电弱相变与粒子物理

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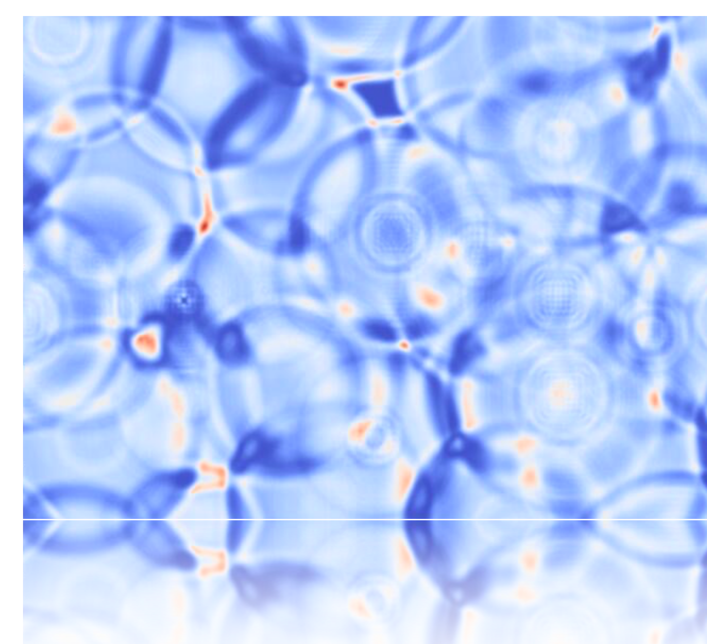
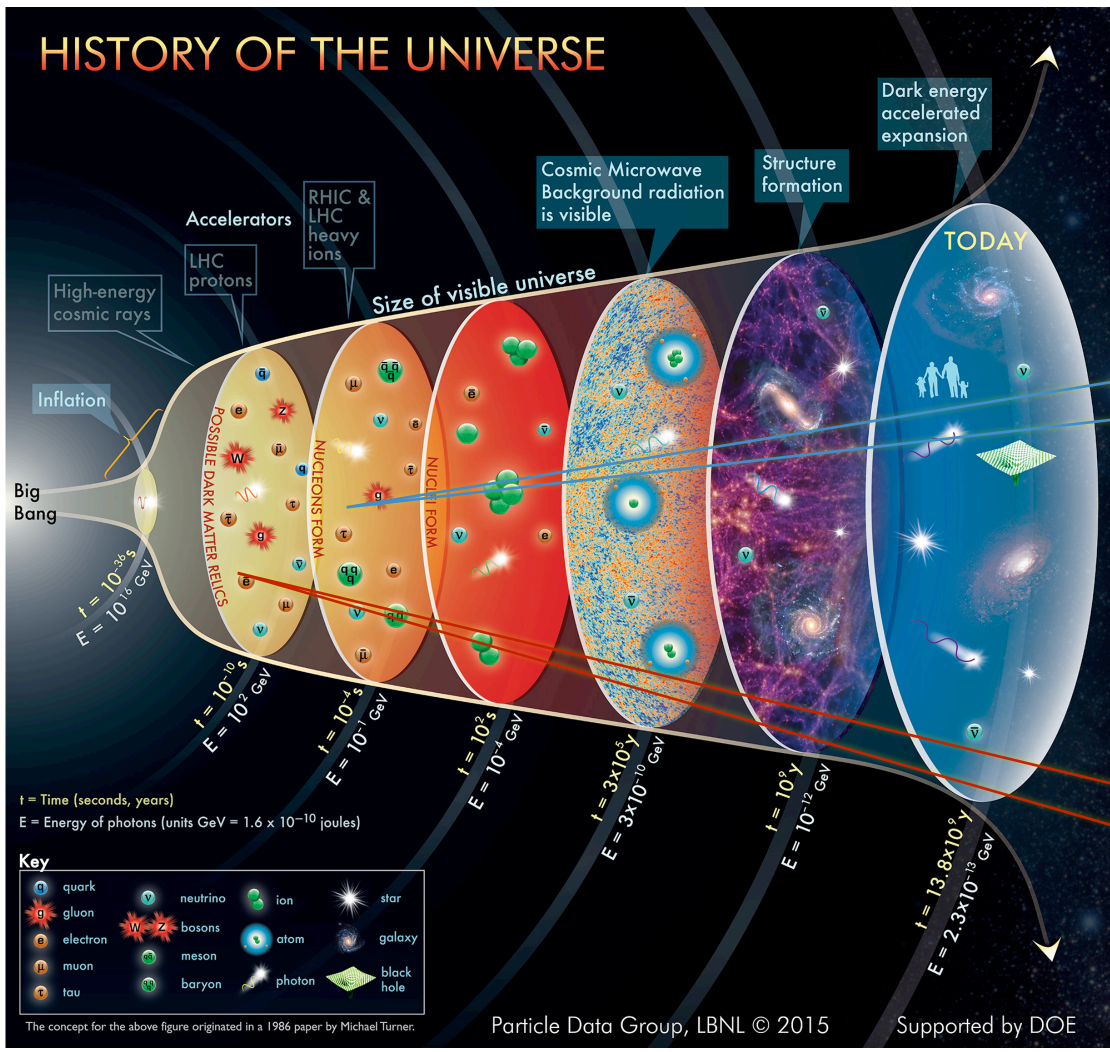
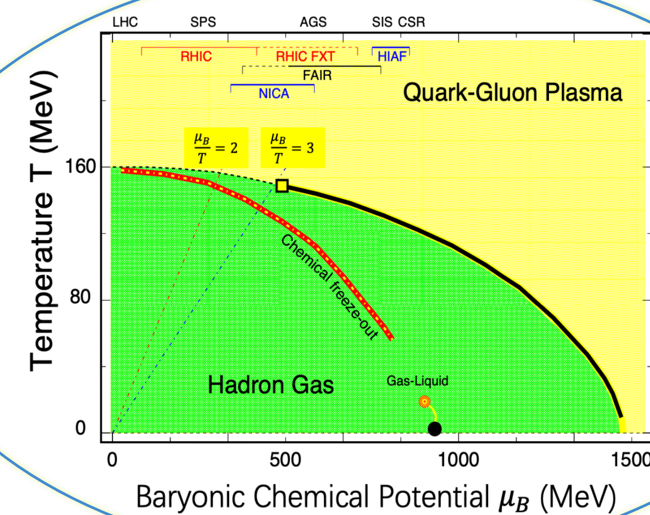


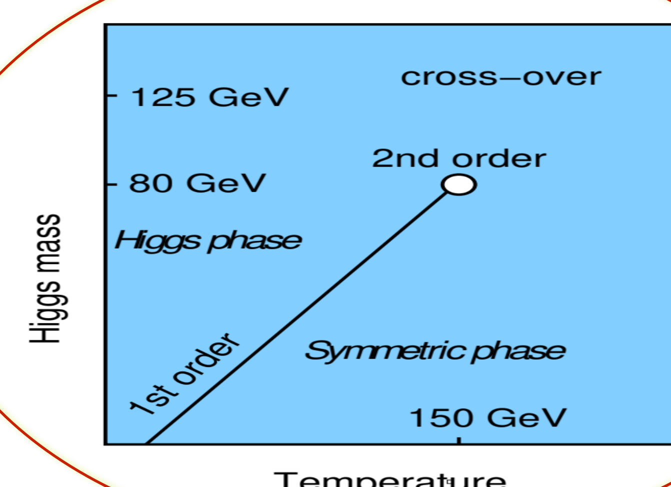
研究背景与动机

对称展示宇宙之美，不对称生成宇宙之实！——李政道





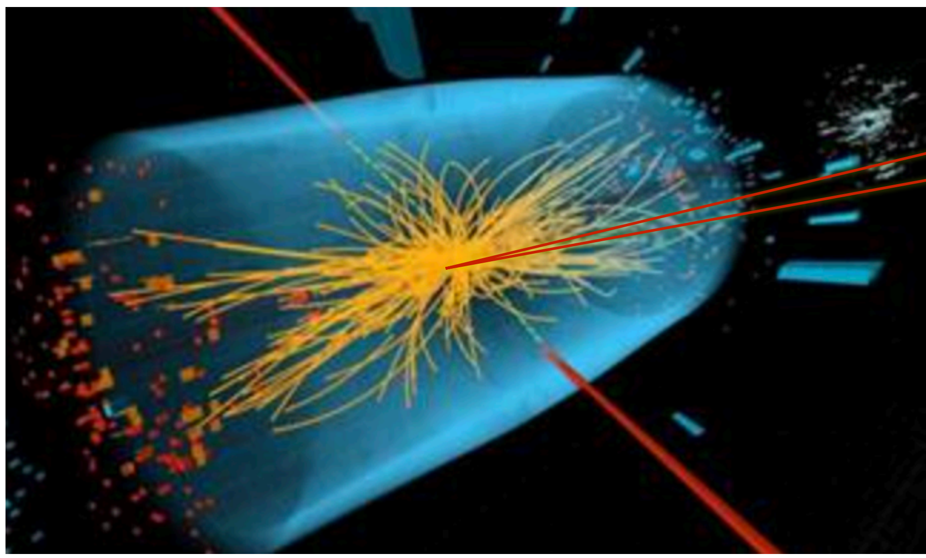
QCD
相图

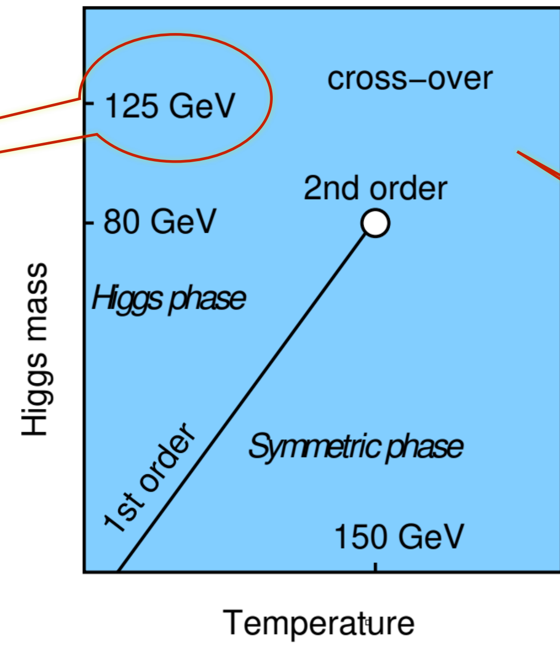


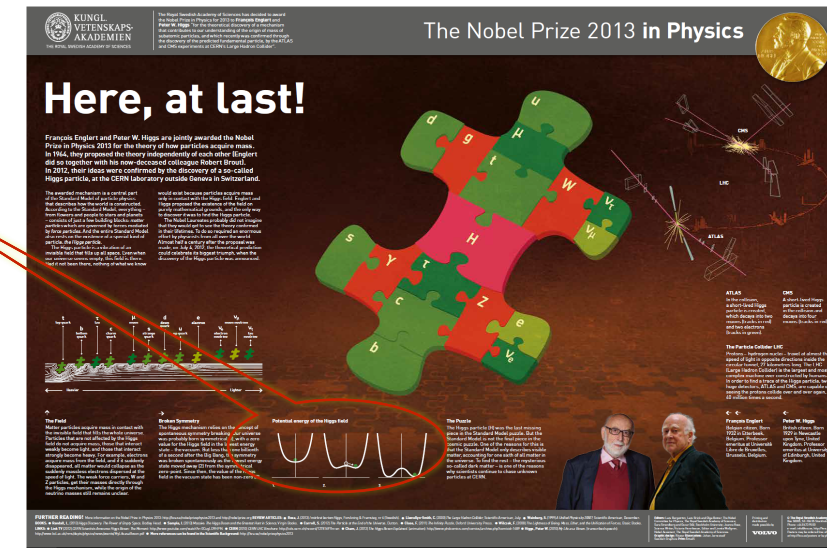
电弱
相变

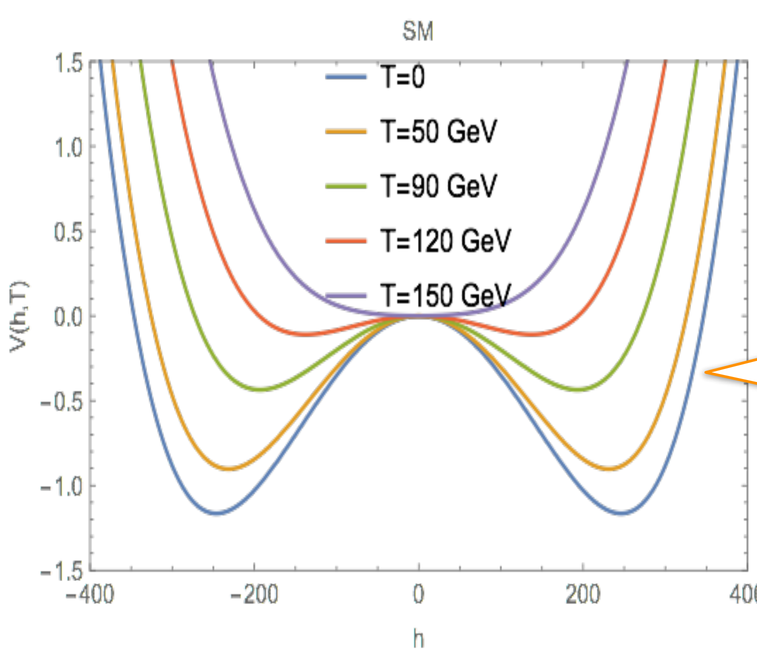
研究背景与动机

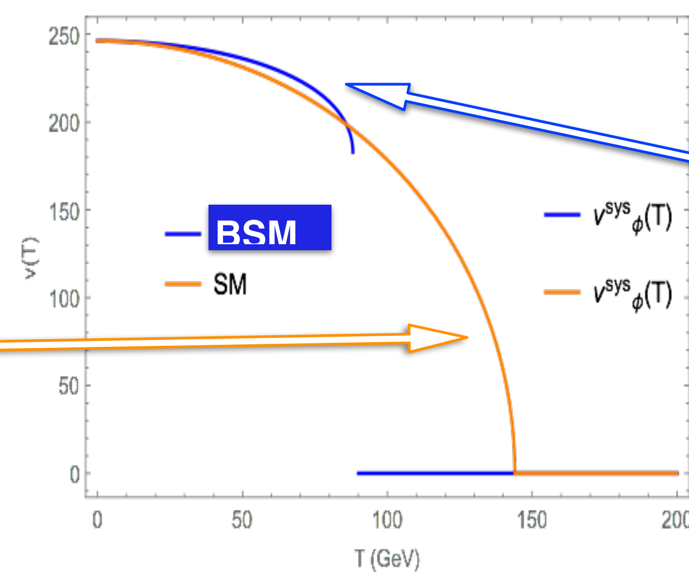
电弱对称性破缺的热历史是什么?

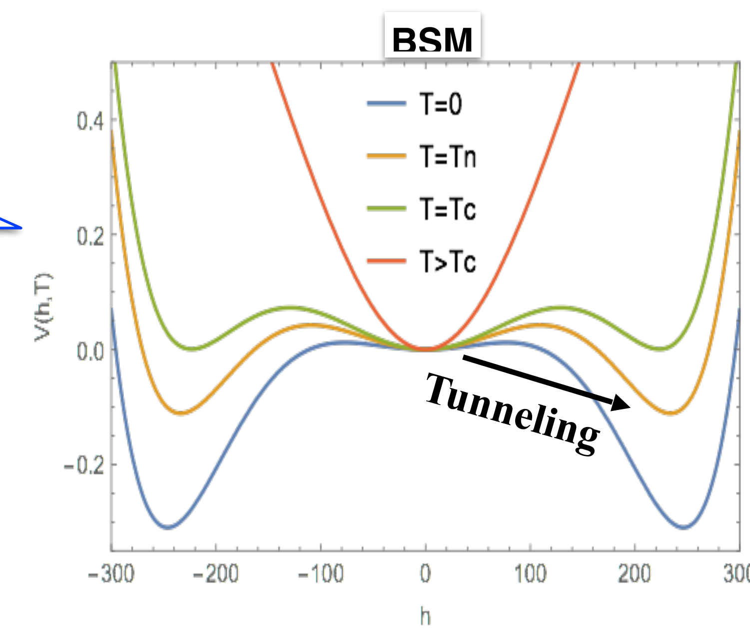








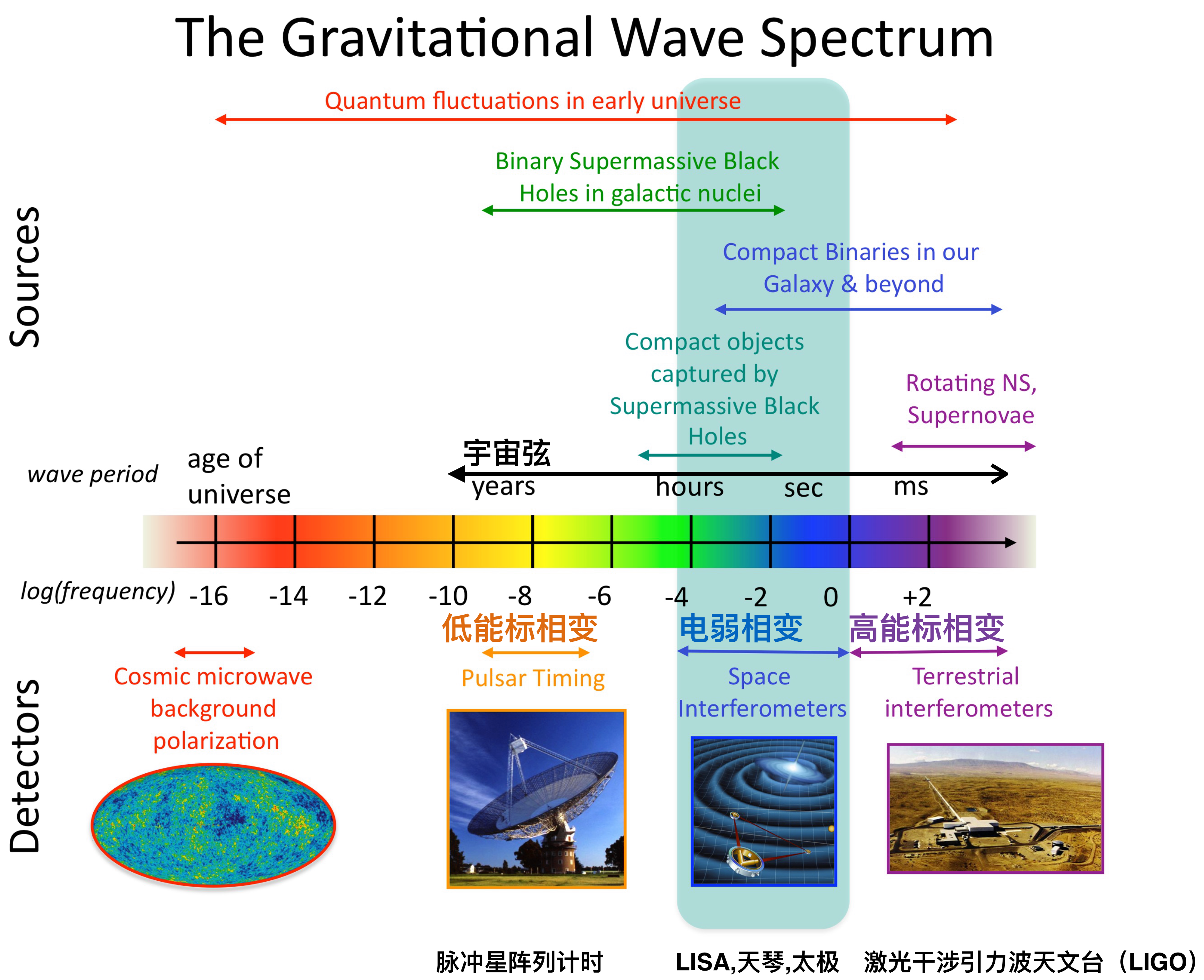




平滑过度

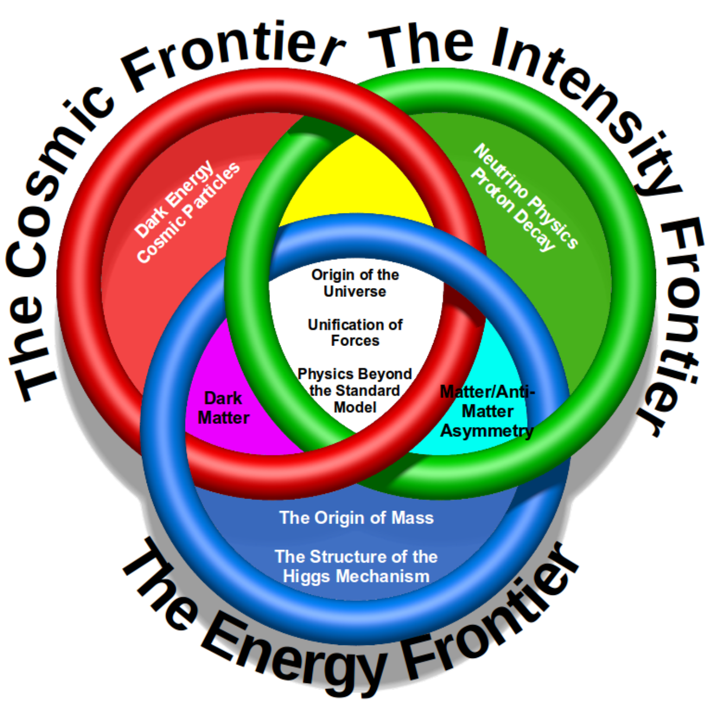
一阶相变

随机引力波探测开启了探索早期宇宙背后新物理的一个新的窗口



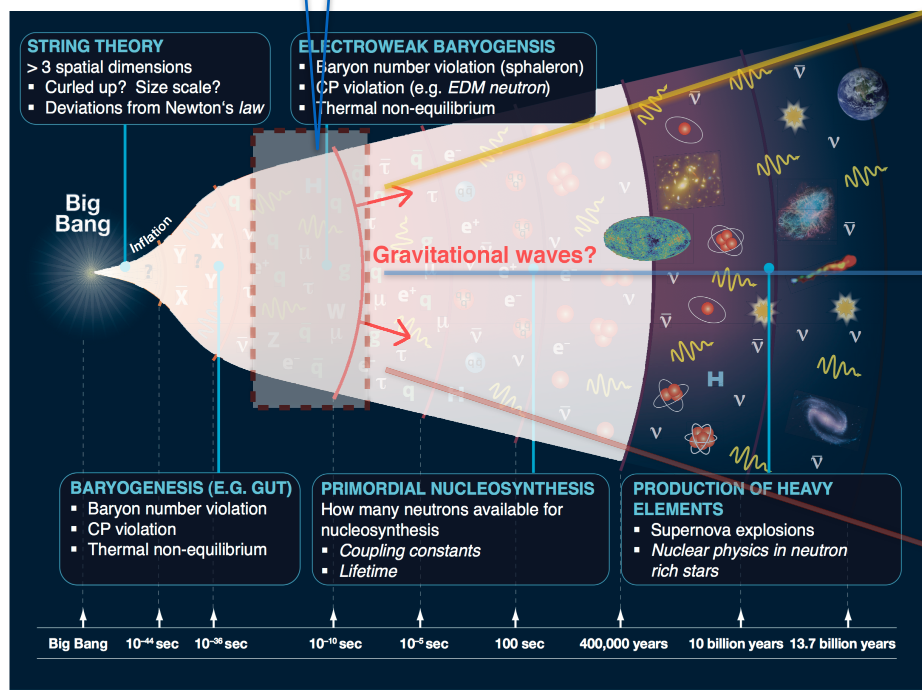
研究背景与动机

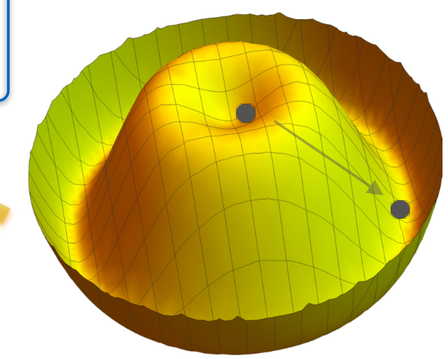
粒子物理三个前沿



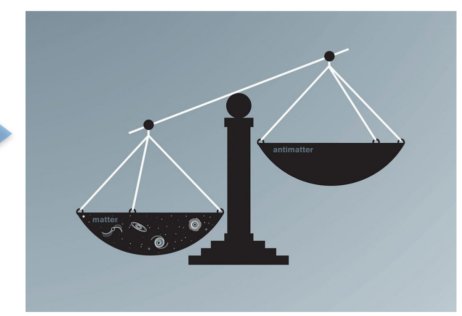
强一阶电弱相变

相关粒子物理

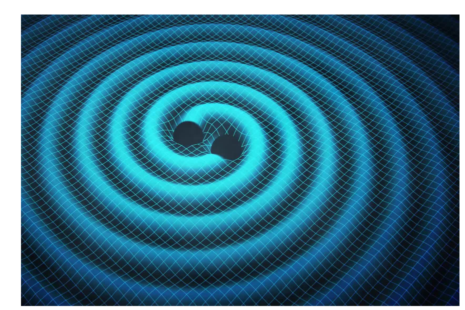




电弱对称性破缺，基本粒子质量起源



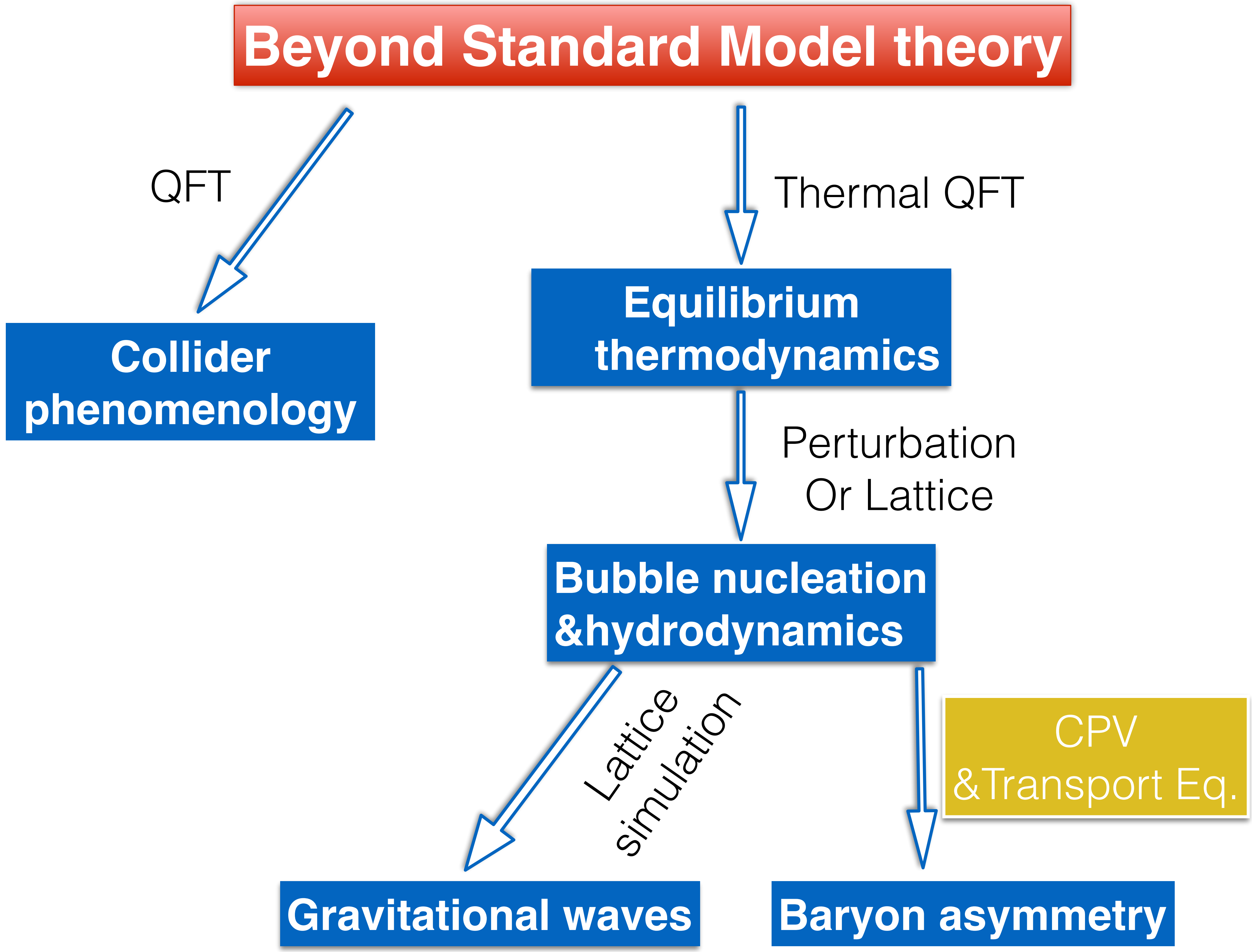
宇宙正反物质不对称



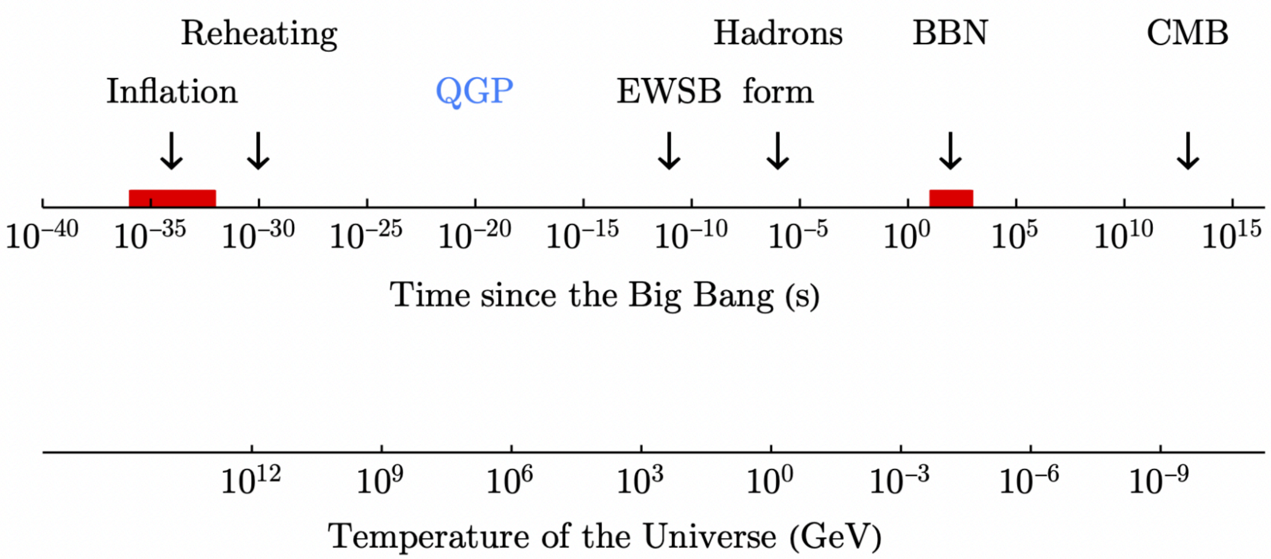
相变引力波，检验爱因斯坦广义相对论

粒子物理描述早期宇宙演化

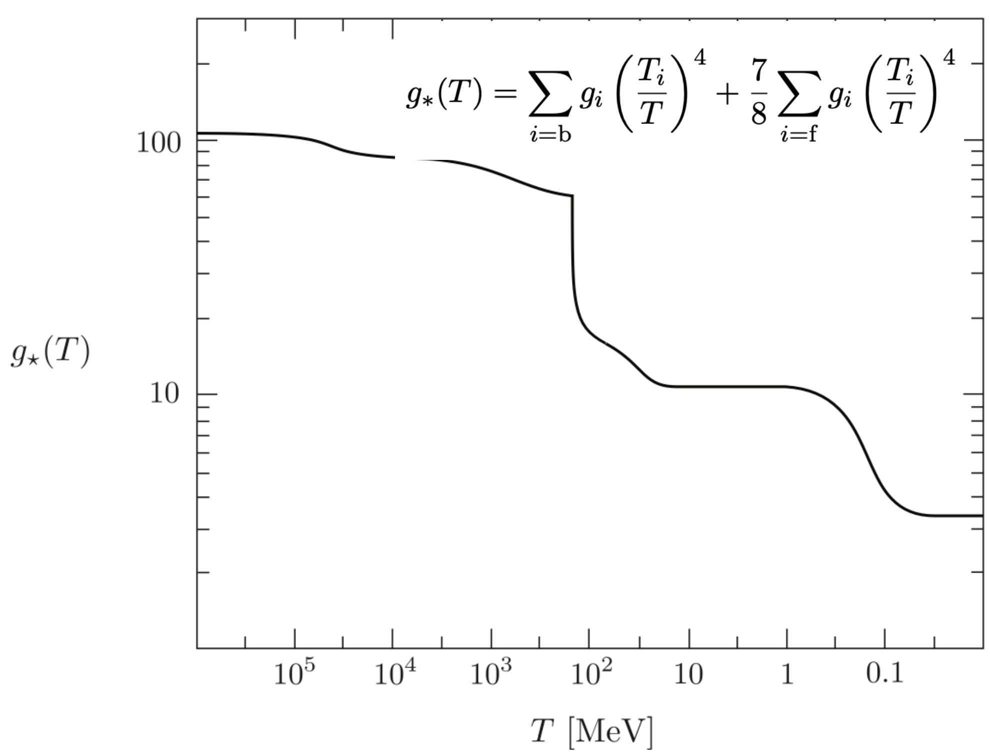
电弱相变相关粒子物理宇宙学



Key Events in the early Universe



Radiation dominate Universe: $H = \frac{1}{2t} = \sqrt{\frac{8\pi}{3}\frac{\rho_{\rm rad}}{M_p^2}} = \sqrt{\frac{8\pi}{3}\frac{\pi^2}{30}g_*\frac{T^4}{M_p^2}} = \sqrt{\frac{8\pi^3}{90}g_*}\frac{T^2}{M_p}$



