

Analog Quantum Simulation Based on Superconducting Quantum Processors

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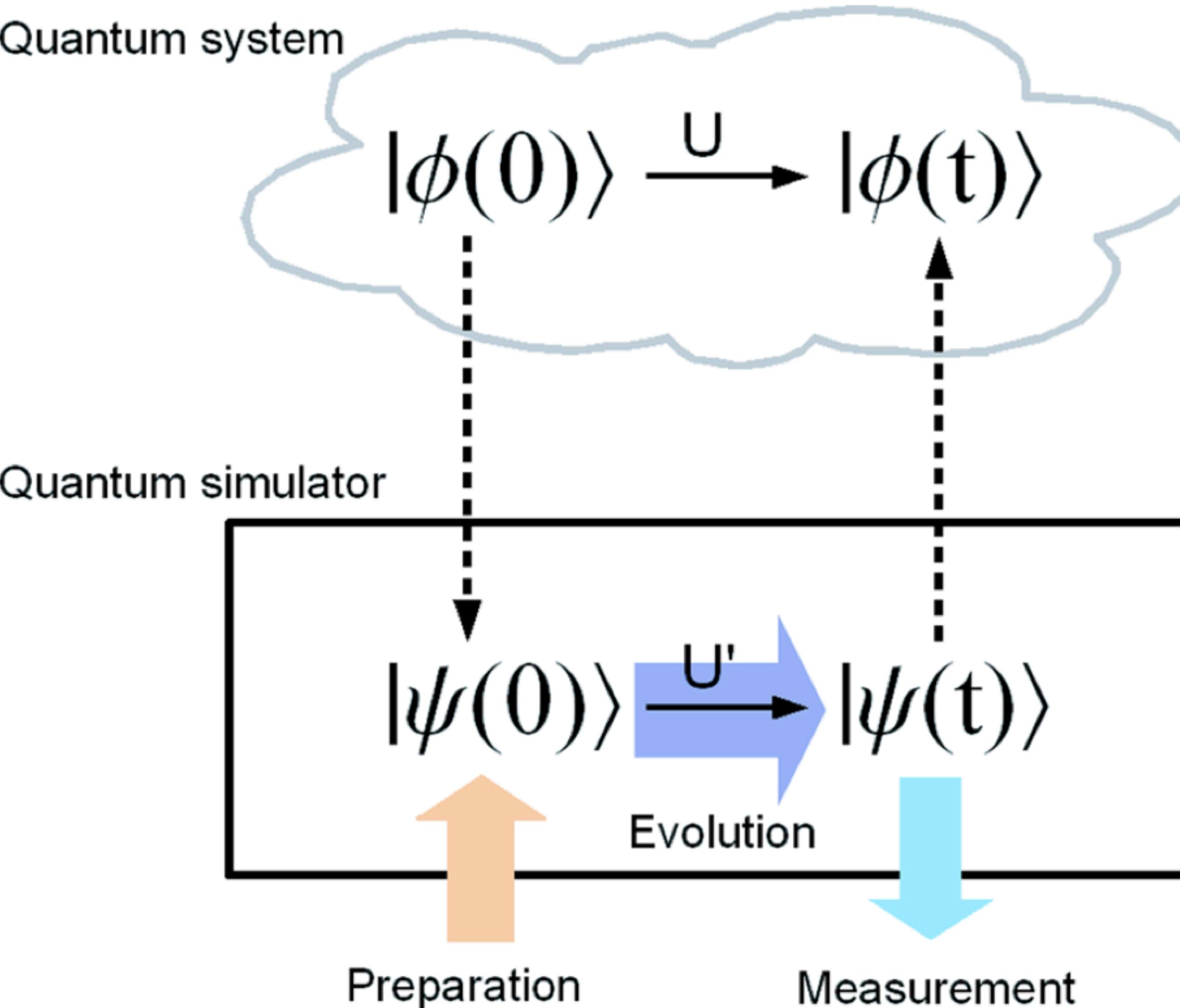
Catalogue

- 1. Concepts of quantum simulation**
- 2. Superconducting quantum processors**
- 3. Analog quantum simulation of dynamical phase transitions**
- 4. Analog quantum simulation of strong and weak thermalization**
- 5. Analog quantum simulation of energy-resolved many-body localization**

1. Concepts of quantum simulation

Rev. Mod. Phys. **86**, 153 (2014)

Nature **534**, 516-519 (2016)



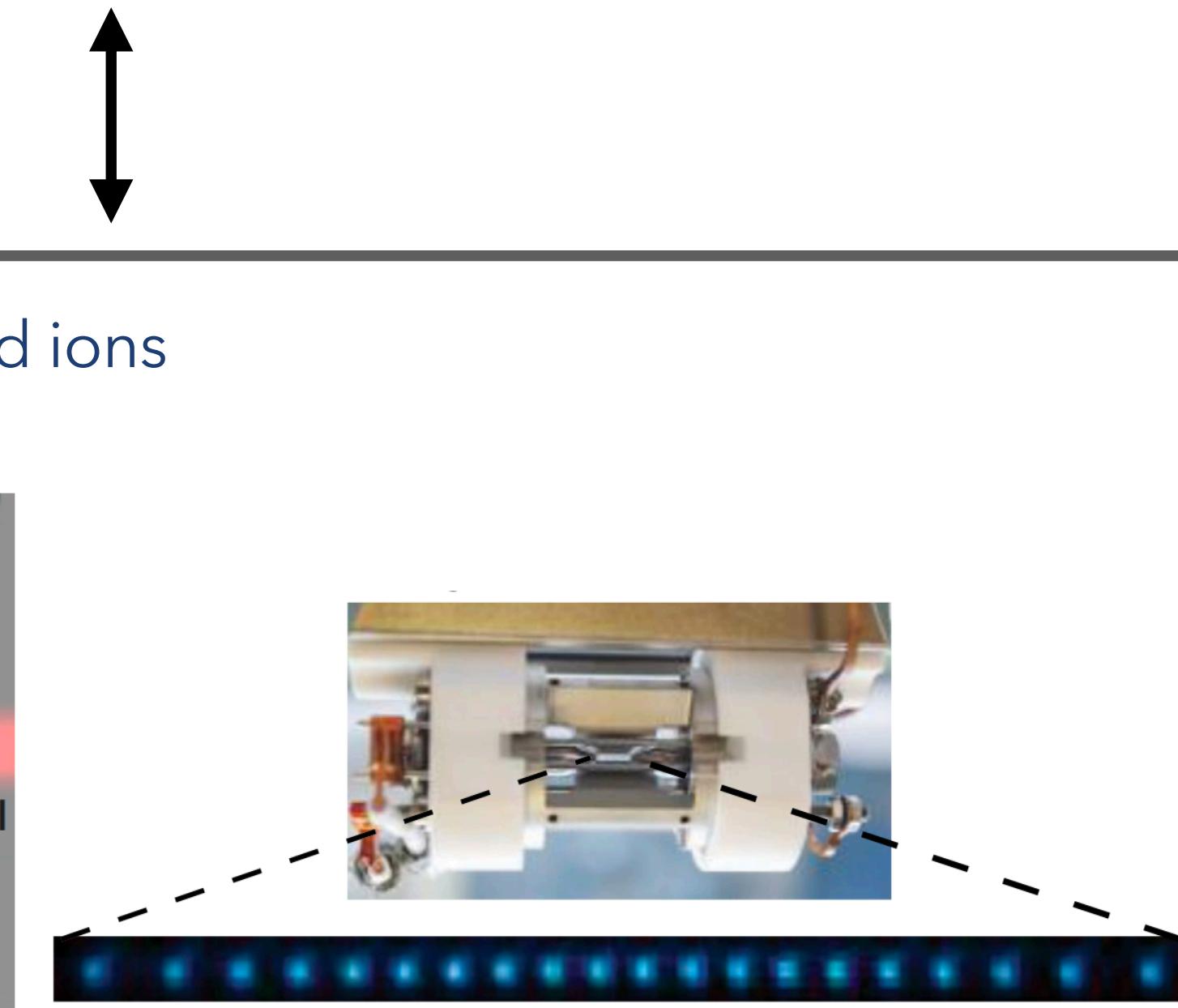
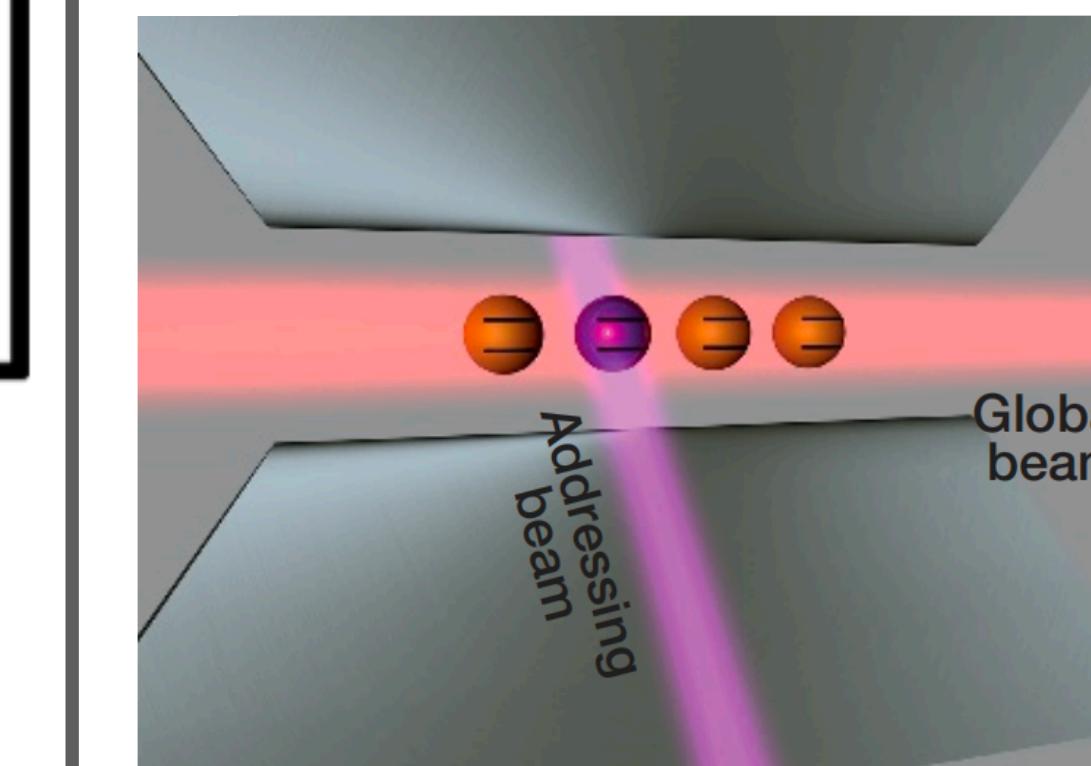
Quantum system:

Kogut-Susskind Hamiltonian formulation of the Schwinger model

$$\hat{H}_{\text{lat}} = -iw \sum_{n=1}^{N-1} [\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.}] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^N (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

$$\hat{H}_S = \frac{m}{2} \sum_{n=1}^N (-1)^n \hat{\sigma}_n^z + w \sum_{n=1}^{N-1} [\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{h.c.}] + J \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^n [\hat{\sigma}_m^z + (-1)^m] \right]^2$$

Quantum simulator: trapped ions



1.1 Analog quantum simulation

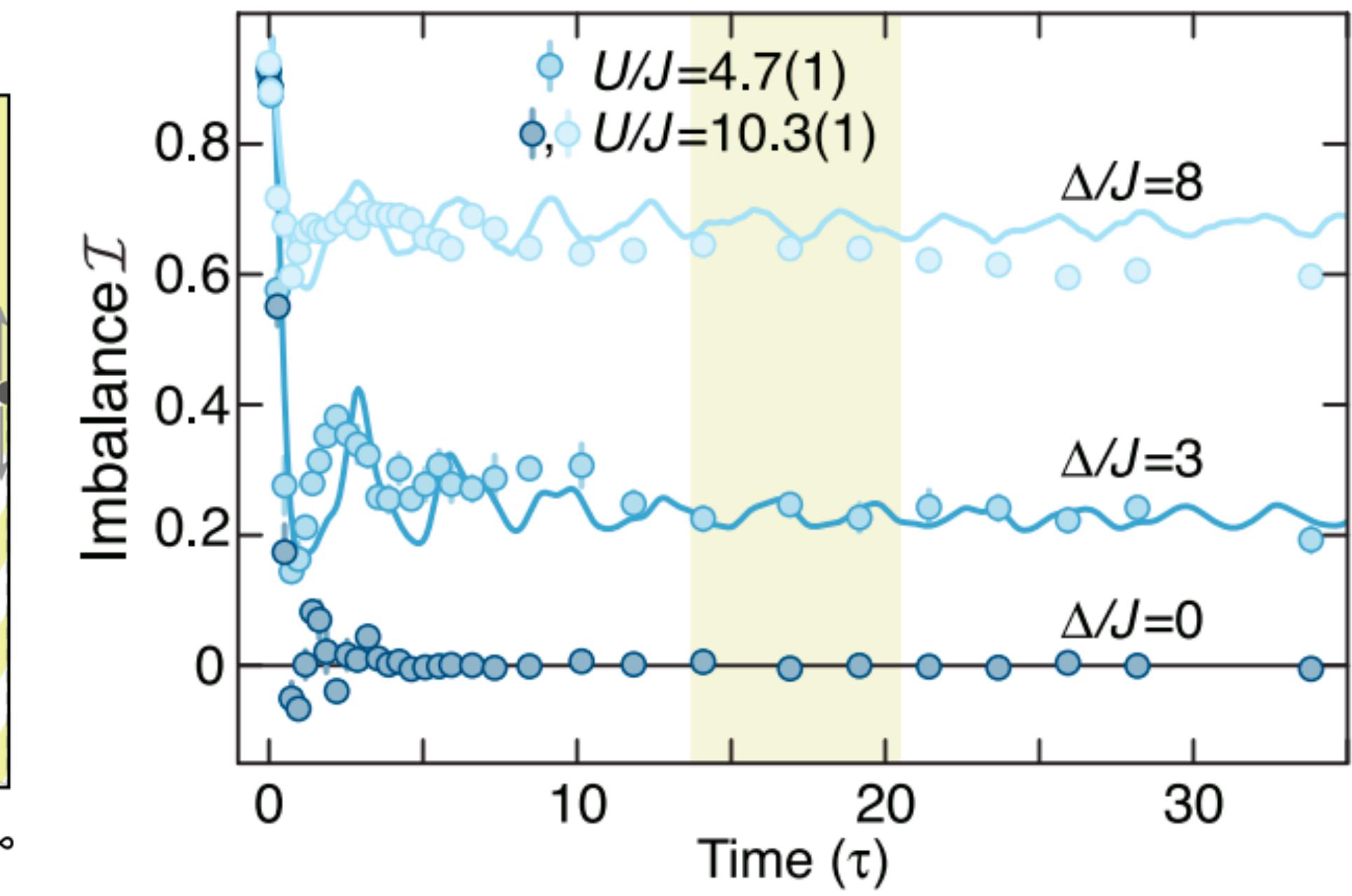
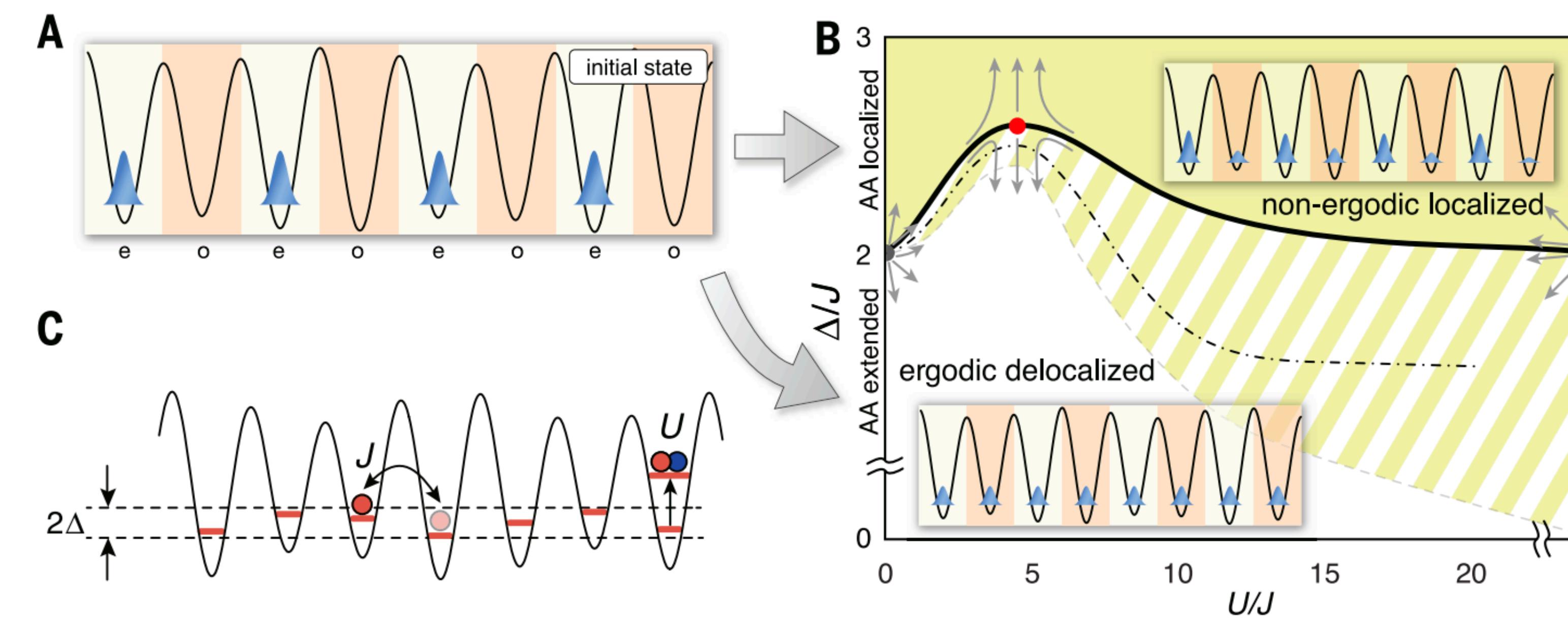
Science 349, 842-845 (2015)

