Introduction to Soft-Collinear Effective Theory

Outline

I. Sudakov Problem in QCD

II. Construction of SCET Lagrangian

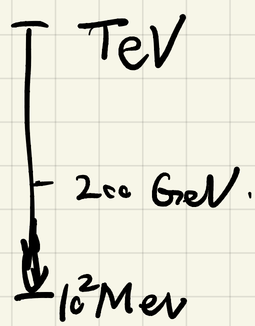
III. Matching and Running in SCET

Ref : Becher et.al. 1410.1892

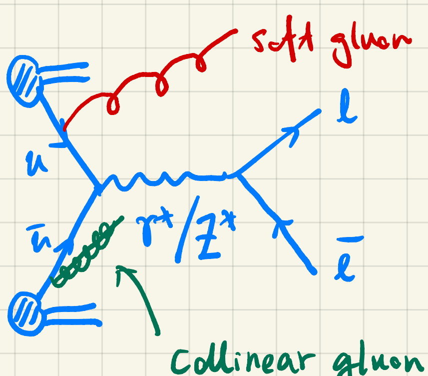
compare with J. Collins hep-ph/0312336

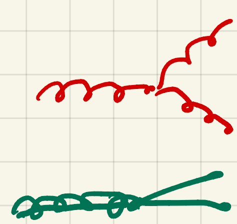
I. Motivation.

LHC. $pp \to X$.

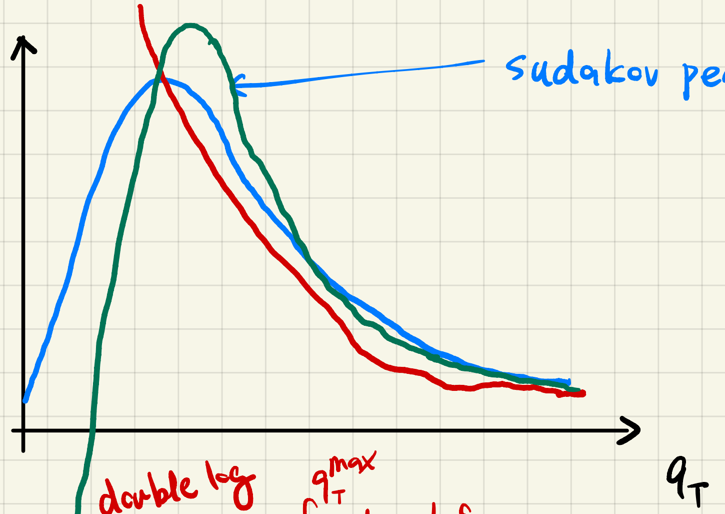


* Drell-Yan $\underline{pp} \to \ell^+\ell^- + \underline{X}$





$$\frac{d\sigma}{dq_T}$$



$q_T = (\bar{\ell}\ell)\ p_T.$

$$\sigma(\alpha_s) = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \cdots$$

double log $\quad \int^{q_T^{max}} dq_T \frac{\ln q_T}{q_T} \sim \ln^2 q_T^{max}$

$$\frac{d\sigma}{dq_T} \overset{q_T \to 0}{\sim} \alpha_s \cdot \left[-\frac{\ln q_T}{q_T} + \frac{1}{q_T} + \cdots \right]$$

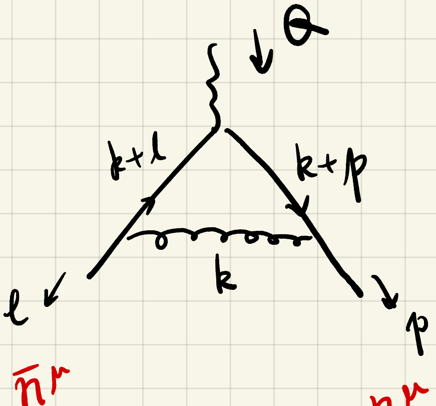
$$\alpha_s^2 \left[\frac{\ln^3 q_T}{q_T} + \frac{\ln^2 q_T}{q_T} + \cdots \right]$$

$$\alpha_s^3 \left[\frac{\ln^5 q_T}{q_T} + \frac{\ln^4 q_T}{q_T} + \cdots \right]$$

resummation

double scaling $\begin{cases} \alpha_s \to 0 \\ \alpha_s \cdot \ln q_T \sim \text{finite.} \end{cases}$

Sudakov form factor



$$\boxed{0 < -\ell^2 \sim -p^2 \ll Q^2}$$

Euclidean kinematics.

$$I^{(1)} = i\pi^{-d/2} \mu^{4-d} \int \frac{d^4k}{(2\pi)^4} \frac{N}{k^2 (k+\ell)^2 (k+p)^2}$$

$$\overset{\frac{\ell^2}{Q^2} \to 0}{=\!=} \frac{-1}{Q^2} \left[\ln\frac{Q^2}{-\ell^2} \ln\frac{Q^2}{-p^2} + \frac{\pi^2}{3} + \cdots \right]$$

* Light-cone Coordinate.

