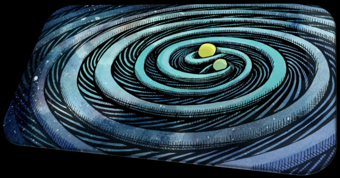
*That would be one of the most fascinating things man could do, because it would tell you very much how the universe started.*

*— Rainer Weiss*



北京大學
PEKING UNIVERSITY

# Gravitational Waves and Gravity Tests

Kavli Institute for Astronomy and Astrophysics

**Lijing Shao (邵立晶)**

暑期学校·济南

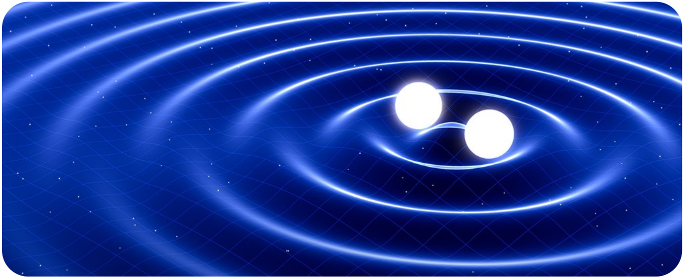
# Plan of Lectures

- **I** **Overview**
  - Lecture duration $\sim$ 45 min
- **II** **What are GWs?**
  - Lecture duration $\sim$ 45 min
- **III** **Gravity Tests with GWs**
  - Lecture duration $\sim$ 90 min

**Contact:** lshao@pku.edu.cn

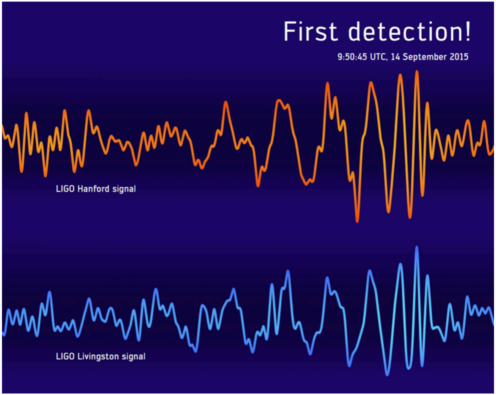
# References


- M. Bailes, *et al.*, Nature Rev. Phys. **3** (2021) 344 [DOI]
- A. Buonanno, Les Houches Lecture Notes (2006) [arXiv:0709.4682]
- B. S. Sathyaprakash & B. F. Schutz, Living Rev. Rel. **12** (2009) 2 [arXiv:0903.0338]
- E. Barausse, COST Action Summer School (2022) [arXiv:2303.11713]

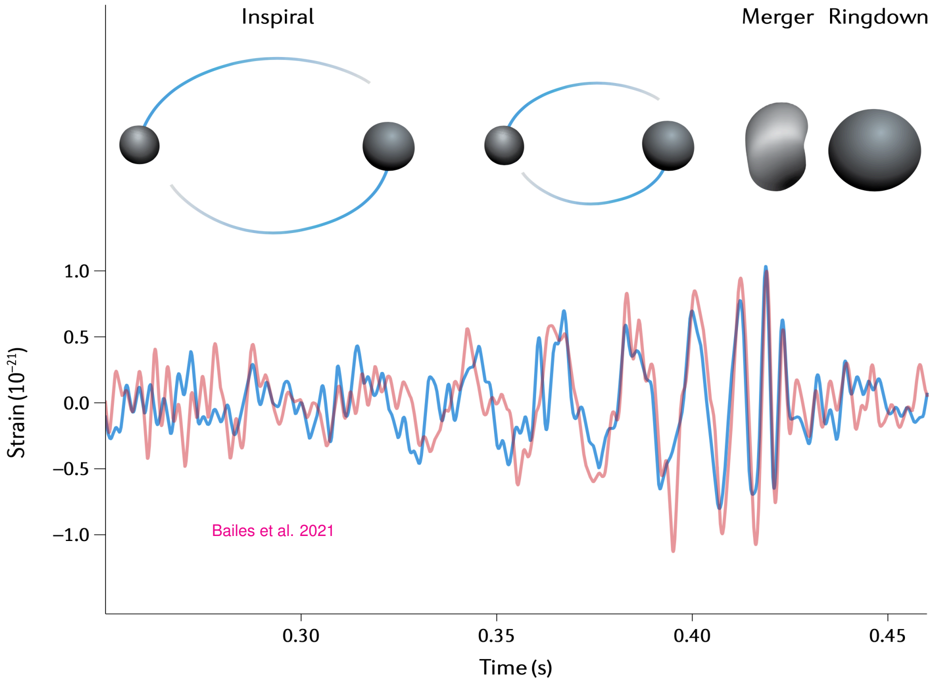



# GW150914: Binary Black Hole

- **September 14, 2015**: Advanced Laser Interferometer Gravitational-Wave Observatory (AdvLIGO)




LIGO/Virgo 2016 [1602.03837]


Inspiral
Merger
Ringdown
1.0
0.5
0.0
−0.5
−1.0
Strain ($10^{-21}$)
Bailes et al. 2021
0.30
0.35
0.40
0.45
Time (s)

# GW170817: Binary Neutron Star

- **August 17, 2017**: Advanced LIGO & Advanced Virgo

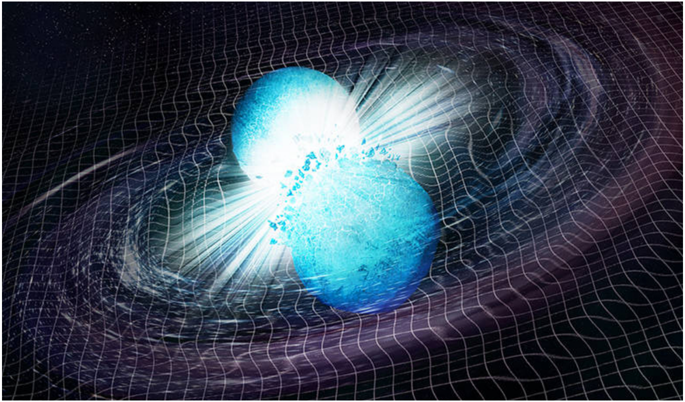


Figure Credit: M. Weiss

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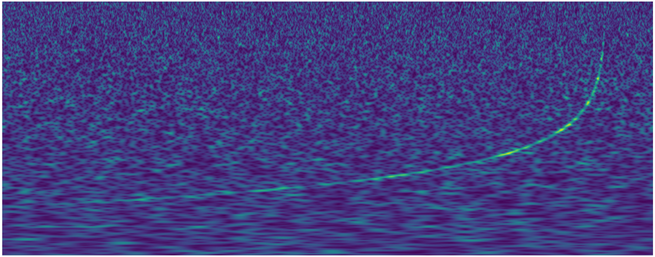
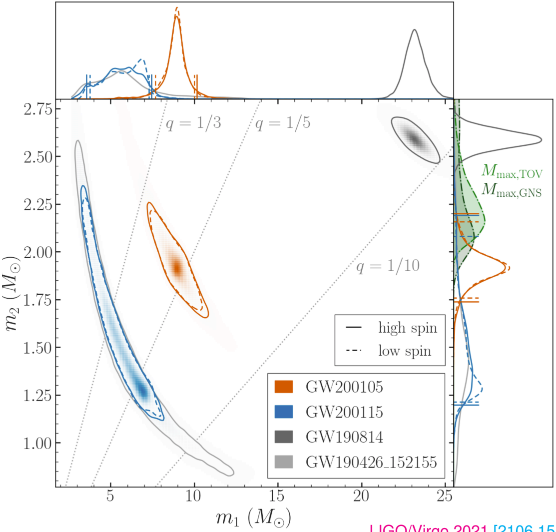


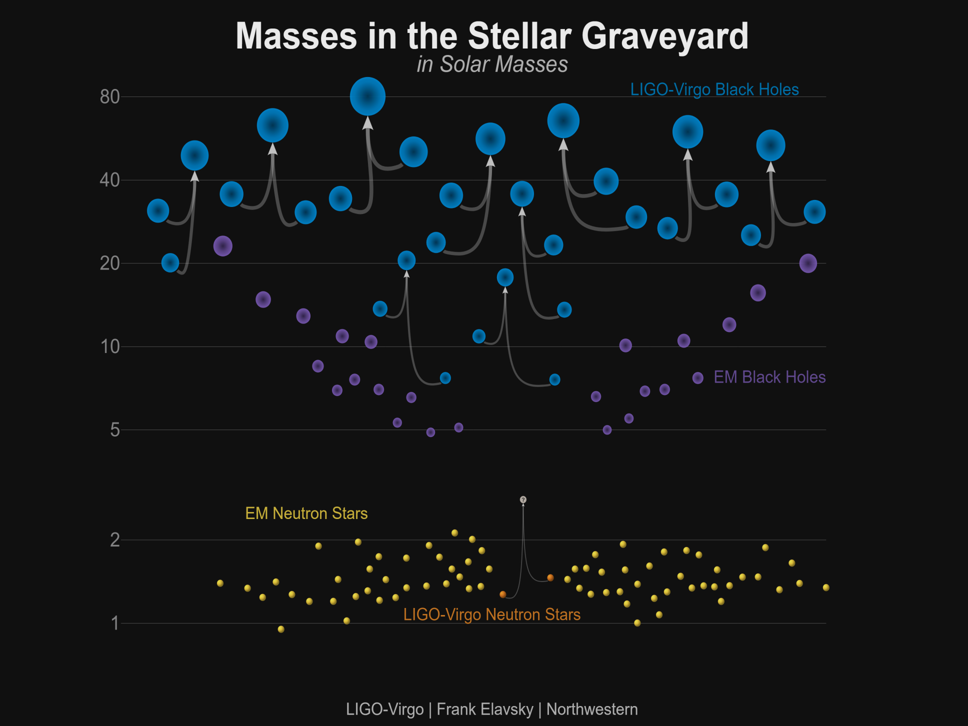
How do data tell stories?

LIGO/Virgo 2017 [1710.05832]

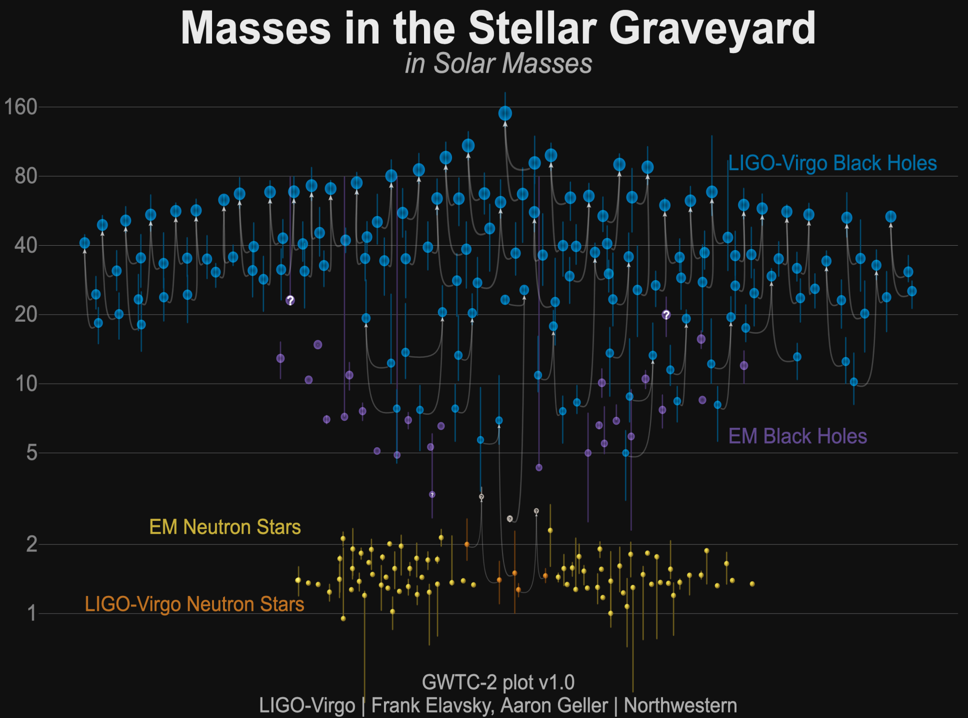
# GW200105 & GW200115: BH-NS Binaries

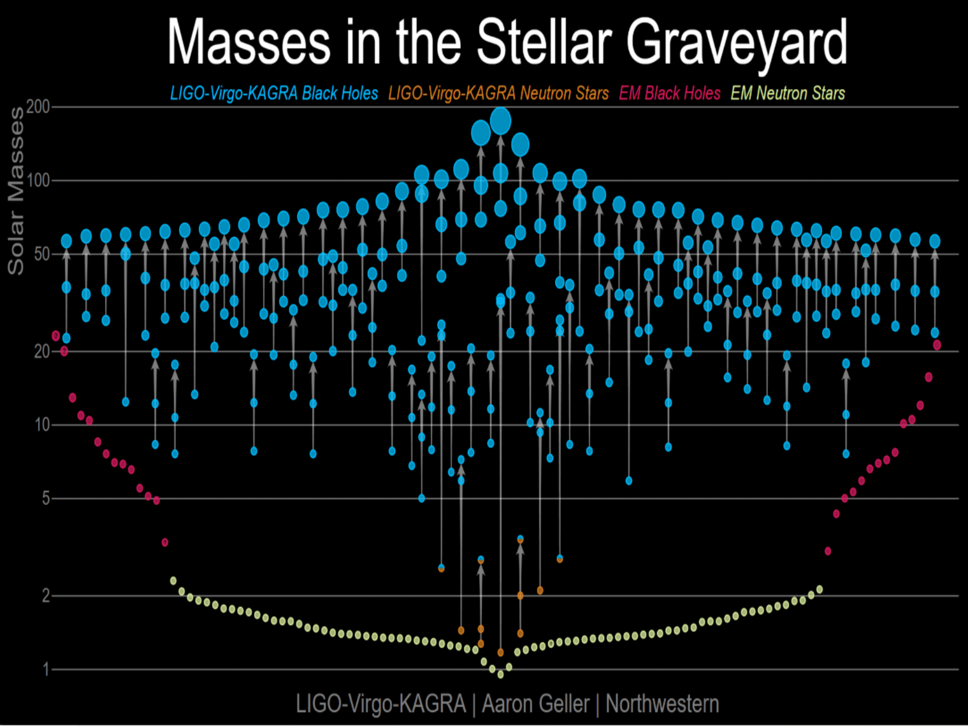



LIGO/Virgo 2021 [2106.15163]



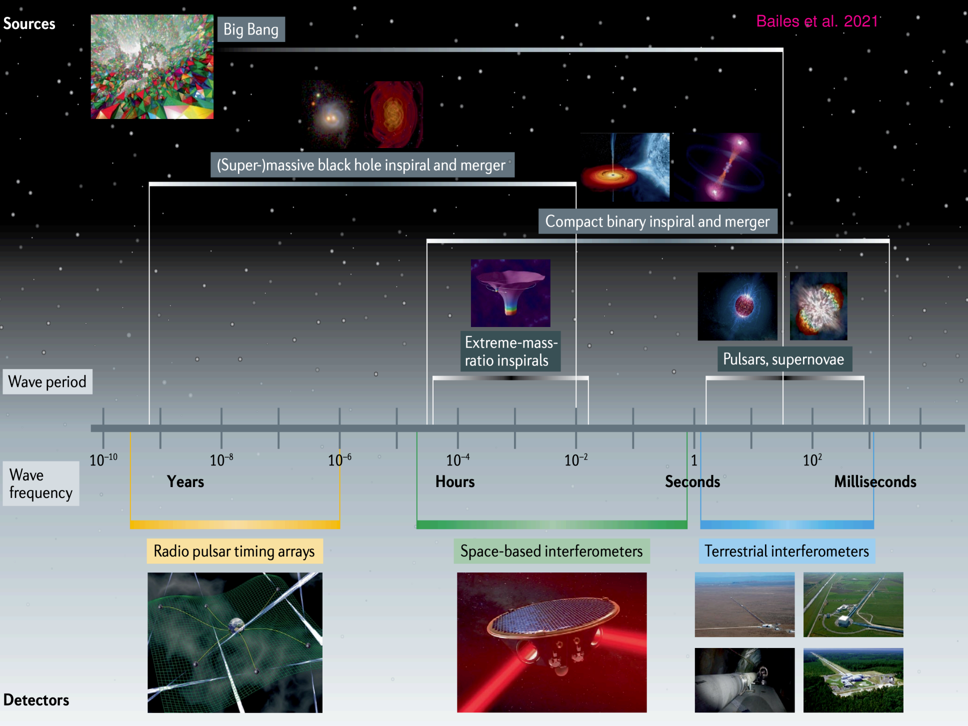
Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
?
LIGO-Virgo | Frank Elavsky | Northwestern


Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern



Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

# What is the next?



Sources
Big Bang
Bailes et al. 2021
(Super-)massive black hole inspiral and merger
Compact binary inspiral and merger
Extreme-mass-ratio inspirals
Pulsars, supernovae
Wave period
$10^{-10}$
$10^{-8}$
$10^{-6}$
$10^{-4}$
$10^{-2}$
1
$10^{2}$
Wave frequency
Years
Hours
Seconds
Milliseconds
Radio pulsar timing arrays
Space-based interferometers
Terrestrial interferometers
Detectors

# GW Roadmap in the 2020s and 2030s

*Past*: BH & NS binaries by LIGO/Virgo

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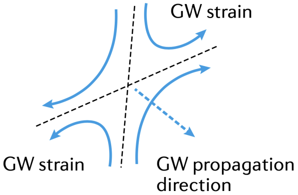
- **PTAs (nHz to $\mu$Hz)** $\Leftarrow$ past mergers of SMBHBs

# Michelson Interferometer




- Quadrupolar $h \sim \delta L/L$
  - "$+$" and "$\times$" modes

Bailes et al. 2021

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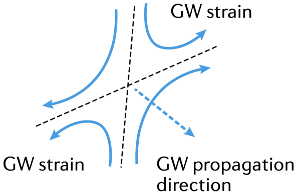
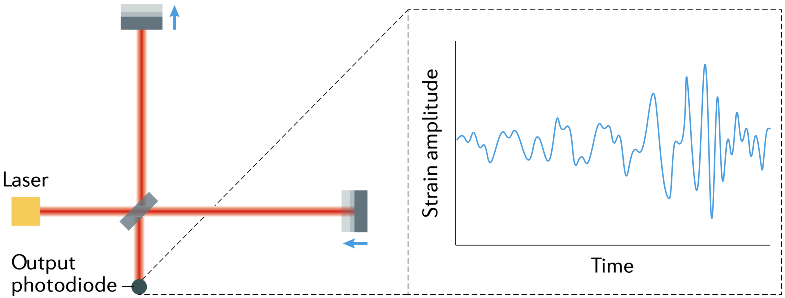



Bailes et al. 2021

# Optics

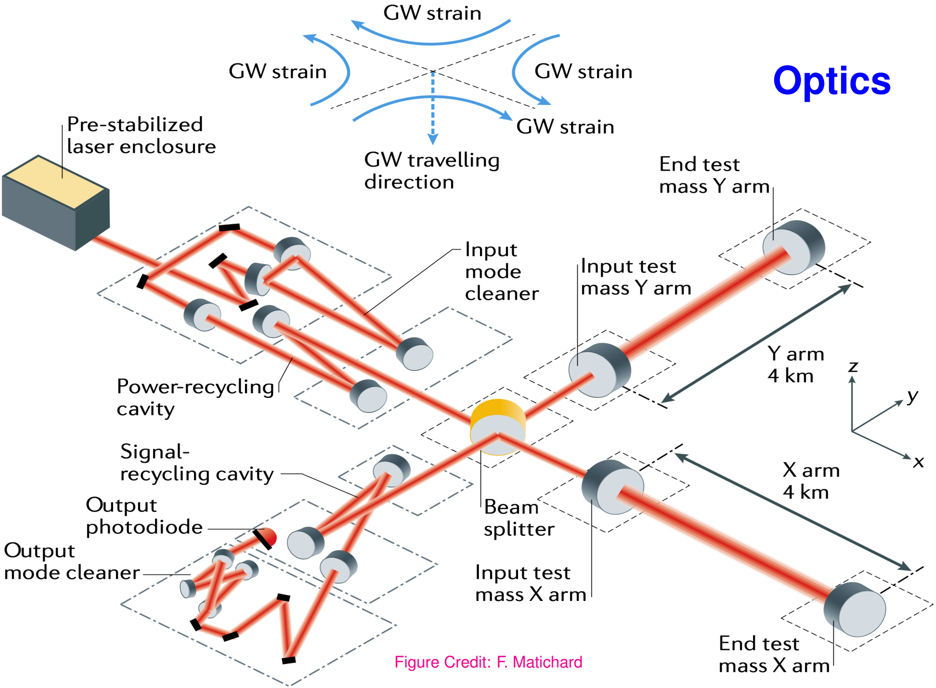
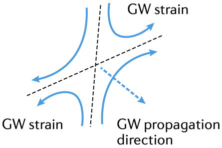
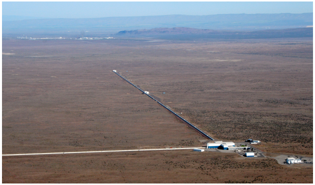



