1. Fermi 相互作用，β衰变.

$$n \to p^+ e^- \bar{\nu}_e$$

$$\mathcal{L}_F = \frac{1}{\sqrt{2}} G_F \cdot \bar{\Psi}_n \Gamma_1 \Psi_p \bar{\Psi}_\nu \Gamma_2 \Psi_e \qquad [G_F] = M^{-2}$$

幺正性.

$$|out\rangle = S|in\rangle$$

$|in\rangle$ 归一化. $\langle in|in\rangle = 1$.

$|out\rangle$ 亦可归一化. $\langle out|out\rangle = 1$.

S 是 $\mathcal{H}_{in} \to \mathcal{H}_{out}$ 的幺正变换. 否则. 我们遗漏了一些 $\mathcal{H}_{out}$ 中的态.

$S^\dagger S = \mathbb{1}$. $\because$ $S = \mathbb{1} + iT$ T为跃迁振幅.

$\therefore$ $S^\dagger S = (1 - iT^\dagger)(1 + iT) = \mathbb{1} + i(T - T^\dagger) + T^\dagger T$

$\Rightarrow$ $i(T^\dagger - T) = T^\dagger T$

$T = A + iB$ A, B均为自伴算子. 则

$i(T^\dagger - T) = i(A^\dagger - iB^\dagger - A - iB) = i(A - iB - A - iB) = 2B$

$T^\dagger T = (A - iB)(A + iB) = A^2 + B^2 + i[A, B]$

对于 $|p_1 p_2\rangle \to |p_1 p_2\rangle$ 散射

$$\overset{e^+}{\rightarrow} \quad \overset{e^-}{\leftarrow} \quad \longrightarrow \quad \overset{e^-}{\leftarrow} \quad \overset{e^+}{\rightarrow}$$

$i\langle p_1 p_2|T^\dagger|p_1 p_2\rangle - i\langle p_1 p_2|T|p_1 p_2\rangle$

$= \langle p_1 p_2|T^\dagger T|p_1 p_2\rangle = \sum\!\!\!\!\!\!\int_f d\Pi_f \langle p_1 p_2|T^\dagger|f\rangle\langle f|T|p_1 p_2\rangle$

$= \sum\!\!\!\!\!\!\int_f d\Pi_f\, |\langle f|T|p_1 p_2\rangle|^2$

$$\langle f|T|p_1p_2\rangle = (2\pi)^4\delta^4(p_1+p_2-p_f)\mathcal{M}(p_1p_2\to f)$$

$$\therefore\ i\mathcal{M}^*(p_1p_2\to p_1p_2) - i\mathcal{M}(p_1p_2\to p_1p_2) \geq \int d\Pi_f\,(2\pi)^4\delta^4(p_1+p_2-p_f)\,|\mathcal{M}(p_1p_2\to f)|^2$$

而 对于两体末态 f_2

$$\sigma(p_1p_2\to f_2) = \frac{1}{4E_{cm}|\vec{p}_i|}\int d\Pi_2\,(2\pi)^4\delta^4(p_1+p_2-p_{f_2})\,|\mathcal{M}(p_1p_2\to f_2)|^2$$

$$\therefore\ \mathrm{Im}\,\mathcal{M}(p_1p_2\to p_1p_2) \geq 2E_{cm}|\vec{p}_i|\,\sigma(p_1\ p_2\to k_1\ k_2)$$

对所有可能的末态求和 $\Rightarrow$ $\boxed{\mathrm{Im}\,\mathcal{M}(p_1p_2\to p_1p_2) = 2E_{cm}|\vec{p}_i|\sum_f\sigma(p_1p_2\to f)}$

光学定理.

特别地, 选 $f = p_1p_2$ 则

$$\mathrm{Im}\,\mathcal{M}(p_1p_2\to p_1p_2) \geq 2E_{cm}|\vec{p}_i|\,\sigma(p_1p_2\to p_1p_2)$$

可以证明, 上述结果对所有的 l 分波都分别成立.

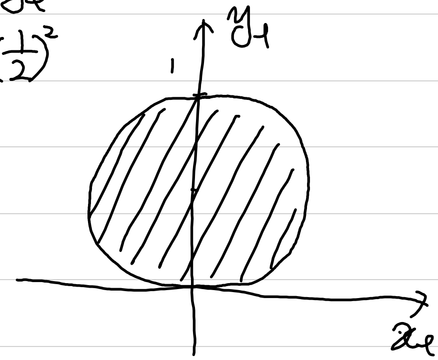
$$\therefore\quad \mathrm{Im}\,a_l \geq \frac{2|\vec{p}_i|}{E_{cm}}|a_l|^2$$

其中 $\mathcal{M} = 16\pi\sum_{l=0}^{\infty} a_l(2l+1)P_l(\cos\theta)$, a_l 为分波振幅

高能极限 $E_{cm} = \sqrt{s} \gg m_1, m_2$ $\quad\therefore\ |\vec{p}_i| \approx E_{cm}/2$

$$\mathrm{Im}\,a_l \geq |a_l|^2 \qquad a_l = x_l + iy_l$$

$$\Rightarrow\ y_l \geq x_l^2 + y_l^2 \quad\Rightarrow\quad x_l^2 + \left(y_l - \tfrac{1}{2}\right)^2 \leq \left(\tfrac{1}{2}\right)^2$$



非弹性散射 $\mathcal{M}_X = 16\pi \sum_{l=0}^{+\infty} b_l (2l+1) P_l(\cos\theta)$

$\because \ \mathrm{Im}\,\mathcal{M}(p_1 p_2 \to p_1 p_2) \geq 2E_{CM} |\vec{p}_i|\, \sigma(p_1 p_2 \to X)$

$\therefore \quad \mathrm{Im}\, a_l \geq |b_l|^2$

$\therefore$ High Energy limit

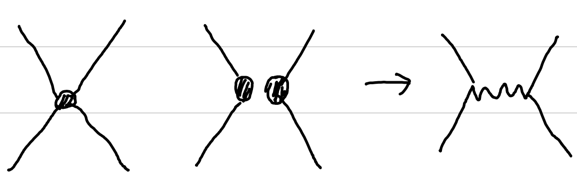
$$\sigma_l = \frac{1}{32\pi s} \int |\mathcal{M}_{Xl}(\cos\theta)|^2 d\cos\theta$$
$$= \frac{8\pi}{s} \int |b_l|^2 (2l+1)^2 P_l(\cos\theta)^2 d\cos\theta$$
$$= \frac{16\pi(2l+1)}{s} |b_l|^2 \leq \frac{16\pi(2l+1)}{s} \mathrm{Im}\, a_l \leq \frac{16\pi(2l+1)}{s}$$

考虑 $n + \nu_e \to p + e^-$ （或 $p + e^- \to n + \nu_e$, ‥）

$$\sigma_l = \frac{G_F^2 s}{\pi} \delta_{l0}$$

对 0-分波. $\frac{G_F^2 s}{\pi} \geq \frac{16\pi}{s}$ $\Rightarrow$ 高能散射"破坏幺正性"

引入 $W^\pm$



$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J_H^{-\mu} J_{L\mu}^{+} + h.c.$$
$$J_H^{-\mu} = \bar{\psi}_p \gamma^\mu (1-\gamma_5) \psi_n$$
$$J_L^{+\mu} = \bar{\psi}_e \gamma^\mu (1-\gamma_5) \psi_\nu$$

$$\to \mathcal{L} = -\frac{g}{2\sqrt{2}} J_H^{-\mu} W_\mu^+ - \frac{g}{2\sqrt{2}} J_L^{+\mu} W_\mu^- + h.c$$

$$\mathcal{L} = -\frac{1}{2} W^{\mu\nu} W_{\mu\nu} + m_W^2 W^{+\mu} W_\mu^-$$

$$W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu$$

$$\mathcal{M} = \frac{g^2 J_H^{-\mu} J_L^{+\nu}}{8(p^2-m_W^2+i\varepsilon)}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2}\right)$$

$$\xrightarrow{p^2 \ll m_W^2} -\frac{g^2}{8m_W^2} J_H^{-\mu} J^+_{L\mu} \qquad \therefore \frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}$$

$$4\sqrt{2}\, m_W^2 G_F^2 = g^2$$

$$\mathcal{L}^{\nu e} = -\frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu\, \bar{e}\gamma_\mu(g_{LV}^{\nu e} - g_{LA}^{\nu e}\gamma_5)e$$

$$\mathcal{L}^{\nu h} = -\frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu \sum_q \left[g_{LL}^{\nu q}\, \bar{q}\gamma_\mu(1-\gamma_5)q + g_{LR}^{\nu q}\, \bar{q}\gamma_\mu(1+\gamma_5)q\right]$$

$$\mathcal{L}^{ee} = \frac{G_F}{\sqrt{2}} g_{AV}^{ee}\, \bar{e}\gamma^\mu\gamma_5 e\, \bar{e}\gamma_\mu e$$

$$\mathcal{L}^{eh} = \frac{G_F}{\sqrt{2}} \sum_q \left[g_{AV}^{eq}\, \bar{e}\gamma^\mu\gamma_5 e\, \bar{q}\gamma_\mu q + g_{VA}^{eq}\, \bar{e}\gamma^\mu e\, \bar{q}\gamma_\mu\gamma_5 q\right]$$