$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$

$n\cdot\bar{n} = 2 \qquad n^2 = \bar{n}^2 = 0$

$p^\mu = (p^0, p^1, p^2, p^3) = \underbrace{(p^+, p^-, p_\perp^\mu)}$

$$p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp^\mu = \frac{n^\mu}{2} p^- + \frac{\bar{n}^\mu}{2} p^+ + p_\perp^\mu$$

$\bar{n}\cdot p \equiv p^- = p^0 + p^3 \qquad n\cdot p \equiv p^+ = p^0 - p^3$

$p^2 = p^+p^- + p_\perp^2 \qquad 2\,l\cdot p = p^+l^- + p^-l^+ + 2\,p_\perp\cdot l_\perp$

* Method of Region

$$I^{(1)} = i\pi^{-d/2}\mu^{4-d}\int\frac{d^4k}{(2\pi)^4}\,\frac{N}{k^2(k+l)^2(k+p)^2}$$

$$Q^\mu \rightarrow \underset{\bar{n}^\mu}{l^\mu} + \underset{n^\mu}{p^\mu}$$

$\underbrace{\frac{l^2}{Q^2},\ \frac{p^2}{Q^2}}$ leading Power

Non-vanishing momentum region.

① hard : $k^\mu \sim Q(\overset{k^+}{1}, \overset{k^-}{1}, \overset{k_\perp}{1})$ $\qquad k^2 \sim Q^2$

② collinear : $k^\mu \sim Q(\lambda^2, 1, \lambda)$ $\qquad k^\mu // p^\mu \quad k^2 \sim \lambda^2$

③ anti-coll : $k^\mu \sim Q(1, \lambda^2, \lambda)$ $\qquad 0 < \lambda \ll 1$

④ Soft : $k^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$p^\mu = Q(\lambda^2, 1, \lambda)$

$l^\mu = Q(1, \lambda^2, \lambda)$

① hard region.

$$I_h = \frac{1}{k^2(k+l)^2(k+p)^2} = \frac{1}{\underbrace{k^2}_{Q^2} \cdot [k^2 + \underbrace{k^+l^-}_{\cancel{\lambda^2 Q^2}} + \underbrace{k^- l^+}_{Q^2} + \underbrace{2k_\perp \cdot l_\perp}_{\cancel{\lambda Q^2}} + \underbrace{l^2}_{\cancel{\lambda^2 Q^2}}]} \frac{1}{(k+p)^2}$$

$$I_h^{(Q)} = \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 [k^2 + k^- l^+][k^2 + k^+ p^-]} \qquad d = 4 - 2\epsilon$$

$$= \frac{\Gamma(1+\epsilon)}{Q^2} \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{Q^2} + \frac{1}{2} \ln^2 \frac{\mu^2}{Q^2} - \frac{\pi^2}{6} \right)$$

$$Q^2 = (l+p)^2 = l^+ p^- + l^- p^+ + 2 l_\perp \cdot p_\perp + l^2 + p^2$$

$$\approx l^+ p^-$$

② Collinear region. $k^\mu \sim Q(\lambda^2, 1, \lambda)$ $\qquad k^2 \sim \lambda^2 Q^2$

$$I_n = \frac{1}{k^2 \cdot [\underbrace{k^2}_{\lambda^2} + \underbrace{k^+ l^-}_{\lambda^4} + \underbrace{k^- l^+}_{1} + \underbrace{2k_\perp \cdot l_\perp}_{\lambda^2} + \underbrace{l^2}_{\lambda^2}][\underbrace{k^2}_{\lambda^2} + \underbrace{k^+ p^-}_{\lambda^2} + \underbrace{k^- p^+}_{\lambda^2} + \underbrace{2k_\perp \cdot p_\perp}_{\lambda^2} + \underbrace{p^2}_{\lambda^2}]}$$

$$\int d^4k = \frac{1}{k^2 [k^- l^+][k+p]^2}$$

$$I_n = \frac{\Gamma(1+\epsilon)}{Q^2} \left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{\mu^2}{-p^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{-p^2} + \frac{\pi^2}{6} \right]$$

