The Standard Model Effective Field Theory
(SMEFT)

- why EFT? (why SMEFT?)
- examples of EFT
- SMEFT d6 · basis · RG-running · ~~d8 pos~~?
- phenomenology

useful refs.

Manohar's lectures on EFT 1804.05863 or TASI 2022

Skiba's TASI lecture notes (a bit old) 1006.2142

warsaw basis 1008.4884

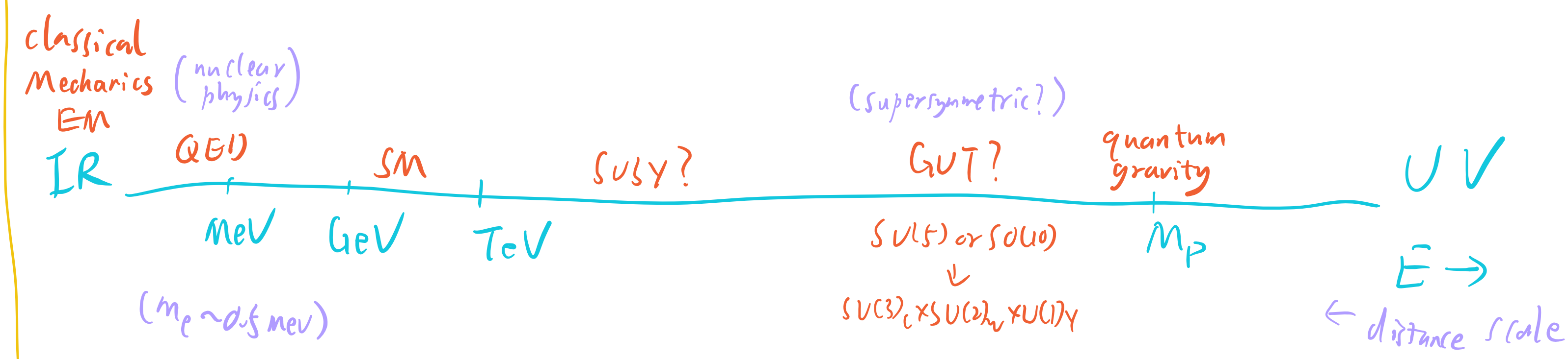
Higgs+EW SMEFT 1308.1879 (Barcelona)

SMEFTsim 3.0 Ilaria Brivio 2012.11343

Why EFT?

(We think) Every theory is an effective theory!

natural units: 1 unit $[E] \sim [L]^{-1}$



↳ A more fundamental theory will appear at a higher energy / smaller scale.

Stops somewhere? ↑

(Or maybe there is some ultimate theory?)

Quantum gravity ⇐ space time quantized?
notion of energy / distance breaks down?

Key ingredient: Locality (many definitions, here it means)

Measurements at large distance / low energy should not be sensitive to the physics at small distance / high energy.

Engineers don't need to learn QFT to build bridges / car!

Classical Mechanics is replaced by QFT / QM / Relativity at small scale / high energy does not mean it's wrong, it's an effective theory at large scale / low energy.

Why SMEFT ?

SM is incomplete (dark matter, gravity, matter anti-matter asymmetry ...)

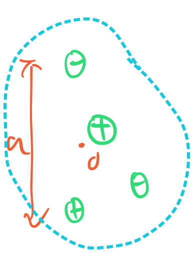
There must be BSM New physics but we don't know what it is. (some people think they know ....)

light particle	heavy particle ($M \gg v$)
very weak coupling	SMEFT ✓
~~SMEFT~~	

(since we don't know what it is)

- bottom-up approach: Be agnostic about the UV physics and try to systematically parameterize its effects at low energies by writing down higher dimension operators. (top-down: model building ....)
- useful even if we know the UV model

example 1 Multipole Expansion in Electrostatics



$a \sim$ scale of the charge distribution

$\vec{r}$. $V(\vec{r})$ electric potential

locality: For $r \gg a$, this looks like a point charge!

$$V(\vec{r}) = \frac{1}{r} \sum_{l,m} b_{lm} \frac{1}{r^l} \underbrace{Y_{lm}(\theta, \varphi)}_{\text{spherical harmonics}}$$

$$= \frac{1}{r} \sum_{l,m = 0,1,\ldots} c_{lm} \left(\frac{a}{r}\right)^l Y_{lm} \qquad b_{lm} \equiv c_{lm} a^l$$

c_{lm} s are dimensionless parameters (usually of order 1).

separation of scales ($\frac{a}{r} \ll 1$) $\Rightarrow$ The expansion is useful,

i.e. we only need to keep a few terms in the l expansion to get a good approximation (more terms $\Rightarrow$ better accuracy)

	distance	energy	
IR	r	$E \sim 1/r$	collider energy (or v)
UV	a	$\Lambda \sim 1/a$	scale of new physics
expansion parameter	$\frac{a}{r}$	$\frac{E}{\Lambda}$	(or $\frac{v}{\Lambda}$)

- There is no precise definition of a. One could only measure the combination $b_{lm} = c_{lm} a^l$. $\left(\frac{c_{lm}}{\Lambda^l}\right)$
- If we know the charge distribution (UV theory), we can do the expansion to find out all the b_{lm} (c_{lm}). This is called matching.