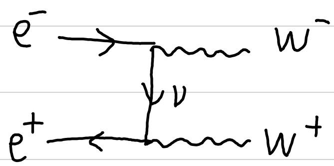
引入带电重矢量玻色 $W^\pm$ 后的 Lagrangian

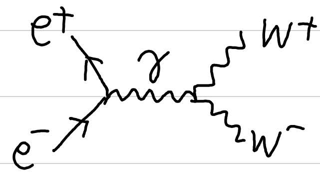
$$\mathcal{L} = \sum_f \bar{\psi}_f(i\not{\partial} - m_f)\psi_f - \frac{1}{2}W^{+\mu\nu}W^-_{\mu\nu} + m_W^2 W^{+\mu}W^-_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- \sum_f eQ_f \bar{\psi}_f\gamma^\mu\psi_f A_\mu - \frac{g}{2\sqrt{2}}\left[\bar{\psi}_u\gamma^\mu(1-\gamma_5)\psi_d W^+_\mu + h.c.\right]$$
$$- \frac{g}{2\sqrt{2}}\left[\bar{\psi}_\nu\gamma^\mu(1-\gamma_5)\psi_e W^+_\mu + h.c.\right]$$

其中 $W^\pm_{\mu\nu} = D_\mu W^\pm_\nu - D_\nu W^\pm_\mu = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \mp ie(A_\mu W^\pm_\nu - A_\nu W^\pm_\mu)$

W_μ 传播子. $-\dfrac{i}{p^2-m^2+i\varepsilon}\left(g_{\mu\nu} - \dfrac{k_\mu k_\nu}{m^2}\right)$

$e^+e^- \to W^+W^-$





$\frac{d\sigma}{d\cos\theta} \sim G_F^2 s(1-\cos^2\theta) \quad \propto s$ 破坏幺正性.

$\varepsilon_\mu(\uparrow) = \frac{1}{\sqrt{2}}(0,1,i,0) \quad \varepsilon_\mu(\downarrow) = \frac{1}{\sqrt{2}}(0,1,-i,0)$

$\varepsilon_\mu(L) = (0,0,0,1)$

$\xrightarrow{W\ \text{boost}} \quad \varepsilon_\mu(L) \to (\sqrt{\gamma^2-1}, 0, 0, \gamma) \qquad \gamma = E/m$

高能行为 $\varepsilon_\mu(L) \xrightarrow{\simeq} p_\mu/m$

$\therefore$ 纵极化 $VV \to X$ 的高能行为

$\mathcal{M} = \varepsilon_\mu J^\mu \propto p_\mu J^\mu. \qquad \therefore J^\mu \lesssim O(\frac{1}{s^{3/2}})$

上述例子显示 $J^\mu \lesssim O(s^{-3/2})$ 不易获得

但确有一类理论 s.t $p_\mu J^\mu = 0. \quad \partial_\mu J^\mu = 0.$

$\longrightarrow$ 规范理论.

小群的 E_2 平移变换

$$\varepsilon_\mu \to (0,1,\pm i,0)\begin{pmatrix} \gamma & 0 & \sqrt{2\gamma-2} & \gamma-1 \\ 0 & 1 & 0 & 0 \\ \sqrt{2\gamma-2} & 0 & 1 & \sqrt{2\gamma-2} \\ 1-\gamma & 0 & -\sqrt{2\gamma-2} & 2-\gamma \end{pmatrix}$$

$$= \varepsilon_\mu \pm i p_\mu$$

$\Rightarrow$ 0质量矢量场耦合的流. J^μ 必须满足

$$\partial_\mu J^\mu = 0 \quad \leftarrow \text{规范不变}.$$

2. 规范冗余, 规范场

$$[T^a, T^b] = (-1)^{S_1}\, i f_{abc} T^c$$

$$U(g)\psi_i U(g^{-1}) = \exp\left((-1)^{S_2} i\theta_a T^a\right)_i{}^j \psi_j$$

$$\mathcal{L} = \bar{\psi}_j (i\delta_{jk}\not{\partial} - m_{jk})\psi_k.$$

对于不可约表示, $m_{jk} = m\delta_{jk}$.

☆ 同一表示的粒子质量相同!!!

红 u、绿 u, 蓝 u, 质量相同

局域对称变换 (纯被动观点. 不是新的对称性)

$$\mathcal{L}' = \mathcal{L} + \bar{\psi}\, e^{-(-1)^{S_2} i\theta_a T^a} \cdot i\not{\partial}\left(e^{(-1)^{S_2} i\theta_b T^b}\psi\right)$$

$$\rightarrow \mathcal{L} - (-1)^{S_2}\bar{\psi}\gamma^\mu T^a\psi\, \partial_\mu\theta_a + \mathcal{O}(\theta^2)$$

引入辅助场 $\tilde{A}_{\mu,a}$

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} + (-1)^{S_3}\gamma^\mu \tilde{A}_{\mu,a}T^a - m)\psi.$$

$$\tilde{A}_{\mu,a} \to \tilde{A}_{\mu,a} + (-1)^{S_2-S_3}\partial_\mu\theta_a + C^b_{a,\mu}\theta_b$$

$\Rightarrow \bar{\Psi}(i\not{\partial} + (-1)^{S_3}\gamma^\mu\tilde{A}_{\mu,a}T^a)\psi$ 变化量为

$$(-1)^{S_3}\bar{\Psi}\gamma^\mu T^a[-(-1)^{S_1+S_2}f_{abc}\tilde{A}_{\mu,b}\theta_c + C^b_{a,\mu}\theta_b]\psi.$$

为令 $\mathcal{L}$ 形式不变，此式 $=0$

$$\Rightarrow (-1)^{S_1+S_2}f_{abc}\tilde{A}_{\mu,b}\theta_c = C^b_{a,\mu}\theta_b$$

$$\therefore \tilde{A}_{\mu,a} \to \tilde{A}_{\mu,a} + (-1)^{S_2-S_3}\partial_\mu\theta_a + (-1)^{S_1+S_2}f_{abc}\tilde{A}_{\mu,b}\theta_c$$

对于有限大变换 $\underline{\tilde{A}_\mu} \equiv \tilde{A}_{\mu,a}T^a$

$$\underline{\tilde{A}_\mu} \to U\underline{\tilde{A}_\mu}U^{-1} - i(-1)^{S_3}(\partial_\mu U)U^{-1}$$

if $\quad \mathcal{L} = \bar{\Psi}(i\not{\partial} + \gamma^\mu\underline{\tilde{A}_\mu})\psi.$

$\tilde{A}_{\mu,a}$ 的运动方程 $\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)} = \frac{\partial\mathcal{L}}{\partial A_\nu} \Rightarrow \frac{\partial\mathcal{L}}{\partial A_\nu} = 0$

$$\therefore \begin{cases} \bar{\Psi}\gamma^\mu T^a\psi \equiv 0 & \cdots ① \\ (i\not{\partial} - m + \gamma^\mu\tilde{A}_{\mu,a}T^a)\psi = 0 & \cdots ② \end{cases}$$ 规范固定.

$$\underline{\tilde{E}_{\mu\nu}} = \partial_\mu\underline{\tilde{A}_\nu} - \partial_\nu\underline{\tilde{A}_\mu} - i(-1)^{S_3}[\underline{\tilde{A}_\mu}, \underline{\tilde{A}_\nu}]$$

$$\underline{\tilde{E}_{\mu\nu}} \to U\underline{\tilde{E}_{\mu\nu}}U^{-1}$$

tr$(\tilde{\underline{F}}^{\mu\nu}\tilde{\underline{F}}_{\mu\nu})$ 为 G 不变动能.

$$\tilde{A}_{\mu,a} = g A_{\mu,a}$$

$$[T^a, T^b] = (-1)^{s_1} i f_{abc} T^c$$
$$\varphi \to e^{(-1)^{s_2} i\theta_a T^a}\varphi$$
$$\partial_\mu \to D_\mu = \partial_\mu - i(-1)^{s_3} g A_{\mu,a}T^a$$
$$\underline{A}_\mu = A_{\mu,a}T^a \to U\underline{A}_\mu U^{-1} - (-1)^{s_3}\frac{i}{g}(\partial_\mu U)U^{-1}$$
$$A_{\mu,a} \to A_{\mu,a} + (-1)^{s_2-s_3}\frac{1}{g}\partial_\mu\theta_a + (-1)^{s_1+s_2} f_{abc}A_{\mu,b}\theta_c + O(\theta^2)$$
$$\underline{F}_{\mu\nu} \equiv F_{\mu\nu,a}T^a = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu - ig(-1)^{s_3}[\underline{A}_\mu, \underline{A}_\nu]$$
$$F_{\mu\nu,a} = \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} + (-1)^{s_1+s_3} g f_{abc} A_{\mu,b}A_{\nu,c}$$
$$\underline{F}_{\mu\nu} \to U\underline{F}_{\mu\nu}U^{-1}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu,a}F^{\mu\nu}{}_a + \bar{\psi}(i\not{D} - m)\psi + (D_\mu\varphi)^\dagger(D^\mu\varphi) - V(\varphi)$$

$$A_{\mu,a} \to A_{\mu,a} + (-1)^{s_2-s_3}\frac{1}{g}\partial_\mu\theta_a + (-1)^{s_1+s_2} f_{abc}A_{\mu,b}\theta_c$$

选 $\theta_\alpha \propto \delta_{a\alpha}$ ∵ $f_{abc} = -f_{cba}$ ∴ $A_{\mu,a} \to A_{\mu,a} + (-1)^{s_2-s_3}\frac{1}{g}\partial_\mu\theta_a$

不变.