Example: cold atoms in optical lattice

$$\hat{H} = -J \sum_{i,\sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{i+1,\sigma} + \text{h.c.}) +$$

$$\Delta \sum_{i,\sigma} \cos(2\pi\beta i + \phi) \hat{c}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

$$|\psi(t, U, \Delta)\rangle_f = \exp[-i\hat{H}(U, \Delta)t]$$



1.2 Digital quantum simulation

PRXQuantum 3, 020324 (2022)

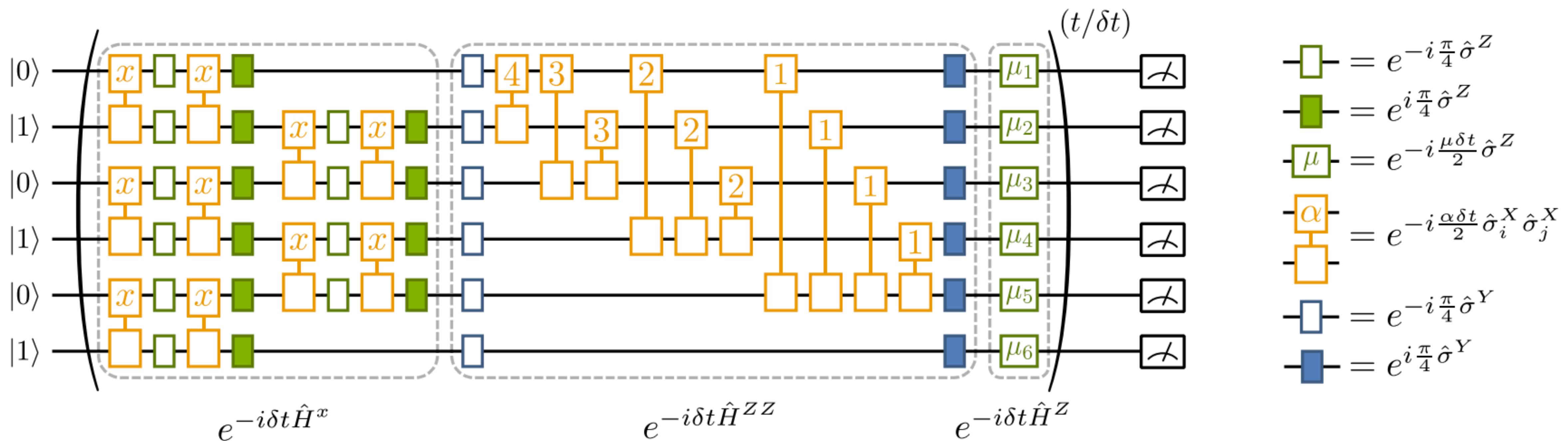
The Schwinger model:

$$\begin{aligned}\hat{H} &= x \sum_{n=1}^{N-1} (\hat{\sigma}_n^X \hat{\sigma}_{n+1}^X + \hat{\sigma}_n^Y \hat{\sigma}_{n+1}^Y) \\ &+ \frac{1}{4} \sum_{n=1}^{N-1} \left[\sum_{m=1}^n \left(\hat{\sigma}_m^Z + (-1)^m \right) \right]^2 + \mu \sum_{n=1}^N (-1)^n \frac{\hat{\sigma}_n^Z + 1}{2} \\ &\equiv \hat{H}^x + \hat{H}^{ZZ} + \hat{H}^Z + \text{const.}\end{aligned}$$

$$e^{-i\hat{H}t} = (e^{-i\hat{H}\delta t})^M \quad t = M\delta t$$

Suzuki-Trotter decomposition:

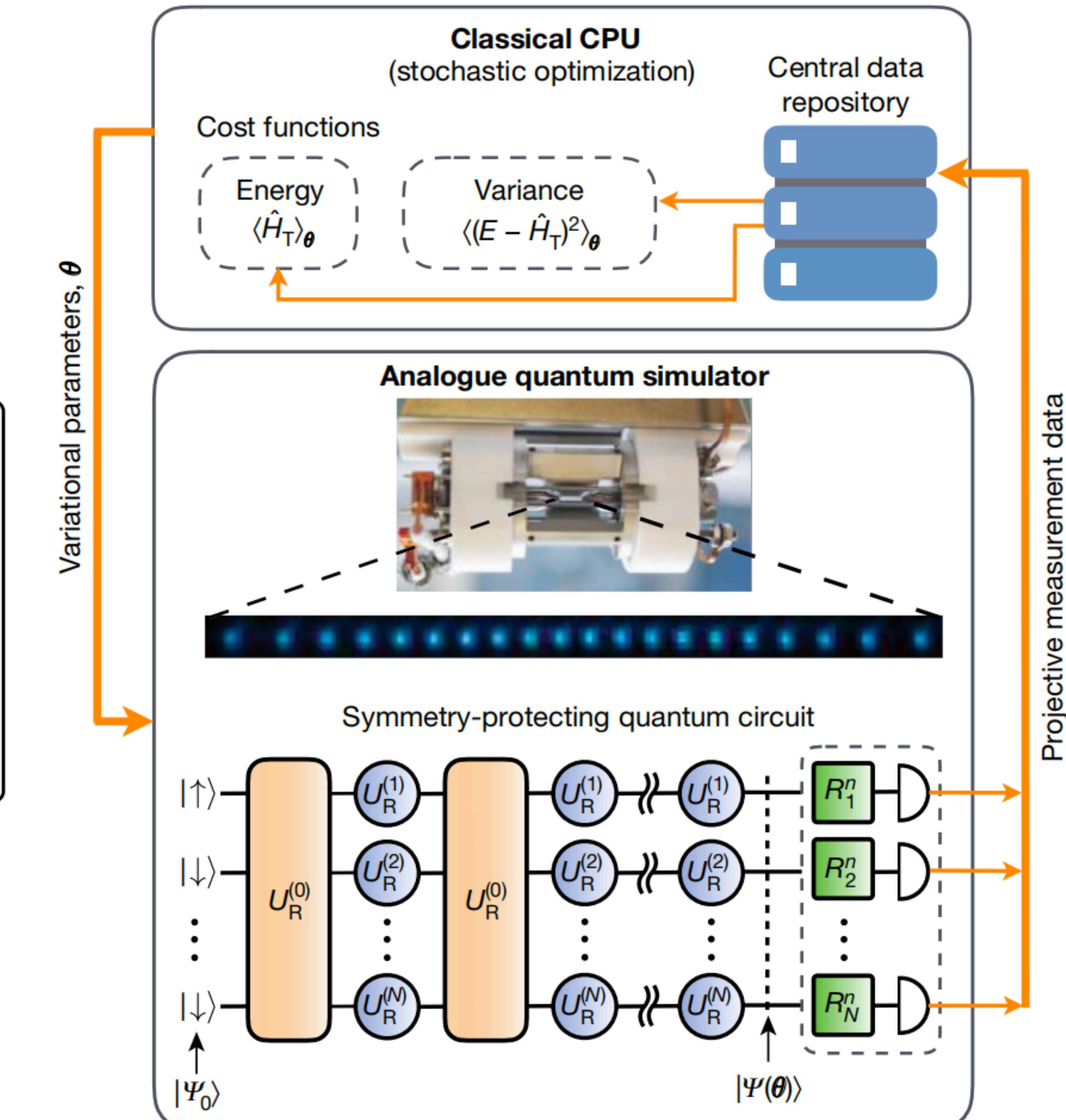
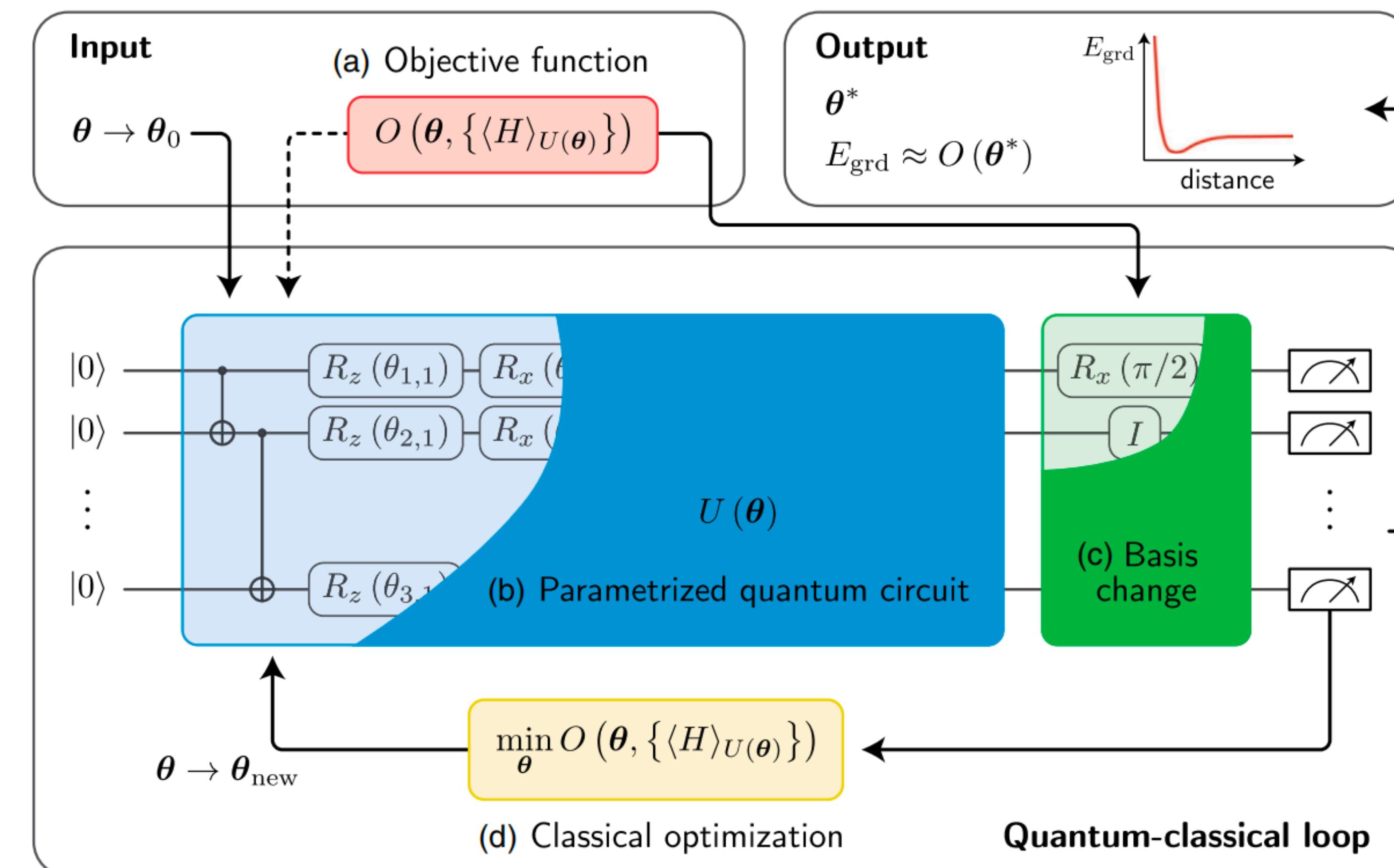
$$e^{-i\hat{H}\delta t} = e^{-i(\hat{H}^x + \hat{H}^{ZZ} + \hat{H}^Z)\delta t} \simeq e^{-i\hat{H}^x\delta t} e^{-i\hat{H}^{ZZ}\delta t} e^{-i\hat{H}^Z\delta t}$$



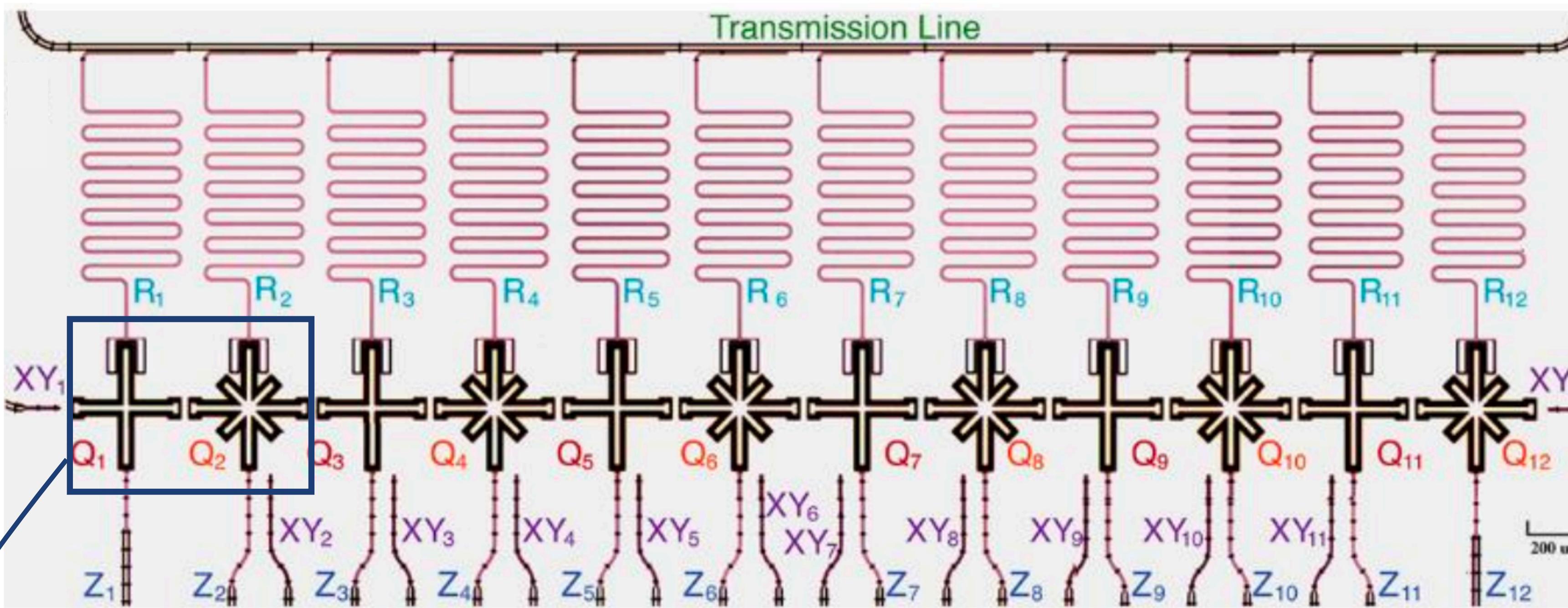
1.3 Variational quantum simulation

Rev. Mod. Phys. 94, 015004 (2022)