  If we don't know . . . . . . we can treat all c_{lm} s as free parameters and try to measure them experimentally.

- After truncating the series (throwing away terms with $l > l_{max}$) there are a finite number of parameters. If we make enough measurements we can constrain all parameters (C_{lm}).

- To precisely determine the values of C_{lm} we can either
  - make very precise measurement at large r (low energy)
  - make measurements at small r (high energy)

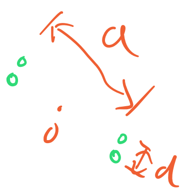
energy vs. precision (or both!)

Important aspects for colliders.

If $r \approx a$, the expansion breaks down!

High energy is always good, but EFT may not be valid!

multiple scales



$\vec{r}$.

$$r \gg a \gg d$$

$$(\Lambda_{EW} \ll \Lambda_{SUSY} \ll \Lambda_{GUT})$$

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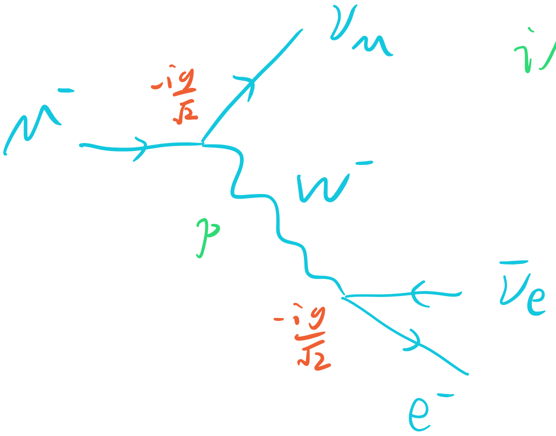
example 2 Fermi's theory

muon decay



$$i\mathcal{M} = \left(\frac{-ig}{\sqrt{2}}\right)^2 (\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma^\nu \nu_e) \cdot \frac{-ig_{\mu\nu}}{p^2 - M_W^2}$$

(ignore W width since $p^2 \ll M_W^2$)

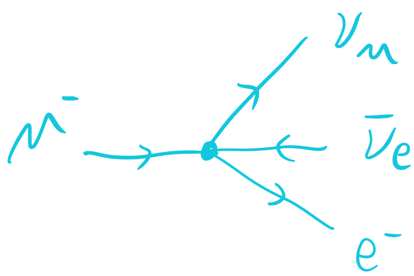
For $p^2 \ll M_W^2$, we can expand the operator

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$$

Keeping only the 1st term we have

$$i\mathcal{M} = \frac{-ig^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e) + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

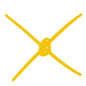
4-fermion contact interaction



which can be produced by the local Lagrangian

$$\mathcal{L} = -\frac{g^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e) + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

dimension-6 operator, what Fermi wrote down.

EFT:  $\sim \frac{E^2}{\Lambda^2}$ $(\Lambda \sim M_W)$, breaks down at large E!

If we keep more terms in the Lagrangian we'll generate higher dimensional operators, e.g. the $\frac{1}{M_W^4}$ term corresponds to dim-8 operators.

(Higher dimensional operators may look very complicated, it's actually much easier to use on-shell amplitudes!)

(amplitude)

This is the simplest example of the matching between the full model (SM) and the low-energy effective field theory (Fermi's theory).

- For $p^2 \ll M_W^2$, the 4F operator gives a very good approximation of the full theory. This is the case for muon decay. ($p^2 < m_\mu^2$ $\quad \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$)

- The coefficient of the 4F operator is $-\frac{g^2}{2M_W^2} \sim \frac{1}{v^2}$.
Measuring muon decay only tells us the value of v (or $G_F \equiv \frac{1}{\sqrt{2}v^2}$) but not M_W, which depends on g.

- M_W is the scale at which the EFT breaks down!

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$$

breaks down at $p^2 \sim M_W^2$!

In our world, $g \simeq 0.65$.

If g is $\begin{cases} \text{very small}, & W, Z \text{ would be much lighter} \\ \text{very large}, & \ldots \quad \ldots \quad \ldots \text{ heavier.} \end{cases}$

(but if $g \gtrsim 4\pi$, the theory becomes non-perturbative)

- In this simple example, if we also measure the dim-8 coefficient ($\sim \frac{g^2}{m_W^4}$) we can derive the W mass. In more complicated cases (with multiple heavy particles) it is in general not possible.

- global tran ....

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example 3 (bottom up) a complex scalar EFT (with U(1) symmetry)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \qquad [O] \leq 4$$

$$+ \sum_i \frac{C_i^{(5)}}{\Lambda} O_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \cdots\cdots$$

(the coefficients $\frac{C_i^{(n)}}{\Lambda^{n-4}}$) has dimension $4-n$

$[\mathcal{L}] = 4, \quad [O^{(n)}] = n, \quad [C] = 0, \quad [\Lambda] = 1$

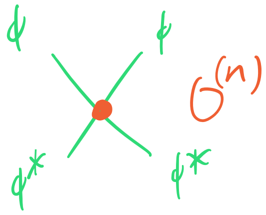
$\Lambda \sim$ scale of the new physics (mass of the new particles)

Large $\Lambda \Rightarrow$ The EFT expansion is good.