Event	time t	redshift z	temperature T
Inflation	10^{-34} s	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.7 Gyr	0	0.24 meV

Effective potential at zero temperature

Action $$S[\phi] = \int d^4x \mathcal{L}\{\phi(x)\}$$

The generating functional (vacuum-to-vacuum amplitude):

$$Z[j] = \langle 0_{\rm out} \mid 0_{\rm in}\rangle_j \equiv \int d\phi \exp\{i(S[\phi] + \phi j)\} \qquad \phi j \equiv \int d^4x \phi(x) j(x)$$

The connected generating functional W[j] defined as:

$$Z[j] \equiv \exp\{iW[j]\}$$

The effective action $\Gamma[\phi]$ as the Legendre transformation:

$$\Gamma[\overline{\phi}] = W[j] - \int d^4x \frac{\delta W[j]}{\delta j(x)} j(x) \qquad \overline{\phi}(x) = \frac{\delta W[j]}{\delta j(x)}$$

$$\Longrightarrow \quad \frac{\delta \Gamma[\overline{\phi}]}{\delta \overline{\phi}} = \frac{\delta W[j]}{\delta j}\frac{\delta j}{\delta \overline{\phi}} - j - \overline{\phi}\frac{\delta j}{\delta \overline{\phi}} = -j$$

$$\left.\frac{\delta \Gamma[\overline{\phi}]}{\delta \overline{\phi}}\right|_{j=0} = 0$$

▸ Effective potential at zero temperature

Expand Z[j] (W[j]) in a power series of j, to obtain its representation in terms of Green functions $G_{(n)}$ (connected Green functions $G_{(n)}{}^{c}$)

$$Z[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots d^4x_n j(x_1) \ldots j(x_n) G_{(n)}(x_1, \ldots, x_n)$$

$$iW[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots d^4x_n j(x_1) \ldots j(x_n) G^c_{(n)}(x_1, \ldots, x_n)$$

The effective action can be expanded as

$$\Gamma[\overline{\phi}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \overline{\phi}(x_1) \ldots \overline{\phi}(x_n) \Gamma^{(n)}(x_1, \ldots, x_n)$$

$\Gamma^{(n)}$ are the one-particle irreducible (1PI) Green functions

Fourier transformation

$$\Gamma^{(n)}(x) = \int \prod_{i=1}^{n} \left[\frac{d^4p_i}{(2\pi)^4} \exp\{ip_i x_i\} \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \Gamma^{(n)}(p)$$

$$\tilde{\phi}(p) = \int d^4x e^{-ipx} \overline{\phi}(x)$$

$$\Gamma[\overline{\phi}] = \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} \left[\frac{d^4p_i}{(2\pi)^4} \tilde{\phi}(-p_i) \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \Gamma^{(n)}(p_1, \ldots, p_n) \qquad (1)$$

▸ Effective potential at zero temperature

Translationally invariant theory, with ϕc being constant $\overline{\phi}(x) = \phi_c$

Define the effective potential $V_{eff}(\phi_c)$ as

$$\Gamma[\phi_c] = -\int d^4x V_{\text{eff}}(\phi_c) \qquad (2)$$

Using the definition of Dirac δ-function

$$\delta^{(4)}(p) = \int \frac{d^4x}{(2\pi)^4} e^{-ipx}$$

We get

$$\tilde{\phi}_c(p) = (2\pi)^4 \phi_c \delta^{(4)}(p).$$

Inserting into EQ.(1), the effective action for constant field configurations recast the form of

$$\Gamma(\phi_c) = \sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n (2\pi)^4 \delta^{(4)}(0) \Gamma^{(n)}(p_i = 0) = \sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0) \int d^4x \qquad (3)$$

(2,3) ⇒

$$V_{\text{eff}}(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0) \qquad (4)$$

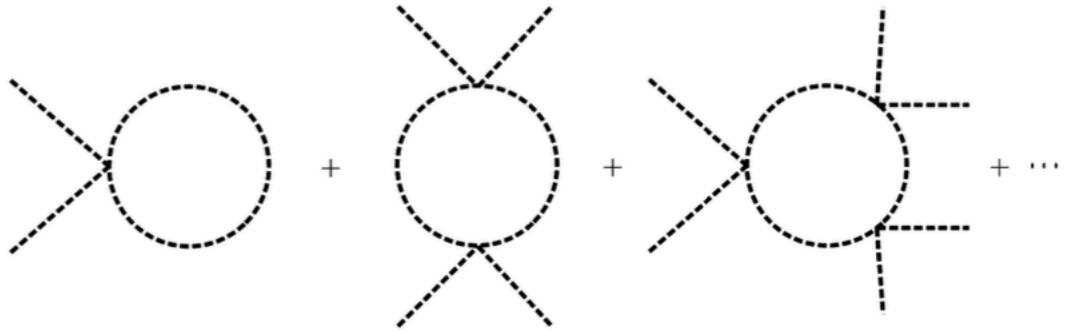
Expanding in powers of momentum, about the point where all external momenta vanish

$$\Gamma[\overline{\phi}] = \int d^4x \left[-V_{\text{eff}}(\overline{\phi}) + \frac{1}{2} (\partial_\mu \overline{\phi}(x))^2 Z(\overline{\phi}) + \cdots \right]$$

▸ Effective potential at zero temperature

An example

Tree-level potential $$V_0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$



In momentum space the scalar field is $$\phi_c(p) = (2\pi)^4\phi_c\delta^4(p)$$

Recall EQ.(4), we get one-loop potential:

$$\begin{aligned} V_1(\phi_c) &= i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left[\frac{\lambda\phi_c^2/2}{p^2-m^2+i\epsilon}\right]^n \\ &= -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\log\left[1-\frac{\lambda\phi_c^2/2}{p^2-m^2+i\epsilon}\right] \end{aligned}$$

After Wick rotation: $$V_1(\phi_c) = \frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\log\left[1+\frac{\lambda\phi_c^2/2}{p_E^2+m^2}\right]$$

$$p^0 = ip_E^0,\ p_E = (-ip^0,\vec{p}\,),\ p^2 = (p^0)^2 - \vec{p}^{\,2} = -p_E^2$$

1-loop effective potential is $$V_1(\phi_c) = \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\log\left[p^2+m^2(\phi_c)\right] \quad (5)$$

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) + V_1(\phi_c)$$

shifted mass : $$m^2(\phi_c) = m^2 + \frac{1}{2}\lambda\phi_c^2 = \frac{d^2V_0(\phi_c)}{d\phi_c^2}$$

Effective potential at zero temperature

An example

With dimensional regularization

$$V_1(\phi_c) = \frac{1}{2}(\mu^2)^{2-\frac{n}{2}} \int \frac{d^n p}{(2\pi)^n} \log\left[p^2 + m^2\right].$$

We calculate the one-loop correction to the effective potential by first calculating it with respect to the mass and then integrating.

$$\frac{\partial V_1}{\partial m^2} = \frac{1}{2}(\mu^2)^{2-\frac{n}{2}} \int \frac{d^n p}{(2\pi)^n} \frac{1}{p^2 + m^2}$$

$$V_1 = \frac{m^4}{64\pi}\left(-\left[\frac{1}{\epsilon} - \gamma_E + \log 4\pi\right] + \log\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

The derivative is just a single disconnected bubble.

Subtracting the $1/\varepsilon - \gamma - \log 4\pi$ term, we get

$$V_1 = \frac{1}{64\pi^2} m^4\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$m^2 = d^2V/d\phi^2$

▸ Effective potential at finite temperature-imaginary time

Feynman rules for the different fields in the imaginary time formalism:

$$\text{Boson propagator} \quad : \quad \frac{i}{p^2 - m^2};\ p^\mu = [2ni\pi\beta^{-1}, \vec{p}\,]$$

$$\text{Fermion propagator} \quad : \quad \frac{i}{\gamma \cdot p - m};\ p^\mu = [(2n+1)i\pi\beta^{-1}, \vec{p}\,]$$

$$\text{Loop integral} \quad : \quad \frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3}$$

$$\text{Vertex function} \quad : \quad -i\beta(2\pi)^3 \delta_{\sum \omega_i} \delta^{(3)}(\sum_i \vec{p}_i)$$

With above FR EQ.(5) becomes

$$V_1^\beta(\phi_c) = \frac{1}{2\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \log(\omega_n^2 + \omega^2) \qquad (6)$$

with $\omega^2 = \vec{p}^{\,2} + m^2(\phi_c)$

Define

$$v(\omega) = \sum_{n=-\infty}^{\infty} \log(\omega_n^2 + \omega^2)$$

We have

$$v(\omega) = 2\beta \left[\frac{w}{2} + \frac{1}{\beta} \log\left(1 - e^{-\beta\omega}\right) \right] + \omega - \text{independent} \quad \text{terms}$$

Substituting into EQ.(6) we get

$$V_1^\beta(\phi_c) = \int \frac{d^3 p}{(2\pi)^3} \left[\frac{\omega}{2} + \frac{1}{\beta} \log\left(1 - e^{-\beta\omega}\right) \right] \qquad (7)$$

$$\frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \omega = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \log[p^2 + m^2(\phi_c)]$$

$$\frac{1}{\beta} \int \frac{d^3 p}{(2\pi)^3} \log\left(1 - e^{-\beta\omega}\right) = \frac{1}{2\pi^2 \beta^4} J_B[m^2(\phi_c)\beta^2]$$

▸ Effective potential at finite temperature-real time

Propagators for scalar fields can be written as

$$G(p) \equiv \begin{pmatrix} G^{(11)}(p) & G^{(12)}(p) \\ G^{(21)}(p) & G^{(22)}(p) \end{pmatrix}$$

$$\begin{aligned} G^{(11)}(p) &= \Delta(p) + 2\pi n_B(\omega_p)\delta(p^2 - m^2) \\ G^{(22)}(p) &= G^{(11)*} \\ G^{(12)} &= 2\pi e^{\beta\omega_p/2} n_B(\omega_p)\delta(p^2 - m^2) \\ G^{(21)} &= G^{(12)} \end{aligned}$$

$$n_B(\omega) = \frac{1}{e^{\beta\omega} - 1}$$

The propagators for fermion fields can be written as

$$S(p)_{\alpha\beta} \equiv \begin{pmatrix} S^{(11)}_{\alpha\beta}(p) & S^{(12)}_{\alpha\beta}(p) \\ S^{(21)}_{\alpha\beta}(p) & S^{(22)}_{\alpha\beta}(p) \end{pmatrix}$$

$$\begin{aligned} S^{(11)}(p) &= (\gamma \cdot p + m)\left(\Delta(p) - 2\pi n_F(\omega_p)\delta(p^2 - m^2)\right) \\ S^{(22)}(p) &= S^{(11)*} \\ S^{(12)} &= -2\pi(\gamma \cdot p + m)[\theta(p^0) - \theta(-p^0)] e^{\beta\omega_p/2} n_F(\omega_p)\delta(p^2 - m^2) \\ S^{(21)} &= -S^{(12)} \end{aligned}$$

$$n_F(\omega) = \frac{1}{e^{\beta\omega} + 1}$$

Δ(p) is the boson/fermion propagator at zero temperature

The main feature of the real time formalism is that the propagators come in two terms:

1. one which is the same as in the zero temperature field theory(Δ(p)), and a second one where all the temperature dependence is contained.

2. (12), (21) and (22) components are unphysical since one of their time arguments has an imaginary component.

▸ Effective potential at finite temperature-real time

Disconnected bubble diagrams

$$\frac{dV_1^\beta}{dm^2(\phi_c)} = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\left[\frac{-i}{-p^2+m^2(\phi_c)-i\epsilon} + 2\pi n_B(\omega)\delta(p^2-m^2(\phi_c))\right] \quad (8)$$

After integration on $m^2(\varphi c)$, the first part contributes to the effective potential as

$$-\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\log(-p^2+m^2(\phi_c)-i\epsilon)$$

Considering

$$-\frac{i}{2}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\log(-x^2+\omega^2-i\epsilon) = \frac{\omega}{2} + \text{constant}$$

Performing the p^0 integral, we get

$$\int \frac{d^3p}{(2\pi)^3}\frac{\omega}{2} \quad (9)$$

Using the identity

$$\delta(p^2-m^2) = \frac{1}{2\omega_p}\left[\delta(p^0+\omega_p)+\delta(p^0-\omega_p)\right]$$

Integration over p^0 in the β-dependent of the EQ 8, we get

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega}n_B(\omega) \quad (10)$$

Upon integration over $m^2(\varphi c)$ leads to the second term of EQ (7)

hep-ph/9901312
1701.01554

1-loop thermal mass at Finite temperature

Euclidean four-momenta $P \equiv (\omega_n, \mathbf{p})$

bosonic Matsubara frequency is $\omega_n = 2\pi\, n\, T$

Fermionic… $\omega_n = 2\pi\, (n+1)\, T$

$$\sum\!\!\!\!\!\!\!\int_P \equiv T \sum_{\omega_n} \int_p , \qquad \int_p \equiv \left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi} \right)^{\epsilon} \int \frac{\mathrm{d}^d p}{(2\pi)^d} ,$$

$$\sum\!\!\!\!\!\!\!\int_P{}' \equiv T \sum_{\omega_n \neq 0} \int_p , \qquad n_{\mathrm{B/F}}(E_p, T) \equiv \frac{1}{e^{E_p/T} \mp 1} .$$

$D = d + 1 = 4 - 2\epsilon$

$$\equiv \sum\!\!\!\!\!\!\!\int_P \frac{1}{P^2 + m^2} = \left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi} \right)^{\epsilon} \int \frac{\mathrm{d}^D p}{(2\pi)^D} \frac{1}{p^2 + m^2} + \int_p \frac{n_{\mathrm{B}}(E_p, T)}{E_p}$$

T =0, UV-div. $T \neq 0$, UV-finite, IR-sensitive

$$= \int_p \frac{T}{p^2 + m^2} + \sum\!\!\!\!\!\!\!\int_P{}' \frac{1}{P^2 + m^2}$$

UV-finite, IR-sensitive   UV-div., IR-safe

$$m_T^2 = m^2 + \# g^2 T^2$$

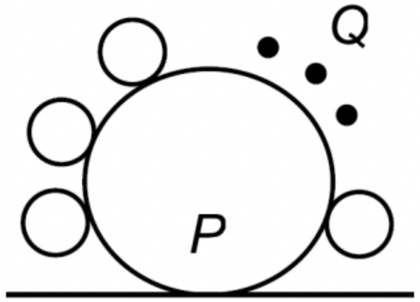
thermal correction

effective thermal mass for zero-mode/thermally corrected mass

Linde's IR problem and Resummation

$$g^2 \to g^2 n_{\rm B}(E,T) = \frac{g^2}{{\rm e}^{E/T}-1} \approx \frac{g^2 T}{E} \geq \frac{g^2 T}{m}$$

the perturbative expansion breaks down for the bosonic zero mode when m $\ll g^2$T （Linde's IR problem）



$$\propto g^{2N}\left[\int_p \frac{T}{(p^2+m_T^2)^N}\right]\left[\sum\!\!\!\!\!\!\int_Q{}' \frac{1}{Q^2}\right]^N \propto m_T^3 T\left(\frac{gT}{m_T}\right)^{2N}$$

$$m_T^2 = m^2 + \# g^2 T^2$$

thermal correction

$$\int_p \equiv \left(\frac{\bar{\mu}^2 e_E^{\gamma}}{4\pi}\right)^{\epsilon} \int \frac{{\rm d}^d p}{(2\pi)^d}, \qquad \text{and} \qquad \sum\!\!\!\!\!\!\int_P{}' \equiv T \sum_{\omega_n \neq 0} \int_p$$

$\sim\mathcal{O}(g^3)$, when P = $(0, \mathbf{p})$, Q=$(\omega_n, \mathbf{q})$ with $\omega_n \neq 0$

IR-divergent for N ≥ 2 for m=0 when thermal correction are not included

Phys. Rev. D 9 (1974) 3320
arXiv:hep-ph/9204216
Phys.Lett.B 96 (1980) 289-292

Finite temperature potential and free energy

The grand canonical partition function

$$\mathcal{Z}(T) \equiv \mathrm{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}], \quad \text{where} \quad \beta \equiv \frac{1}{T} \qquad \mu_B/T \ll 1$$

$$\phi(x) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{ik\cdot x}\phi(k) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{i(\omega_k\tau-\mathbf{k}\cdot\mathbf{x})}\phi(k)$$

$$\omega_n = 2n\pi T \qquad\qquad k = (\omega_n, \mathbf{k})$$

$$\mathcal{Z}(T) = \int \mathcal{D}\phi \exp\left(-T\sum\int_{\mathbf{k}}\frac{1}{2}\left(\mathbf{k}^2+\omega_n^2+m^2\right)|\phi(k)|^2\right)$$

$$= \exp\left[-\frac{V}{T}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)\right]$$

The free energy

$$F = -T\ln\mathcal{Z}$$

$$\lim_{V\to\infty}\frac{F}{V} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)$$
$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$
$$\equiv J_0(m)+\tilde{J}_B(m,T) \qquad \tilde{J}_i = T^4/2\pi^2 J_i.$$

$$\left(\lim_{V\to\infty}\frac{F}{V}\right)_{\text{fermions}} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1+e^{-\omega/T}\right)\right)$$
$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1+e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$
$$\equiv J_0(m)+\tilde{J}_F(m,T)$$

$$\tilde{J}_B(m,T) = T\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$
$$= \frac{T}{2\pi^2}\int d|\mathbf{k}|\,\mathbf{k}^2\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$
$$= \frac{T^4}{2\pi^2}\int dx\,x^2\ln\left(1-e^{-\sqrt{(m/T)^2+x^2}}\right)$$

$$\tilde{J}_F = \frac{T^4}{2\pi^2}\left(-\frac{7\pi^4}{360}+\frac{\pi^2m^2}{24T^2}-\frac{m^4}{32T^4}\left[\ln\left(\frac{e^{\gamma_E}}{\pi^2}\frac{m^2}{T^2}\right)-\frac{3}{2}\right]+\mathcal{O}\left(\frac{m^6}{T^6}\right)\right)$$

high-T expansion m ≪ T

2307.00068

▸ 1-loop Effective potential at finite temperature

1-loop finite-T thermal effective potential

$$V_{\rm eff}(\phi, T) = V_{\rm tree} + V_{\text{1-loop}}$$

1-loop

$$V_{\text{1-loop}} = \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$= \frac{1}{2}\left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int \frac{d^D p}{(2\pi)^D}\ln(p^2+m^2) - \int_p T\ln\left(1 \mp n_{B/F}(E_p, T)\right)$$

$V_{CW}(m)$ $\qquad$ $V_T \sim J_{T,b/f}\left(\frac{m^2}{T^2}\right)$

$$= \frac{T}{2}\int_p \ln(p^2+m^2) + \frac{1}{2}\sum\!\!\!\!\!\!\!\int'_{P/\{P\}} \ln(P^2+m^2)$$

$V_{\rm soft}(m)$ $\qquad$ $V_{\rm hard}(m)$

Daisy/ring resummation $\quad V_{daisy} = V_{soft}^{resummed} - V_{soft}$

$$V_{\rm soft}(m) = -\frac{T}{12\pi}(m^2)^{\frac{3}{2}} \longrightarrow V_{soft}^{resummed} = -\frac{T}{12\pi}(m^2 + \Pi_T)^{\frac{3}{2}}$$

Arnold-Espinosa eff potential $\quad V_{\rm eff}^{\text{A-E res.}}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm CW} + V_T + V_{\rm daisy}$

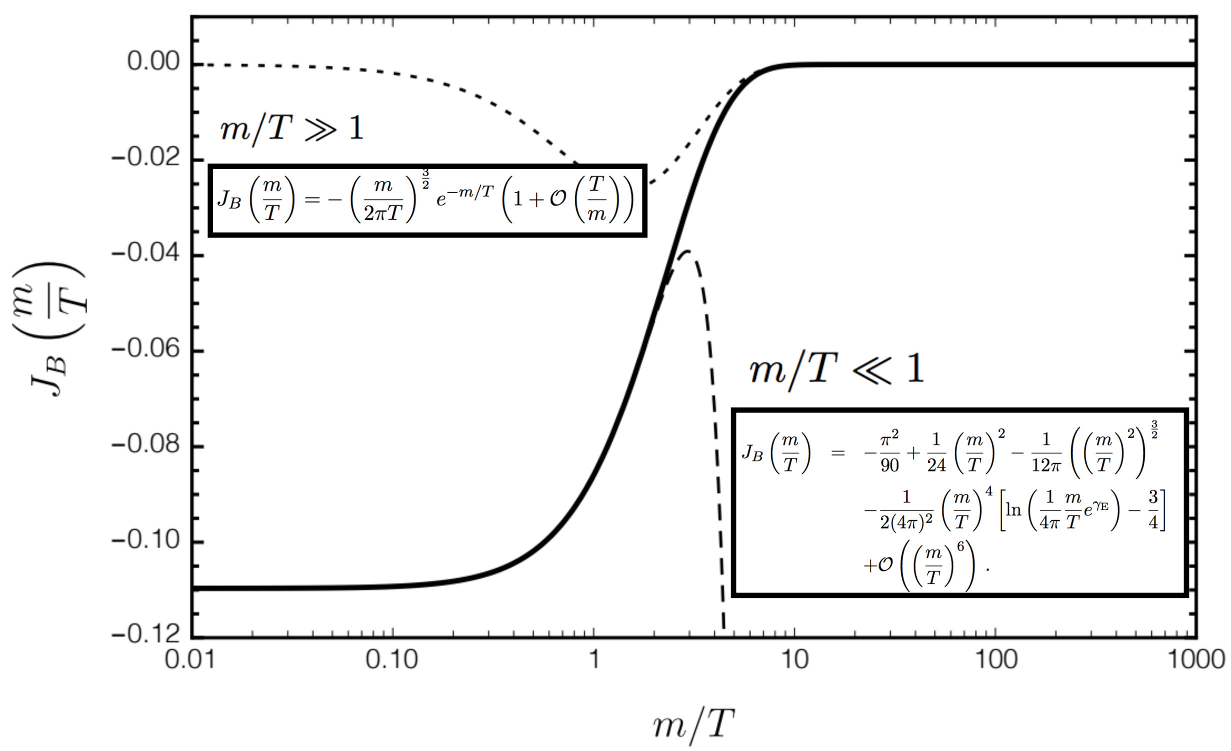
$$V_{\rm eff}^{\rm resummed}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm soft}^{\rm resummed} + V_{\rm hard}$$

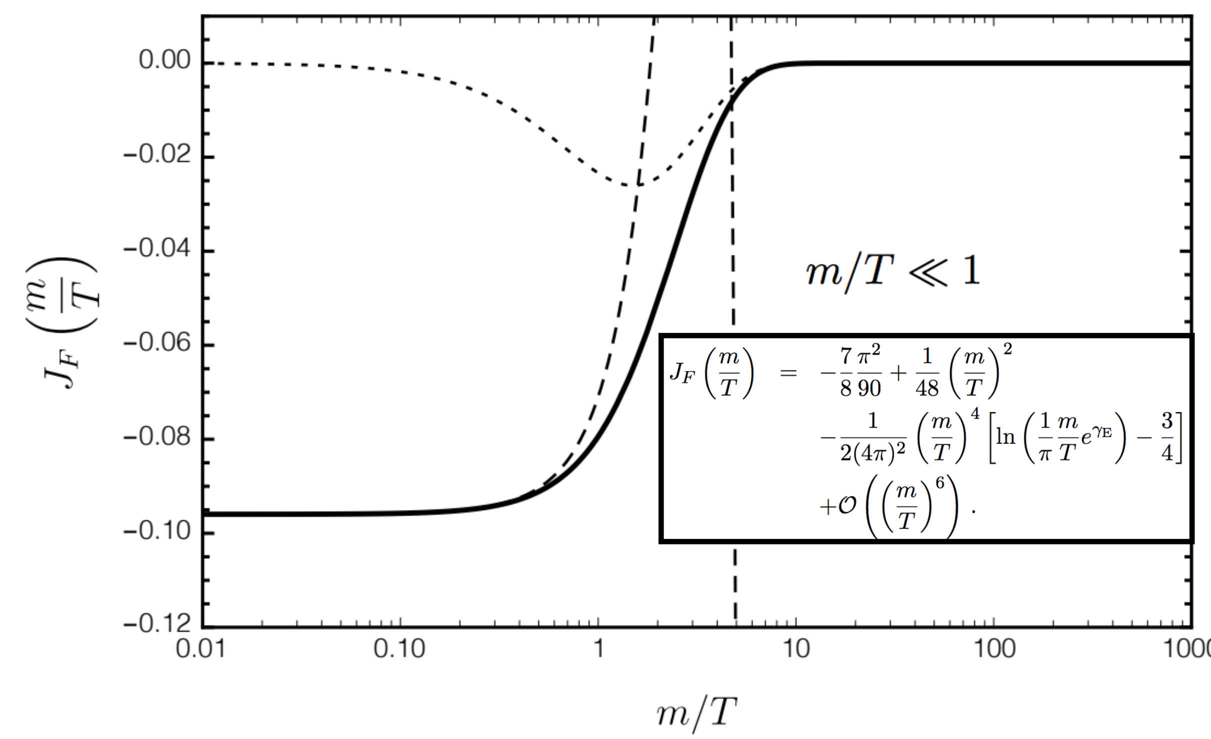
Phys. Rev. D47 (1993) 3546 [hep-ph/9212235]
See also Parwani method in Phys. Rev. D45 (1992) 4695 [hep-ph/9204216]

Thermal effective scalar potential for PT study

$$V_T(\phi, T) = V_0(\phi) + T^4\left[\sum_B J_B\left(\frac{M_B}{T}\right) + \sum_F J_F\left(\frac{M_F}{T}\right)\right]$$

all fermions F and bosons B that are relativistic at temperature T





High-T expansion

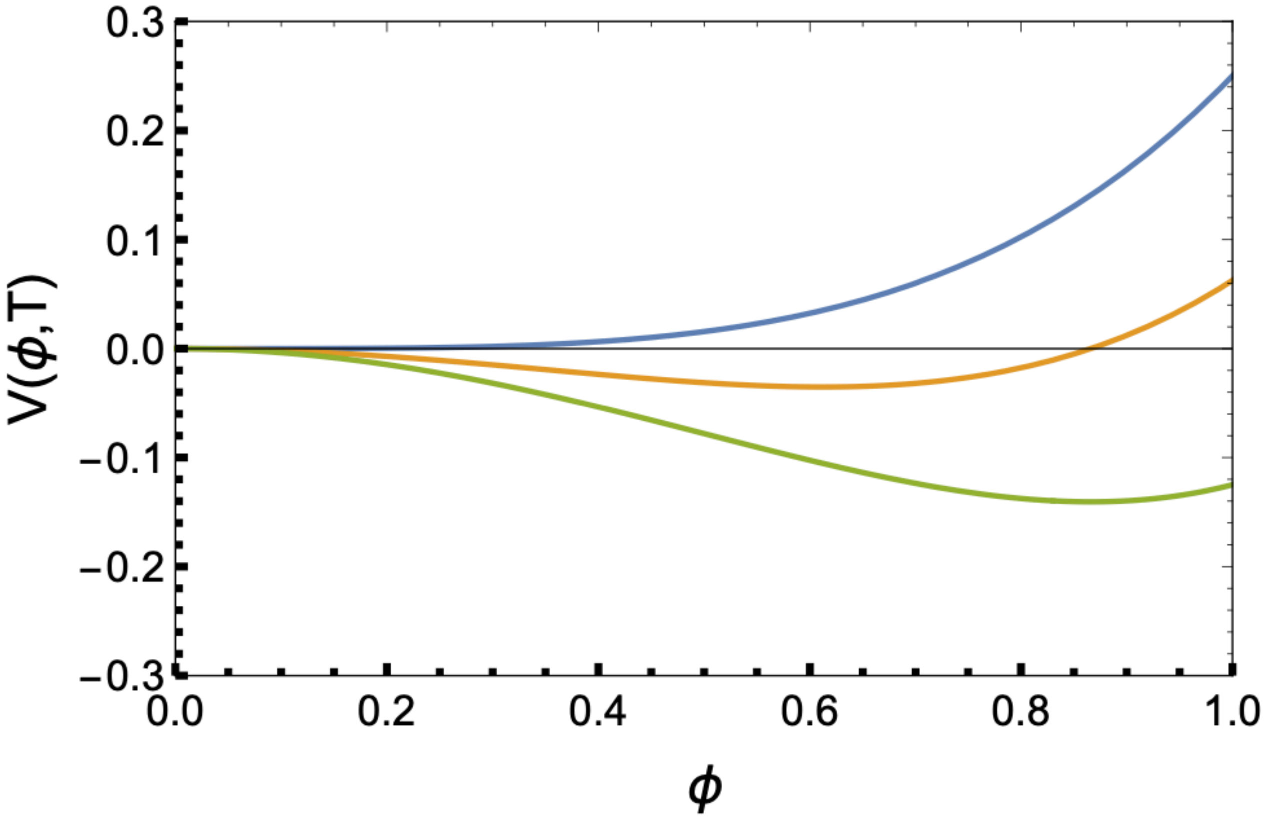
$m/T \ll 1$

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24}\left(\sum_S M_S^2(\phi) + 3\sum_V M_V^2(\phi) + 2\sum_F M_F^2(\phi)\right) - \frac{T}{12\pi}\left(\sum_S \left(M_S^2(\phi)\right)^{\frac{3}{2}} + \sum_V \left(M_V^2(\phi)\right)^{\frac{3}{2}}\right) + \text{higher order terms}\,.$$

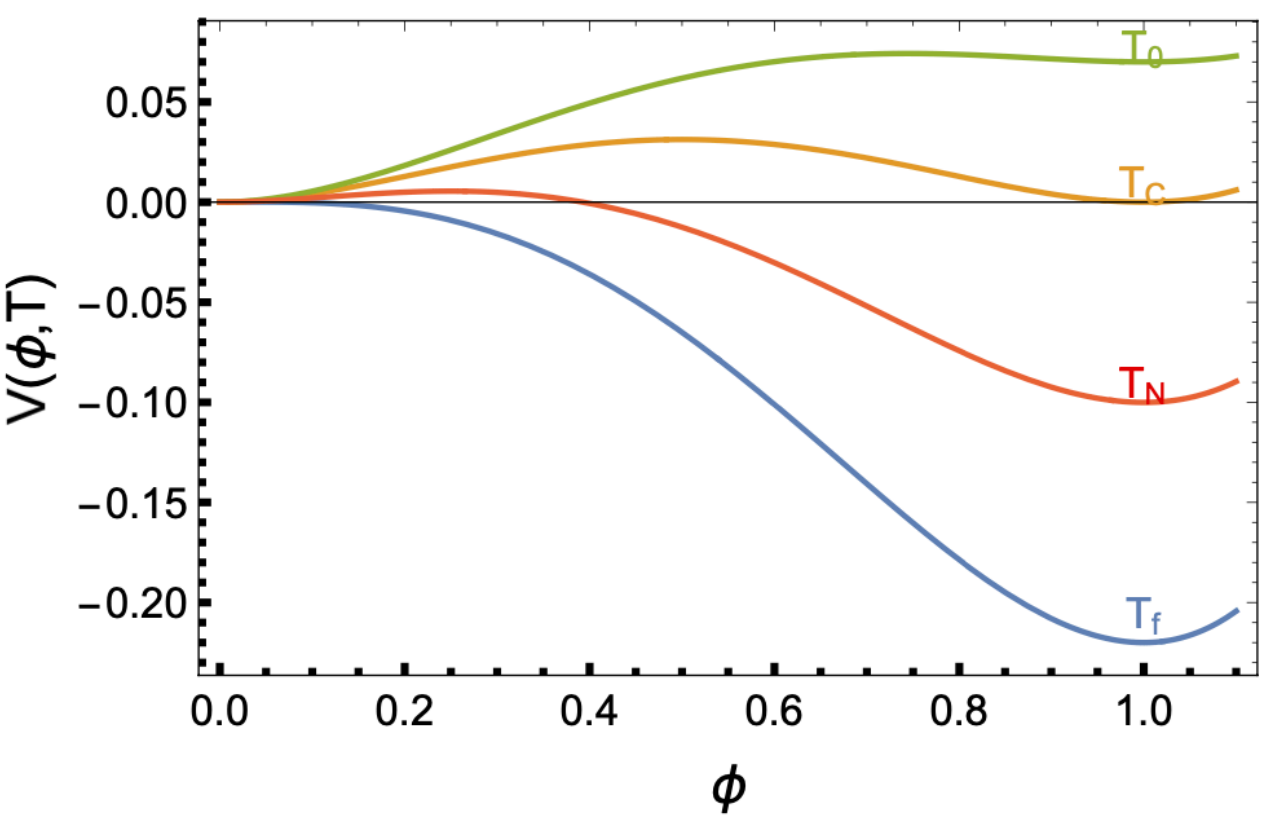
MS, MV , MF are the masses of the scalar fields S, vector fields V and fermonic fields F

