulation for High Energy Physics

Christian W. Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[5] Junyu Liu,[14,15] Mikhail Lukin,[16] Yannick Meurice,[17] Christopher Monroe,[18,19,20,21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23] Enrico Rinaldi,[24,25,26] Alessandro Roggero,[27,28] David I. Santiago,[29,30] Martin J. Savage,[31] Irfan Siddiqi,[29,30,32] George Siopsis,[33] David Van Zanten,[5] Nathan Wiebe,[34,35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

$$\langle x | e^{-iHt} | y \rangle = \int \mathcal{D}\phi \, e^{iS}$$

[arXiv:2204.03381]

Quantum Simulation for High Energy Physics

Christian W. Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[5] Junyu Liu,[14,15] Mikhail Lukin,[16] Yannick Meurice,[17] Christopher Monroe,[18,19,20,21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23] Enrico Rinaldi,[24,25,26] Alessandro Roggero,[27,28] David I. Santiago,[29,30] Martin J. Savage,[31] Irfan Siddiqi,[29,30,32] George Siopsis,[33] David Van Zanten,[5] Nathan Wiebe,[34,35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

[Snowmass 2021 LOI]

Practical Quantum Advantages in High Energy Physics

Marcela Carena,[1,2,3,*] Henry Lamm,[1,†] Scott Lawrence,[4,‡] Ying-Ying Li,[1,§] Joseph D. Lykken,[1,¶] Lian-Tao Wang,[2,**] and Yukari Yamauchi[5,††]

[1] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[2] *Enrico Fermi Institute, University of Chicago, Chicago, Illinois, 60637, USA*
[3] *Kavli Institute for Cosmological Physics, University of Chicago, Chicago, Illinois, 60637, USA*
[4] *1Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
[5] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

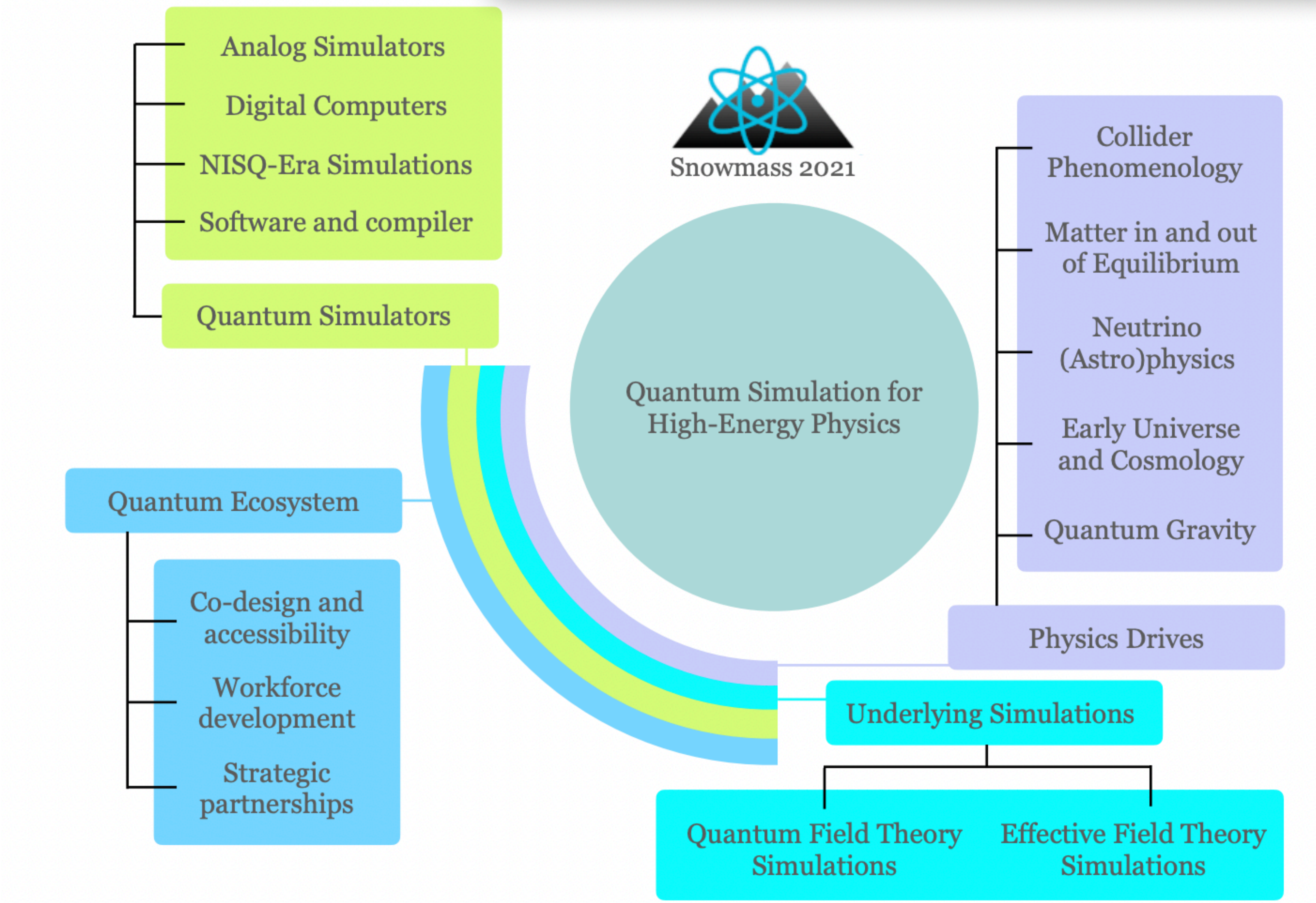
In this LOI, we discuss open questions in cosmology and particle physics whose solutions would demonstrate practical quantum advantage – solving a problem of *interest* using quantum hardware that is impractical for classical resources. Arriving at these calculation will require theoretical developments in nonperturbative and nonequilibrium physics along side improved quantum algorithms.

[arXiv:2204.03381]

Quantum Simulation for High Energy Physics

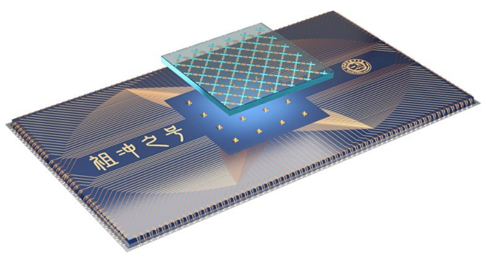
Christian W. Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[5] Junyu Liu,[14,15] Mikhail Lukin,[16] Yannick Meurice,[17] Christopher Monroe,[18,19,20,21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23] Enrico Rinaldi,[24,25,26] Alessandro Roggero,[27,28] David I. Santiago,[29,30] Martin J. Savage,[31] Irfan Siddiqi,[29,30,32] George Siopsis,[33] David Van Zanten,[5] Nathan Wiebe,[34,35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