④ soft region.

$$\frac{1}{k^2 (k^- l^+)(k^+ p^-)}$$

$$I_s = \frac{1}{\underbrace{k^2}_{\lambda^2}\,(\underbrace{k^- l^+ + l^2}_{\lambda^2})(\underbrace{k^+ p^- + p^2}_{\lambda^2})}$$

$$= \frac{\Gamma(1+\epsilon)}{Q^2}\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2 Q^2}{(-p^2)(-l^2)} + \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{(-p^2)(-l^2)} + \frac{\pi^2}{6}\right]$$

$$I_h + I_n + I_{\bar{n}} + I_s = I_{tot}.$$

exercise: $k_G \sim Q(\lambda^2, \lambda^2, \lambda)$ Glauber region

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Method of region

* need to identify all region
  Mathematica: asy.m
* Regularization
* double counting.
* complicated at higher loops

Soft-Collinear EFT.

* built into axiom
* Gauge symmetry.
* zero-bin.
* Renormalization Group.

λ^0	Q^2	hard	
λ^2	p^2, l^2	collinear, anti-coll	$\downarrow \ln\frac{Q^2}{\mu^2}$
λ^4	$\frac{p^2 \cdot l^2}{Q^2}$	soft	$\downarrow \ln\frac{Q^2}{p^2}$

$C(\mu) O(\mu)$

$$\frac{dC}{d\ln\mu} = \gamma\, C(\mu)$$

$C(\mu = Q)$

$$e^{\alpha \ln\frac{\mu^2}{Q^2}}$$

SCET

top - down

QCD

SCET

HQET

NRQCD

$\lambda^0 + \lambda^2 + \lambda^4 + \cdots$

leading power

non-local operators

SMEFT

GUT / String.

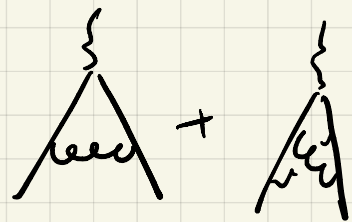
SMEFT

$c_6 O_6 + O_8 +$

$\phi^6(x)$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

II. Construction of SCET Lagrangian.



$$\alpha^n \ln^{2n}\frac{Q^2}{\ell^2}$$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu,a} F^{\mu\nu,a} + \bar{\psi} i \not{D} \psi$$

$$\Downarrow$$

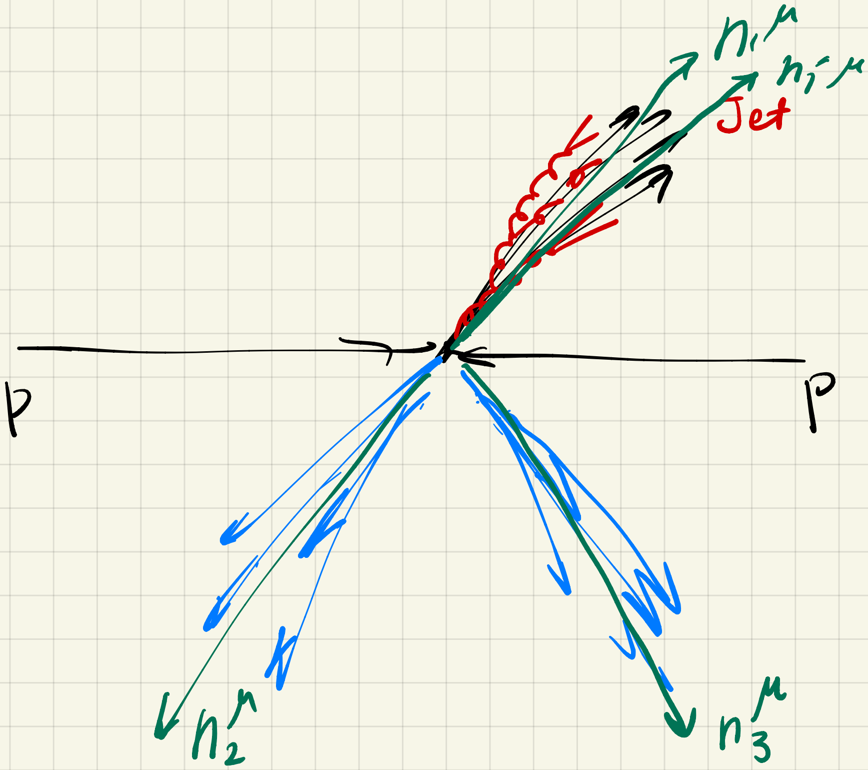
$$\boxed{\mathcal{L}_{SCET}^{(m)} = \sum_n \mathcal{L}_n^{(m)} + \mathcal{L}_s^{(m)}}$$ our goal.