Figure Credit: F. Matichard

# Ground-based Detectors

- **Frequency**: 10 Hz to 10 kHz



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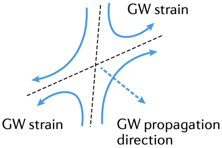


- **Frequency**: 10 Hz to 10 kHz
- **Sources**: $h \sim 10^{-21}$ and $\delta L \sim 10^{-18}$ m
  - Stellar-mass compact sources: BHs and NSs
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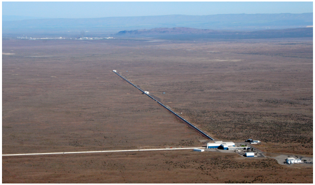
  - Isolated NSs



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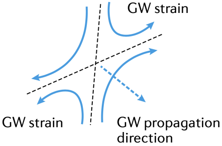


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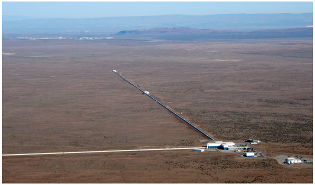
- **Detectors**: can be effectively treated as in **free fall** (i.e. local inertial frame) in the direction of light propagation **(Why?)**



# Ground-based Detectors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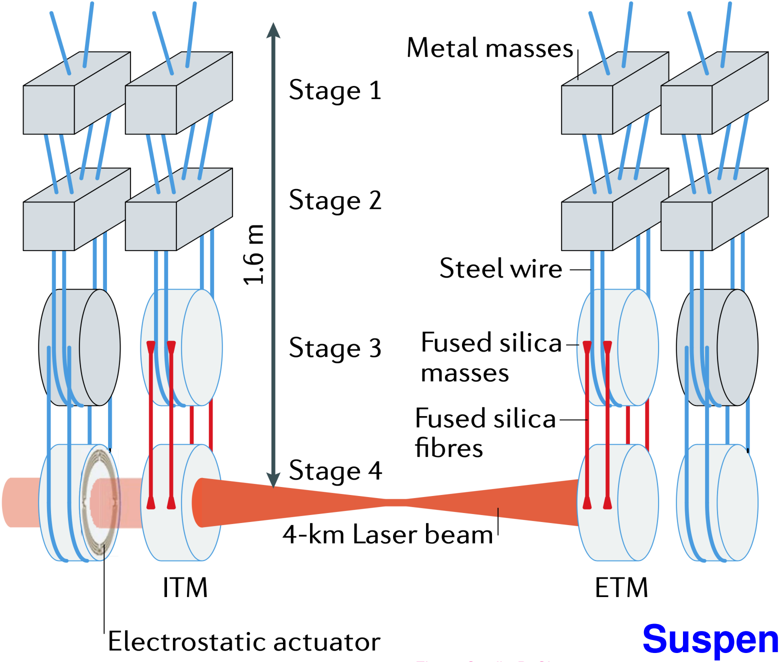
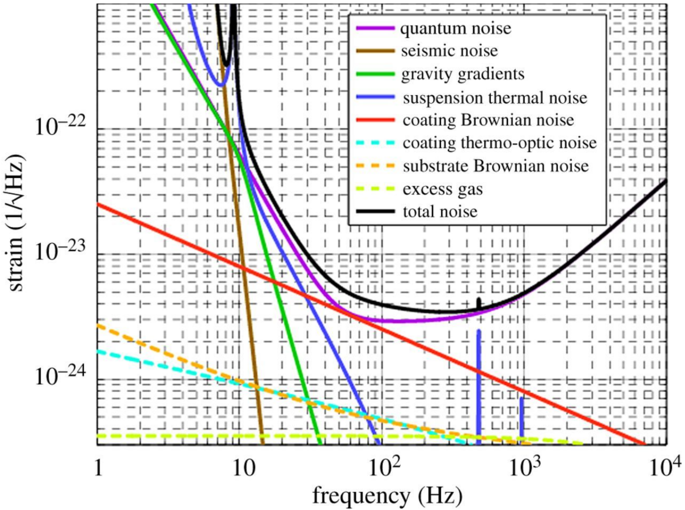
  - **Others**: laser frequency and intensity noises, acoustically and seismically driven scattered light noises, sensor and actuator noises, stochastic forces from electrical and magnetic fields, energy deposited by energetic particles, etc.




## Suspension

Figure Credit: D. Sigg

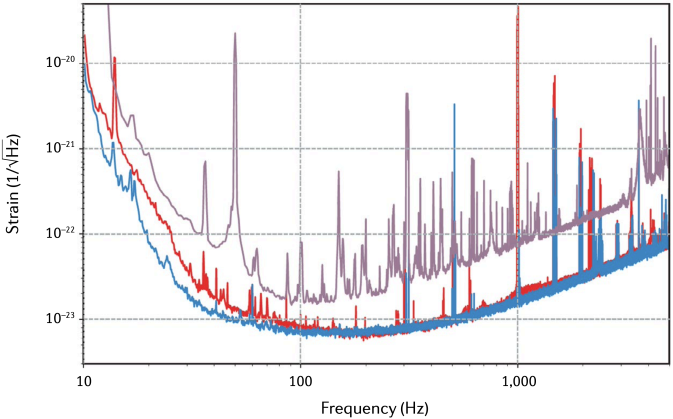
# Ground-based Detectors




van Veggel, Phil. Trans. Roy. Soc. Lond. A 376 (2018) 20170281

# Ground-based Detectors

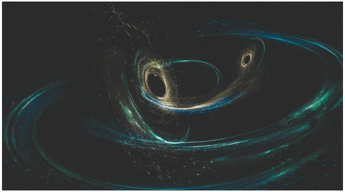
- **O2 Noise Curve**




LIGO/Virgo 2019 [1811.12907]

# Ground-based Detectors

- **What have we learned? (BBHs)**



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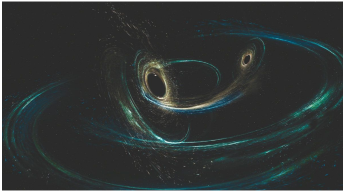

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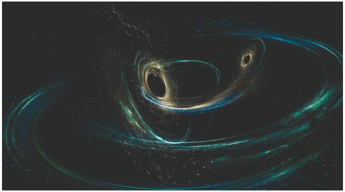

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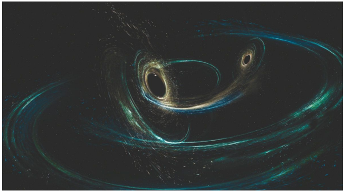
  3. Properties of the observed BHs are entirely consistent with GR to within current measurement limits



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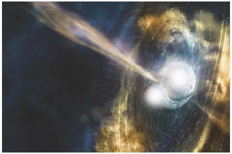

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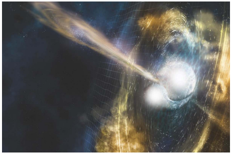
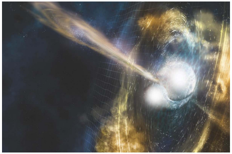


- **What have we learned? (BNSs)**
- 1$^{\rm st}$ demonstration of **GW-EM multi-messenger astronomy**

# Ground-based Detectors



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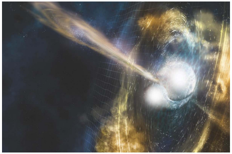

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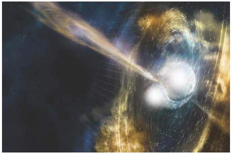

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  3. conclusive spectroscopic proof that BNS mergers produce heavy elements through ***r*-process nucleosynthesis**

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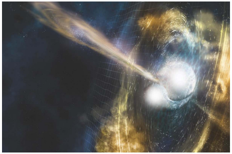

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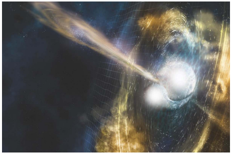

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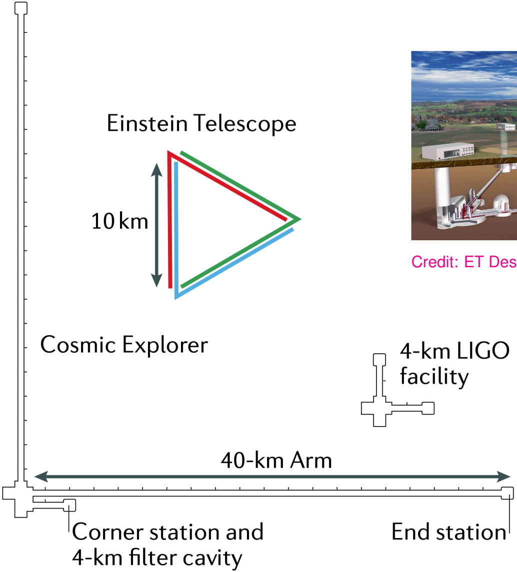
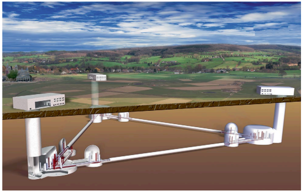
  5. an independent method for measuring the **Hubble constant** using detected GWs as a **"standard siren"**

# 3G Ground-based GW Detectors

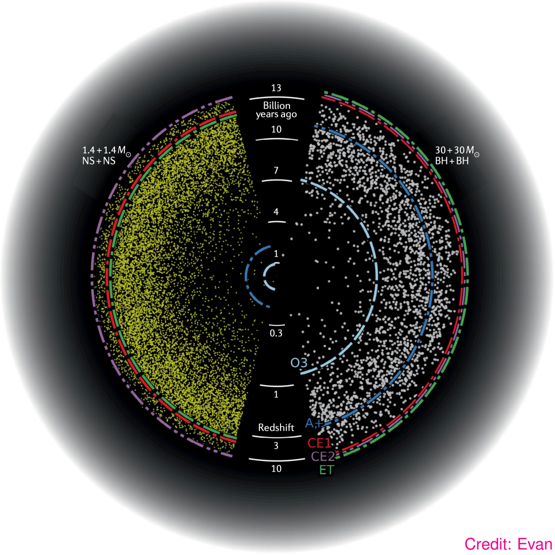


Credit: ET Design Study Team

Bailes et al. 2021

# 3G Ground-based GW Detectors

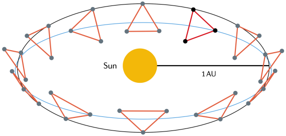


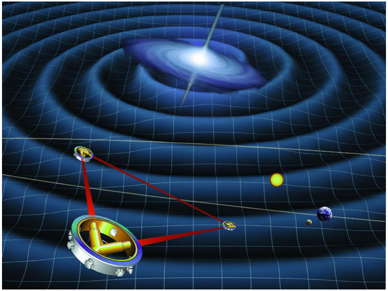

- Cosmic Explorer
- Einstein Telescope

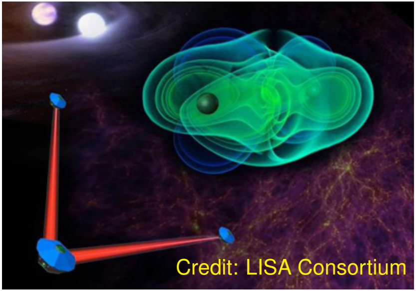
Credit: Evan Hall

# Space-based Detectors

- **LISA**: 100 $\mu$Hz–100 mHz, $2.5 \times 10^9$ m
  - seed BHs @ $z \sim 20$
  - IMBHs and SMBHs: $10^2$–$10^7$ $M_\odot$
  - EMRIs: extreme mass ratio inspirals
  - Galactic binaries: mapping Milky Way

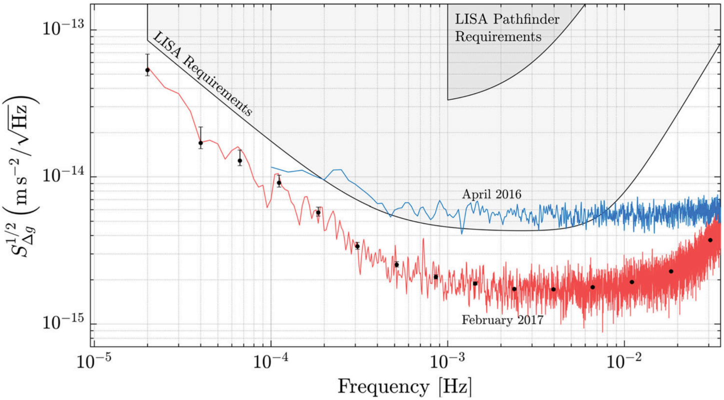



Credit: LISA Consortium

# Space-based Detectors

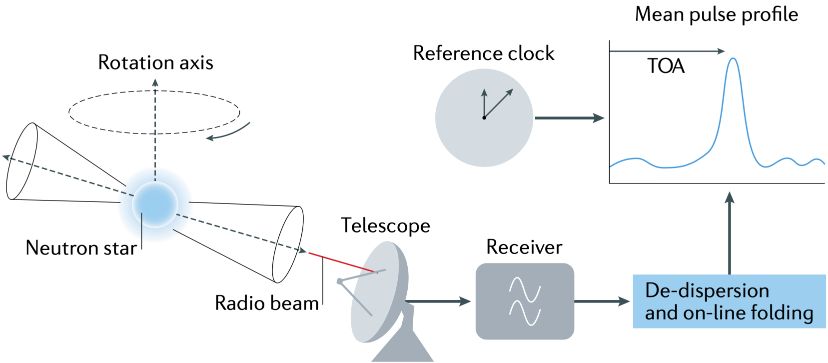
- **LISA Pathfinder**: 2015 – mid-2017



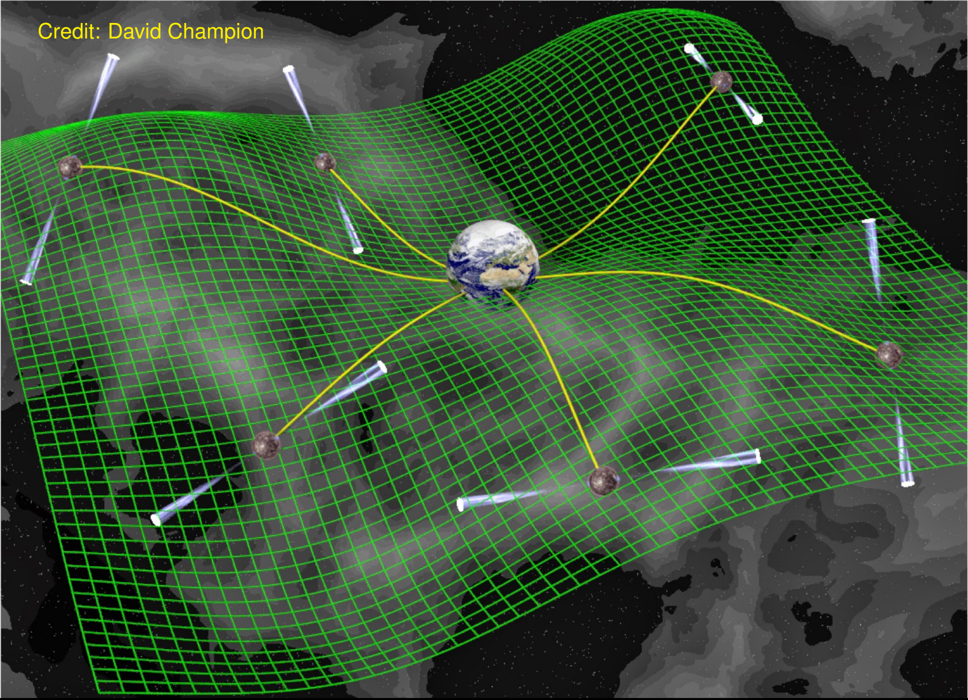

Armano et al., PRL 120 (2018) 061101

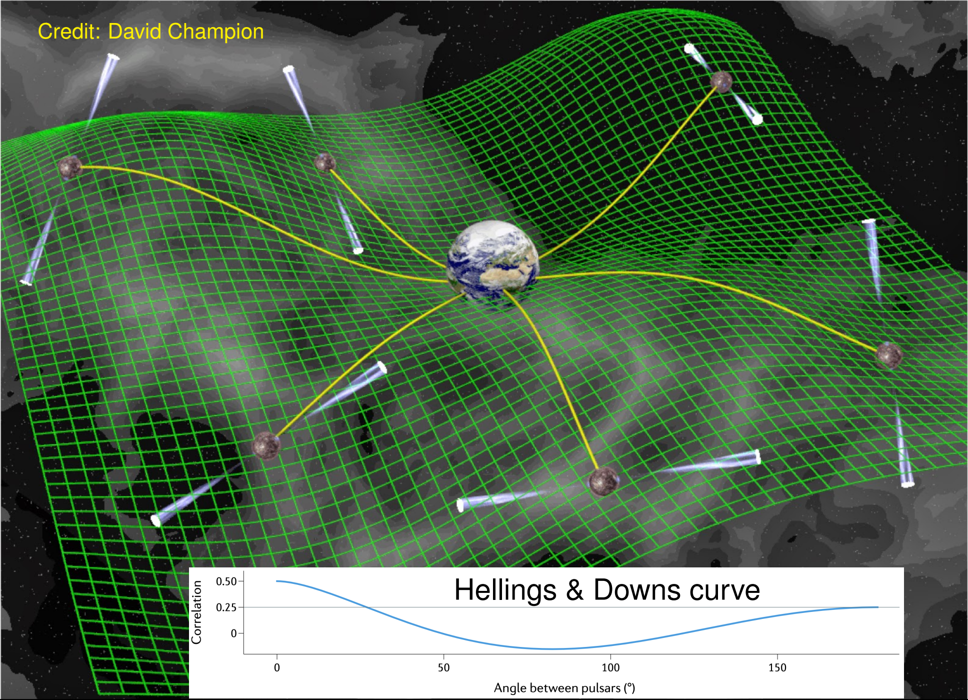
# Pulsar Timing Arrays

- **Pulsars**: magentized rotating NSs $\Rightarrow$ lighthouse
  - TOAs: time of arrivals ($\sigma \lesssim 1\,\mu$s)




Bailes et al. 2021

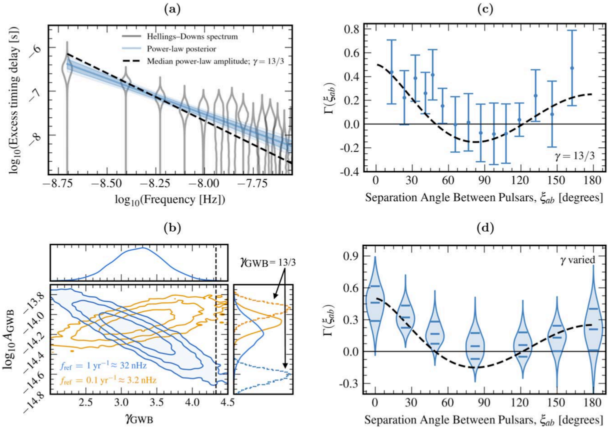

Credit: David Champion


Credit: David Champion
Hellings & Downs curve
Correlation
0.50
0.25
0
0
50
100
150
Angle between pulsars (°)


Credit: Shami Chatterjee

# Key Evidence

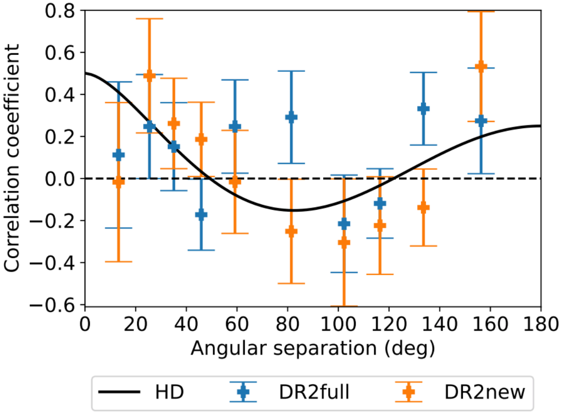
- **NANOGrav PTA** $\approx 3\sigma$




Agazie et al. 2023, ApJL [2306.16213]

# Key Evidence

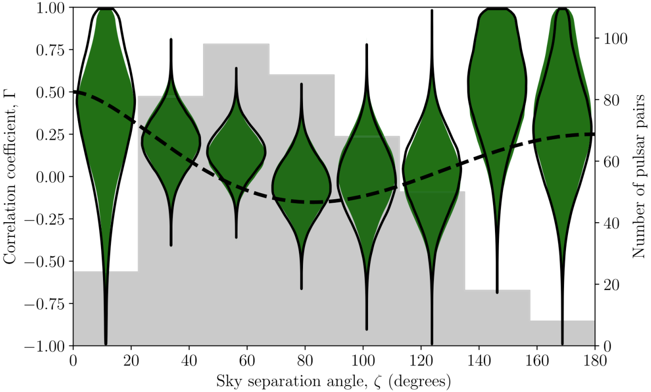
- **European PTA** $\gtrsim 3\sigma$ [inconsistency?]