$$\langle x|e^{-iHt}|y\rangle = \int \mathcal{D}\phi e^{iS}$$

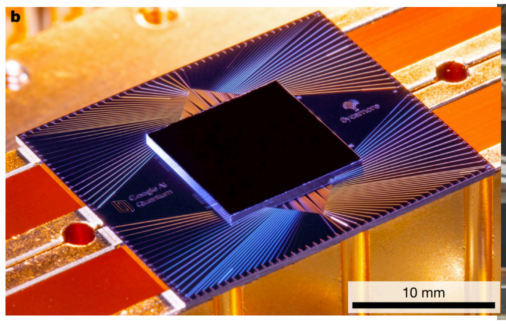


A range of quantum simulators with varying capacity and capability

Superconducting Processor



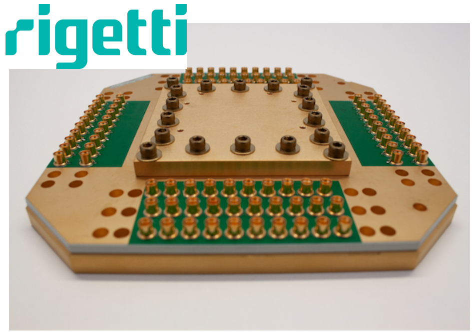
66 qubits



54 qubits

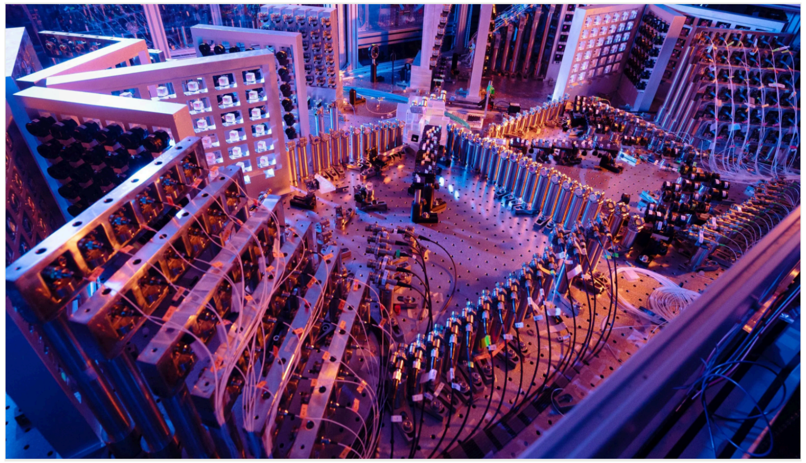
127 qubits

multi-chip quantum processor



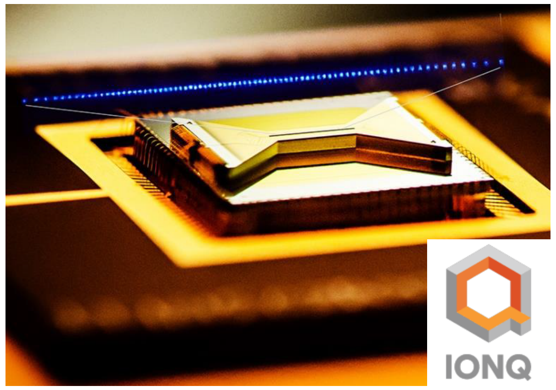
80 qubits

Photon qubits



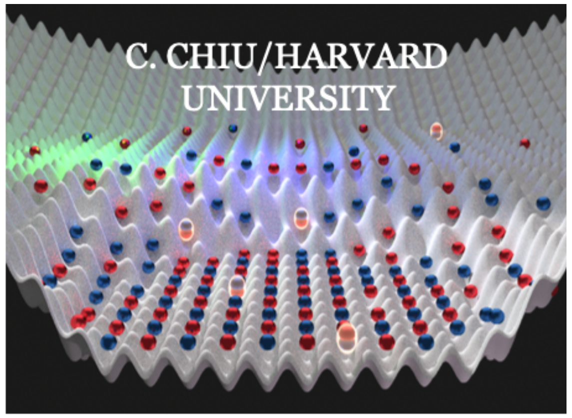
九章- 76 qubits

trapped ion qubits

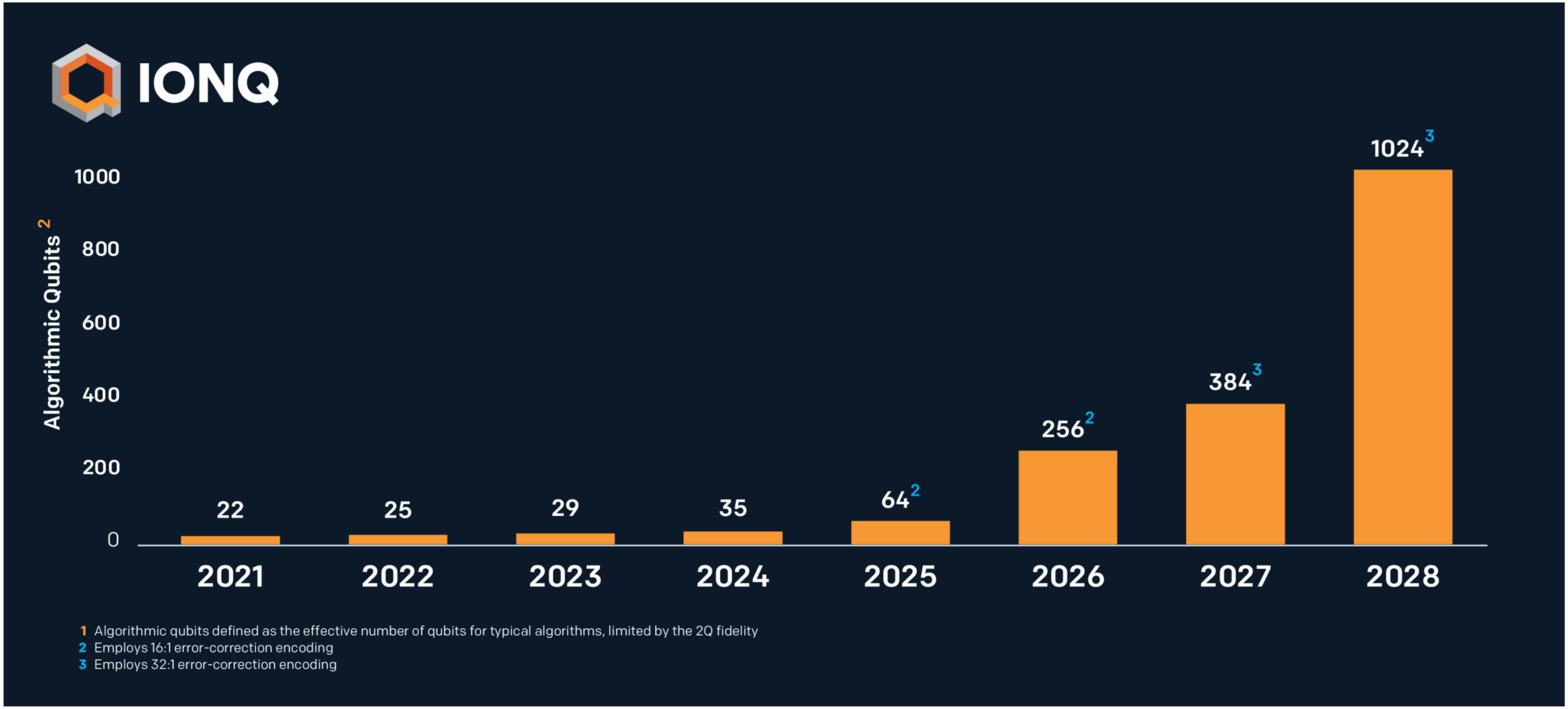


22 qubits

Analog quantum simulator



A range of quantum simulators with varying capacity and capability



rapid development
still limited resources

History of LQCD

From Henry Lamm's talk

quantize a gauge field theory on a discrete lattice in Euclidean space-time,

(1970s) Formulate the problem

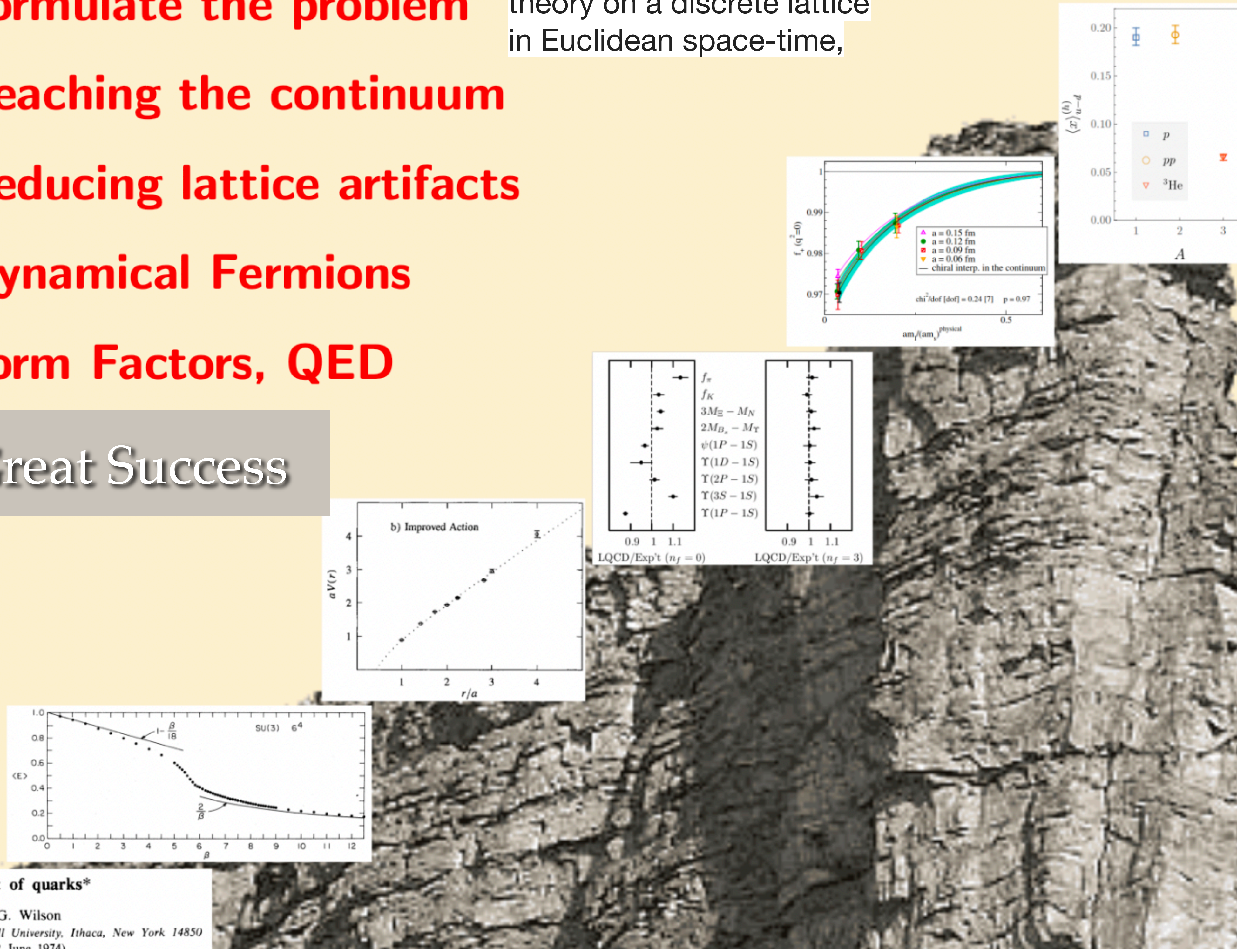
(1980s) Reaching the continuum

(1990s) Reducing lattice artifacts

(2000s) Dynamical Fermions

(2010s) Form Factors, QED

Now: Great Success



History of LQCD

From Henry Lamm's talk

(1970s) Formulate the problem

quantize a gauge field theory on a discrete lattice in Euclidean space-time,

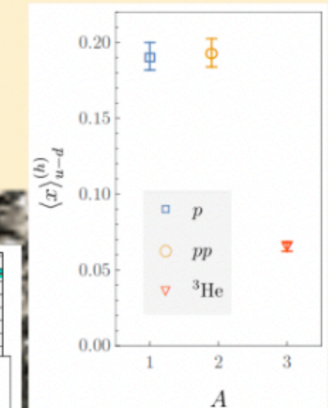
(1980s) Reaching the continuum

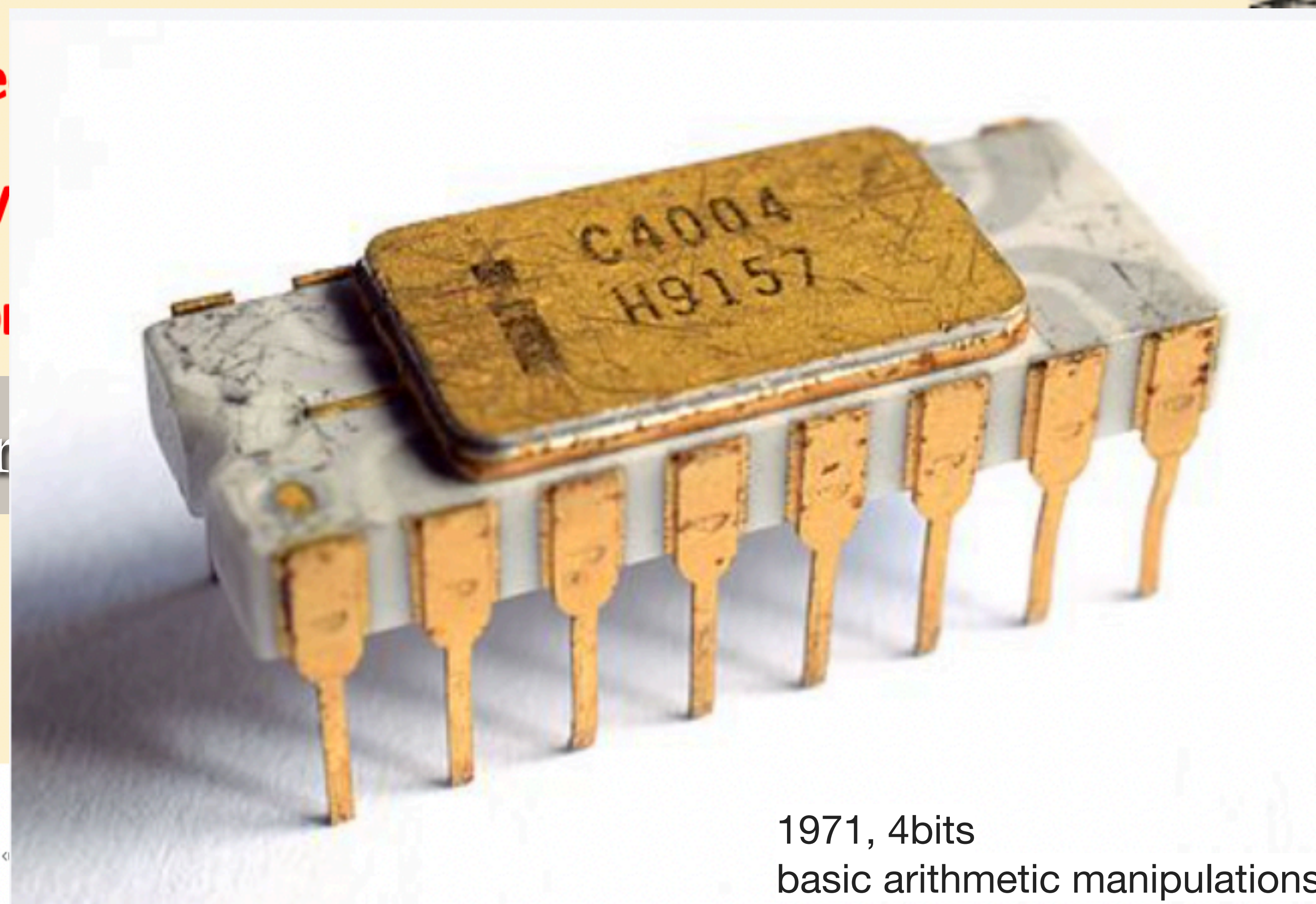
(1990s) Re

(2000s) Dy

(2010s) Fo

Now: Gr





1971, 4bits
basic arithmetic manipulations

Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

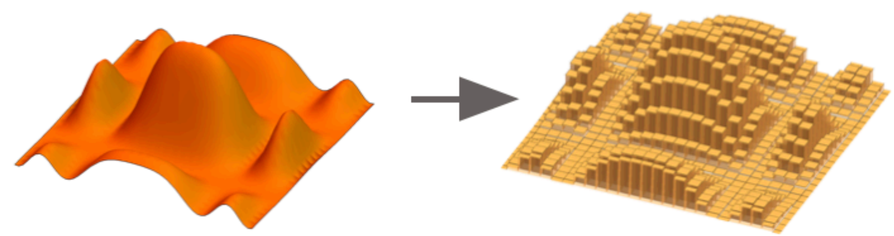
Quantum Simulation for Quantum Field Theory

Bosonic and fermionic DOF,
Dynamical and coupled global and local (gauge) symmetries,
Relativistic - particle number non-conservation,
Nontrivial vacuum state in strongly interacting theories

Quantum Simulation for Quantum Field Theory

Bosonic and fermionic DOF,
Dynamical and coupled global and local (gauge) symmetries,
Relativistic - particle number non-conservation,
Nontrivial vacuum state in strongly interacting theories

"Champagne Problems"

Discretization		*KS Hamiltonian*
Digitization	$\|q\rangle^N \to \|G\rangle$	*truncation, discrete subgroup, …*
	↓	
Initialization	$\mathcal{U}\,\|G\rangle^L \to \|\psi_0\rangle$	*ground/thermal/particle state prep*
	↓	
Propagation	$\mathcal{U}\,\|\psi_0\rangle \to \|\psi(t)\rangle$	*Trotterization, variational approach, Taylor series, …*
	↓	
Evaluation	$\langle\mathcal{O}\rangle$	*Parton distribution function, particle decay, …*
Error mitigation		

See references in [M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

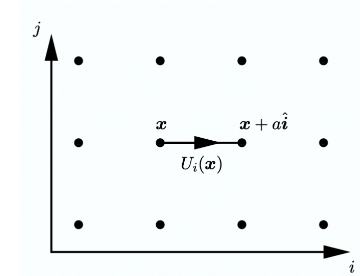
Discretization

infinities in QFT

$$H = \int d^d x \mathrm{Tr}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$U_\square = \exp\left\{ig \oint_\square A \cdot dx\right\}$$

gauge invariance



$$U_i(x) = e^{ig \int_a^0 dt A_i(x + t\hat{i})}$$

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t \sum_{\langle xy \rangle} s_{xy} \left(\psi_x^\dagger U_{xy} \psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m \sum_x s_x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2} \sum_\square \mathrm{Tr}\left(U_\square + U_\square^\dagger\right)$$

fermion kinetic | fermion mass | energy of electric field | energy of magnetic field

Discretization

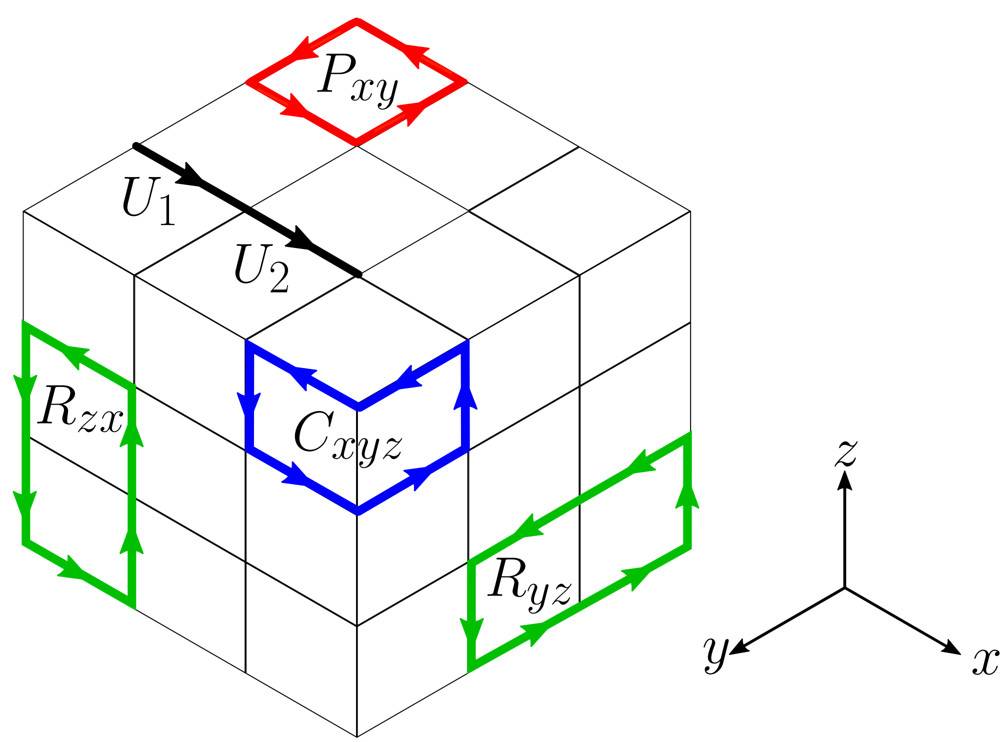
[J. Carlsson, et al, hep-lat/0105018]

$$P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\exp\left\{ig\oint_{\Box} A\cdot dx\right\} \approx \frac{g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{g^2a^6}{12N}\mathrm{Tr}\left\{F_{ij}(x)(D_i^2+D_j^2)F_{ij}(x)\right\} + \ldots$$

deviations from the continuum, starts from a^2 error, classical
computational resources proportional a^{-k} to for Wilson action

$$R_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{\,\square_{i}^{\,j}\,\right\} = \frac{4g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{4g^2a^6}{24N}\mathrm{Tr}\left\{F_{ij}(x)(4D_i^2+D_j^2)F_{ij}(x)\right\} + \ldots$$

deviations from the
continuum starts from
a^2g^2 at quantum level



[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Discretization 

[M. Alford, et al, hep-lat/9507010]
[…, …]

With improved action, for Euclidean spacetime at the same error level, simulations can be done with a lattice spacing of at least 2 larger

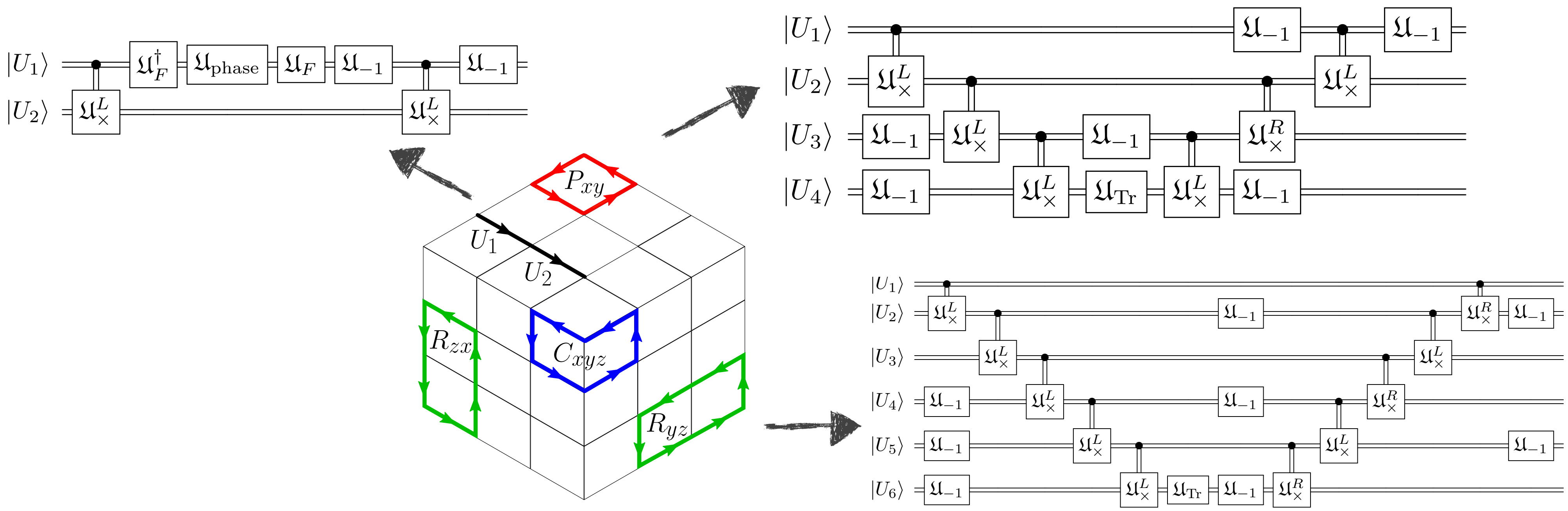
Qubits required

$$N_q \sim \left(\frac{L}{a}\right)^d$$

Only count qubits:
saving us at least 3 years for 3+1d,
assuming number of qubits increases by a factor of 2 each year on hardware

circuits for improved Hamiltonian need to be designed

Discretization 



Gate	$N[\hat{K}_{KS}+\hat{V}_{KS}]$	$N[\hat{K}_{2\text{L}}+\hat{V}_{\text{rect}}]$
$\mathfrak{U}_F$	2	2
$\mathfrak{U}_{\text{phase}}$	1	1
$\mathfrak{U}_{\text{Tr}}$	$\frac{d-1}{2}$	$d-1$
$\mathfrak{U}_{-1}$	$3(d-1)$	$2+8(d-1)$
$\mathfrak{U}_\times$	$6(d-1)$	$4+20(d-1)$

- # of Gates here for a single trotter is increasing only multiplicatively, could be compensated by the decreasing of links.
- Larger trotter steps, instead could be used for improved Hamiltonian.

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x\psi_x^\dagger\psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_{\Box}\mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_\Box \mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

rapid development with its own pros and cons

rate of convergences to the infinite-dimensional theory, resource requirements, local and global gauge symmetry

Casimir dynamics, Natalie et al

LSH formalism, Mathur et al, Anishetty et al

Group-element basis and discrete subgroups, Erez et al, Lamm et al, Carena et al

Magnetic or dual representations, Mathur et al, Bauer et al

Tensor renormalization group (character expansion, Fourier series), Meurice et al

Light-front quantization (light-cone instead of fixed time-slicing), Mannheim et al

Quantum link models/qubit regularization (critical point), Brower et al

Matrix models (dimension reduction), Shen et al

Digitization $\quad |q\rangle^N \rightarrow |G\rangle$

infinities in QFT

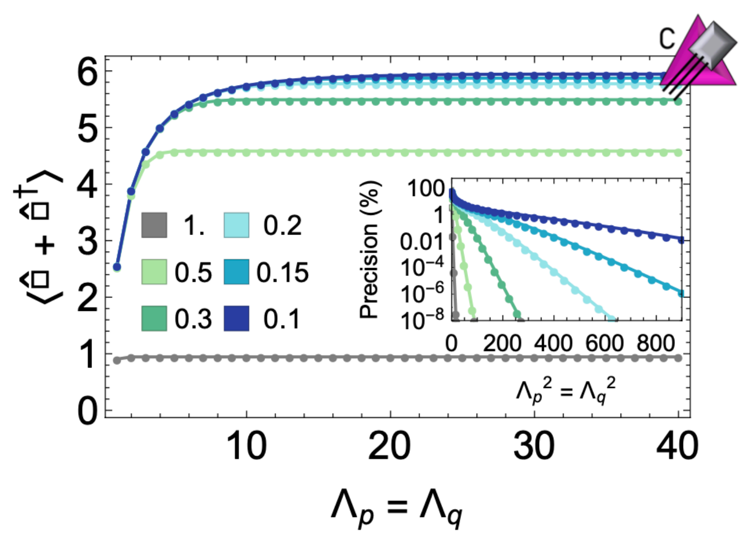
Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x\psi_x^\dagger\psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_\square \mathrm{Tr}\left(U_\square + U_\square^\dagger\right)$$

continuous field variables

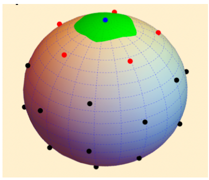
In angular momentum basis,
truncated with cut-off
SU(3) for one plaquette

$$\sum_b |\hat{\mathbf{E}}^{(b)}|^2\, |p,q\rangle = \frac{p^2+q^2+pq+3p+3q}{3}\, |p,q\rangle$$



Ciavarella, Klco, and Savage,
arXiv:2101.10227 [quant-ph]

group element basis -
truncated with discrete subgroup



$\xi_{1-\mathrm{loop}}$	ξ	
	$\mathbb{BI}$	$SU(2)$ [111]
2.097	2.099(1)	...
4.278	...	4.35(19)
4.207	...	4.22(11)
1.351	1.369(19)	...
4.136	...	4.08(9)
1.351	1.36(1)	...