▸ Phase transition types

Second order



First order

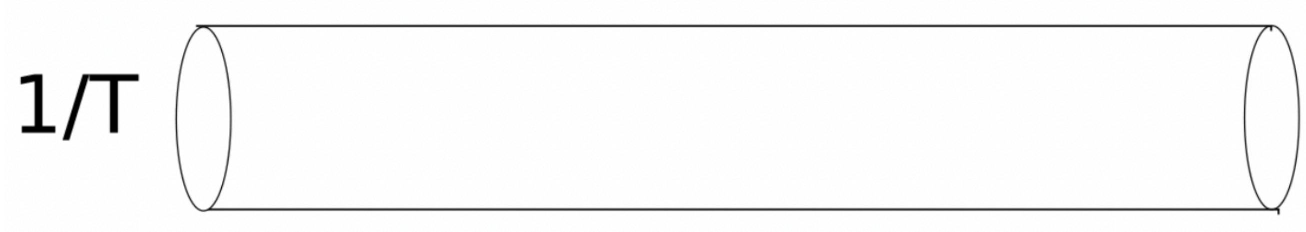


Finite temperature EFT for the 3d Phase transition study

Matsubara decomposition

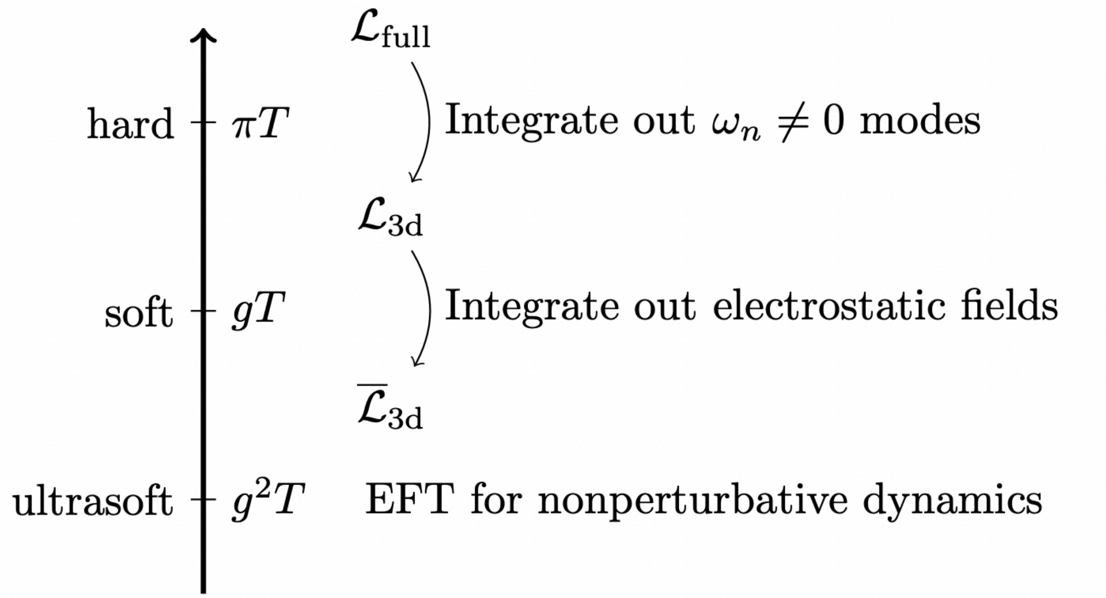
$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\mathbf{p}) e^{i\omega_n \tau}, \ \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

$\omega_n \neq 0$ modes are heavy and decouple at distances ≫ 1/T, and can be integrated out



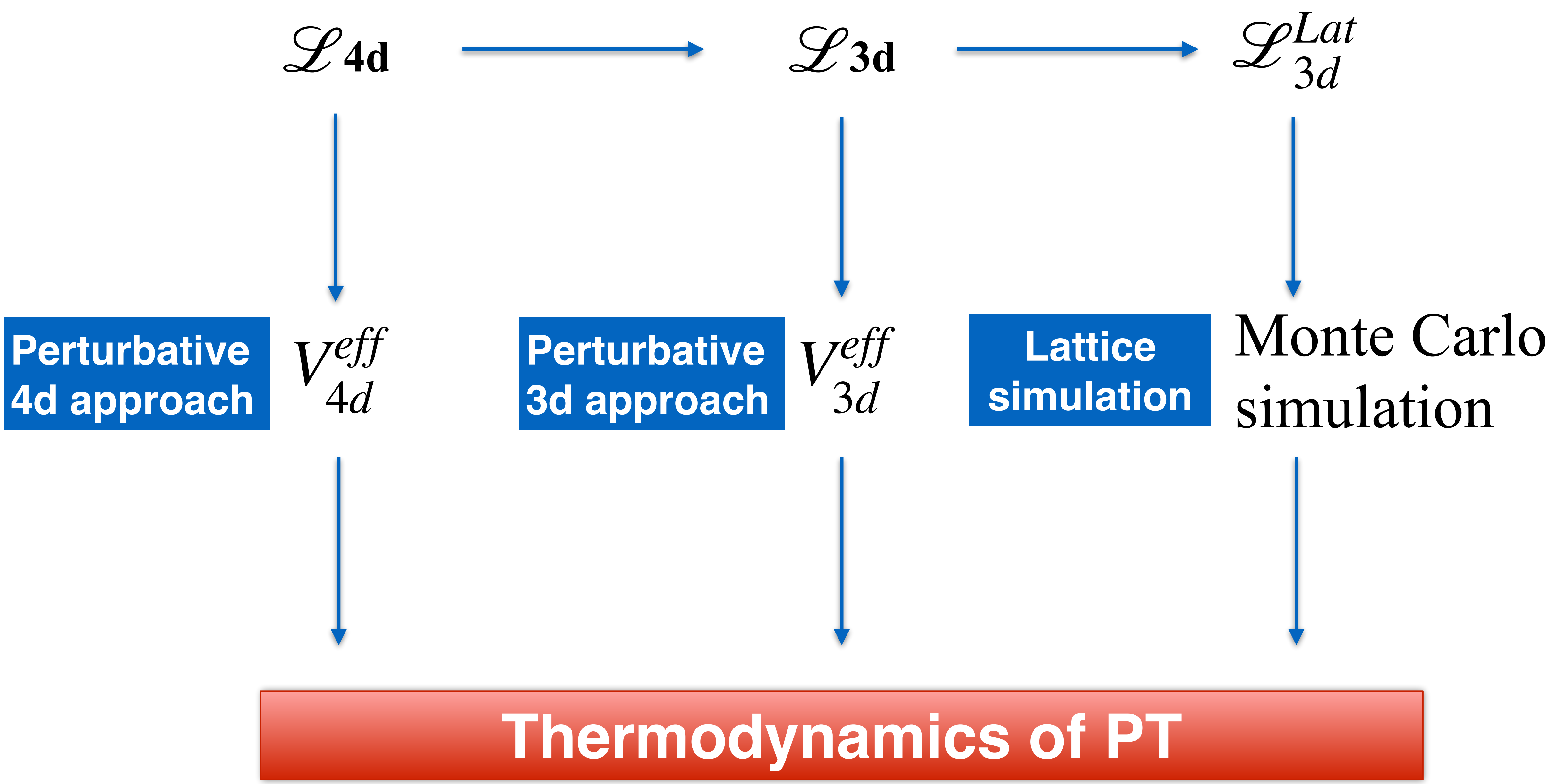
$$S = \int d^4x \, [\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}]$$

Procedure

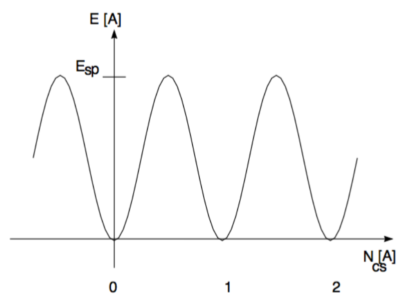


$$S_{\text{3d}} = \int d^3x \left[\frac{1}{4} F^a_{ij} F^a_{ij} + (D_i\phi)^\dagger (D_i\phi) + \bar{m}^2 \phi^\dagger \phi + \bar{\lambda}(\phi^\dagger \phi)^2 + \mathcal{L}_{\text{BSM}} \right.$$
$$\left. + \text{higher-order operators} \right]$$

Methods for PT dynamics study



BNPC, v/T and EW sphaleron



$$\partial_\mu J_B^\mu = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}),$$

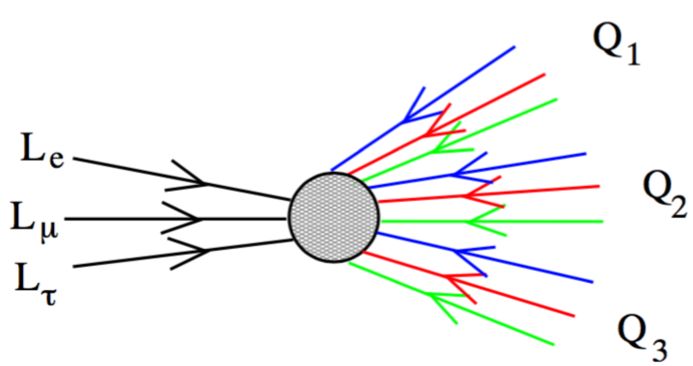
$$N_{\rm CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,{\rm Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right],$$

$$n_{\rm CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\, \epsilon^{ijk}\,\partial_i B_j B_k,$$

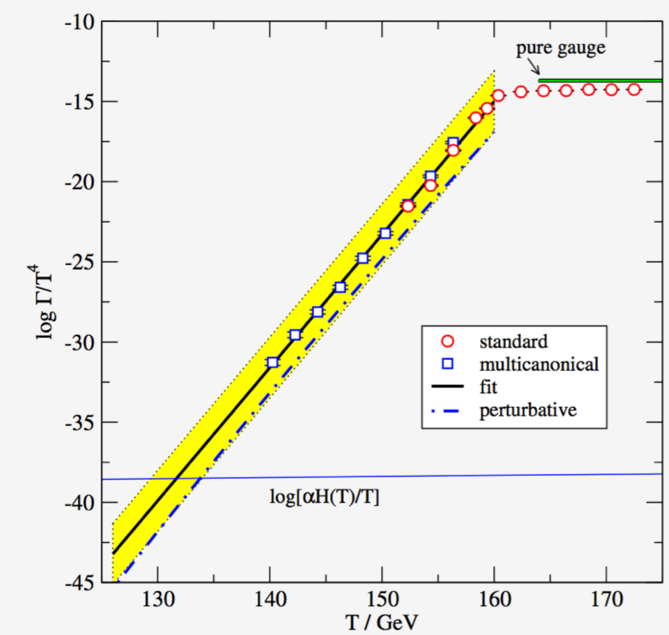
$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1},$$

$$\delta N_{\rm CS} = \frac{1}{24\pi^2}\int d^3x\,{\rm Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}.$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleros" is Greek for "ready to fall").



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



Lattice result, $T_C = (159.5 \pm 1.5)$GeV, Phys.Rev.Lett,113, 141602 (2014).

$$\Gamma^{\rm sym} \approx 6\times(18\pm3)\alpha_W^5 T^4, \qquad \Gamma^{\rm brok} \sim T^4 \exp\left(-\frac{E_{\rm sph}}{T}\right)$$

Washout avoidance,BNPC

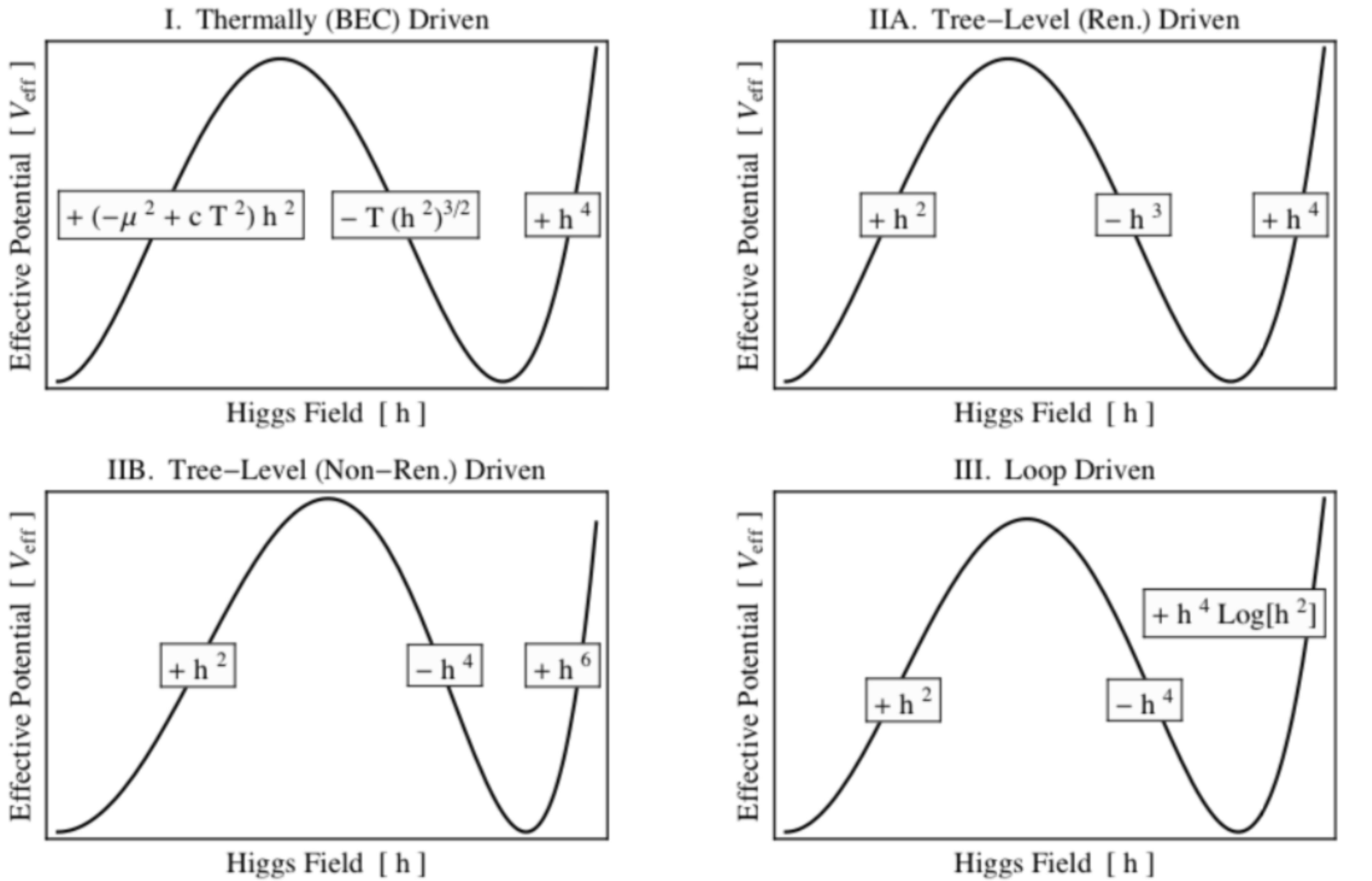
$$\Gamma_{\rm sph} = A_{\rm sph}(T)\exp[-E_{\rm sph}(T)/T] < {\rm H(T)}$$

$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ {\rm GeV}} \qquad PT_{sph} > (35.9 - 42.8)$$

$$E_{\rm sph}(T) \approx E_{\rm sph,0}\frac{v(T)}{v} \qquad \frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\rm sph,0}}{1.916\times 4\pi v/g}\right)^{-1}$$

Hiren H. Patel and Michael J. Ramsey-Musolf, 15'   Xucheng Gan, Andrew J. Long, Lian-Tao Wang, 17'

Model classes for one-step FOPT



Chung, Long, Wang, Phys.Rev.D 87 (2013) 2, 023509

▶ Thermal driven Class-I

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(-\mu^2 + cT^2)h^2 - \frac{eT}{12\pi}(h^2)^{3/2} + \frac{\lambda}{4}h^4$$

$$e \sim \sum_{\text{light bosonic fields}} (\text{degrees of freedom}) \times (\text{coupling to Higgs})^{3/2}.$$

$$\frac{v(T_c)}{T_c} \approx \frac{e}{6\pi\lambda}$$

TABLE I. Examples of models in the Thermally (BEC) Driven class. The expressions for e are calculated in the limit that the field-independent contributions to $m^2_{\rm eff}(h,T)$ are negligible (e.g., the thermal mass tuning has been performed). Here, the symbol $\tilde{A}_t$ is $\tilde{A}_t = A_t - \mu/\tan\beta$ and g_s is the number of real scalar singlet degrees of freedom coupling to the Higgs.

Model	$-\Delta\mathcal{L}$	c	e
SM [43]		$c_{\rm SM} = \frac{6m_t^2+6m_W^2+3m_Z^2+\frac{3}{2}m_H^2}{12v^2}$	$e_{\rm SM} = \frac{6m_W^3+3m_Z^3}{v^3}$
MSSM [41]		$c_{\rm SM} + \frac{6m_t^2}{12v^2}\left(1-\frac{\tilde{A}_t^2}{m_Q^2}\right)$	$e_{\rm SM} + \frac{6m_t^3}{v^3}\left(1-\frac{\tilde{A}_t^2}{m_Q^2}\right)^{3/2}$
Colored scalar [20]	$M_X^2\lvert X\rvert^2 + \frac{K}{6}\lvert X\rvert^4 + Q\lvert H\rvert^2\lvert X\rvert^2$	$c_{\rm SM} + \frac{6}{24}\frac{Q}{2}$	$e_{\rm SM} + 6(\frac{Q}{2})^{3/2}$
Singlet scalar [43,44]	$M^2\lvert S\rvert^2 + \lambda_S\lvert S\rvert^4 + 2\zeta^2\lvert H\rvert^2\lvert S\rvert^2$	$c_{\rm SM} + \frac{g_S}{24}\zeta^2$	$e_{\rm SM} + g_S\zeta^3$
Singlet Majoron [45]	$\mu_s^2\lvert S\rvert^2 + \lambda_s\lvert S\rvert^4 + \lambda_{hs}\lvert H\rvert^2\lvert S\rvert^2 + \frac{1}{2}y_i S\nu_i\nu_i$ + H.c.	$c_{\rm SM} + \frac{2}{24}\frac{\lambda_{hs}}{2}$	$e_{\rm SM} + 2(\frac{\lambda_{hs}}{2})^{3/2}$
Two-Higgs doublets [46]	$\mu_D^2 D^\dagger D + \lambda_D(D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D + \lambda_4\lvert H^\dagger D\rvert^2 + (\lambda_5/2)[(H^\dagger D)^2 + {\rm H.c.}]$	$c_{\rm SM} + \frac{2\lambda_3+\lambda_4}{12}$	$e_{\rm SM} + 2(\frac{\lambda_3}{2})^{3/2} + (\frac{\lambda_3+\lambda_4-\lambda_5}{2})^{3/2} + (\frac{\lambda_3+\lambda_4+\lambda_5}{2})^{3/2}$

▸ Tree driven-Class IIA

$$V_{\text{eff}}(\varphi, T) \approx \frac{1}{2}(m^2 + cT^2)\varphi^2 - \mathcal{E}\varphi^3 + \frac{\lambda}{4}\varphi^4$$

$$T_c \approx \sqrt{\frac{m^2}{c}}\sqrt{\frac{2\mathcal{E}^2}{\lambda m^2} - 1}, \qquad \frac{v(T_c)}{T_c} \approx \sqrt{\frac{2c}{\lambda}} \frac{1}{\sqrt{1 - \frac{\lambda m^2}{2\mathcal{E}^2}}} \cos\alpha.$$

TABLE II. Examples of models that fall into Class IIA. For the non-SUSY models, corrections to the SM Lagrangian are shown, whereas for the SUSY models only the superpotential corrections are given.

Model	$\Delta\mathcal{L}$
xSM [53–56]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_3}{3}S^3 + \frac{b_4}{4}S^4 + \frac{a_1}{2}H^\dagger HS^2 + \frac{a_2}{2}H^\dagger HS^2]$
$\mathbb{Z}_2$xSM [14,57]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_4}{4}S^4 + \frac{a_2}{2}H^\dagger HS^2]$
Two-Higgs doublets [58]	$\mu_D^2\lvert D\rvert^2 + \lambda_D\lvert D\rvert^4 + \lambda_3\lvert H\rvert^2\lvert D\rvert^2 + \lambda_4\lvert H^\dagger D\rvert^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$

Model	ΔW
NMSSM [59–61]	$\lambda H_1H_2N - \frac{\kappa}{3}N^3 + rN$
nMSSM [62]	$\lambda H_1H_2S + \frac{m_{12}^2}{\lambda}S$
$\mu\nu$MSSM [63]	$-\lambda_i H_1H_2\nu_i^c + \frac{\kappa_{ijk}}{3}\nu_i^c\nu_j^c\nu_k^c + Y_\nu^{ij}H_2L_i\nu_j^c$

Class IIA (1) no extra EWSB: xSM

For the "xSM" model, the gauge invariant finite temperature effective potential is found to be:

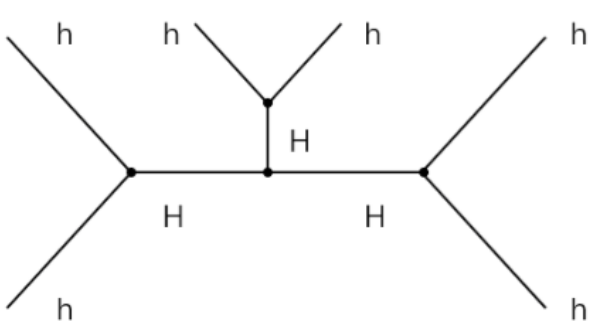
$$V(h,s,T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2$$
$$+\frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad \text{(C1)}$$

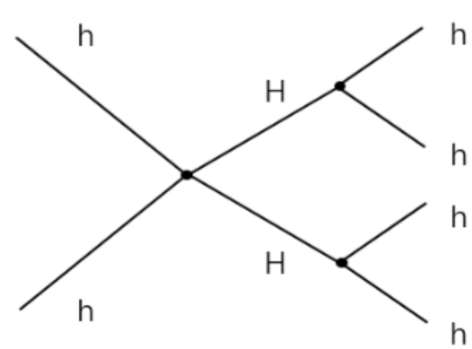
with the thermal masses given by

$$\Pi_h(T) = \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24}\right)T^2,$$
$$\Pi_s(T) = \left(\frac{a_2}{6} + \frac{b_4}{4}\right)T^2, \quad \text{(C2)}$$

PT strength

$$v^{\rm xSM}/T \equiv \frac{v_h(T)}{T} = \frac{\sqrt{v_h^2(T)+v_s^2(T)}\cos\theta(T)}{T},$$
$$\cos\theta(T) \equiv \frac{v_h(T)}{\sqrt{v_h^2(T)+v_s^2(T)}},$$





□

For small mixing limit between the extra Higgs and the SM Higgs, one have

$$c_4^{xSM} = -\frac{a_1^2 - 8b_2\lambda}{32b_2} + \frac{\theta^2(a_1^2(6b_2-\mu^2) - 8a_1b_2b_3 + 8b_2^2(a_2-2\lambda))}{32b_2^2} + O(\theta^3)$$

$$c_6^{xSM} = -\frac{a_1^2(a_1b_3 - 3a_2b_2)}{192b_2^3} - \frac{\theta^2 a_1}{256b_2^4}(a_1^3b_2 + 4a_1^2b_3(\mu^2-3b_2)$$
$$+4a_1b_2(a2(11b_2-2\mu^2) - 6b_2(b4+\lambda) + 4b_3^2) - 32a_2b_2^2b_3) + O(\theta^3)$$

$$c_8^{xSM} = \frac{a_1^4 b_4}{1024b_2^4} + \frac{a_1^3\theta^2}{1024b_2^5}(a_1(a_2b_2 + 4b_4(\mu^2-3b_2)) + 16b_2b_3b_4) + O(\theta^3)$$

Class IIA (1) with extra EWSB: **GM model**

The most general scalar potential $V(\Phi,\Delta)$ invariant under $SU(2)_L \times SU(2)_R \times U(1)_Y$ is given by

extra EWSB

$$v_\phi^2+8v_\xi^2 \equiv v^2 \approx (246\mathrm{GeV})^2$$

$$\begin{aligned}
V(\Phi,\Delta) =& \frac{1}{2}m_1^2 \mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2 \mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1\left(\mathrm{tr}[\Phi^\dagger\Phi]\right)^2 \\
&+ \lambda_2\left(\mathrm{tr}[\Delta^\dagger\Delta]\right)^2 + \lambda_3 \mathrm{tr}\left[\left(\Delta^\dagger\Delta\right)^2\right] + \lambda_4 \mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] \\
&+ \lambda_5 \mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right]\mathrm{tr}[\Delta^\dagger T^a \Delta T^b] \\
&+ \mu_1 \mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right](P^\dagger\Delta P)_{ab} + \mu_2 \mathrm{tr}[\Delta^\dagger T^a \Delta T^b](P^\dagger \Delta P)_{ab}\,, \qquad (3)
\end{aligned}$$

Custodial symmetry

$$v_\chi = \sqrt{2} v_\xi$$

$$\Phi \equiv (\varepsilon_2\phi^*, \phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad \Delta \equiv (\varepsilon_3\chi^*, \xi, \chi) = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}, \qquad (1)$$

with

$$\varepsilon_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad (2)$$

where the phase convention for the scalar field components is: $\chi^{--} = \chi^{++*}$, $\chi^- = \chi^{+*}$, $\xi^- = \xi^{+*}$, $\phi^- = \phi^{+*}$. Φ and Δ are transformed under $SU(2)_L \times SU(2)_R$ as $\Phi \to U_{2,L}\Phi U_{2,R}^\dagger$ and $\Delta \to U_{3,L}\Delta U_{3,R}^\dagger$ with $U_{L,R} = exp(i\theta^a_{L,R}T^a)$ and T^a being the $SU(2)$ generators.

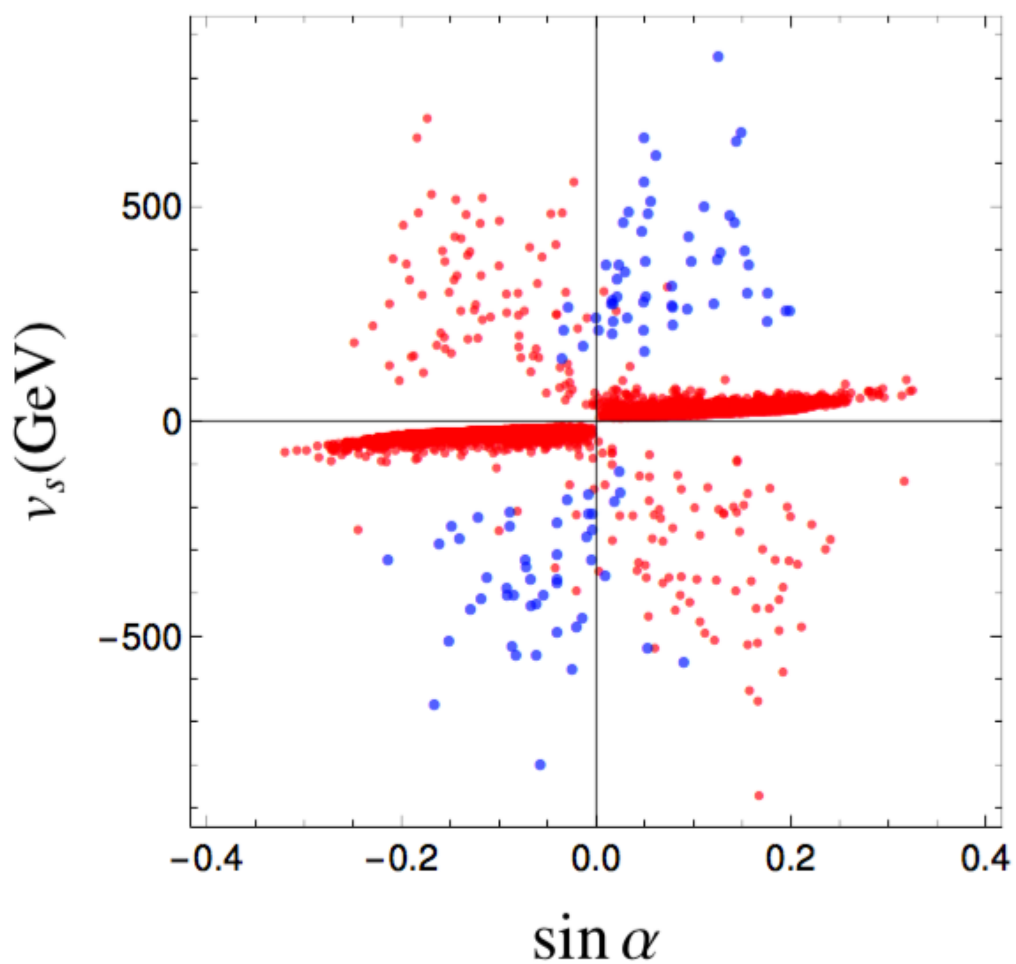
where summations over $a,b = 1,2,3$ are understood, σ's and T's are the 2×2 (Pauli matrices) 3×3 matrix representations of the $SU(2)$ generators, respectively

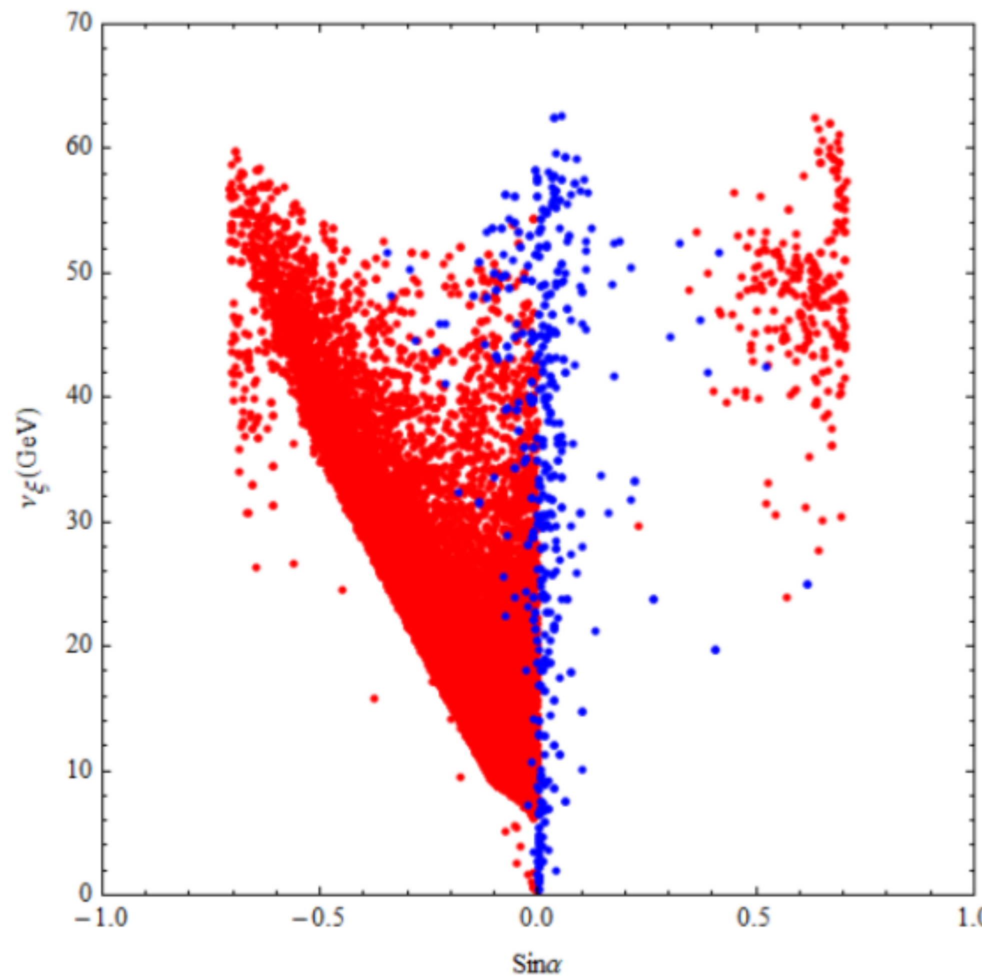
$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