∴ $A_\mu J^\mu$ 的 J^μ 满足 $\partial_\mu J^\mu = 0$

量子水平必须仍然保持！ Ward 恒等式 $Z_g = Z_A^{-1/2}$

反常相消

2. 自发对称性破缺，Higgs机制，和 SM Lagrangian

(1). 中性流，SU(2) 对称性（规范）

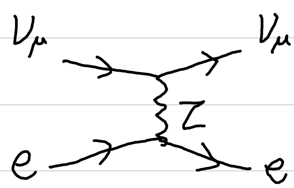
$Q^2 \ll (100\,\mathrm{GeV})^2$.

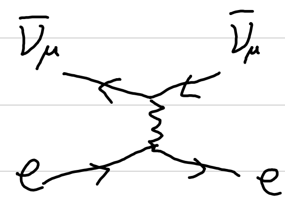
$$\mathcal{L}^{\nu e} = -\frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \cdot \bar{e}\gamma^\mu(g_{LV}^{\nu e} - g_{LA}^{\nu e}\gamma^5)e$$

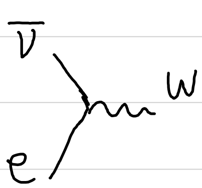
中微子散射. $\nu_\mu e \to \nu_\mu e$ $\quad \bar{\nu}_\mu e \to \bar{\nu}_\mu e$

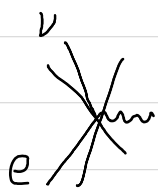
$$\sigma = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(g_V^{\nu e} \pm g_A^{\nu e})^2 + \frac{1}{3}(g_V^{\nu e} \mp g_A^{\nu e})^2\right]$$

$R \equiv \sigma_{\nu_\mu e} / \sigma_{\bar{\nu}_\mu e}$









不可能为带电流.

$J^{0\mu}$ — 中性流 $\quad \bar{\nu}\gamma^\mu(1-\gamma_5)\nu$

$e\gamma^\mu(g_{LV}^{\nu e} - g_{LA}^{\nu e}\gamma_5)e$

....

~~~ W^+

~~~ W^-

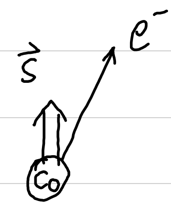
~~~ Z（中性玻色子）

3个生成元.

SU(2)

$\sigma^1 \quad \sigma^2 \quad \sigma^3$
~~~

(2) 左手相互作用. 宇称破坏

"$\theta-\tau$"疑难. 3π, 2π 衰变模式 $\theta^+ \to \pi^+\pi^0$

$P_\pi = -1$ $\tau^+ \to \pi^+\pi^+\pi^-$

$${}^{60}_{27}Co \to {}^{60}_{28}Ni + e^- + \bar{\nu}_e + 2\gamma$$



β衰变(带电流)是纯左手相互作用.

$$\begin{pmatrix} e_L^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu_L^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau_L^- \\ \nu_\tau \end{pmatrix} \quad e_R^- \quad \mu_R^- \quad \tau_R^-$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix} \quad \begin{matrix} c_R \\ s_L \end{matrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$$

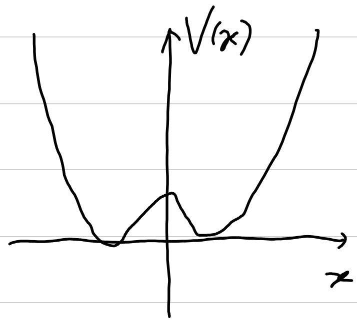
$$\mathcal{L} \ni \bar{\psi}_f (i\not{D} - m_f)\psi - \frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu}$$

(1) $\bar{\psi}_f m_f \psi$ 不是 SU(2) 不变的. 明显破坏

(2) $W^\pm$, Z^0 的质量非零 ($\sim 100GeV$)

自发对称性破缺与 Higgs 机制

(1) 自发对称性破缺



→ 量子力学基态唯一

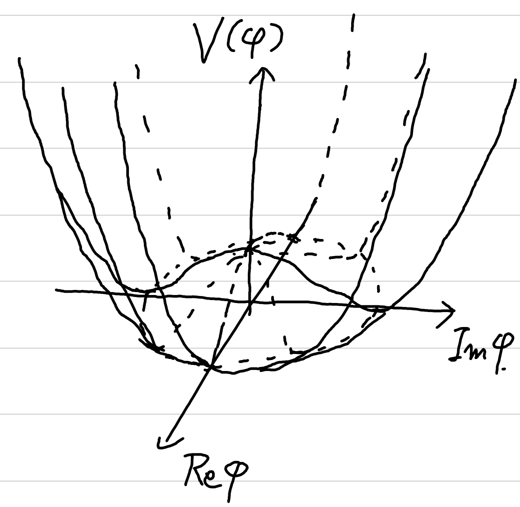
tunneling eff, instanton

(2) Nambu - Goldstone 定理

either $\langle\phi\rangle = 0$, or massless pseudo-scalar

~~H+D~~

例: $U(1)$ 对称性



$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - V(\varphi)$$

$$V(\varphi) = \frac{\lambda}{4}(\varphi^*\varphi)^2 + m^2 \varphi^*\varphi$$

if $m^2 < 0$ $\quad \mu^2 \equiv -m^2 > 0$

$$\mathcal{L} = \partial_\mu\varphi^*\partial^\mu\varphi + \mu^2\varphi^*\varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2$$

$$\langle \varphi^*\varphi\rangle = 2\mu^2/\lambda \equiv v^2 \qquad \mu^2 = \lambda v^2/2$$

$$\langle\varphi\rangle = v e^{i\alpha}$$

$$\therefore \quad \varphi(x) = (\rho(x) + v)\, e^{i[\alpha + \theta(x)]}$$

$$\mathcal{L} = [\partial_\mu \rho\, e^{-i(\alpha+\theta)} - i(\rho+v)\partial_\mu\theta \cdot e^{-i(\alpha+\theta)}][\partial_\mu\rho \cdot e^{i(\alpha+\theta)} + i(\rho+v)\partial_\mu\theta e^{i(\alpha+\theta)}]$$
$$+ \mu^2(v^2 + 2v\rho + \rho^2) - \frac{\lambda}{4}(v^4 + 4v^3\rho + 6v^2\rho^2 + 4v\rho^3 + \rho^4)$$
$$= \partial_\mu\rho\partial^\mu\rho + (\rho+v)^2\partial_\mu\theta\partial^\mu\theta + \frac{\lambda}{2}v^4 + \lambda v^3\rho + \frac{\lambda v^2}{2}\rho^2$$
$$- \frac{\lambda}{4}v^4 - \lambda v^3\rho - \frac{3}{2}\lambda v^2\rho^2 - \lambda v\rho^3 - \frac{\lambda}{4}\rho^4$$

$$\therefore \mathcal{L} = \partial_\mu \rho \partial^\mu \rho + (v+\rho)^2 \partial_\mu \theta \partial^\mu \theta + \frac{\lambda}{4} v^4$$
$$- \lambda^2 v^2 \rho^2 - \lambda v \rho^3 - \frac{\lambda}{4} \rho^4$$

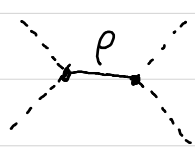
$$\rho \to \frac{1}{\sqrt{2}} \rho \qquad a = \frac{1}{\sqrt{2}} v \theta$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - \frac{1}{2} (\lambda v)^2 \rho^2 - \frac{\lambda v}{2\sqrt{2}} \rho^3 - \frac{\lambda}{16} \rho^4$$
$$+ \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{\rho}{v} \partial_\mu a \partial^\mu a + \frac{\rho^2}{2v^2} \partial_\mu a \partial^\mu a$$

ρ — 质量为 λv 的实标量粒子.

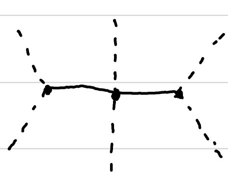
a — 零质量 赝标量粒子.