Carena, Gustafson, Lamm,
YYL, Liu, PRD **106**, 114504 (2022)

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x\psi_x^\dagger\psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_\Box \mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

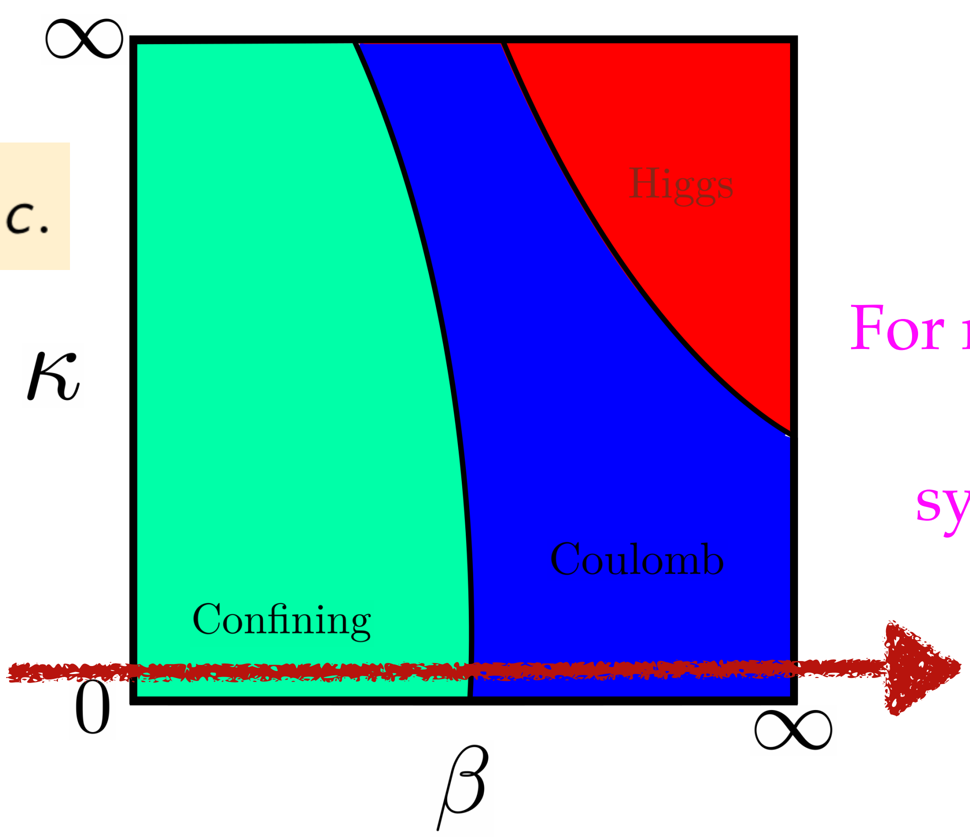
Gauge field truncations with discrete group

$$S = \beta\sum \mathrm{Re\,Tr}\,U_P + \kappa\sum \phi\cdot D\{U_P\}\cdot\phi^\dagger + c.c.$$

discrete group

continuous group + Higgs

continuum limit for U(1)



For non-abelian gauge symmetry?

[Fradkin, Shenker, PRD. 19. 3682]

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x\psi_x^\dagger\psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_{\Box}\mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

Gauss's law operator $G^a(x) = -E_L^a(x) + E_R^a(x-1) + \psi^\dagger(x)T^a\psi(x)$ $\qquad G_x^a|P\rangle = 0$

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

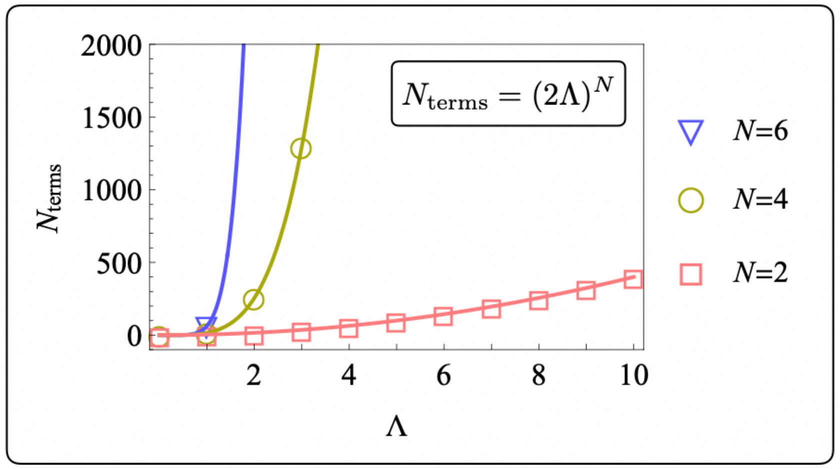
Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_\Box \mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

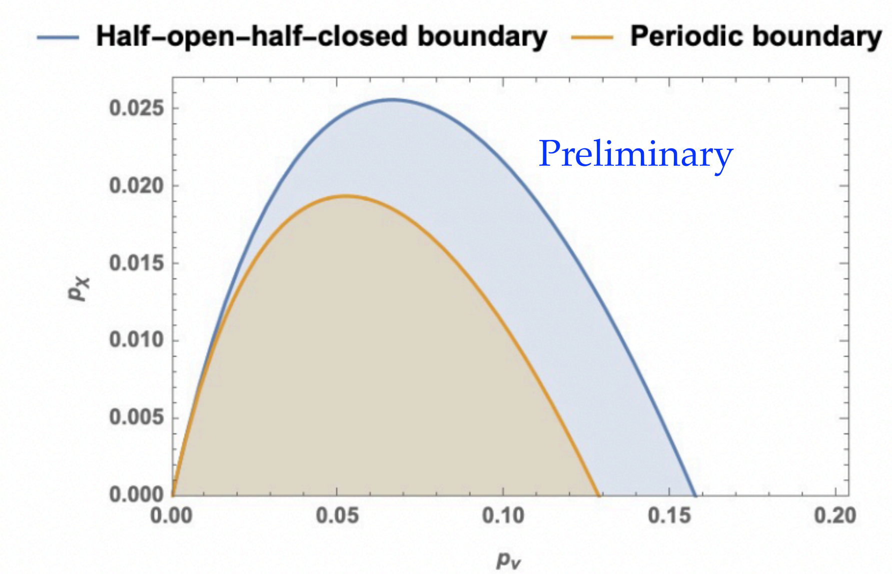
Gauss's law operator $G^a(x) = -E_L^a(x) + E_R^a(x-1) + \psi^\dagger(x)T^a\psi(x)$ $\quad G_x^a|P\rangle = 0$

To present a physical state in the angular momentum basis



Davoudi, Raychowdhury, and Shaw, arXiv:2009.11802 [hep-lat]

Gauge fixing in higher dimensions, and resilience to quantum errors?

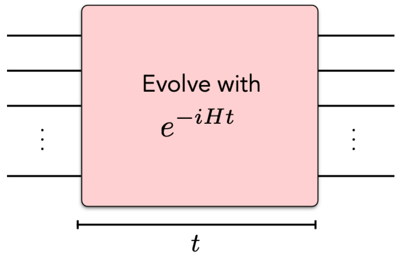


M. Carena, H. Lamm, **YYL**, W. Liu

Gauge symmetry used for error corrections, see Halimeh, et al. Lamm, et al. …

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

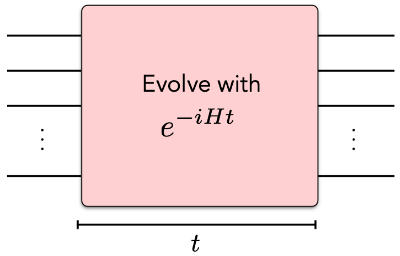
ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
...

Propagation $\mathcal{U}|\psi_0\rangle \to |\psi(t)\rangle$

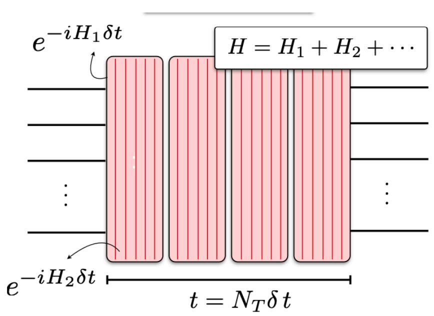
ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
…

DIGITAL

superconducting qubit/trapped-ion system



building blocks:
one-qubit/two-qubit gate set

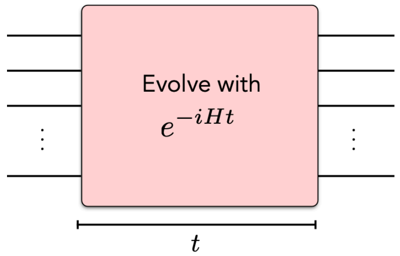
$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

optimal asymptotically?
overload of resources?
easy implementation?

[Bauer et al, arXiv:2204.03381]

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

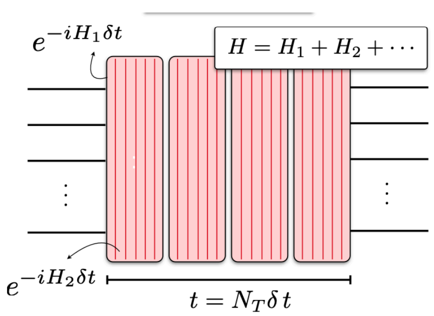
ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
…

DIGITAL

superconducting qubit / trapped-ion system



building blocks:
one-qubit / two-qubit gate set

$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

optimal asymptotically?
overload of resources?
easy implementation?

Trotter-Suzuki decomposition

$$H = \sum_{l=1}^{\Gamma} H^{(l)}$$

$$\mathcal{U} = \left[\prod_{l=1}^{\Gamma} e^{-itH^{(l)}/r}\right]^r$$

p-th order trotterization: $\mathcal{O}\left((\frac{t}{r})^p\right)$

Errors depends on t and r
No ancillary overhead
Simpler implementation

Taylor series expansion (LCU)

$$e^{-iHt} = (e^{-iHt/r})^r \equiv V^r$$

$$V \approx \tilde{V} = \sum_{k=0}^{K} \frac{1}{k!}\left(\frac{-iHt}{r}\right)^k$$

$$\mathcal{U} = \tilde{V}^r$$

$$||\tilde{V} - V|| < \epsilon/r$$

K values depends on the aimed errors
Ancillary qubits are needed
Complex circuits implementation

Quantum singular value transformation

$$e^{-iHt} = \cos(Ht) - i\sin(Ht)$$

$$e^{i\phi_0\sigma_z}\prod_{j=1}^{k}\left(W(x)e^{i\phi_j\sigma_z}\right) = \begin{bmatrix} P(x) & iQ(x)\sqrt{1-x^2} \\ iQ^*(x)\sqrt{1-x^2} & P^*(x) \end{bmatrix}$$

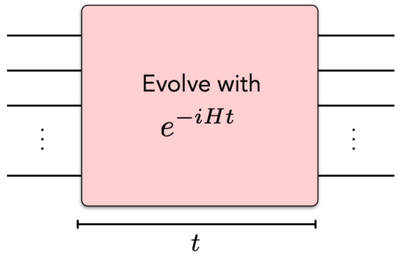
$$W(x) := \begin{bmatrix} x & i\sqrt{1-x^2} \\ i\sqrt{1-x^2} & x \end{bmatrix}$$

Jacobi-Anger expansion for cos and sin

error: truncation order of the expansion
Ancillary qubits are needed
Complex circuits implementation

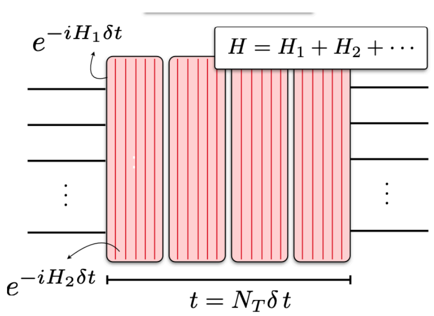
Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
…

DIGITAL superconducting qubit/trapped-ion system



building blocks:
one-qubit/two-qubit gate set

$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

optimal asymptotically?
overload of resources?
easy implementation?

Trotter-Suzuki decomposition

$$H = \sum_{l=1}^{\Gamma} H^{(l)}$$

$$\mathcal{U} = \left[\prod_{l=1}^{\Gamma} e^{-itH^{(l)}/r}\right]^r$$

p-th order trotterization: $\mathcal{O}\left((\frac{t}{r})^p\right)$

Errors depends on t and r
No ancillary overhead
Simpler implementation

Taylor series expansion (LCU)

$$e^{-iHt} = (e^{-iHt/r})^r \equiv V^r$$

$$V \approx \tilde{V} = \sum_{k=0}^{K} \frac{1}{k!}(\frac{-iHt}{r})^k$$

$$\mathcal{U} = \tilde{V}^r$$

$$||\tilde{V} - V|| < \epsilon/r$$

K values depends on the aimed errors
Ancillary qubits are needed
Complex circuits implementation

Quantum singular value transformation

$$e^{-iHt} = \cos(Ht) - i\sin(Ht)$$

$$e^{i\phi_0\sigma_z}\prod_{j=1}^{k}\left(W(x)e^{i\phi_j\sigma_z}\right) = \begin{bmatrix} P(x) & iQ(x)\sqrt{1-x^2} \\ iQ^*(x)\sqrt{1-x^2} & P^*(x) \end{bmatrix}$$

$$W(x) := \begin{bmatrix} x & i\sqrt{1-x^2} \\ i\sqrt{1-x^2} & x \end{bmatrix}$$

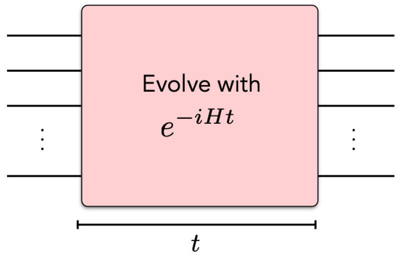
Jacobi-Anger expansion for cos and sin

error: truncation order of the expansion
Ancillary qubits are needed
Complex circuits implementation

Quantum signal processing, blocking encoding, off-diagonal Hamiltonian expansion, etc… [Bauer et al, arXiv:2204.03381]

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

ANALOG



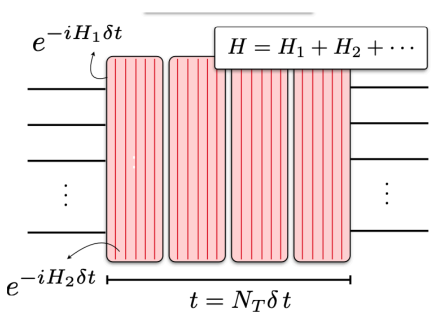
Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics

Superconducting circuits

...

DIGITAL

superconducting qubit / trapped-ion system



building blocks:

one-qubit / two-qubit gate set

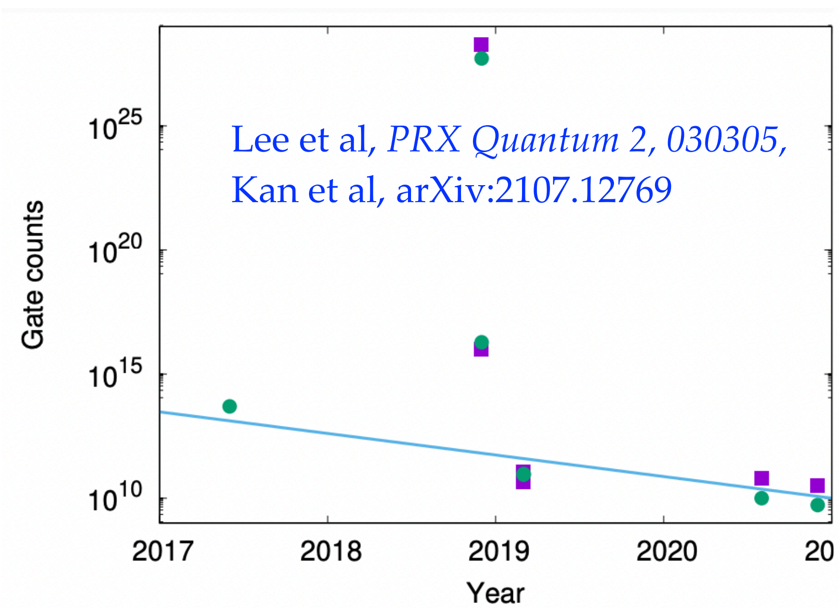
$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