Suppose we have only 1 collinear direction n^μ

$$p^\mu \sim Q(\lambda^2, 1, \lambda)$$

$$[\psi] = \frac{3}{2} \qquad [A^\mu] = 1. \qquad [\phi] = 1$$

How to power counting ψ, A^μ, ϕ, in λ.



RPI

* Power counting of fields in SCET

$$p^2 \sim \lambda^2 \cdot Q^2$$

$$\psi(x) \Rightarrow \psi_c(x) + \psi_s(x)$$

$$\psi_{n_1} + \psi_{n_2} + \cdots \psi_{n_k}.$$

$$\psi_c(x) = \xi(x) + \eta(x)$$

$$\xi(x) = P_+ \psi_c = \frac{\not{n}\bar{\not{n}}}{4}\psi_c$$

$$\eta(x) = P_- \psi_c = \frac{\bar{\not{n}}\not{n}}{4}\psi_c$$

$$P_+ + P_- = \frac{1}{4}(\not{n}\bar{\not{n}} + \bar{\not{n}}\not{n}) = \underline{\mathbb{1}}. \quad \Rightarrow \quad P_+^2 = P_+ \quad P_-^2 = P_-$$

$$P_+ P_- = P_- P_+ = 0$$

$$\langle 0|T\{\xi(x)\bar{\xi}(0)\}|0\rangle = \frac{\not{n}\not{\bar{n}}}{4}\langle 0|\psi_c(x)\bar{\psi}_c(0)|0\rangle\frac{\not{\bar{n}}\not{n}}{4}$$

$$= \underbrace{\int\frac{d^4p}{(2\pi)^4}}\, e^{-ipx}\,\frac{\not{n}\not{\bar{n}}}{4}\,\underbrace{\frac{\not{p}}{p^2}}\,\frac{\not{\bar{n}}\not{n}}{4}$$

$$\int d^4p \sim \int dp^+dp^-d^2p_\perp \qquad \frac{\not{n}\not{\bar{n}}}{4}\,\frac{(\not{n}p^-+\not{\bar{n}}p^++\not{p}_\perp)}{p^2}\,\frac{\not{\bar{n}}\not{n}}{4}$$

$$\lambda^4 \times \lambda^{-2} \times \lambda^0 = \lambda^2$$

$p^\mu \sim Q(\lambda^2, 1, \lambda)$

n^μ

$$\frac{\not{\bar{n}}\not{n}}{4}\,(\not{n}p^-+\underbrace{\not{\bar{n}}p^+}+\not{p}_\perp)\,\frac{\not{n}\not{\bar{n}}}{4}$$

$$\lambda^4 \times \lambda^{-2} \times \lambda^2$$

$$\boxed{[\xi] = \lambda \qquad [\eta] = \lambda^2 \qquad [\psi_s] = \lambda^3}$$

$$A^\mu(x) \Rightarrow A_c^\mu(x) + A_s^\mu(x)$$

Covariant gauge.

$$\langle 0|T\{A_c^\mu(0)A_c^\nu(x)\}|0\rangle = \underbrace{\int\frac{d^4p}{(2\pi)^4}}_{\lambda^4}\, e^{-ipx}\,\underbrace{\frac{1}{p^2}}_{\lambda^{-2}}\left[g^{\mu\nu}+k\frac{p^\mu p^\nu}{p^2}\right]$$

$$\frac{p^\mu p^\nu}{\lambda^{-2}}$$

$$\boxed{\begin{aligned}[A_c^\mu] &\sim [p^\mu] \sim (\lambda^2, 1, \lambda)\\ [A_s^\mu] &\sim [p^\mu] \sim (\lambda^2, \lambda^2, \lambda^2)\end{aligned}}$$

$\not{n}\xi = 0$

$\not{n}\frac{\not{n}\bar{\not{n}}}{4}\psi_c$

* Effective Lagrangian

$$\mathcal{L}_c = \bar{\Psi}_c i\not{D}\Psi_c = \bar{\Psi}_c\left(i\partial_\mu\gamma^\mu + gA_\mu\gamma^\mu\right)\Psi_c$$

$$= (\bar{\xi}+\bar{\eta})\left[i\frac{\bar{\not{n}}}{2}D^+ + i\frac{\not{n}}{2}D^- + i\not{D}_\perp\right](\xi+\eta)$$

$$\boxed{= \bar{\xi}\frac{i}{2}\bar{\not{n}}D^+\xi + \bar{\eta}\, i\frac{\not{n}}{2}D^-\eta + \bar{\xi}\, i\not{D}_\perp\eta + \bar{\eta}\, i\not{D}_\perp\xi}$$