Antoniadis et al. 2023, A&A [2306.16214]

# Key Evidence

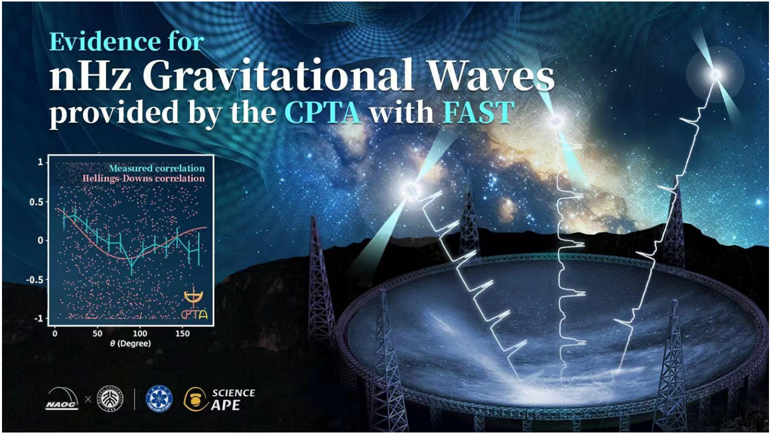
- **Parkes PTA** $\approx 2\sigma$




Reardon et al. 2023, ApJL [2306.16215]

# Key Evidence

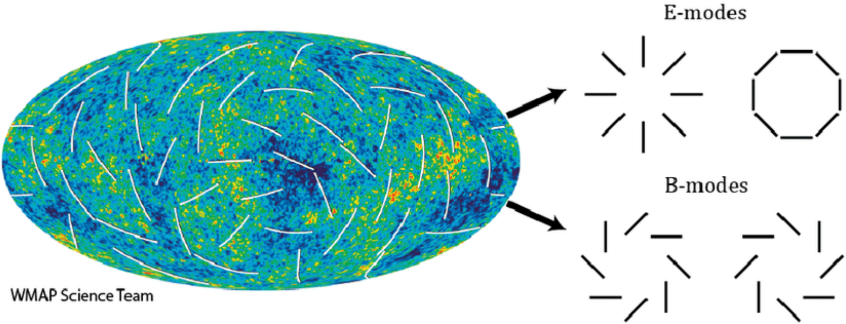
- **Chinese PTA** 4.6$\sigma$ [one single frequency?]



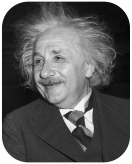

Xu et al. 2023, RAA [2306.16216]

# CMB Polarization

- B-mode polarization: down to $10^{-18}$ Hz
  - remnant primordial GWs



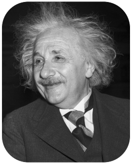
# Fundamental Physics



- Testing GR and modified theories of **gravity**
  - information loss, contradicting quantum, singularity, late-time acceleration $\Rightarrow$ quantum gravity?

Sathyaprakash & Schutz 2009 [0903.0338]; Bailes et al. 2021

# Fundamental Physics



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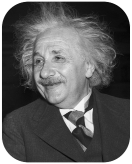

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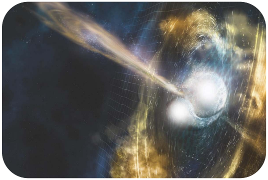
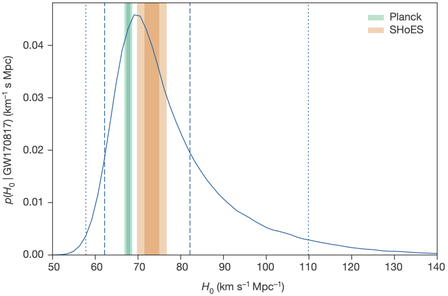
- **Equation of state** of ultra-high density matter
  - low-energy QCD ⇒ nonperturbative
  - phase transition?
- Exploring **dark matter** properties with GW observations
  - WIMPs, axions ⇒ superradiance, primordial BHs

Sathyaprakash & Schutz 2009 [0903.0338]; Bailes et al. 2021

# Cosmology



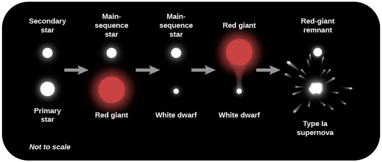
- **Standard Sirens**
  - Hubble constant
  - dark energy equation of state
  - energy Universe ⇐ stochastic backgrounds

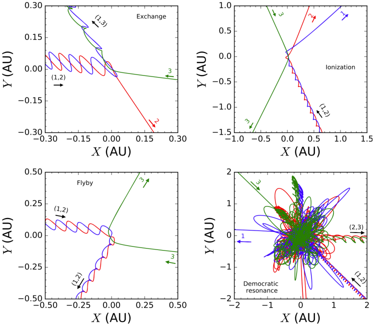



LIGO/Virgo + EM Groups 2017 [1710.05835]

# Astrophysics

- Formation and evolution of compact stars
  - BH-BH, BH-NS, NS-NS, supernovae, etc.
- SMBH growth and evolution

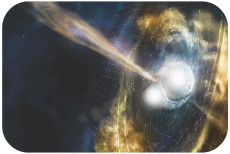


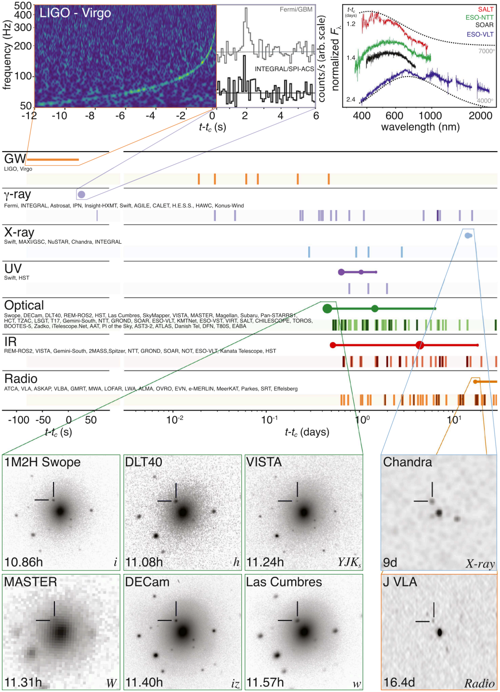

Zhang, Shao, Zhu 2019 [1903.02685]

# Multi-messenger

- Gravitational waves
  - $\gamma$-ray, X-ray
  - UV, optical, IR
  - Optical
- $\gamma$-ray bursts
- kilonovae
- afterglows



Abbott et al. 2017 [1710.05833]




Thanks!!!
Any question?

# Plan of Lectures

I **Overview**
- Lecture duration $\sim$ 45 min

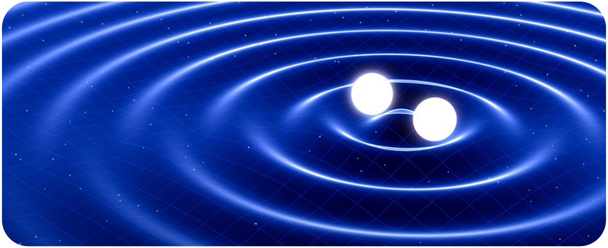
II **What are GWs?**
- Lecture duration $\sim$ 45 min

III **Gravity Tests with GWs**
- Lecture duration $\sim$ 90 min

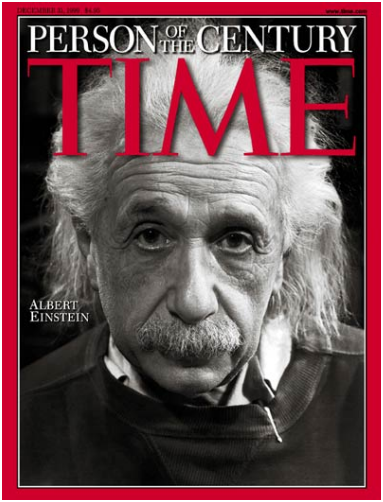
**Contact:** lshao@pku.edu.cn

# References

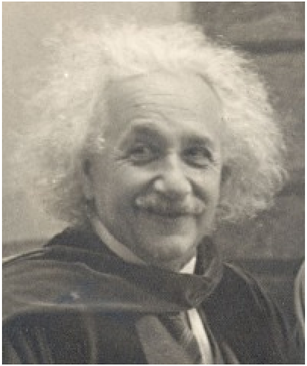

- M. Maggiore, *Gravitational Waves* (Volume 1: Theory and Experiments), Oxford University Press (2008)
- M. Maggiore, *Gravitational Waves* (Volume 2: Astrophysics and Cosmology), Oxford University Press (2018)
- A. Buonanno, Les Houches Lecture Notes (2006) [arXiv:0709.4682]

# Person of the Century



# General Relativity



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

*"Matter tells spacetime how to curve, and spacetime tells matter how to move."*

# Einstein Field Equations

## Einstein Field Equations in a Nutshell

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

where

$$\begin{aligned}
R &= g^{\mu\nu} R_{\mu\nu} \\
R_{\mu\nu} &= g^{\rho\sigma} R_{\rho\mu\sigma\nu} \\
R^{\nu}{}_{\mu\rho\sigma} &= \Gamma^{\nu}{}_{\mu\sigma,\rho} - \Gamma^{\nu}{}_{\mu\rho,\sigma} + \Gamma^{\nu}{}_{\lambda\rho}\Gamma^{\lambda}{}_{\mu\sigma} - \Gamma^{\nu}{}_{\lambda\sigma}\Gamma^{\lambda}{}_{\mu\rho} \\
\Gamma^{\mu}{}_{\nu\rho} &= \frac{1}{2} g^{\mu\lambda} \left( g_{\lambda\nu,\rho} + g_{\lambda\rho,\nu} - g_{\nu\rho,\lambda} \right)
\end{aligned}$$

# Linearized Gravity

# Perturbation of $g_{\mu\nu}$

- In order to study GWs, we assume there exists a coordinate system where the spacetime of interests has

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \left|h_{\mu\nu}\right| \ll 1$$

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- Consider a Lorentz transformation $x^\mu \to \Lambda^\mu{}_\nu x^\nu$, we have

$$g_{\mu\nu} \to g'_{\mu\nu}(x') = \Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu g_{\rho\sigma} = \eta_{\mu\nu} + \Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu h_{\rho\sigma}(x) = \eta_{\mu\nu} + h'_{\mu\nu}(x')$$

where we have used $\Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu \eta_{\rho\sigma} = \eta_{\mu\nu}$

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  where we have used $\Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu \eta_{\rho\sigma} = \eta_{\mu\nu}$

- Therefore, $h_{\mu\nu}$ can be viewed as a tensor field in a flat spacetime

# Perturbation of $g_{\mu\nu}$

- Now consider a coordinate transformation

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x), \quad \left|\partial_\mu \xi_\nu\right| \leq \left|h_{\mu\nu}\right|$$

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- Keeping leading-order terms,

$$g'_{\mu\nu} = \eta_{\mu\nu} - \partial_{\nu}\xi_{\mu} - \partial_{\mu}\xi_{\nu} + h_{\mu\nu} + \mathcal{O}\left(\xi^{2}\right)$$

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- Therefore, $h_{\mu\nu}$ satisfies

$$h'_{\mu\nu} = h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}\,, \quad \left|h'_{\mu\nu}\right| \ll 1$$

# Perturbation of $g_{\mu\nu}$

- Keeping the leading-order terms of $h_{\mu\nu}$, we have[1]

$$\Gamma^{\nu}{}_{\mu\rho} = \frac{1}{2}\eta^{\nu\lambda}\left(\partial_\rho h_{\lambda\mu} + \partial_\mu h_{\lambda\rho} - \partial_\lambda h_{\mu\rho}\right)$$

$$R^{\nu}{}_{\mu\rho\sigma} = \partial_\rho \Gamma^{\nu}{}_{\mu\sigma} - \partial_\sigma \Gamma^{\nu}{}_{\mu\rho} + \mathcal{O}\left(h^2\right)$$

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_{\rho\nu} h_{\mu\sigma} + \partial_{\sigma\mu} h_{\nu\rho} - \partial_{\rho\mu} h_{\nu\sigma} - \partial_{\sigma\nu} h_{\mu\rho}\right)$$

[1] Homework ;-)

# Perturbation of $g_{\mu\nu}$

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R_{\mu\nu\rho\sigma} &= \frac{1}{2}\left(\partial_{\rho\nu} h_{\mu\sigma} + \partial_{\sigma\mu} h_{\nu\rho} - \partial_{\rho\mu} h_{\nu\sigma} - \partial_{\sigma\nu} h_{\mu\rho}\right)
\end{aligned}
$$

- A direct calculation shows that, under the change of $h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$, the Rieman tensor does not change

[1] Homework ;-)

# Equation of GWs

- Define a trace-reverse tensor,

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$$

which satisfies $h = \eta_{\alpha\beta} h^{\alpha\beta}$ and $\bar{h} = -h$

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- With a linearized metric, the Einstein field equations become

$$\Box \bar{h}_{\nu\sigma} + \eta_{\nu\sigma}\partial^\rho \partial^\lambda \bar{h}_{\rho\lambda} - \partial^\rho \partial_\nu \bar{h}_{\rho\sigma} - \partial^\rho \partial_\sigma \bar{h}_{\rho\nu} + \mathcal{O}\left(h^2\right) = -\frac{16\pi G}{c^4} T_{\nu\sigma}$$

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- Introduce Lorenz gauge (a.k.a. harmonic gauge, De Donder gauge)

$$\partial_\nu \bar{h}^{\mu\nu} = 0$$

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- We finally obtain a wave equation

$$\Box \bar{h}_{\nu\sigma} = -\frac{16\pi G}{c^4} T_{\nu\sigma}$$

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- If $\bar{h}^{\mu\nu}$ does not satisfy Lorenz gauge, namely

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- **Lorenz gauge** reduces the d.o.f.s of $h_{\mu\nu}$ from **10** to **6**

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- In vacuum, $T_{\mu\nu} = 0$, therefore

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thus, GWs propagate with the speed of light

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  - Now $\bar{h}_{\mu\nu}$ becomes

$$\bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} + \xi_{\mu\nu}$$

where $\xi_{\mu\nu} = \eta_{\mu\nu}\partial_{\rho}\xi^{\rho} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$, satisfying $\Box \xi_{\mu\nu} = 0$

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- As for now, we know from the $\mu = 0$ component of the Lorenz gauge $\partial_\nu \bar{h}^{\mu\nu} = \partial_\nu h^{\mu\nu} = 0$ that $\partial_0 h^{00} = 0$ (we take $h^{00} = 0$)

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- Overall, we call the following transverse-traceless gauge

$$h^{00} = h^{ii} = h^{0i} = 0, \quad \partial_i h^{ij} = 0$$

We denote GWs in TT gauge as $h_{ij}^{\mathrm{TT}}$

# $h_{ij}^{\rm TT}$

- For a plane wave, $\partial_i h^{ij} = 0$ means $\hat{n}^i h_{ij}^{\rm TT} = 0$, where $\hat{\boldsymbol{n}} = \boldsymbol{k}/k$ is the propagating direction

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- For a plane wave, $\partial_i h^{ij} = 0$ means $\hat{n}^i h_{ij}^{\rm TT} = 0$, where $\hat{\boldsymbol{n}} = \boldsymbol{k}/k$ is the propagating direction
- Without losing generality, we consider GWs propagating along $z$-axis, and we have

$$h_{ij}^{\rm TT}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos\left[\omega\left(t - \frac{z}{c}\right)\right]$$

where $h_+$ and $h_\times$ are two independent **polarizations**

# $h_{ij}^{\mathrm{TT}}$

- If we rotate about $z$ axis by an angle $\psi$,

$$h_\times \pm ih_+ \to e^{\mp 2i\psi} (h_\times \pm ih_+)$$

Therefore, gravitons are **spin-2** particles

## $h_{ij}^{\mathrm{TT}}$

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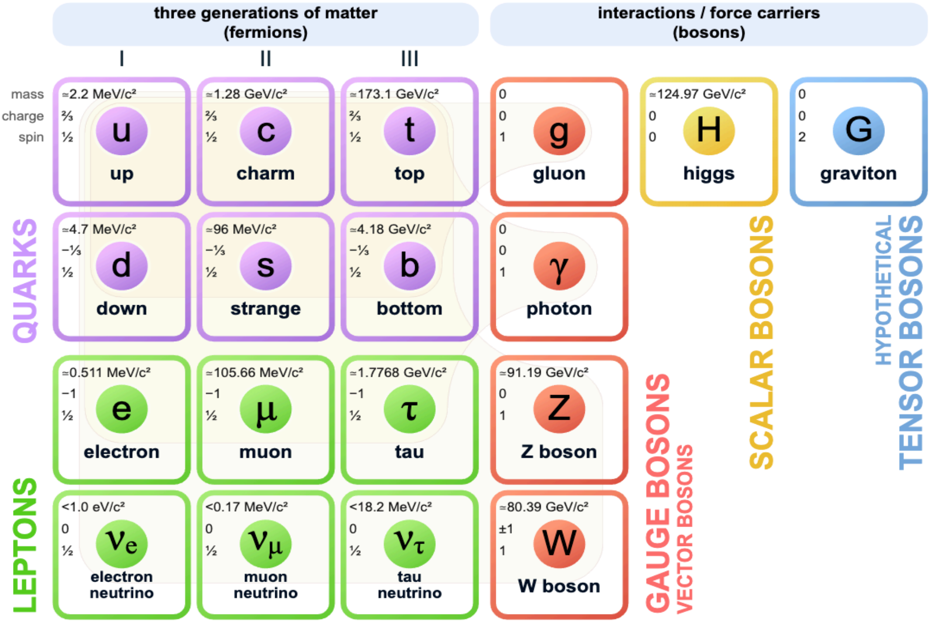
  Therefore, gravitons are **spin-2** particles

- **What are gravitons?**

# Standard Model of Elementary Particles and Gravity



# Projection Operators

- For a given direction $\hat{\boldsymbol{n}}$, introduce

$$
\begin{aligned}
P_{ij}(\hat{\boldsymbol{n}}) =& \delta_{ij} - \hat{n}_i \hat{n}_j \\
\Lambda_{ijkl}(\hat{\boldsymbol{n}}) =& P_{ik} P_{jl} - \frac{1}{2} P_{ij} P_{kl}
\end{aligned}
$$

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$$\Lambda_{ijkl}(\hat{\boldsymbol{n}}) = P_{ik} P_{jl} - \frac{1}{2} P_{ij} P_{kl}$$

- If $h_{kl}$ describes GWs in **Lorenz gauge** (not necessarily **TT guage**), then

$$h_{ij} \equiv \Lambda_{ijkl} h_{kl}$$

satisfies TT gauge

# GW Detections

- Now we consider a local free fall (FF) coordinate (note: not a TT gauge!)