The P matrix, which is the similarity transformation relating the generators in the triplet and adjoint representations, is given by

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

xSM: without extra EWSB   GM: with extra EWSB





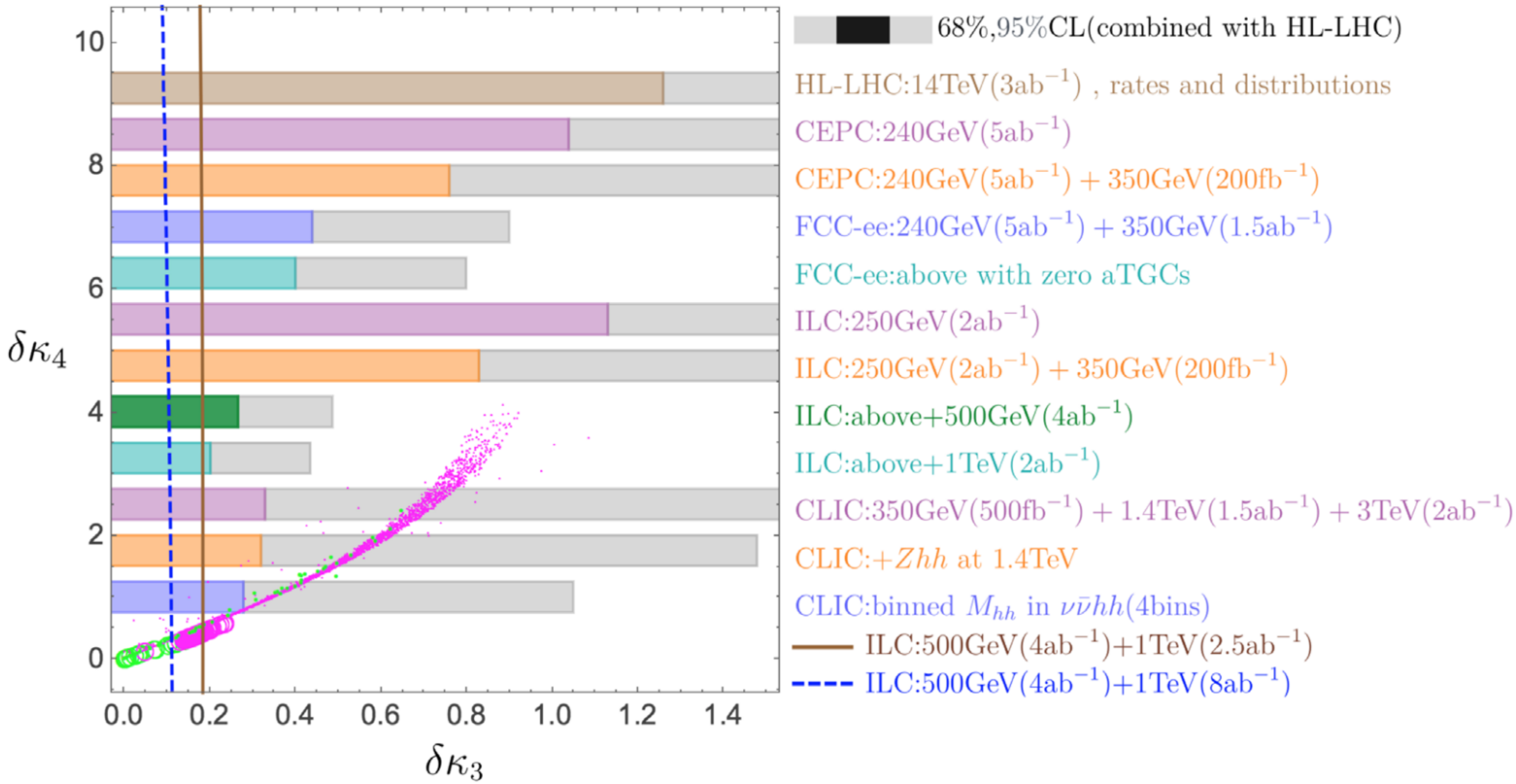
$$g_{hxx} = \cos\alpha\, g^{SM}_{hxx}$$

$$g_{hf\bar{f}} = \cos\alpha / \cos\theta_H g^{SM}_{hf\bar{f}},\ g_{hVV} = (\cos\alpha\cos\theta_H - \sqrt{\frac{8}{3}}\sin\alpha\sin\theta_H) g^{SM}_{hf\bar{f}}\,,$$

$$g_{Hf\bar{f}} = \sin\alpha / \cos\theta_H g^{SM}_{hf\bar{f}},\ g_{HVV} = (\sin\alpha\cos\theta_H + \sqrt{\frac{8}{3}}\cos\alpha\sin\theta_H) g^{SM}_{hVV}\,.$$

Collider & GW complementary search

SNR > 10 points for two-step and one-step SFOEWPT



Circles and the dotted points for the GM and xSM scenarios

$$\delta\kappa_3^{\rm xSM} = \alpha_H^2\left[-\frac{3}{2}+\frac{2m_H^2-2b_3v_s-4b_4v_s^2}{m_h^2}\right]+\mathcal{O}(\alpha_H^3),$$

$$\delta\kappa_4^{\rm xSM} = \alpha_H^2\left[-3+\frac{5m_H^2-4b_3v_s-8b_4v_s^2}{m_h^2}\right]+\mathcal{O}(\alpha_H^3).$$

$$\delta\kappa_3^{GM} = -\alpha_H\frac{\sqrt{3}\mu_1 v}{2m_h^2}+\frac{\alpha_H v^2(4\alpha_H-\sqrt{6}\theta_H)(2\lambda_4+\lambda_5}{2m_h^2} - \frac{(3\alpha_H^2+\theta_H^2)}{2}+\mathcal{O}(\alpha_H^3,\theta_H^3),$$

$$\delta\kappa_4^{GM} = -2\alpha_H^2\left(1-\frac{2(2\lambda_4+\lambda_5)v^2}{m_h^2}\right)+\mathcal{O}(\alpha_H^3).$$

▸ Tree-level driven-Class II B

< 0 causes the potential to turn over

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8\Lambda^2}h^6$$

stabilizes the EW-broken vacuum

$$\lambda = \frac{m_H^2}{2v^2}\left(1 - \frac{\Lambda_{\rm max}^2}{\Lambda^2}\right), \qquad \Lambda_{\rm max} \equiv \sqrt{3}v^2/m_H \qquad T_c = \sqrt{\frac{\mu^2}{c}}\sqrt{\frac{\lambda^2\Lambda^2}{4\mu^2} - 1},$$

$$\mu^2 = \frac{m_H^2}{2}\left(\frac{\Lambda_{\rm max}^2}{2\Lambda^2} - 1\right), \qquad \Lambda < \Lambda_{\rm max} \qquad \frac{v(T_c)}{T_c} = \sqrt{\frac{c}{-\lambda}}\frac{2}{\sqrt{1 - \frac{4\mu^2}{\lambda^2\Lambda^2}}}.$$

$$\lambda_{HHH} \equiv \frac{m_H^2}{v}\left(1 + 2\frac{\Lambda_{\rm min}^2}{\Lambda^2}\right) \qquad \Lambda_{\rm min} = v^2/m_H$$

Model	Couplings	Wilson coefficient of H^6
$\mathbb{R}$ Singlet	$-\frac{1}{2}\lambda_{HS}\|H\|^2S^2 - g_{HS}H^\dagger HS$	$-\frac{\lambda_{HS}}{2}\frac{g_{HS}^2}{M^4}$
$\mathbb{C}$ Singlet	$-g_{HS}\|H\|^2\Phi - \frac{\lambda_{H\Phi}}{2}\|H\|^2\Phi^2 - \frac{\lambda'_{H\Phi}}{2}H^\dagger H\|\Phi\|^2 + h.c.$	$-\frac{\|g_{HS}\|^2\lambda'_{H\Phi}}{2M^4} - \frac{{\rm Re}[g_{HS}^2\lambda_{H\Phi}]}{M^4}$
2HDM	$-Z_6\|H_1\|^2H_1^\dagger H_2 - Z_6^*\|H_1\|^2H_2^\dagger H_1$	$\frac{\|Z_6\|^2}{M^2}$
$\mathbb{R}$ triplet	$gH^\dagger\tau^aH\Phi^a - \frac{\lambda_{H\Phi}}{2}\|H\|^2\|\Phi^a\|^2$	$-\frac{g^2}{M^4}\left(\frac{\lambda_{H\Phi}}{8} - \lambda\right)$
$\mathbb{C}$ triplet	$gH^Ti\sigma_2\tau^aH\Phi^a - \frac{\lambda_{H\Phi}}{2}\|H\|^2\|\Phi^a\|^2$	$-\frac{g^2}{M^4}\left(\frac{\lambda_{H\Phi}}{4} + \frac{\lambda'}{8} - 2\lambda\right)$
	$-\frac{\lambda'}{4}H^\dagger\tau^a\tau^bH\Phi^a(\Phi^b)^\dagger + h.c.$	
$\mathbb{C}$ 4−plet	$-\lambda_{H3\Phi}H_i^*H_j^*H_k^*\Phi^{ijk} + h.c.$	$\frac{\|\lambda_{H3\Phi}\|^2}{M^2}$

1705.02551

Class IIB

Dim. six operator, SMEFT

Higgs potential

$$V(H) = -m^2(H^\dagger H) + \lambda (H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

Finite temperature potential

$$V_T(h,T) = V(h) + \frac{1}{2} c_{hT} h^2$$

Thermal correction

$$c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$$

Electroweak minimum being the global one

$$\Lambda \geq v^2/m_h$$

Potential barrier requirement

$$\Lambda < \sqrt{3} v^2/m_h$$

▸ Loop driven-Class III

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{\kappa}{4}h^4 \ln\frac{h^2}{M^2}$$

$$\lambda = \frac{m_H^2}{2v^2} - \kappa\left(\ln\frac{v^2}{M^2} + \frac{3}{2}\right),$$

$$\mu^2 = -\frac{m_H^2}{2} + \kappa v^2.$$

$$T_c \approx \frac{m_H}{2\sqrt{c}}\sqrt{\epsilon}\left(1 + \frac{1}{8}\epsilon + \frac{37}{384}\epsilon^2 + \cdots\right),$$

$$\frac{v(T_c)}{T_c} \approx \frac{2v\sqrt{c}}{m_H}\frac{1}{\sqrt{\epsilon}}\left(1 - \frac{3}{8}\epsilon - \frac{103}{384}\epsilon^2 + \cdots\right).$$

$$\epsilon = 1 - \kappa v^2/m_H^2$$

TABLE III. Examples of models in the Loop Driven class.

Model	$-\Delta\mathcal{L}$
Singlet scalars [12,72]	$\sum_i^N M^2\lvert S_i\rvert^2 + \lambda_S\lvert S_i\rvert^4 + 2\zeta^2\lvert H\rvert^2\lvert S_i\rvert^2$
Singlet Majoron [73,74]	$\mu_s^2\lvert S\rvert^2 + \lambda_s\lvert S\rvert^4 + \lambda_{hs}\lvert H\rvert^2\lvert S\rvert^2 + \frac{1}{2}y_i S\nu_i\nu_i + \text{H.c.}$
Two-Higgs doublets [75–78]	$\mu_D^2 D^\dagger D + \lambda_D(D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D + \lambda_4\lvert H^\dagger D\rvert^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$

Class III 2HDM Finite-T potential in 2HDM

$$V(h_1,h_2,T) = V_0(h_1,h_2) + V_{\rm CW}(h_1,h_2) + V_{\rm CT}(h_1,h_2) + V_{\rm th}(h_1,h_2,T) + V_{\rm daisy}(h_1,h_2,T)$$

Tree-level

$$V_0(h_1,h_2) = \frac{1}{2} m_{12}^2 t_\beta \left(h_1 - h_2 t_\beta^{-1}\right)^2 - \frac{v^2}{4}\frac{\lambda_1 h_1^2 + \lambda_2 h_2^2 t_\beta^2}{1+t_\beta^2} - \frac{v^2}{4}\frac{\lambda_{345}(h_1^2 t_\beta^2 + h_2^2)}{1+t_\beta^2} + \frac{1}{8}\lambda_1 h_1^4 + \frac{1}{8}\lambda_2 h_2^4 + \frac{1}{4}\lambda_{345} h_1^2 h_2^2$$

One-loop at zero temperature:

$$V_{\rm CW}(h_1,h_2) = \sum_i (-1)^{2s_i} n_i \frac{\hat{m}_i^4(h_1,h_2)}{64\pi^2}\left[\ln\left(\frac{\hat{m}_i^2(h_1,h_2)}{Q^2}\right) - C_i\right]$$

[Coleman, Weinberg '73]

One-loop at finite temperature:

$$V_{\rm th}(h_1,h_2,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{B,F}\left(\frac{m_i^2(h_1,h_2)}{T^2}\right)$$

[Dolan, Jackiw '74]

$$J_{B,F}(y) = \mp \sum_{l=1}^{\infty} \frac{(\pm 1)^l y}{l^2} K_2(\sqrt{y}\, l)$$

[Anderson, Halle '92]

$$V_{\rm daisy}(h_1,h_2,T) = -\frac{T}{12\pi}\sum_i n_i \left[\left(M_i^2(h_1,h_2,T)\right)^{\frac{3}{2}} - \left(m_i^2(h_1,h_2)\right)^{\frac{3}{2}}\right]$$

[Carrington '92; Arnold, Espinosa '93; Delaunay, Grojean, Wells '07]

Beyond SM models for FOPT

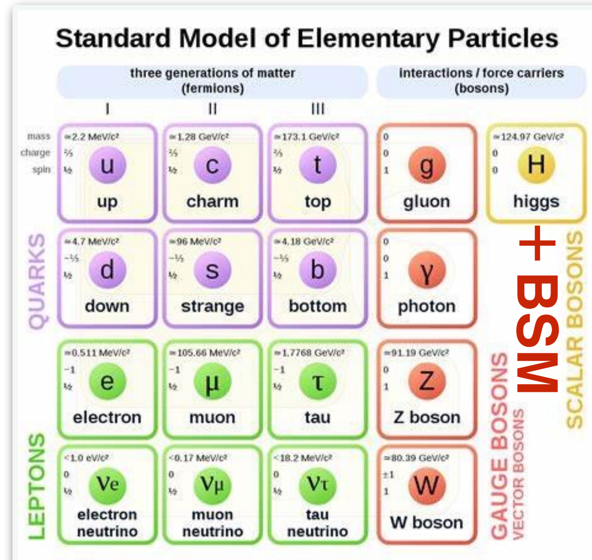
Higgs&GWs

- ☑ **SM+Scalar Singlet**

  Bian, Huang, Shu 15， Cheng, **Bian** 17, **Bian**, Tang 18,Chen, Li, Wu, **Bian**,19
- ☑ **SM+Scalar Doublet**

  Bernon, **Bian,** Jiang 17, **Bian**, Liu 18
- ☑ **SM + Scalar Triplet**

  Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19，Zhou, **Bian**,Du,22



- ☑ **Composite Higgs**

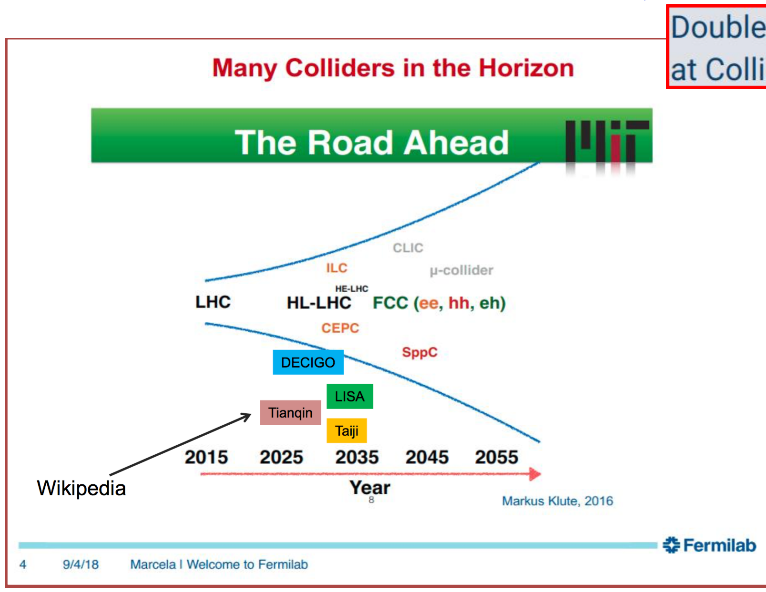
  Bian,Wu,Xie 19， **Bian**,Wu,Xie 20
- ☑ **NMSSM**

  Bi, **Bian**, Huang, Shu, Yin 15, **Bian**, Guo, Shu 17
- ☑ **SMEFT**

  Zhou, **Bian**, Guo 19

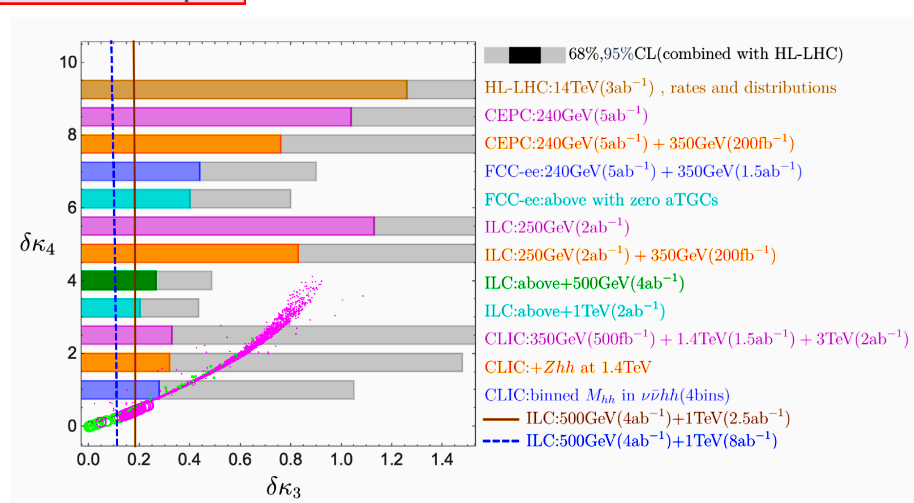
Symmetry breakdown

Symmetry breaking process

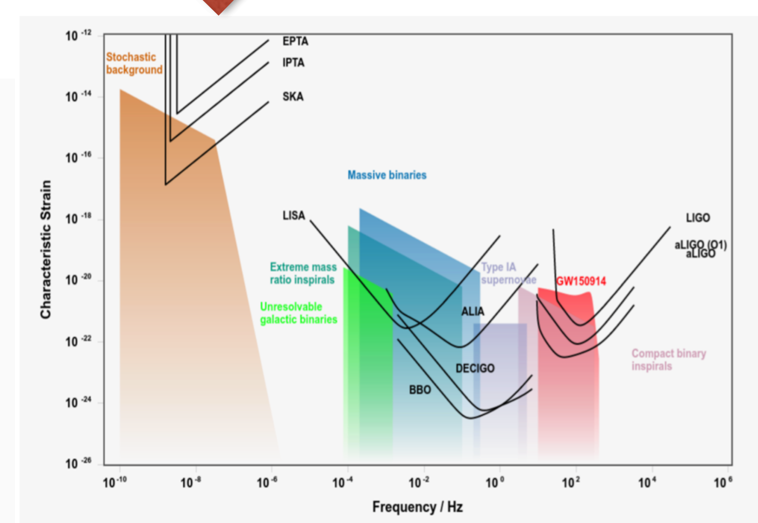


Double Higgs Production at Colliders Workshop

$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$



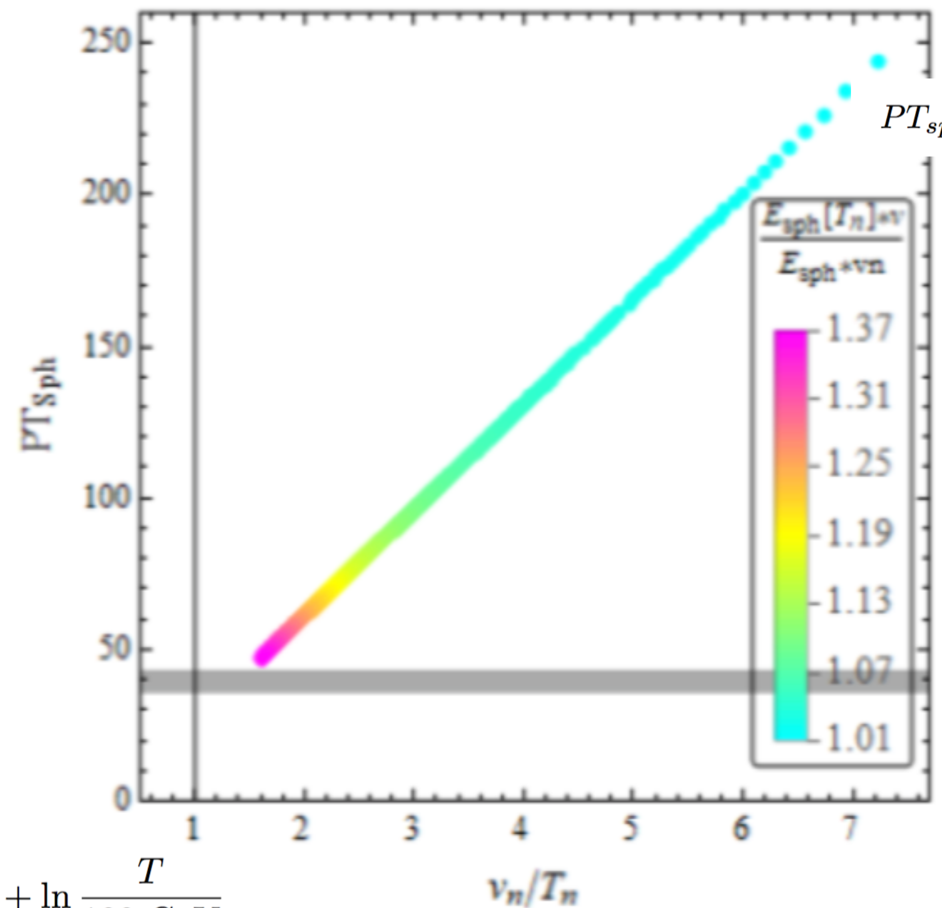
SNR > 10 for two-step and one-step SFOEWPT

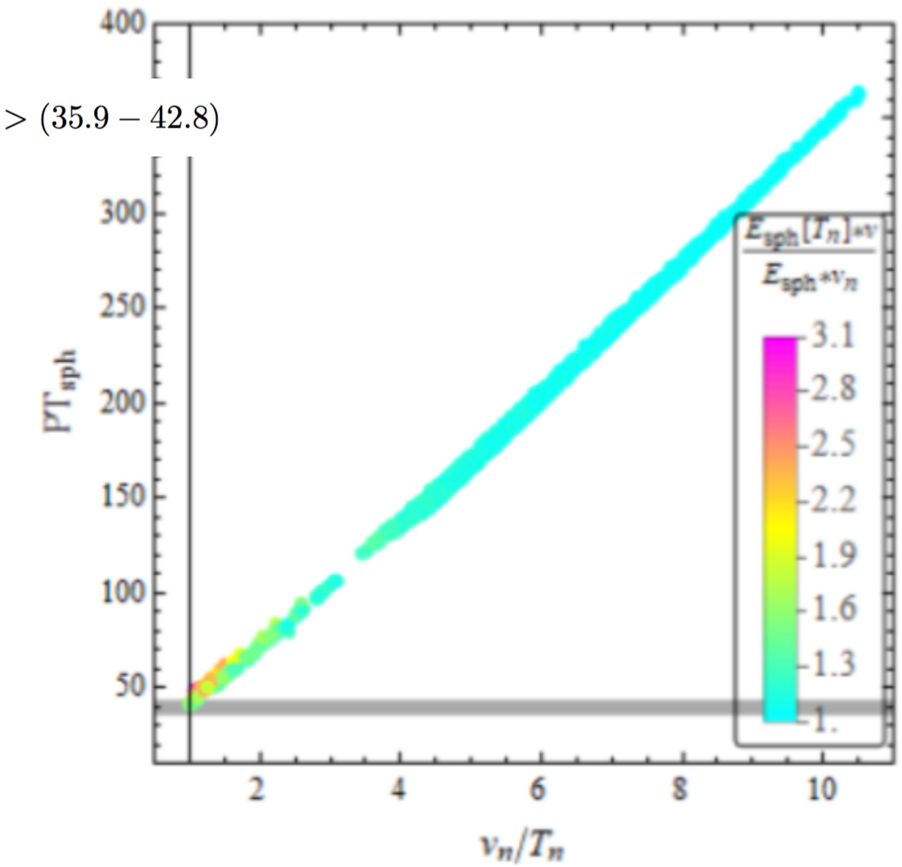


C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

PTA,LISA,TianQin,Taiji,LIGO,...

Sphaleron energy and SFOEWPT condition



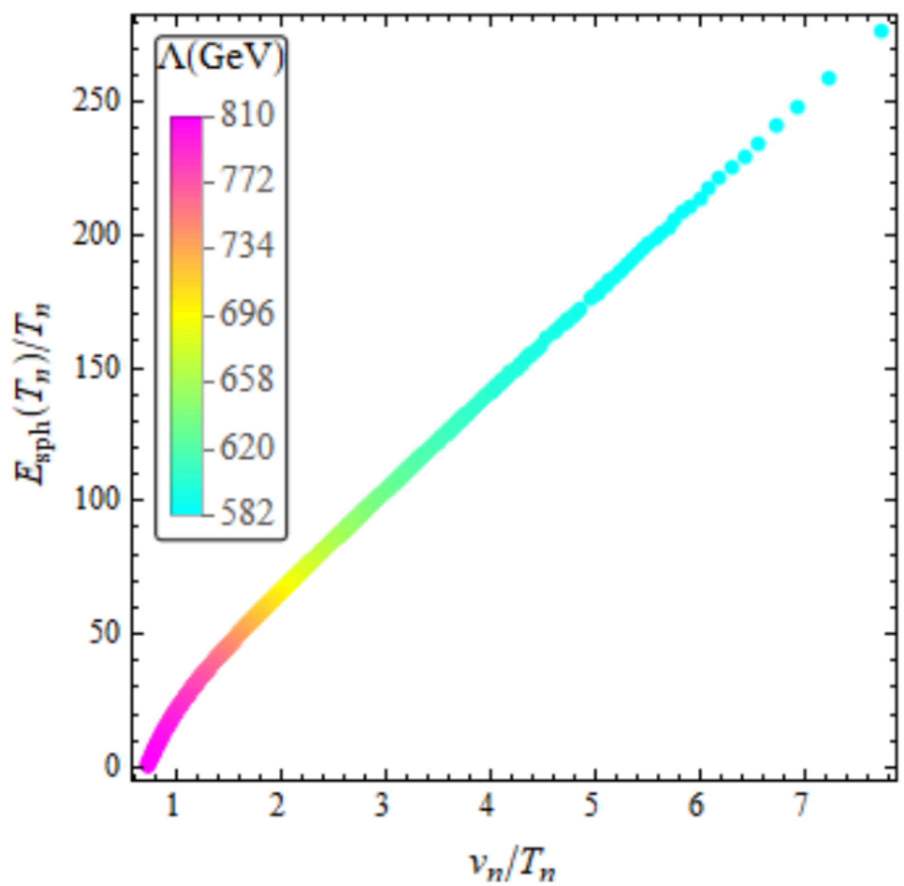


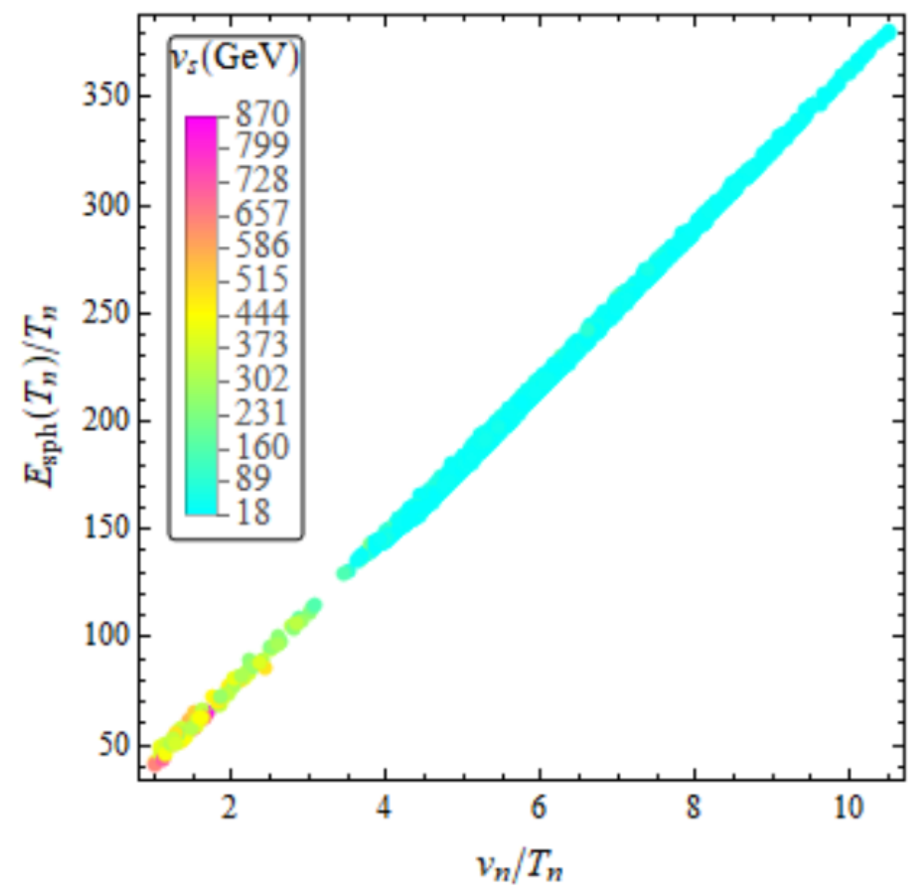
$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7 \ln \frac{v(T)}{T} + \ln \frac{T}{100 \text{ GeV}}$$