Operators in $\mathcal{L}$ are divided into 3 classes based on their mass dimensions

		coupling dimension	
$[O] < 4$	relevant	> 0	$\lvert\phi\rvert^2$
$[O] = 4$	marginal	$= 0$	$\lvert\phi\rvert^4$
$[O] > 4$	irrelevant	< 0	

irrelevant → Small effects at low energy



$$\mathcal{M} \sim \left(\frac{E}{\Lambda}\right)^{n-4}$$

Small effects at low energy

EFT breaks down at high energy! With $[O] > 4$, the theory cannot be a complete theory ⇒ why it's called an EFT.

Now let's try to write down the higher dim. operators ...

Each $O_i^{(n)}$ need to be invariant under Lorentz / U(1) symmetry.

No odd dimension operator!

$\phi^* \phi \phi^* \partial_\mu \phi$ not Lorentz invariant

$\lvert\phi\rvert^4 \phi$ not U(1) invariant

Large Λ $\Rightarrow$ leading contribution: $O_i^{(6)}$!

Bottom-up approach: write down all possible $O_i^{(6)}$!

Not all of them are independent!

Operator redundancy

Operators are related by

- Integration by parts (IBP)
- Equation of Motion (EOM) ← why EOM works beyond the classical level
- Fierz identity ....

(more generally, field redefinition)

Let's try to write down all possible d6 operators:

$|\phi|^6$

$(\partial^\mu \phi^* \partial_\mu \phi)\, \phi^* \phi$, $(\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi$, $c\left((\partial^\mu \phi^*)\phi\right)^2$ + h.c., ...

$(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)$, ...

only 1 indep operator

IBP:

$$(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi) = \partial^\mu[(\partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)] - \partial_\mu \phi^* \partial^\mu \partial^\nu \partial_\nu \phi$$

total derivative

......

EOM:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0 \quad (\phi \leftrightarrow \phi^*)$$

$$-m^2 \phi^* - \frac{\lambda}{2} \phi^* \phi^* \phi - \partial_\mu \partial^\mu \phi^* + \underbrace{\cdots\cdots}_{d \geq 5} = 0$$

$$\partial_\mu \partial^\mu \phi^* = -m^2 \phi^* - \frac{\lambda}{2} \phi^* \phi^* \phi + \underbrace{\cdots\cdots}_{d \geq 5}$$

$$\partial_\mu \partial^\mu \phi = -m^2 \phi - \frac{\lambda}{2} \phi \phi \phi^* + \overbrace{\cdots\cdots}^{d \geq 5}$$

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things get more complicated if we go beyond d6!

$$\frac{c}{\Lambda^2}(\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi$$

$$= -\frac{c}{\Lambda^2} m^2 |\phi|^4 - \frac{c}{\Lambda^2}\frac{\lambda}{2} |\phi|^6 + \overbrace{\cdots\cdots}^{d \geq 8\text{, ignore}}$$

We can eliminate $(\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi$ in favor of $|\phi|^6$

$(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)$

total derivative

$$\partial^\mu\left((\partial_\mu\phi^*)\phi\,\phi^*\phi\right)$$

$$= (\partial^\mu\partial_\mu\phi^*)\phi\,\phi^*\phi + 2(\partial_\mu\phi^*\partial^\mu\phi)\phi^*\phi$$

$$+ \left((\partial_\mu\phi^*)\phi\right)^2$$

~~$+\left(\partial_\mu\phi^*)\phi\,\phi^*(\partial^\mu\phi\right)$~~

can eliminate

2 independent d6 operators!

We can choose $|\phi|^6$, $|\phi|^2\partial_\mu\phi^*\partial^\mu\phi$.

This is called choosing a basis. (choose which redundant operators to eliminate)

Of course we can choose a different basis.

Why not just keep redundant operators? < convenience, global fit

☆ Physics are basis-independent.

Physicists are basis-dependent!

HEFT workshop
吵架……

- rest of ϕ theory
- 讲义 share?
- SMEFT
- phenomenology?