[2] It can be obtained from geodesic equation; see Sec. 3.3 in arXiv:0709.4682

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  - In FF coordinate, we have $g_{\mu\nu}(P) = \eta_{\mu\nu}$ and $\Gamma^{\rho}{}_{\mu\nu}(P) = 0$
  - LIGO/Virgo/KAGRA are obviously not in a FF state

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- Without proof,[2] we denote that for two nearby particles,

$$\frac{\mathrm{d}^2 \xi^j}{\mathrm{d}t^2} = \frac{1}{2} \ddot{h}_{jk}^{\mathrm{TT}} \xi^k$$

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# GW Detections

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- According to the equation on the previous slide, we obtain[3]

$$\delta x(t) = \frac{h_+}{2} x_0 \sin \omega t$$
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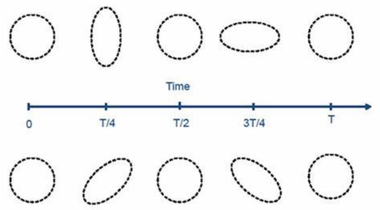
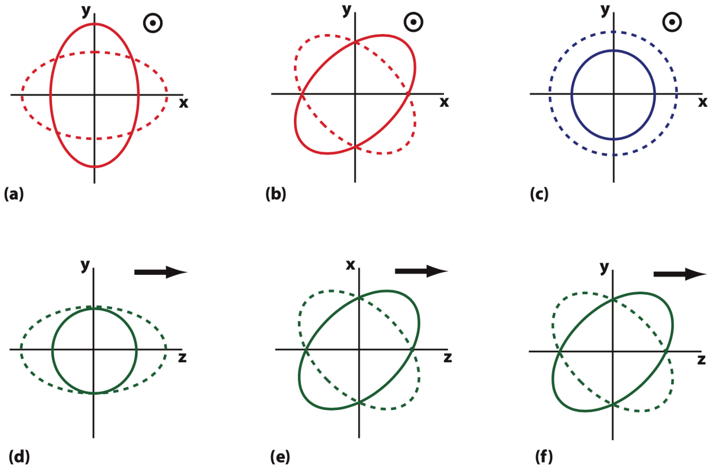
- Therefore, we have the positions of particles as a function of time,



# GW Polarizations in Alternative Gravity




Eardley et al., PRD 8 (1973) 3308;  Will 2014 [1403.7377]

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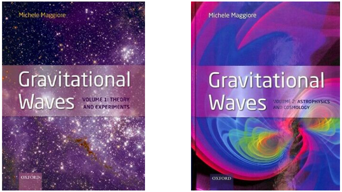

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- We make weak-field & slow-motion assumptions, and get

$$h_{ij}^{\mathrm{TT}}(t, \mathbf{x}) = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ijkl}(\hat{\mathbf{n}})\ddot{M}^{kl}\left(t - \frac{r}{c}\right)$$

where

$$M^{ij} = \frac{1}{c^2}\int \mathrm{d}^3 x T^{00}(t, \mathbf{x}) x^i x^j$$

is mass quadrupole moment in Newtonian approximation

# GW Generation

- Take $\hat{\mathbf{n}} = (\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta)$ and insert into the projection operator $\Lambda_{ijkl}(\hat{\mathbf{n}})$, then[4]

$$
\begin{aligned}
h_+ = & \frac{G}{rc^4} \Big\{ \ddot{M}_{11} \left( \sin^2\varphi - \cos^2\theta \cos^2\varphi \right) \\
& + \ddot{M}_{22} \left( \cos^2\varphi - \cos^2\theta \sin^2\varphi \right) - \ddot{M}_{33} \sin^2\theta \\
& - \ddot{M}_{12} \sin 2\varphi \left(1 + \cos^2\theta\right) + \ddot{M}_{13} \cos\varphi \sin 2\theta + \ddot{M}_{23} \sin 2\theta \sin\varphi \Big\} \\
h_\times = & \frac{2G}{rc^4} \Big\{ \frac{1}{2} \left( \ddot{M}_{11} - \ddot{M}_{22} \right) \cos\theta \sin 2\varphi - \ddot{M}_{12} \cos\theta \cos 2\varphi \\
& - \ddot{M}_{13} \sin\theta \sin\varphi + \ddot{M}_{23} \cos\varphi \sin\theta \Big\}
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$$

[4] Don't be afraid & take it homework ;-)

# Binary Systems

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- Then the mass quadrupole moments are

$$\begin{aligned} M_{11} &= \frac{1}{2}\mu R^2 (1 + \cos 2\omega t) \\ M_{22} &= \frac{1}{2}\mu R^2 (1 - \cos 2\omega t) \\ M_{12} &= \frac{1}{2}\mu R^2 \sin 2\omega t \end{aligned}$$

# Binary Systems

- Insert into expressions of $h_+$ & $h_\times$, we have

$$
\begin{aligned}
h_+(t) =& \frac{1}{r}\frac{4G}{c^4}\mu R^2\omega^2\frac{\left(1+\cos^2\theta\right)}{2}\cos(2\omega t)\\
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- These are the leading-order GW formuae that we frequently use

# GW Radiation

- As GWs carry energy, the GW radiation reduces the binary's orbital energy $\Leftarrow$ energy balance equation

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# GW Radiation

- As GWs carry energy, the GW radiation reduces the binary's orbital energy $\Leftarrow$ energy balance equation
- The orbital size becomes **smaller**, and the orbital frequency becomes **larger**
- At leading order,[5] with $\nu \equiv \mu/M$ and $\mathcal{M} = \mu^{3/5}M$,

$$
\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu\left(\frac{GM\omega}{c^3}\right)^{5/3}
$$

$$
\dot{f}_{\rm GW} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f_{\rm GW}^{11/3}
$$

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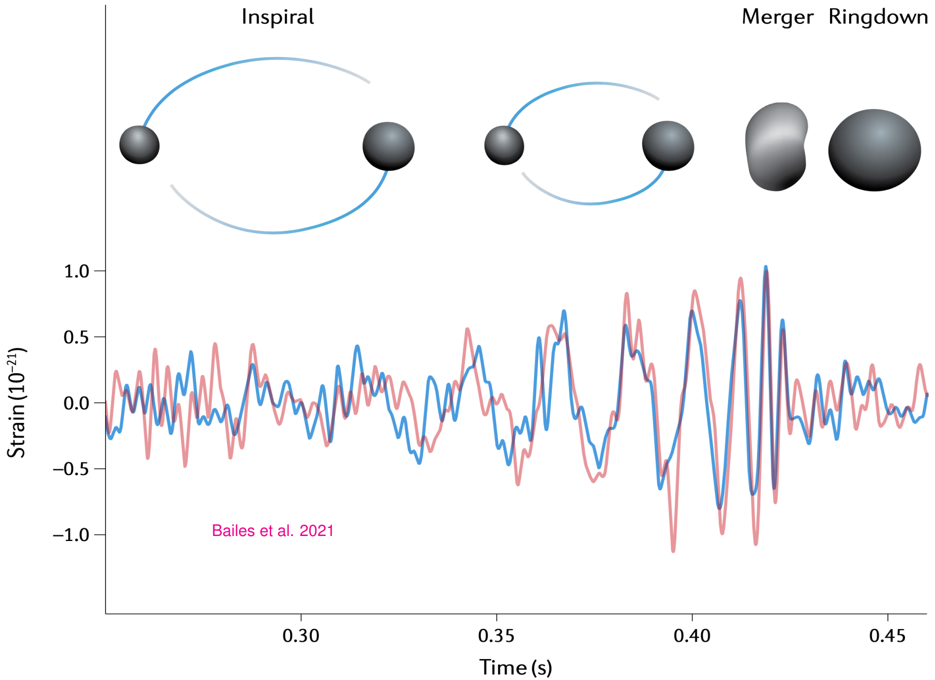
Inspiral
Merger
Ringdown
1.0
0.5
0.0
−0.5
−1.0
Strain (10$^{-21}$)
0.30
0.35
0.40
0.45
Time (s)
Bailes et al. 2021

# GWs on a curved spacetime

# Separation of GWs from the Background

- On a **curved**, **dynamical** background metric

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x), \quad |h_{\mu\nu}| \ll 1$$

such that satisfying (short-wave expansion) $\lambdabar \ll L_B$ or $f \gg f_B$

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- **Ricci tensor**: $\mathcal{O}(1)$, $\mathcal{O}(h)$, and $\mathcal{O}(h^2)$

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  - $R^{(1)}_{\mu\nu}$: only **high** frequency
  - $R^{(2)}_{\mu\nu}$: **mixture of both**

# Separation of GWs from the Background

- **Master equations**

$$\bar{R}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\mathrm{Low}} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\mathrm{Low}}$$

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- low-frequency equation $\Rightarrow$ energy-stress tensor of GWs
- high-frequency equation $\Rightarrow$ propagating equation of GWs

# Low-frequency Equation

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- If curvature is determined by GWs $\Rightarrow h \sim \frac{\lambda}{L_B}$

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- If curvature is determined by GWs $\Rightarrow h \sim \frac{\lambda}{L_B}$
- If curvature is determined by matter fields $\Rightarrow h \ll \frac{\lambda}{L_B}$
- These two conclusions will be important for a later context

# Low-frequency Equation

- Difficulties with **localized** **energy-stress tensor** in GR

# Low-frequency Equation

- Difficulties with **localized** **energy-stress tensor** in GR
- Learn from **renormalization group**

**"coarse-grained" form of the Einstein equation**

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{8\pi G}{c^4}\left(\bar{T}_{\mu\nu} + t_{\mu\nu}\right)$$

where

$$t_{\mu\nu} \equiv -\frac{c^4}{8\pi G}\left\langle R^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(2)}\right\rangle$$

# Low-frequency Equation

- In Lorentz gauge and with $h = 0$, one has

$$t_{\mu\nu} = \frac{c^4}{32\pi G} \left\langle \partial_\mu h_{\alpha\beta} \partial_\nu h^{\alpha\beta} \right\rangle$$

[6] Use $x'^\mu = x^\mu + \xi^\mu$

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- Explicit calculations show that,[6] $t_{\mu\nu}$ only depends on the physical modes $h_{ij}^{\mathrm{TT}}$ $\Rightarrow$ namely, gauge invariant
- Energy-momentum exchange between matters and GWs

$$\bar{D}^\mu \left( \bar{T}_{\mu\nu} + t_{\mu\nu} \right) = 0$$

[6] Use $x'^\mu = x^\mu + \xi^\mu$

# Low-frequency Equation

- With **energy-stress tensor** for GWs, we can discuss many aspects of GWs, e.g.,

$$t^{00} = \frac{c^2}{16\pi G} \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

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$$t^{00} = \frac{c^2}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

- Similarly to the electromagnetism, we have

$$\frac{\mathrm{d}E}{\mathrm{d}A\mathrm{d}t} = +ct^{00}$$

$$\frac{\mathrm{d}P^k}{\mathrm{d}A\mathrm{d}t} = +t^{0k}$$

# Low-frequency Equation

- We can have the energy radiation rate and the momentum taken away by GWs,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{c^3 r^2}{32\pi G}\int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}} \dot{h}_{ij}^{\mathrm{TT}} \right\rangle$$

$$\frac{dP^k}{dt} = -\frac{c^3}{32\pi G} r^2 \int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}} \partial^k h_{ij}^{\mathrm{TT}} \right\rangle$$

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- as well as the energy spectrum

$$\frac{\mathrm{d}E}{\mathrm{d}f} = \frac{\pi c^3}{2G} f^2 r^2 \int d\Omega \left( \left|\tilde{h}_+(f)\right|^2 + \left|\tilde{h}_\times(f)\right|^2 \right)$$

and so on

# High-frequency Equation

$$R^{(1)}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\text{High}} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}}$$

- If $T^{\mu\nu} = 0$,[7] one has $R^{(1)}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\text{High}}$

$$R^{(1)}_{\mu\nu} \sim \partial^2 h \sim \frac{h}{\lambda\!\!\!^{-2}} \sim \frac{1}{\epsilon}$$

$$R^{(2)}_{\mu\nu} \sim \partial^2 h^2 \sim \frac{h^2}{\lambda\!\!\!^{-2}} \sim 1$$

[7] Now, according to the low-frequency equation, one has $h \sim \lambda\!\!\!^{-}/L_B$

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- At leading order, $R^{(1)}_{\mu\nu} = 0 \Rightarrow \Box\bar{h}_{\mu\nu} = 0$ in Lorenz gauge

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# High-frequency Equation

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- If $T^{\mu\nu} \neq 0$,[8] one simply has $R^{(1)}_{\mu\nu} = 0$

[8] Now, according to the low-frequency equation, one has $h \ll \lambda\!\!\!/ / L_B$; also, $\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}} = \mathcal{O}\left(h/L_B^2\right)$

# High-frequency Equation

$$R^{(1)}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\text{High}} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}}$$

- If $T^{\mu\nu} \neq 0$,[8] one simply has $R^{(1)}_{\mu\nu} = 0$
  - Imposing a generalized "Lorenz gauge" $\bar{D}^{\nu}\bar{h}_{\mu\nu} = 0$, one has a wave equation in curved spacetime

$$\bar{D}^{\rho}\bar{D}_{\rho}\bar{h}_{\mu\nu} = 0$$

[8] Now, according to the low-frequency equation, one has $h \ll \lambda\!\!\!^{-}/L_B$; also, $\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}} = \mathcal{O}\left(h/L_B^2\right)$


THANK YOU FOR LISTENING
ANY QUESTIONS?

# Plan of Lectures

- I **Overview**
  - Lecture duration $\sim$ 45 min
- II **What are GWs?**
  - Lecture duration $\sim$ 45 min
- III **Gravity Tests with GWs**
  - Lecture duration $\sim$ 90 min

**Contact:** lshao@pku.edu.cn

# References

## Matched Filter

- Finn 1992 [gr-qc/9209010]; Cutler & Flanagan 1994 [gr-qc/9402014]; Cutler 1998 [gr-qc/9703068]

## LIGO/Virgo Collaboration

- GW150914 [1602.03841]; GW170817 [1811.00364]; GWTC-1 [1903.04467]; GWTC-2 [2010.14529]; GWTC-3 [2112.06861]

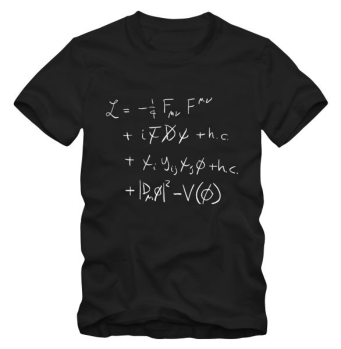
## Review Papers

- Berti *et al.* 2015 [1501.07274]; Yunes, Yagi, & Pretorius 2016 [1603.08955]; Yunes & Siemens 2013 [1304.3473]; Gair, Vallisneri, Larson, & Baker 2013 [1212.5575]

# Modern Physics Landscape

## ▪ Standard Model

quantum field theory




## ▪ General Relativity

gravitation and spacetime

# How the Universe is Ruled

- **Particles of strong, weak, electromagnetic interactions**

$$\mathcal{L}_{\rm lepton} = \frac{1}{2} i e e_a^{\mu} \left[ \bar{L}_A \gamma^a \overleftrightarrow{D}_\mu L_A + \bar{R}_A \gamma^a \overleftrightarrow{D}_\mu R_A \right]$$

$$\mathcal{L}_{\rm quark} = \frac{1}{2} i e e_a^{\mu} \left[ \bar{Q}_A \gamma^a \overleftrightarrow{D}_\mu Q_A + \bar{U}_A \gamma^a \overleftrightarrow{D}_\mu U_A + \bar{D}_A \gamma^a \overleftrightarrow{D}_\mu D_A \right]$$

$$\mathcal{L}_{\rm Yukawa} = -e \left[ (G_L)_{AB} \bar{L}_A \varphi R_B + (G_U)_{AB} \bar{Q}_A \varphi^c U_B + (G_D)_{AB} \bar{Q}_A \varphi D_B \right] + \mathrm{h.c.}$$

$$\mathcal{L}_{\rm Higgs} = -e \left[ (D_\mu \varphi)^\dagger D^\mu \varphi - \mu^2 \varphi^\dagger \varphi + \frac{\lambda}{3!} \left( \varphi^\dagger \varphi \right)^2 \right]$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{2} e \left[ \mathrm{Tr} \left( G_{\mu\nu} G^{\mu\nu} \right) + \mathrm{Tr} \left( W_{\mu\nu} W^{\mu\nu} \right) + \frac{1}{2} B_{\mu\nu} B^{\mu\nu} \right]$$

- **Spacetime of gravitational interaction**

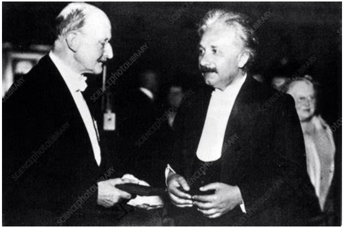
$$S_{\rm gravity} = \frac{1}{2\kappa} \int \mathrm{d}^4 x \, e (R - 2\Lambda + \cdots)$$

# Absence of Quantum Gravity

- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime

# Absence of Quantum Gravity

- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime
- However, QFT and GR are **Not Compatible** at their face values!