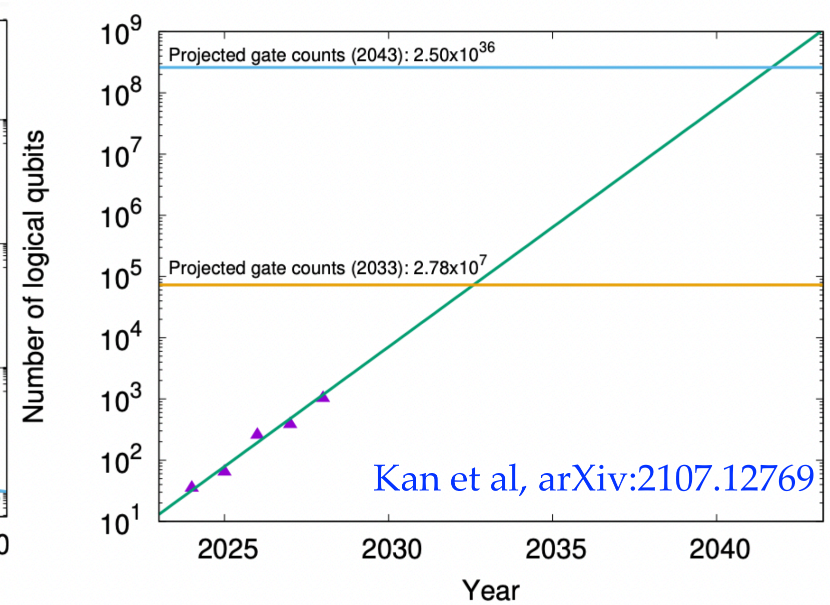
Trotter-Suzuki decomposition

$$H = \sum_{l=1}^{\Gamma} H^{(l)}$$

$$\mathcal{U} = \left[\prod_{l=1}^{\Gamma} e^{-itH^{(l)}/r}\right]^r$$

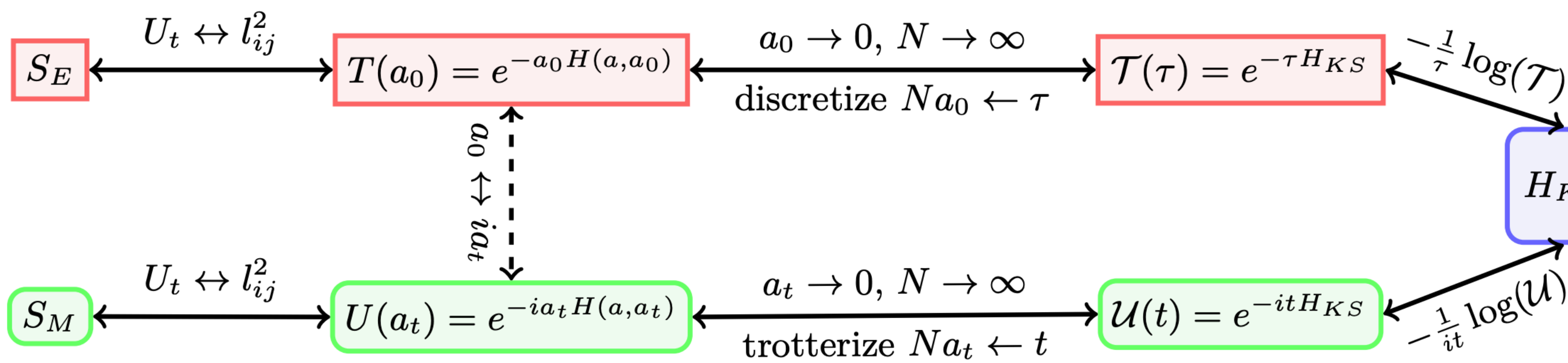
RESOURCE ESTIMATION AND CIRCUITS CONSTRUCTION IMPROVEMENT

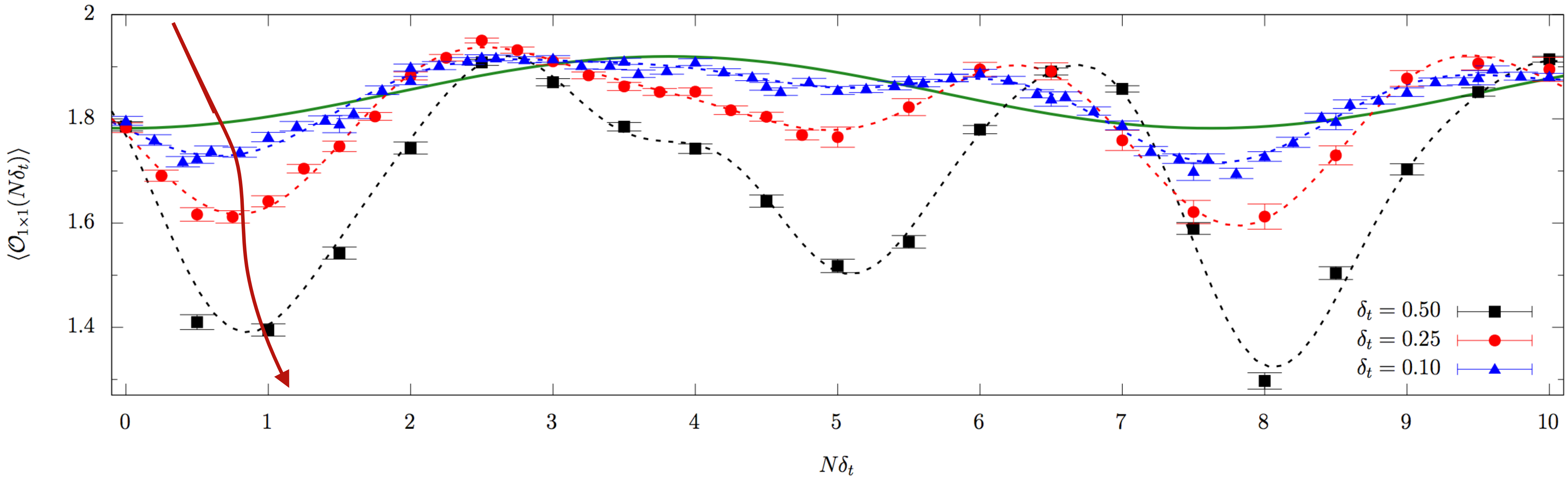




Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

RENORMALIZATION EFFECTS FOR TROTTERIZATION





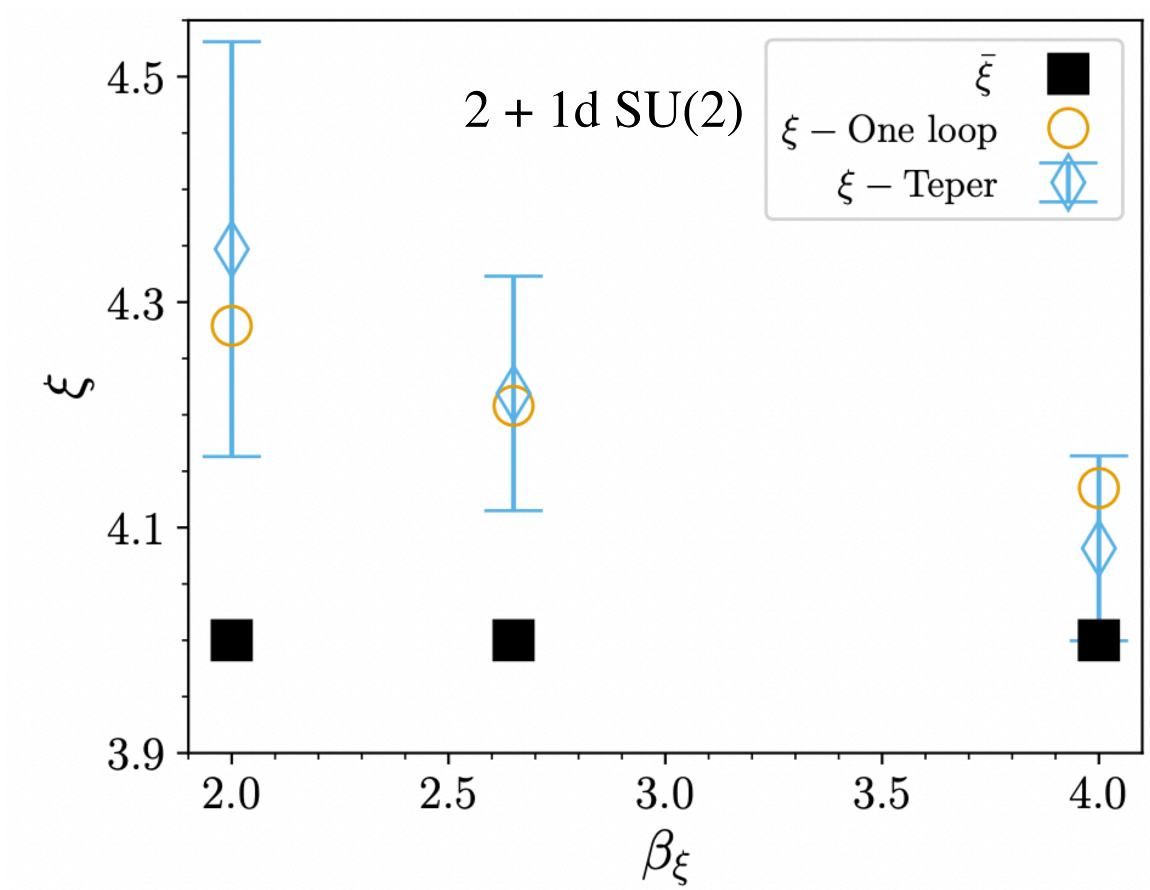
$$T(\delta_\tau, g_H^2) = e^{-\delta_\tau \overline{H}_V/2} e^{-\delta_\tau \overline{H}_K} e^{-\delta_\tau \overline{H}_V/2} \longleftrightarrow U(\delta_t, g_H^2) = e^{-i\delta_t \overline{H}_V/2} e^{-i\delta_t \overline{H}_K} e^{-i\delta_t \overline{H}_V/2}$$

[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

Anisotropic Parameter $\xi = a/a_t$ *Renormalization*

- numerical results is pretty tedious—saving measurement on the Euclidean side
- Preferred for analytical continuation
- Determine the fixed anisotropic trajectory
- Continuous group agrees quite well with their discrete subgroups

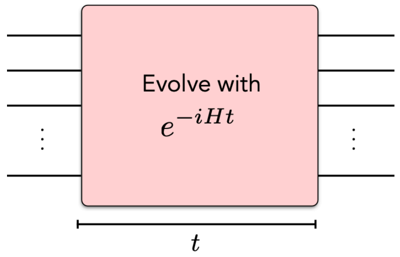


β_ξ	N_s	N_t	$\bar{\xi}$	$\xi_{1-\mathrm{loop}}$	ξ	
					$\mathbb{BI}$	$SU(2)$ [111]
$D=3$						
2.00	36	72	2.00	2.097	2.099(1)	⋯
2.00	12[a]	60[a]	4.00	4.278	⋯	4.35(19)
2.65	16[a]	64[a]	4.00	4.207	⋯	4.22(11)
3.00	36	72	1.33	1.351	1.369(19)	⋯
4.00	24[a]	96[a]	4.00	4.136	⋯	4.08(9)
$D=4$						
3.0	36	72	1.33	1.351	1.36(1)	⋯

[M. Carena, E. Gustafson, H. Lamm, **YYL**, W. Liu, PRD 106 11, 114504]

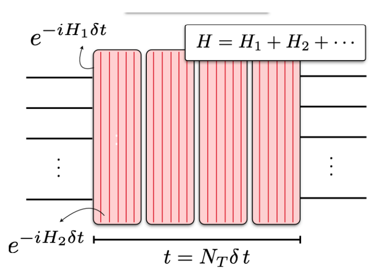
Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
…

DIGITAL superconducting qubit/trapped-ion system



building blocks:
one-qubit/two-qubit gate set

$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

optimally asymptotically?
overload of resources?
easy implementation?

HYBRID METHOD

Casanova et al (2011), Davoudi et al (2021) [trapped ion]

Harmalkar et al (2022) classical preprocessing

Zohar et al (2017), Bender et al (2018) effective interactions

Klco et al (2018), Kokail et al (2019), Atas et al (2021) state preparations

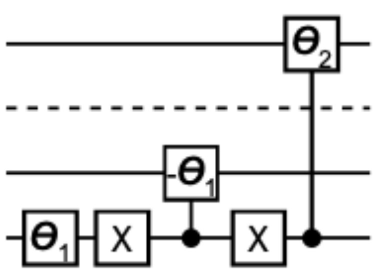
Peruzzo et al (2014), Farhi et al (2014) optimization methods

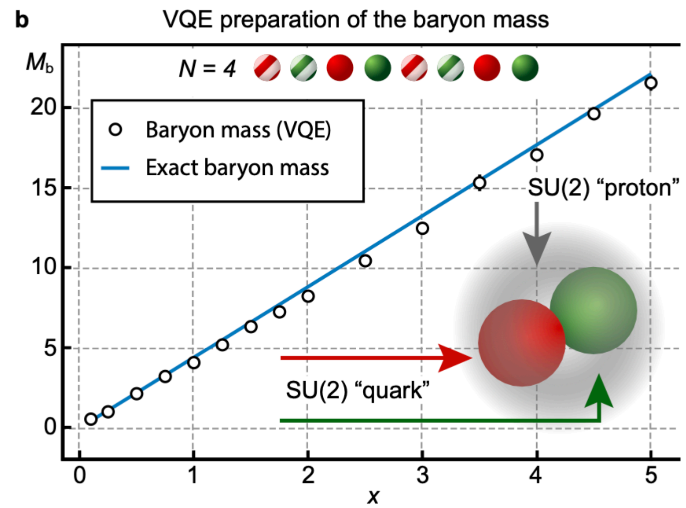
To reach the observables — How to do …

INITIAL STATE PREP

hadronic state, topological vacuum state, thermal state, etc

Hybrid methods with VQE protocols



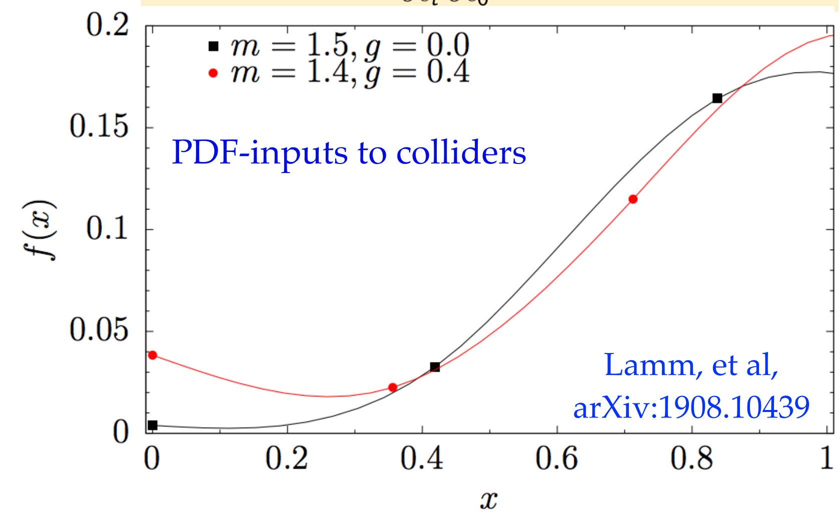


Atas et al, Nat Commun 12, 6499 (2021)

MEASUREMENTS

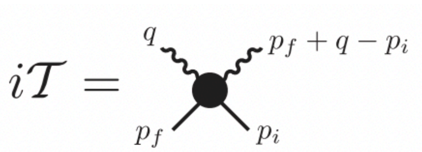
time-separated correlators, exponentially suppressed process, entanglement, etc

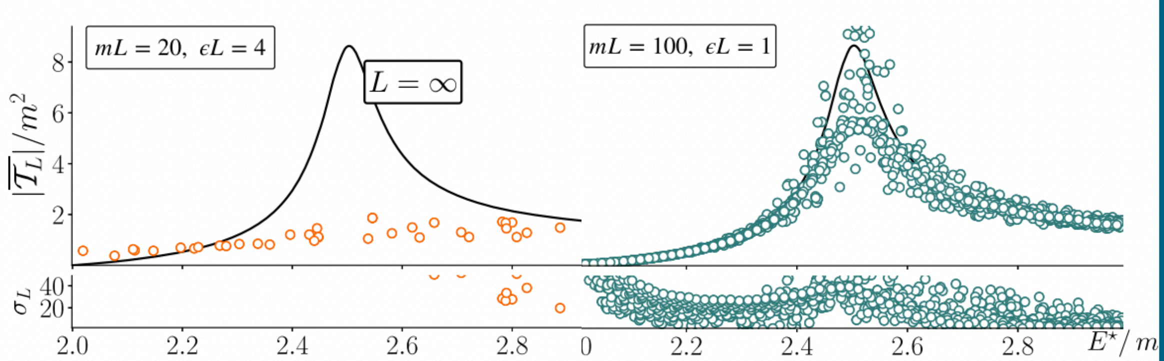
$$\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{\partial}{\partial\epsilon_t}\frac{\partial}{\partial\epsilon_0}\langle\Psi|e^{-iHt}e^{-i\mathcal{O}\epsilon_t}e^{iHt}e^{i\mathcal{O}\epsilon_0}|\Psi\rangle$$



SYSTEMATIC UNCERTAINTIES

finite volume effects, truncation errors, convergence rate



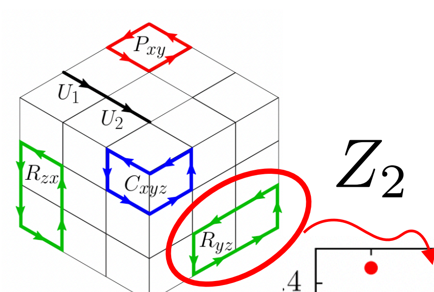


Briceno, PRD 103, 014506 (2021)

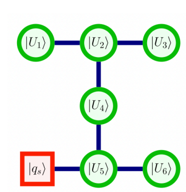
ERROR CORRECTIONS

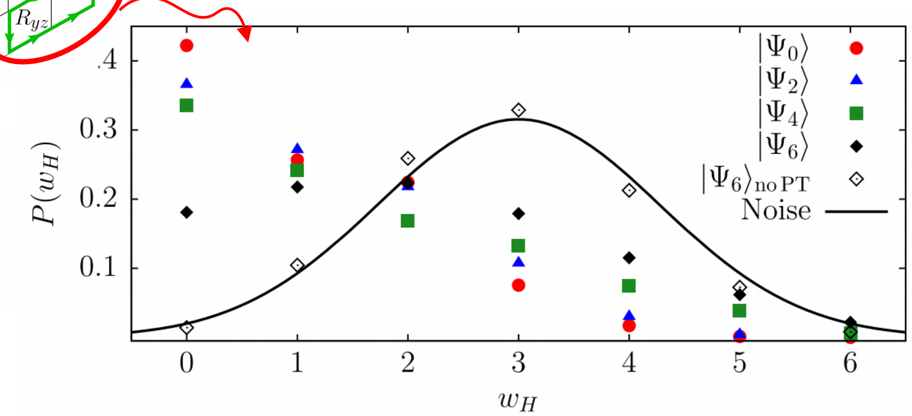
gate error - stochastic and coherent errors

readout errors



$$Z_2 \quad \left[\prod_i (\sigma_i^{b_i})^{\otimes}\right]\text{CNOT}\otimes 1_4\left[\prod_i (\sigma_i^{a_i})^{\otimes}\right] = \text{CNOT}\otimes 1_4$$



