Noise-resilient phase estimation with randomized compiling

Gu Yanwu (顾炎武)

Beijing Academy of Quantum Information Sciences

2023/03/31

arXiv:2208.04100

Overview

1. Quantum phase estimation
   - Phase estimation based on quantum Fourier transform
   - Control-free phase estimation
2. Noise-resilient control-free phase estimation
   - A benign type of noise for phase estimation
   - The effect of randomized compiling
3. Simulated experiments
   - Floquet quasi-energies estimation
   - Order finding

Quantum phase estimation

Given a unitary operator U, how to get the phases of its eigenvalues? That is, given a circuit implementation of

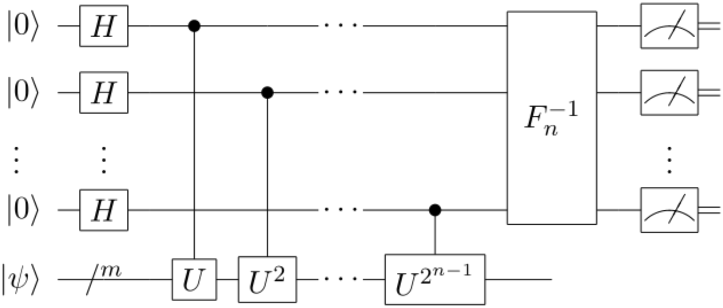
$$U = \sum_a e^{i\lambda_a} |\phi_a\rangle\langle\phi_a|\,,$$

how to efficiently get phases λ_a?

The applications of phase estimation:

- Estimate the eigenvalues of a Hamiltonian.
- Factorization or order-finding.
- Linear systems of equations.

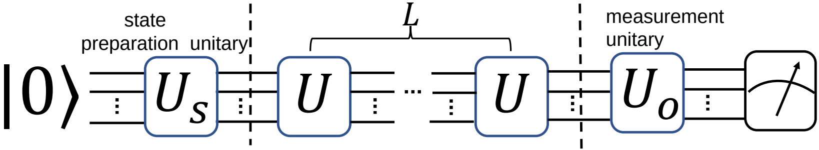
Phase estimation based on quantum Fourier transform



Drawbacks (in NISQ era):

- Require ancillary qubits and control version of U.
- Require quantum error correction to combat noise.

Control-free phase estimation



Phys. Rev. A, 92(6), 062315 (2015)

The expectation value of the operator O is

$$
\begin{aligned}
\langle O \rangle_L &= \langle \psi | (U^\dagger)^L O U^L | \psi \rangle \\
&= \sum_{a,b} c_a c_b^* \langle \phi_b | O | \phi_a \rangle e^{i(\lambda_a - \lambda_b)L}
\end{aligned}
$$

where $|\psi\rangle = \sum_a c_a |\phi_a\rangle$.

This work introduces an error mitigation method for control-free phase estimation.

A benign type of noise

Assume a noisy channel $\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$ is appended to the unitary channel $\mathcal{U}$, and the resultant noisy gate is $\widetilde{\mathcal{U}} = \mathcal{E}\,\mathcal{U}$. The measured signals $\langle O \rangle_L$ are a damping oscillating function. Control-free phase estimation actually measures the eigenvalues of $\widetilde{\mathcal{U}}$ under noise.

Theorem

If every Kraus operator E_k of a noise $\mathcal{E}$ is Hermitian, then the noisy version $\widetilde{\mathcal{U}} = \mathcal{E}\,\mathcal{U}$ of a unitary channel $\mathcal{U}$ keeps the phases unchanged up to the first-order correction.

A short proof

The perturbation matrix is

$$\Delta = \widetilde{\mathcal{U}} - \mathcal{U} = (\mathcal{E} - \mathcal{I})\mathcal{U}\,.$$

Then for a non-degenerate eigenvalue $e^{i(\lambda_a - \lambda_b)}$ with eigen-operator $|\phi_a\rangle\langle\phi_b|$, the first order correction is

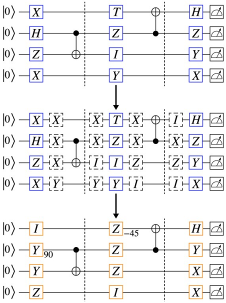
$$\begin{aligned}
\epsilon &= \mathrm{tr}\left\{(|\phi_a\rangle\langle\phi_b|)^\dagger \Delta(|\phi_a\rangle\langle\phi_b|)\right\} \\
&= e^{i(\lambda_a - \lambda_b)}\left[\mathrm{tr}\{|\phi_b\rangle\langle\phi_a|\mathcal{E}(|\phi_a\rangle\langle\phi_b|)\} - 1\right] \\
&= e^{i(\lambda_a - \lambda_b)}\left[\mathrm{tr}\left\{|\phi_b\rangle\langle\phi_a|\sum_k E_k|\phi_a\rangle\langle\phi_b|E_k^\dagger\right\} - 1\right] \\
&= e^{i(\lambda_a - \lambda_b)}\left[\sum_k \langle\phi_a|E_k|\phi_a\rangle\langle\phi_b|E_k^\dagger|\phi_b\rangle - 1\right]\,.
\end{aligned}$$

Noisy eigenvalue under the first order correction is

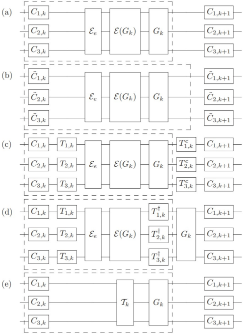
$$e^{i(\lambda_a - \lambda_b)} + \epsilon = e^{i(\lambda_a - \lambda_b)}\left[\sum_k \langle\phi_a|E_k|\phi_a\rangle\langle\phi_b|E_k^\dagger|\phi_b\rangle\right]\,.$$

Randomized compiling

RC can convert the noise in each cycle of a circuit into stochastic Pauli noise with depth of the circuit unchanged.