SMEFT

xSM

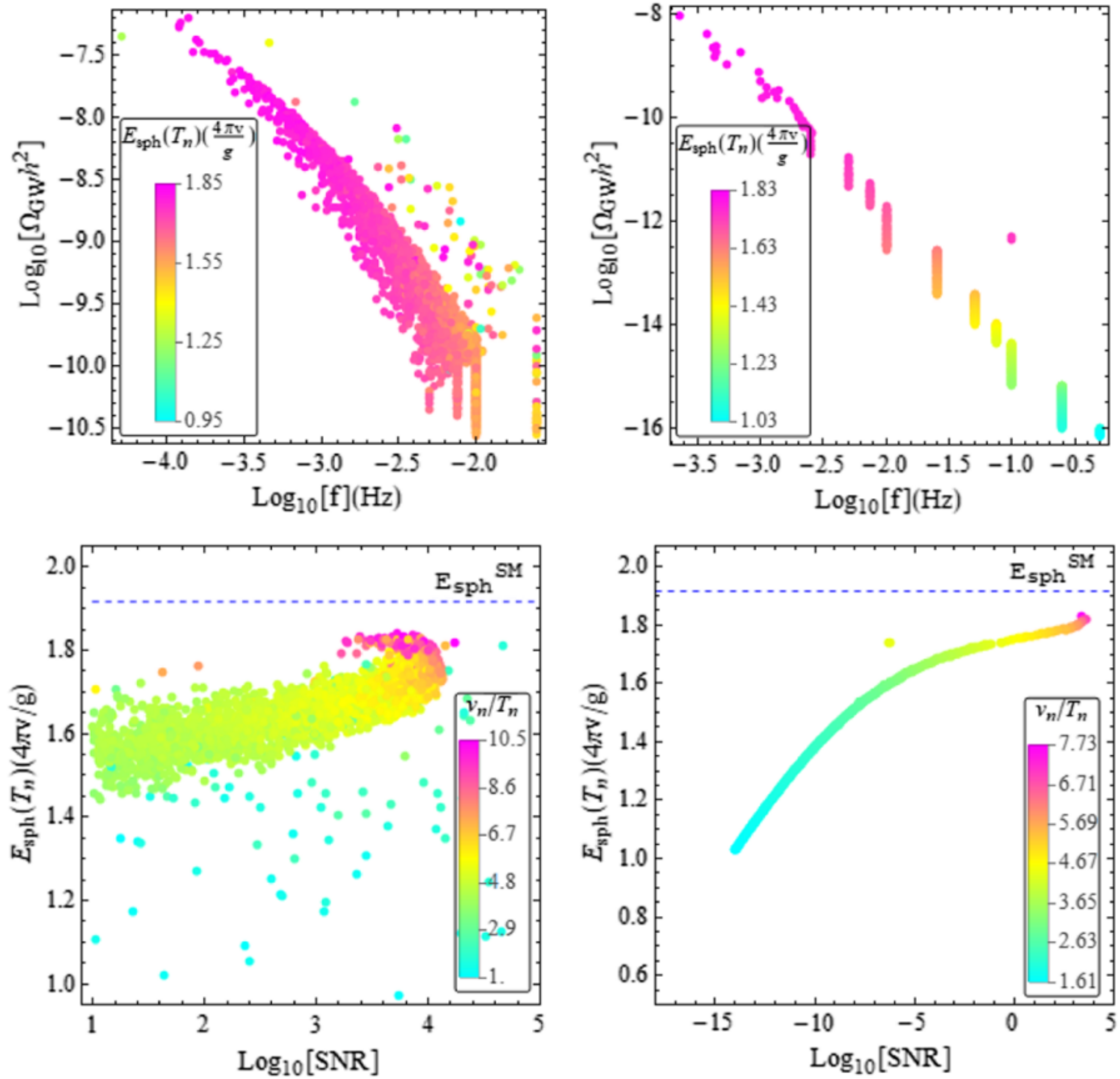
$$PT_{sph} > (35.9 - 42.8)$$





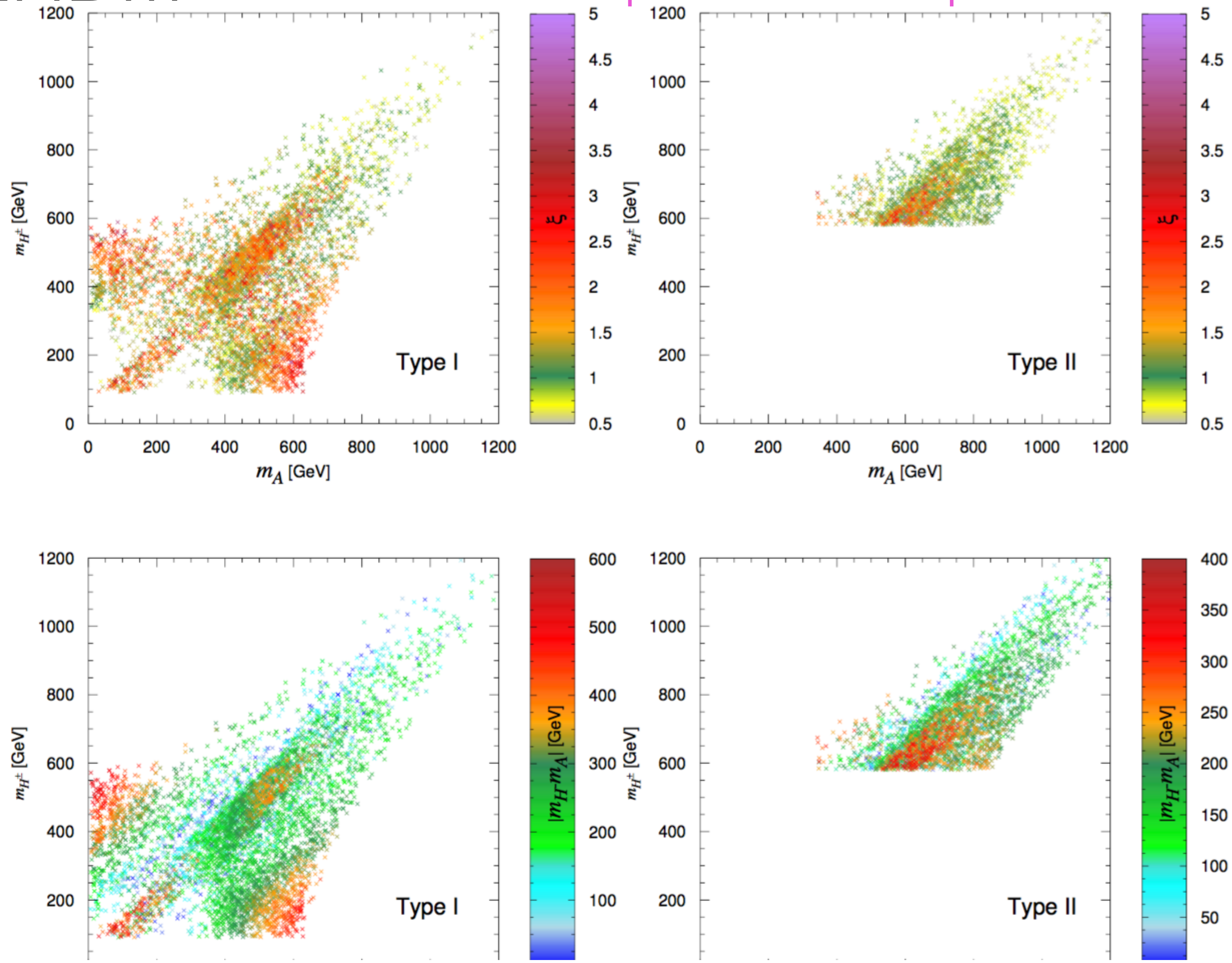
Sphaleron Energy & GW

xSM & SMEFT



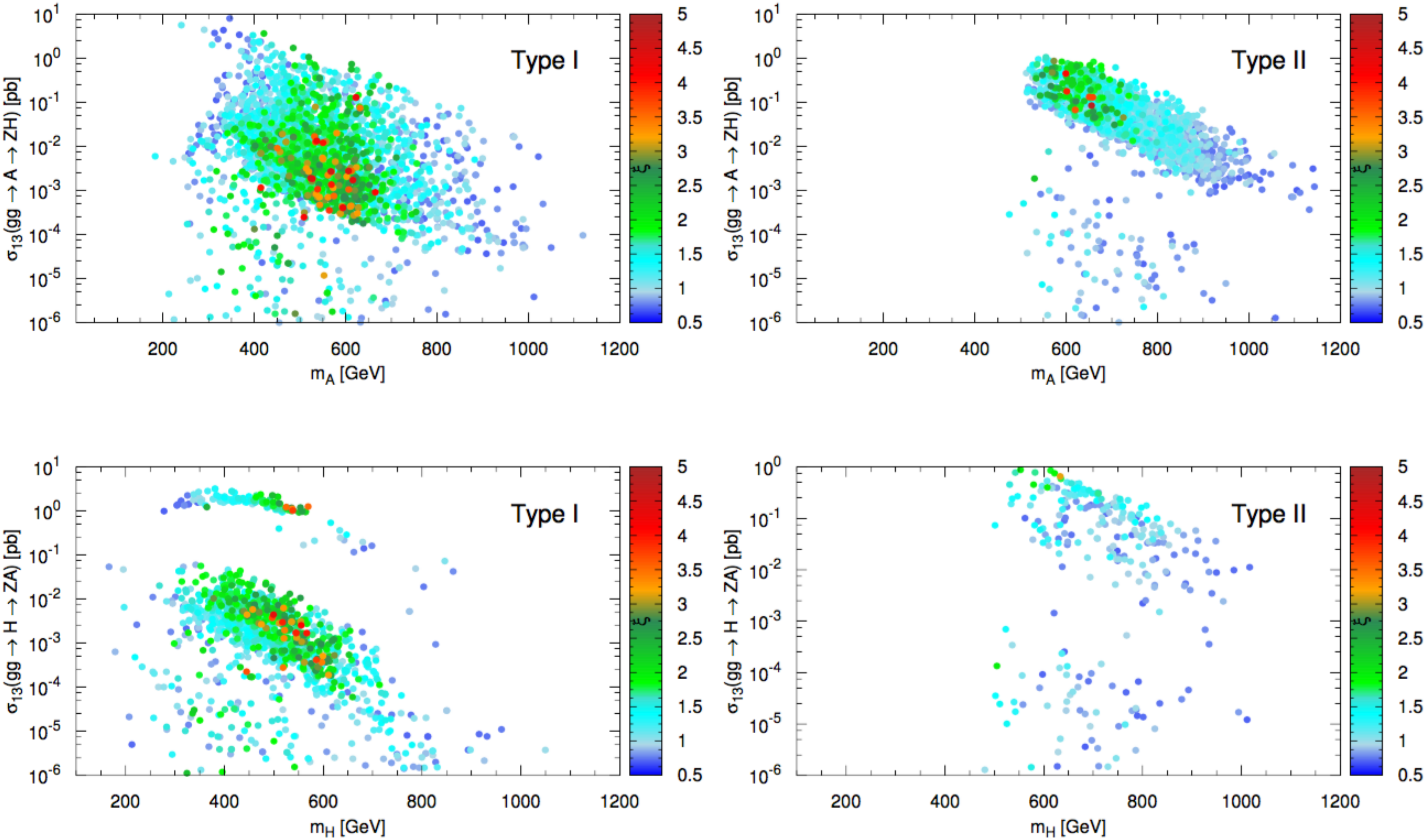
Zhou, **Bian***, Guo*, Phys.Rev.D 101 (2020) 091903(R)

Class III 2HDM SFOEWPT parameter spaces

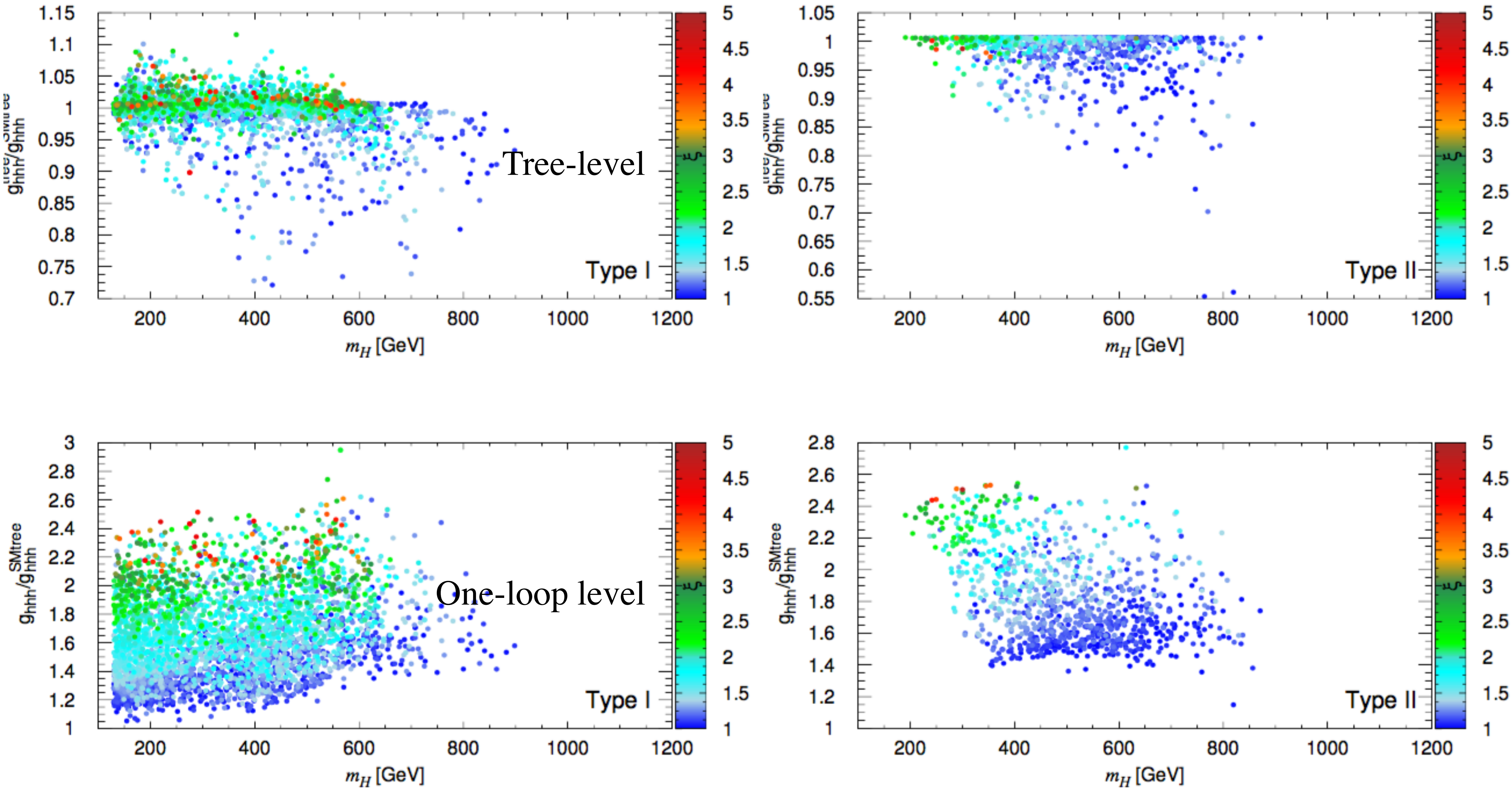


Sce.	m_H [GeV]	m_A [GeV]	$m_{H^\pm}$ [GeV]	Type	Main H/A decays
A	130 – 300	400 – 600	100 – 300	I	$A \to W^- H^+ (60\%), ZH(25\%)$
B	400 – 600	10 – 200	400 – 600	I, II	$H \to ZA(50 - 75\%)$
C	130 – 200	450 – 800	450 – 800	I, II	$A \to ZH(\sim 100\%)$
D	400 – 600	10 – 250	100 – 250	I	$H \to W^- H^+ (60\%), ZA(25\%)$
E	300 – 350	300 – 350	300 – 350	I	$A \to Zh(\sim 100\%),\ H \to W^+ W^- (\gtrsim 40\%)$

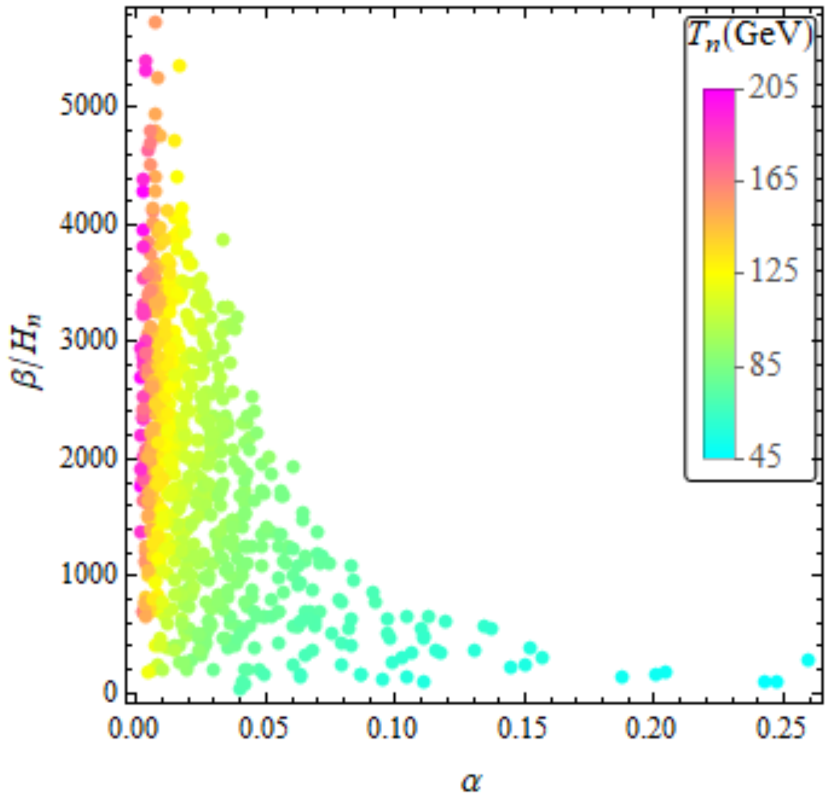
Class III 2HDM **13 TeV cross sections at the LHC**

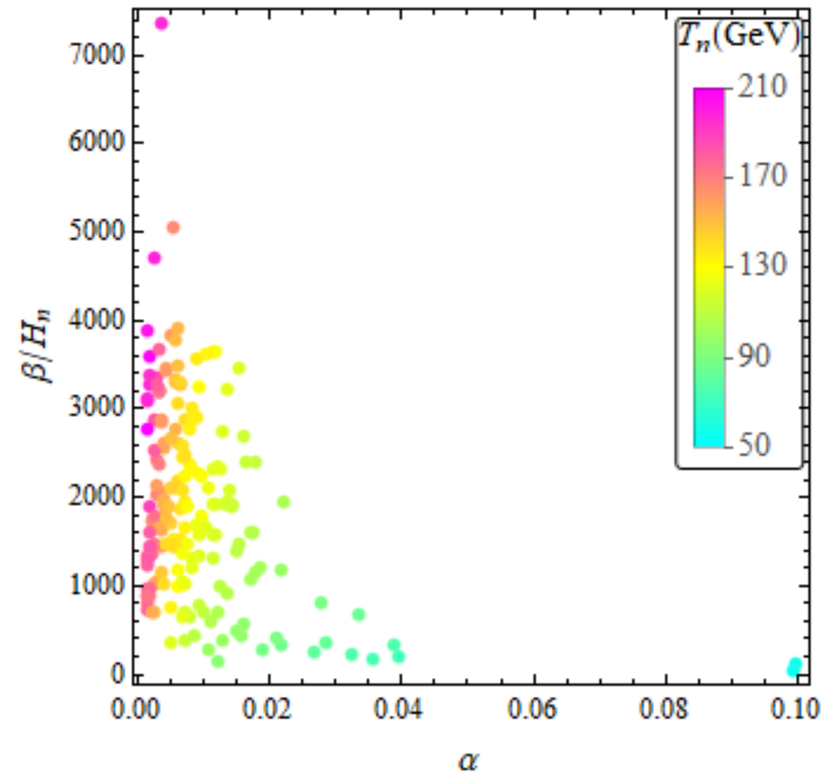


Class III 2HDM **Triple Higgs coupling**



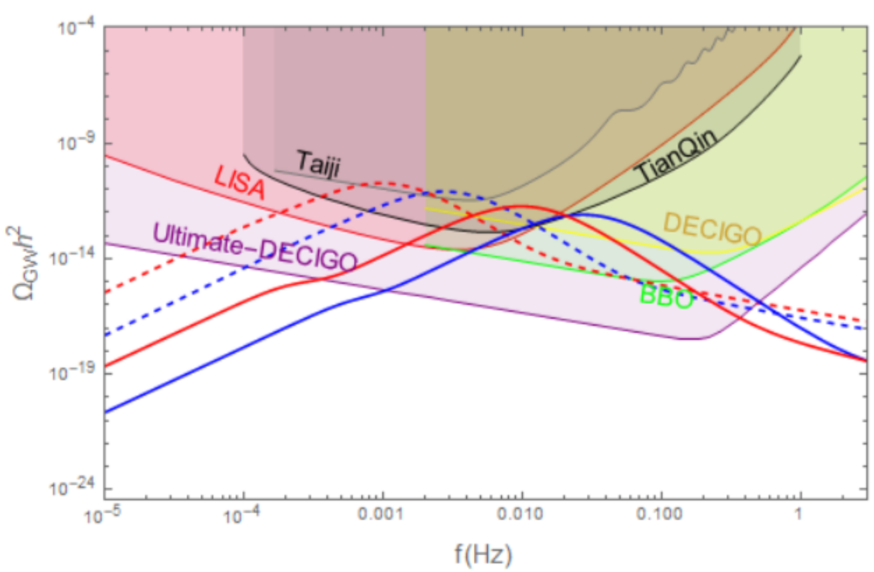
Class III   2HDM   **GW parameters**

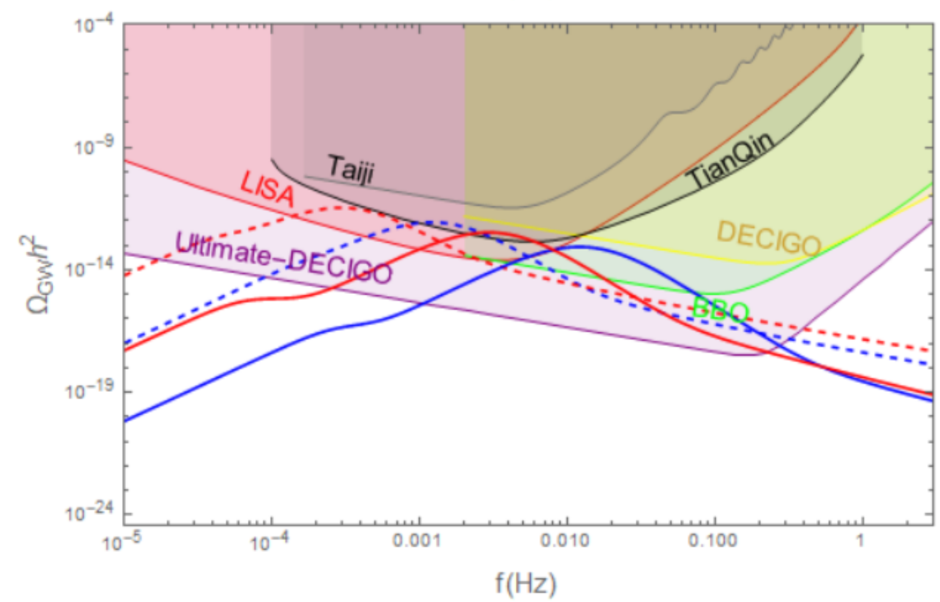




type I

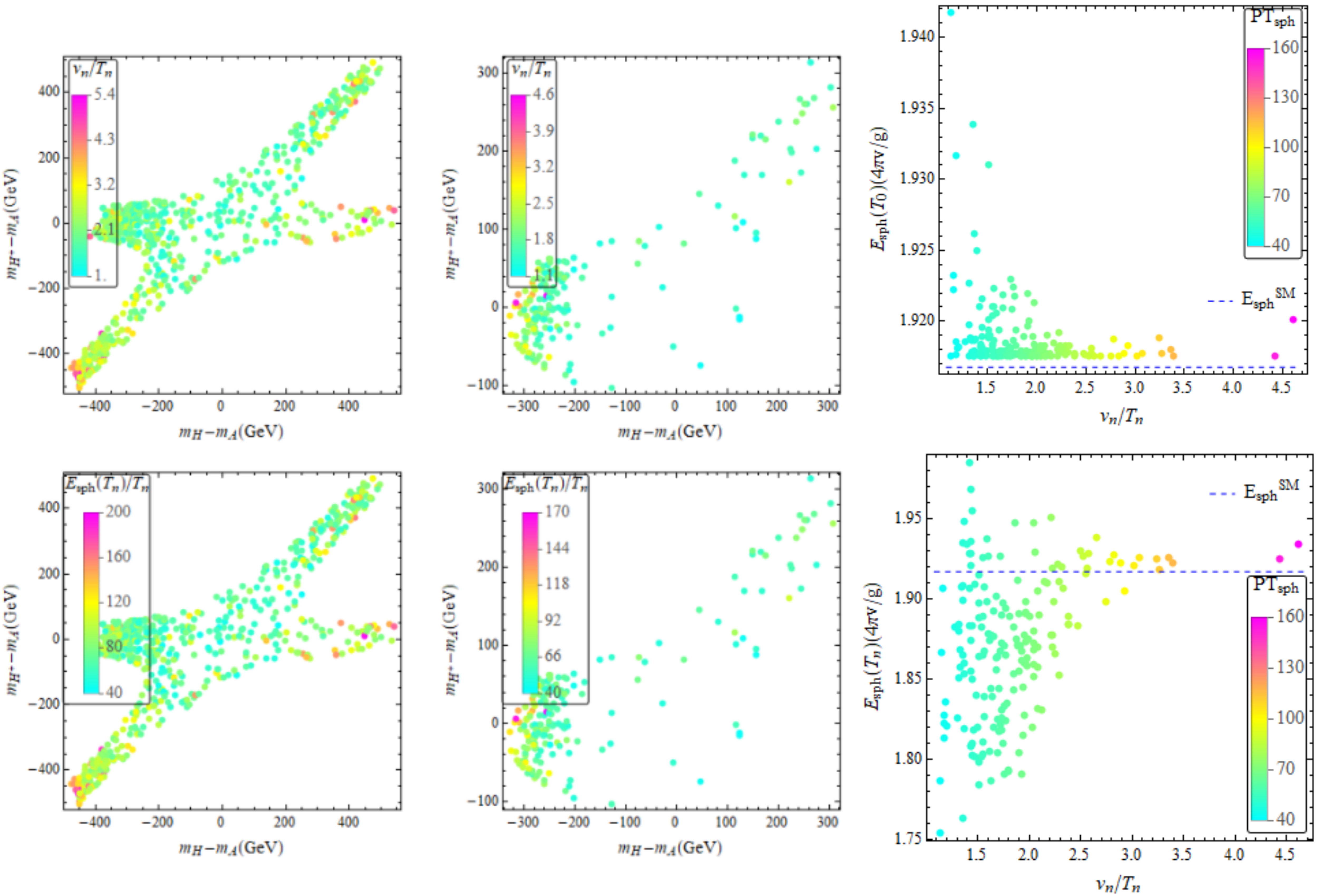
type II





Dashed (solid) line depicts the $v_w = 0.1(1)$ scenarios.

Class III 2HDM Sphaleron energy and SFOEWPT condition

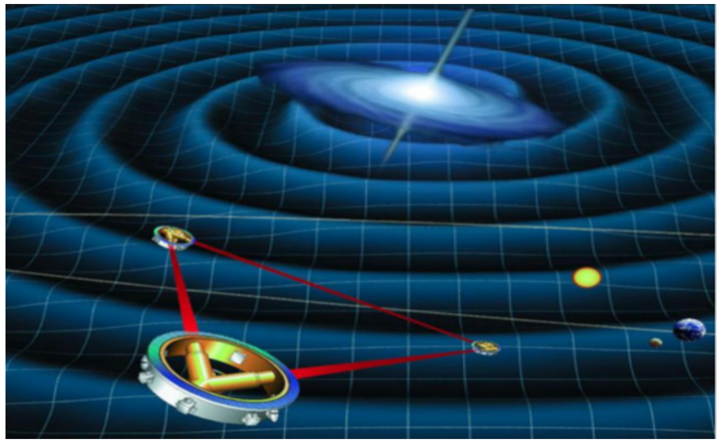


新物理&相变引力波

超出粒子物理模型
新物理模型

重要的引力波源，主要科学目标之一

PTA,LIGO,LISA,天琴,太极,…



Particle physics model

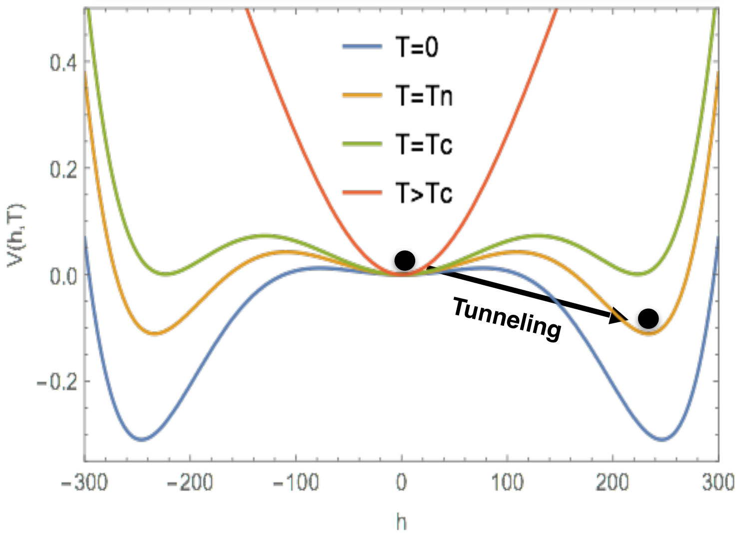
PT parameters

Effective action → β, H_*

Energy budget → α, $\kappa(\alpha, v_w)$

Bubble wall dynamics → v_w

有限温场论计算



GW power spectrum

Numerical simulations →

$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

格点场论模拟

LISA sensitivity

Configuration + noise level →

$h^2\Omega_{sens}(f)$

Signal-to-noise ratio

SNR

Chiara Caprini *et al* JCAP03(2020)024

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0\ , \qquad \frac{d\phi_b}{dr}\Big|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

PT strength

$$\alpha \equiv \frac{1}{\rho_r}\left(\Delta V_{\rm eff}(\phi, T) - \frac{T}{4}\Delta\frac{\partial V_{\rm eff}(\phi, T)}{\partial T}\right)$$

Phase transition inverse duration

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}\Big|_{T=T_n}$$

GW parameters and FOPT

The probability, that a randomly chosen point is still in the false vacuum, given by

$$P(t) = e^{-I(t)} \qquad I(t) = \frac{4\pi}{3}\int_{t_c}^{t} dt'\Gamma(t')a(t')^3 r(t,t')^3$$

The fraction of the space which has already been converted to the broken phase

$$r(t,t') = \int_{t'}^{t} \frac{v_w(\tilde{t})d\tilde{t}}{a(\tilde{t})}$$

$r(t,t')$: the comoving radius of a bubble nucleated at t' propagated until a subsequent time t

a(t): the scale factor, $v_w(t)$: the wall velocity.