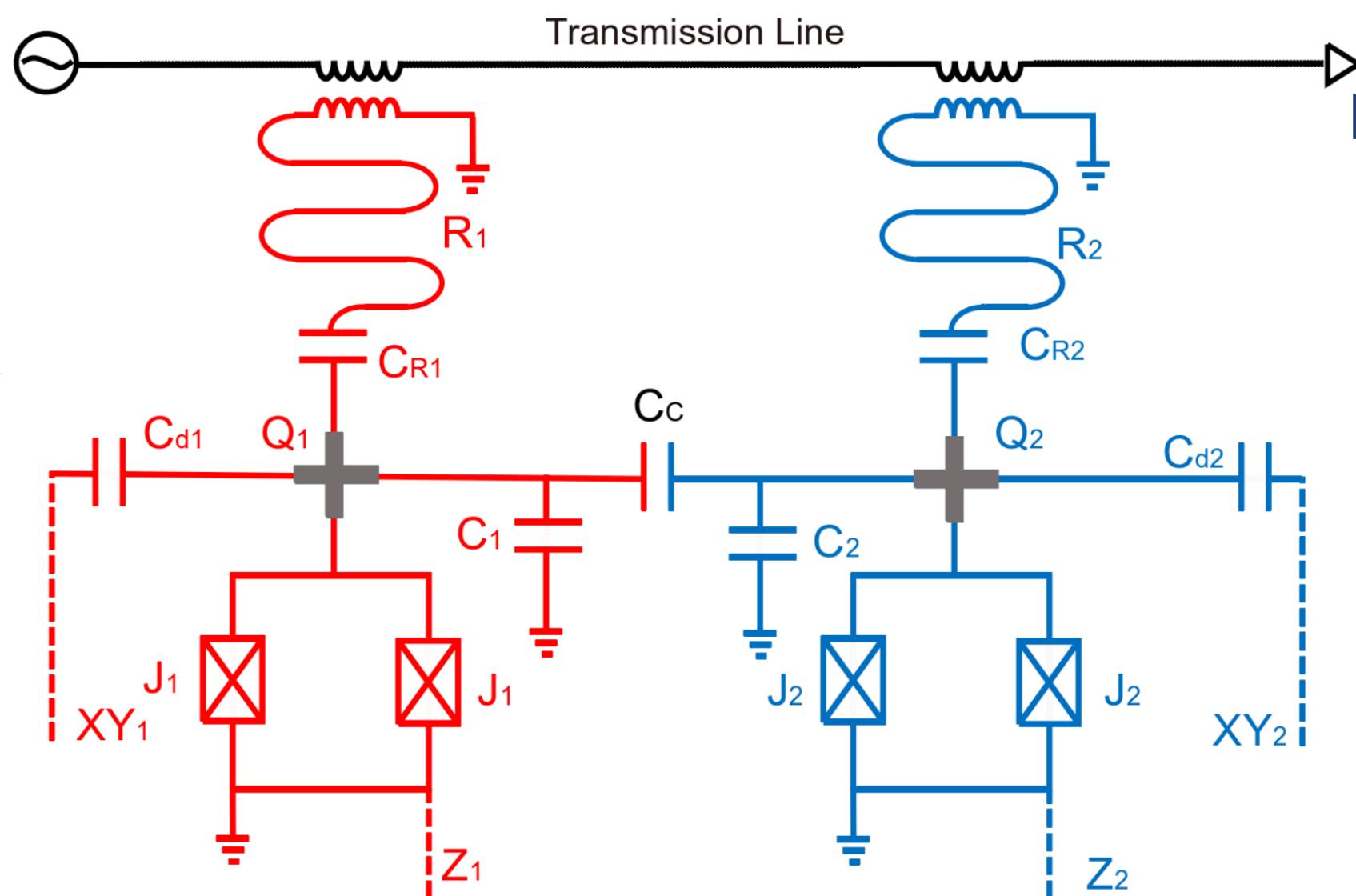
Nature 569, 355-360 (2019)



2. Superconducting quantum processors



Science 364, 753-756 (2019)



Bose-Hubbard (BH) model: $H = J \sum_{j=1}^{11} (\hat{a}_j^\dagger \hat{a}_{j+1} + h.c.) + \frac{U}{2} \sum_{j=1}^{12} \hat{n}_j (\hat{n}_j - 1) + \sum_{j=1}^{12} h_j \hat{n}_j$

$J/2\pi \simeq 12\text{MHz}$

$U/2\pi \simeq -240\text{MHz}$

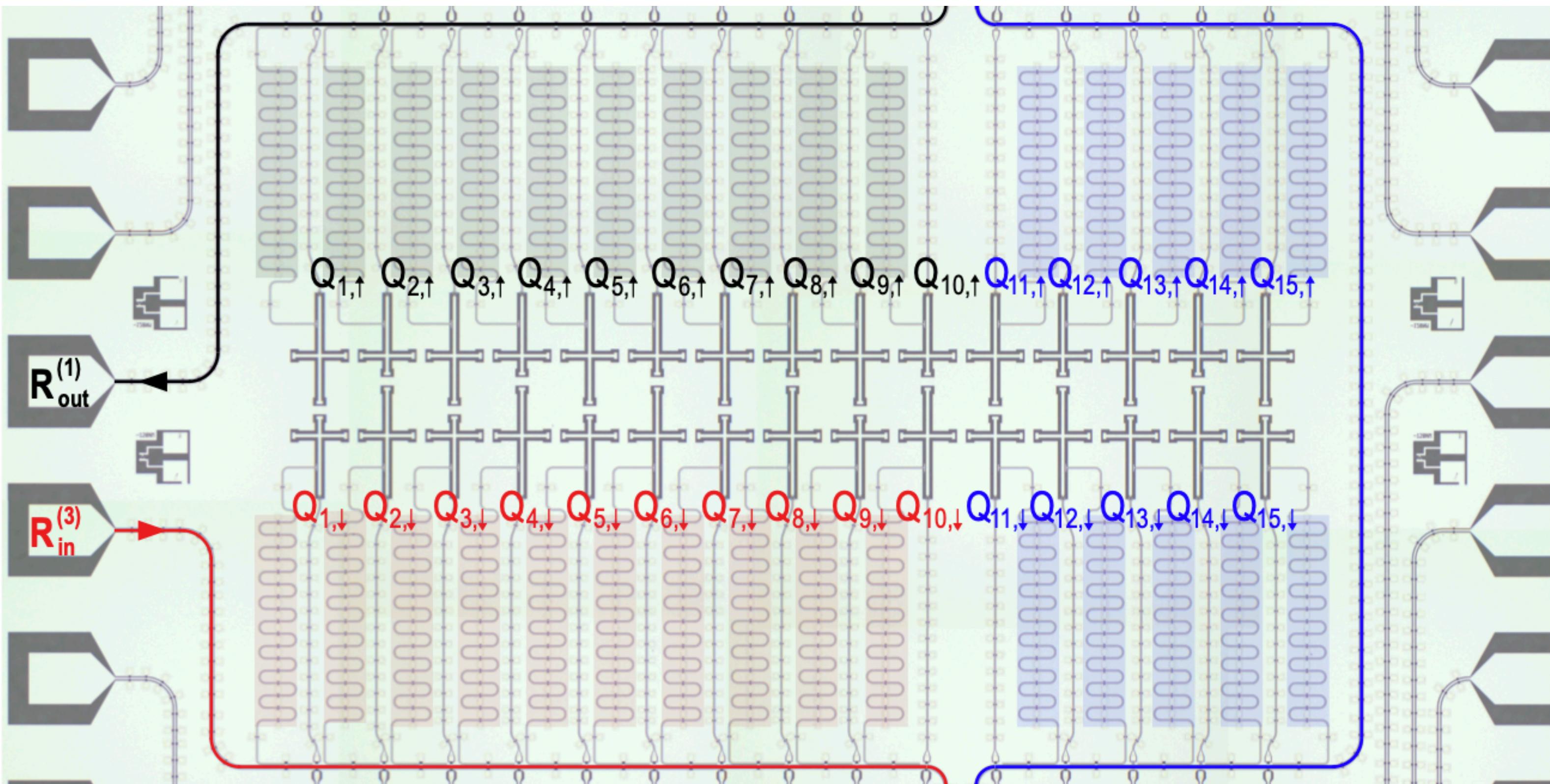


$|U|/J \sim 20$

Hard-core BH model: $H = J \sum_{i=1}^{N-1} (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + h.c.) + \sum_{i=1}^N h_i \hat{\sigma}_i^+ \hat{\sigma}_i^-$

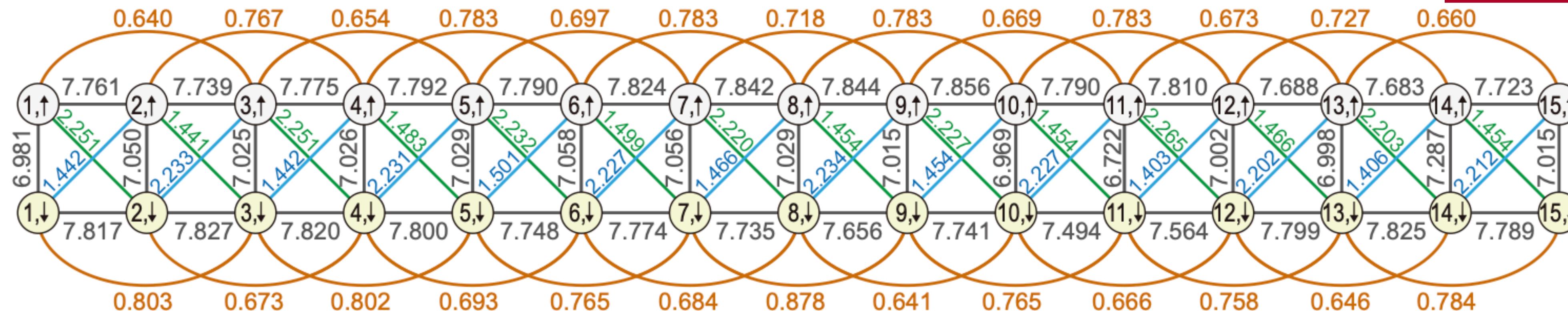
Example1: ladder-type circuit

arXiv: 2207.11797



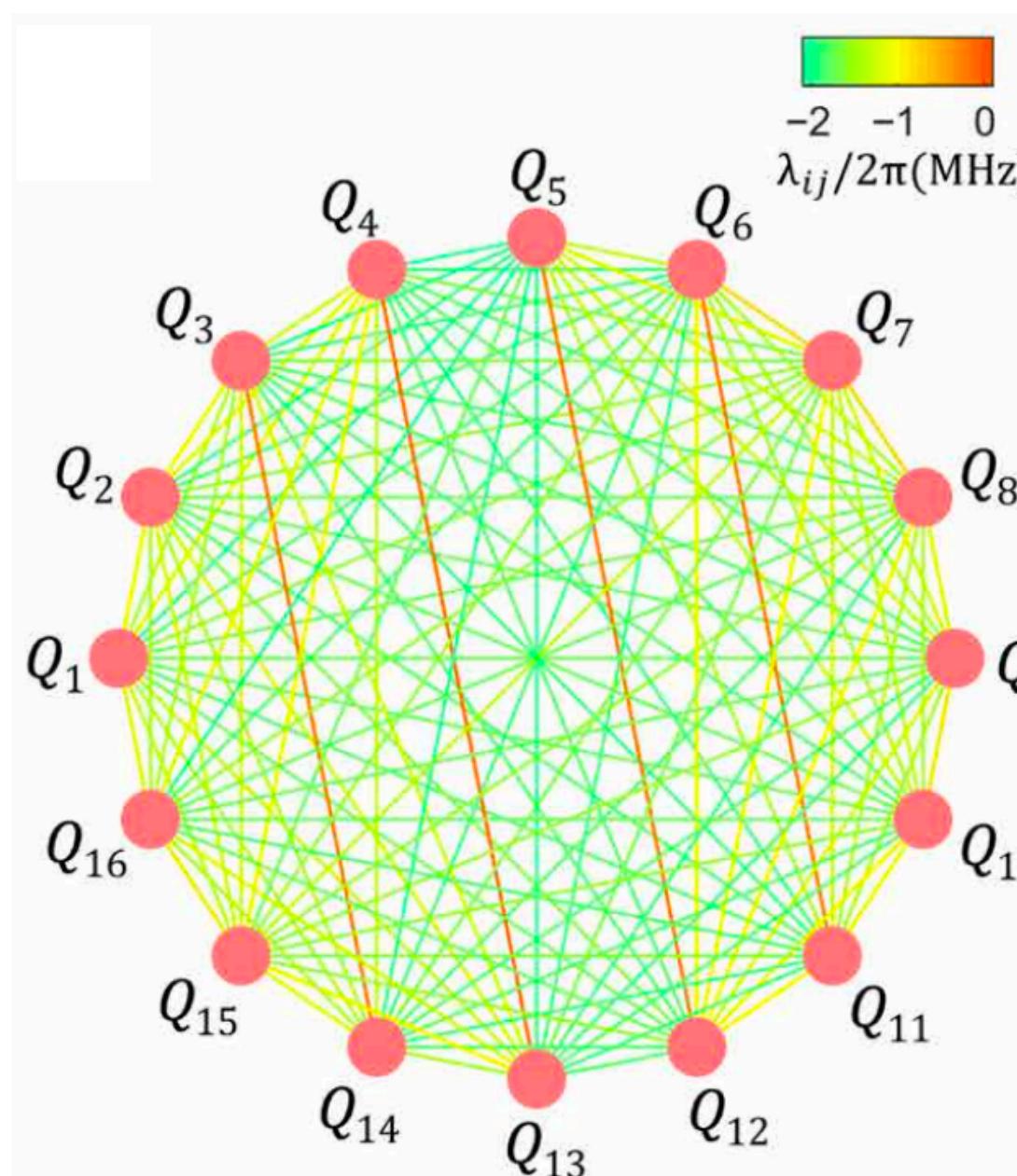
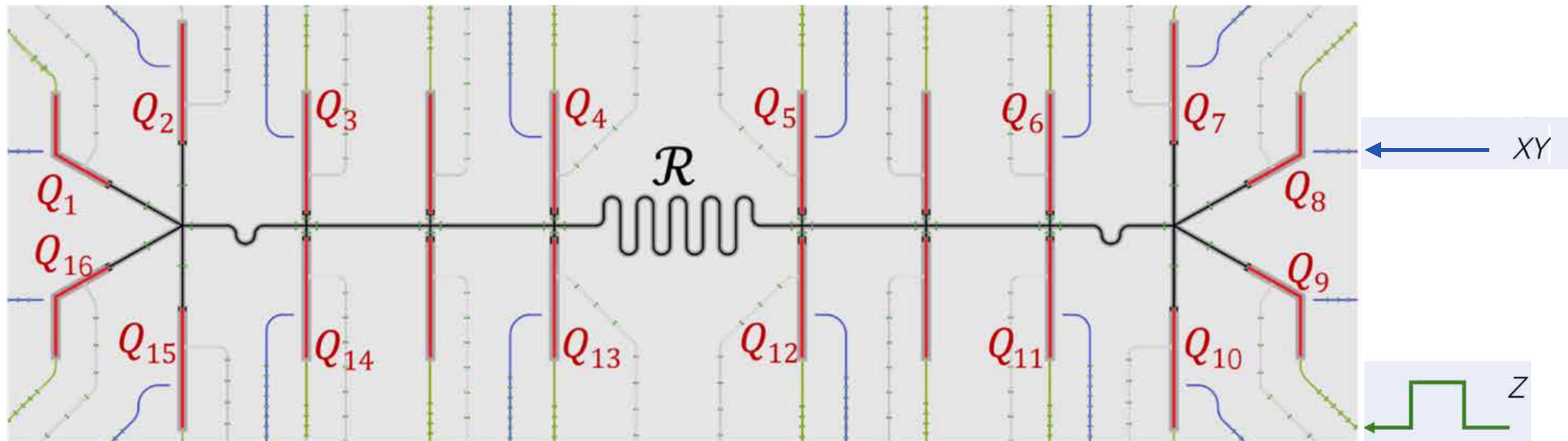
b Coupling strength for the 30-qubit experiment (MHz) $J_{ij}/2\pi$

$$J_{ij}(\hat{\sigma}_i^+ \hat{\sigma}_j^- + h.c.)$$



Example2: the circuit with all-to-all connectivity

Sci. Adv. 6, eaba4935 (2020)