[Planck & Einstein]

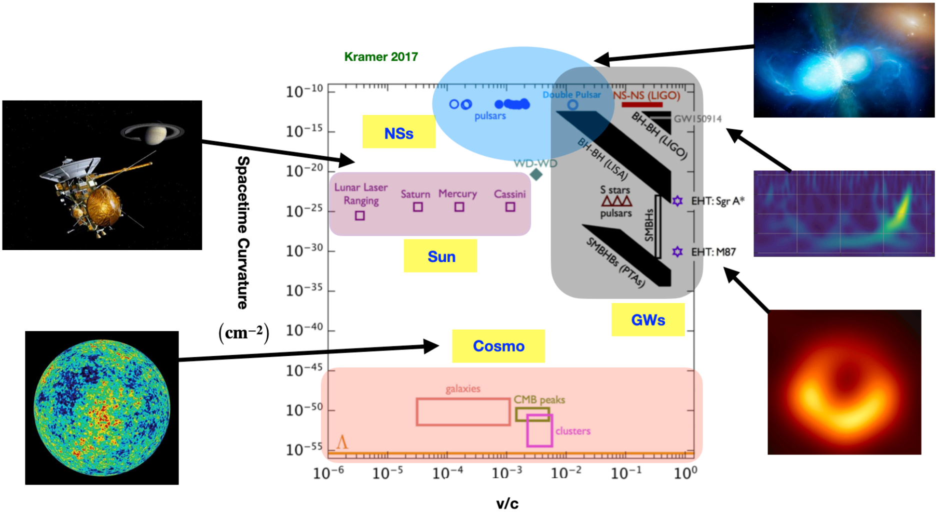
# Theoretical physics is beautiful, but not yet complete

Theoretical physics is beautiful,

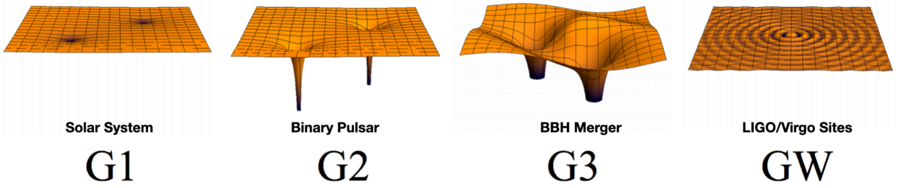
but not yet complete

**Gravity may be holding the key**

# Parameter Space in Gravity Tests



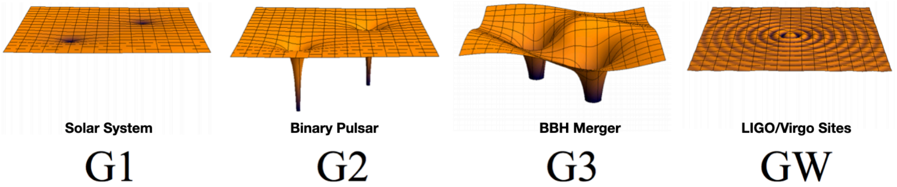
# Parameter Space in Gravity Tests




Wex 2014 [1402.5594]

# Parameter Space in Gravity Tests

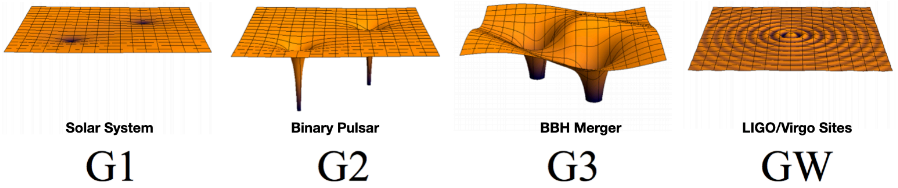
- **G1**: Quasi-stationary weak-field regime




Wex 2014 [1402.5594]

# Parameter Space in Gravity Tests

- **G1**: Quasi-stationary weak-field regime
- **G2**: Quasi-stationary strong-field regime




Wex 2014 [1402.5594]

# Parameter Space in Gravity Tests

- **G1**: Quasi-stationary weak-field regime
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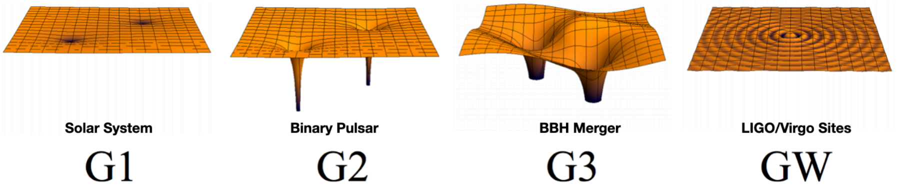
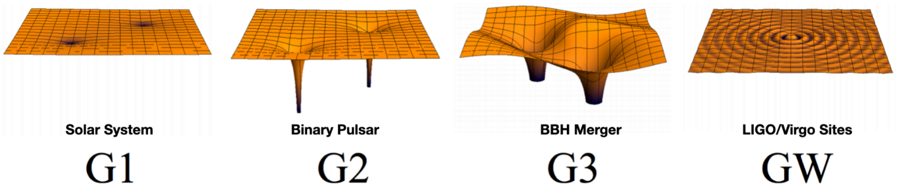
- **G3**: Highly dynamical strong-field regime




Wex 2014 [1402.5594]

# Parameter Space in Gravity Tests

- **G1**: Quasi-stationary weak-field regime
- **G2**: Quasi-stationary strong-field regime
- **G3**: Highly dynamical strong-field regime
- **GW**: Radiation regime

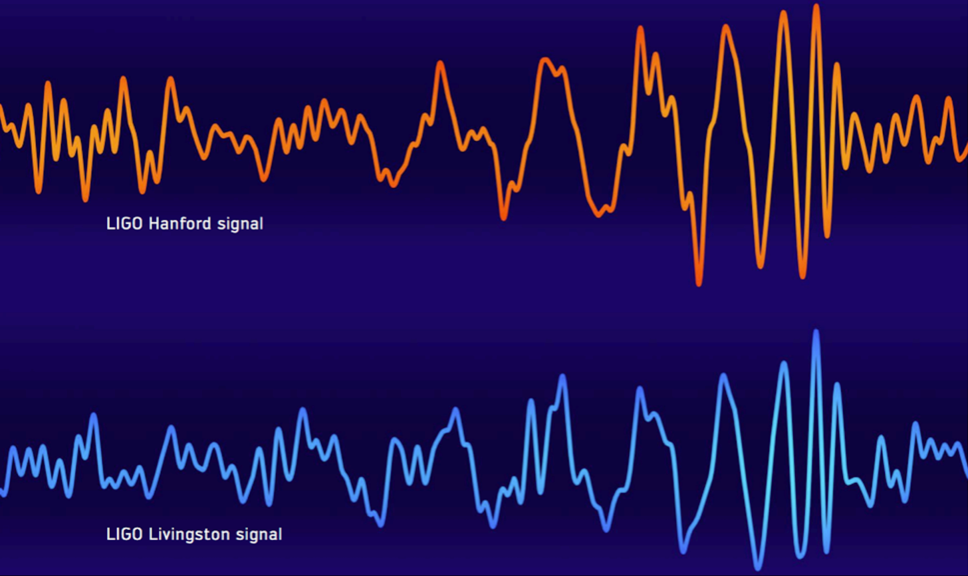



Wex 2014 [1402.5594]

# Gravitational-wave Data

## First detection!

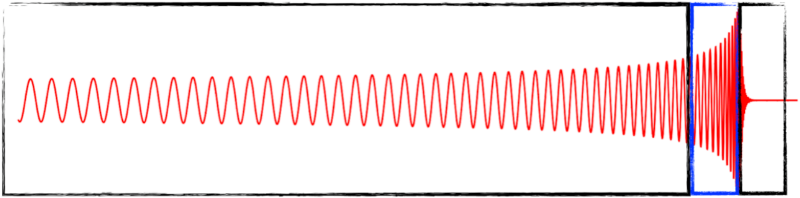
9:50:45 UTC, 14 September 2015



# Gravitational Waveform (Time Domain)

**"Inspiral"**
**post-Newtonian method**

**"Ringdown"**
**BH perturbation**



**"Merge"**
**Numerical relativity**

Bohé, Shao, Taracchini et al. 2017 [1611.03703]

# Gravitational Waveform (Time Domain)

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“Inspiral”
post-Newtonian method

“Ringdown”
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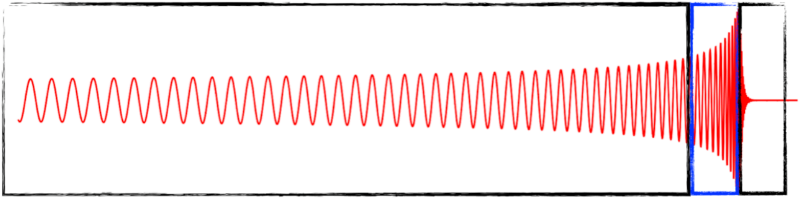

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Numerical relativity

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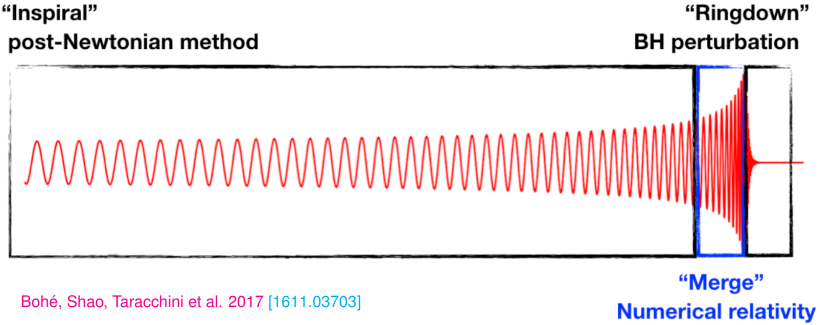
# Gravitational Waveform (Time Domain)

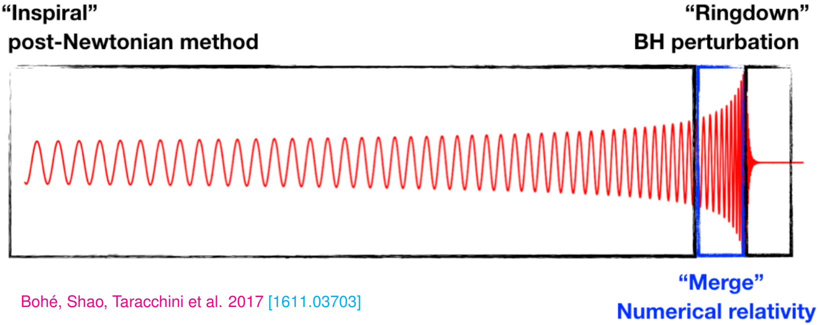
- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity




Bohé, Shao, Taracchini et al. 2017 [1611.03703]

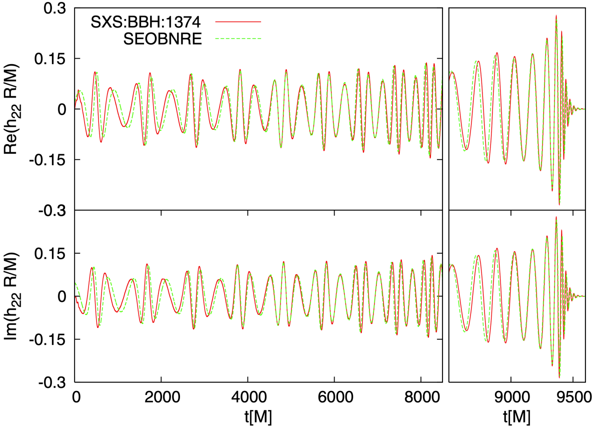
# Gravitational Waveform (Time Domain)

- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity
- **Ringdown**: black hole perturbation




Bohé, Shao, Taracchini et al. 2017 [1611.03703]

# Eccentric Waveform (Time Domain)

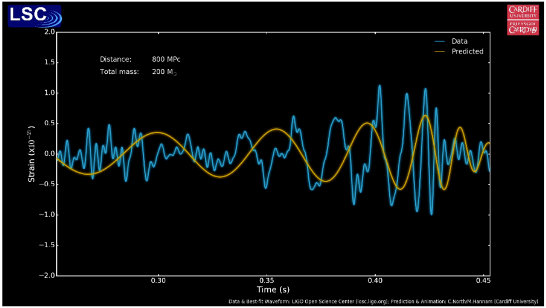



Cao & Han 2017 [1708.00166]; Liu, Cao, Shao 2020 [1910.00784]; Liu, Cao, Zhu 2021 [2102.08614]; Liu, Cao, Shao 2023 [2306.15277]

# Matched Filter

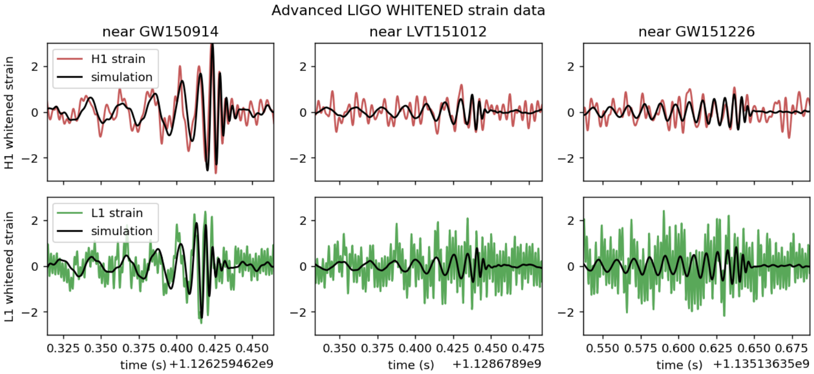
- **Matched fitlering** is a standard analysis method for **wideband** time series data    Finn 1992 [gr-qc/9209010]

$$(\mathbf{g}|\,\mathbf{k}) \equiv 2\int_0^\infty \frac{\tilde{g}^*(f)\tilde{k}(f)+\tilde{g}(f)\tilde{k}^*(f)}{S_n(f)}\mathrm{d}f$$



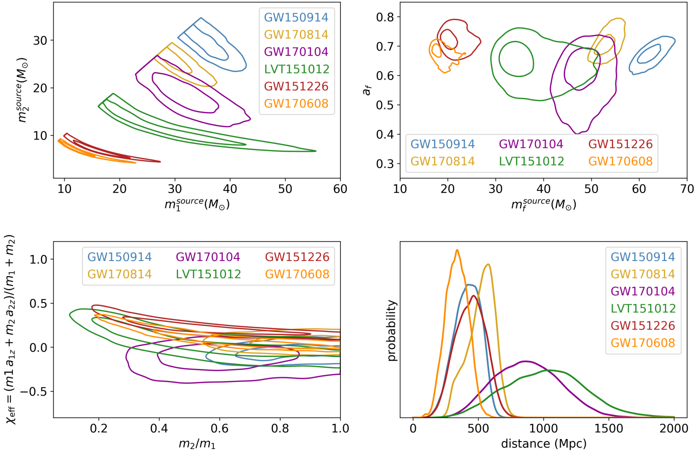
# Matched Filter

- The power of **matched fitlering** lays in its ability/sensitivity to the **phase** of time-series data




Credit: Vivien Raymond / Cardiff U.

# Parameter Estimation

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# Parameter Estimation: GW150914

- GW data encode plenty of information of GW sources
  - Apply matched filter to data & theory

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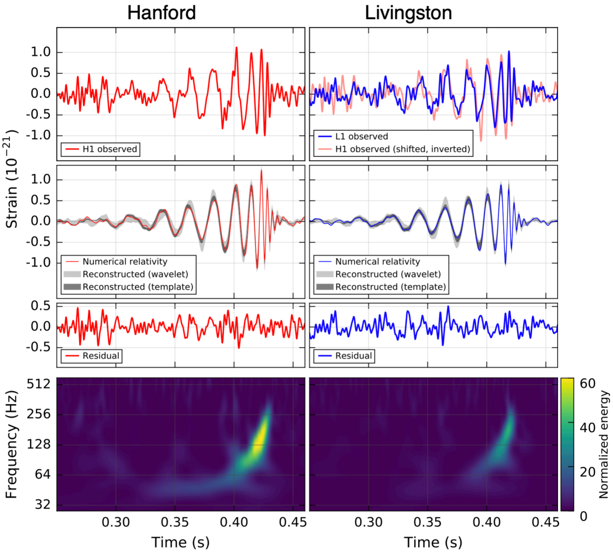
- GW data encode plenty of information of GW sources
  - Apply matched filter to data & theory

| | |
|---|---|
| Primary black hole mass | $36^{+5}_{-4} M_\odot$ |
| Secondary black hole mass | $29^{+4}_{-4} M_\odot$ |
| Final black hole mass | $62^{+4}_{-4} M_\odot$ |
| Final black hole spin | $0.67^{+0.05}_{-0.07}$ |
| Luminosity distance | $410^{+160}_{-180}$ Mpc |
| Source redshift $z$ | $0.09^{+0.03}_{-0.04}$ |

LIGO/Virgo 2016 [1602.03837]

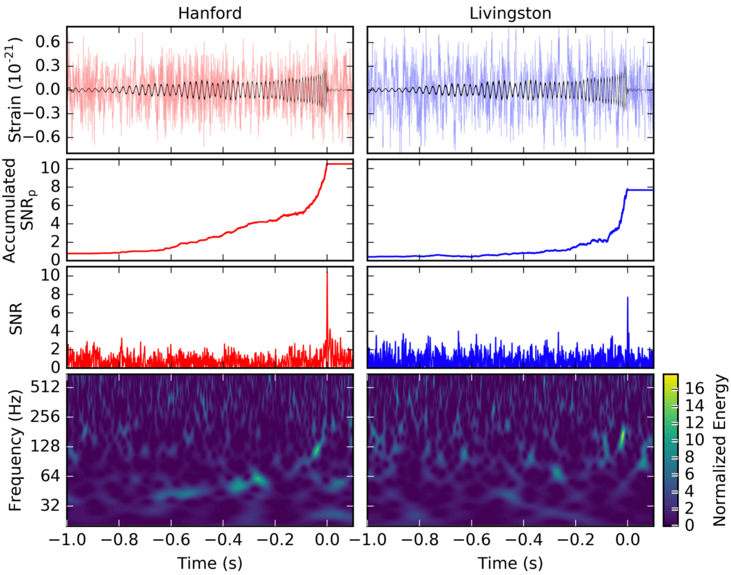
# GW150914 (LIGO/Virgo 2016)

$36 + 29\,M_\odot$: 0.2 sec, SNR=23



# GW151226 (LIGO/Virgo 2016)

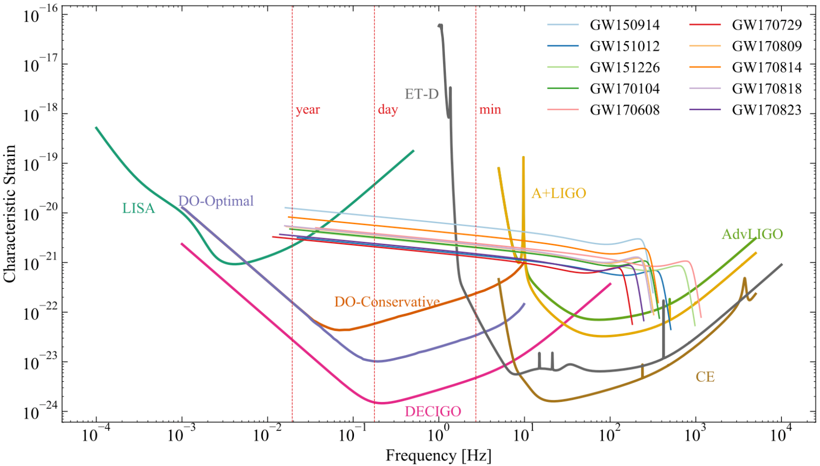
$14 + 8\,M_\odot$: 1 sec, SNR=13



# GW Transient Catalog GWTC-1 (LIGO/Virgo 2019)

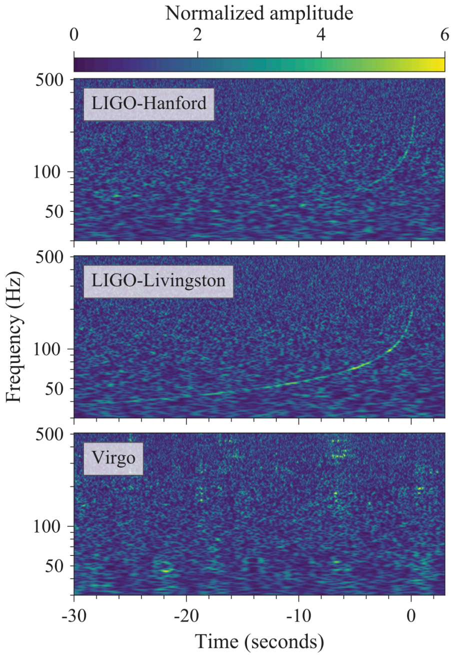
| | Type | $m_1$ $[M_\odot]$ | $m_2$ $[M_\odot]$ | $d_L$ [Mpc] | Redshift $z$ |
|---|---|---|---|---|---|
| GW150914 | BBH | $35.6^{+4.8}_{-3.0}$ | $30.6^{+3.0}_{-4.4}$ | $430^{+150}_{-170}$ | $0.09^{+0.03}_{-0.03}$ |
| GW151012 | BBH | $23.3^{+14.0}_{-5.5}$ | $13.6^{+4.1}_{-4.8}$ | $1060^{+540}_{-480}$ | $0.21^{+0.09}_{-0.09}$ |
| GW151226 | BBH | $13.7^{+8.8}_{-3.2}$ | $7.7^{+2.2}_{-2.6}$ | $440^{+180}_{-190}$ | $0.09^{+0.04}_{-0.04}$ |
| GW170104 | BBH | $31.0^{+7.2}_{-5.6}$ | $20.1^{+4.9}_{-4.5}$ | $960^{+430}_{-410}$ | $0.19^{+0.07}_{-0.08}$ |
| GW170608 | BBH | $10.9^{+5.3}_{-1.7}$ | $7.6^{+1.3}_{-2.1}$ | $320^{+120}_{-110}$ | $0.07^{+0.02}_{-0.02}$ |
| GW170729 | BBH | $50.6^{+16.6}_{-10.2}$ | $34.3^{+9.1}_{-10.1}$ | $2750^{+1350}_{-1320}$ | $0.48^{+0.19}_{-0.20}$ |
| GW170809 | BBH | $35.2^{+8.3}_{-6.0}$ | $23.8^{+5.2}_{-5.1}$ | $990^{+320}_{-380}$ | $0.20^{+0.05}_{-0.07}$ |
| GW170814 | BBH | $30.7^{+5.7}_{-3.0}$ | $25.3^{+2.9}_{-4.1}$ | $580^{+160}_{-210}$ | $0.12^{+0.03}_{-0.04}$ |
| **GW170817** | **BNS** | $1.46^{+0.12}_{-0.10}$ | $1.27^{+0.09}_{-0.09}$ | $40^{+10}_{-10}$ | $0.01^{+0.00}_{-0.00}$ |
| GW170818 | BBH | $35.5^{+7.5}_{-4.7}$ | $26.8^{+4.3}_{-5.2}$ | $1020^{+430}_{-360}$ | $0.20^{+0.07}_{-0.07}$ |
| GW170823 | BBH | $39.6^{+10.0}_{-6.6}$ | $29.4^{+6.3}_{-7.1}$ | $1850^{+840}_{-840}$ | $0.34^{+0.13}_{-0.14}$ |