$E \ll \lambda v = M$

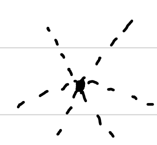


$$\frac{1}{v} \cdot \frac{i}{E^2 - M^2} \cdot \frac{1}{v} \sim -\frac{1}{v^2 M^2} \sim \frac{1}{v^4}$$
$$\Rightarrow \frac{1}{v^4} (\partial_\mu a \partial^\mu a)^2$$

$$\frac{E^4}{v^4}$$

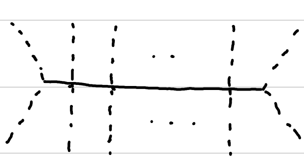


$$\frac{1}{v} \frac{i}{E^2 - M^2} \cdot \frac{1}{2v^2} \cdot \frac{i}{E^2 - M^2} \cdot \frac{1}{v} \sim \frac{1}{v^4 M^4} \sim \frac{1}{v^8}$$
$$\Rightarrow \frac{1}{v^8} (\partial_\mu a \partial^\mu a)^3$$

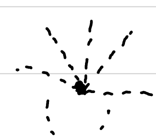


$$\frac{E^6}{v^8}$$

$\vdots$



$$\frac{1}{v^{4(N-1)}} (\partial_\mu a \partial^\mu a)^N$$



$$\frac{E^{2N}}{v^{4(N-1)}}$$

$a \; a \longrightarrow (N-2)\, a$

$$\mathcal{M} \sim \frac{E^{2N}}{v^{4(N-1)}}$$

Gauged symmetry. (Higgs mechanism)

引入 SU(2) 复标量 2 重态

$$H(x)=\begin{pmatrix}\varphi_1+i\varphi_2\\ \varphi_3+i\varphi_4\end{pmatrix}$$

在 SM 中，有 $SU(2)_L\times U(1)_Y$ 规范群 $\longrightarrow W^\pm, Z^0, \gamma$

今天我们知道. H 在 $SU(2)_L\times U(1)_Y$ 变换下满足.

$$H\to e^{i\alpha_i\cdot\frac{\sigma_i}{2}}\, e^{i\alpha/2 I} H$$

其中 $\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ $\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

Dim ≤ 4 的标量势 $V(H)=\frac{\lambda}{4}(H^\dagger H)^2-\mu^2H^\dagger H$

真空位置 $\langle H^\dagger H\rangle=2\mu^2/\lambda=v^2$

$\therefore \langle\varphi_1\rangle^2+\langle\varphi_2\rangle^2+\langle\varphi_3\rangle^2+\langle\varphi_4\rangle^2=v^2$ 并且 $\partial_\mu\langle\varphi_i\rangle\equiv 0.$

$\therefore$ 总可以找到整体 $SU(2)_L\times U(1)_Y$ 转动，使得 $\langle H\rangle=\begin{pmatrix}0\\v\end{pmatrix}$

不妨选择转动后的基底. 因而不失一般性.

$$\langle H\rangle=\begin{pmatrix}0\\v\end{pmatrix}\qquad H=\begin{pmatrix}\varphi_1(x)+i\varphi_2(x)\\ v+\varphi_3(x)+i\varphi_4(x)\end{pmatrix}$$

$\because$ $SU(2)_L\times U(1)_Y$ 是局域对称性. 对每一时空点都可以选不同的 α_i 和 α, $\alpha_i(x)$, $\alpha(x)$ 场

固定 $\begin{pmatrix}0\\v\end{pmatrix}$ 后，只有 $e^{i\theta\cdot\frac{\sigma_3}{2}}\cdot e^{i\theta\frac{I}{2}}$ 转动是不改变 $\begin{pmatrix}0\\v\end{pmatrix}$ 的. 这意味着 σ_3+I 对称性没有被破坏. 同时意味着 $\varphi_1\cdots\varphi_4$ 中有 3 个可以通过 α_i,α 实现

例: $\mathcal{L}=(D_\mu\varphi)^*(D^\mu\varphi)+\mu^2\varphi^*\varphi-\frac{\lambda}{4}(\varphi^*\varphi)^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$$D_\mu\varphi=\partial_\mu\varphi-igA_\mu\varphi.$$

$$\varphi\to(v+\rho)e^{i\alpha(x)}$$

但是此时，$U(1)$为规范对称性. 所以对任意的 $\varphi(x)=(v+\rho(x))e^{i\alpha(x)}$，总可以做规范变换.

$e^{-i\alpha(x)}$!

Goldstone 玻色子自由度 $\alpha(x)$"消失"了!

$$D_\mu[(v+\rho)e^{i\alpha}]=e^{i\alpha}[\partial_\mu\rho-ig(A_\mu+\frac{\partial_\mu\alpha}{g})(\rho+v)]$$

同时. 规范自由度也消失了. —— 幺正规范

对于 $\varphi=(v+\rho)e^{i\alpha}$，总作变换 $\varphi\to v+\rho$. $A_\mu\to A_\mu+\frac{\partial_\mu\alpha}{g}$

此后不再能做规范变换.

$$C_\mu\equiv A_\mu+\partial_\mu\alpha/g.$$

$$\partial_\mu C_\nu-\partial_\nu C_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu=F_{\mu\nu}$$

$$\begin{aligned}(D_\mu\varphi)^*(D^\mu\varphi)&=[\partial_\mu\rho+igC_\mu(\rho+v)][\partial^\mu\rho-igC^\mu(\rho+v)]\\&=\partial_\mu\rho\,\partial^\mu\rho+g^2C_\mu C^\mu(v^2+2v\rho+\rho^2)\\&=\partial_\mu\rho\,\partial^\mu\rho+2vg^2\rho C_\mu C^\mu+g^2\rho^2C_\mu C^\mu\\&\quad+g^2v^2C_\mu C^\mu\end{aligned}$$

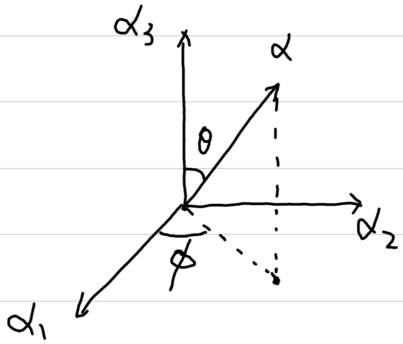
对于 $SU(2)_L \times U(1)_Y$ 的 Higgs 场

$$H = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \qquad \langle H^\dagger H \rangle = v^2$$

$$\Rightarrow \langle \varphi_1 \rangle^2 + \langle \varphi_2 \rangle^2 + \langle \varphi_3 \rangle^2 + \langle \varphi_4 \rangle^2 = v^2$$

选择规范（幺正规范），使得时空每一点的 $\langle H \rangle \equiv \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$e^{\frac{i}{2}\alpha_i \sigma_i} = \begin{pmatrix} \cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\cos\theta & ie^{-i\phi}\sin\frac{\alpha}{2}\sin\theta \\ ie^{i\phi}\sin\frac{\alpha}{2}\sin\theta & \cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}\cos\theta \end{pmatrix}$$



$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}$$
$$\sin\theta = \sqrt{\alpha_1^2 + \alpha_2^2}/\alpha$$
$$\cos\theta = \alpha_3/\alpha$$
$$e^{i\phi} = (\alpha_1 + i\alpha_2)/\sqrt{\alpha_1^2 + \alpha_2^2}$$

$$e^{\frac{i}{2}\alpha_i\sigma_i} H$$

$$= \begin{bmatrix} \varphi_1\cos\frac{\alpha}{2} - \varphi_2\sin\frac{\alpha}{2}\cos\theta + \varphi_3\sin\frac{\alpha}{2}\sin\theta\sin\phi - \varphi_4\sin\frac{\alpha}{2}\sin\theta\cos\phi \\ \quad + i\left(\varphi_1\sin\frac{\alpha}{2}\cos\theta + \varphi_2\cos\frac{\alpha}{2} + \varphi_3\sin\frac{\alpha}{2}\sin\theta\cos\phi + \varphi_4\sin\frac{\alpha}{2}\sin\theta\sin\phi\right) \\ -\varphi_1\sin\frac{\alpha}{2}\sin\theta\sin\phi - \varphi_2\sin\frac{\alpha}{2}\sin\theta\cos\phi + \varphi_3\cos\frac{\alpha}{2} + \varphi_4\sin\frac{\alpha}{2}\cos\theta \\ \quad + i\left(\varphi_1\sin\frac{\alpha}{2}\sin\theta\cos\phi - \varphi_2\sin\frac{\alpha}{2}\sin\theta\sin\phi - \varphi_3\sin\frac{\alpha}{2}\cos\theta + \varphi_4\cos\frac{\alpha}{2}\right) \end{bmatrix}$$

令 $\varphi = \sqrt{\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2}$

$$\begin{cases} \varphi_1 = -\varphi\sin\frac{\alpha}{2}\sin\theta\sin\phi \\ \varphi_2 = -\varphi\sin\frac{\alpha}{2}\sin\theta\cos\phi \\ \varphi_3 = \varphi\cos\frac{\alpha}{2} \\ \varphi_4 = \varphi\sin\frac{\alpha}{2}\cos\theta \end{cases} \Rightarrow \begin{array}{l} \cos\frac{\alpha}{2} = \varphi_3/\varphi \\ \sin\frac{\alpha}{2}\cos\theta = \varphi_4/\varphi \\ e^{i\varphi}\sin\frac{\alpha}{2}\sin\theta = -\varphi_2 - i\varphi_1 \end{array}$$