Equation of motion for $\bar{\eta}, \eta$.

$$0 = \frac{\delta\mathcal{L}_c}{\delta\bar{\eta}} = \partial_\mu\frac{\partial\mathcal{L}_c}{\partial(\partial_\mu\bar{\eta})} - \frac{\partial\mathcal{L}}{\partial\bar{\eta}} = \underline{0 - i\frac{\not{n}}{2}D^-\eta - i\not{D}_\perp\xi = 0}$$

$$\eta = -\frac{1}{D^-}\not{D}_\perp\frac{\bar{\not{n}}}{2}\xi \qquad \bar{\eta} = -\bar{\xi}\overleftarrow{\not{D}}_\perp\frac{\bar{\not{n}}}{2\overleftarrow{D}^-}$$

non-local

$$\frac{\not{n}\bar{\not{n}}}{4}\xi = \xi$$

$$\mathcal{L}_c = \bar{\xi}\frac{i}{2}\bar{\not{n}}D^+\xi + \bar{\xi}\overleftarrow{\not{D}}_\perp\frac{\bar{\not{n}}}{2\overleftarrow{D}^-}\, i\frac{\not{n}}{2}D^-\frac{1}{D^-}\not{D}_\perp\frac{\bar{\not{n}}}{2}\xi$$
$$- \bar{\xi}\, i\not{D}_\perp\frac{1}{D^-}\not{D}_\perp\frac{\bar{\not{n}}}{2}\xi - \bar{\xi}\overleftarrow{\not{D}}_\perp\frac{\bar{\not{n}}}{2\overleftarrow{D}^-}\, i\not{D}_\perp\xi$$

$$\boxed{\mathcal{L}_{c,\psi} = \underbrace{\bar{\xi}}_{\lambda}\frac{i}{2}\underbrace{\bar{\not{n}}D^+}_{\lambda^2}\underbrace{\xi}_{\lambda} - \bar{\xi}\, i\not{D}_\perp\frac{1}{D^-}\not{D}_\perp\frac{\bar{\not{n}}}{2}\xi} \qquad \lambda^4$$

$$\mathcal{L}_{c,A} = -\frac{1}{4}F^{a}_{\mu\nu,c}F^{\mu\nu,a}_{c}$$

$\mathcal{L}_{s,\psi} = \bar{\psi}_s i\not{D}\psi_s \qquad \mathcal{L}_{s,A} = -\frac{1}{4}F^{a}_{\mu\nu,s}F^{\mu\nu,a}_{s}$

$\mathcal{L} \quad (\bar{\psi}_c + \bar{\psi}_s)\, i\not{D}\, (\psi_c + \psi_s)$

$\bar{\psi}_s\, i\not{D}\, \psi_c$

$= \bar{\psi}_s\, i\left(\frac{\not{n}}{2}D^- + \frac{\bar{\not{n}}}{2}D^+ + \not{D}_\perp\right)(\xi + \eta) \qquad \not{n}\xi = 0$

$\lambda^3 \qquad \lambda^0 + \lambda^2 + \lambda \qquad (\lambda + \cancel{\lambda^2})$

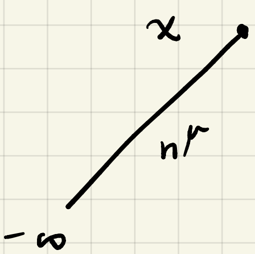
* soft-collinear decouple

$$\boxed{\bar{\xi}\,\frac{i}{2}D^+\bar{\not{n}}\,\xi = \bar{\xi}\Big[\underbrace{in\cdot\partial}_{\lambda^2} + \underbrace{g\,n\cdot A_c}_{\lambda^2} + \underbrace{g\,n\cdot A_s}_{\lambda^2}\Big]\frac{\bar{\not{n}}}{2}\xi}$$