Toolbox

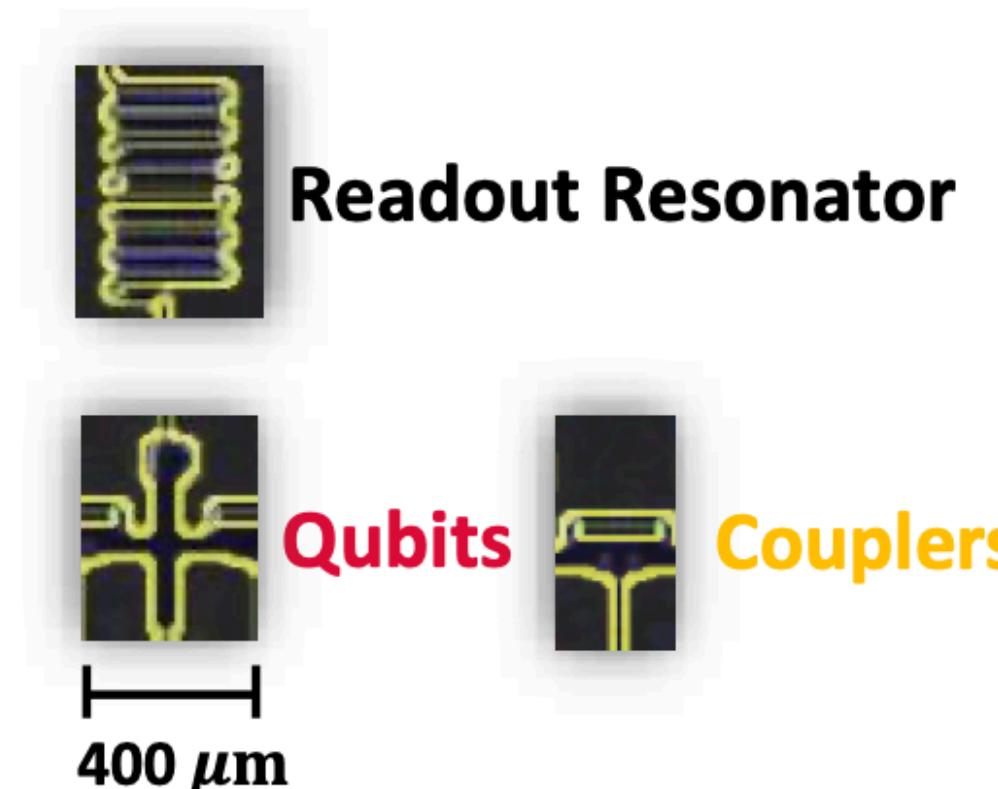
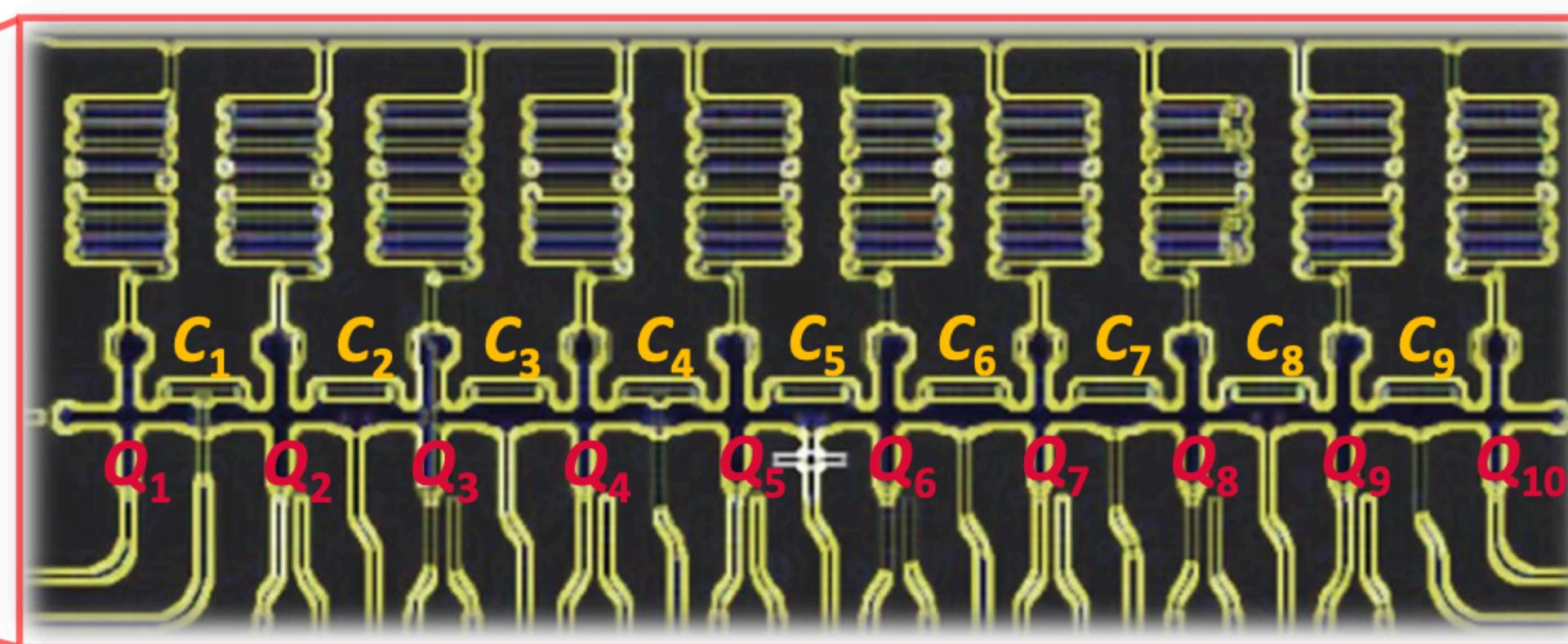
Controllability of single qubits:

Z-control $H_Z = \sum_{i=1}^N h_i \hat{\sigma}_i^+ \hat{\sigma}_i^-$ $\{h_1, h_2, \dots, h_N\}$

XY-control $H_T = \sum_{j=1}^N \Omega_j (e^{-i\phi_j} \hat{\sigma}_j^+ + e^{i\phi_j} \hat{\sigma}_j^-)$

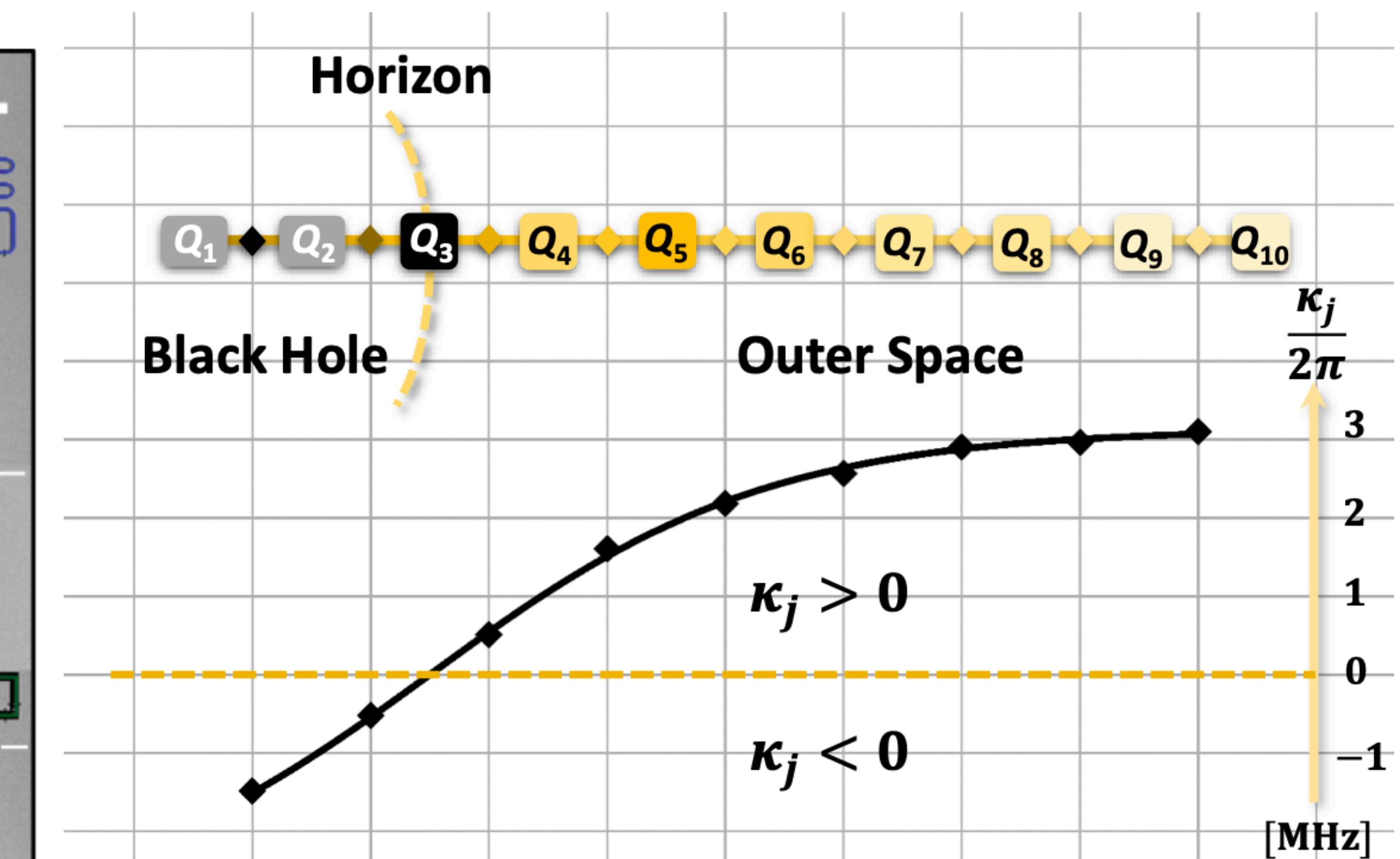
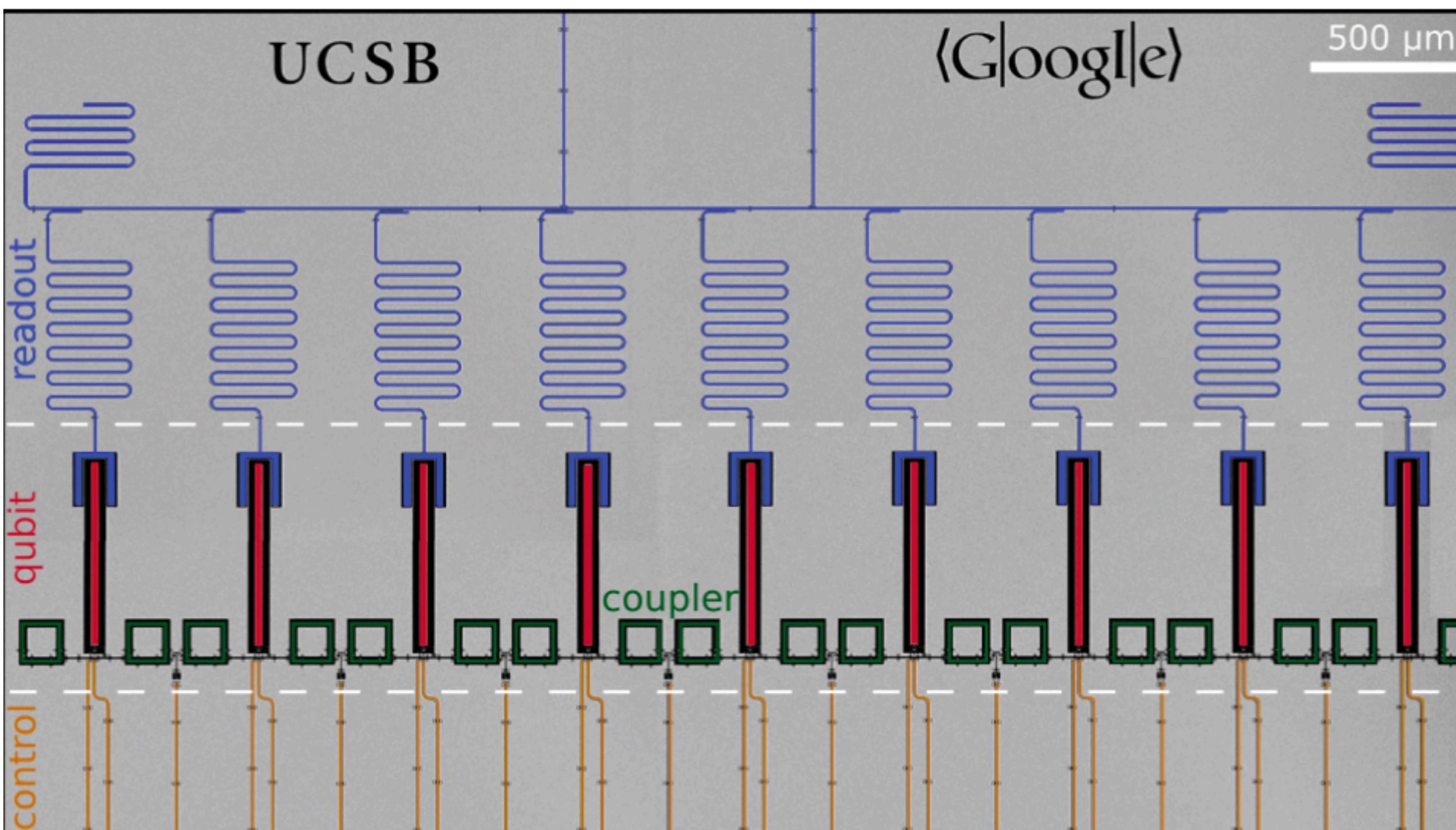
$\{\Omega_1, \Omega_2, \dots, \Omega_N\}$ $\{\phi_1, \phi_2, \dots, \phi_N\}$

Example3: the circuit with couplers between qubits



Science 360, 195-199 (2018)

arXiv: 2111.11092



3. Analog quantum simulation of dynamical phase transitions

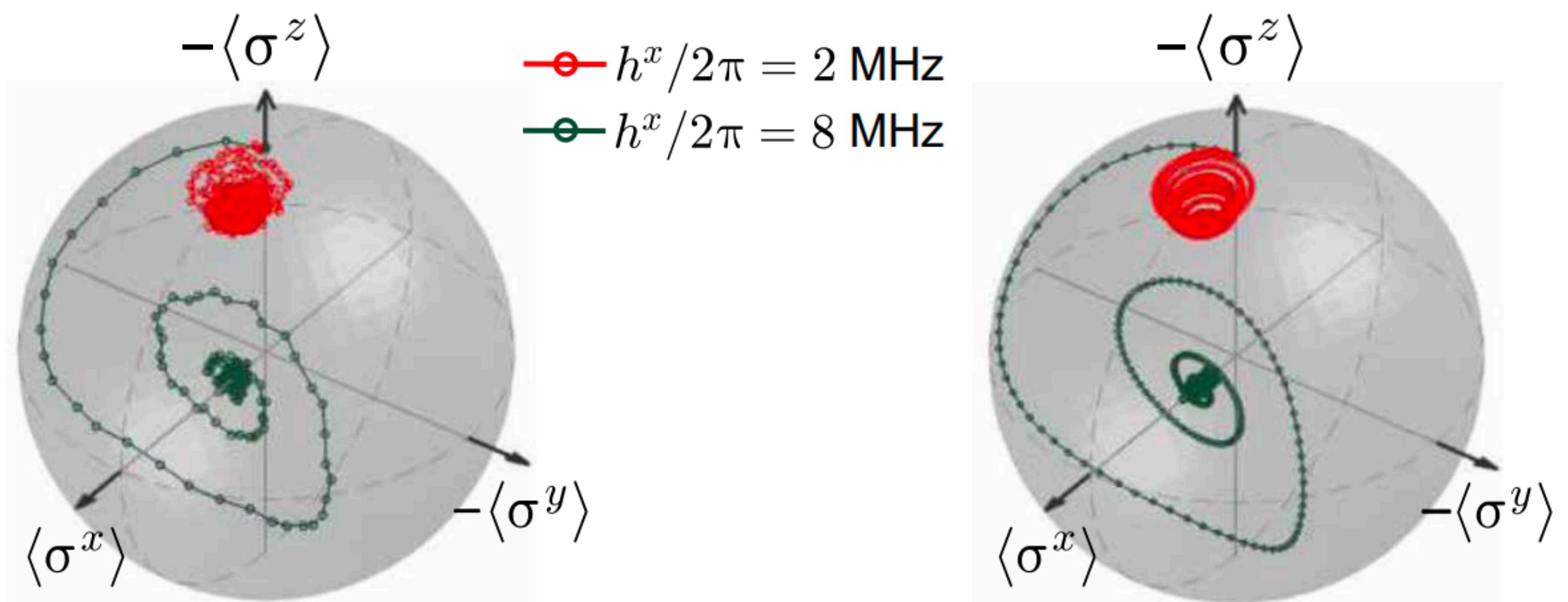
Hamiltonian of the superconducting circuit with all-to-all connectivity: $H_1/\hbar = \sum_{i<j}^N \lambda_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + h^x \sum_{j=1}^N \sigma_j^x$