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4$$

$$+ \frac{c_1}{\Lambda^2} |\phi|^6 + \frac{c_2}{\Lambda^2} |\phi|^2 \partial_\mu \phi^* \partial^\mu \phi + \cdots$$

↑ choose α_1, α_2 to cancel

Field redefinition

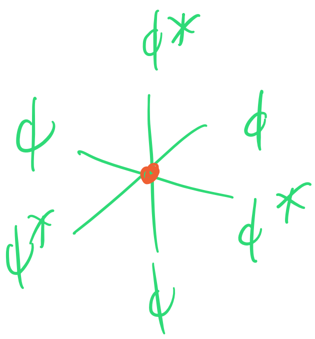
$$\phi \to \phi + \alpha_1 \frac{\phi^* \phi \phi}{\Lambda^2} + \alpha_2 \frac{\partial_\mu \partial^\mu \phi}{\Lambda^2}$$

does not change kinetic, mass terms

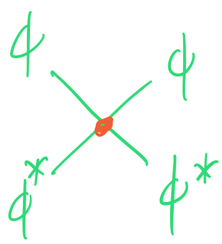
complex?

indep. operators ⟺ (massless) on-shell amplitude

$$[A] = 4 - n$$



$\sim \frac{c_1}{\Lambda^2}$



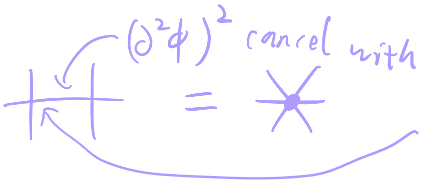
$\sim \frac{c_2}{\Lambda^2}(s+u)$

general:

$$\alpha s + \beta t + \gamma u \quad (s+t+u=0)$$

(with βt crossed out)

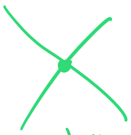
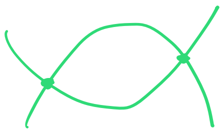
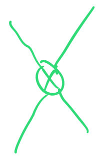
$s \Longleftrightarrow u$ symmetric $\Rightarrow \alpha = \gamma$



$$\frac{1}{p^2\left(1+\frac{c\phi^2}{\Lambda^2}\right)} = \frac{1}{p^2}\left(1 - \frac{c\phi^2}{\Lambda^2} + \cdots\right)$$

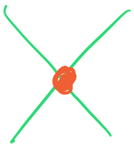
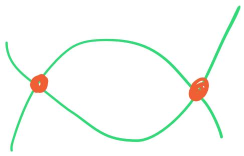
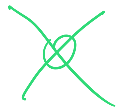
What about renormalizability?

Operators with $d > 4$ are non-renormalizable?

 $\sim \lambda$  $\sim \lambda^2$  counter term $\sim |\phi|^4$

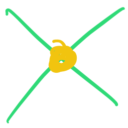
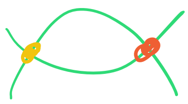
$|\phi|^4$

no need to add extra terms in $\mathcal{L}$ $\Rightarrow$ renormalizable

 $\sim \frac{C_{(6)}}{\Lambda^2}$  $\sim \frac{C_{(6)}^2}{\Lambda^4}$  counter term d8 $|\partial_\mu \phi|^4$

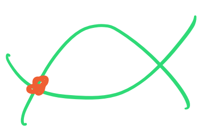
$|\phi|^2 (\partial_\mu \phi^* \partial^\mu \phi)$

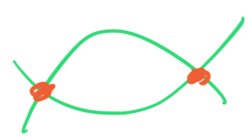
need to add ∞ many terms in $\mathcal{L}$ $\Rightarrow$ non-re..!

need to add  $\sim \frac{C_{(8)}}{\Lambda^4}$ $\Rightarrow$  $\sim \frac{C_{(6)} C_{(8)}}{\Lambda^6}$

$\Rightarrow$ need d10 counter term! ......

Correct argument: We only work up to a fixed order in the EFT expansion and discard all higher order terms

d6: keep  $\sim \frac{1}{\Lambda^2}$ counter term also d6

discard  $\sim \frac{1}{\Lambda^4}$

SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{(7)} + \dots$$

$[\mathcal{L}] = 4, \quad [\mathcal{O}^{(n)}] = n, \quad [\Lambda] = 1$

Note: each $\mathcal{O}_i$ needs to be invariant under Lorentz and gauge transformations $SU(3) \times SU(2) \times U(1)$

HEFT: only $SU(3) \times U(1)$. (linear vs. non-linear ...)

bottom up approach: write down all possible operators!
(not all operators are independent ....)

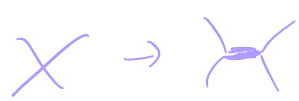
dim 5: only 1 type of operators $\sim LLHH$ (Weinberg operator)

HW: write down the exact form of the Weinberg operator

neutrino majorana mass

$$\mathcal{L} \sim \frac{c}{\Lambda} LLHH \to c \frac{v^2}{\Lambda} \nu\nu \qquad \left. \begin{matrix} c \sim 1 \\ \Lambda \sim \Lambda_{GUT} \end{matrix} \right\} \Rightarrow m_\nu \sim 10^{-2} eV$$

Seesaw mechanism: large $\Lambda \Rightarrow$ naturally explains why m_ν is small!

several possible UV completions: type I, II, III, ... 

Furthermore, all odd dimension operators violate Baryon (B) or Lepton (L) numbers.

B, L, charge of $U(1)_B$, $U(1)_L$ global symmetry

	B	L
q	$\frac{1}{3}$	0
$\bar{q}$	$-\frac{1}{3}$	0
l	0	1
$\bar{l}$	0	-1

$\not{B}$ $\not{L}$ effects are usually strongly constrained (e.g. proton decay).