Using temperature T instead of time variable t, we have

$$I(T) = \frac{4\pi}{3}\int_{T}^{T_c} \frac{dT'}{H(T')}\Gamma(T')\frac{r(T,T')^3}{T'^4}$$

The transition completes when $P(t) \approx 0.7$, which leads to a percolation temperature T_p when

$$I(T_p) = 0.34.$$

GW spectrum from FOPT

- **Bubble collisions**

$$\Omega_{\rm col}h^2 = 1.67\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_b^3}{0.42+v_b^2}\right)\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}$$

peak frequency: $$f_{\rm env} = 16.5\times 10^{-6}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$

- **Sound Wave**

$$\Omega h^2_{\rm sw}(f) = 2.65\times 10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1} v_b\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

phase transition duration: $$\tau_{sw} = min\left[\frac{1}{H_*},\frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma: $$\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$$

peak frequency: $$f_{\rm sw} = 1.9\times 10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$$

- **MHD turbulence**

$$\Omega h^2_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b\frac{(f/f_{\rm turb})^3(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

peak frequency: $$f_{\rm turb} = 2.7\times 10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$$

GW sources

$$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW1}} & \text{for } f < f_*, \\ \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW2}} & \text{for } f > f_*, \end{cases}$$

Table 1. Cosmological GW sources

source	$n_{\rm GW1}$	$n_{\rm GW2}$	f_* [Hz]	$\Omega_{\rm GW}$
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{\rm PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times10^{-4}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times10^{-6}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{\rm pl}}\right)^2$
Cosmic strings (loops 1)	$[1, 2]$	$[-1, -0.1]$	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	$[-1, -0.1]$	0	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	$[0, 0.2]$	$[0, 0.2]$	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1{\rm TeV}^3}\right)^2\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Magnetic fields	3	$\alpha_B + 1$	$\sim 10^{-6}\left(\frac{T_*}{10^2{\rm GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}{\rm G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi - 6)(\epsilon - \eta)$	—	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times10^5\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{1/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{-4/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times10^{-2}\left(\frac{M_{\rm PBH}}{10^{20}\ {\rm g}}\right)^{-1/2}$	$\sim 7\times10^{-9}\left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3 - 2\mu$	—	$\sim 1.4\times10^{-6}\left(\frac{H_s}{0.15M_{\rm pl}}\right)^4$

1807.00786

▶ One-step FOPT

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Magnetic Field and Gravitational Waves from the First-Order Phase Transition

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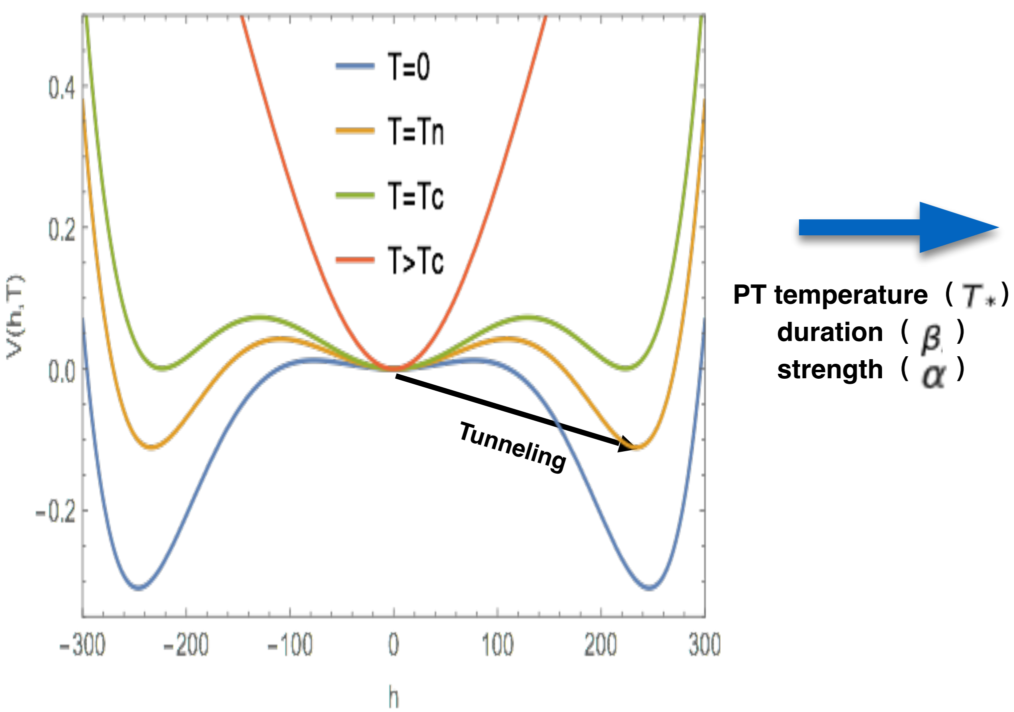
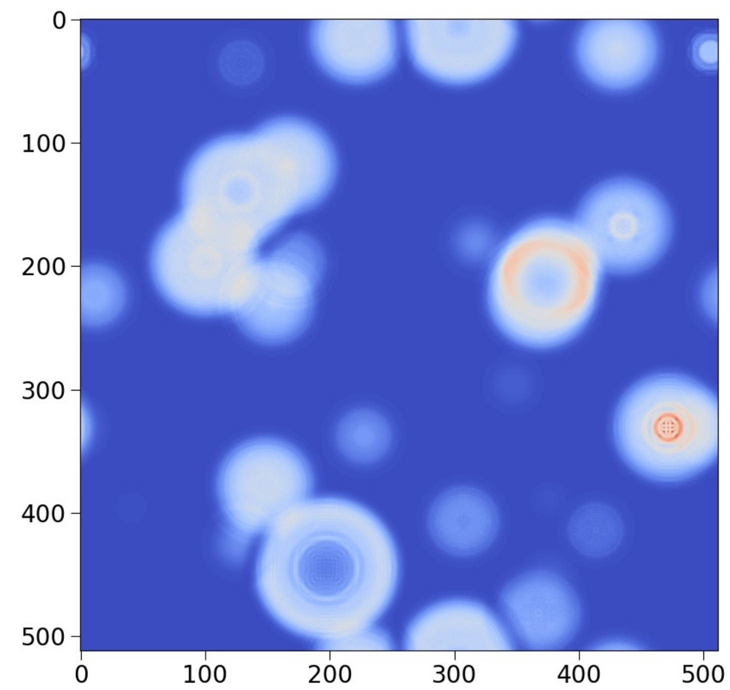
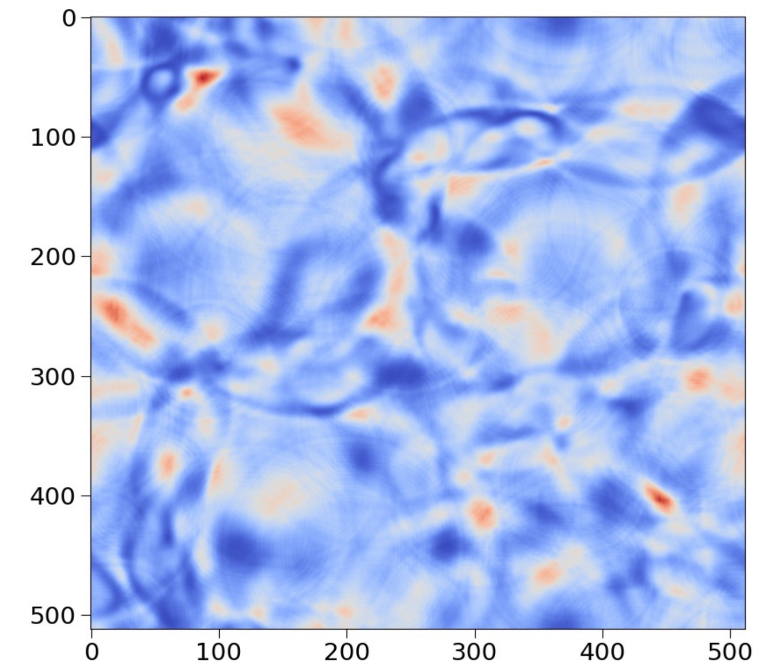
Finite-T Veff



Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

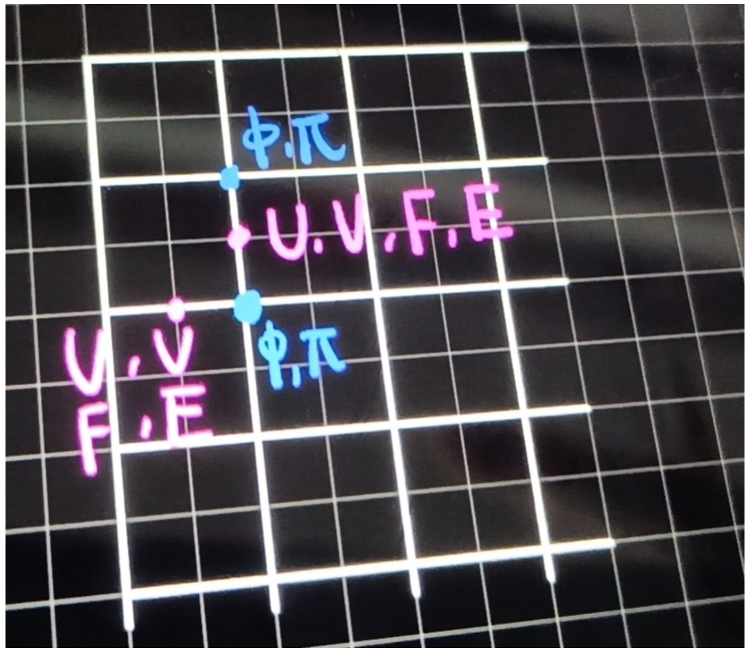
Lattice EW field foundation

$\Phi(t, x)$: Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2,x) V_i(t,x)$$
$$U_i(t+\Delta t,x) = g \Delta x \Delta t F_i(t+\Delta t/2,x) U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

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PT process simulation

Field basis

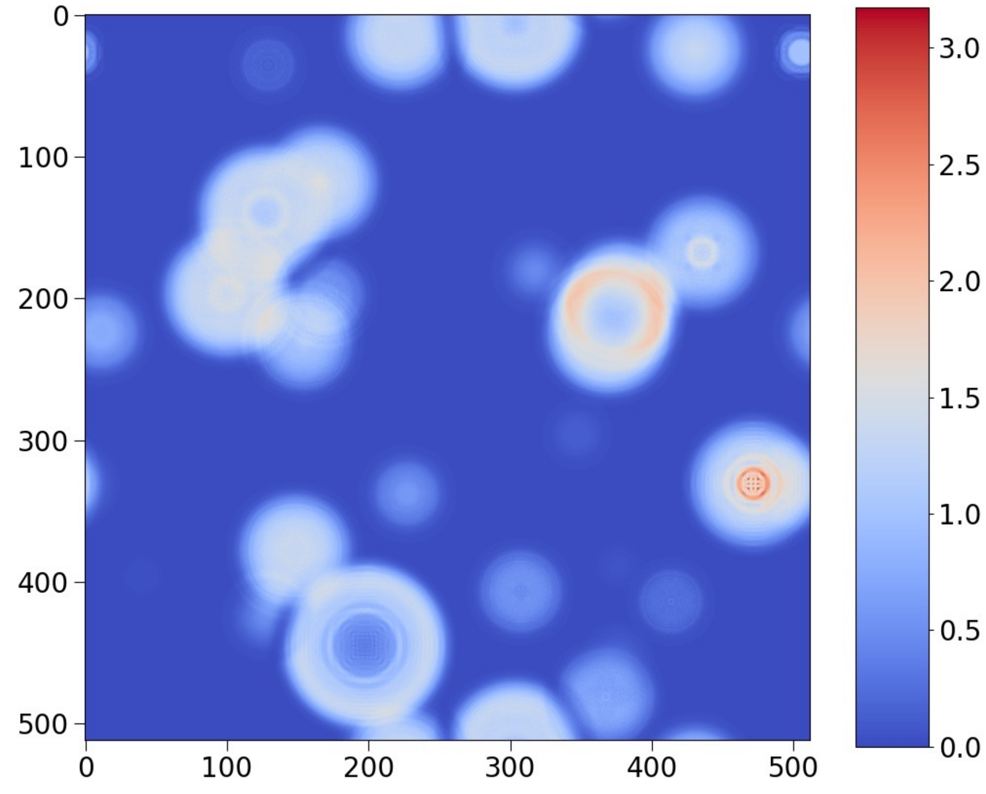
$$
\begin{aligned}
\partial_0^2 \Phi =& D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi - 3(\Phi^\dagger \Phi)^2 \Phi/\Lambda^2, \\
\partial_0^2 B_i =& -\partial_j B_{ij} + g' \, \mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a =& -\partial_k W_{ik}^a - g\, \epsilon^{abc} W_k^b W_{ik}^c + g\, \mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi].
\end{aligned}
$$

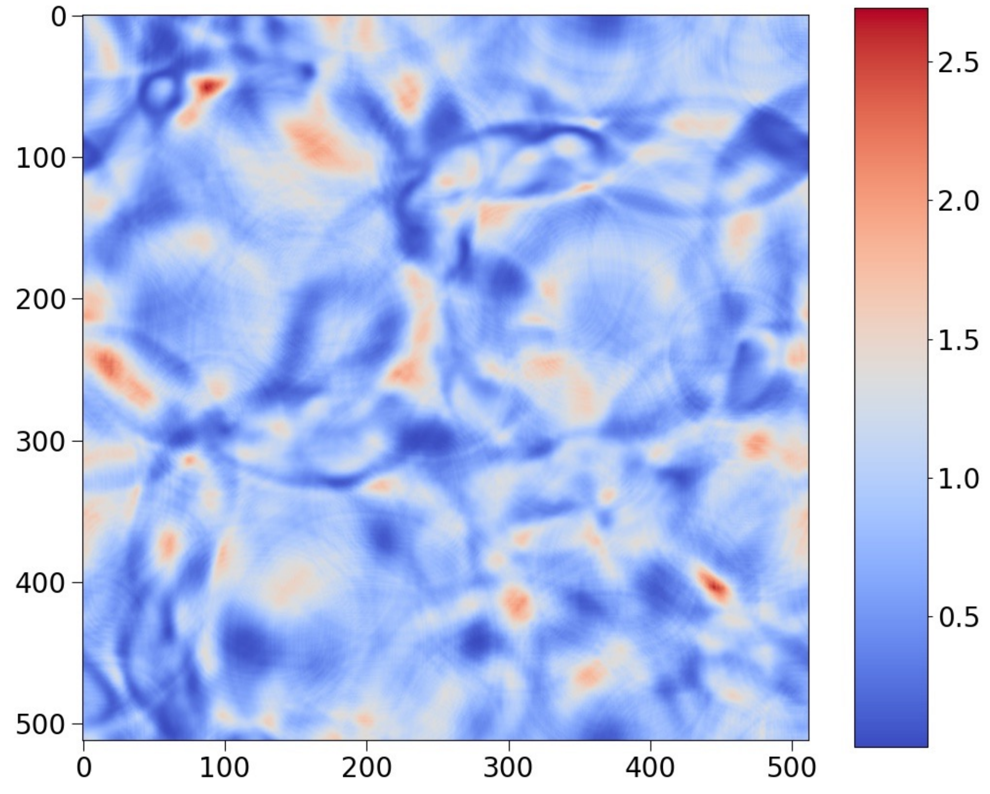
$$
\partial_0 \partial_j B_j - g' \, \mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0,
$$

$$
\partial_0 \partial_j W_j^a + g\, \epsilon^{abc} W_j^b \partial_0 W_j^c - g\, \mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.
$$

Lattice implementation

$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t \Big\{ \frac{1}{\Delta x^2} \sum_i \big[U_i(t,x) V_i(t,x) \Phi(t, x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t, x-i) V_i^\dagger(t, x-i) \Phi(t, x-i) \big] - \frac{\partial U}{\partial \Phi^\dagger} \Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2, x)] =& \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g'}{\Delta x} \mathrm{Im}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) \Phi(t,x)] \\
& - \frac{2}{g' \Delta x^3} \sum_i \mathrm{Im}[V_k(t,x) V_i(t, x+k) V_k^\dagger(t, x+i) V_i^\dagger(t,x) \\
& + V_i(t, x-i) V_k(t,x) V_i^\dagger(t, x+k-i) V_k^\dagger(t, x-i)] \Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g}{\Delta x} \mathrm{Re}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) i\sigma^m \Phi(t,x)] \\
& - \frac{1}{g \Delta x^3} \sum_i \mathrm{Tr}[i\sigma^m U_k(t,x) U_i(t, x+k) U_k^\dagger(t, x+i) U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x) U_i^\dagger(t, x+k-i) U_k^\dagger(t, x-i) U_i(t, x-i)] \Big\},
\end{aligned}
$$





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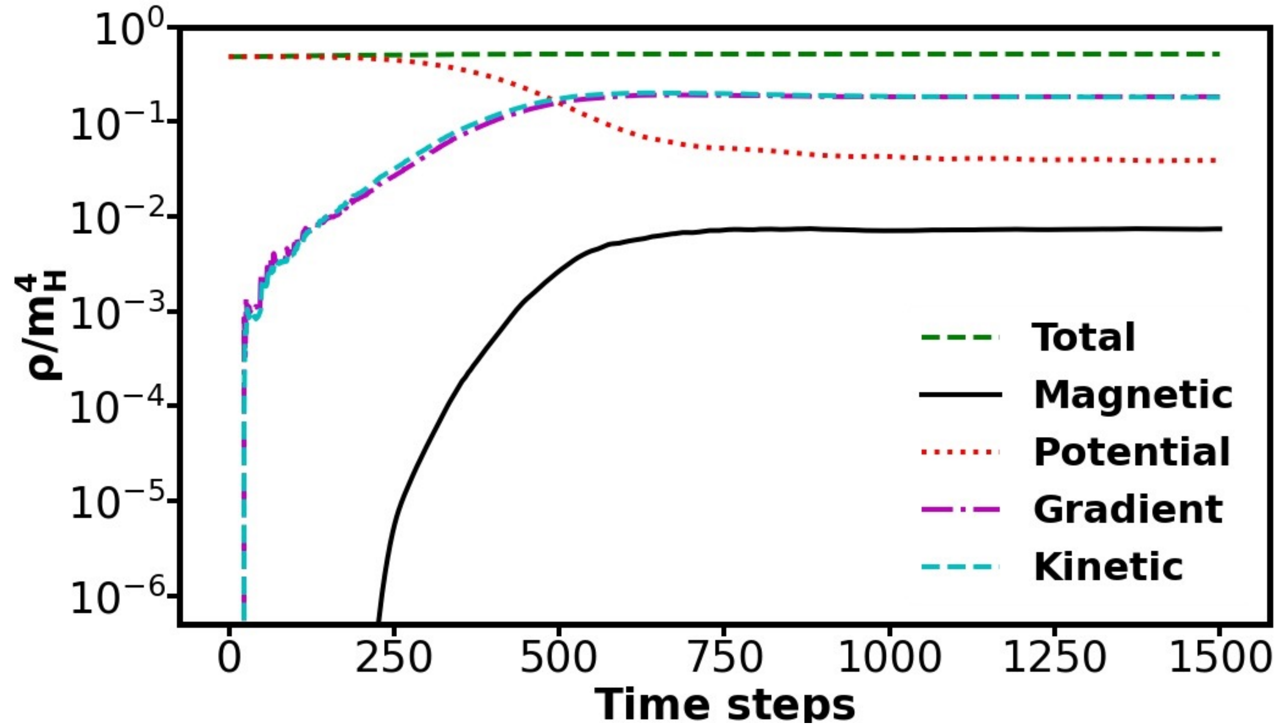
GW from Bubble collisions

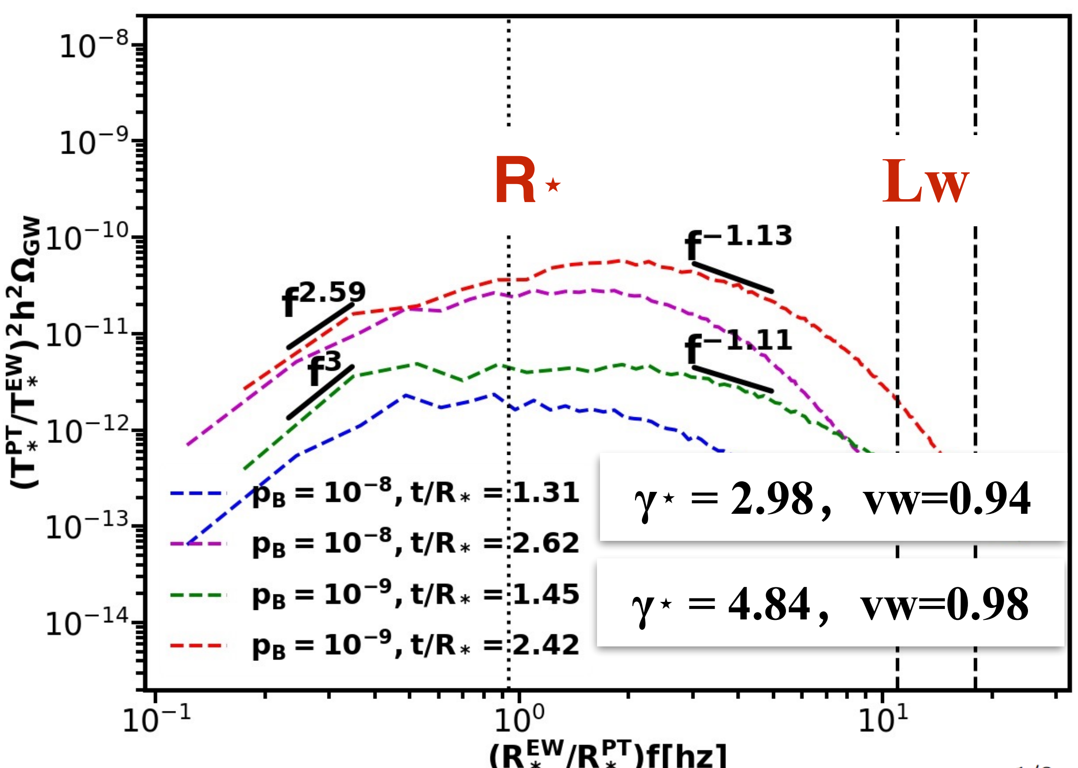
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\rm TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}{\rm Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

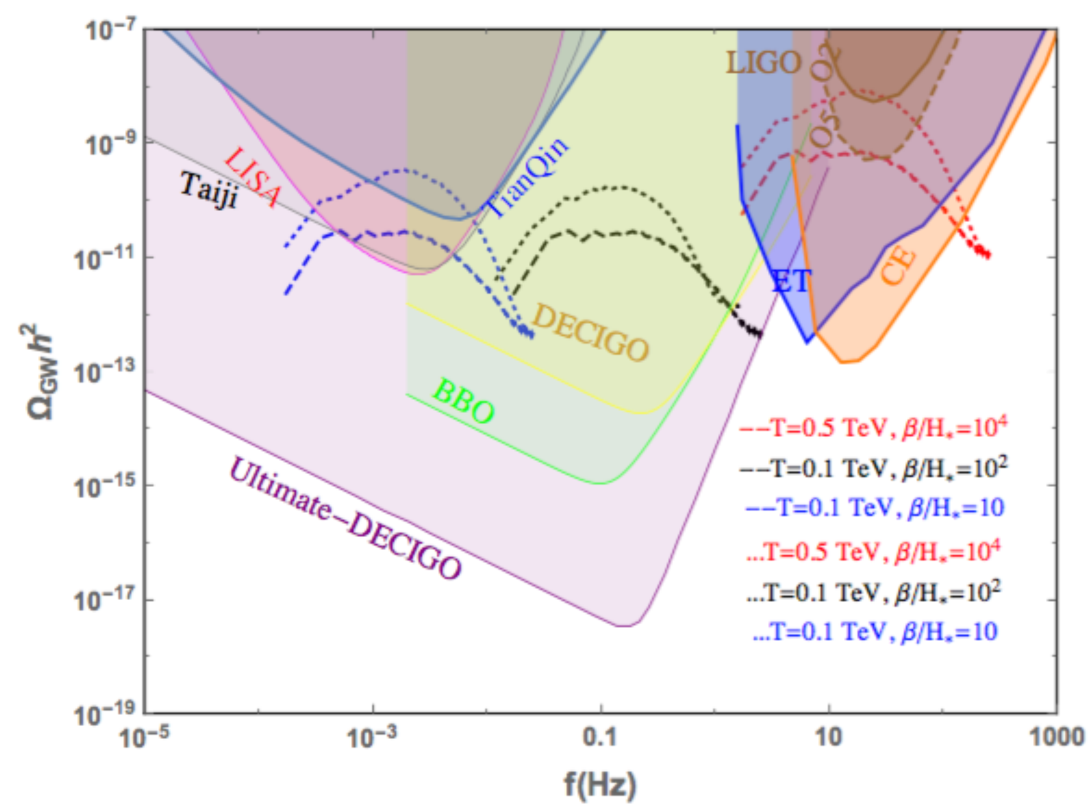
$$\frac{d\Omega_{\rm gw}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





$\gamma_\star = R_\star/(2R_c)$

$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



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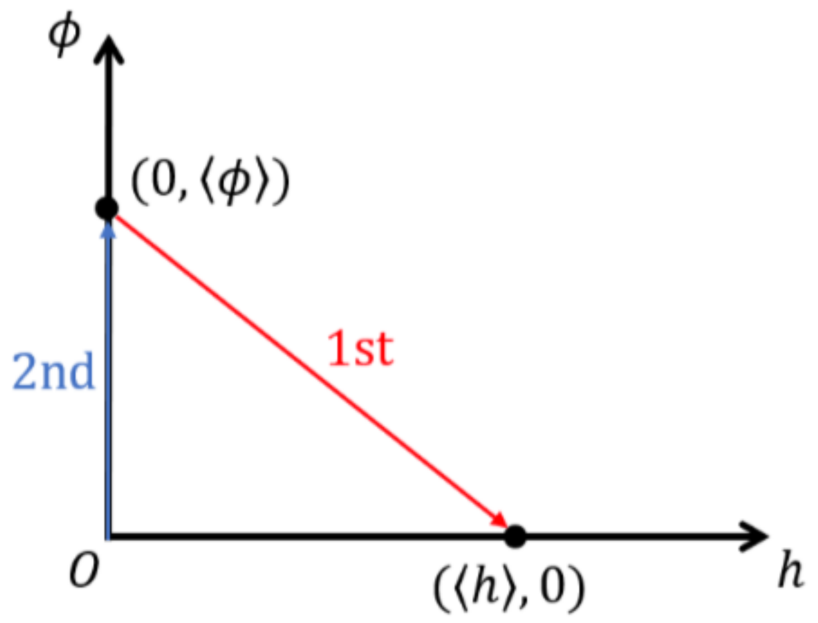
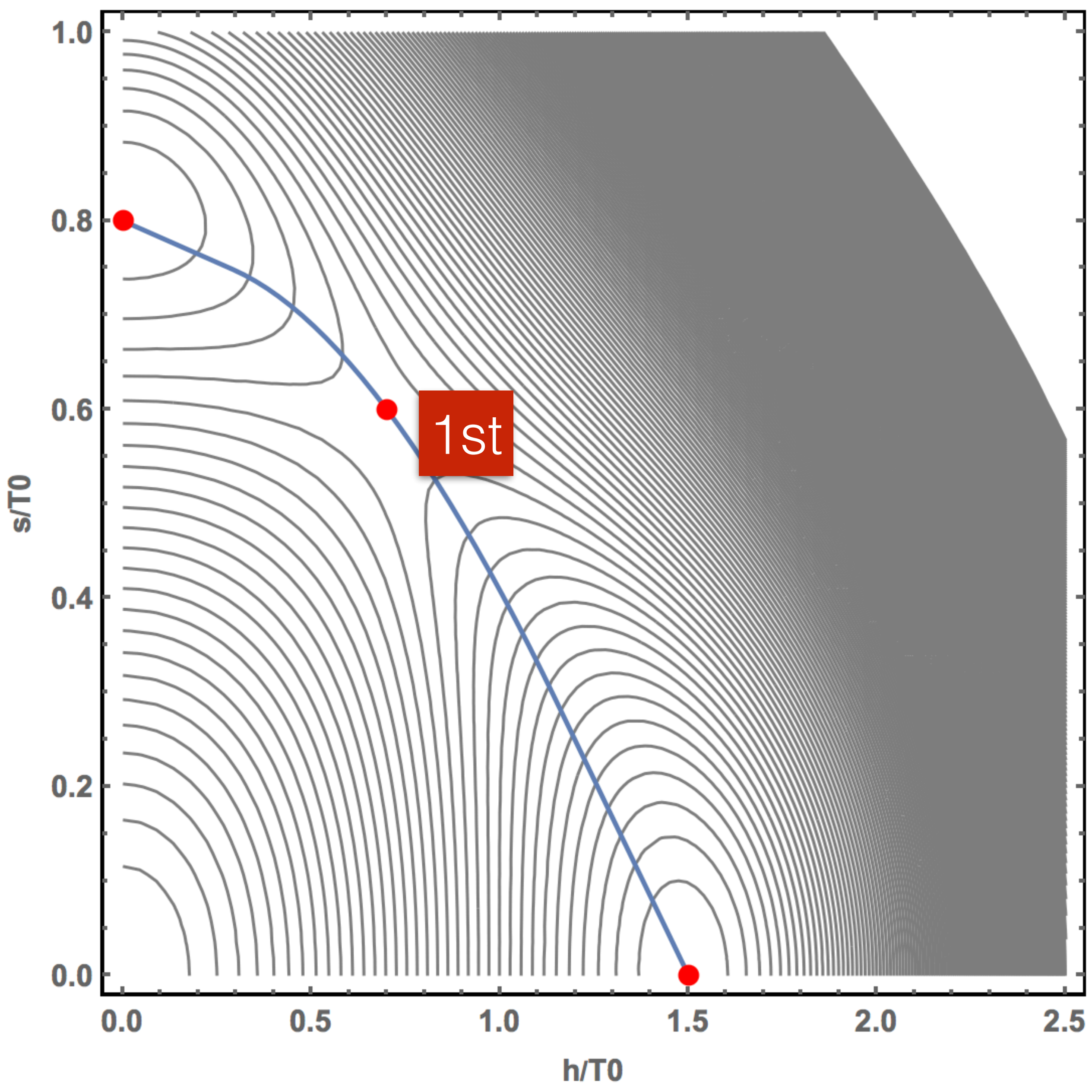
▸ Two-step FOPT potential

Type-a





$$V_a(\phi, h, T) = \frac{1}{2}(\mu_\phi^2 + c_\phi T^2)\phi^2 + \frac{1}{2}\lambda_{h\phi}h^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}(-\mu_h^2 + c_h T^2)h^2 + \frac{1}{4}\lambda_h h^4$$

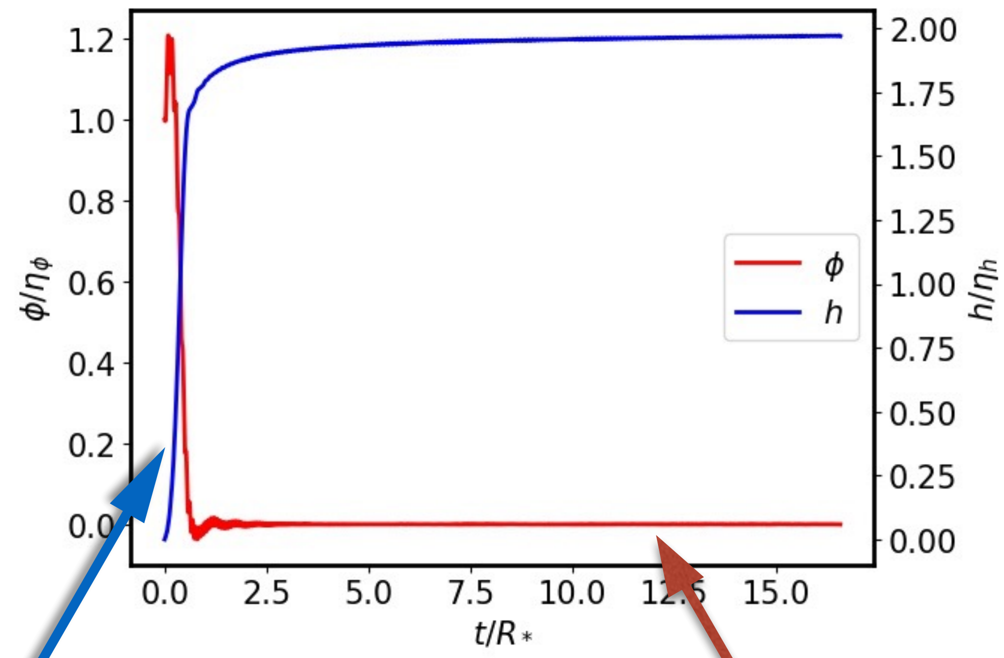
$c_\phi = \lambda_\phi/4 + \lambda_{h\phi}/3$

$c_h = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_{h\phi}/12$

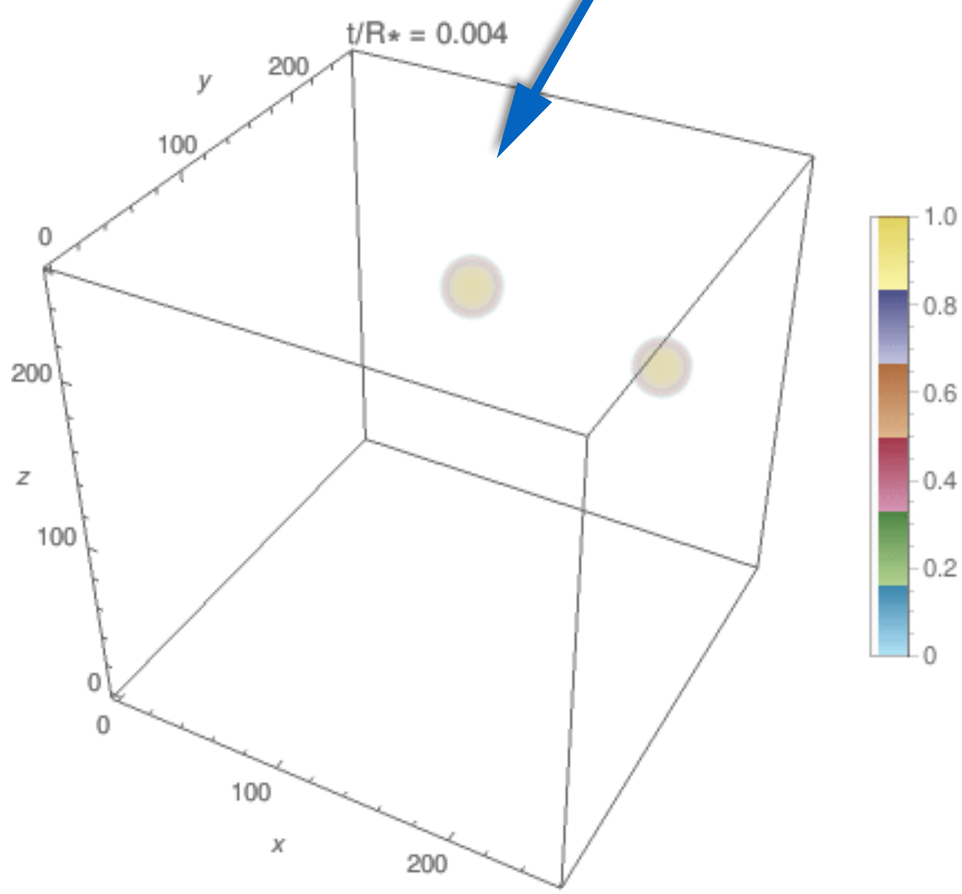
Motivated for DM&EWBG, see:1804.06813,1702.06124,1609.07143, 1605.08663, 1605.08663,etc