因此，对于 $H(x)=\begin{pmatrix}\varphi_1(x)+i\varphi_2(x)\\ \varphi_3(x)+i\varphi_4(x)\end{pmatrix}$

总可以做 SU(2) 转动

$$U(x)=\begin{pmatrix}\varphi_3+i\varphi_4 & -\varphi_1-i\varphi_2\\ \varphi_1-i\varphi_2 & \varphi_3-i\varphi_4\end{pmatrix}\cdot\frac{1}{\sqrt{\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2}}$$

使得 $H(x)=\begin{pmatrix}0\\ \sqrt{\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2}\end{pmatrix}$

选此种（幺正）规范

$$H(x)=\begin{pmatrix}0\\ v+\frac{1}{\sqrt2}h(x)\end{pmatrix}$$

$$\therefore\quad D_\mu H=\partial_\mu H-ig'\frac{y}{2}B_\mu H+ig\frac{\sigma_i}{2}W_\mu^i H$$

$$=\begin{pmatrix}\partial_\mu-ig'\frac{y}{2}B_\mu+\frac{i}{2}gW_\mu^3 & \frac{ig}{2}(W_\mu^1-iW_\mu^2)\\ \frac{ig}{2}(W_\mu^1+iW_\mu^2) & \partial_\mu-ig'\frac{y}{2}B_\mu-\frac{i}{2}gW_\mu^3\end{pmatrix}\begin{pmatrix}0\\ v+\frac{1}{\sqrt2}h\end{pmatrix}$$

$$=\begin{pmatrix}\frac{ig}{2}(W_\mu^1-iW_\mu^2)(v+\frac{1}{\sqrt2}h)\\ \frac{1}{\sqrt2}\partial_\mu h-\frac{i}{2}(yg'B_\mu+gW_\mu^3)(v+\frac{1}{\sqrt2}h)\end{pmatrix}$$

定义. $W_\mu^\pm=(W_\mu^1\mp iW_\mu^2)/\sqrt2$

$$D_\mu H=\begin{pmatrix}\frac{ig}{\sqrt2}W_\mu^+(v+\frac{1}{\sqrt2}h)\\ \frac{1}{\sqrt2}\partial_\mu h-\frac{i}{2}(yg'B_\mu+gW_\mu^3)(v+\frac{1}{\sqrt2}h)\end{pmatrix}$$

$$\therefore (D_\mu H)^\dagger(D^\mu H)=\frac{g^2}{2}(v+\frac{1}{\sqrt2}h)^2\ W_\mu^+W^{-\mu}$$

$$+\frac12\partial_\mu h\,\partial^\mu h+\frac14(v+\frac{1}{\sqrt2}h)^2(yg'B_\mu+gW_\mu^3)(yg'B^\mu+gW^{3\mu})$$

定义. $Z_\mu = (y g' B_\mu + g W_\mu^3)/\sqrt{y^2 g'^2 + g^2}$

$$\Rightarrow (D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 v^2}{2} W_\mu^+ W^{-\mu} + \frac{1}{4}(y^2 g'^2 + g^2) v^2 Z_\mu Z^\mu$$
$$+ \frac{g^2 v}{\sqrt{2}} h W_\mu^+ W^{-\mu} + \frac{1}{2\sqrt{2}}(y^2 g'^2 + g^2) v h Z_\mu Z^\mu + \frac{g^2}{4} h^2 W_\mu^+ W^{-\mu}$$
$$+ \frac{1}{8}(y^2 g'^2 + g^2) h^2 Z_\mu Z^\mu$$

$$\mathcal{L}_{KG} = -\frac{1}{4} W_{\mu\nu}^1 W^{1\mu\nu}_{K} - \frac{1}{4} W_{\mu\nu}^2 W^{2\mu\nu}_{K} - \frac{1}{4} W_{\mu\nu}^3 W^{3\mu\nu}_{K} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$-\frac{1}{4} W_{\mu\nu}^1 W^{1\mu\nu}_{K} = -\frac{1}{8}(W_{\mu\nu}^+ + W_{\mu\nu}^-)(W^{+\mu\nu} + W^{-\mu\nu})$$
$$= -\frac{1}{8} W_{\mu\nu}^+ W^{+\mu\nu} - \frac{1}{4} W_{\mu\nu}^- W^{+\mu\nu} - \frac{1}{8} W_{\mu\nu}^- W^{-\mu\nu}.$$

$$-\frac{1}{4} W_{\mu\nu}^2 W^{2\mu\nu}_{K} = +\frac{1}{8}(W_{\mu\nu}^- - W_{\mu\nu}^+)(W^{-\mu\nu} - W^{+\mu\nu})$$
$$= \frac{1}{8} W_{\mu\nu}^- W^{-\mu\nu} - \frac{1}{4} W_{\mu\nu}^- W^{+\mu\nu} + \frac{1}{8} W_{\mu\nu}^+ W^{+\mu\nu}$$

定义. $A_\mu = (g B_\mu - y g' W_\mu^3)/\sqrt{y^2 g'^2 + g^2}$

则 $B_\mu = (g A_\mu + y g' Z_\mu)/\sqrt{y^2 g'^2 + g^2}$

$W_\mu^3 = (-y g' A_\mu + g Z_\mu)/\sqrt{y^2 g'^2 + g^2}$

则 $-\frac{1}{4} W_{\mu\nu}^3 W^{3\mu\nu}_{K} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} = -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

$Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \varepsilon_{ijk} W^j{}_\mu W^k{}_\nu$$

$$\therefore W_{\mu\nu}^i W^{i,\mu\nu} = (\partial_\mu W_\nu^i - \partial_\nu W_\mu^i)(\partial^\mu W^{i,\nu} - \partial^\nu W^{i,\mu})$$
$$+ 2g \varepsilon_{ijk} (\partial_\mu W_\nu^i - \partial_\nu W_\mu^i) W^{j,\mu} W^{k,\nu}$$
$$+ g^2 \varepsilon_{ijk} \varepsilon_{iab} W^j{}_\mu W^k{}_\nu W^{a,\mu} W^{b,\nu}$$

$$W_\mu = \begin{pmatrix} \frac{1}{2}W^3_\mu & \frac{1}{\sqrt{2}}W^+_\mu \\ \frac{1}{\sqrt{2}}W^-_\mu & -\frac{1}{2}W^3_\mu \end{pmatrix} = W^3_\mu \cdot \frac{\sigma^3}{2} + \frac{1}{\sqrt{2}}W^+_\mu \sigma^+ + \frac{1}{\sqrt{2}}W^-_\mu \sigma^-$$

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\therefore \left[\frac{\sigma^+}{\sqrt{2}}, \frac{\sigma^-}{\sqrt{2}}\right] = \frac{\sigma^3}{2} \qquad t_+ = \frac{\sigma^+}{\sqrt{2}}, \quad t_- \equiv \frac{\sigma^-}{\sqrt{2}}, \quad t_3 \equiv \frac{\sigma^3}{2}$$

$$\left[\frac{\sigma^3}{2}, \frac{\sigma^+}{\sqrt{2}}\right] = \frac{\sigma^+}{\sqrt{2}}$$

$$\left[\frac{\sigma^-}{\sqrt{2}}, \frac{\sigma^3}{2}\right] = \frac{\sigma^-}{\sqrt{2}}$$

$$\Rightarrow [t_+, t_-] = t_3$$
$$[t_-, t_3] = t_-$$
$$[t_3, t_+] = t_+$$

注意 $tr(\quad)$

$\to t_+t_- + t_-t_+$

$$\therefore f_{+-}{}^3 = -f_{-+}{}^3 = 1 \qquad f_{+-3} = -f_{-+3} = 1$$
$$f_{-3}{}^- = -f_{3-}{}^- = 1 \Rightarrow f_{-3+} = -f_{3-+} = 1$$
$$f_{3+}{}^+ = -f_{+3}{}^+ = 1 \qquad f_{3+-} = -f_{+3-} = 1$$

$\therefore f_{ab}{}^c \to f_{abc}$

$$\Rightarrow f_{3+-} = -f_{3-+} = 1$$
$$f_{+-3} = -f_{+3-} = 1$$
$$f_{-3+} = -f_{-+3} = 1$$

$$\therefore 2g f_{abc} (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) W^{b,\mu} W^{c,\nu}$$
$$= 2g\{(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(W^{-,\mu}W^{3,\nu} - W^{3\mu}W^{-\nu})$$
$$+ (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)(W^{3,\mu}W^{+\nu} - W^{+\mu}W^{3\nu})$$
$$+ (\partial_\mu W^3_\nu - \partial_\nu W^3_\mu)(W^{+,\mu}W^{-,\nu} - W^{-\mu}W^{+\nu})\}$$
$$= 2g\, W^{3,\mu}\{ W^{-,\nu} W^+_{K,\nu\mu} - W^{-,\nu}W^+_{K,\mu\nu} + W^{+,\nu}W^-_{K,\mu\nu} - W^{+\nu}W^-_{K,\nu\mu}\}$$
$$+ 2g\, W^3_{K,\mu\nu}(W^{+\mu}W^{-\nu} - W^{-\mu}W^{+\nu})$$
$$= 4g\, W^{3,\mu}\{W^{+,\nu}W^-_{K,\mu\nu} - W^{-,\nu}W^+_{K,\mu\nu}\} + 4g\, W^3_{K,\mu\nu}W^{+\mu}W^{-\nu}$$

$$(D_\mu H)^\dagger(D^\mu H)=\frac{1}{2}\partial_\mu h\partial^\mu h+\frac{g^2v^2}{2}W_\mu^+W^{-\mu}+\frac{1}{4}(y^2g'^2+g^2)v^2Z_\mu Z^\mu$$
$$+\frac{g^2v}{\sqrt{2}}hW_\mu^+W^{-\mu}+\frac{1}{2\sqrt{2}}(y^2g'^2+g^2)vhZ_\mu Z^\mu+\frac{g^2}{4}h^2W_\mu^+W^{-\mu}$$
$$+\frac{1}{8}(y^2g'^2+g^2)h^2Z_\mu Z^\mu$$

∴ $W^\pm$ 的质量. $m_W^2=g^2v^2$

Z^0 的质量 $m_Z^2=(y^2g'^2+g^2)v^2$

A ，零质量. $\longrightarrow$ 光子自由度.