Uniform-coupling assumption: $\lambda \sum_{i<j}^{16} (\sigma_i^+ \sigma_j^- + \text{H.c.}) = (J/N) [S^2 - (S^z)^2] \quad S^\alpha = \frac{1}{2} \sum_{i=1}^N \sigma_i^\alpha \quad \alpha = x, y, z$

Initial state: $|\psi_0\rangle = |00\dots 0\rangle$ as the eigenstate of $S^2 = (S^x)^2 + (S^y)^2 + (S^z)^2 \quad [S^2, S^\alpha] = 0$

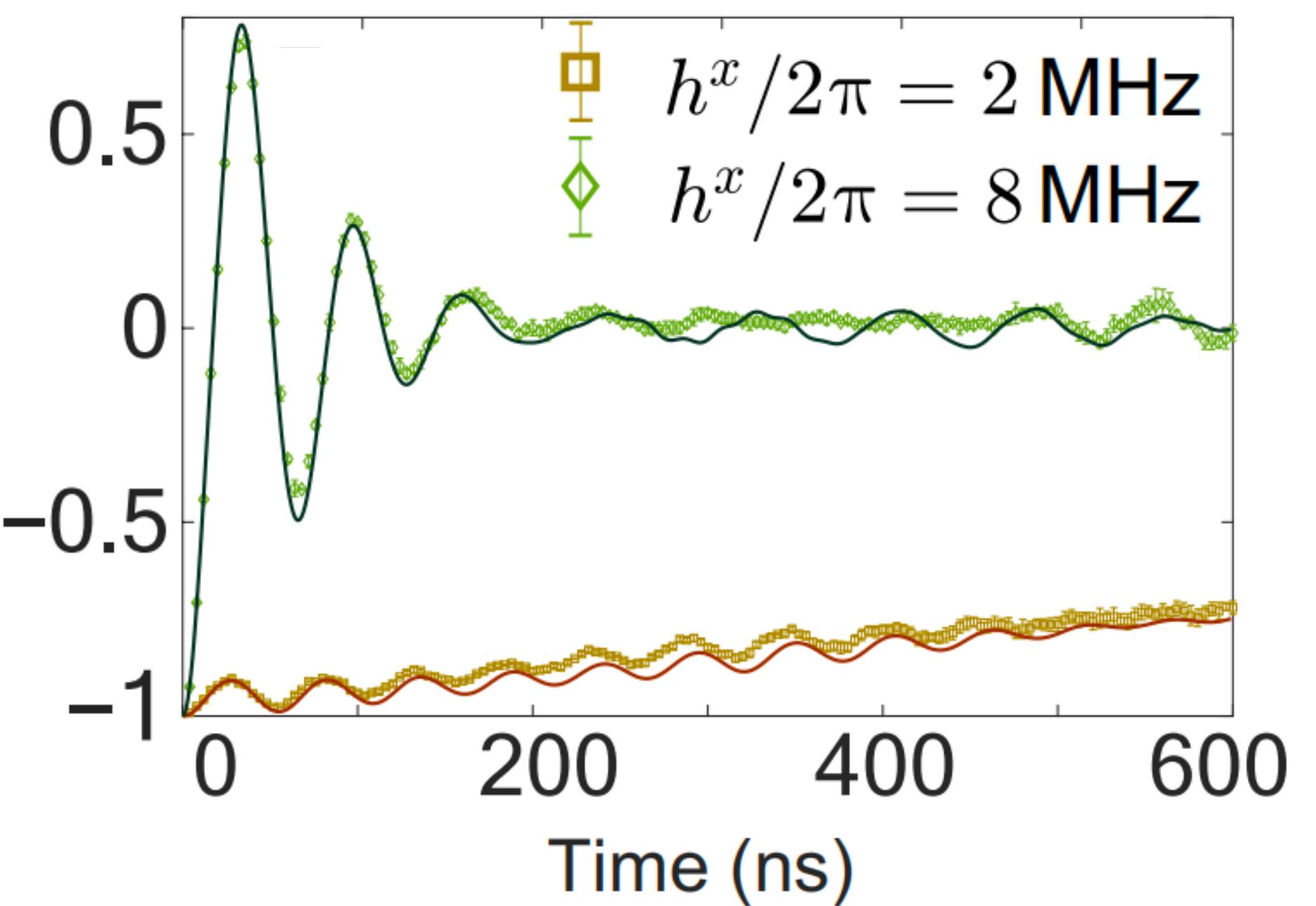
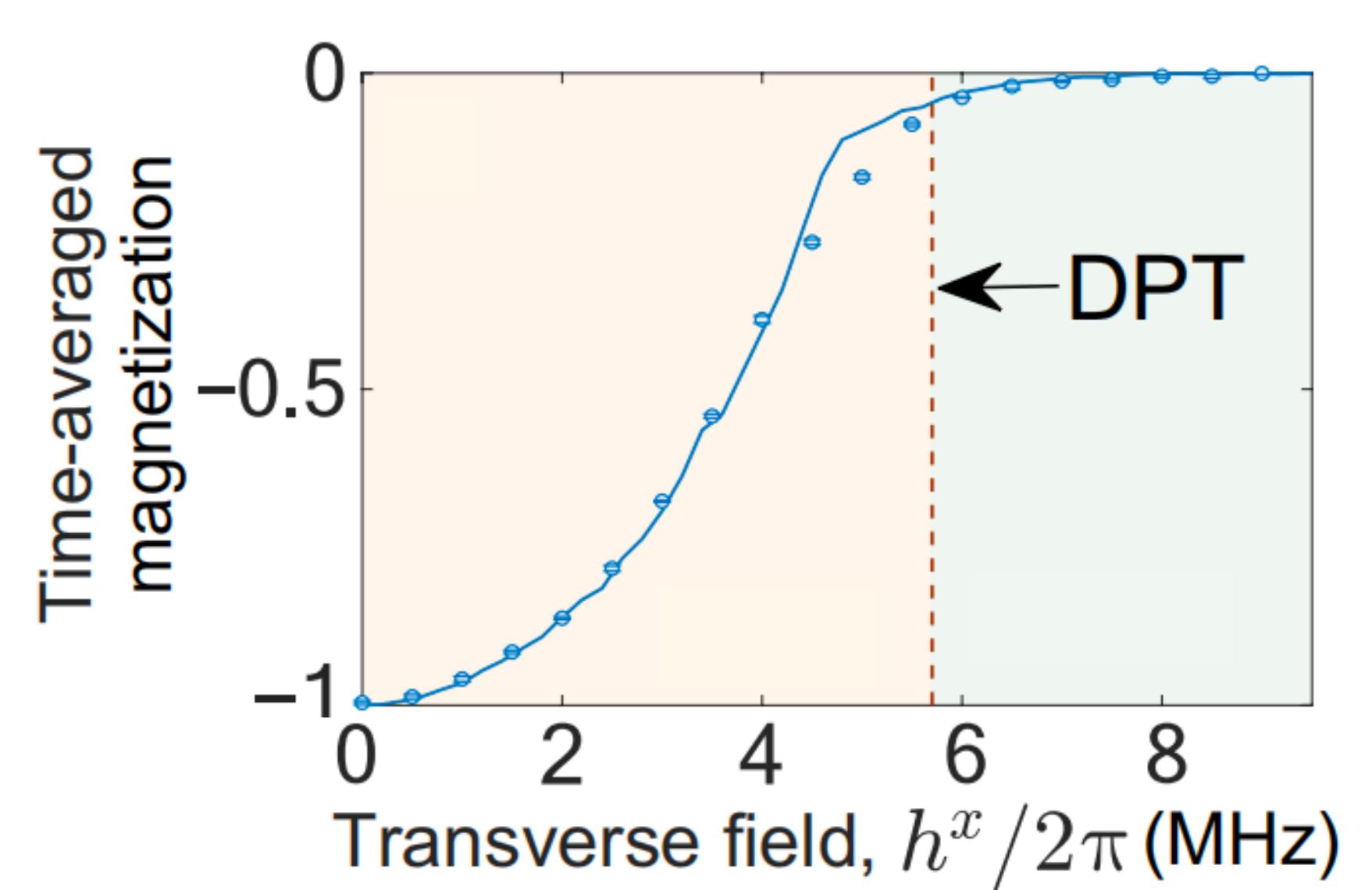
$$\exp[-i(H_1/\hbar)t] |00\dots 0\rangle \propto \exp(-iH_{\text{LMG}}t) |00\dots 0\rangle \quad H_{\text{LMG}} = -(J/N)(S^z)^2 + \mu S^x$$

$$J = N\lambda \quad \mu = 2h^x \quad \text{critical point: } h_c^x = N\lambda/4$$

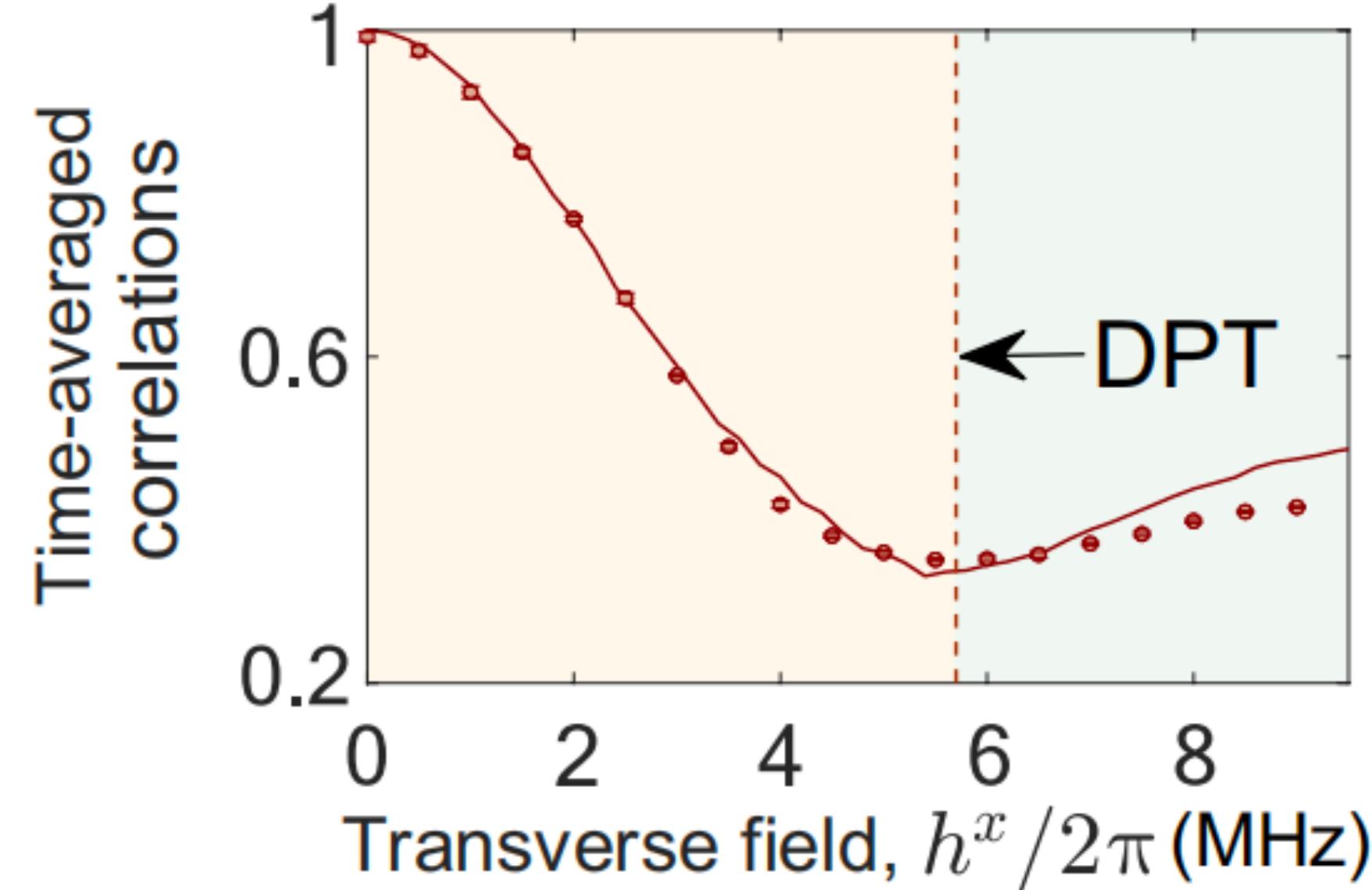


Global Z_2 symmetry : $\sigma^{z,y} \rightarrow -\sigma^{z,y}$

$$\overline{\langle \sigma^z \rangle} \equiv (1/t_f) \int_0^{t_f} dt \langle \sigma^z(t) \rangle$$



$$\overline{C_{zz}} \equiv (1/t_f) \int_0^{t_f} dt \sum_{ij} \langle \sigma_i^z(t) \sigma_j^z(t) \rangle / N^2$$

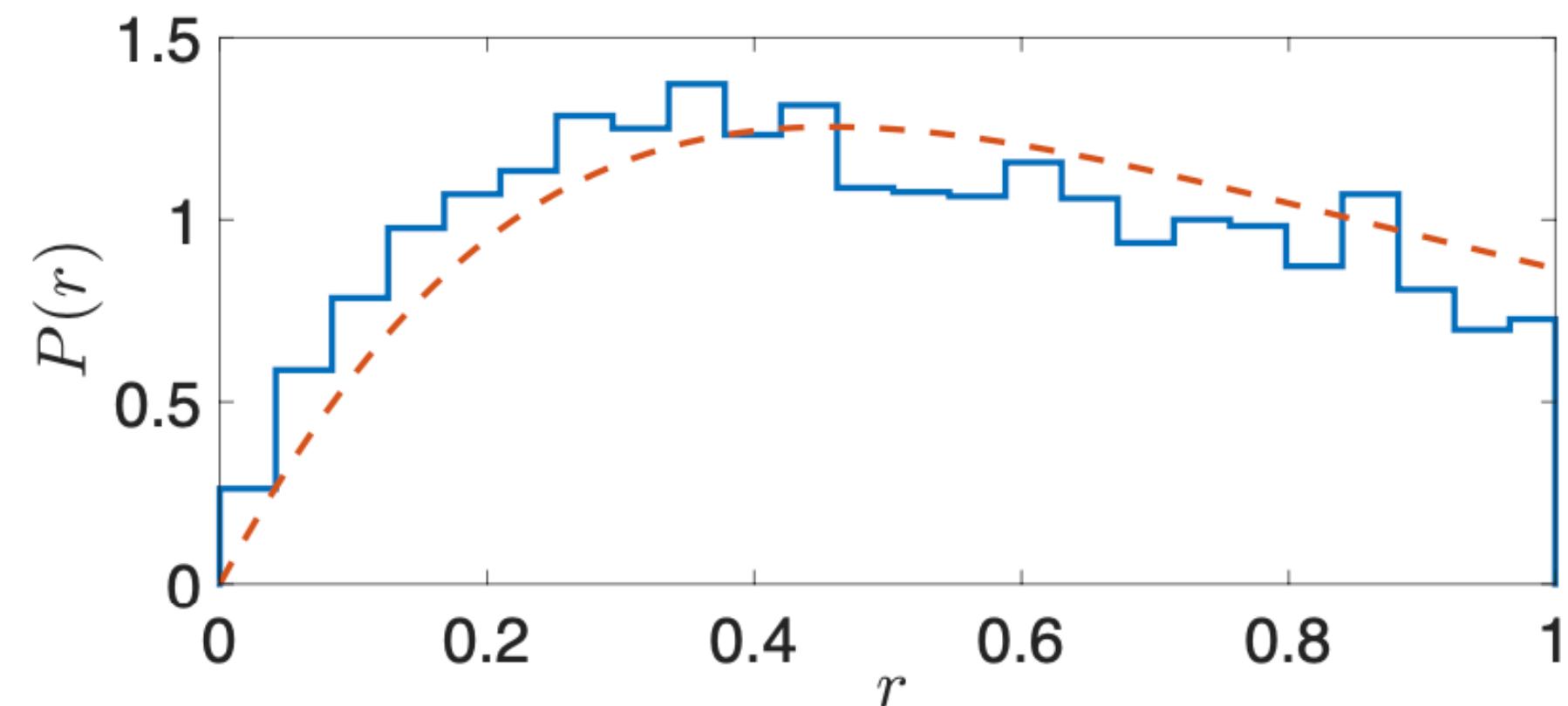


4. Analog quantum simulation of strong and weak thermalization

System Hamiltonian:

$$\hat{H} = \lambda \sum_{j=1}^{11} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y) + g \sum_{j=1}^{12} \hat{\sigma}_j^y,$$