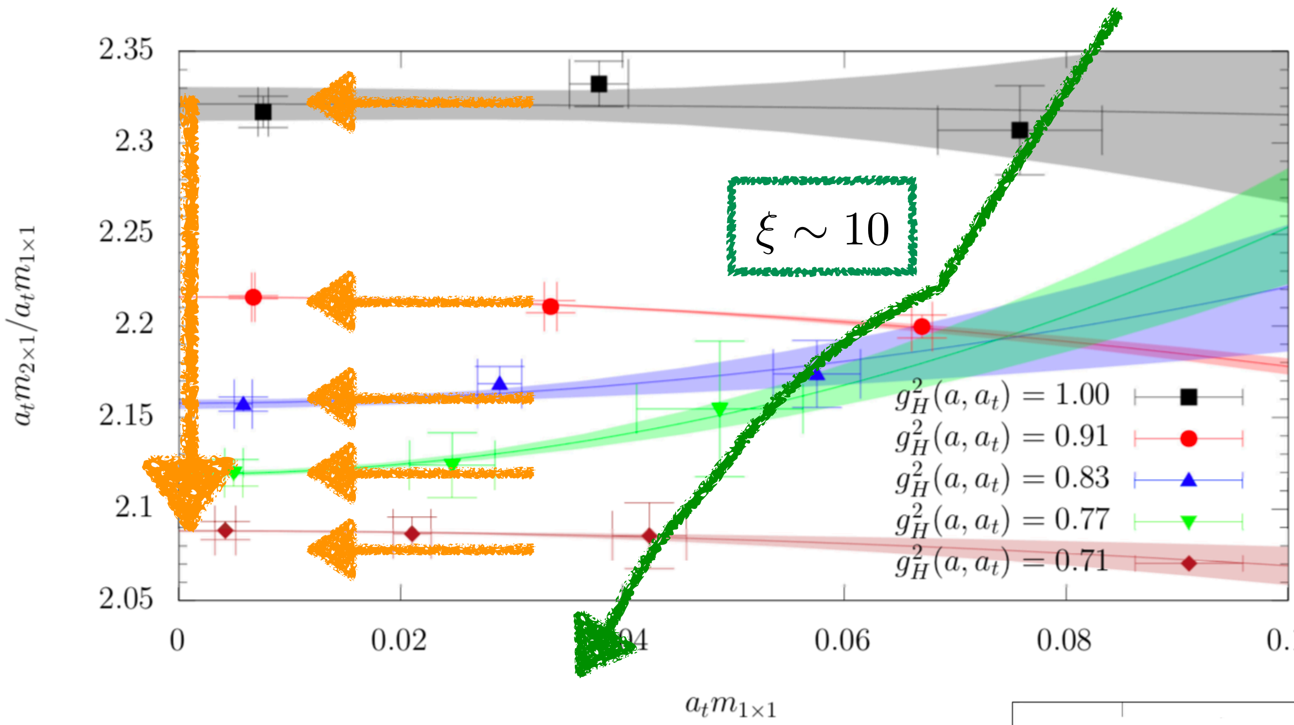


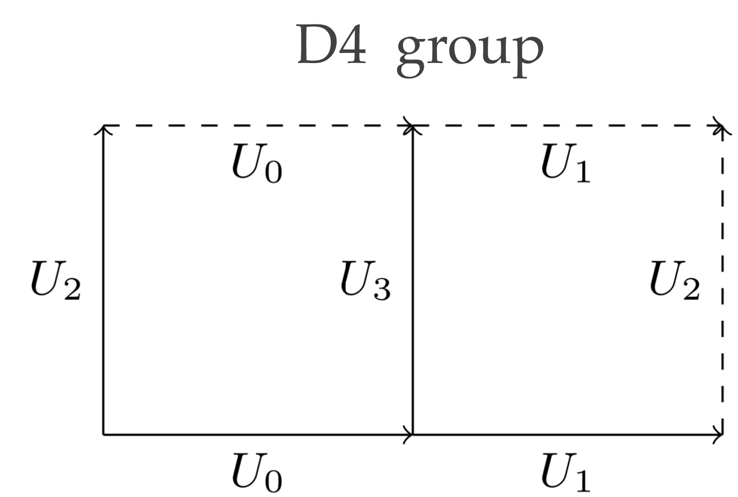
Carena, Lamm, **YYL**, Liu, PRL. **129**, 051601

To reach observables in the continuum limit

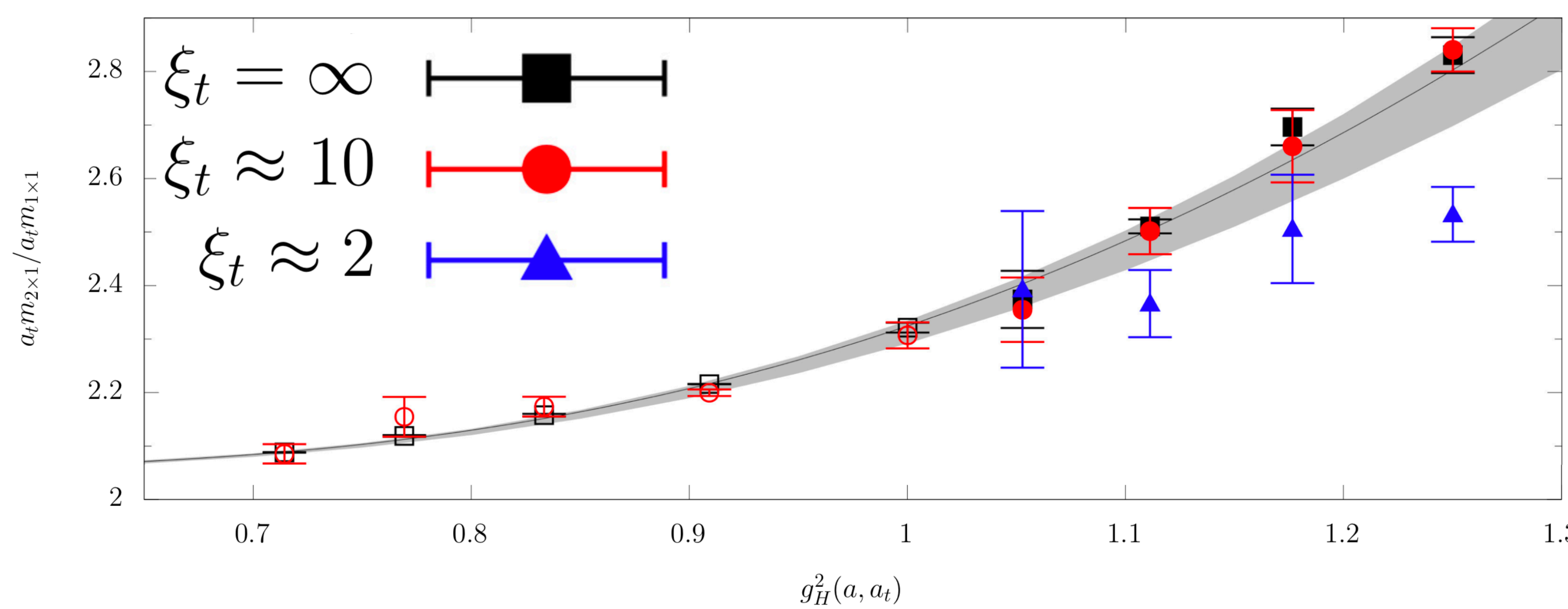
TRAJECTORY TO THE CONTINUUM LIMIT

continuum limit: extrapolation to the continuum at $\xi = a/a_t$





[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

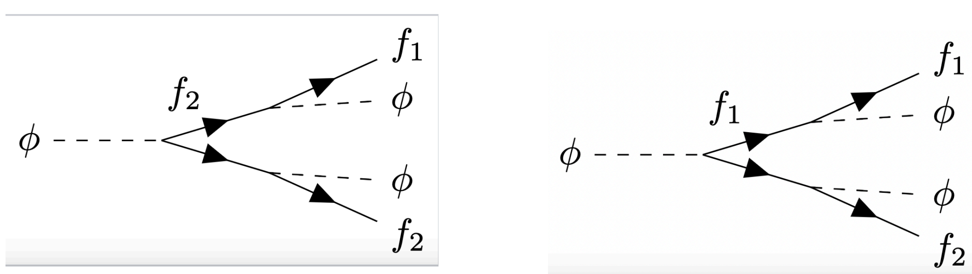


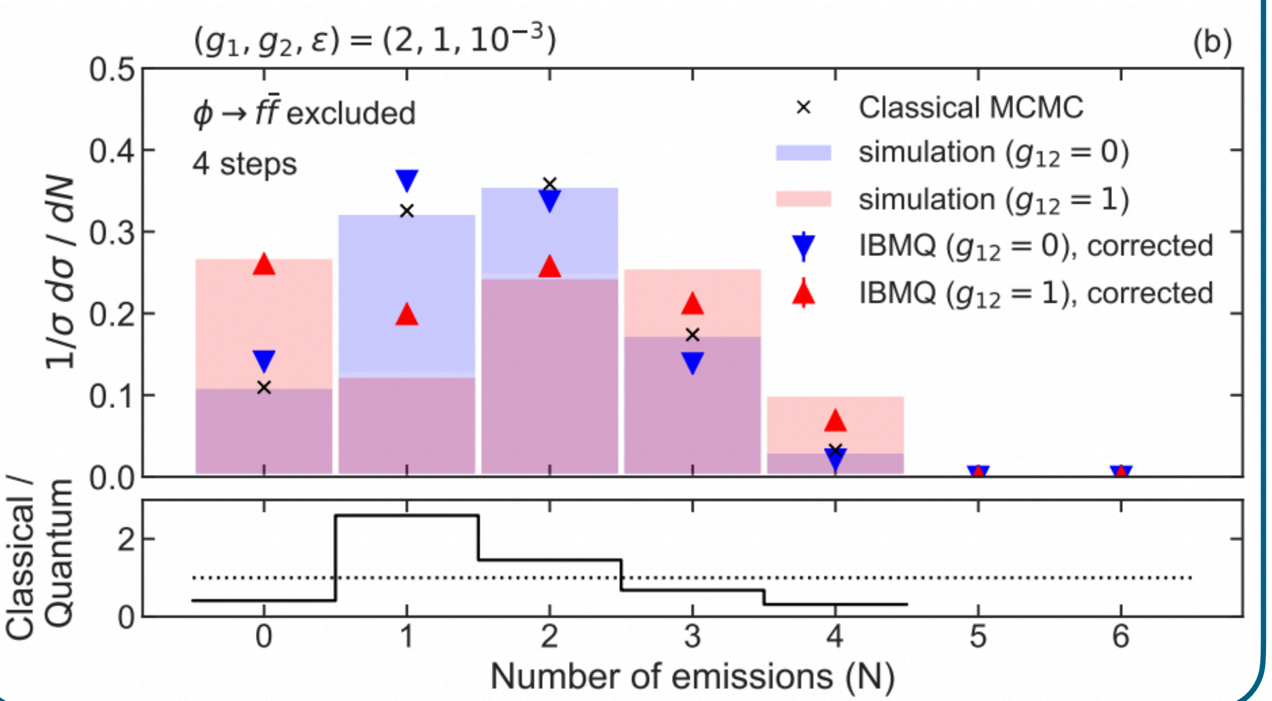
$g_H^2 \propto 1/\log(a) \to 0$

Benchmarks

PARTON SHOWERING [arXiv:2102.05044, PRD 103, 076020, PRD 106, 056002,…]

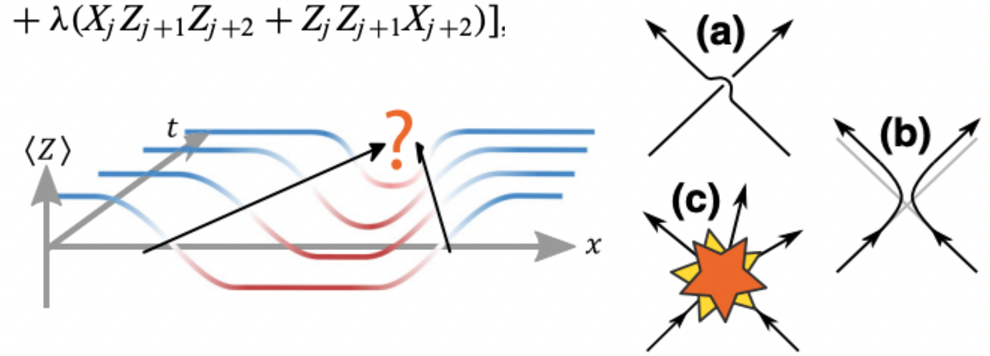
$$\mathcal{L} = \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2 + g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]\phi$$

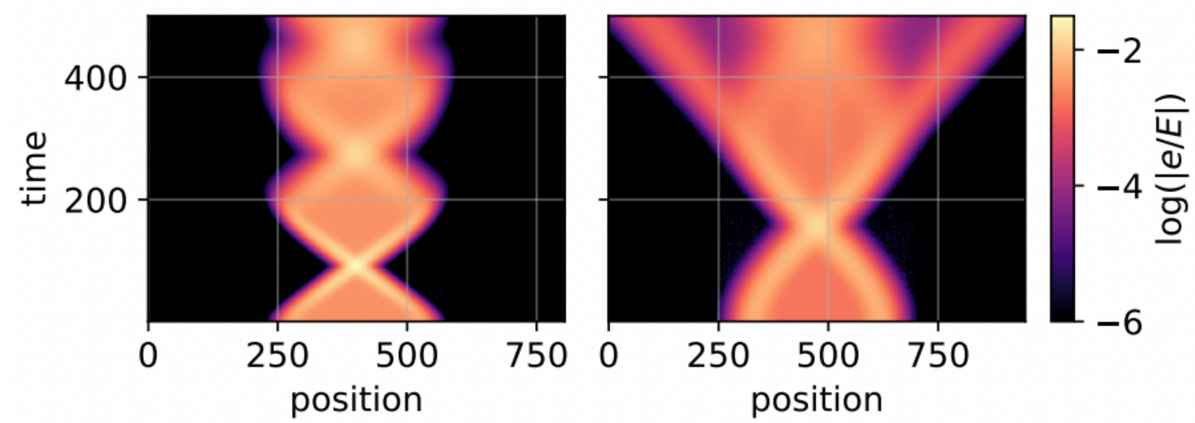


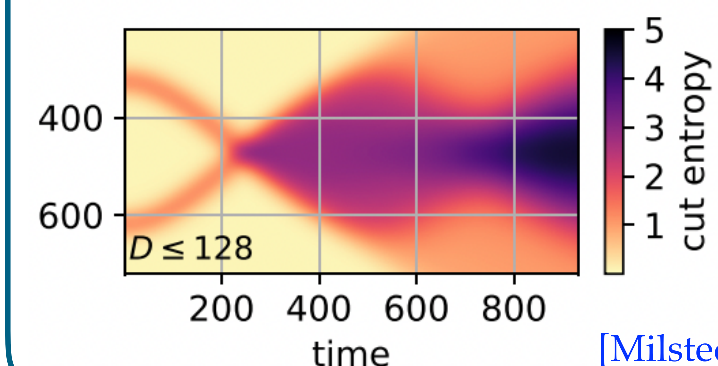


BUBBLE COLLISION

$$H = \sum_{j=1}^{N}[-Z_j Z_{j+1} - gX_j - hZ_j + \lambda(X_j Z_{j+1} Z_{j+2} + Z_j Z_{j+1} X_{j+2})],$$







[Milsted, et al, PRXQuantum.3.020316]

Quantum Machine Learning

computational complexity improvements, computational speed-ups

Supervised Learning—better separation power?

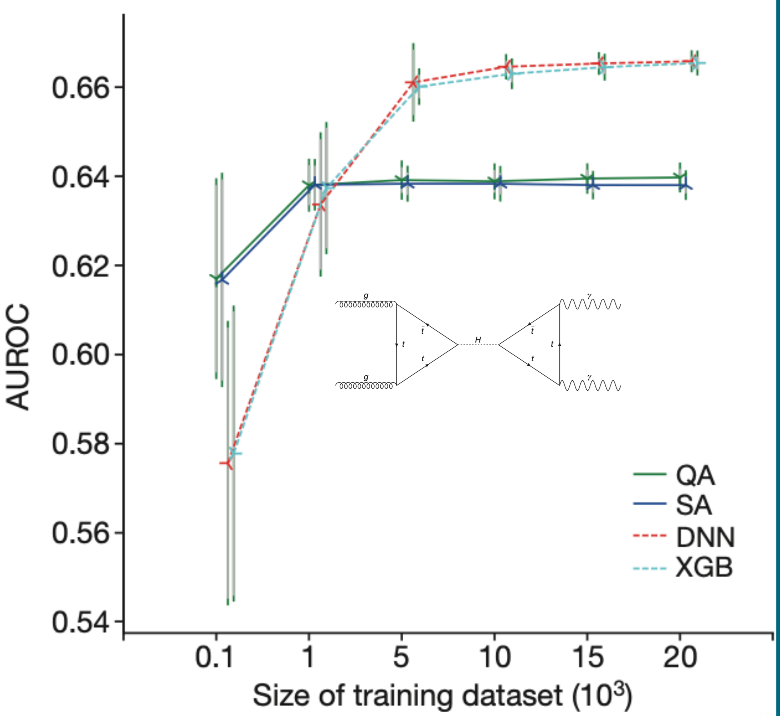
Quantum variational circuits, quantum annealing, QSVM, etc.