* BPS field redifinition. Bauer - P - Stewart

def $Y_n(x) = P\exp\left[ig\int_{-\infty}^{0} ds\; n\cdot A_s(x+ns)\right]$

$= P\exp\left[ig\int_{-\infty}^{x} d\gamma\; n\cdot A_s(\gamma)\right]$



$\partial_\mu Y_n(x) = Y_n(x)\partial_\mu + ig\,n\cdot A_s(x)\,Y_n$

$\xi(x) \to Y(x)\,\xi^{(0)}(x), \qquad \bar{\xi}(x) \to \bar{\xi}^{(0)}(x)\,Y^\dagger(x)$

$\bar{\xi}\,Y_n^\dagger\,(in\cdot\partial + g\,n\cdot A_c + g\,n\cdot A_s)\,\frac{\bar{\not{n}}}{2}\,Y_n\,\xi \qquad Y^\dagger Y = \mathbb{1}$

$$\bar{\xi}^{(0)}\left[\cancel{Y_n^\dagger}\,\cancel{Y_n} i\partial_\mu + i\cancel{Y_n^\dagger}\, ig\, n\cdot A_s \cancel{Y_n} + g\, n\cdot A_c + g\, n\cdot A_s\right]$$

$$\times \frac{\bar{n}\!\!\!/}{2}\xi^{(0)}$$

$$\xi^{(0)}\left[\underbrace{i\partial^+ + g\, n\cdot A_c}_{iD_c^+}\right]\frac{\bar{n}\!\!\!/}{2}\xi^{(0)}$$

$$\mathcal{L}_c^{(0)} = \bar{\xi}^{(0)} iD_c^+ \frac{\bar{n}\!\!\!/}{2}\xi^{(0)} + \bar{\xi}^{(0)} iD\!\!\!/_{\perp,c}\frac{1}{D_c^-}D\!\!\!/_{\perp,c}\frac{\bar{n}\!\!\!/}{2}\xi^{(0)}$$

$$\mathcal{L}_{SCET}^{(0)} = \sum_n \mathcal{L}_n^{(0)} + \mathcal{L}_s + \boxed{J^\mu_{SCET} A_{cl\,\mu} + \cdots}$$

Feynman Rule for Soft Wilson Line.

$$\text{def}\quad Y_n(x) = P\exp\left[ig\int_{-\infty}^0 ds\; n\cdot A_s(x+ns)\right]$$

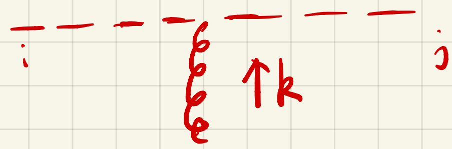
$$= \mathbb{1} + ig\int_{-\infty}^0 n\cdot A_s(x^\mu + n^\mu s) + (ig)^2\int_{-\infty}^0 ds_1\, n\cdot A_s(x+ns_1)$$

$$\times\int_{-\infty}^{s_1} ds_2\, n\cdot A_s(x+ns_2) + \cdots$$

mom space

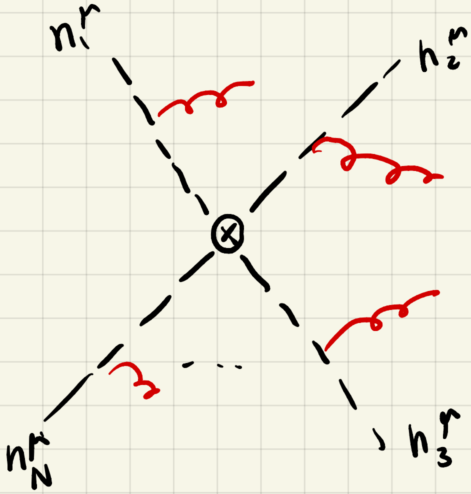
$$ig\int_{-\infty}^0 ds\, n^\mu \int\frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x+ns)}\, A_\mu(k)$$

$$\frac{ig n^\mu}{n\cdot k + i\varepsilon}\, t^a_{ij}$$

$=$ 

$$\frac{i\varepsilon\cdot n}{n\cdot k+i\epsilon}\, t^a_{ji}$$

N- collinear direction

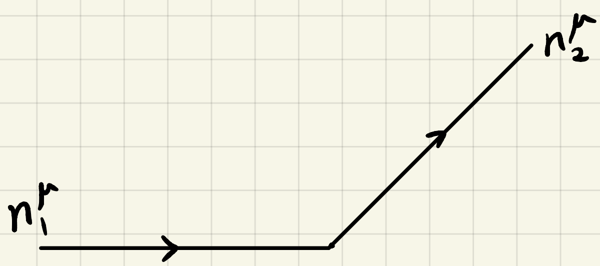


$$Y_{n_1} Y_{n_2} Y_{n_3} \cdots Y_{n_N}$$

$$\boxed{\sum_{i=1}^{N} \frac{n_i\cdot\varepsilon(k)}{n_i\cdot k}\, T_i}$$

Weinberg's soft theorem.