# Signals of GW Events (Frequency Domain)




Liu, Shao, Zhao, Gao 2020 [2004.12096]

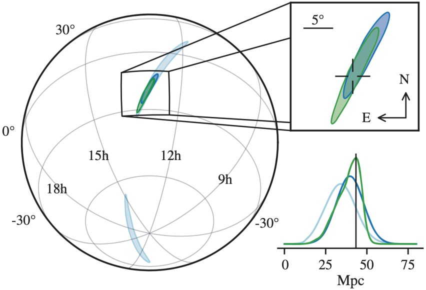
# GW170817 (LIGO/Virgo 2017)

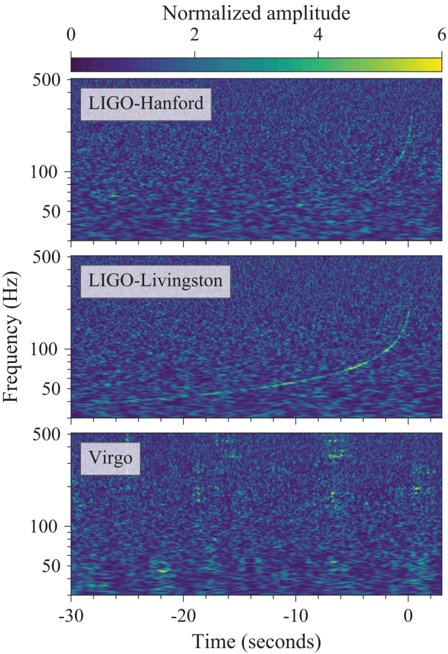



1 min, SNR=32

3000 cycles from 30 Hz

# GW170817 (LIGO/Virgo 2017)

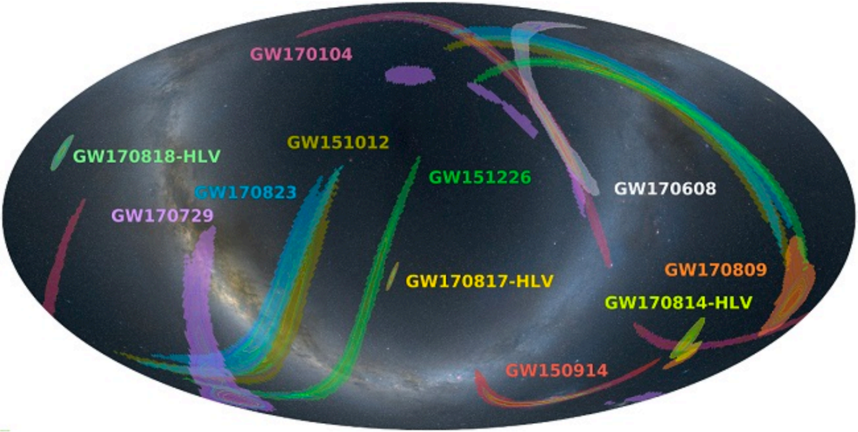



1 min, SNR=32

3000 cycles from 30 Hz

# GWTC-1: Sky Position (LIGO/Virgo 2019)

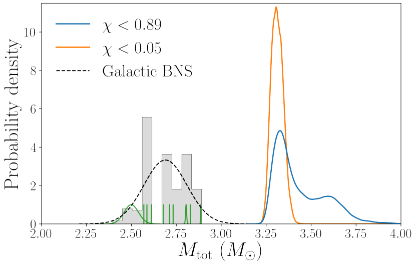


# New Events from O3 (LIGO/Virgo 2020)

- **GW190412**: Observation of a Binary-Black-Hole Coalescence with Asymmetric Masses
  - $30\,M_\odot + 8\,M_\odot$; higher multipole modes

# New Events from O3 (LIGO/Virgo 2020)

- **GW190412**: Observation of a Binary-Black-Hole Coalescence with Asymmetric Masses
  - $30\,M_\odot + 8\,M_\odot$; higher multipole modes
- **GW190425**: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4\,M_\odot$



# New Events from O3 (LIGO/Virgo 2020)

- **GW190521**: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$
  - $85\,M_\odot + 66\,M_\odot \Rightarrow 142\,M_\odot$
  - Intermediate mass black hole?

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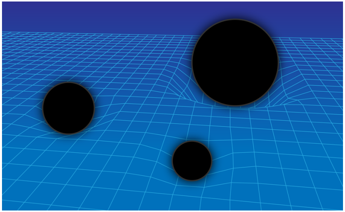
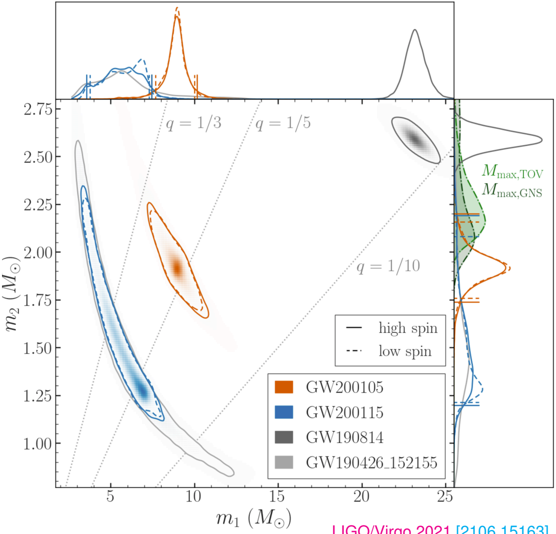
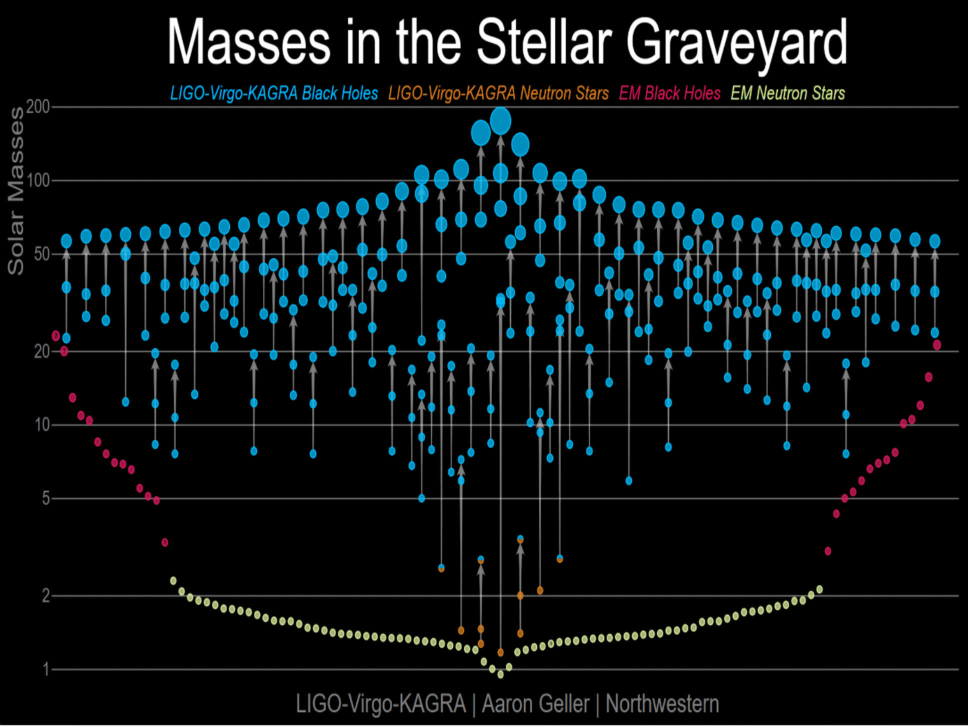
- **GW190521**: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$
  - $85\,M_\odot + 66\,M_\odot \Rightarrow 142\,M_\odot$
  - Intermediate mass black hole?
- **GW190814**: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 $M_\odot$ Compact Object
  - Mass gap: either the lightest black hole or the heaviest neutron star ever discovered



# GW200105 & GW200115: BH-NS Binaries




LIGO/Virgo 2021 [2106.15163]


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

# Testing Gravity with BBHs

- Residual tests (RT)
- Inspiral-merger-ringdown consistency tests (IMR)
- Parameterized tests: inspiral & post-inspiral (PI & PPI)
- Modified dispersion relation (MDR)

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- Residual tests (RT)
- Inspiral-merger-ringdown consistency tests (IMR)
- Parameterized tests: inspiral & post-inspiral (PI & PPI)
- Modified dispersion relation (MDR)

| Event | Properties: $D_L$ [Mpc] | Properties: $M_{tot}$ [$M_\odot$] | Properties: $M_f$ [$M_\odot$] | Properties: $a_f$ | SNR | GR tests performed: RT | IMR | PI | PPI | MDR |
|---|---|---|---|---|---|---|---|---|---|---|
| **GW150914**[b] | $430^{+150}_{-170}$ | $66.2^{+3.7}_{-3.3}$ | $63.1^{+3.3}_{-3.0}$ | $0.69^{+0.05}_{-0.04}$ | $25.3^{+0.1}_{-0.2}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| GW151012[b] | $1060^{+550}_{-480}$ | $37.3^{+10.6}_{-3.9}$ | $35.7^{+10.7}_{-3.8}$ | $0.67^{+0.13}_{-0.11}$ | $9.2^{+0.3}_{-0.4}$ | ✓ | – | – | ✓ | ✓ |
| **GW151226**[b,c] | $440^{+180}_{-190}$ | $21.5^{+6.2}_{-1.5}$ | $20.5^{+6.4}_{-1.5}$ | $0.74^{+0.07}_{-0.05}$ | $12.4^{+0.2}_{-0.3}$ | ✓ | – | ✓ | – | ✓ |
| **GW170104** | $960^{+440}_{-420}$ | $51.3^{+5.3}_{-4.2}$ | $49.1^{+5.2}_{-4.0}$ | $0.66^{+0.08}_{-0.11}$ | $14.0^{+0.2}_{-0.3}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| **GW170608** | $320^{+120}_{-110}$ | $18.6^{+3.1}_{-0.7}$ | $17.8^{+3.2}_{-0.7}$ | $0.69^{+0.04}_{-0.04}$ | $15.6^{+0.2}_{-0.3}$ | ✓ | – | ✓ | ✓ | ✓ |
| GW170729[d] | $2760^{+1380}_{-1340}$ | $85.2^{+15.6}_{-11.1}$ | $80.3^{+14.6}_{-10.2}$ | $0.81^{+0.07}_{-0.13}$ | $10.8^{+0.4}_{-0.5}$ | ✓ | ✓ | – | ✓ | ✓ |
| **GW170809** | $990^{+320}_{-380}$ | $59.2^{+5.4}_{-3.9}$ | $56.4^{+5.2}_{-3.7}$ | $0.70^{+0.08}_{-0.09}$ | $12.7^{+0.2}_{-0.3}$ | ✓ | ✓ | – | ✓ | ✓ |
| **GW170814** | $580^{+160}_{-210}$ | $56.1^{+3.4}_{-2.7}$ | $53.4^{+3.2}_{-2.4}$ | $0.72^{+0.07}_{-0.05}$ | $17.8^{+0.3}_{-0.3}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| GW170818 | $1020^{+430}_{-360}$ | $62.5^{+5.1}_{-4.0}$ | $59.8^{+4.8}_{-3.8}$ | $0.67^{+0.07}_{-0.08}$ | $11.9^{+0.3}_{-0.4}$ | ✓ | ✓ | – | ✓ | ✓ |
| **GW170823** | $1850^{+840}_{-840}$ | $68.9^{+9.9}_{-7.1}$ | $65.6^{+9.4}_{-6.6}$ | $0.71^{+0.08}_{-0.10}$ | $12.1^{+0.2}_{-0.3}$ | ✓ | ✓ | – | ✓ | ✓ |

# Residual Tests (LIGO/Virgo 2019)

- **Model**: best fitted model
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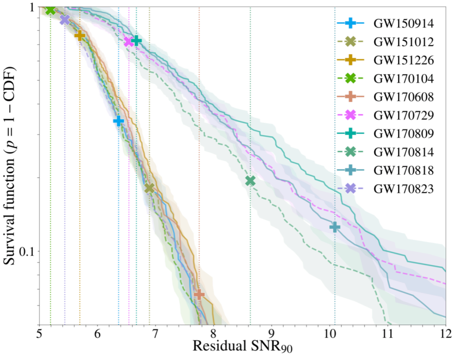
- **Model**: best fitted model
- Residual = Data − Model
- Residual tests: consistent with noise distribution!

| Event | IFOs | Residual $SNR_{90}$ | Fitting factor | $p$-value |
|---|---|---|---|---|
| GW150914 | HL | 6.4 | $\geq 0.97$ | 0.34 |
| GW151012 | HL | 6.9 | $\geq 0.81$ | 0.18 |
| GW151226 | HL | 5.7 | $\geq 0.91$ | 0.76 |
| GW170104 | HL | 5.2 | $\geq 0.94$ | 0.97 |
| GW170608 | HL | 7.8 | $\geq 0.90$ | 0.07 |
| GW170729 | HLV | 6.5 | $\geq 0.87$ | 0.72 |
| GW170809 | HLV | 6.7 | $\geq 0.91$ | 0.73 |
| GW170814 | HLV | 8.6 | $\geq 0.90$ | 0.19 |
| GW170818 | HLV | 10.1 | $\geq 0.78$ | 0.13 |
| GW170823 | HL | 5.4 | $\geq 0.92$ | 0.89 |



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- Parameter estimation *separately* with inpsiral and merger + ringdown

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- Parameter estimation *separately* with inpsiral and merger + ringdown
- Check consistency!

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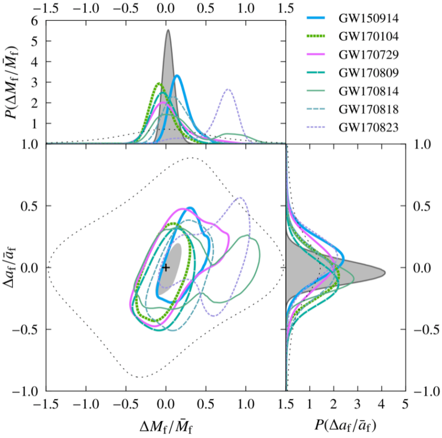
- Parameter estimation *separately* with inpsiral and merger + ringdown
- Check consistency!

| Event | $f_c$ [Hz] | $\rho_{\rm IMR}$ | $\rho_{\rm insp}$ | $\rho_{\rm post-insp}$ | GR quantile [%] |
|---|---|---|---|---|---|
| GW150914 | 132 | 25.3 | 19.4 | 16.1 | 55.5 |
| GW170104 | 143 | 13.7 | 10.9 | 8.5 | 24.4 |
| GW170729 | 91 | 10.7 | 8.6 | 6.9 | 10.4 |
| GW170809 | 136 | 12.7 | 10.6 | 7.1 | 14.7 |
| GW170814 | 161 | 16.8 | 15.3 | 7.2 | 7.8 |
| GW170818 | 128 | 12.0 | 9.3 | 7.2 | 25.5 |
| GW170823 | 102 | 11.9 | 7.9 | 8.5 | 80.4 |



# Parameterized Tests (LIGO/Virgo 2019)

$$\psi \sim \frac{3}{128\eta}(\pi f M)^{-5/3} \sum_{i=0}^{n} \varphi_i^{\mathrm{GR}} (\pi f M)^{i/3}$$

$$\varphi_i \to (1 + \delta\hat{\varphi}_i)\, \varphi_i^{\mathrm{GR}}$$

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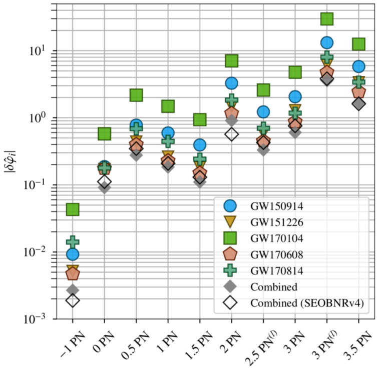
$$\Psi \sim \frac{3}{128\eta}(\pi f M)^{-5/3} \sum_{i=0}^{n} \varphi_i^{\rm GR} (\pi f M)^{i/3}$$

$$\varphi_i \rightarrow (1 + \delta\hat{\varphi}_i)\, \varphi_i^{\rm GR}$$



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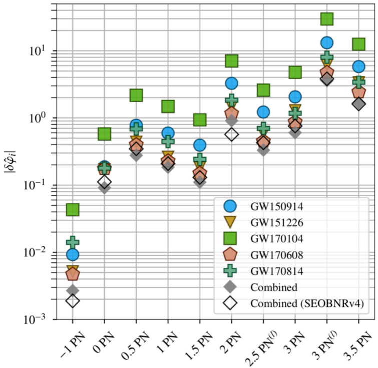


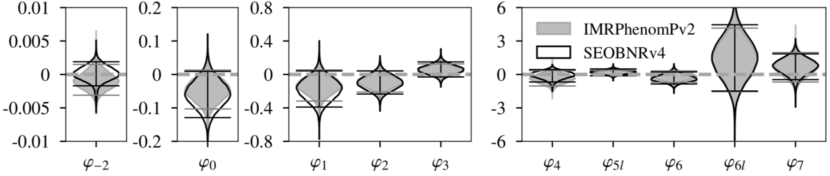

# Graviton Dispersion Relation

- **GR**: massless spin-2 metric field $\Rightarrow \quad E = p$

Will 1998 [gr-qc/9709011]

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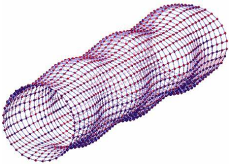

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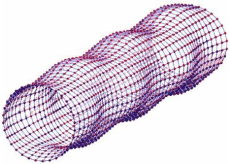
  - In a FRW spacetime, one has

$$\Delta t_a = (1+z)\left[\Delta t_e + \frac{D}{2\lambda_g^2}\left(\frac{1}{f_e^2} - \frac{1}{f_e'^2}\right)\right]$$

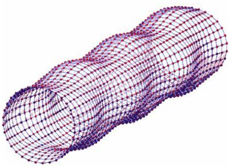


Will 1998 [gr-qc/9709011]

# Propagation of GWs

- The extra time delay results in a phase shift in $h(f) \propto e^{i\Psi(f)}$

$$\Psi(f) = \Psi_{\rm GR}(f) - \frac{\pi^2 D\mathcal{M}}{\lambda_g^2(1+z)}(\pi\mathcal{M}f)^{-1}$$



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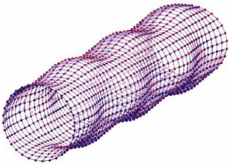

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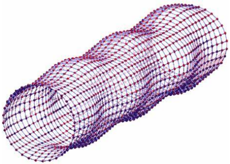
- On the other hand, the waveform is *totally* calculable and deterministic in GR
- Therefore, GWs provide an observational window to the dispersion relation of graviton



# Propagation of GWs w/ Lorentz Violation

- Lorentz violation occurs in a few quantum gravity candidate theories [Kostelecký & Samuel 1989; Amelino-Camelia 2013]