Assuming B, L are conserved around the TeV scale

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(8)}}{\Lambda^2} O_i^{(8)} + \cdots$$

How many independent parameters do we have?

	1 generation		3 generations	
dim-6	76	1008.4884	2499	Manohar et al. 1312.2014
dim-8	895		36971	2005.00008 理论所组 2005.00059 Murphy

Hilbert series: Murayama et al. 1512.03433

Warsaw basis 1008.4884

- first to write down a complete d6 basis
- try to eliminate operators with more derivatives in favor of operators with more fields.

Buchmüller & Wyler almost did it in 1986.... (why no one completed it in 24 years?)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi \widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi \widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi \widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

4+3+3

8 × 3

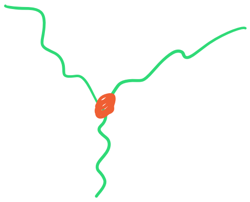
briefly explain each type of operators ...

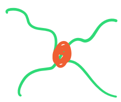
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$
$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating			
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$				

5+7+8

5

= 59 operators

1)  (transverse) anomalous triple gauge coupling aTGC

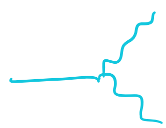
 & (quartic GC) aQGC

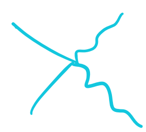
2) $\varphi \leftrightarrow H$ $|H|^6 \rightarrow$  h^3 modifies h^3 & h^4 couplings

$(\partial |H|^2)^2$ modify $\partial_\mu h \partial^\mu h$, h wave function renormalization

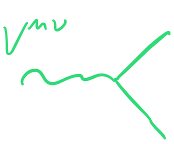
shift Higgs couplings

3) $\psi^2 \varphi^3 \rightarrow$ modify Yukawa coupling (relation between m & y) ~~$y = \frac{v}{\sqrt{2}}$~~

4) $|H|^2 V_{\mu\nu} V^{\mu\nu}$ < $h V_{\mu\nu} V^{\mu\nu}$ 

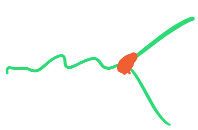
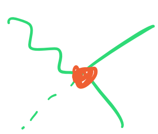
$hh V_{\mu\nu} V^{\mu\nu}$ 

different from $h Z^\mu Z_\mu$ $h W^\mu W_\mu$

5) $H \to V$ dipole 

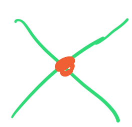
real magnetic

imaginary electric

6) $H \to V$  $\Longleftrightarrow$ 

modifies SM $Vf\bar{f}$ coupling contact interaction

7) 4f interaction



1 generation:

59 operators
- 17 non hermitian ⇒ complex coefficient
- 42 hermitian ⇒ real coefficient

$42 + 17 \times 2 = 76$ parameters

3 generations: 2499 parameters!

(many of them are 4f operators)

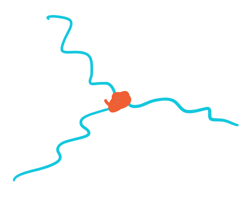
In other bases, we sometimes keep operators with more derivatives.

e.g. $O_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2 \quad O_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$

useful in describing universal contributions to 4f interactions.

$O_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$

$O_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$



useful for describing (longitudinal) anomalous triple gauge couplings (aTGCs)

RG running

1308.2627
1310.4838 Alonso, Jenkins, Manohar, Trott
1312.2014

running couplings $\Longleftrightarrow$ resum large logs

tree level calculation + running couplings give a reasonably accurate prediction.

suppose vev $\to 0$, $[g_{SM}] = 0$

let $C_i^{(6)} = \frac{c_i^{(6)}}{\Lambda^2}$ $\quad [C_i] = -2$

d6 RGE $\beta_{C_i} \equiv \mu \frac{d}{d\mu} C_i = \gamma_{ij} C_j$ ← still holds!

↑ anomalous dimension matrix depends on g_{SM}

This is the only term allowed by dimensional analysis!

How about vev $\neq 0$?

C_i contribute to $\beta_{g_{SM}} = \mu \frac{d}{d\mu} g_{SM} = \cdots + \gamma_i v^2 C_i g_{SM} + \cdots$ (with v^2 of dimension 2 and C_i of dimension −2)

C_i^2 $C_i^{(8)}$ contribute to β_{C_i}

d6^2 d8 ← can ignore since they are higher order

(d8 RGEs are more complicated.)