Quantum Annealing

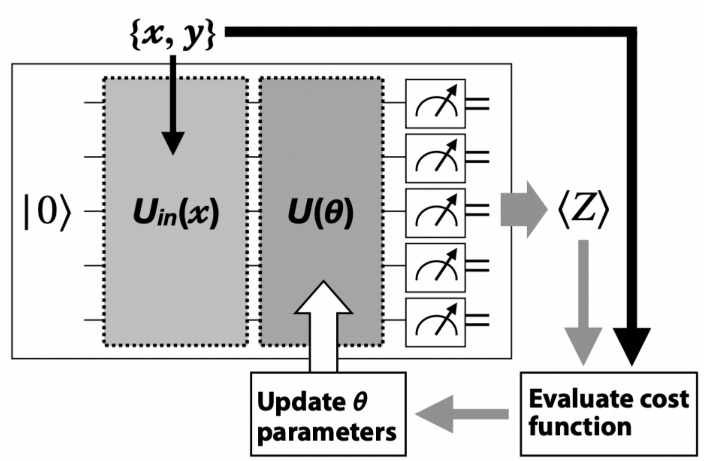
$$C_{ij} = \sum_{\tau} c_i(\boldsymbol{x}_\tau) c_j(\boldsymbol{x}_\tau), \quad C_i = \sum_{\tau} c_i(\boldsymbol{x}_\tau) y_\tau$$

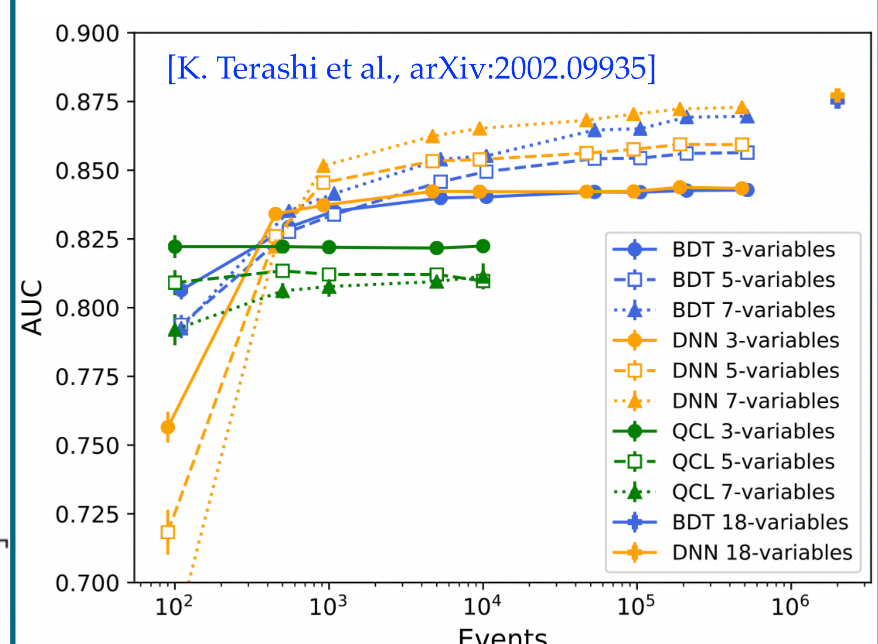
$$H = \sum_{i,j} J_{ij} s_i s_j + \sum_i h_i s_i$$

$$R(\boldsymbol{x}) = \sum_i s_i^g c_i(\boldsymbol{x}) \in [-1, 1]$$



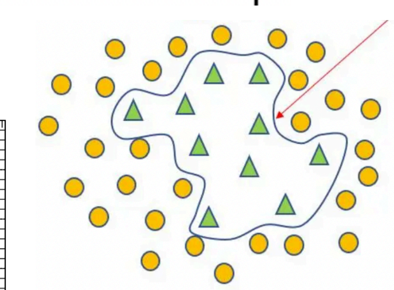
Variational Quantum Approach

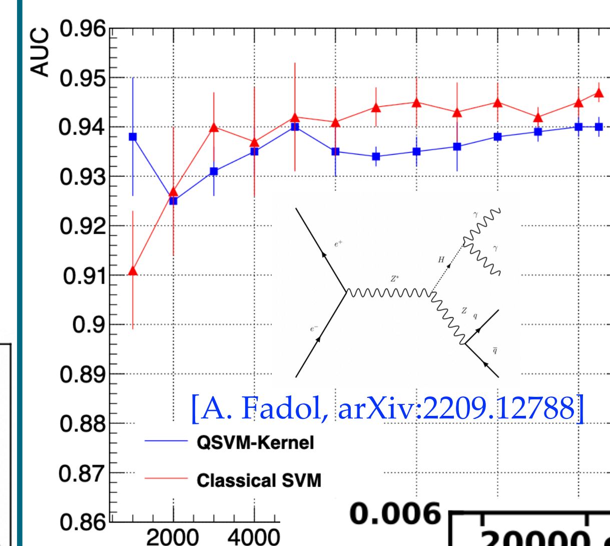


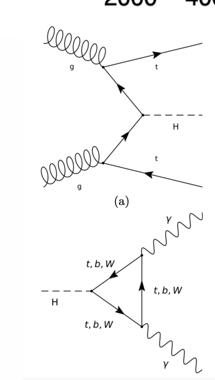


Quantum Support Vector Machine

$$k(\vec{x}_i, \vec{x}_j) = \left| \left\langle 0^{\otimes N} \right| \mathcal{U}^{\dagger}_{\Phi(\vec{x}_i)} \mathcal{U}_{\Phi(\vec{x}_j)} \left| 0^{\otimes N} \right\rangle \right|^2$$

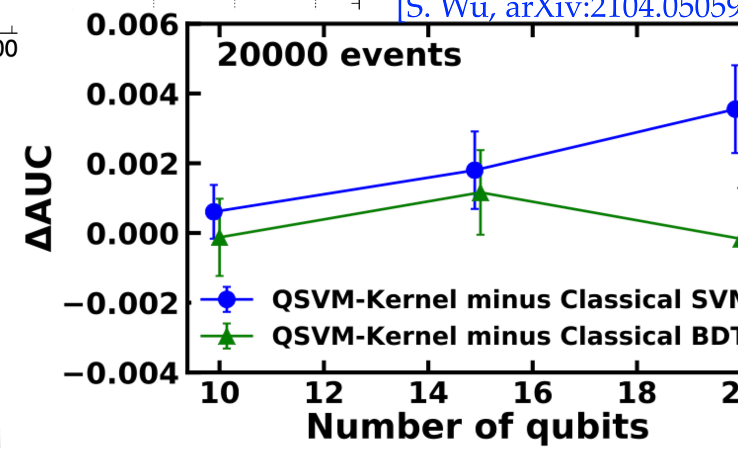






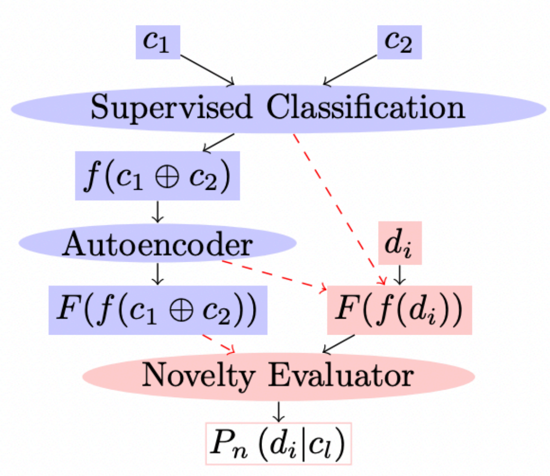
larger sample size?

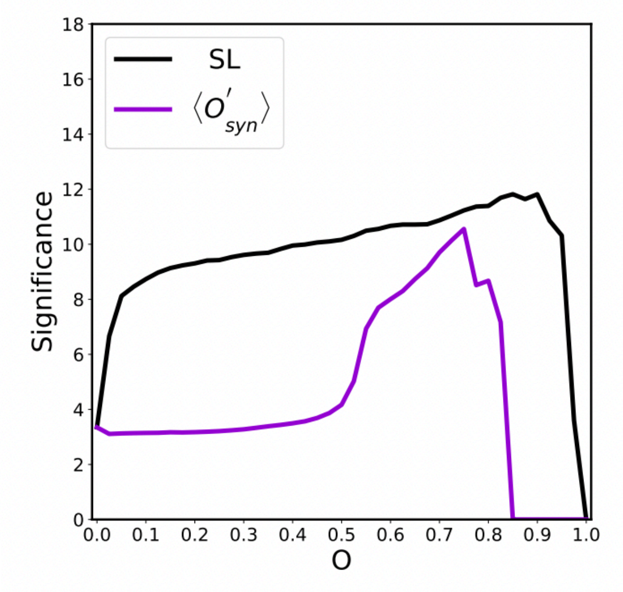
[S. Wu, arXiv:2104.05059]



Quantum Machine Learning - Anomaly detection

ANOMALY DETECTION





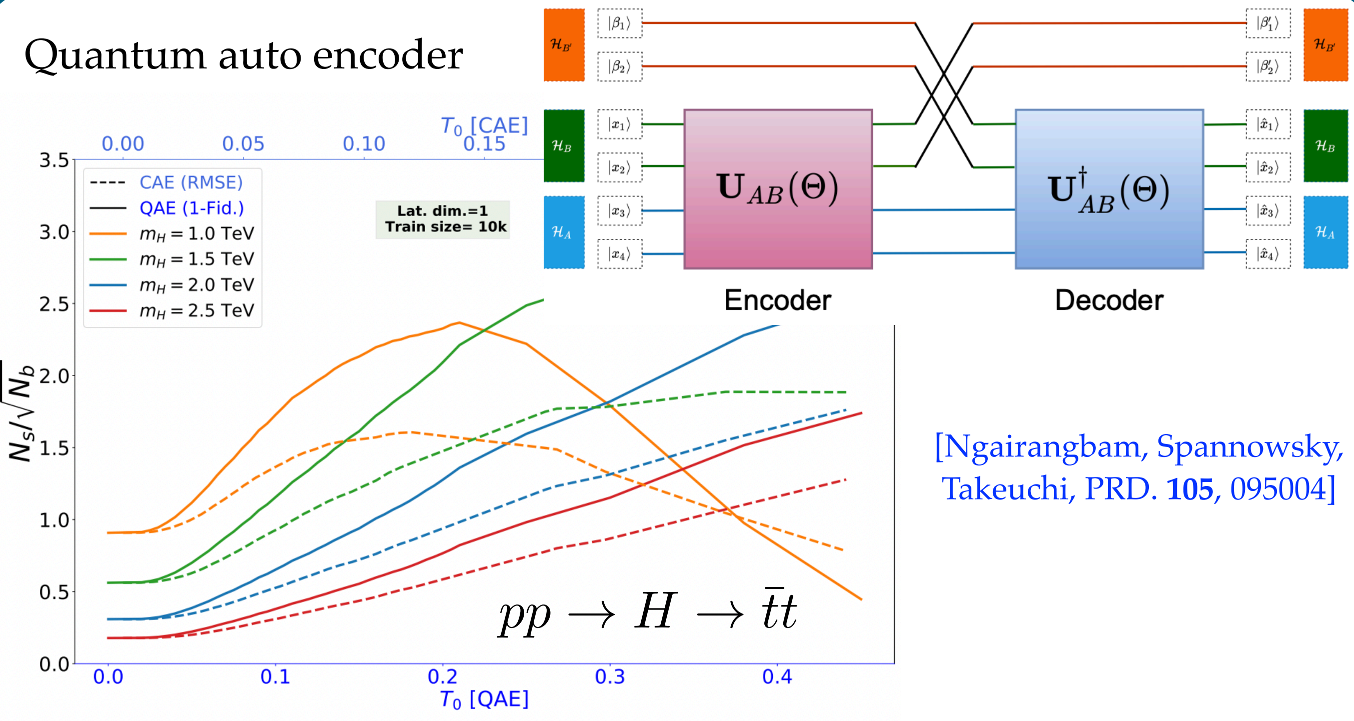
(a) $t\bar{t}h$

[J. Hajer, **YYL**, T. Liu, H. Wang, *PRD* 101 7, 076015]
[X.-H. Jiang, **YYL**, A. Juste, T. Liu, JHEP 10 (2022) 085]

- arXiv:1808.08992: ``Searching for New Physics with Deep Autoencoders'', Marco Farina, Yuichiro Nakai, and David Shih
- arXiv:1808.08992: ``QCD or What?'', Theo Heimel, Gregor Kasieczka, Tilman Plehn, and Jennifer M Thompson
- arXiv:1811.10276, ``Variational Autoencoders for New Physics Mining at the Large Hadron Collider'', Olmo Cerri, Thong Q. Nguyen, Maurizio Pierini, Maria Spiropulua and Jean-Roch Vlimant
- arXiv:1903.02032, ``A robust anomaly finder based on autoencoder'', Tuhin S. Roy and Aravind H. Vijay
- arXiv:1905.10384, ``Adversarially-trained autoencoders for robust unsupervised new physics searches'', Andrew Blance, Michael Spannowsky, and Philip Waite
- … … … …

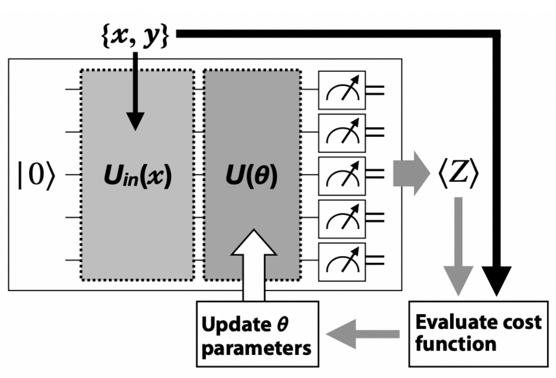
QUANTUM ANOMALY DETECTION

Quantum auto encoder



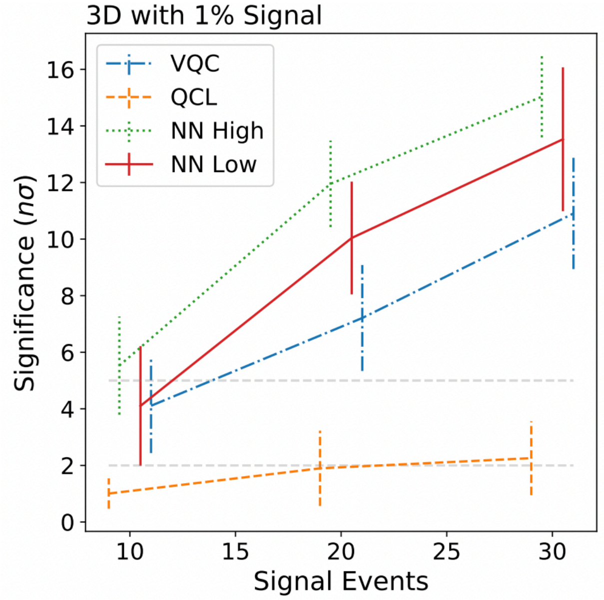
[Ngairangbam, Spannowsky, Takeuchi, PRD. **105**, 095004]

Quantum weakly-supervised learning (bg VS bg + ϵ signal)



[Terashi et al, arXiv:2002.09935]

$pp \to A \to B(\to e^+e^-)C(\to \mu^+\mu^-)$



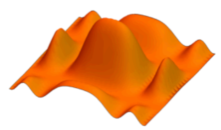
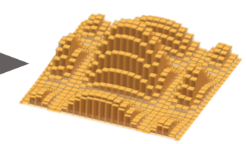
[Alvi, Bauer, Nachman, arXiv:2206.08391]

"It is time to go"

SUMMARY and OUTLOOK

Quantum computing can access to quantities in high energy physics which are intractable with classical methods

So many things to do, … and lots should be done to before scalable noise-resilient ones are available.

Discretization  →  *KS Hamiltonian*

Digitization $|q\rangle^N \to |G\rangle$ *truncation, discrete subgroup, …*

↓

Initialization $\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$ *ground/thermal/particle state prep*

↓

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$ *Trotterization, variational approach, Taylor series, …*

↓

Evaluation $\langle\mathcal{O}\rangle$ *Parton distribution function, particle decay, …*

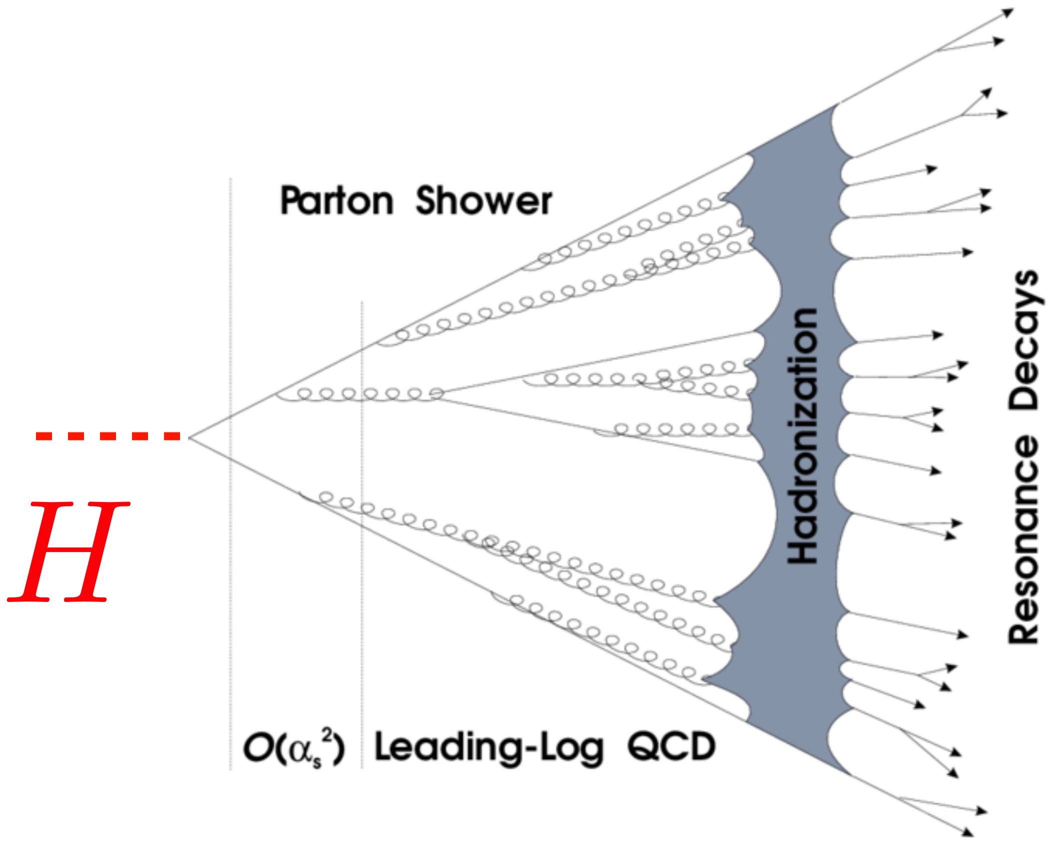
Error mitigation

Theory investigations, algorithmic developments, benchmark study, hardware co-design,…