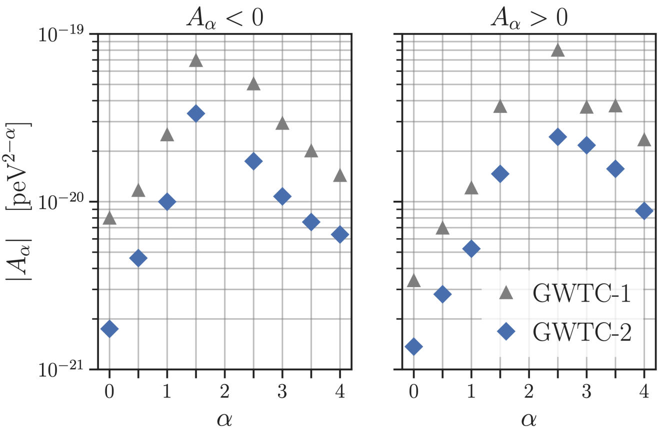
# Propagation of GWs w/ Lorentz Violation

- Lorentz violation occurs in a few quantum gravity candidate theories [Kostelecký & Samuel 1989; Amelino-Camelia 2013]
- Dispersion relation of GWs with isotropic Lorentz violation [Mirshekari, Yunes, Will 2012]

$$E^2 = p^2 c^2 + m_g^2 c^4 + \mathbb{A} p^\alpha c^\alpha$$

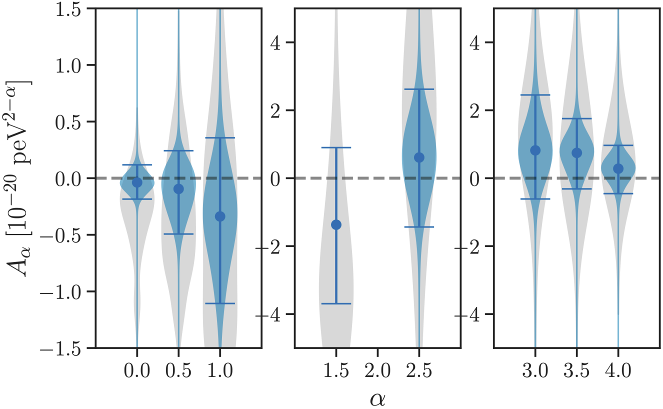
where $m_g$ is the graviton mass; $\mathbb{A}$ and $\alpha$ are two Lorentz-violating parameters

# Lorentz-violating Propagation of GWs



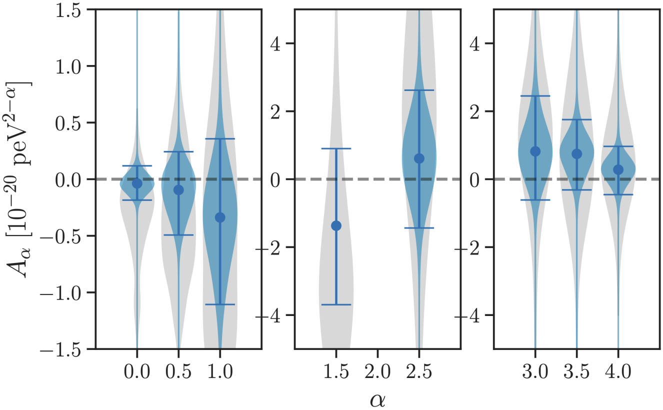

LIGO/Virgo 2021 [2010.14529]

# Lorentz-violating Propagation of GWs




LIGO/Virgo 2021 [2010.14529]

# Lorentz-violating Propagation of GWs




But... such a combination is **problematic** in general

LIGO/Virgo 2021 [2010.14529]

# Standard-model Extension

- The most generic linearized gravity has the Lagrangian

[Kostelecký & Mewes 2018]

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma i_1 i_2 \cdots i_{d-2}} \partial_{i_1} \partial_{i_2} \cdots \partial_{i_{d-2}}$

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- It predicts a modified dispersion relation for GWs

$$\omega = \left(1 - \zeta^0 \pm \sqrt{\left(\zeta^1\right)^2 + \left(\zeta^2\right)^2 + \left(\zeta^3\right)^2}\right) p$$

# Standard-model Extension

$$\omega = \left(1 - \zeta^0 \pm \sqrt{\left(\zeta^1\right)^2 + \left(\zeta^2\right)^2 + \left(\zeta^3\right)^2}\right) p$$

$$\zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(I)jm}$$

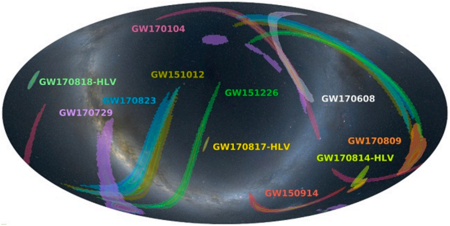
$$\zeta^1 \mp i\zeta^2 = \sum_{djm} \omega^{d-4}_{\pm 4} Y_{jm}(\hat{\boldsymbol{n}}) \left[k^{(d)}_{(E)jm} \pm i k^{(d)}_{(B)jm}\right]$$

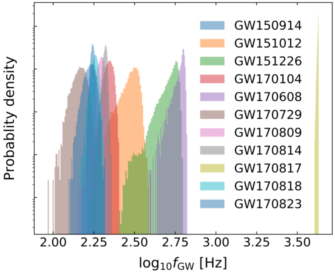
$$\zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(V)jm}$$

- Therefore, gravitons of different **polarization** or **frequency**, coming from different **directions** have different velocity

# GWTC-1 Events

**A simplified/naive approach**: $|\omega_{\rm GW}\Delta t| \leq 2\pi/\rho$

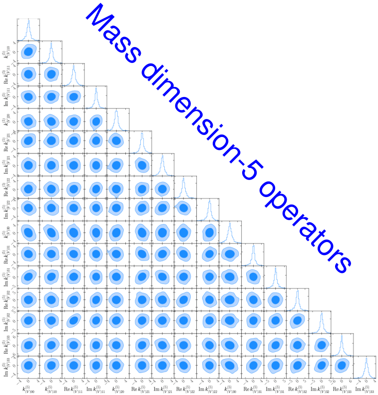


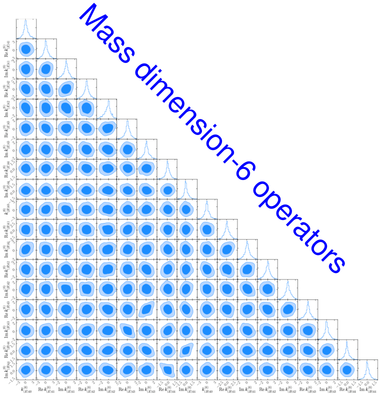




We have all the information available to perform the test

Shao 2020 [2002.01185]

# Anisotropic Birefringence Combined Search

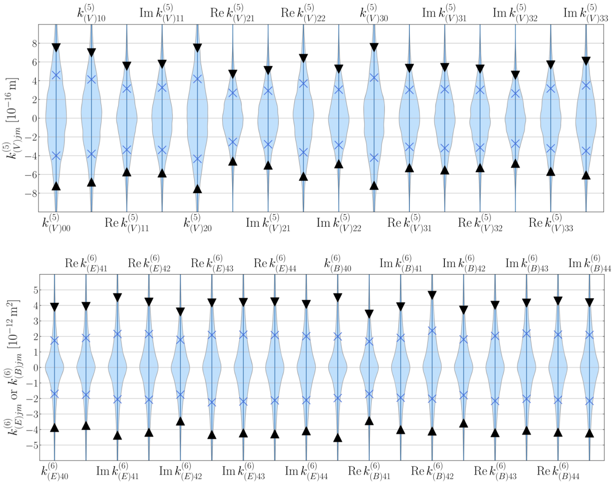



We have all the information available to perform the test

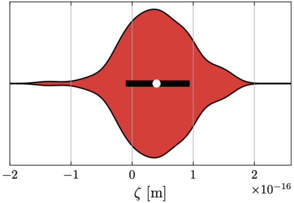
Shao 2020 [2002.01185]

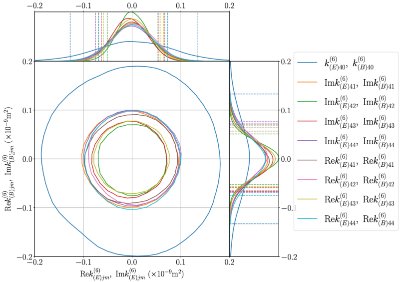
# Combined Results from GWTC-2




Wang, **Shao**, Liu 2021, ApJ [2108.02974]

# Combined Results from GWTC-3



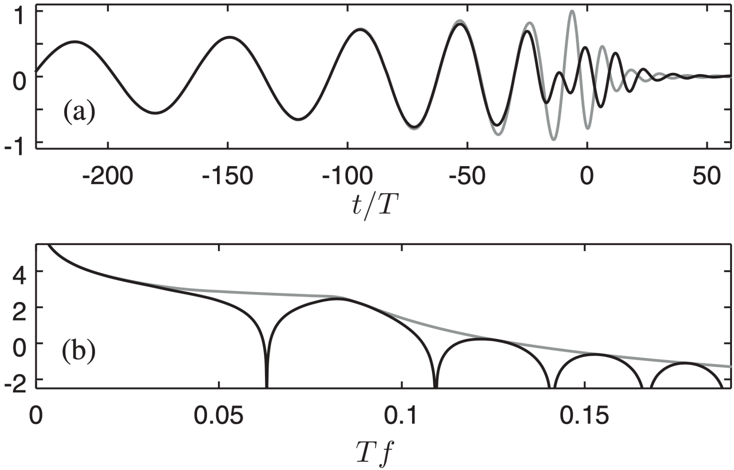

Isotropic violation

Anisotropic violation

Zhao, Cao, Wang, ApJ [2201.02813]

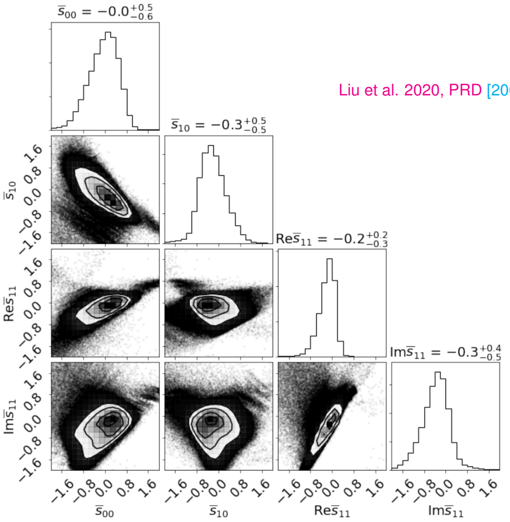
Niu, Zhu, Zhao [2202.05092]

# Matched Filter Analysis




Mewes 2019, PRD [1905.00409]

# Monte Carlo Markov-Chain Runs




Liu et al. 2020, PRD [2005.03121]

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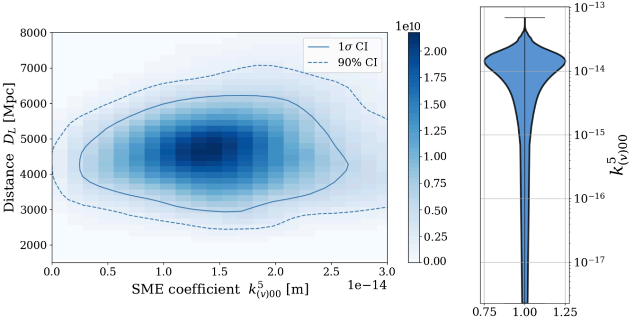
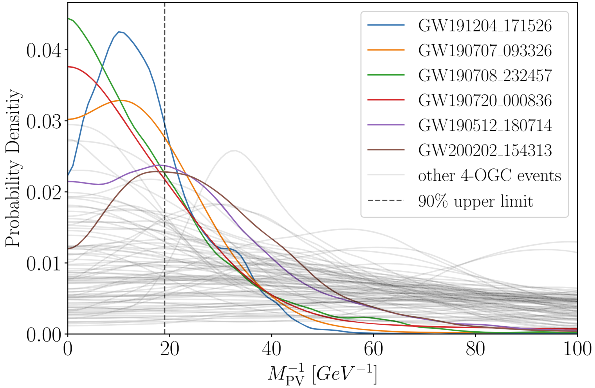



A simulated 50 $M_\odot$–50 $M_\odot$ BBH at 5 Gpc

O'Neal-Ault, et al. 2021, Universe [2108.06298]

# Monte Carlo Markov-Chain Runs

Analysis with isotropic parity violation




Wang, Brown, **Shao**, Zhao 2022, PRD [2109.09718]

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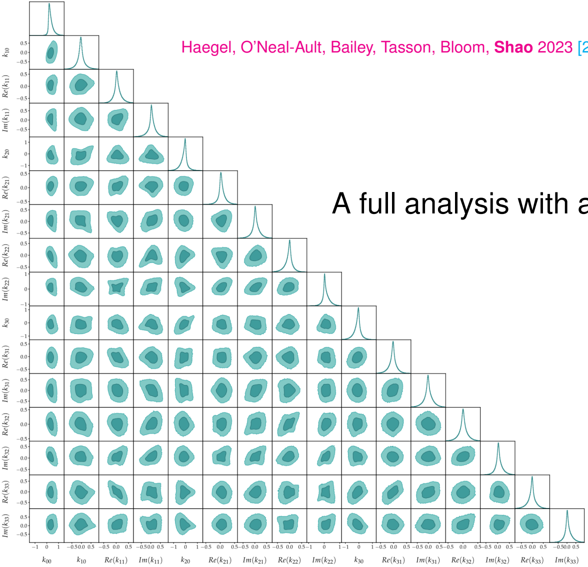
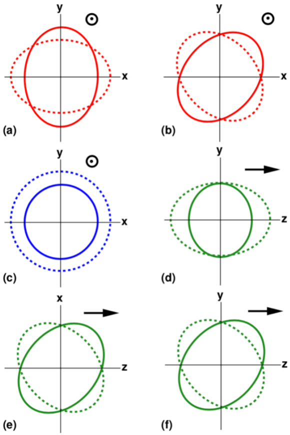



Haegel, O'Neal-Ault, Bailey, Tasson, Bloom, **Shao** 2023 [2210.04481]

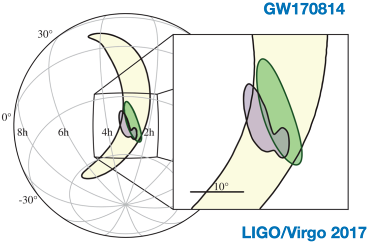
A full analysis with anisotropy

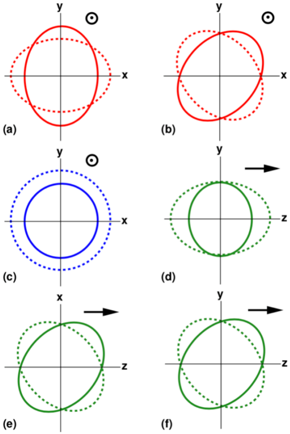
# Polarization Tests (LIGO/Virgo 2019)



# Polarization Tests (LIGO/Virgo 2019)

- **Triple** detections
  - GW170729, GW170809, GW170814, GW170818



# Polarization Tests (LIGO/Virgo 2019)

- **Triple** detections
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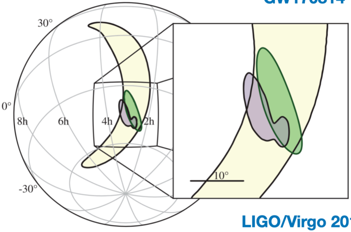
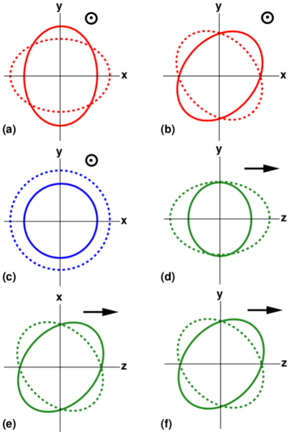
- Bayes factors: $10^1$–$10^2$
  - tensor vs vector
  - tensor vs scalar

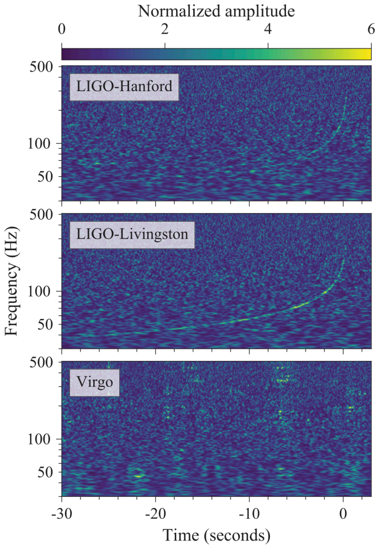
GW170814




LIGO/Virgo 2017



# Waveform: tidal deformability (LIGO/Virgo 2017)



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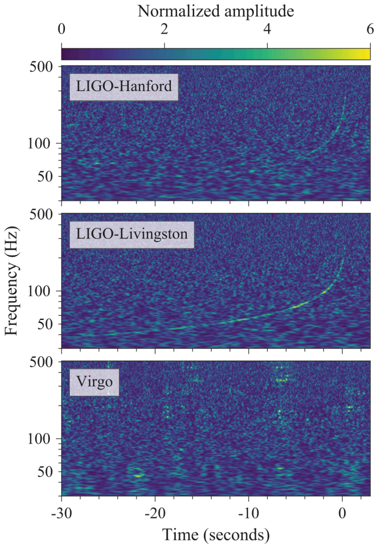
- **SEOBNRv4T**
  - tidal deformability
  - equation of state



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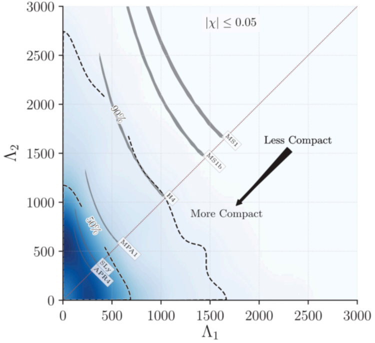
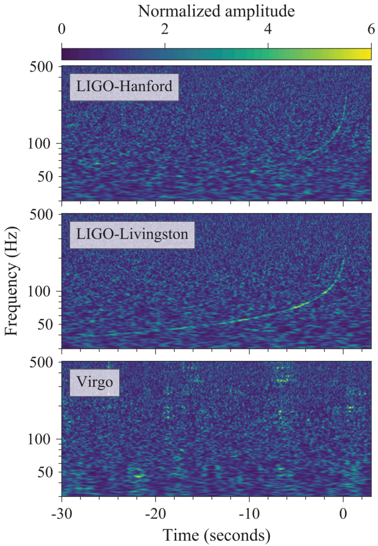
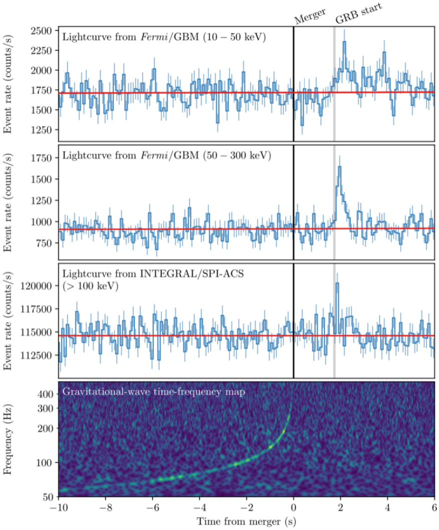


# Speed of Gravity (LIGO/Virgo 2017)

- The famous 1.7 sec

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\rm EM}} \leqslant +7 \times 10^{-16}$$