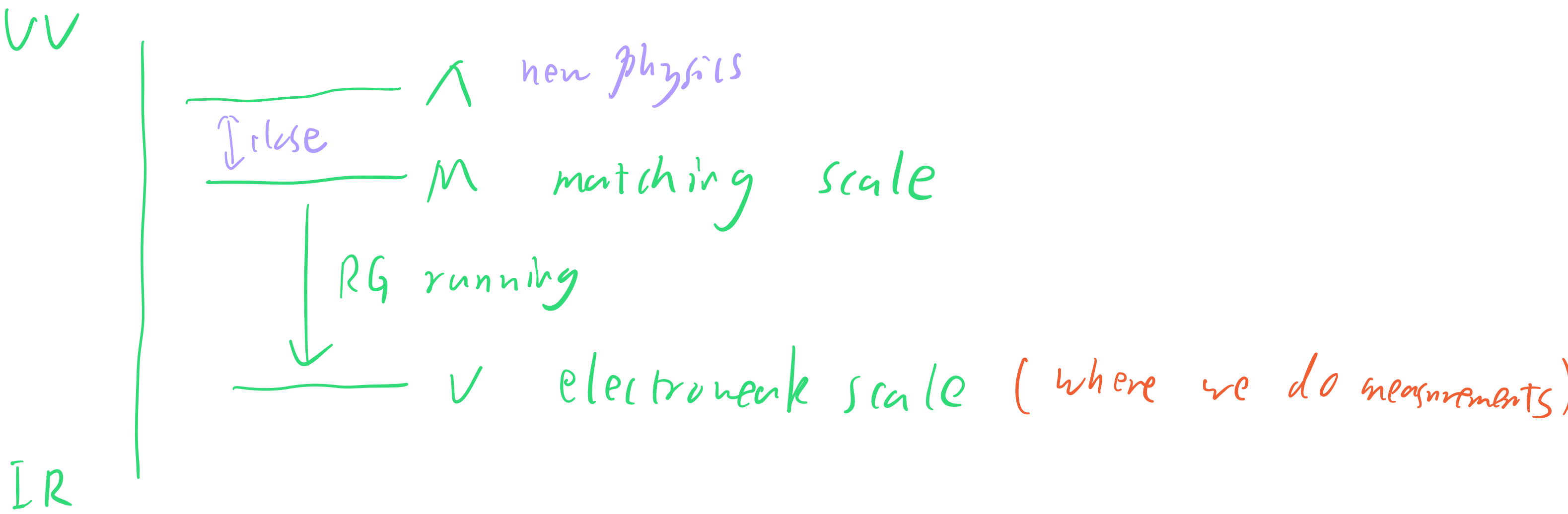
Solve RGE. If expand to 1st loop order (not resumed)

$$C_i(\mu) = C_i(\mu_0) + \log\frac{\mu_1}{\mu_0}\,\gamma_{ij}\,C_j$$

note: important to resum! generally solve numerically.

RGE implies:
- running
- mixing (off-diagonal terms in γ_{ij})

Matching & Running



mixing: Operators not generated at matching scale can be generated at a lower scale via running!

running effects can be significant if $v \ll \Lambda$!!

See examples in Manohar / Skiba 's lecture notes,

Schwartz 31.3

RGE ⟺ on-shell amplitude   see e.g. 1505.01844 Cheung & Shen

Phenomenology

$\mathcal{L}_{SMEFT} \longrightarrow \mathcal{M} \longrightarrow \frac{d\sigma}{d\Omega}, \frac{dP}{d\Omega}$

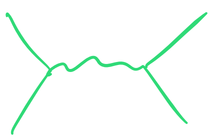
(amplitude: $\mathcal{M}$; observable: $\frac{d\sigma}{d\Omega}, \frac{dP}{d\Omega}$)

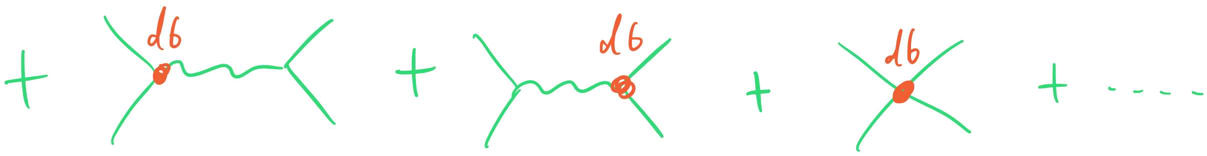
expand in terms of $\frac{1}{\Lambda}$

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_6 + \mathcal{L}_8 + \cdots$$