但是 $SU(2)_L\times U(1)_Y\to U(1)_{em}$. 中的 $U(1)_{em}\neq U(1)_Y$.

$$A_\mu=(gB_\mu-yg'W_\mu^3)/\sqrt{y^2g'^2+g^2}\neq B_\mu.$$

~~费~~米子.

$Q_{1L}=\begin{pmatrix}u_L\\d_L\end{pmatrix}$ $Q_{2L}=\begin{pmatrix}c_L\\s_L\end{pmatrix}$ $Q_{3L}=\begin{pmatrix}t_L\\b_L\end{pmatrix}$ $\begin{matrix}u_R\\d_R\end{matrix}$ $\begin{matrix}c_R\\s_R\end{matrix}$ $\begin{matrix}t_R\\b_R\end{matrix}$

$L_{1L}=\begin{pmatrix}\nu_{eL}\\e_L\end{pmatrix}$ $L_{2L}=\begin{pmatrix}\nu_{\mu L}\\\mu_L\end{pmatrix}$ $L_{3L}=\begin{pmatrix}\nu_{\tau L}\\\tau_L\end{pmatrix}$ $\begin{matrix}?\\e_R\end{matrix}$ $\begin{matrix}?\\\mu_R\end{matrix}$ $\begin{matrix}?\\\tau_R\end{matrix}$

$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$: $\quad D_\mu\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}\partial_\mu - i\frac{g'}{2}y_f B_\mu + i\frac{g}{2}W^3_\mu & \frac{ig}{\sqrt{2}}W^+_\mu \\ \frac{ig}{\sqrt{2}}W^-_\mu & \partial_\mu - i\frac{g'}{2}y_f B_\mu - i\frac{g}{2}W^3_\mu\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$

$$=\begin{pmatrix}\partial_\mu - i(g'\frac{y_f}{2}B_\mu - gI_3^{(1)}W^3_\mu) & \frac{ig}{\sqrt{2}}W^+_\mu \\ \frac{ig}{\sqrt{2}}W^-_\mu & \partial_\mu - i(g'\frac{y_f}{2}B_\mu - gI_3^{(2)}W^3_\mu)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$B_\mu=(gA_\mu+g'y_H Z_\mu)/\sqrt{g^2+y_H^2 g'^2}$$
$$W^3_\mu=(gZ_\mu-g'y_H A_\mu)/\sqrt{g^2+y_H^2 g'^2}$$

$$g'\frac{y_f}{2}B_\mu - gI_3W^3_\mu=\frac{1}{\sqrt{g^2+y_H^2g'^2}}\Big[\Big(\frac{g'y_f}{2}g+gg'I_3y_H\Big)A_\mu$$
$$+\Big(g'^2\cdot\frac{y_fy_H}{2}-g^2I_3\Big)Z_\mu\Big]$$
$$=\frac{gg'}{\sqrt{g^2+y_H^2g'^2}}\Big(\frac{y_f}{2}+I_3\cdot y_H\Big)A_\mu+\frac{1}{\sqrt{g^2+y_H^2g'^2}}\Big(\frac{g'^2y_fy_H}{2}-g^2I_3\Big)Z_\mu.$$

Higgs 场是二重态，$\therefore I_3=-\frac{1}{2}$ 的分量电荷为 0.

对于费米子二重态 $I_3=\pm\frac{1}{2}$

对于右手费米子单态 $I_3=0.$

选取. $g'\to y_Hg' \to$ (等效于重新定义 $y_H=+1$)

$y_f\to y_f\cdot y_H$

上式变为: $\frac{gg'}{\sqrt{g^2+g'^2}}\Big(I_3+\frac{y_f}{2}\Big)A_\mu+\frac{1}{\sqrt{g^2+g'^2}}\Big(g'^2\cdot\frac{y_f}{2}-g^2I_3\Big)Z_\mu.$

定义. $\quad e=\frac{gg'}{\sqrt{g^2+g'^2}} \qquad Q_f=I_3+\frac{y_f}{2}$

$g'/\sqrt{g^2+g'^2}=\sin\theta_W$

$$\because \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W \qquad \frac{g'}{\sqrt{g^2+g'^2}} = \sin\theta_W \qquad \Rightarrow m_W = m_Z \cos\theta_W$$

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} \Rightarrow g = \frac{e}{\sin\theta_W} \qquad g' = \frac{e}{\cos\theta_W}$$

$$\frac{gg'}{\sqrt{g^2+g'^2}}\left(I_3+\frac{y_f}{2}\right)A_\mu + \frac{1}{\sqrt{g^2+g'^2}}\left(g'^2\cdot\frac{y_f}{2} - g^2 I_3\right)Z_\mu.$$

$$= eQ_f A_\mu + \left(\frac{e\sin\theta_W}{2\cos\theta_W}y_f - \frac{e\cos\theta_W}{\sin\theta_W}I_3\right)Z_\mu$$

$$\because I_3 + \frac{y_f}{2} = Q_f \quad . \quad y_f = 2(Q_f - I_3)$$

$$\therefore \rightarrow eQ_f A_\mu + \left((Q_f - I_3)\cdot\frac{e\sin\theta_W}{\cos\theta_W} - \frac{e\cos\theta_W}{\sin\theta_W}I_3\right)Z_\mu$$

$$= eQ_f A_\mu + \left(Q_f\cdot\frac{e\sin\theta_W}{\cos\theta_W} - I_3 e\left(\frac{1}{\cos\theta_W\sin\theta_W}\right)\right)Z_\mu$$

$$= eQ_f A_\mu - \frac{g}{\cos\theta_W}\left(I_3 - Q_f\sin^2\theta_W\right)Z_\mu$$

$$\therefore D_\mu\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\partial_\mu - ieQ_f A_\mu + \frac{ig}{\cos\theta_W}(I_3 - Q_f\sin^2\theta_W)Z_\mu & \frac{ig}{\sqrt{2}}W_\mu^+ \\ \frac{ig}{\sqrt{2}}W_\mu^- & \partial_\mu - ieQ_f A_\mu + \frac{ig}{\cos\theta_W}(I_3 - Q_f\sin^2\theta_W)Z_\mu\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\therefore i(\bar\psi_1\ \bar\psi_2)\not{D}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \bar\psi_1\left(i\not\partial + eQ_1 A_\mu\gamma^\mu - \frac{g}{\cos\theta_W}(I_3 - Q_1\sin^2\theta_W)Z_\mu\gamma^\mu\right)\psi_1$$

$$+ \bar\psi_2\left(i\not\partial + eQ_2 A_\mu\gamma^\mu - \frac{g}{\cos\theta_W}(I_3 - Q_2\sin^2\theta_W)Z_\mu\gamma^\mu\right)\psi_2$$

$$- \frac{g}{\sqrt{2}}\bar\psi_1\gamma^\mu\psi_2 W_\mu^+ - \frac{g}{\sqrt{2}}\bar\psi_2\gamma^\mu\psi_1 W_\mu^-$$

$$Q_2 = Q_1 - 1$$

对于右手，$I_3 = 0$

$$D_\mu \psi = \partial_\mu \psi - i g' \frac{y_f}{2} B_\mu \psi$$

$$= \partial_\mu \psi - i e Q_f A_\mu \psi - \frac{i g Q_f}{\cos\theta_W} \sin^2\theta_W Z_\mu \psi$$

$$\therefore i \bar{\psi} \not{D} \psi = \bar{\psi} i \not{\partial} \psi + e Q_f \bar{\psi} \gamma^\mu \psi A_\mu + \frac{g Q_f}{\cos\theta_W} \sin^2\theta_W \bar{\psi} \gamma^\mu \psi Z_\mu .$$

$$Q_f = y_f / 2$$

$$\therefore \mathcal{L}_f = i \bar{\psi}_L \not{D}_L \psi_L + i \bar{\psi}_R \not{D}_R \psi_R$$

$$= i \bar{\psi}_1 \not{\partial} \psi_1 + i \bar{\psi}_2 \not{\partial} \psi_2 + e Q_1 \bar{\psi}_1 \gamma^\mu \psi_1 A_\mu$$

$$+ e Q_2 \bar{\psi}_2 \gamma^\mu \psi_2 A_\mu - \frac{g}{2\sqrt{2}} \bar{\psi}_1 \gamma^\mu (1-\gamma_5) \psi_2 W_\mu^+$$

$$- \frac{g}{2\sqrt{2}} \bar{\psi}_2 \gamma^\mu (1-\gamma_5) \psi_1 W_\mu^-$$

$$- \sum_{i=1,2} \frac{g}{2\cos\theta_W} Z_\mu \bar{\psi}_i \gamma^\mu \left[(1-\gamma_5)(I_{3_i} - Q_i \sin^2\theta_W) - Q_i \sin^2\theta_W (1+\gamma_5) \right] \psi_i$$

$$\downarrow$$

$$- \frac{g}{2\cos\theta_W} \sum_{i=1,2} Z_\mu \bar{\psi}_i \gamma^\mu \left[(I_{3_i} - 2 Q_i \sin^2\theta_W) - I_{3_i} \gamma_5 \right] \psi_i$$

Yukawa Interaction, CKM matrix

Fermion mass.