Thank you

BACK UP

Theoretical inputs to colliders



Parton Shower

Long-distance dynamics - dominated by massless modes, high multiplicity final states

$$\sigma = H \otimes J_1 \otimes \cdots \otimes J_n \otimes S$$

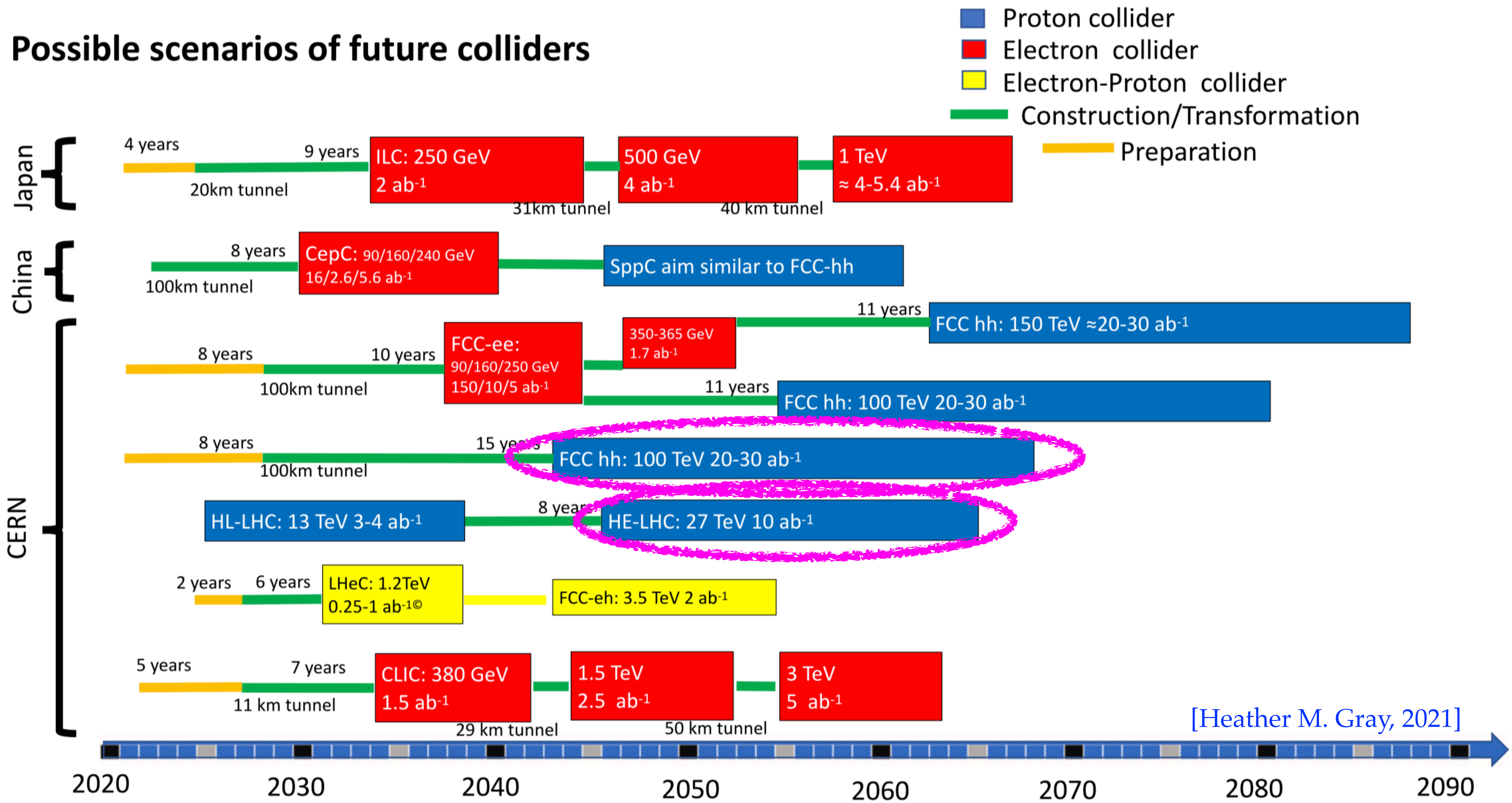
collinear

soft

Lattice: in principle, sign problem
State-of-art tech (MCMC):
probability level—interference not properly included

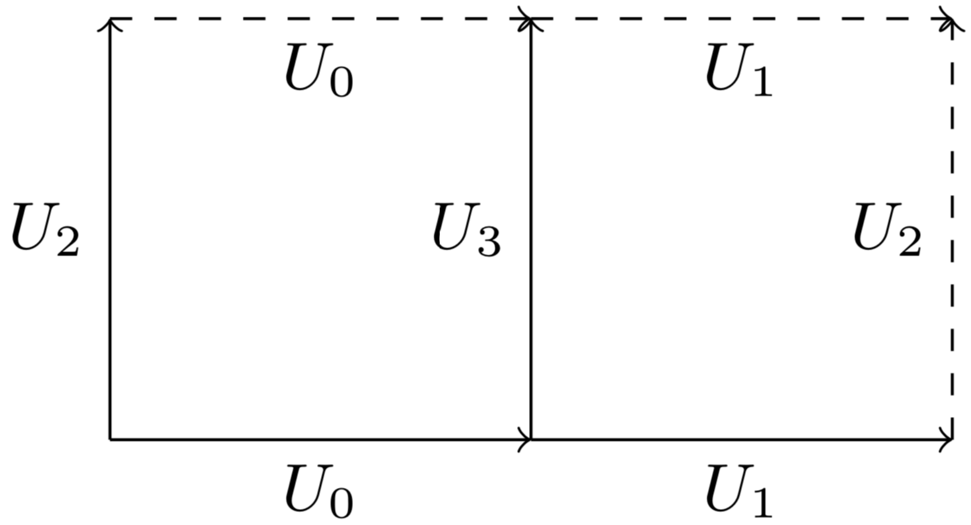
[arXiv:2102.05044, arXiv: 1904.03196, PRD 103, 076020, PRD 106, 056002,…]

-Now-: precision measurement



Propagation– Discretization in Time: Demonstration

D4 group Wilson action



In the position basis

$$
\begin{aligned}
H = &\,\mathrm{Re\,Tr}\left[U_2^\dagger(t)U_0^\dagger(t)U_3(t)U_0(t)\right] \\
&+ \mathrm{Re\,Tr}\left[U_3^\dagger(t)U_1^\dagger(t)U_2(t)U_1(t)\right] \\
&- \sum_{i=0..3} \log T_K^{(1)}(i)
\end{aligned}
$$

$$\beta_t = \frac{1}{g_H^2}$$

$$\langle \tilde{g}|\, T_K^{(1)}\, |g\rangle = e^{\beta_t \mathrm{ReTr}\left[\rho^\dagger(\tilde{g})\rho(g)\right]}$$

group element g: $|abc\rangle$

$$\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]^a \left[\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\right]^{2b+c}$$

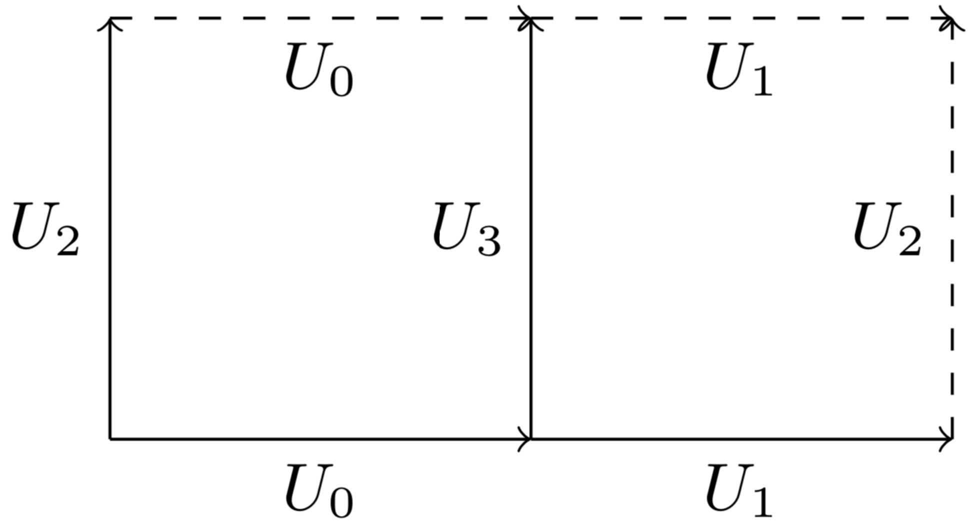
12 physical qubits

3 qubits for ancillary group register

[H. Lamm, et al, arXiv:1903.08807]

D4 group Wilson action

D4 group Wilson action

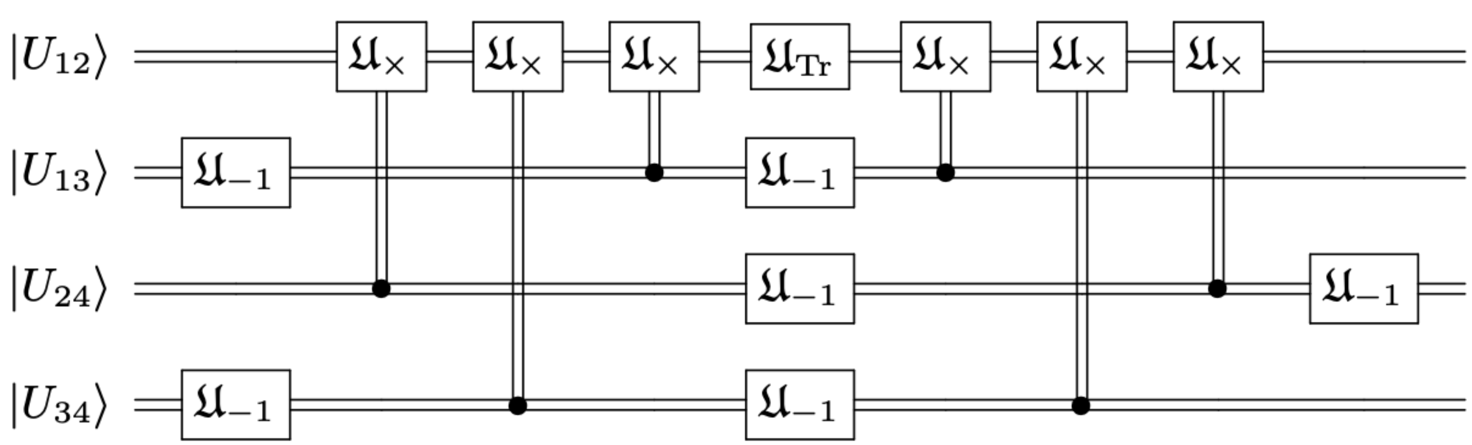


In the position basis

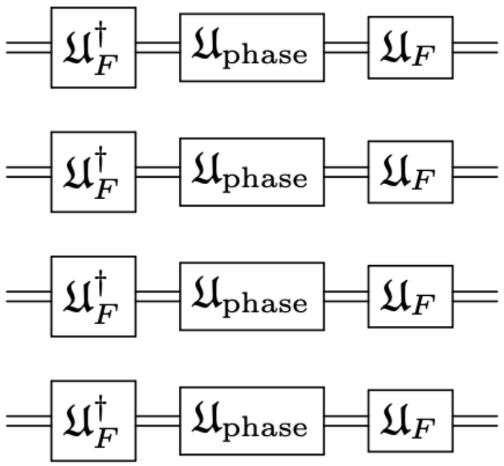
$$H = \mathrm{Re\,Tr}\left[U_2^\dagger(t)U_0^\dagger(t)U_3(t)U_0(t)\right] + \mathrm{Re\,Tr}\left[U_3^\dagger(t)U_1^\dagger(t)U_2(t)U_1(t)\right] - \sum_{i=0..3}\log T_K^{(1)}(i)$$

$$\langle\tilde{g}|\,T_K^{(1)}\,|g\rangle = e^{\beta_t \mathrm{ReTr}\left[\rho^\dagger(\tilde{g})\rho(g)\right]}$$

$$\mathrm{ReTr}U_{13}^\dagger U_{34}^\dagger U_{24}U_{12}$$



$$\log T_K^{(1)}$$



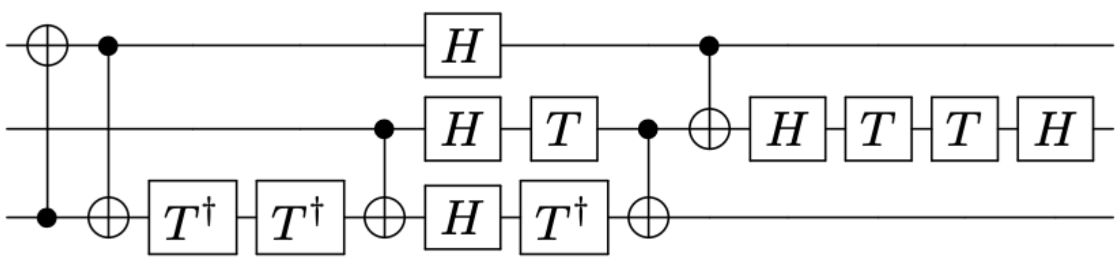
2 additional ancillary qubits

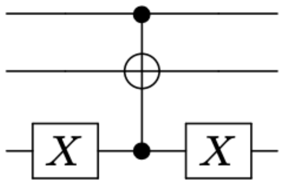
~200 gates per trotter step [H. Lamm, et al, arXiv:1903.08807]

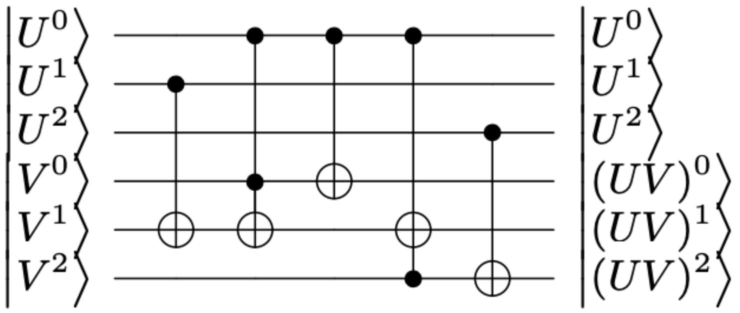
Gate counting for D4

TABLE I. Gate requirements for the propagation of a lattice with N_P plaquettes and L links, for a time T with time-steps of size Δt

Gate	Number
$\mathfrak{U}_F$	$2L\frac{T}{\Delta t}$
$\mathfrak{U}_{\text{phase}}$	$L\frac{T}{\Delta t}$
$\mathfrak{U}_{-1}$	$6N_P\frac{T}{\Delta t}$
$\mathfrak{U}_{\times}$	$6N_P\frac{T}{\Delta t}$
$\mathfrak{U}_{\text{Tr}}$	$N_P\frac{T}{\Delta t}$



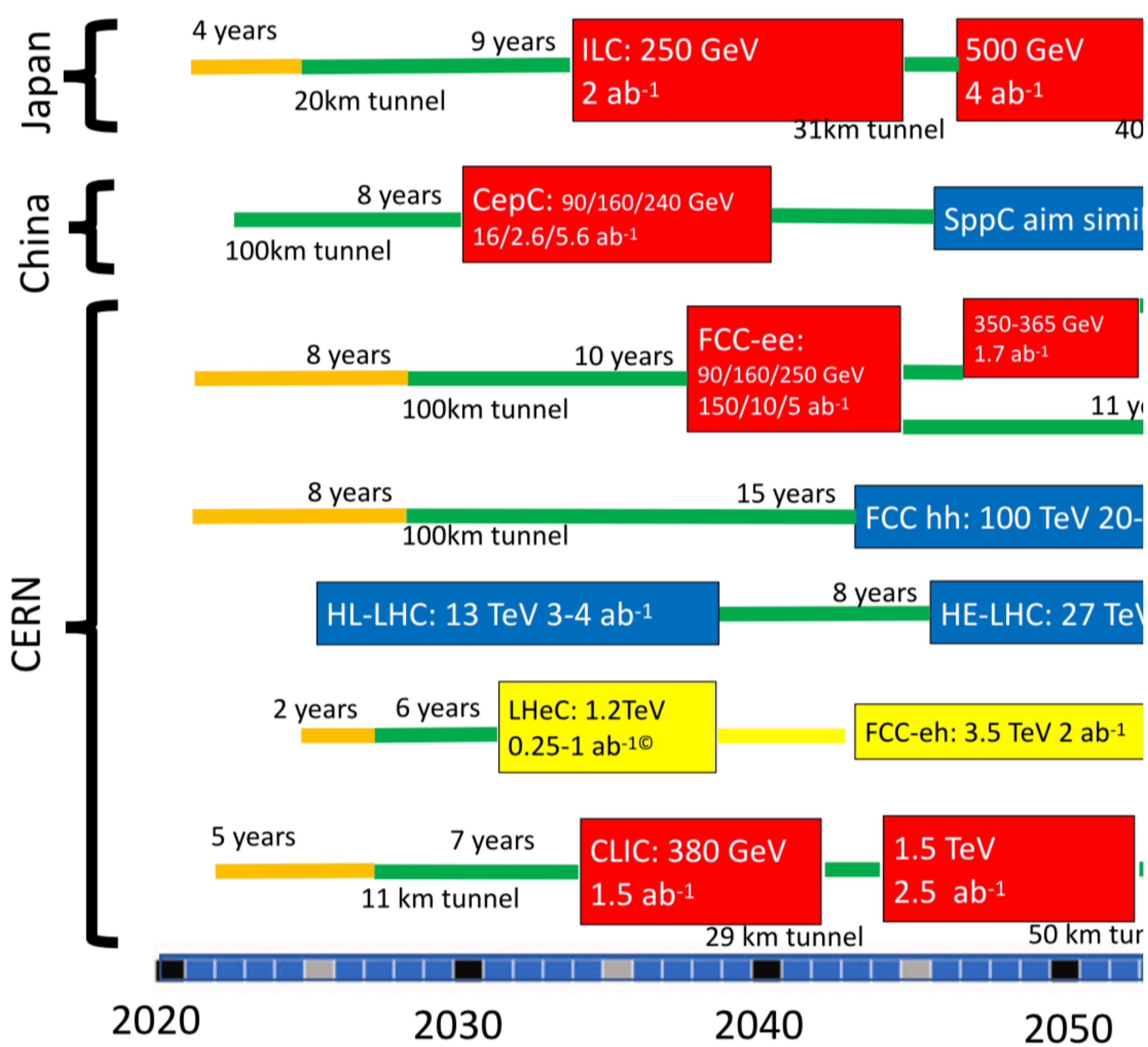




[H. Lamm, et al, arXiv:1903.08807]