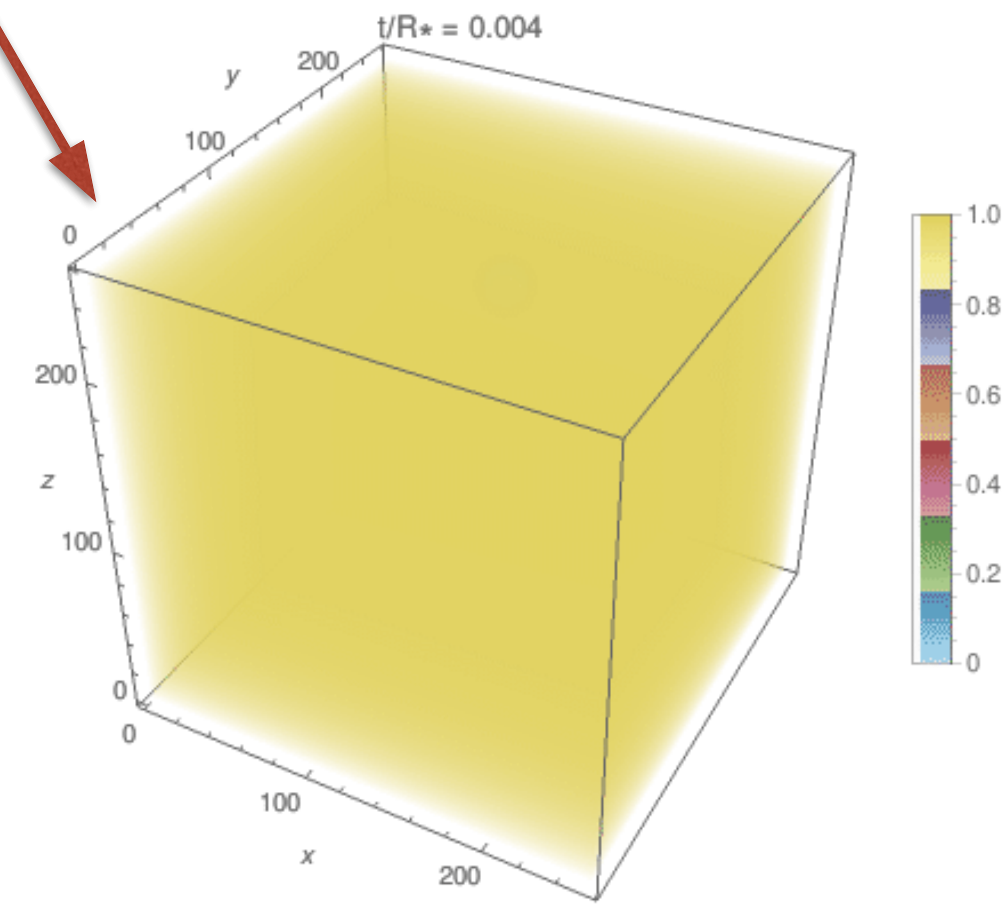
▸ Two-step PT with the second-step being FOPT

Type-a



$$h(t=0,r) = \eta_h/2\left[1-\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$
$$\phi(t=0,r) = \eta_\phi/2\left[1+\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$





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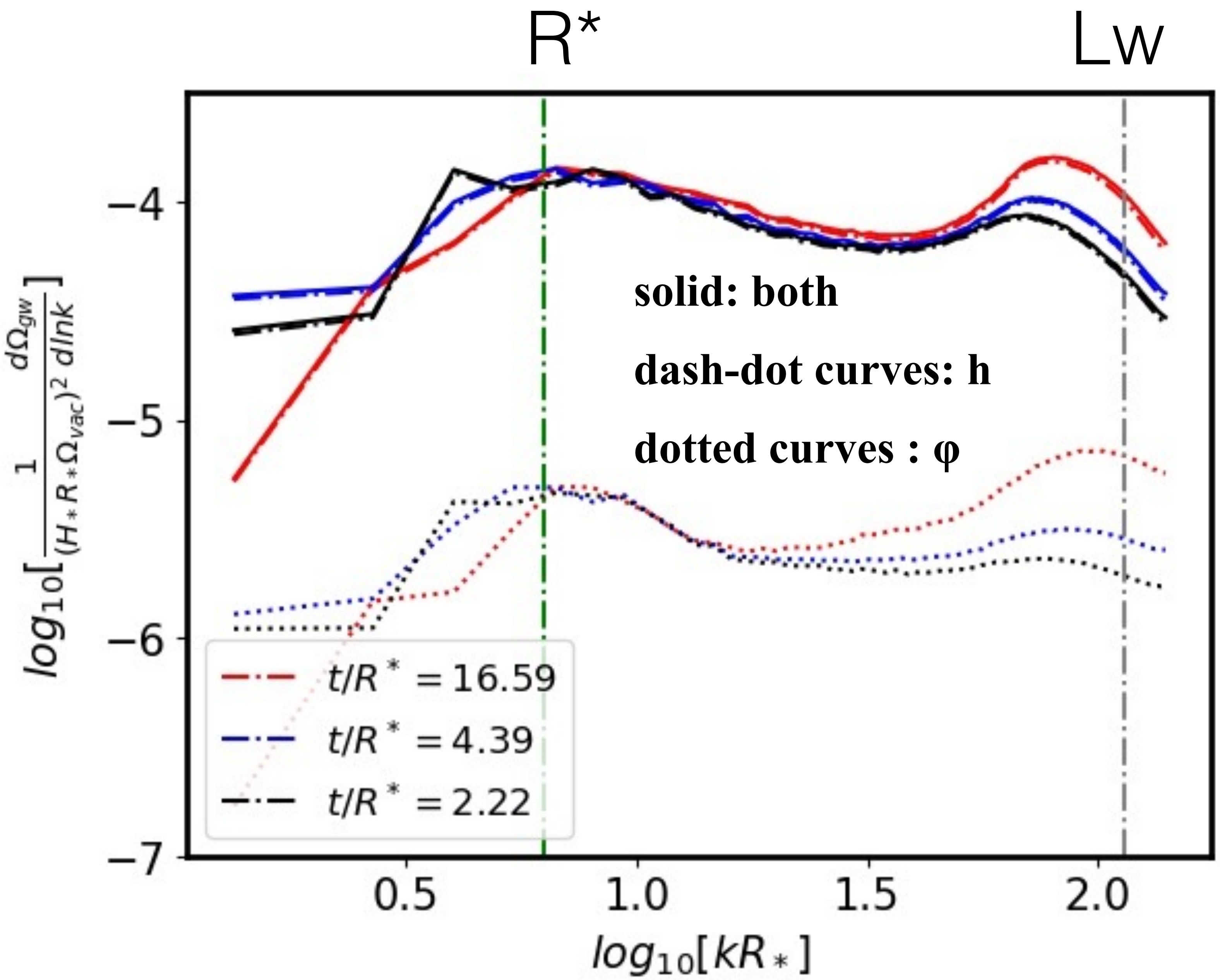
▶ Two-step PT with the second-step being FOPT

Type-a



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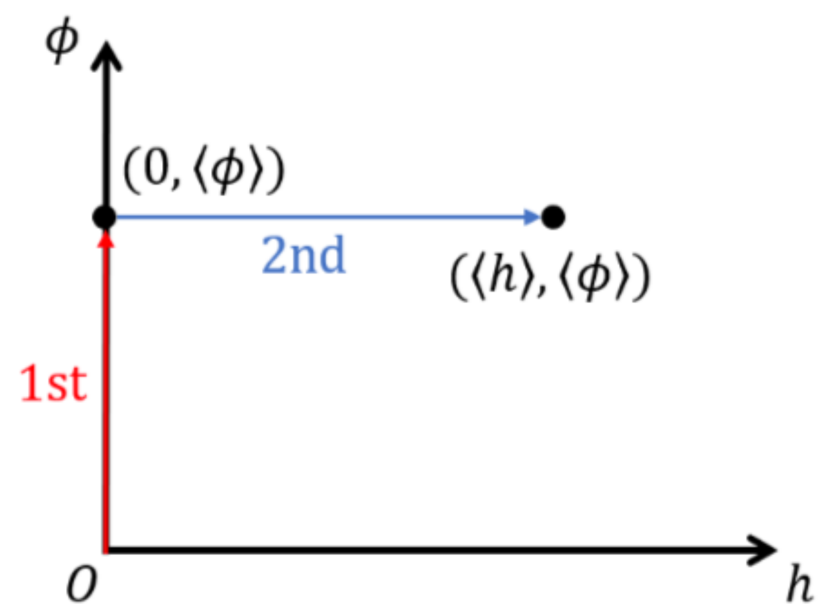
Two-step PT with first-step being FOPT

Type-b Without Global U(1)



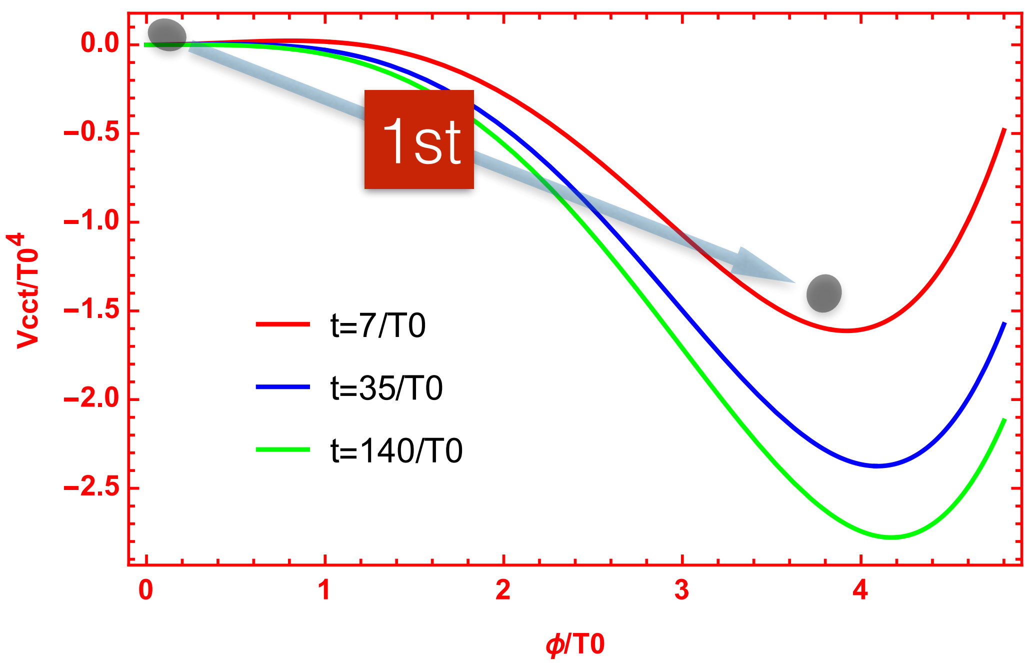
$$V_{cct}(\phi,T) = a\phi^4(\log[|\phi|^2/v_\phi^2] - 1/4) + bT^2|\phi|^2$$

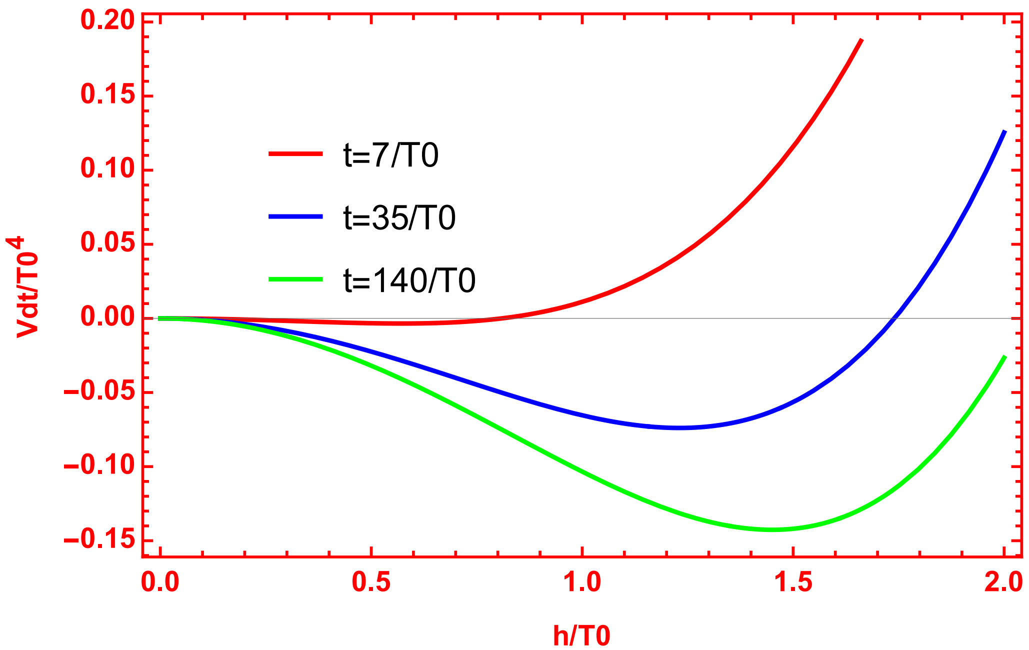
$$V_{dt}(\phi,h,T) = \frac{1}{2}c_h' T^2 h^2 + \frac{1}{4}\lambda_h h^4 - \frac{\lambda_p}{4}h^2\phi^2$$

$$c_h' = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_p/24$$

$$\langle h\rangle = \sqrt{(\lambda_p\eta^2 - 2c_h' T^2)/(2\lambda_h)}$$

Classical conformal + Dimensional transmutation





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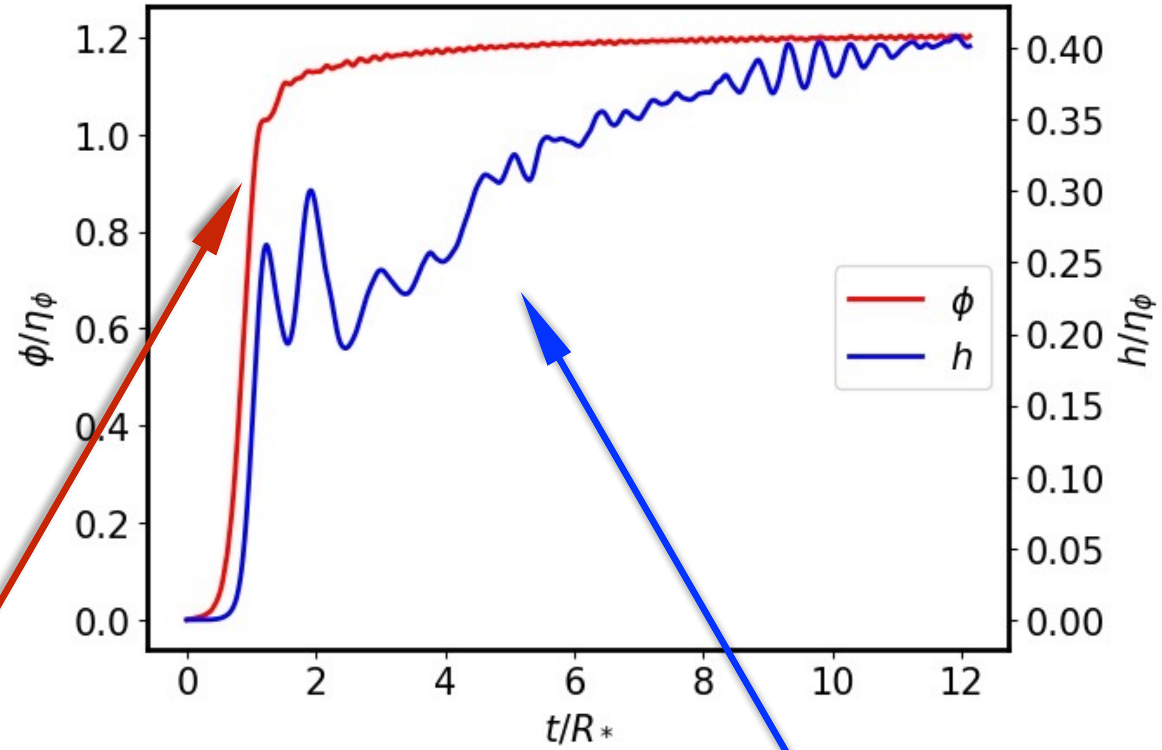
▸ Two-step PT with first-step being FOPT

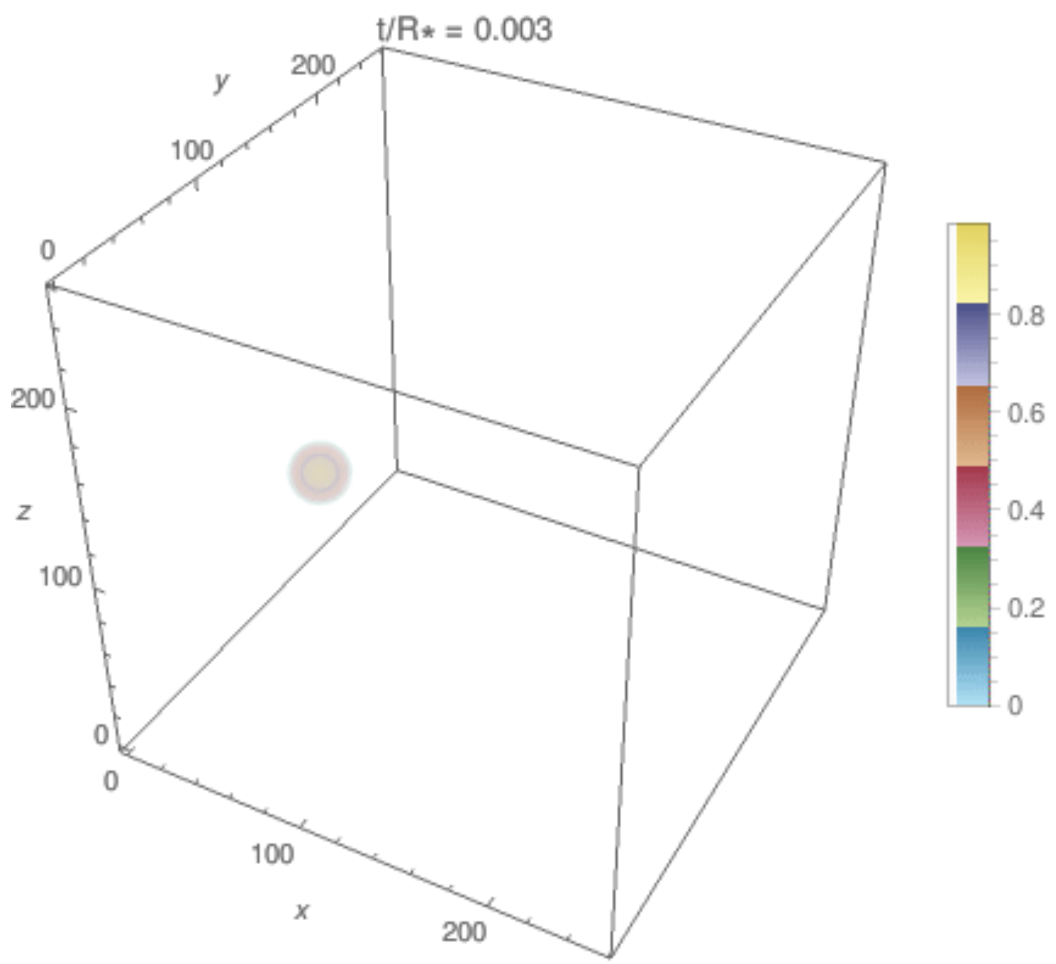
Type-b

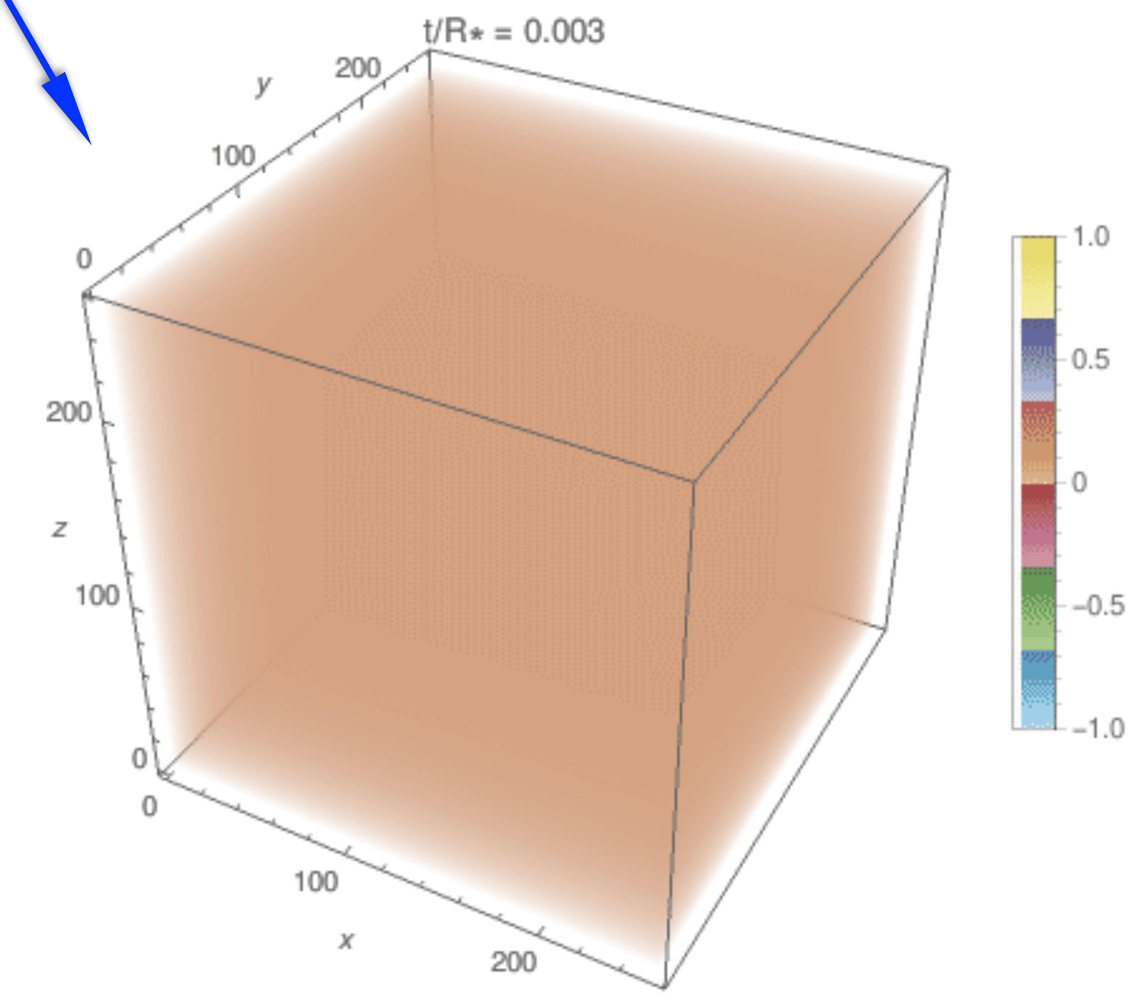
Without Global U(1)



$$\phi(t=0,r) = \eta_\phi/2\left[1-\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

$$\langle h\rangle = \sqrt{(\lambda_p\eta^2 - 2c_h' T^2)/(2\lambda_h)}$$



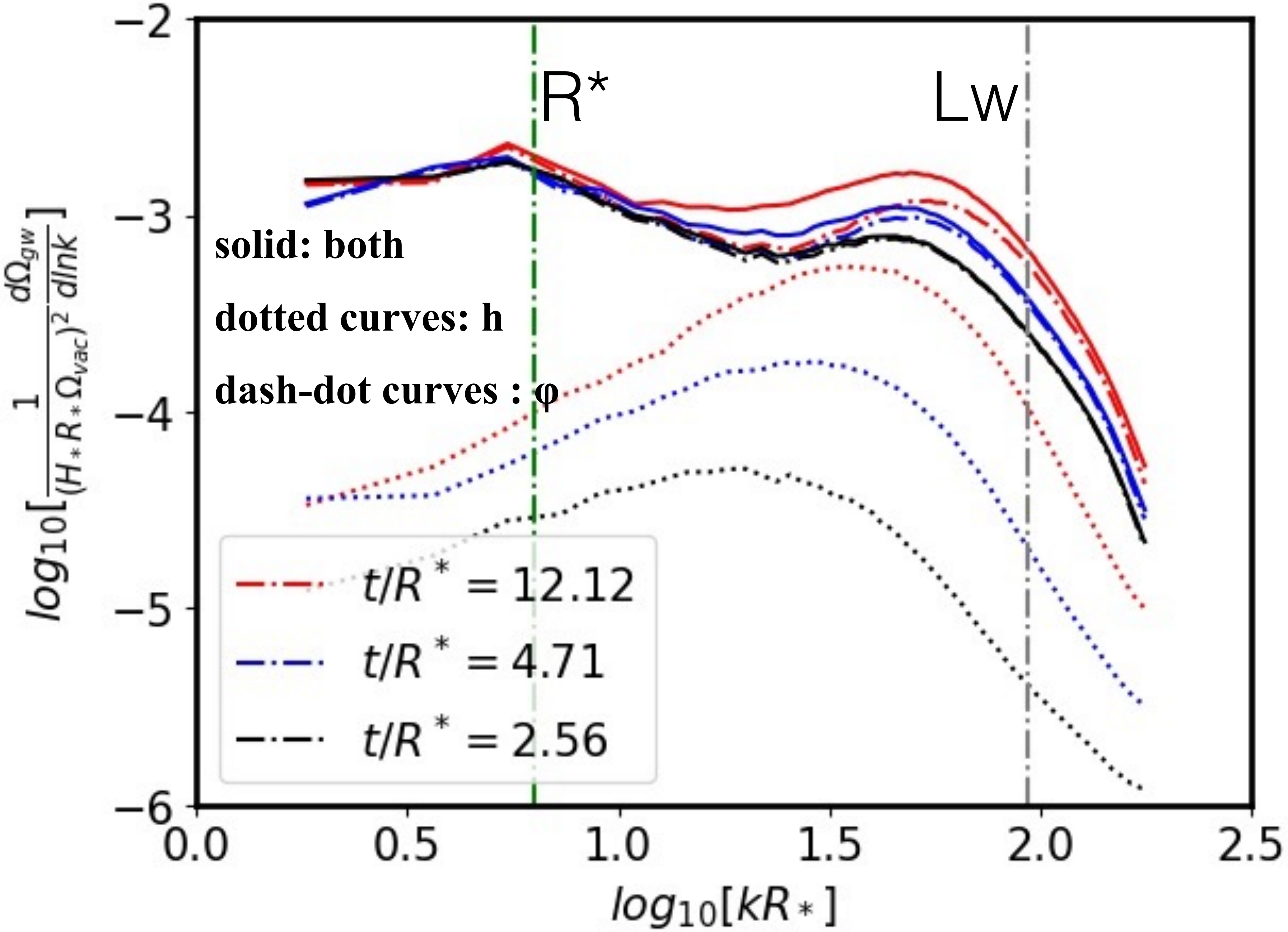


Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with first-step being FOPT

Type-b

Without Global U(1)



Zhao, Di, **Bian**, Cai,2204.04427

scalar field + fluid system

The full energy-momentum tensor into two components, one for the fluid $T^{\mu\nu}$ and one for the Higgs $T^{\mu\nu}$

$$T_{\rm f}^{\mu\nu} = (e+p_1)u^\mu u^\nu + p_1 g^{\mu\nu},$$
$$T_{\phi}^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + V_0(\phi)\right),$$

Fluid pressure is the total contribution from all particles

$$p_1(\phi,T) = \frac{\pi^2}{90} g_{\rm eff} T^4 - V_1(\phi,T) = -\sum_B f_B(m(\phi),T) - \sum_F f_F(m(\phi),T)$$

Thermally corrected potential $V_T(\phi) = V_0(\phi) + V_1(\phi,T)$

$$\partial_\mu T_\phi^{\mu\nu} = +\partial^\nu\phi \frac{\mathrm{d}m^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}} f(p,x).$$

Boltzmann equation with collisions and external forces

$$\Box\phi - V_T'(\phi) = -\frac{\mathrm{d}m^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x)$$

$$f(p,x) = f^{\rm eq}(p,x) + \delta f(p,x)$$

This leaves us with the equation of motion

$$\Box\phi - V_T'(\phi) = -\tilde{\eta}\frac{\mathrm{d}m^2}{\mathrm{d}\phi}u^\mu\partial_\mu\phi$$

where

$$\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x) = \tilde{\eta} u^\mu \partial_\mu \phi\,,$$

Local equilibrium and perfect fluid

SciPost Phys. Lect.Notes 24 (2021)

scalar field + fluid system

With $p(\phi,T)=p_1(\phi,T)-V_0(\phi)$, we have $T_f^{\mu\nu} = (e+p)u^\mu u^\nu + p g^{\mu\nu}$,

The full energy-momentum tensor is conserved $\partial_\mu\left(T_f^{\mu\nu}+T_\phi^{\mu\nu}\right)=0$

which yields

$$\partial_\mu T_f^{\mu\nu} + V_T'(\phi)\partial^\nu\phi = -\partial^\nu\phi \frac{\mathrm{d}m^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x)$$

Consider the scalar product of u with both sides

$$u_\nu\partial_\mu(wu^\mu u^\nu + pg^{\mu\nu}) + V_T'(\phi)u\cdot\partial\phi = \tilde{\eta}(u\cdot\partial\phi)^2$$

Here, $w = e + p = Ts$ is the enthalpy density, and $s = \mathrm{d}p/\mathrm{d}T$ is the entropy density

scalar field + fluid system

Hydrodynamic equations for a single bubble

$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + [1 - \gamma^2 v(\xi - v)]\partial_\xi v\,,$$

$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v.$$

$\xi = r/t$, r is the distance from the center of the bubble and t is the time since nucleation,

$v(\xi_w)$ is the fluid velocity at the location of the bubble wall and $\xi_w = v_w$ is the wall velocity.

Introducing sound speed $c^2_s \equiv (dp/dT)/(de/dT)$, we have

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v,$$

$$\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$$

μ is the fluid velocity at ξ in a frame

that is moving outward at speed ξ

From the conservation of the energy momentum tensor accross the interface, we have the boundary conditions:

$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}\,, \quad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-},$$

+(−) meaning in front (behind) of the bubble wall (in the rest frame of the bubble wall) .

Bubble dynamics and fluid @ FOPT

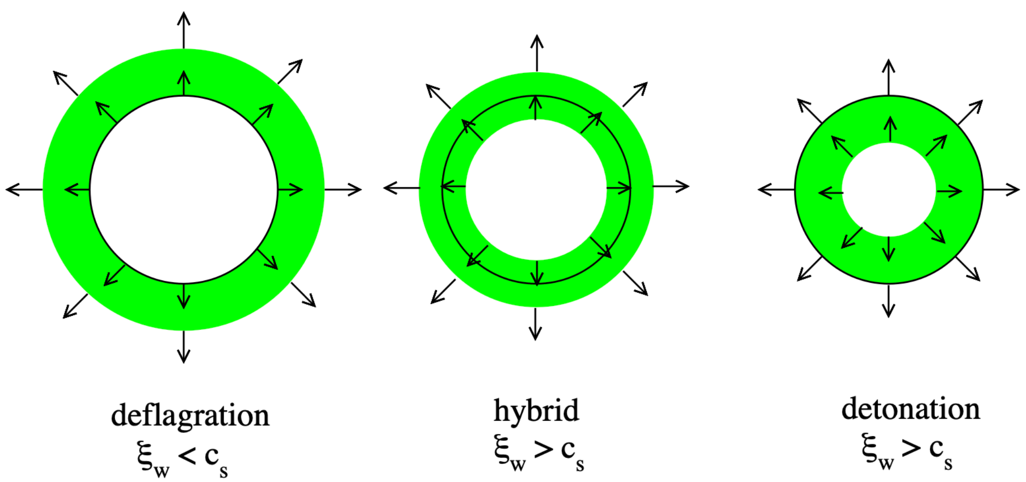
symmetric phase $$p_+ = \frac{1}{3} a_+ T_+^4 - \epsilon\,, \quad e_+ = a_+ T_+^4 + \epsilon\,,$$

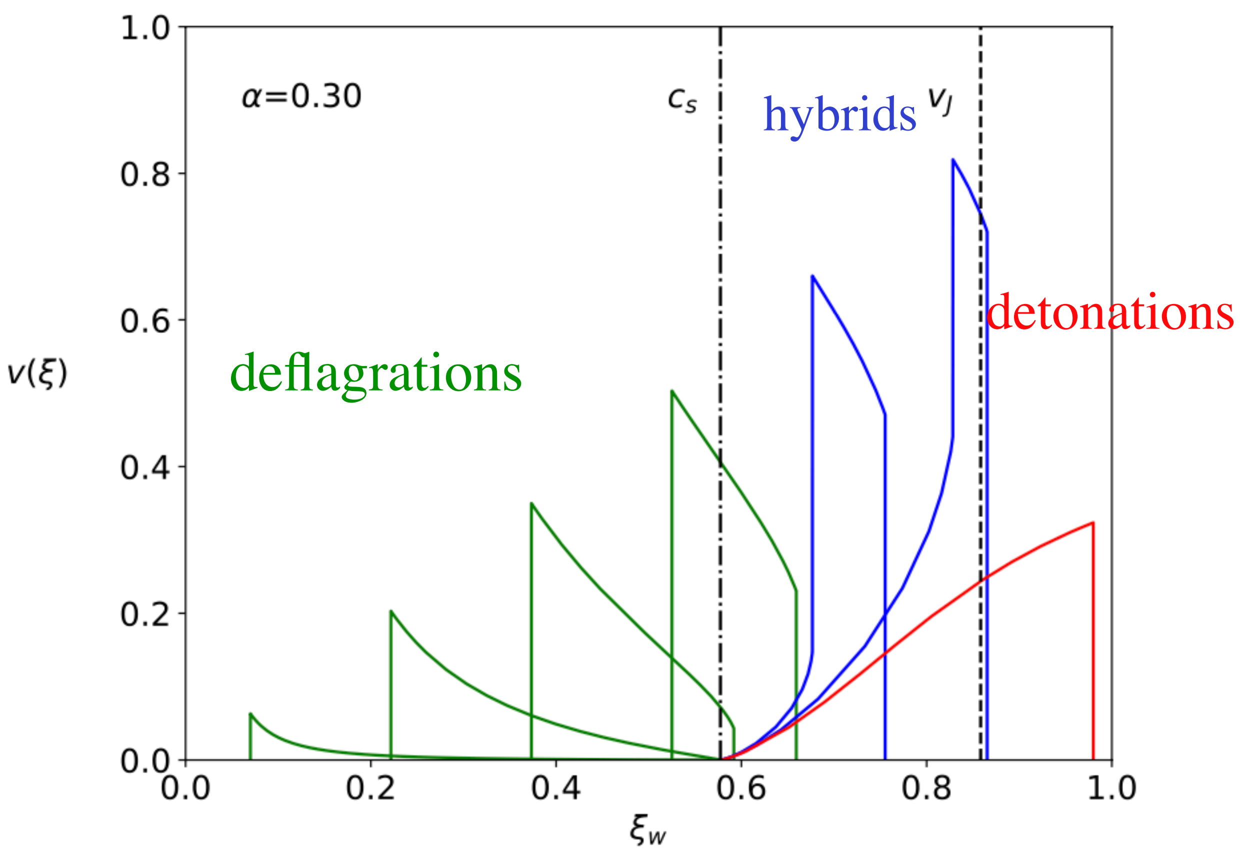
broken phase $$p_- = \frac{1}{3} a_- T_-^4\,, \quad e_- = a_- T_-^4\,,$$

Bag equation of state

the false-vacuum energy resulting from the Higgs potential

Different number of light degrees of freedom across the wall, different values a_+ and a_- (with $a_+ > a_-$) and different temperatures on both sides of the wall





The black circle is the phase interface (bubble wall).