$Y_d \overline{Q}_L \cdot H \cdot d_R = Y_d (\overline{u}_L \ \overline{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} \cdot d_R = Y_d v \overline{d}_L d_R$

$y = (-\frac{1}{3} + 1 - \frac{2}{3}) = 0$

$\therefore -Y_d \overline{Q}_L \cdot H d_R + h.c. = -Y_d v \overline{d} d \qquad Y_d v = m_d$

同理. $Y_e \overline{L}_L \cdot H e_R = Y_e (\overline{\nu}_L, \overline{e}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} e_R = Y_e v \overline{e}_L e_R$

$y = (1 + 1 - 2) = 0$

$\therefore -Y_e \overline{L}_L H e_R + h.c. = -Y_e v \overline{e} e \qquad Y_e v = m_e$

$m_u = ?$

SU(2)群的二维表示, $\xi_\alpha = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$

不变量

(1). $\eta^\dagger_\alpha \xi_\alpha = (\eta_1^*, \eta_2^*) \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$

$\xi \to U\xi, \qquad \eta \to U\eta \qquad \therefore \eta^\dagger \to \eta^\dagger U^\dagger$

$\therefore \eta^\dagger \xi \to \eta^\dagger U^\dagger \cdot U \xi \qquad \because U \in SU(2) \qquad U^\dagger U = 1$

$\therefore \eta^\dagger \xi \to \eta^\dagger \xi$ 是不变量

(2) 考虑 $\begin{pmatrix} \eta_1 & \xi_1 \\ \eta_2 & \xi_2 \end{pmatrix} \equiv (\eta_\alpha, \xi_\alpha)$

$(\eta_\alpha \ \xi_\alpha) \to (U\eta, U\xi) = U \cdot (\eta, \xi)$

$\det(\eta,\xi) \to \det(U\cdot(\eta\ \xi)) = \det U \cdot \det(\eta,\xi) = \det(\eta\ \xi)$

$\therefore$ $\det(\eta\ \xi)$ 是不变量.

$\det(\eta\ \xi) = -\det(\xi\ \eta) = \varepsilon^{\alpha\beta}\eta_\alpha\xi_\beta$ $\quad \varepsilon^{12}=1,\ \varepsilon^{21}=-1,\ \varepsilon^{11}=\varepsilon^{22}=0.$

$\therefore$ $\det(\eta\xi) = \varepsilon^{\alpha\beta}\eta_\alpha\xi_\beta = \eta_1\xi_2 - \eta_2\xi_1$

$\therefore$ 除 $\overline{Q}_L H$ 外，还有另一个不变量：$\det\begin{pmatrix}\overline{Q}_L\\ H^+\end{pmatrix}$

$$-Y_u\det\begin{pmatrix}\overline{u}_L & \overline{d}_L\\ 0 & v\end{pmatrix}\cdot u_R = -Y_u v\,\overline{u}_L u_R$$

$$y = -\frac{1}{3} - 1 + \frac{4}{3}$$

$$\therefore -Y_u\varepsilon^{\alpha\beta}\overline{Q}_{L\alpha}H^*_\beta u_R = -Y_u v\,\overline{u}u$$

为什么 $\varepsilon^{\alpha\beta}\eta_\alpha\xi_\beta$ 能构成不变量？

对于SU(2)的幺正表示 ξ，$\quad\forall\ g\in SU(2) \to \xi\to U(g)\xi$.

ξ 构成2维复线性空间 $V_{\mathbb{C}}$，且 $V_{\mathbb{C}}$ 上有内积（正定非退化二次型）：(η,ξ)，s.t $(U(g)\eta, U(g)\xi) = (\eta,\xi)$

第一种不变量 $\eta^+\xi$ 即此内积

对这一表示，考虑 $U(g)^*$，（复共轭但不转置）.

显然，$U(g_1)^*\cdot U(g_2)^* = (U(g_1)\cdot U(g_2))^* = U(g_1g_2)^*$

$\therefore$ $U(g)^*$ 也是SU(2)群的表示，称为 2^* 表示

$U(g)$ 称为2表示.

Question 2^* 和 2 "等价"吗？

如果能找到不随g变化且总使 $U(g)^* = SU(g)S^{-1}$ 的常变换S，则称2与 2^* 等价. 此时

$$U(g)^*\xi = SU(g)S^{-1}\xi$$

即 $U(g)^*$ 在基底 $\{S\vec{e}_1, S\vec{e}_2\}$ 下的表示矩阵

与 $U(g)$ 在基底 $\{\vec{e}_1, \vec{e}_2\}$ 下的表示矩阵相同

$\because$ 表示矩阵 $U = \begin{pmatrix} a+bi & c+di \\ -c+di & a-bi \end{pmatrix}$ ， $a^2+b^2+c^2+d^2=1$

$\therefore U^* = \begin{pmatrix} a-bi & c-di \\ -c-di & a+bi \end{pmatrix}$

显然 $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a+bi & c+di \\ -c+di & a-bi \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} -c+di & a-bi \\ -a-bi & -c-di \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a-bi & c-di \\ -c-di & a+bi \end{pmatrix} = U^*$$

$\therefore S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma^2 = \varepsilon^{\alpha\beta}$ 满足 $U^* = SUS^{-1}$

显然 U^* 是作用在 ξ^* 上的 $\because U\xi^* = SUS^{-1}\xi^*$

$\therefore S^{-1}\xi^*$ 与 ξ 的变换规律相同 $\therefore (\eta, S^{-1}\xi^*)$ 是内积的合法形式

也即 $(\eta_1^*, \eta_2^*) \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1^* \\ \xi_2^* \end{pmatrix} = -\eta_1^*\xi_2^* + \eta_2^*\xi_1^*$

$\therefore$

这样 对于 up quark，我们也可以由 $\begin{pmatrix} 0 \\ v \end{pmatrix}$ 得到质量项

$$\mathcal{L}_{\text{Yukawa}} = -Y_d \bar{Q}_L H d_R - Y_\ell \bar{L}_L H e_R - Y_u \varepsilon^{\alpha\beta} \bar{Q}_{L\alpha} H_\beta u_R + h.c.$$

$$\therefore \quad = -(\bar{d}_L \ \bar{s}_L \ \bar{b}_L) Y_d v \cdot \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{e}_L \ \bar{\mu}_L \ \bar{\tau}_L) Y_\ell v \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$- (\bar{u}_L \ \bar{c}_L \ \bar{t}_L) Y_u v \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + h.c.$$

定义 $M_d = Y_d v \quad M_u = Y_u v \quad M_\ell = Y_\ell v.$

$$\therefore \mathcal{L}_{\text{Yukawa}} = -\bar{D}_L M_d D_R - \bar{D}_R M_d^\dagger D_L$$
$$- \bar{E}_L M_\ell E_R - \bar{E}_R M_\ell^\dagger E_L$$
$$- \bar{U}_L M_u U_R - \bar{U}_R M_u^\dagger U_L$$

$$D_x \equiv \begin{pmatrix} d_x \\ s_x \\ b_x \end{pmatrix} \qquad E_x \equiv \begin{pmatrix} e_x \\ \mu_x \\ \tau_x \end{pmatrix} \qquad U_x \equiv \begin{pmatrix} u_x \\ c_x \\ t_x \end{pmatrix}$$

定理：任意 $n\times m$ 矩阵 M，一定可以被一对幺正矩阵 U, V 对角化

$$UMV = \text{diag}(m_1, m_2, \cdots, m_n) \quad \text{且} \ m_i \geq 0$$

(Singular Value Decomposition) $\rightarrow$ Schmidt decomposition in Quantum Information

不妨设 $V_u^\dagger M_u U_u$，$V_d^\dagger M_d U_d$，$V_e^\dagger M_e U_e$ 对角化。其非负对角元分别为 (m_u, m_c, m_t)，(m_d, m_s, m_b) (m_e, m_μ, m_τ)

∴ 费米子的质量本征态为 $\begin{pmatrix} V_F F_L \\ U_F F_R \end{pmatrix} = f$

！左手部分与右手部分的转动不同。

$$\Rightarrow \quad F_L = V_F^\dagger \frac{1}{2}(1-\gamma_5) f$$
$$F_R = U_F^\dagger \frac{1}{2}(1-\gamma_5) f$$

例：up 夸克三个质量本征态 u, c, t，与 u_1, u_2, u_3 的关系

$$\psi_u = \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} V_{u1} u_{1L} + V_{u2} u_{2L} + V_{u3} u_{3L} \\ U_{u1} u_{1R} + U_{u2} u_{2R} + U_{u3} u_{3R} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_K = & \bar\psi_u (i\not\partial - m_u)\psi_u + \bar\psi_c (i\not\partial - m_c)\psi_c + \bar\psi_t (i\not\partial - m_t)\psi_t \\ & + \bar\psi_d (i\not\partial - m_d)\psi_d + \bar\psi_s (i\not\partial - m_s)\psi_s + \bar\psi_b (i\not\partial - m_b)\psi_b \\ & + \bar\psi_e (i\not\partial - m_e)\psi_e + \bar\psi_\mu (i\not\partial - m_\mu)\psi_\mu + \bar\psi_\tau (i\not\partial - m_\tau)\psi_\tau \end{aligned}$$

对于 A_μ, Z_μ 耦合的流。(spin-1 !!!)