An array of superconducting qubits
with transverse field



Level spacing:

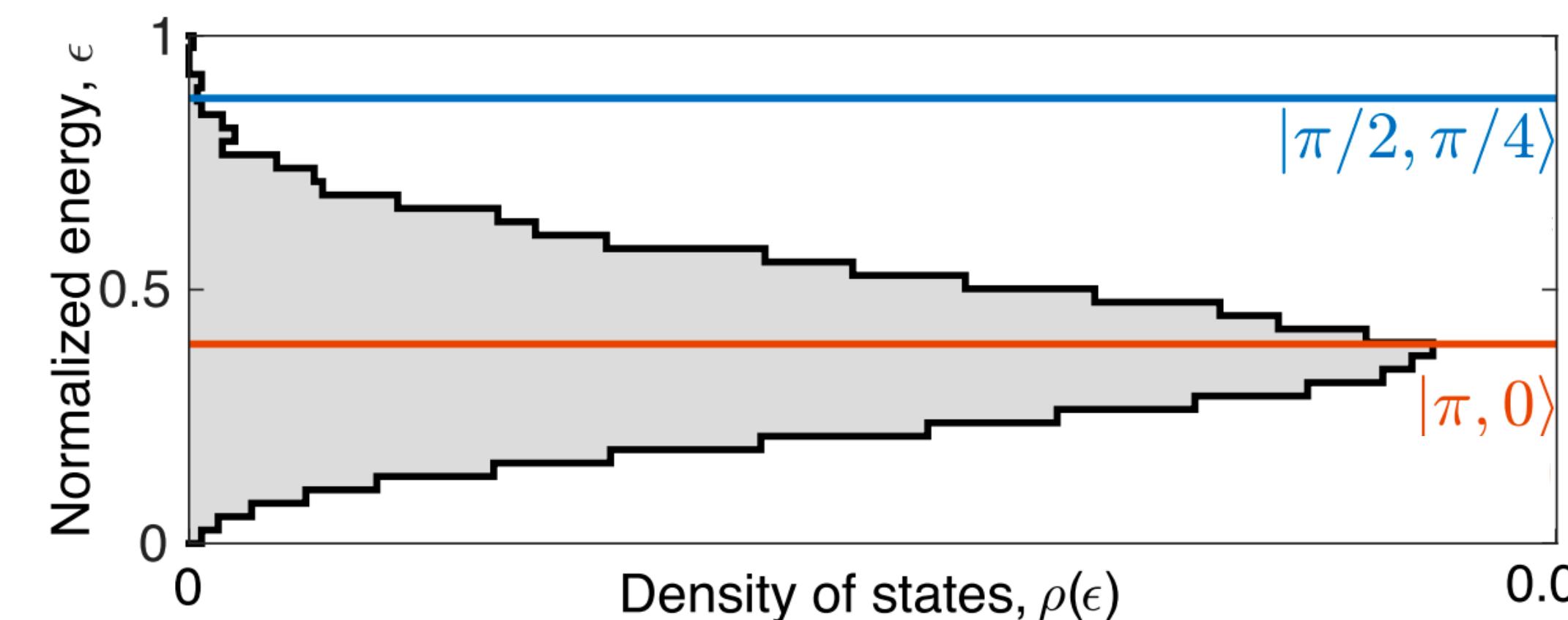
$$r_n = \frac{\min\{s_n, s_{n-1}\}}{\max\{s_n, s_{n-1}\}}, \quad s_n = E_{n+1} - E_n$$

Gaussian orthogonal ensemble :

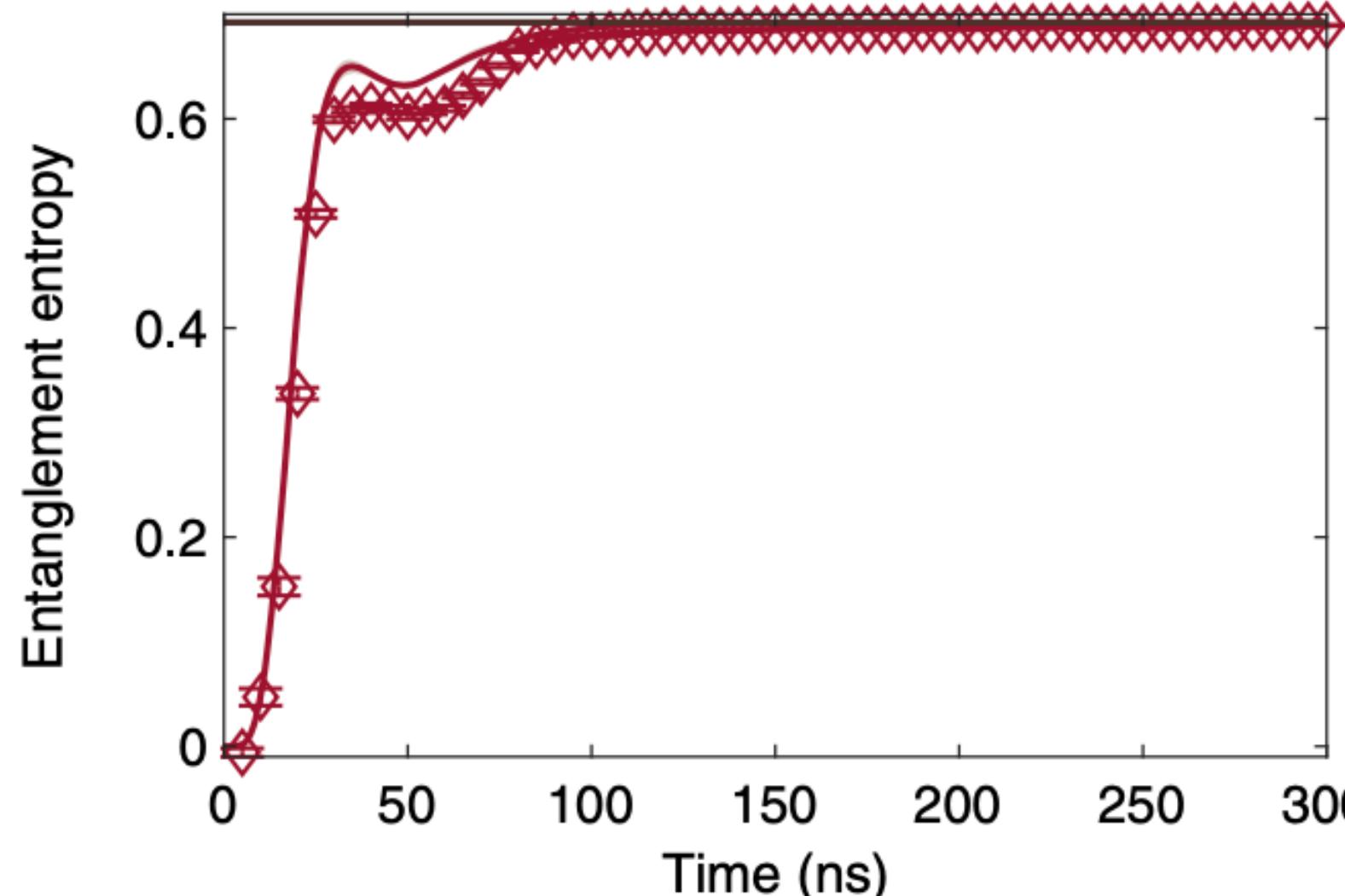
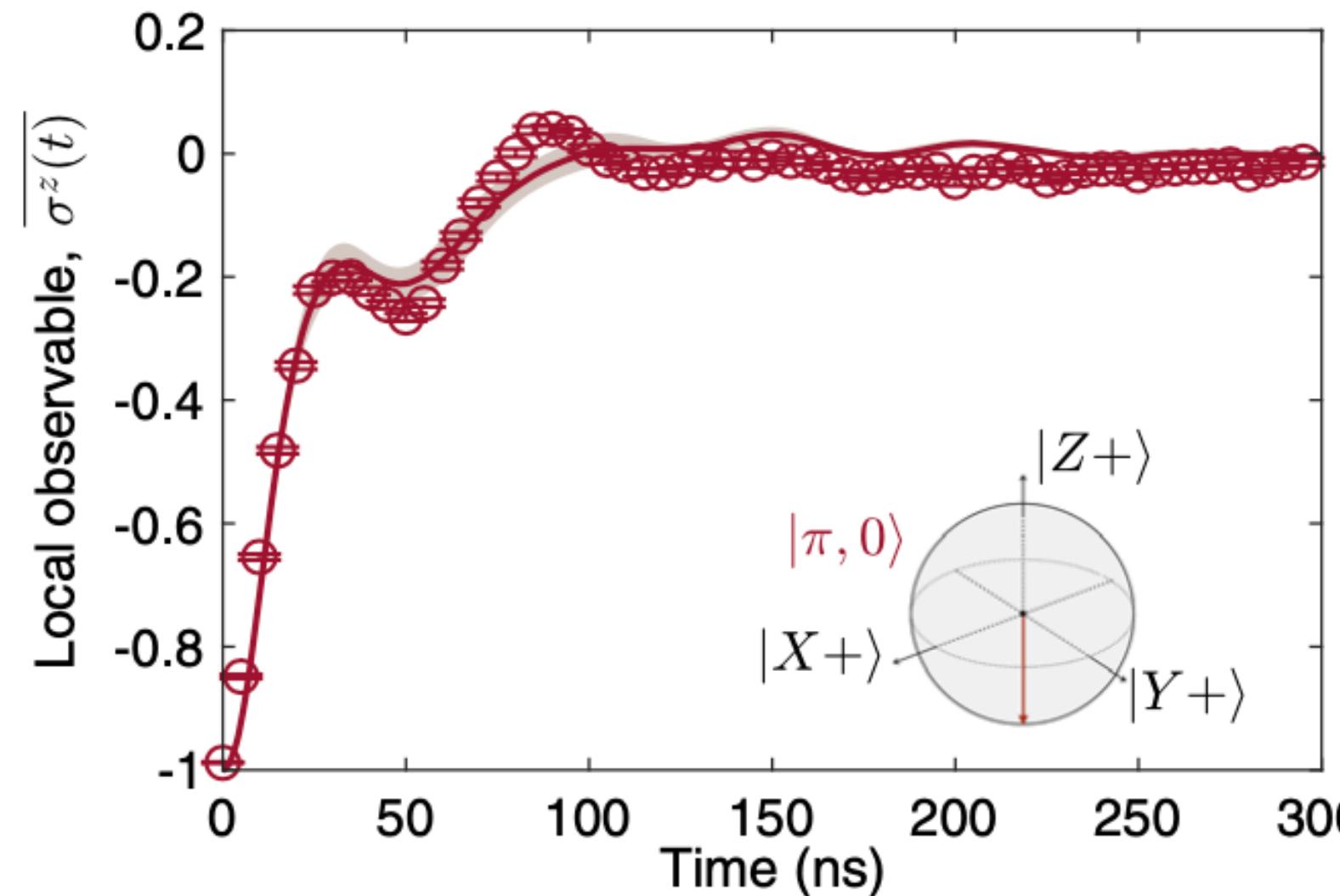
$$P_{\text{GOE}}(r) = \frac{27}{4} \frac{r + r^2}{(1 + r + r^2)^{5/2}}.$$

FIG. S1. **Level statistics of the system Hamiltonian.** The dashed line is the probability distribution of r_n following the GOE. The solid line is the numerics of $P(r)$ of the system (S4).