classical electrodynamics



trajectory

$$x^\mu(\tau) = \begin{cases} n_1^\mu \tau & \tau<0 \\ n_2^\mu \tau & \tau>0 \end{cases}$$

$$A_\mu J^\mu$$

$$J^\mu(x) = e\int d\tau\, \frac{dx^\mu(\tau)}{d\tau}\, \delta^{(4)}(x-x(\tau))$$

$$= e\int d\tau\, \frac{dx^\mu(\tau)}{d\tau} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-x(\tau))}$$

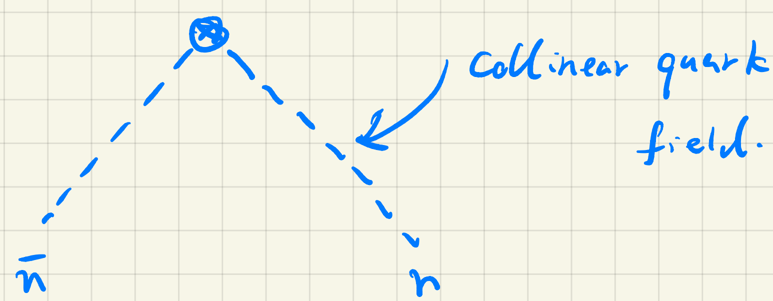
$$= e\left(\int_{-\infty}^{0} d\tau + \int_{0}^{+\infty} d\tau\right) \cdots$$

$$= -ie\int\frac{d^4k}{(2\pi)^4}\left(\frac{n^\mu}{n\cdot k}-\frac{n_2^\mu}{n_2\cdot k}\right)e^{-ikx}\,\varepsilon_\mu$$

$$\text{QCD}\;\Big\downarrow\; \mathcal{L}_{SCET}+\sum_i O_i$$

* Matching from QCD to SCET., Non-local operator, Wilson

$$\xrightarrow[\text{hard mode}]{\text{Integrate out}}$$



QCD current

$$J^\mu(x)=\bar\psi\gamma^\mu\psi$$

SCET current

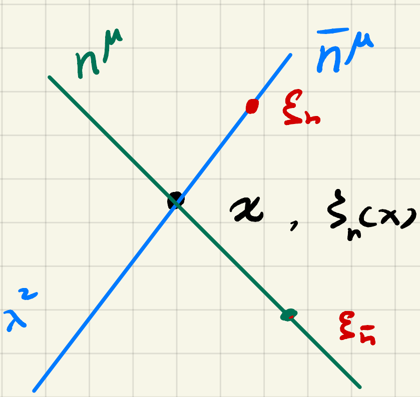
$$J^\mu_{SCET}=\,?$$

* Guess : $J^\mu_{SCET}=\bar\xi_{\bar n}\gamma^\mu\xi_n(x)=\bar\xi_{\bar n}\gamma_\perp^\mu\xi_n$

$$\xi_n(x+\bar n t)=\sum_{m=0}^{\infty}\frac{t^m}{m!}(\bar n\cdot\partial)^m\xi(x)$$

$(\lambda^0)^m$

$n\cdot\partial\sim p^+\sim\lambda^2$



$[\xi_{\bar n}(x+ns)]\sim\lambda^1$

$$J^\mu_{SCET}=\int dt\,ds\;\bar\xi_{\bar n}(x+sn)\gamma_\perp^\mu\xi_n(x+\bar n t)\,C(s,t,\mu)$$

$$\underbrace{\qquad\qquad\qquad}_{\text{non-local}} \qquad \underbrace{\qquad\qquad}_{\substack{\text{consquence of} \\ \text{integrading out hard} \\ \text{mode.}}}$$