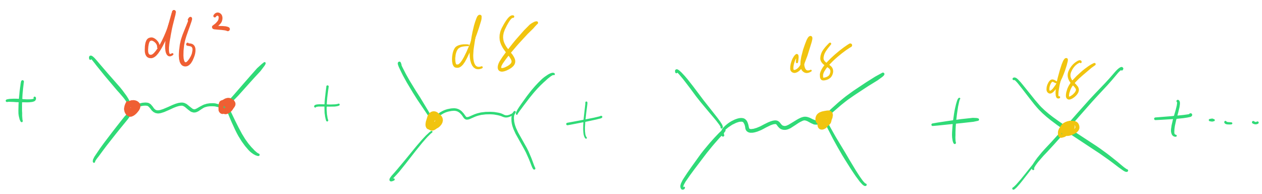
$\mathcal{M}$ $q\bar{q} \to l^+ l^-$

SM





$\frac{1}{\Lambda^2}$



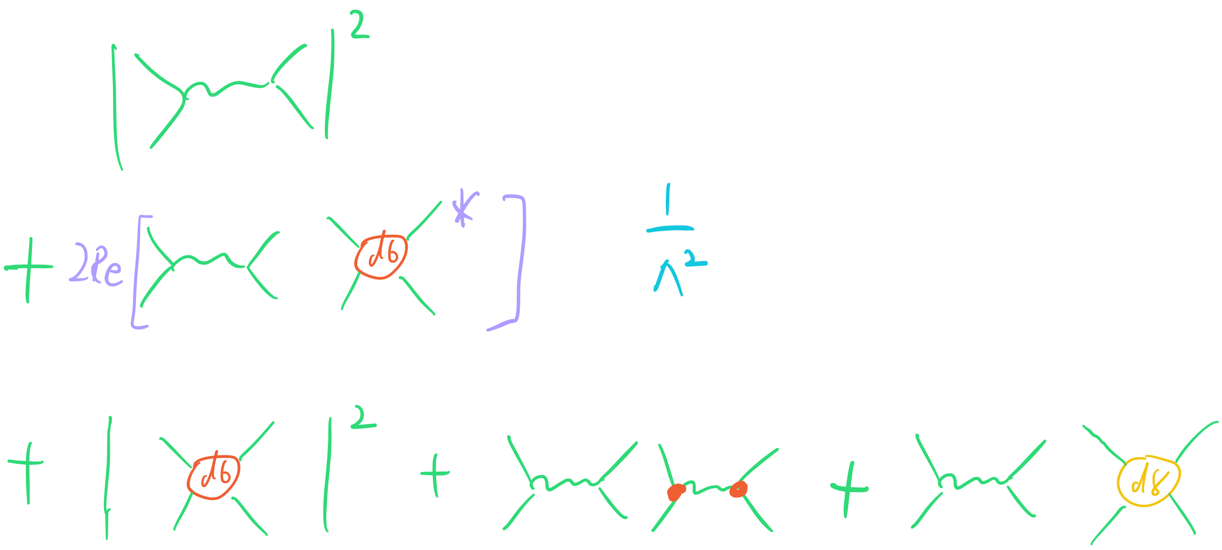
$\frac{1}{\Lambda^4}$

+ .....

($d6^2$ & $d8$ are formally indistinguishable)

Higher dimensional operators can contribute to m, Γ, which appears in dinominators $\frac{1}{p^2-m^2-im\Gamma}$. we don't consider it here (or just expand?)

$\sigma \sim |M|^2$



$\frac{1}{\Lambda^2}$

$\frac{1}{\Lambda^4}$

$+ \cdots$

We can truncate σ at $\frac{1}{\Lambda^2}$ is a very good approximation if $E \ll \Lambda$, $v \ll \Lambda$ (E: collider energy)

$(1+x)^2 \simeq 1+2x$ if x is very small!

$\frac{1}{\Lambda^4}$ strictly speaking, need to calculate d8

~~what if Λ is not that large, shall we keep $d6^2$?~~

Typically:

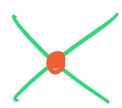
If measurement is very precise $\Rightarrow$ can constrain $\Lambda \gg E$ ✓

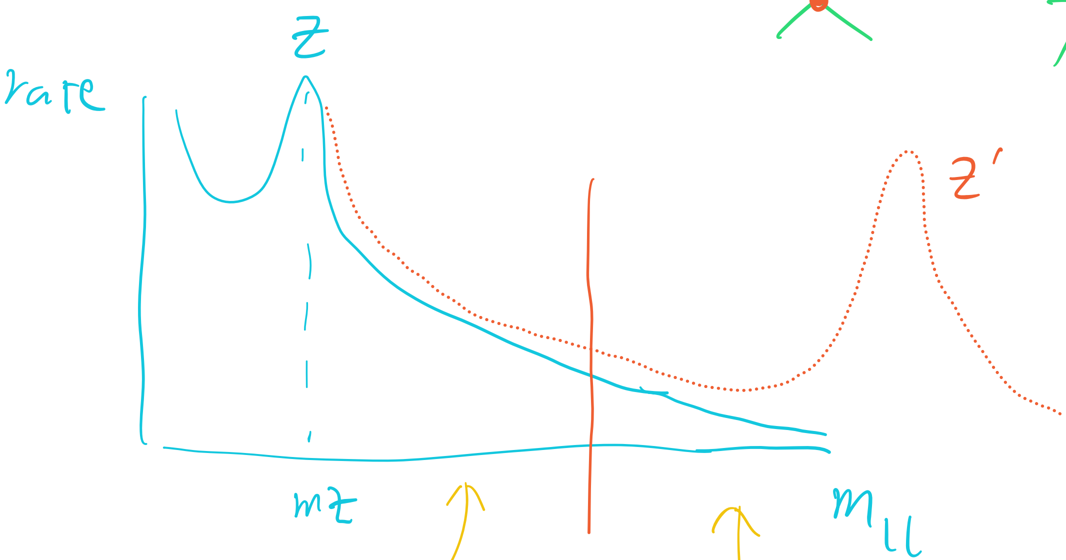
$\Rightarrow$ ok to just keep $\frac{1}{\Lambda^2}$ (ideal case!)