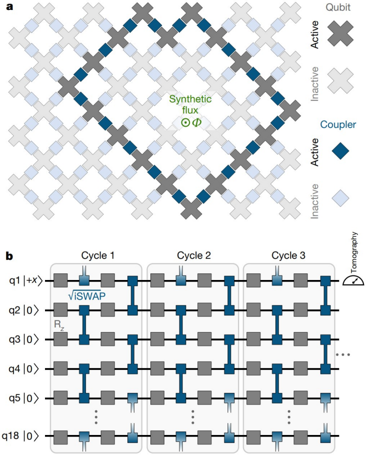
Phys. Rev. X 11, 041039 (2021)



Phys. Rev. A, 94(5), 052325.

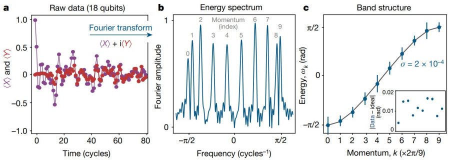
Pauli twirling: $\frac{1}{N}\sum_{n=1}^{N} \mathcal{T}_n^\dagger \mathcal{E}(G_k)\mathcal{E}_e\mathcal{T}_n = \mathcal{E}_p$

Google experiments



Nature 594.7864 (2021): 508-512.

Hamiltonian: $\sum_m \sigma_m^+ \sigma_{m+1}^- + \sigma_m^- \sigma_{m+1}^+$



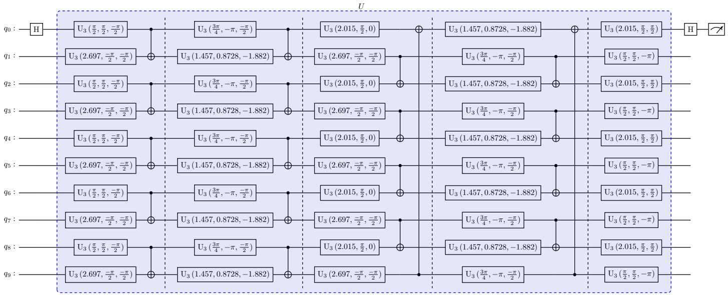
The reasons of high accuracy:

- The problem is insensitive to T_1, T_2.
- Unitary errors are corrected at the level of two qubits.

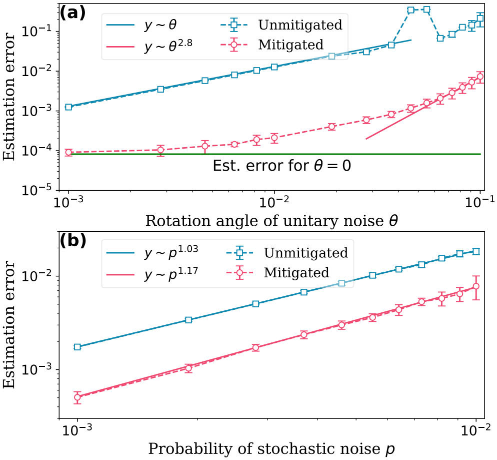
Simulation of Google experiments

Simulation parameters:

- Max num of repetitions of U is $L_{\max} = 50$.
- To keep the resource cost the same, num of shots for each bare circuit is $N_s = 10^5$; and $N_s = 10^5/N_r$ for each randomized circuit, where $N_r = 20$ is the num of randomized circuits for each bare circuit.



Simulation results



Order finding

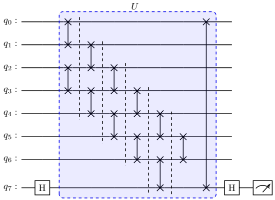
The order finding problem is to find the least positive integer r such that for two specified coprime numbers x and N ($x < N$) we have $x^r = 1 (\mathrm{mod}\, N)$. The factoring problem can be converted to the order finding problem.

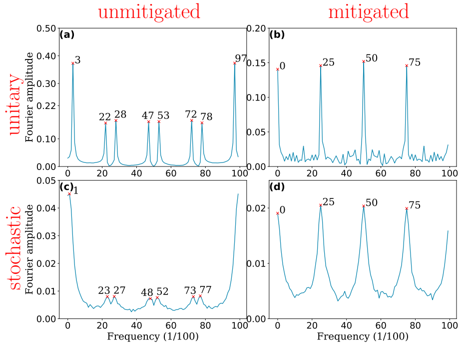
The order finding problem can be solved by finding the phase of the unitary operator U that is defined as

$$U|y\rangle = |xy (\mathrm{mod}\, N)\rangle \,.$$

The phases of U are $0, \frac{1}{r}, \frac{2}{r}, \cdots, \frac{r-1}{r}$.

Simulation of order finding: $x = 4, N = 255$





Summary

- Theoretically identify a benign type of noise for phase estimation.
- Use randomized compiling to get the desired type of noise and achieve a practical error mitigation in a generic noise environment without any quantum resource overhead.