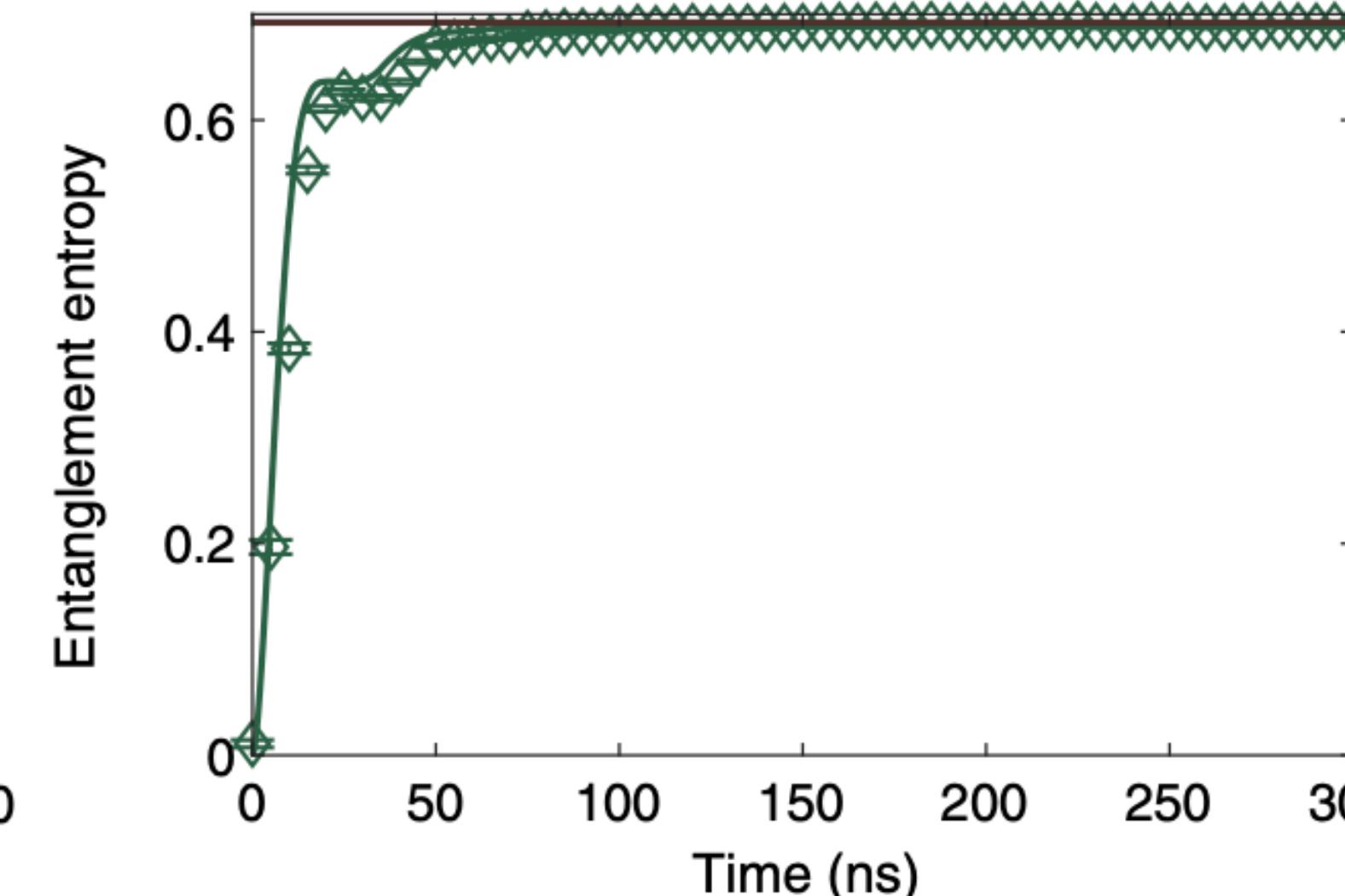
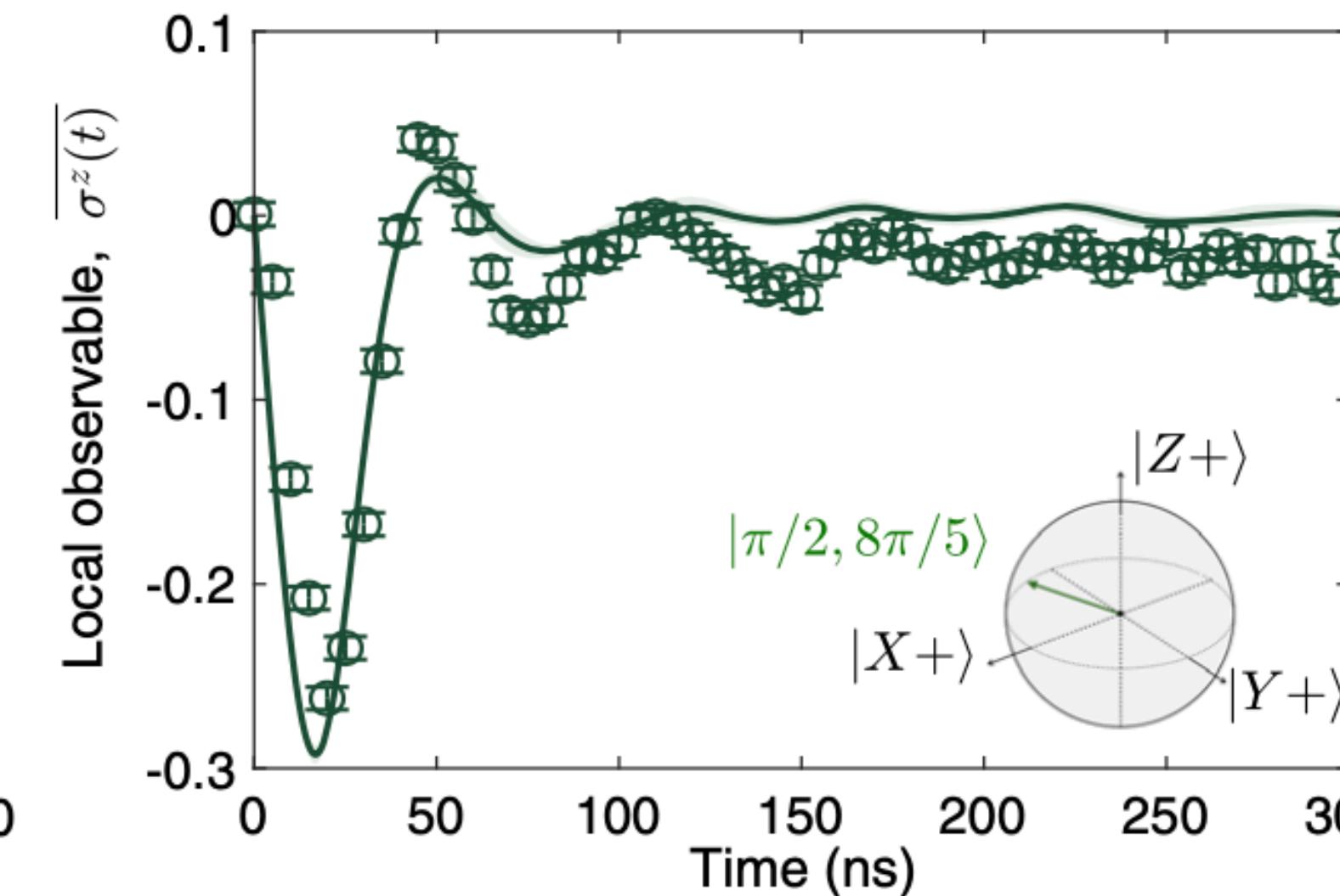
A characterization of non-integrable model



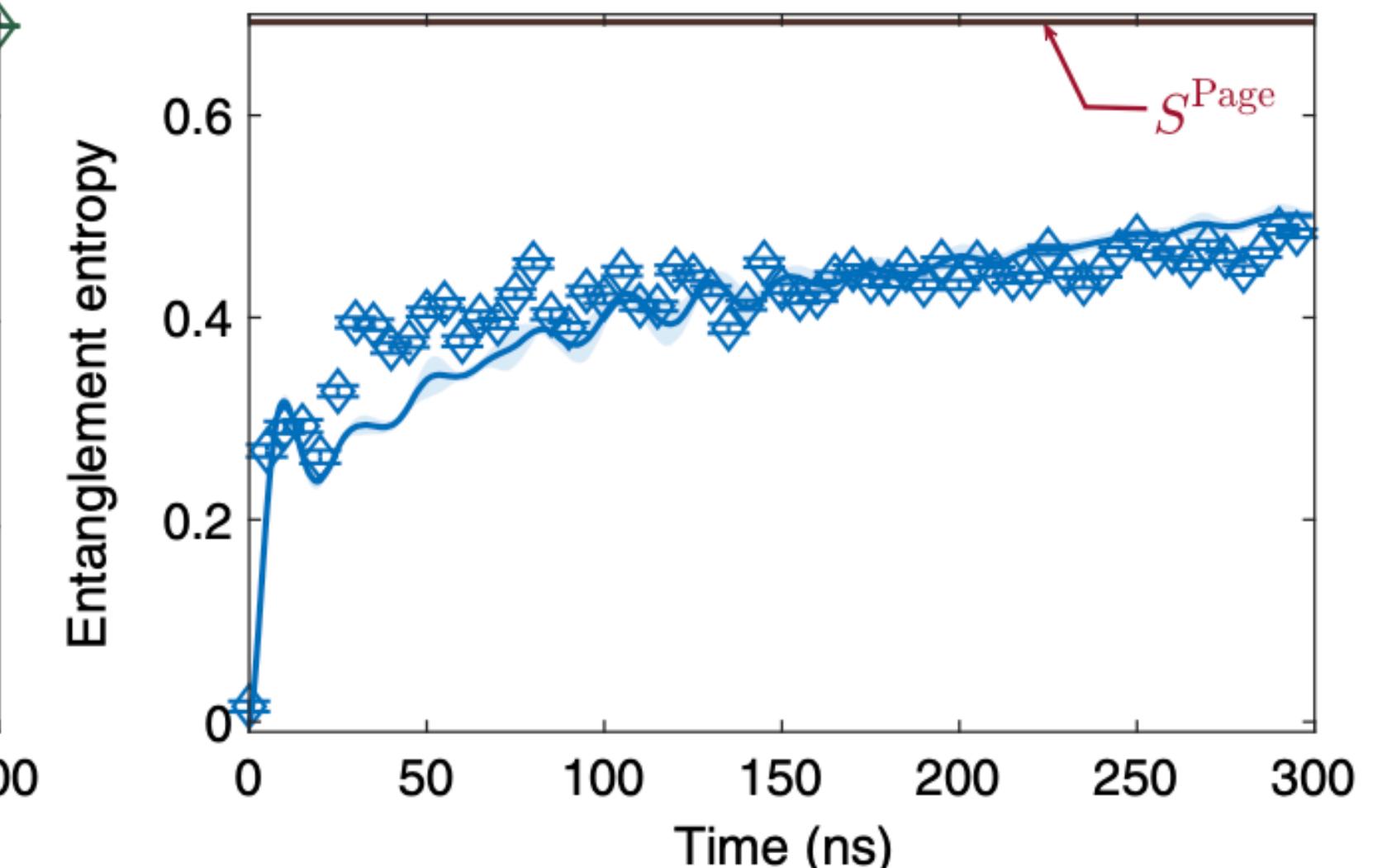
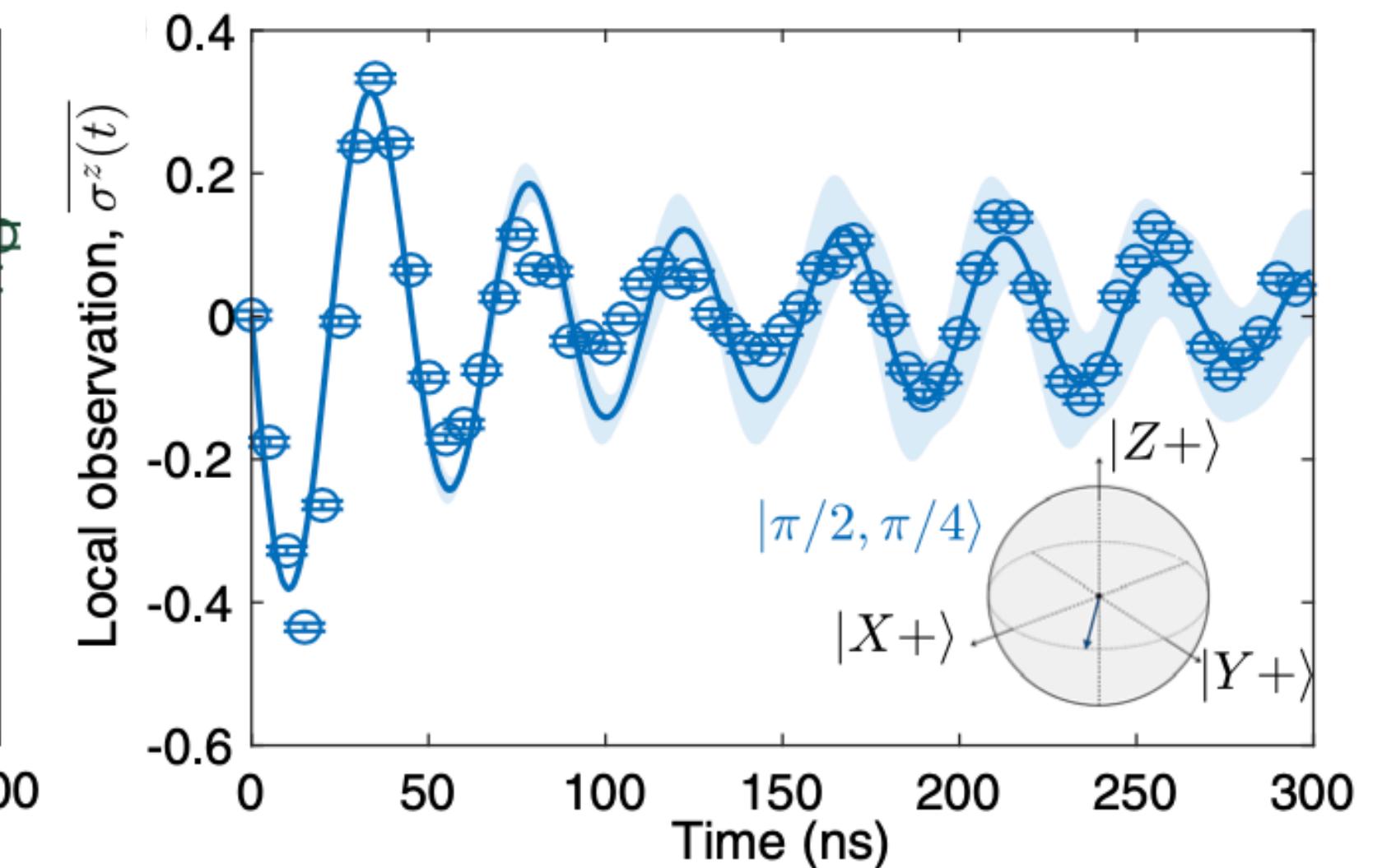
Strong thermalization



Strong thermalization (another initial state)