What if it's not the case?

typical LHC measurement

Drell-Yan $pp \to l^+ l^-$

 $\sim \frac{E^2}{\Lambda^2}$



small E, large statistics

cut here and throw away $\Rightarrow$? EFT valid?

large E, small statistics

(or do something else)

easier to constrain $\Lambda \gg E$

ok to truncate at $\frac{1}{\Lambda^2}$

(Lepton colliders usually don't have this problem.)

Global fit (simplest case)

$\sigma \to X$ for cross section

we'll now use σ to denote the standard deviation!

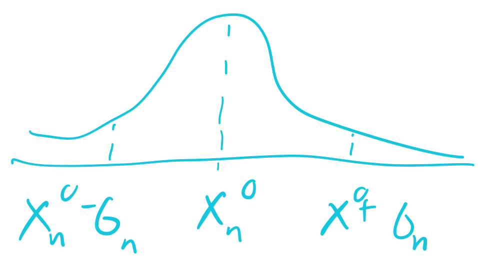
suppose we measure a set of X_n s,

each has a Gaussian error (Posion large N limit)

when we write

$$X_n = X_n^0 \pm \sigma_n$$

we mean



σ_n 1 standard deviation
1 sigma

Gaussian error $\Longrightarrow$ log likelihood $\propto \chi^2$

theory prediction

$$\chi^2 = \sum_n \frac{(X_n - X_n^0)^2}{\sigma_n^2}$$

$$C_i^{(6)} \equiv \frac{c_i^{(6)}}{\Lambda^2}$$

truncate at $\frac{1}{\Lambda^2}$ $\Rightarrow$ $X_n = X_n^{sm} + a_{ni}$ ~~$\frac{c_i^{(6)}}{\Lambda^2}$~~ $C_i^{(6)}$

calculated by hand or MC simulation

minimize χ^2 $\Leftrightarrow$ maximize liklihood $\to$ gives best-fit values for $C_i^{(6)}$

$$\Rightarrow \quad \chi^2 = \sum_{ij} (C_i - C_i^0)\,[\sigma^{-2}]_{ij}\,(C_j - C_j^0) + \chi^2_{min}$$

$$\sigma^{-2} \equiv (\delta\vec{C}\,\rho\,\delta\vec{C})^{-1}$$ inverse covariance matrix

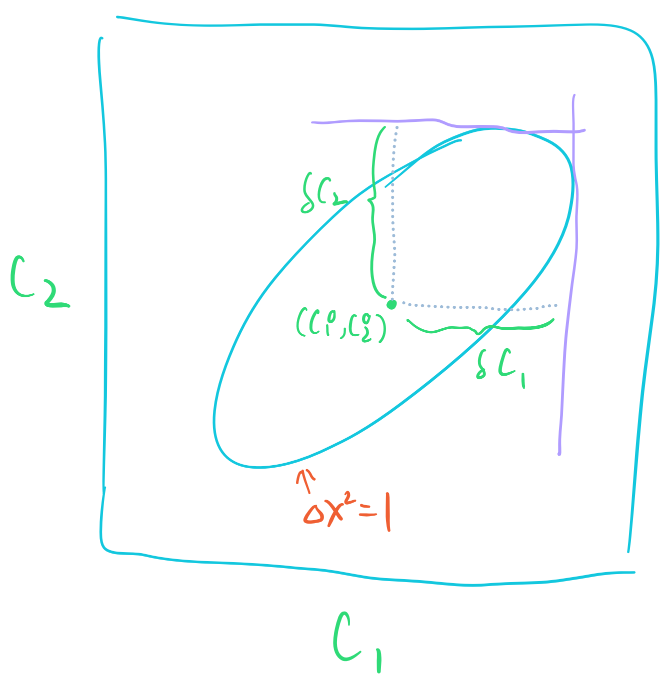
C_i^0 : best-fit values

δC_i : one-sigma precision, ρ_{ij} : correlation matrix

results of the Global fit

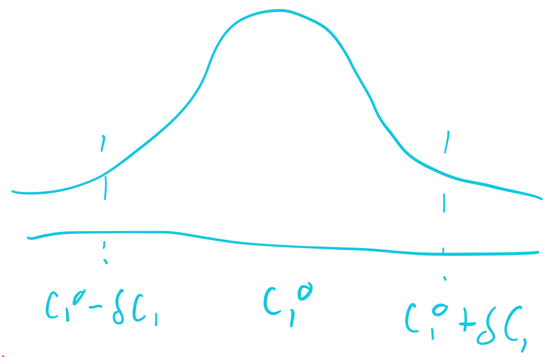
$\Delta\chi^2$ gives a measure of the "goodness of fit".

fixed $\Delta\chi^2$ ⟺ qudratic equations of C_i ⟺ ellipses!



Imagine this in n-dimensional parameter space....

project to 1D



Beyond $\frac{1}{\Lambda^2}$ ⇒ not Gaussian anymore!