∵ 形式均为 $\bar\psi_{iL} \Gamma_L \psi_{iL} + \bar\psi_{iR} \Gamma_R \psi_{iR}$

$i = 1, 2, 3$ 代 ψ_i 为流本征态。

∴ 在质量本征态下，$J \to \bar f_L V \Gamma_L V^\dagger f_L + \bar f_R U \Gamma_R U^\dagger f_R$

$$= \bar f_L \Gamma_L f_L + \bar f_R \Gamma_R f_R$$

$\rightarrow$ GIM 机制

Glashow–Iliopoulos–Maiani

对于带电流，$W_\mu^\pm$

$$\mathcal{L} = -\frac{g}{\sqrt{2}} (\bar{u}_{1L}\ \ \bar{u}_{2L}\ \ \bar{u}_{3L}) \gamma^\mu \begin{pmatrix} d_{1L} \\ d_{2L} \\ d_{3L} \end{pmatrix} W_\mu^+ + h.c.$$

$$= -\frac{g}{2\sqrt{2}} (\bar{u}\ \ \bar{c}\ \ \bar{t}) \cdot V_u\ \gamma^\mu (1-\gamma_5) V_d^\dagger \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^+ + h.c.$$

$$= -\frac{g}{2\sqrt{2}} (\bar{u}\ \bar{c}\ \bar{t}) V_u V_d^\dagger\ \gamma^\mu (1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^+ + h.c.$$

$$V_{CKM} \equiv V_u V_d^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo - Kobayashi - Maskawa matrix

∵ V_u, V_d 为幺正矩阵，∴ V_{CKM} 为 3×3 幺正矩阵

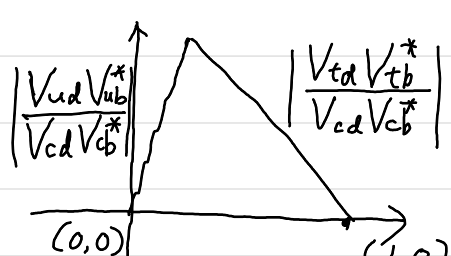
$$\therefore I = \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\therefore |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb}$$
$$= V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0$$

三个幺正三角形

常用 $$0 = \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + 1 + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}$$



V_{CKM} 有多少自由度？

$$(\bar{u}, \bar{c}\ \ \bar{t})\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

V_{ij} 有 9 个复相位.

$$\begin{pmatrix} e^{i\delta_{ud}} & e^{i\delta_{us}} & e^{i\delta_{ub}} \\ e^{i\delta_{cd}} & e^{i\delta_{cs}} & e^{i\delta_{cb}} \\ e^{i\delta_{td}} & e^{i\delta_{ts}} & e^{i\delta_{tb}} \end{pmatrix}$$

$u \to u e^{i\alpha_u}$ ....

$$\to \begin{pmatrix} e^{i(\delta_{ud}-\alpha_u+\alpha_d)} & \cdots & \\ & \cdots & \\ & \cdots & \end{pmatrix}$$

$\delta_{ud} = \alpha_u - \alpha_d$ ✓

$\delta_{us} = \alpha_u - \alpha_s$ ✓

$\to$ $\delta_{ub} = \alpha_u - \alpha_b$ ✓

$\delta_{cd} = \alpha_c - \alpha_d = \alpha_c - \alpha_u + \delta_{ud}$ ✓

$\delta_{cs} = \alpha_c - \alpha_s = \alpha_c - \alpha_u + \delta_{us}$ ×

$\delta_{cb} = \alpha_c - \alpha_b = \alpha_c - \alpha_u + \delta_{ub}$ ×

$\delta_{td} = \alpha_t - \alpha_d = \alpha_t - \alpha_u + \delta_{ud}$ ✓

$\delta_{ts} = \alpha_t - \alpha_s = \alpha_t - \alpha_u + \delta_{us}$ ×

$\delta_{tb} = \alpha_t - \alpha_b = \alpha_t - \alpha_u + \delta_{ub}$ ×

余下 4 个复元素.

∴ 无法通过夸克的相位重定义将 V_{CKM} 中的相位完全消除.

! 2×2 情况，无 CP 相位

可以选取将第一行（一列）的相角消去

$$\Rightarrow V_{CKM} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & V_{cs} & V_{cb} \\ \sin\theta_1\sin\theta_2 & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{1j}^* \cdot V_{2j} = 0 \Rightarrow c_1 s_1 c_2 - V_{cs} s_1 c_3 - V_{cb} s_1 s_3 = 0$$

$$\therefore V_{cb} = \frac{c_1 c_2}{s_3} - \frac{c_3}{s_3} V_{cs}$$

$$V_{1j}^* \cdot V_{3j} = 0 \Rightarrow c_1 s_1 s_2 - V_{ts} s_1 c_3 - V_{tb} s_1 s_3 = 0.$$

$$\therefore V_{tb} = \frac{c_1 s_2}{s_3} - \frac{c_3}{s_3} V_{ts}$$

$$\therefore V_{CKM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & V_{cs} & -\frac{c_3}{s_3} V_{cs} + \frac{c_1 c_2}{s_3} \\ s_1 s_2 & V_{ts} & -\frac{c_3}{s_3} V_{ts} + \frac{c_1 s_2}{s_3} \end{pmatrix}$$

$$V_{j1}^* \cdot V_{j2} = 0 \Rightarrow c_1 c_3 = c_2 V_{cs} + s_2 V_{ts} \Rightarrow V_{ts} = \frac{c_1 c_3}{s_2} - \frac{c_2}{s_2} V_{cs}$$

$$V_{CKM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & V_{cs} & \frac{c_1 c_2}{s_3} - \frac{c_3}{s_3} V_{cs} \\ s_1 s_2 & \frac{c_1 c_3}{s_2} - \frac{c_2}{s_2} V_{cs} & \frac{c_1 s_2}{s_3} - \frac{c_1 c_3^2}{s_2 s_3} + \frac{c_2 c_3}{s_2 s_3} V_{cs} \end{pmatrix}$$

$$\Rightarrow V_{cs} = c_1 c_2 c_3 - s_2 s_3 e^{i\delta}$$

KM matrix

今天，人们习惯用

$$V_{CKM}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

补充：V_{cs} 的确定.

由 $V_{CKM}^{+}V_{CKM}$ 第2行第2列为1，$\Rightarrow$

$$1=s_1^2c_2^2+|V_{cs}|^2+\frac{1}{s_3^2}|c_1c_2-c_3V_{cs}|^2$$

$V_{cs}=x+iy\quad\Rightarrow$

$$1=s_1^2c_2^2+x^2+y^2+s_3^{-2}(c_1c_2-c_3x)^2+s_3^{-2}c_3^2y^2$$

$$\therefore\ s_3^2=s_1^2s_3^2c_2^2+s_3^2x^2+(c_1c_2-c_3x)^2+s_3^2y^2+c_3^2y^2$$

$$\therefore\ y^2+x^2-2c_1c_2c_3x+c_1^2c_2^2+s_1^2c_2^2s_3^2-s_3^2=0$$

$$(x-c_1c_2c_3)^2+y^2+c_1^2c_2^2s_3^2+s_1^2c_2^2s_3^2-s_3^2=0$$

$$s_3^2-c_1^2c_2^2s_3^2-s_1^2c_2^2s_3^2=s_3^2(1-c_1^2c_2^2-s_1^2c_2^2)=s_3^2(1-c_2^2)=s_3^2s_2^2$$

$$\therefore\ (x-c_1c_2c_3)^2+y^2=s_2^2s_3^2$$

$\therefore$ V_{cs} 是圆心在 $c_1c_2c_3$，半径为 s_2s_3 的复平面上圆上的某一点

$$\therefore\ V_{cs}=c_1c_2c_3-s_2s_3e^{i\delta}$$