-Now-: precision measurement

Possible scenarios of future colliders



[Jorge de Blas, et al, arXiv:1907.04311]

μ (%)	Future Circular Colliders		
	CEPC	FCC-ee	
	240 GeV	240 GeV	365 GeV
	unpolarized		
σ_{Zh}	0.005	0.005	0.009
μ_{Zh}^{bb}	[illegible]	[illegible]	0.50
μ_{Zh}^{cc}	[illegible]	[illegible]	.50
$\mu_{Zh}^{\tau\tau}$	[illegible]	[illegible]	.80
$\mu_{Zh}^{\mu\mu}$	17.1	19.0	40.0
μ_{Zh}^{WW}	$0.98^{\dagger}$	1.20	2.60
μ_{Zh}^{ZZ}	$5.09^{\dagger}$	4.40	12.0
$\mu_{Zh}^{Z\gamma}$	15.0	15.9	–
$\mu_{Zh}^{\gamma\gamma}$	6.84	9.00	18.0
μ_{Zh}^{gg}	$1.27^{\dagger}$	1.90	3.50

New physics up to 100 TeV can be probed

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

Kogut and Susskind formulation: [Phys. Rev. D **11**, 395 (1975)]

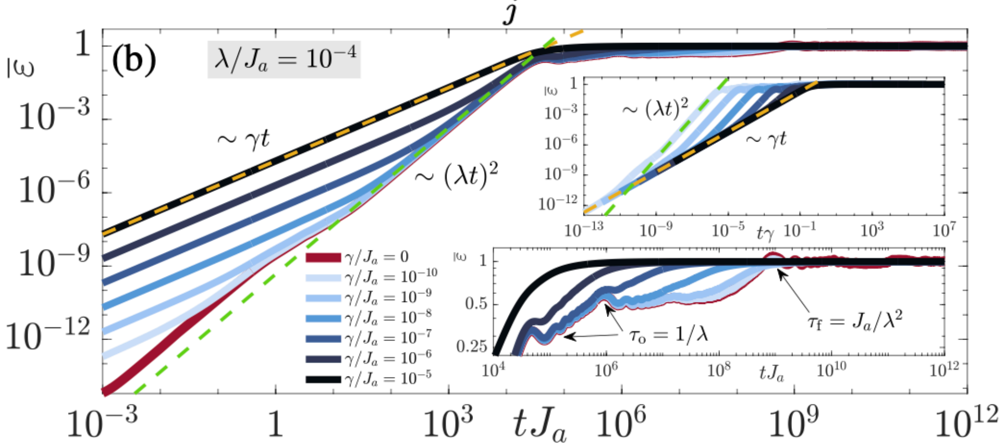
$$H = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x\psi_x^\dagger\psi_x + \frac{g^2}{2}\sum_x \mathbf{E}(x)^2 - \frac{1}{4g^2}\sum_\Box \mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)$$

continuous field variables

Gauss's law operator $G^a(x) = -E_L^a(x) + E_R^a(x-1) + \psi^\dagger(x)T^a\psi(x)$ $\quad G_x^a\,|P\rangle = 0$

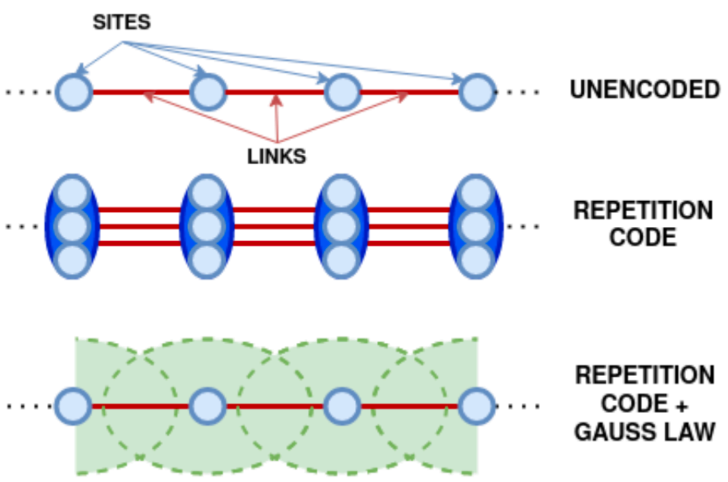
gauge symmetry violation as measures of errors

$$\varepsilon(t) = \frac{1}{N}\mathrm{Tr}\left\{\rho(t)\sum_j\left[G_j - g_j(0)\right]\right\}$$



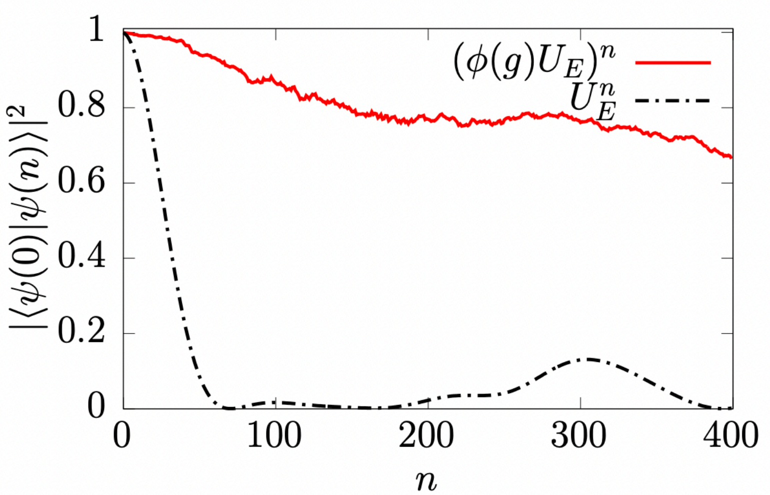
Halimeh, et al. arXiv:2009.07848 [cond-mat]

quantum error correction with gauge symmetry



Rajput, Roggero, and Wiebe, arXiv:2112.05186 [quant-ph]

gauge transformation to suppress coherent gauge drift



Lamm, Lawrence, Yamauchi, arXiv:2005.12688

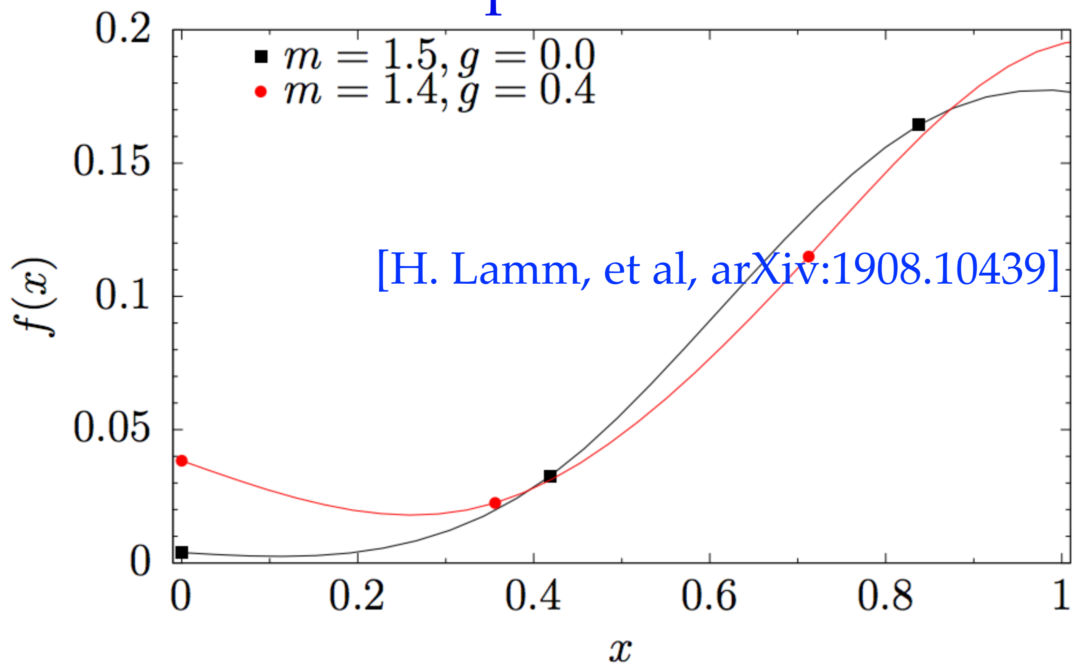
“Champagne Problems”

- Evaluation - how can observables be computed?

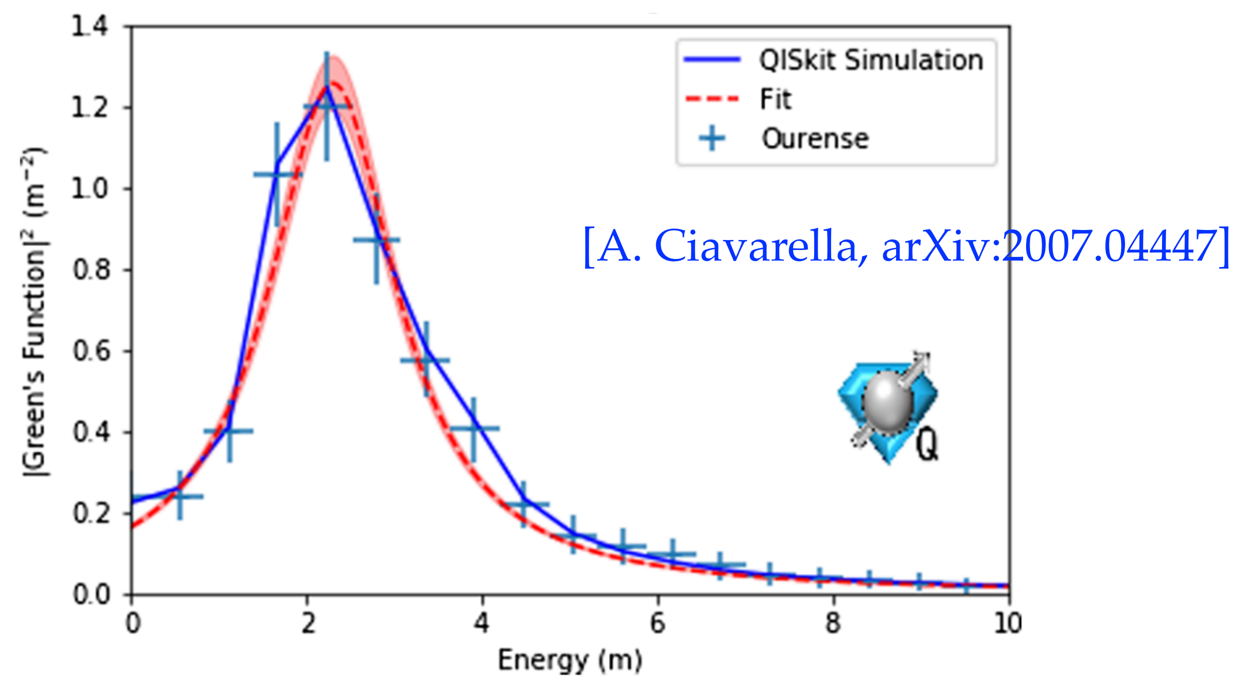
$$\text{instantaneous Hermitian operator} : \langle \mathcal{O}(t) \rangle$$

$$\text{time separated correlators} : \langle \mathcal{O}(t)\mathcal{O}(0) \rangle \quad ?$$

PDF-inputs to colliders



Particle decay width



What about partial width, especially to multi-particles ?

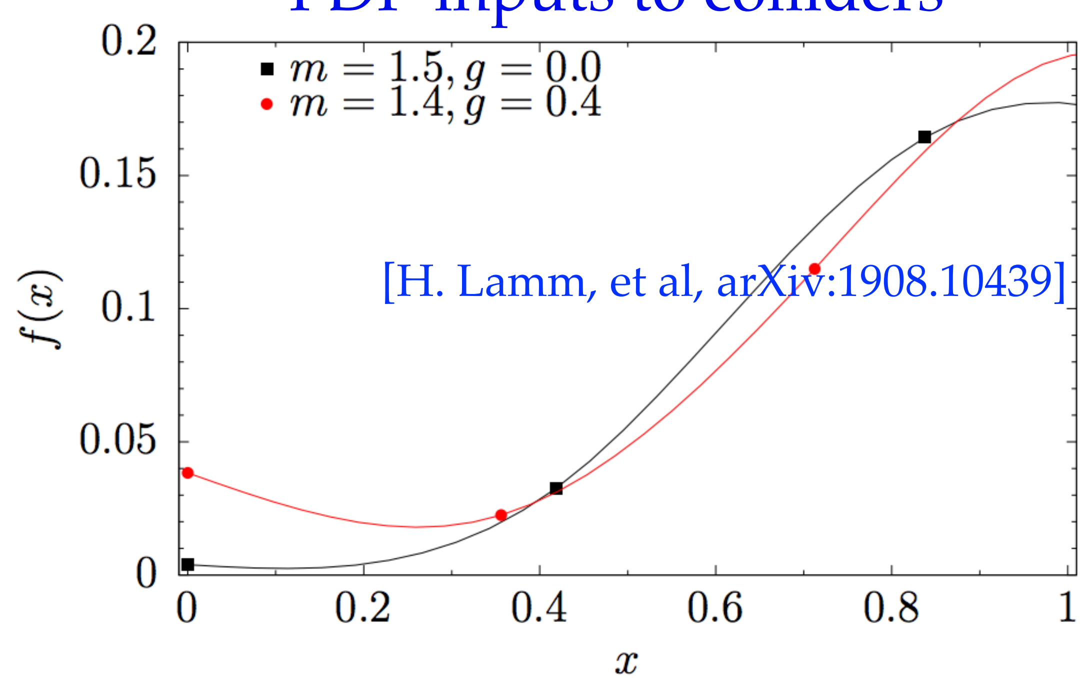
"Champagne Problems"

- Evaluation - how can observables be computed?

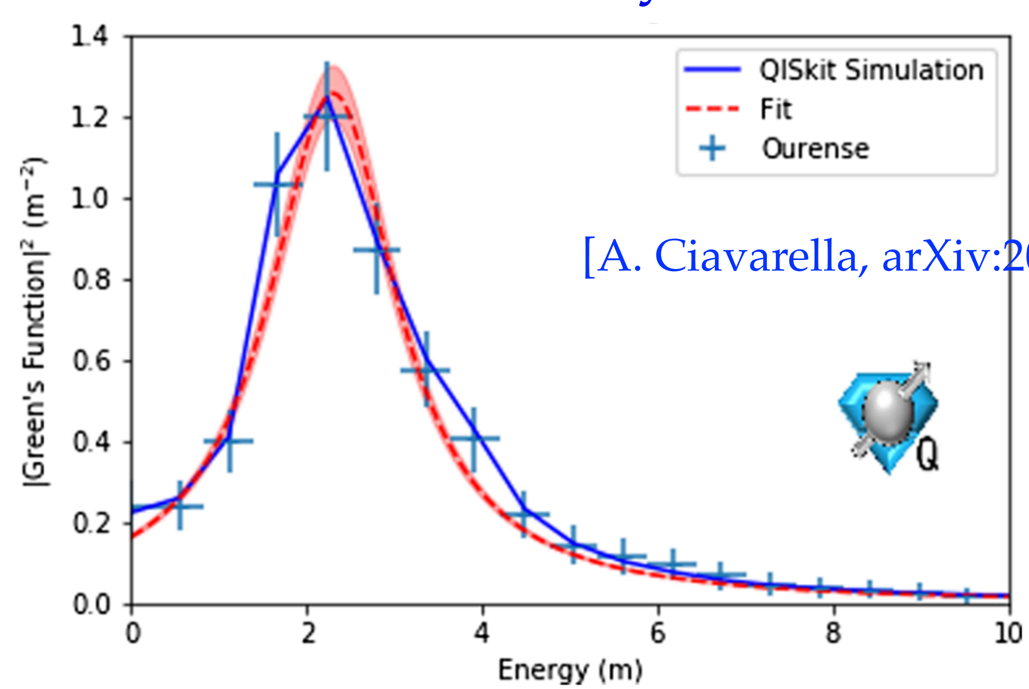
$$\text{instantaneous Hermitian operator} : \langle \mathcal{O}(t) \rangle$$

$$\text{time separated correlators} : \langle \mathcal{O}(t)\mathcal{O}(0) \rangle \quad ?$$

PDF-inputs to colliders



Particle decay width



more efficient methods?

$$\langle \Psi | \mathcal{O}(t)\mathcal{O}(0) | \Psi \rangle = \frac{\partial}{\partial \epsilon_t} \frac{\partial}{\partial \epsilon_0} \langle \Psi | e^{-iHt} e^{-i\mathcal{O}\epsilon_t} e^{iHt} e^{i\mathcal{O}\epsilon_0} | \Psi \rangle$$