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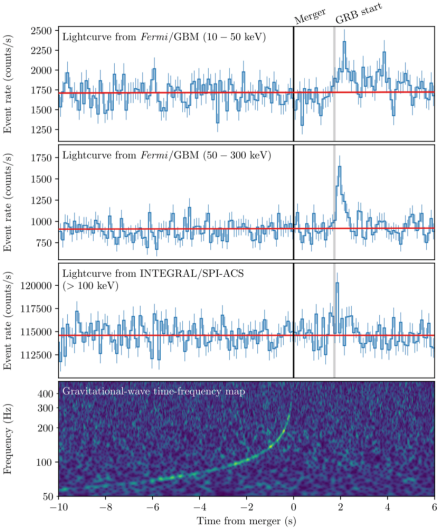
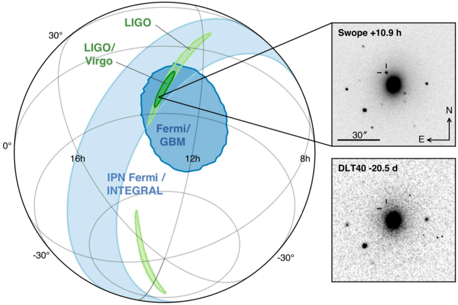
- strong implications on cosmological models
  - ... tons of PRL papers



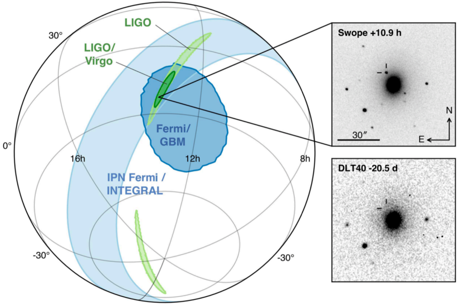
# Polarization Tests (LIGO/Virgo 2019)

- Precise localization: NGC 4993



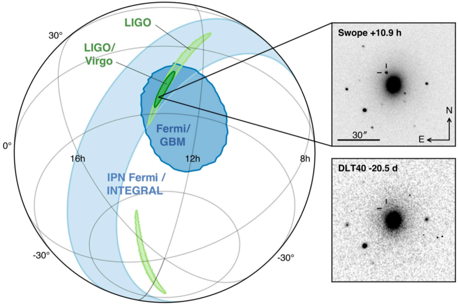
# Polarization Tests (LIGO/Virgo 2019)

- Precise localization: NGC 4993
- Bayes factors
  - tensor vs vector: $10^{21}$
  - tensor vs scalar: $10^{23}$



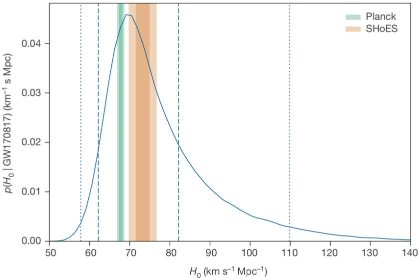
# Polarization Tests (LIGO/Virgo 2019)

- Precise localization: NGC 4993
- Bayes factors
  - tensor vs vector: $10^{21}$
  - tensor vs scalar: $10^{23}$
- **much** tighter than BBHs

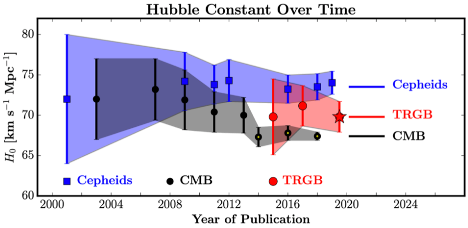


# Hubble Constant (LIGO/Virgo 2017)

- By simultaneously measuring redshift and luminosity distance, GWs provide an independent way to probe cosmological parameters [Schutz 1986]



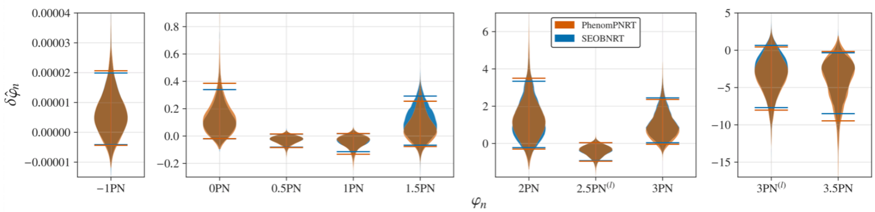

LIGO/Virgo 2017




arXiv:1907.05922 (ApJ, in press)

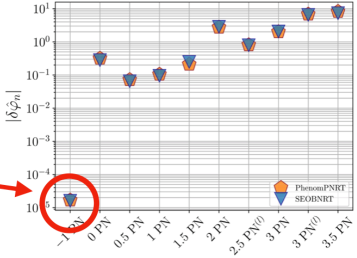
The Carnegie-Chicago Hubble Program

# Parameterized Tests (LIGO/Virgo 2019)




$$\phi \sim \frac{3}{128\eta}(\pi f M)^{-5/3} \sum_{i=0}^{n} \varphi_i^{\mathrm{GR}} (\pi f M)^{i/3}$$

$$\varphi_i \to \left(1 + \delta\hat{\varphi}_i\right) \varphi_i^{\mathrm{GR}}$$




**A tight constraint on dipole radiation**

LIGO/Virgo 2019

# Scalar-Tensor Gravity

$$S = \frac{c^4}{16\pi G_*} \int \frac{\mathrm{d}^4 x}{c} \sqrt{-g_*} \left[ R_* - 2 g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] + S_m \left[ \psi_m ; A^2(\varphi) g^*_{\mu\nu} \right]$$

Damour & Esposito-Farèse 1992; 1993; 1996

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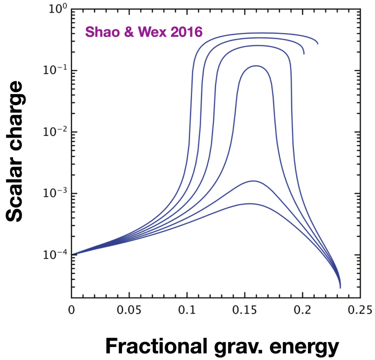
- A class of cosmologically well-motivated scalar-tensor theories, that are solely described by two theory parameters: $\alpha_0$ & $\beta_0$

$$V(\varphi) = 0$$

$$A(\varphi) = \exp\left(\beta_0 \varphi^2 / 2\right), \quad \alpha_0 = \beta_0 \varphi_0$$

Damour & Esposito-Farèse 1992; 1993; 1996

# Scalar-Tensor Gravity

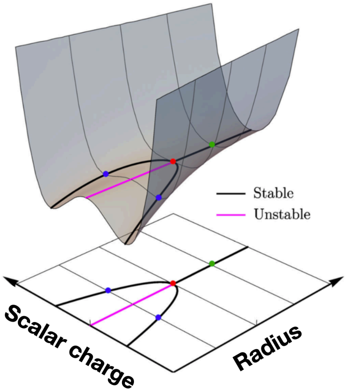



*Nonperturbative* **spontaneous scalarization**

could happen for isolated neutron stars

Damour & Esposito-Farèse 1992; 1993; 1996

# Scalar-Tensor Gravity

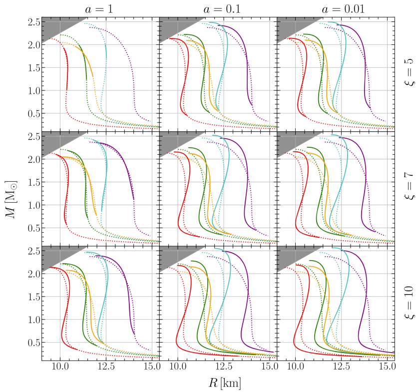



**Strong-field** behavior is analogous to **Landau's phase transition** after a critical point

Damour & Esposito-Farèse 1996; Esposito-Farèse 2004; Sennett, Shao, Steinhoff 2017

# Massive Scalar-Tensor Gravity

- When a mass term is included, say $V(\varphi) \sim m^2\varphi^2$, a Yukawa-type suppression happens for the deviation

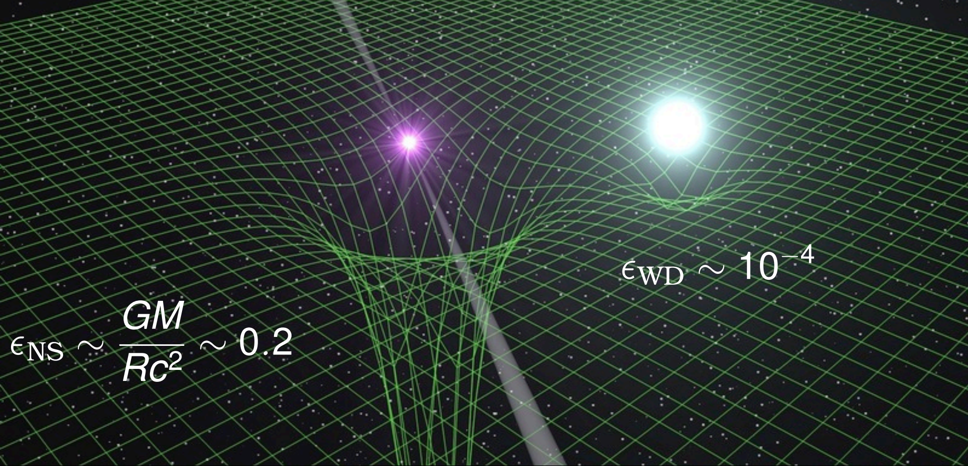



Ramazanoğlu & Pretorius 2016; Xu, Gao, Shao 2020; Hu, Gao, Xu, Shao 2021; Xu, Gao, Shao 2022

**Strong-field** gravity can be **VERY** different from **weak-field** gravity

# Scalar-Tensor Gravity

Due to their **asymmetry**, neutron-star white-dwarf systems provide stringent limits on dipole radiation $\dot{P}_b^{\text{dipole}} \propto (\alpha_{\rm NS} - \alpha_0)^2$

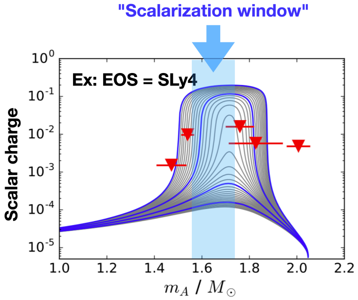


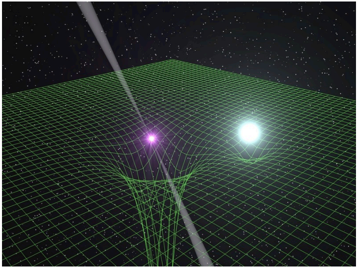
$$\epsilon_{\rm WD} \sim 10^{-4}$$

$$\epsilon_{\rm NS} \sim \frac{GM}{Rc^2} \sim 0.2$$

# Combination of Multiple NS-WD Binaries

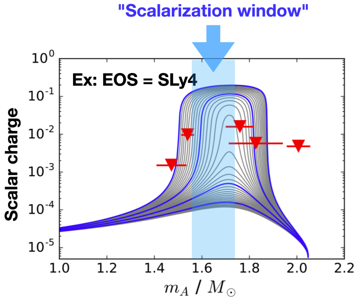
- Strong-field effects could happen at different NS masses for different EOSs [Shibata et al. 2014, PRD 89:084005]

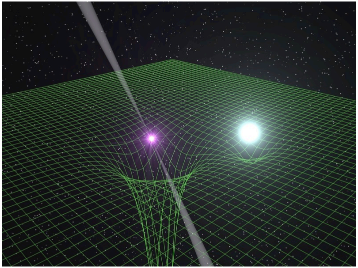


# Combination of Multiple NS-WD Binaries

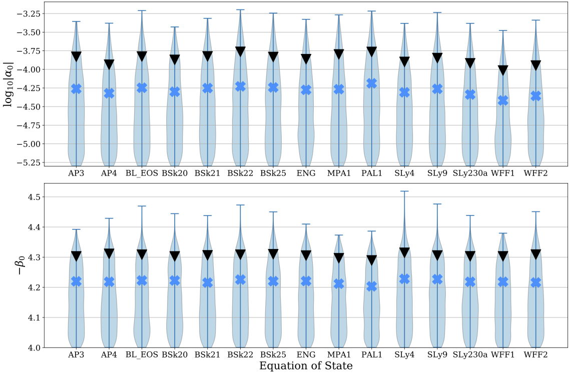
- Strong-field effects could happen at different NS masses for different EOSs [Shibata et al. 2014, PRD 89:084005]
- Combining NS-WDs put the best limits on a class of scalar tensor theories for different EOSs [Shao et al. 2017, PRX 7:041025]



# Combination of Multiple NS-WD Binaries

- Reduced-order surrogate models to speed up Markov-chain Monte Carlo runs: **pySTGROM**,[9] & **pySTGROMX**[10]



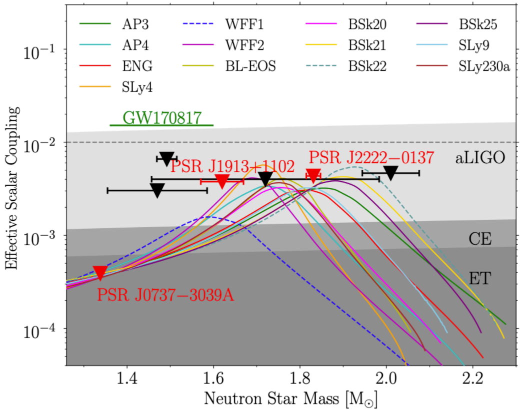

[9] https://github.com/BenjaminDbb/pySTGROM

[10] https://github.com/mh-guo/pySTGROMX

Zhao, Shao, et al. 2019

Guo, Zhao, Shao 2021

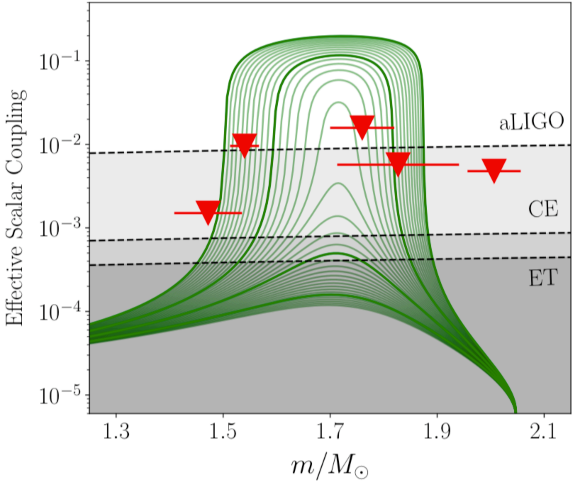
# Combination of Multiple NS-WD Binaries




Scalarization window is closed for $T(\varphi_0, \beta_0)$ theories ($\lesssim 1\%$) with addition of new observations

Zhao et al. 2022 [2201.03771]

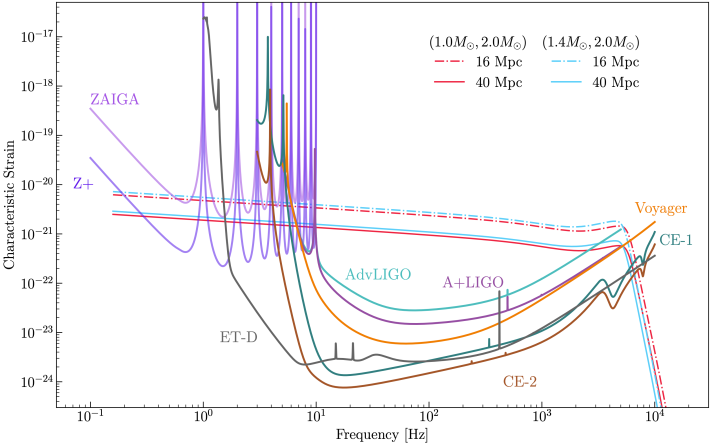
# Gravitational Waves




Will 1994; Damour & Esposito-Farèse 1998; Shao et al. 2017, PRX

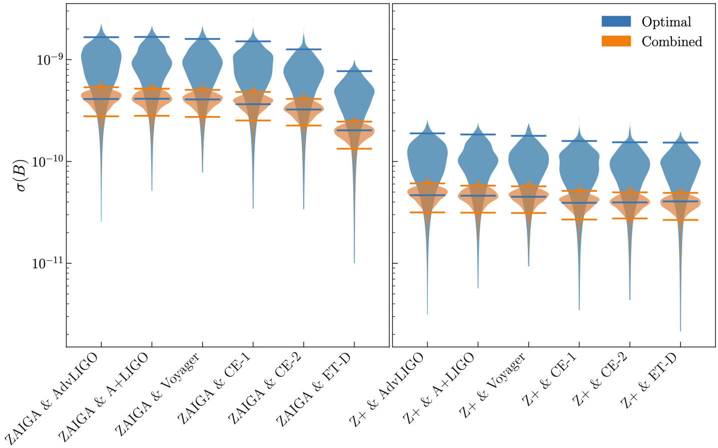
# Gravitational Waves

**laser interferometers & atom interferometers**




Damour & Esposito-Farèse 1998; Zhao, Shao, et al. 2021 [2106.04883]

# Gravitational Waves




Zhao, Shao, et al. 2021 [2106.04883]

# Summary

- **Einstein is still right**
- GWs launch a new era to test gravity
- Hope something new emerges *soon*

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Albert Einstein (1915)

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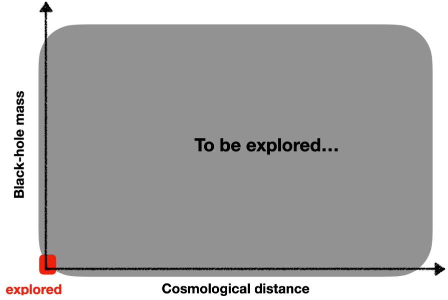


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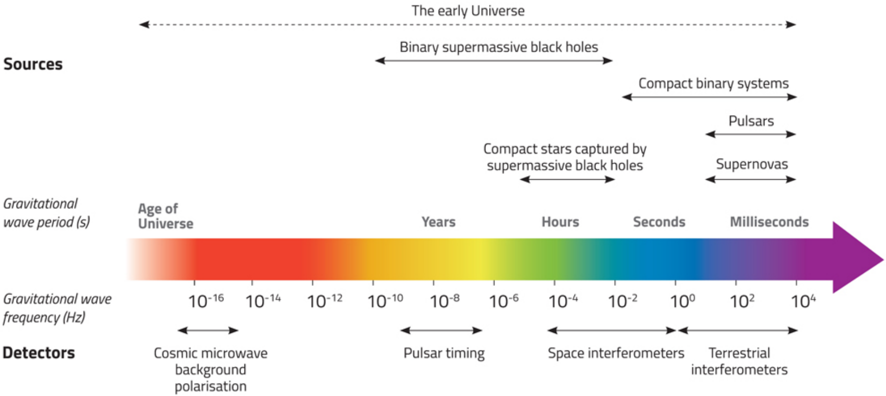
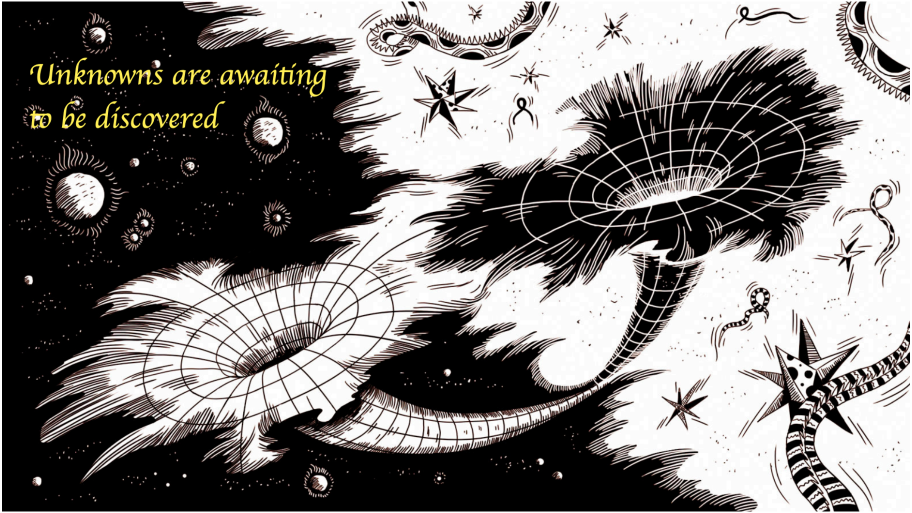



**Only a tiny part of GW spectrum was revealed by now**

**Stay tuned!**




**An exciting era for astronomers & physicists**


THANK YOU FOR LISTENING
ANY QUESTIONS?

# Thank you!