* Collinear Wilson line

$$\bar{\psi}(x_1) \overset{W(x_1,x_2)}{\wedge} \psi(x_2)$$

$$\Downarrow$$

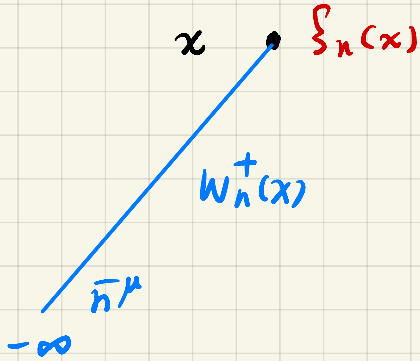
$$\bar{\psi}(x_1)\, \underline{U^{\dagger}(x_1)}\ \underbrace{U(x_2)}\psi(x_2)$$

$$U(x_1) W(x_1,x_2)\, U^{\dagger}(x_2)$$

$$*\quad \xi_n(x) \mapsto \overbrace{W_n^{\dagger}(x)\,\xi_n(x)}^{\sim} \qquad \bar{\xi}_n(x) \to \bar{\xi}_n(x)\, W_n(x)$$

$$W_n(x) = P\exp\left[ig\int_{-\infty}^{0} dt\ \bar{n}\cdot A_n(x+\bar{n}t)\right]$$

Gauge transformation



$$W_n^{\dagger}(x)\,\xi_n(x)$$

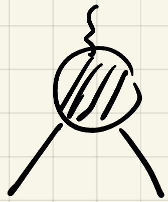
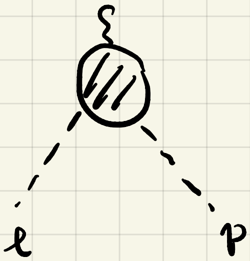
$$\to \underbrace{U(-\infty)}_{1} W_n^{\dagger}(x) \underbrace{U^{\dagger}(x) U(x)}_{1.} \xi_n(x)$$

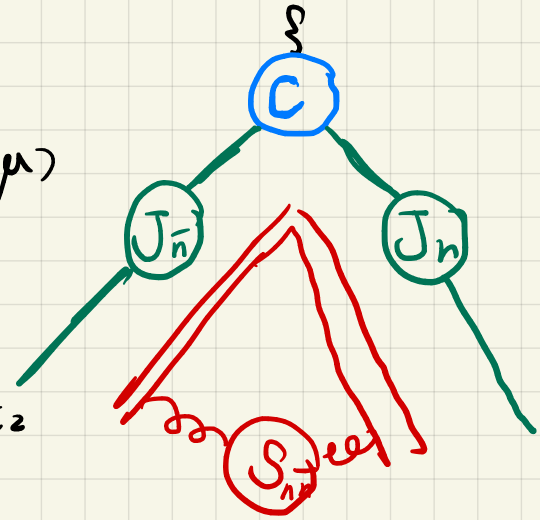
: for gauge function vanish at ∞

$$J^{\mu}_{SCET} = \int ds \int dt\ C(s,t)\ \bar{\xi}_{\bar{n}} W_{\bar{n}}(x+sn)\,\gamma_{\perp}^{\mu}\, W_n^{\dagger}\,\xi_n(x+t\bar{n})$$

※ Solve Sudakov Problem.

SCET Leading Power

 $\cong$  $= G(l, p, \mu)$



$$G(l,p,\mu) = \int d^4x_1 \int d^4x_2 \, e^{-ip\cdot x_1 + il\cdot x_2}$$

$$\times \langle 0| T\{\chi_{\bar n}(x_2) J^{\mu}_{SCET}(0) \bar\chi_n(x_1)\}|0\rangle$$

$$= \int d^4x_1 d^4x_2 e^{-ip\cdot x_1 + il\cdot x_2} \int ds\, dt \boxed{C(s,t)}$$ hard mode

$$\langle 0| T\{\underbrace{\chi^{(0)}_{\bar n}(x_2)\, \bar\chi^{(0)}_{\bar n}(s n)}_{\underline{J_{\bar n}(l^2)}}\, \underbrace{Y^{\dagger}_{\bar n}(sn)\, \gamma^{\mu}_{\perp}\, Y_n(t\bar n)}_{\underline{S_{n\bar n}(0)}}\, \hat{}\, \underbrace{\chi^{(0)}_n(t\bar n)\, \bar\chi^{(0)}_n(x_1)}_{\underline{J_n(p^2)}}\}|0\rangle$$

$|0\rangle\langle 0|$

$$J_n(p^2) = \int d^4x_1 \, e^{-ip\cdot x_1} \langle 0| T\{\chi^{(0)}_n(t\bar n)\, \bar\chi_{\bar n}(x_1)\}|0\rangle$$

$$\mu \frac{dC}{d\mu} = \left[\gamma_{cusp} \ln\frac{Q^2}{\mu^2} + \gamma_v\right] C(Q^2, \mu^2)$$

$$C \sim \exp\left[\frac{1}{2}\ln^2\frac{Q^2}{\underline{\mu^2}}\, \gamma_{cusp}(\alpha_s) + \cdots\right]$$