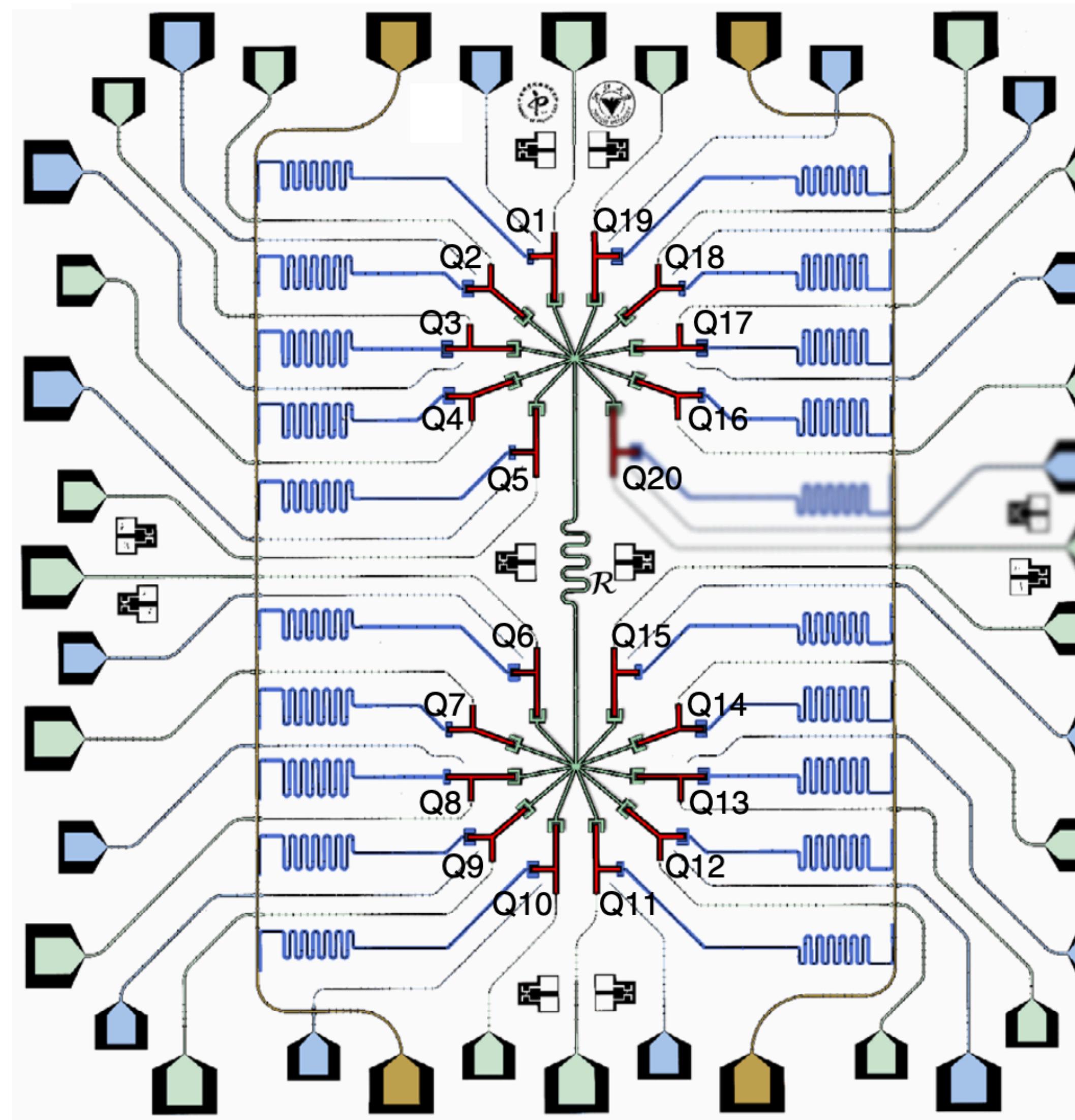
Weak thermalization



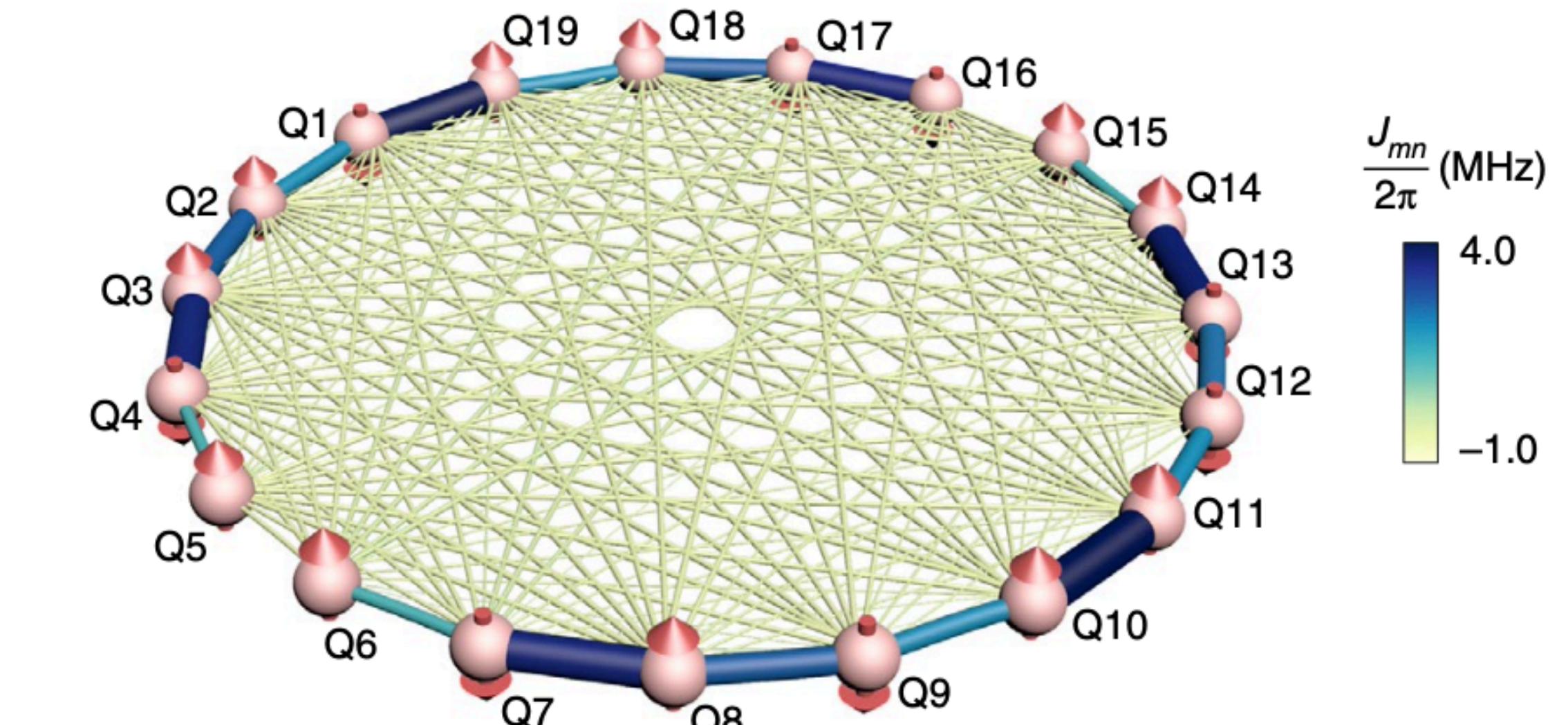
5. Analog quantum simulation of energy-resolved many-body localization

Many-body localization (MBL)

Nat. Phys. 17, 234-239 (2021)



■ Qubit ■ Bus resonator ■ Readout resonator ■ XY line ■ Z line ■ Readout line



Long-range XX model:

$$H / \hbar = \sum_{\{m,n\} \in N} J_{mn} (\sigma_m^+ \sigma_n^- + \sigma_m^- \sigma_n^+) \\ + \sum_m V_m \sigma_m^+ \sigma_m^-$$

$$V_m \in [-V, V]$$

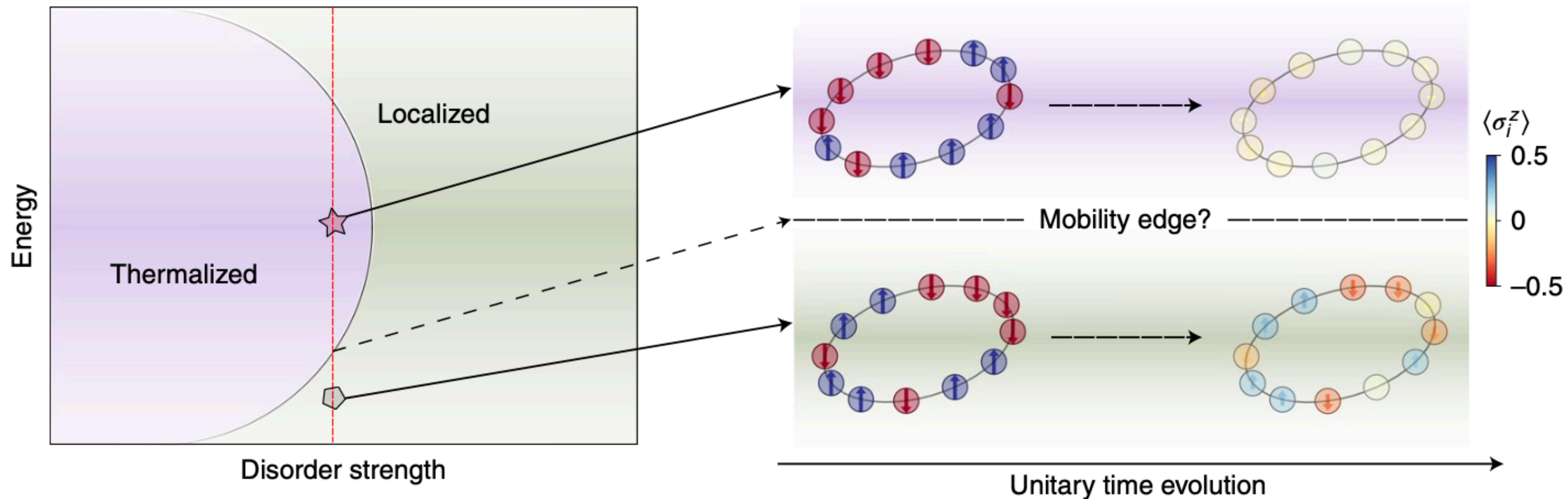
V : strength of disorder

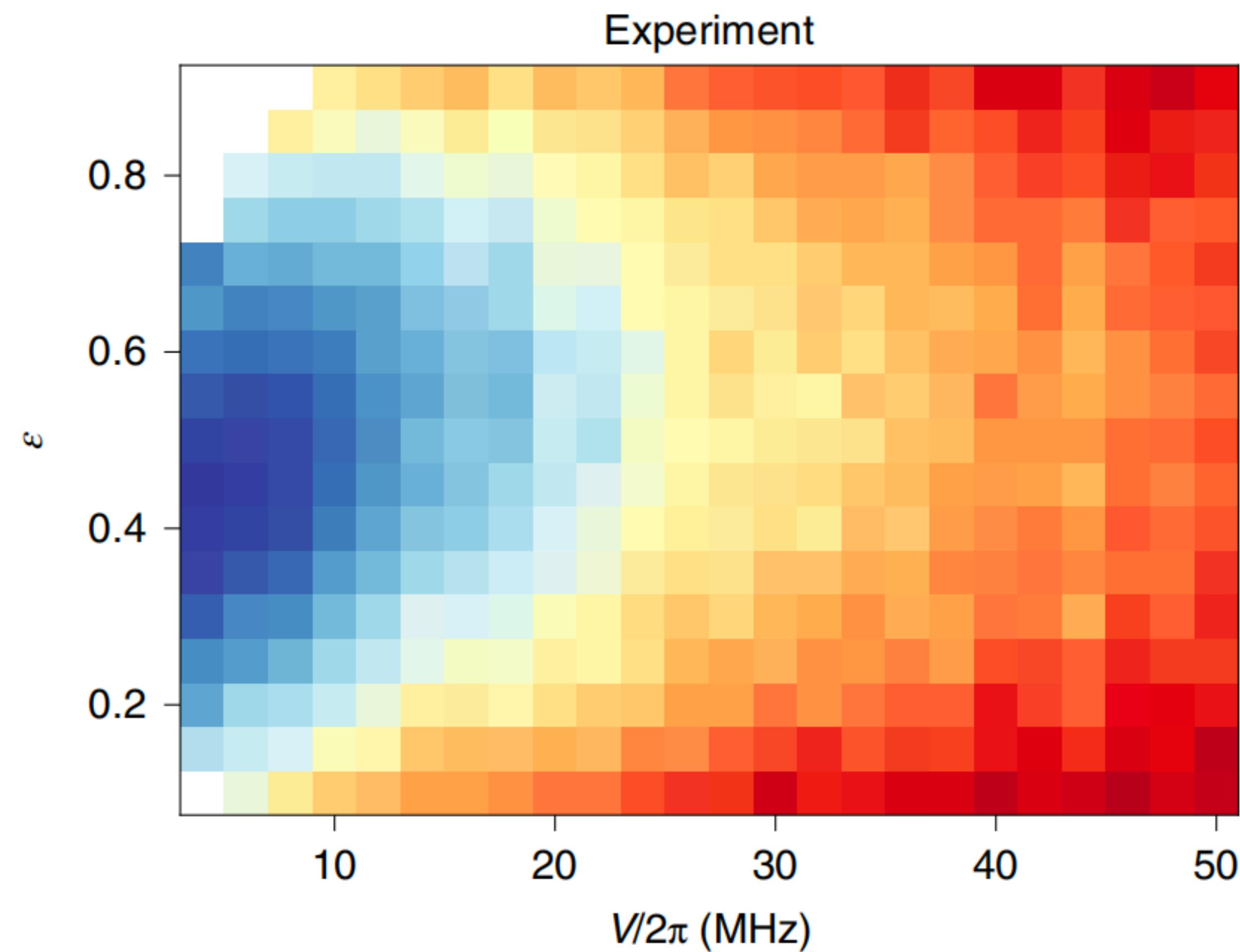
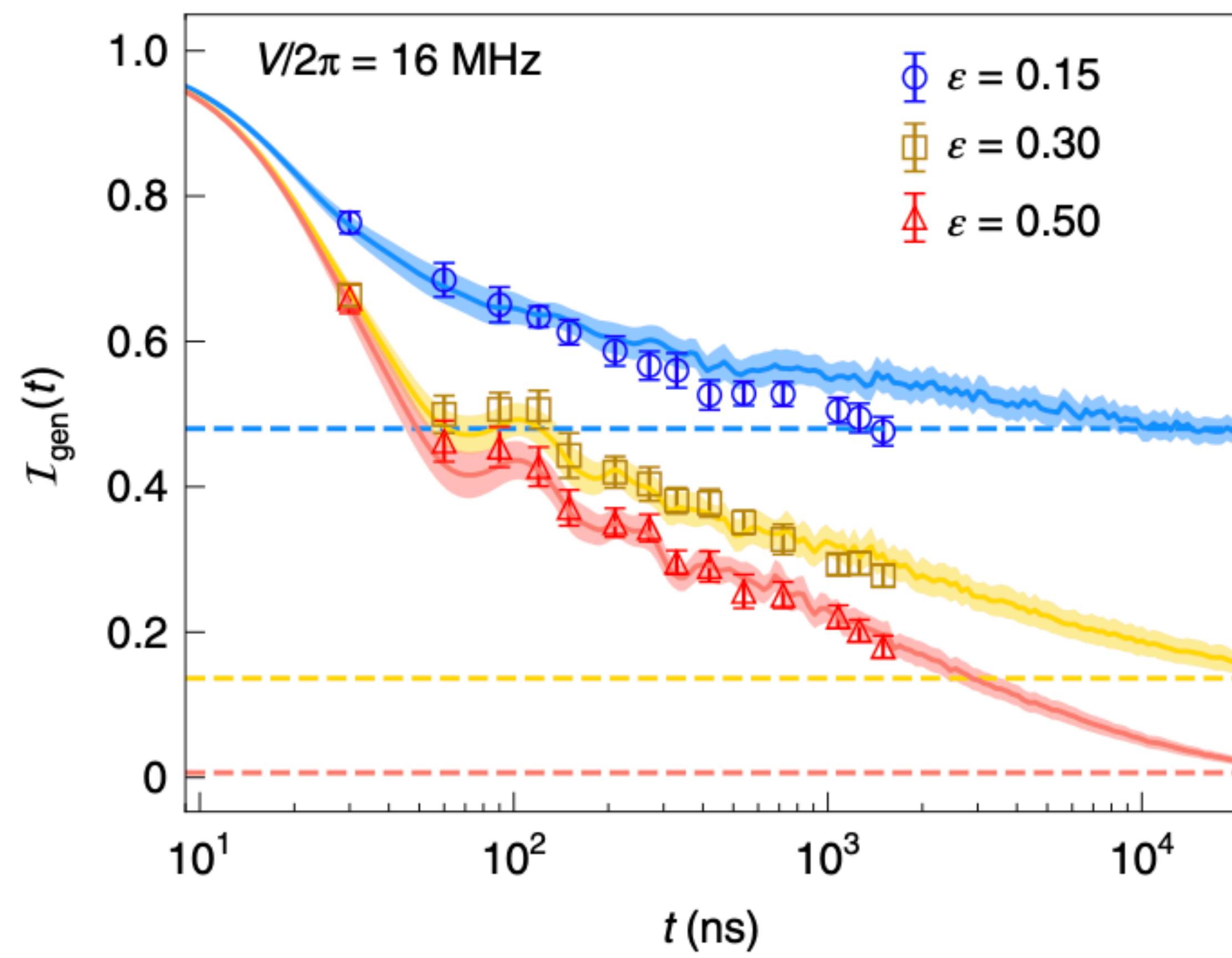
Normalized energy of initial states:

$$\varepsilon = \frac{E - E_{\min}}{E_{\max} - E_{\min}} \quad E = \langle \psi_0 | H | \psi_0 \rangle$$

Nat. Phys. 17, 234-239 (2021)

Mobility edge: the transition between thermalization and MBL is dependent on the normalized energy





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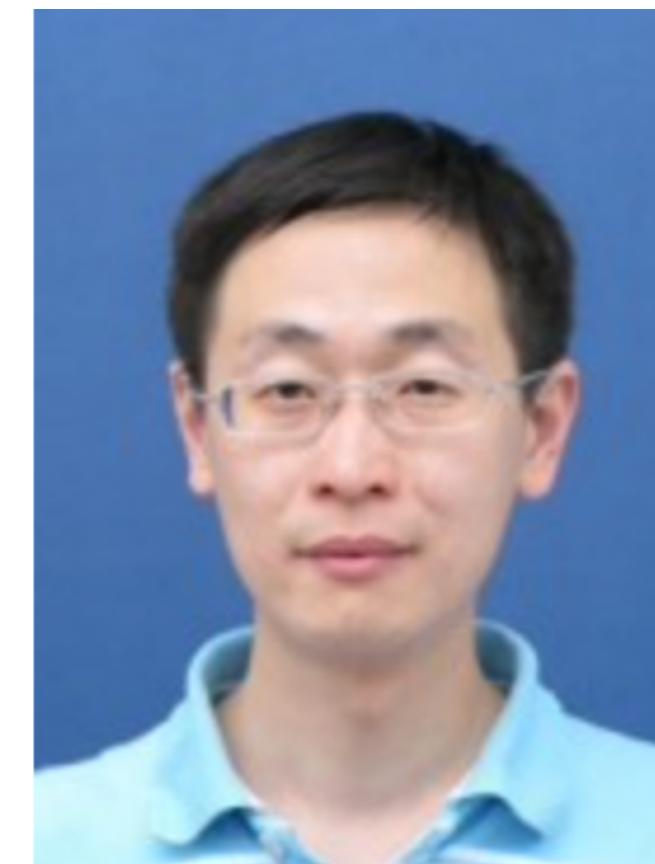
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RIKEN: Prof. Franco Nori, Dr. Yu-Ran Zhang



Prof. Heng Fan



Prof. Haohua Wang



Prof. Xiaobo Zhu



Prof. Franco Nori