Green region is of non-zero fluid velocity

deflagration walls have a shock wave propagating in front of the wall, detonation walls have a rarefaction wave behind it, hybrid walls have both shock and rarefaction waves.

JCAP06(2010)028

真空泡碰撞、合并、流体演化产生引力波

有限温度有效势能

$$V(\phi,T)=\frac{1}{2}\gamma(T^2-T_0^2)\phi^2-\frac{1}{3}AT\phi^3+\frac{1}{4}\lambda\phi^4$$

新物理

标量场-相对论流体运动方程

$$-\ddot{\phi}+\nabla^2\phi-\frac{\partial V}{\partial\phi}=\eta W(\dot{\phi}+V^i\partial_i\phi)$$

η：粒子和真空泡壁相互作用

$$\dot{E}+\partial_i(EV^i)+p[\dot{W}+\partial_i(WV^i)]-\frac{\partial V}{\partial\phi}W(\dot{\phi}+V^i\partial_i\phi)=\eta W^2(\dot{\phi}+V^i\partial_i\phi)^2$$

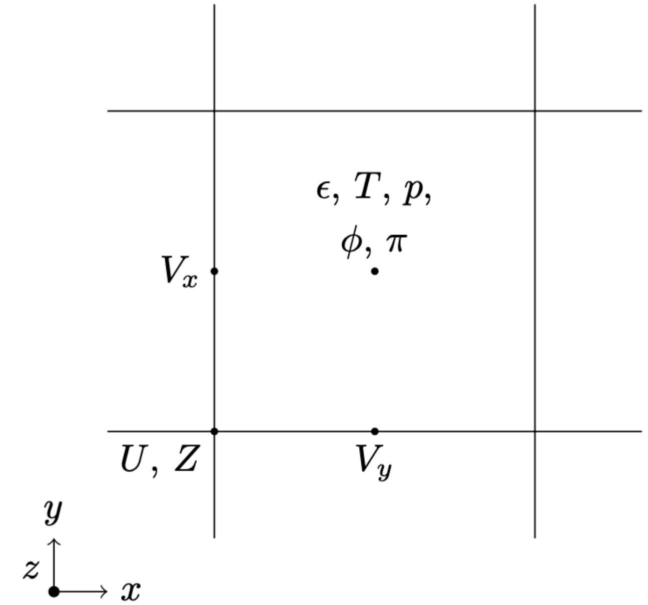
$$\dot{Z}_i+\partial_j(Z_iV^j)+\partial_i p+\frac{\partial V}{\partial\phi}\partial_i\phi=-\eta W(\dot{\phi}+V^j\partial_j\phi)\partial_i\phi$$

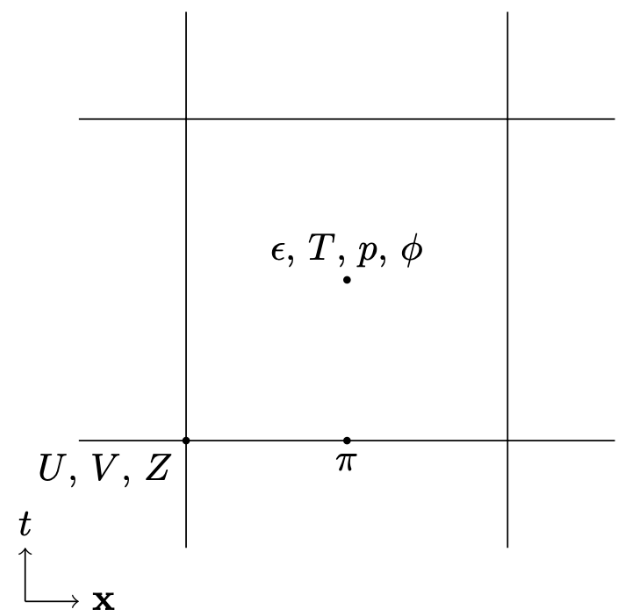
equation of state

$$\epsilon(T,\phi)=3aT^4+V(\phi,T)-T\frac{\partial V}{\partial T},$$
$$p(T,\phi)=aT^4-V(\phi,T)$$

fluid momentum density $Z_i=W(\epsilon+p)U_i$

fluid energy density E = $W\epsilon$

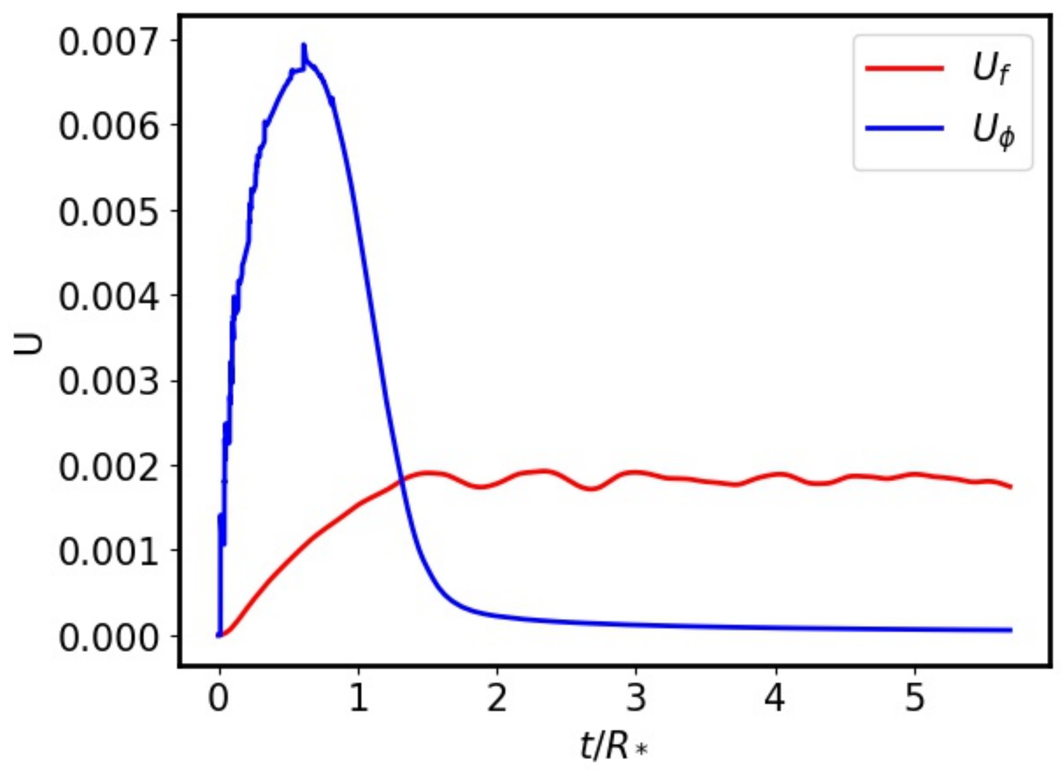




V^i is the fluid 3-velocity

$U^i = WV^i$, W: relativistic γ-factor

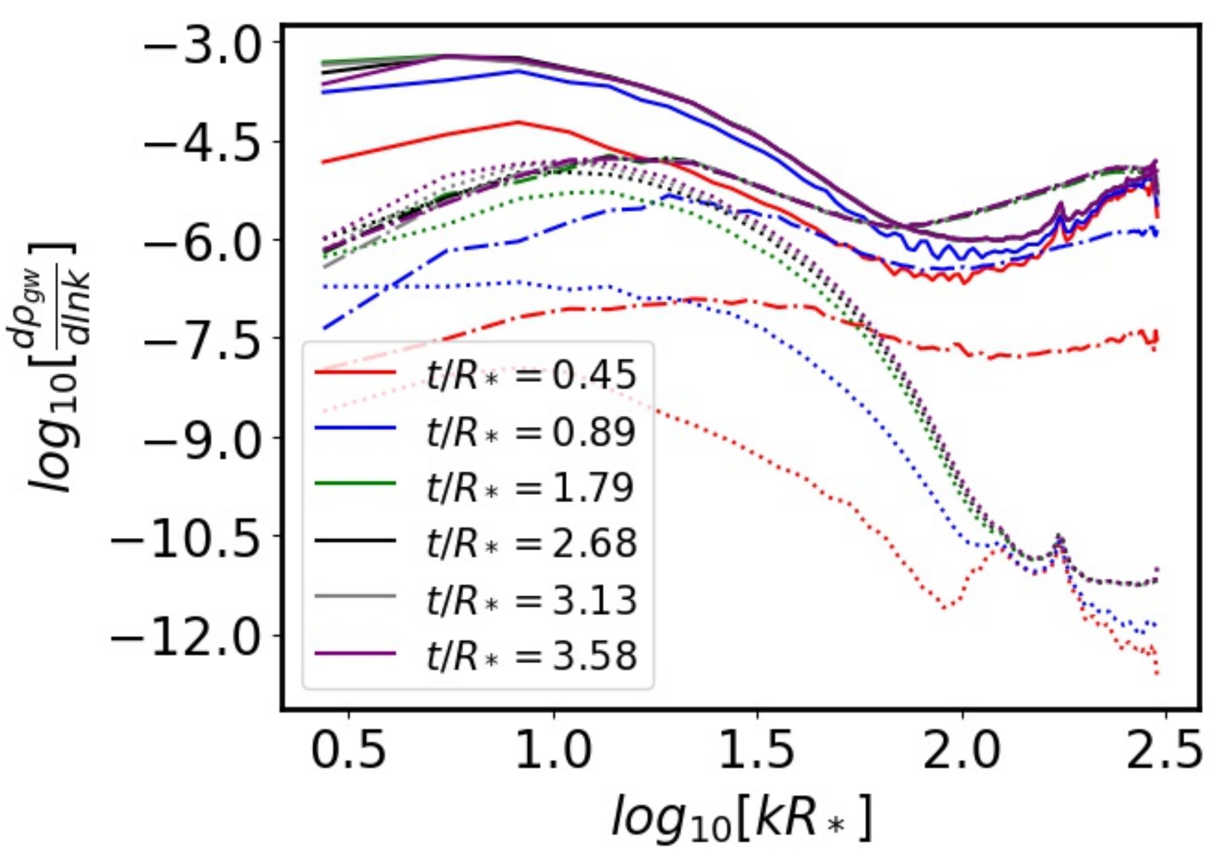
真空泡碰撞、合并、流体演化产生引力波



$$\tau_{ij}^{\phi} = \partial_i\phi\partial_j\phi, \quad \tau_{ij}^{\mathrm{f}} = W^2(\epsilon + p)V_iV_j$$

$$(\bar{\epsilon}+\bar{p})\overline{U}_{\mathrm{f}}^2 = \frac{1}{\mathcal{V}}\int_{\mathcal{V}} d^3x\tau_{ii}^{\mathrm{f}}$$

$$(\bar{\epsilon}+\bar{p})\overline{U}_{\phi}^2 = \frac{1}{\mathcal{V}}\int_{\mathcal{V}} d^3x\tau_{ii}^{\phi}$$

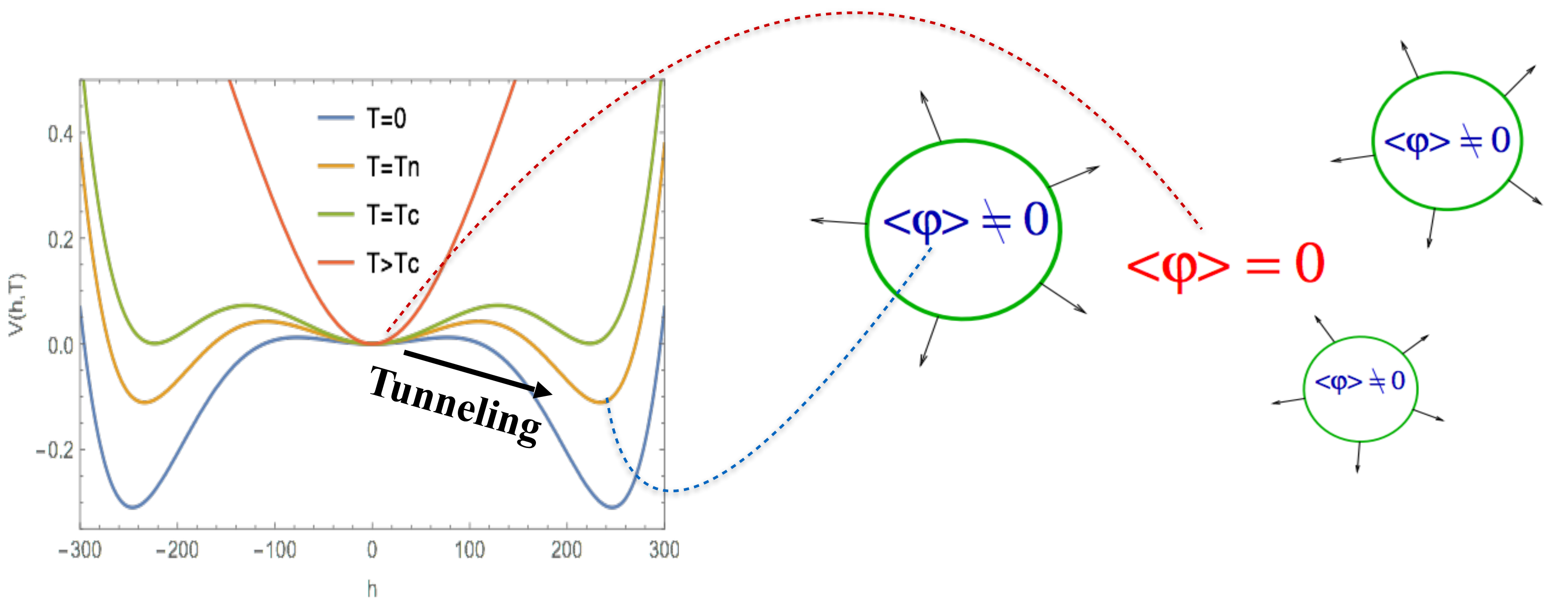


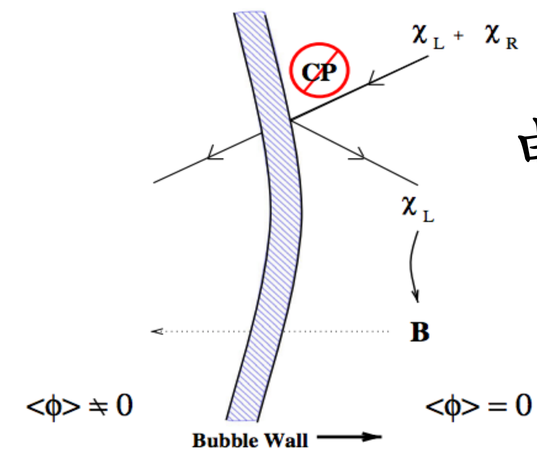
$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G(\tau_{ij}^{\phi} + \tau_{ij}^{\mathrm{f}})$$

$$h_{ij}(\mathbf{k}) = \lambda_{ij,lm}(\hat{\mathbf{k}})u_{lm}(t,\mathbf{k})$$

Bian,Jia,Zhao,Zhu,arxiv:230X.XXXXX

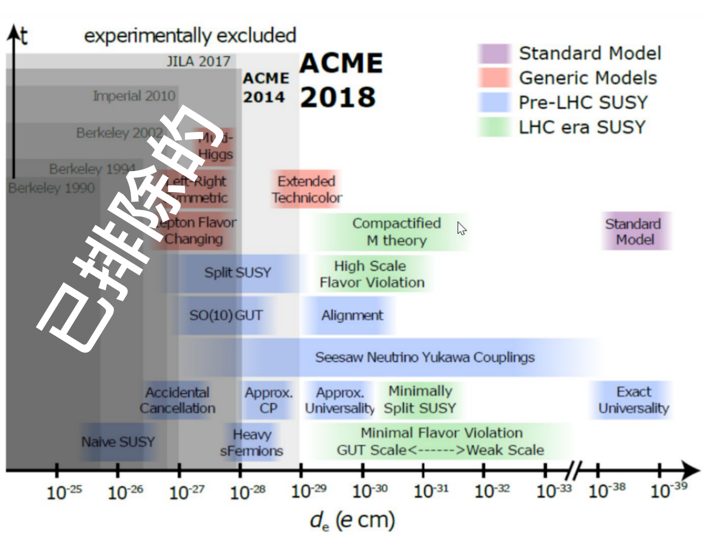
正反物质不对称&强一阶电弱相变

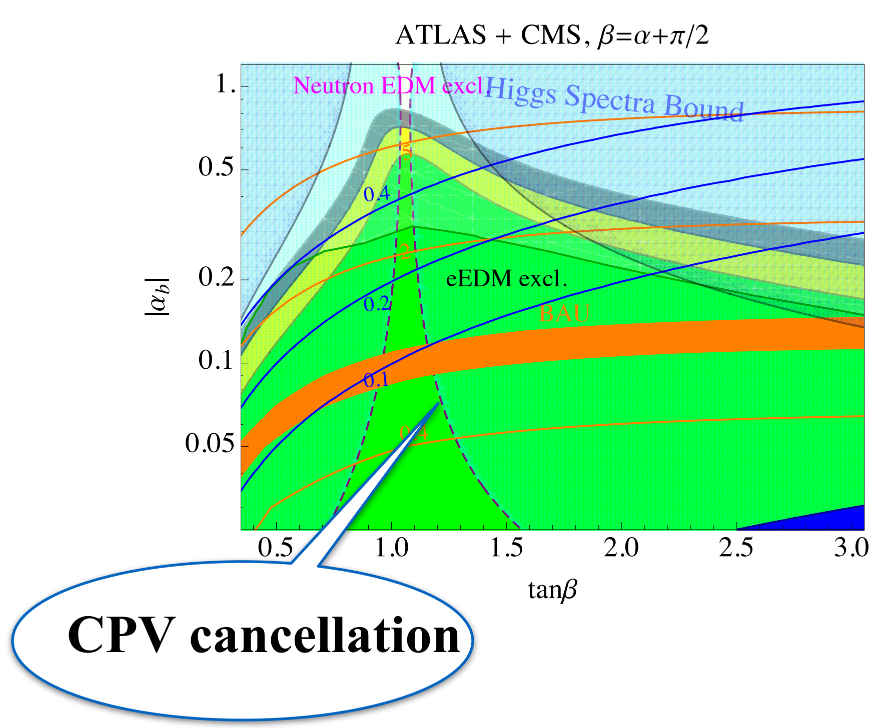


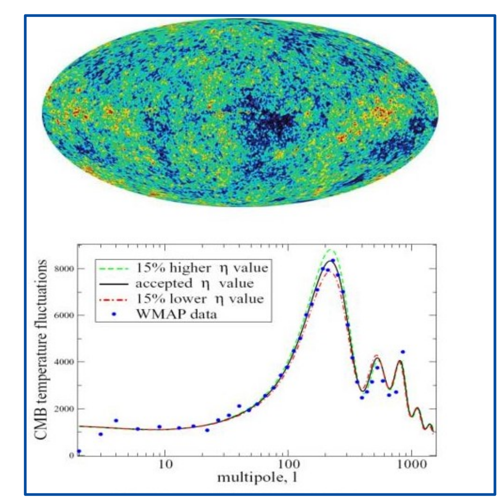


电弱相变中
解释BAU

Chup etal，Rev Mod Phys.91.015001







$$\frac{n_B}{s} = (8.579 \pm 0.109) \times 10^{-11}$$

Bian, Liu, Shu, PRL115 (2015) 021801

Bubble wall velocity with the EW plasma

Wall frame

Boltzmann equation which dictates the time evolution of the particle distribution

$$\frac{df_a}{dt} = \partial_t f_a + \dot{\vec{x}} \cdot \partial_{\vec{x}} f_a + \dot{\vec{p}} \cdot \partial_{\vec{p}} f_a = C[f_a], \qquad \text{(a)}$$

The *fluid ansatz* for the distribution function is written as

$$f \approx f_v - f_v' \delta \bar{X} + \delta f_u + \mathcal{O}(\delta f^2), \qquad f_v = \frac{1}{e^{\beta\gamma(E - v p_z)} \pm 1}, \quad f_v' \equiv \frac{df_v}{d\beta\gamma E},$$

$$\delta \bar{X} = \mu + \beta\gamma\delta\tau(E - v p_z)$$ perturbations from equilibrium

μ: chemical potential, δτ: temperature perturbation, δf_u : the velocity perturbation

The force and group velocity

$$\dot{z} \equiv \frac{\partial \omega}{\partial p_z} = \frac{p_z}{E} + s\frac{m^2\theta'}{2E^2E_z}, \qquad \dot{p_z} \equiv -\frac{\partial\omega}{\partial z} = -\frac{(m^2)'}{2E} + s\frac{(m^2\theta')'}{2EE_z}, \qquad \text{(b)}$$

ω is the energy of the WKB wave packet and $E_z^2 \equiv p_z^2 + m^2$

Inserting the force and group velocity of eq. (b) into the Boltzmann equation (a), we have

$$\left[\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right]\left(f_v - f_v'\delta\bar{X} + \delta f_u\right) = C[f]$$

PHYSICAL REVIEW D 102, 063516 (2020)

Bubble wall velocity with the EW plasma

With $q = (\mu, \delta\tau, u)^T$, the transport equations take the form

$$A_v\vec{q}\,' + \Gamma\vec{q} = S,$$

Γ: the collision term C in the above equation

$$A_v = \begin{pmatrix} C_v^{1,1} & \gamma v C_0^{-1,0} & D_v^{0,0} \\ C_v^{0,1} & \gamma(C_v^{-1,1} - vC_v^{0,2}) & D_v^{-1,0} \\ C_v^{2,2} & \gamma(C_v^{1,2} - vC_v^{2,3}) & D_v^{1,1} \end{pmatrix},$$

Integrals of the particle distribution functions.

$$S = \gamma v \frac{(m^2)'}{2T^2} \begin{pmatrix} C_v^{1,0} \\ C_v^{0,0} \\ C_v^{2,1} \end{pmatrix},$$

Source term

The Higgs EOM in the presence of out of equilibrium particle populations

$$E_h \equiv \Box\phi + \frac{dV_{\text{eff}}(\phi, T)}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{\delta f_i(p,x)}{2E} = 0,$$

$$M_1 \equiv \int dz E_h h' dz = 0,$$

The total pressure on the wall should be zero

$$M_2 \equiv \int dz E_h h' [2h(z) - h_0] dz = 0.$$

An asymmetry in the total pressure between the front and back of the wall should be zero

$$h(z) = \frac{h_0}{2}\left[\tanh\left(\frac{z}{L_h}\right) + 1\right]$$

Bubble profile

EWBG with the EW plasma

Boltzmann equation $(v_g \partial_z + F \partial_{p_z}) f = \mathcal{C}[f]$ $v_g = \frac{p_z}{E_w}$,

CP-violating complex mass term $\hat{m}(z) = m(z) e^{i\gamma^5 \theta(z)}$ $F = -\frac{(m^2)'}{2E_w} + s s_{k_0} \frac{(m^2\theta')'}{2E_w E_{wz}}$,

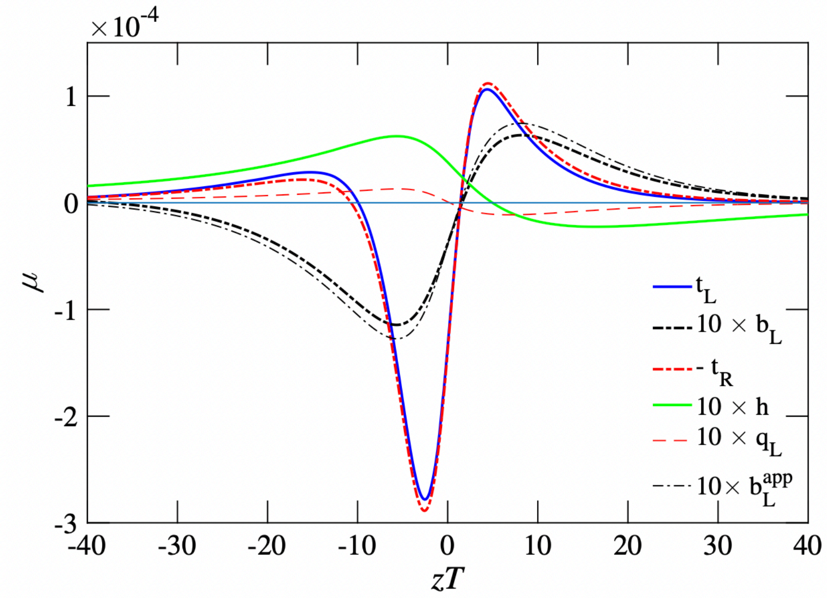
$$\mu \equiv \mu_e + s_{k_0} \mu_o,$$
$$\delta f \equiv \delta f_e + s_{k_0} \delta f_o.$$

Transport equations $Aw' + (m^2)' B w = S + \delta C,$

collision terms

$w=(\mu,u)^T$

$$A = \begin{pmatrix} -D_1 & 1 \\ -D_2 & R \end{pmatrix}, \quad B = \begin{pmatrix} -v_w \gamma_w Q_1 & 0 \\ -v_w \gamma_w Q_2 & \bar{R} \end{pmatrix},$$



Source term $S=(S_1, S_2)^T$ $S^o_{h\ell} = v_w \gamma_w s[(m^2\theta')' Q^{8o}_\ell - (m^2)' m^2 \theta' Q^{9o}_\ell],$

chemical potential for left handed baryon number $\mu_{B_L} = \frac{1}{2}(1 + 4D_0^t)\mu_{t_L} + \frac{1}{2}(1 + 4D_0^b)\mu_{b_L} + 2D_0^t \mu_{t_R}$

Baryon asymmetry $\eta_B = \frac{405\,\Gamma_{\rm sph}}{4\pi^2 v_w \gamma_w g_* T} \int dz\, \mu_{B_L} f_{\rm sph}\, e^{-45\Gamma_{\rm sph}|z|/4v_w},$

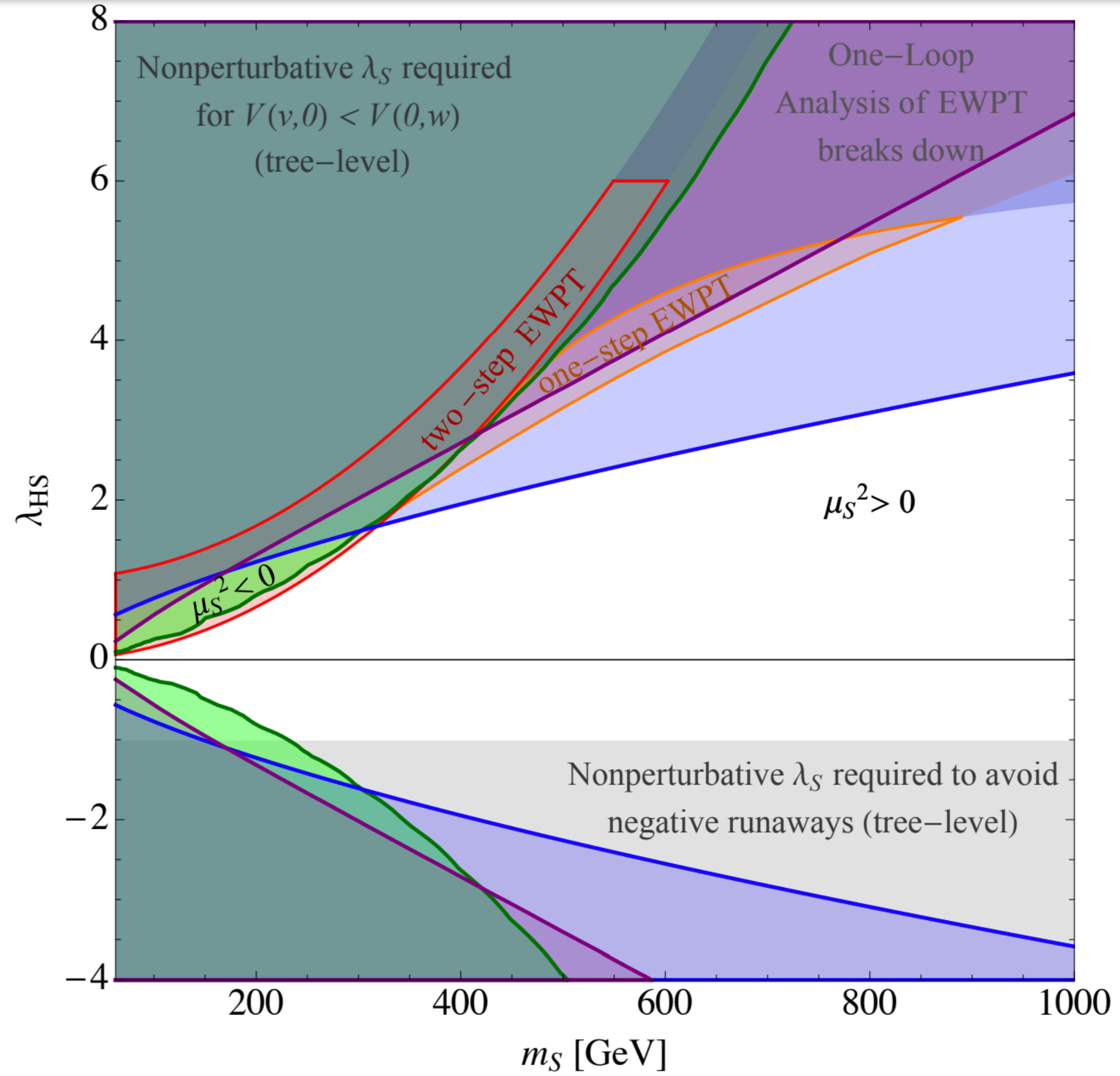
VEV- insertion source tends to predict a larger baryon asymmetry than the WKB source by a factor of ~10.

Phys. Rev. D **101** (2020) 063525

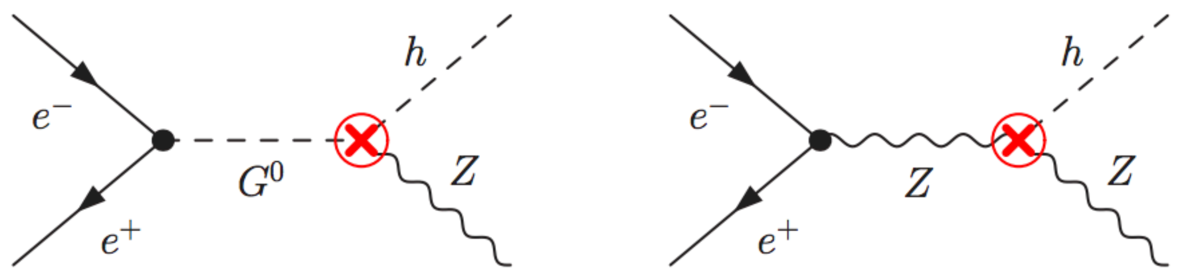
◉ Zh@ILC/CEPC

$$V_0 = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}\mu_S^2 S^2 + \lambda_{HS}|H|^2 S^2 + \frac{1}{4}\lambda_S S^4$$

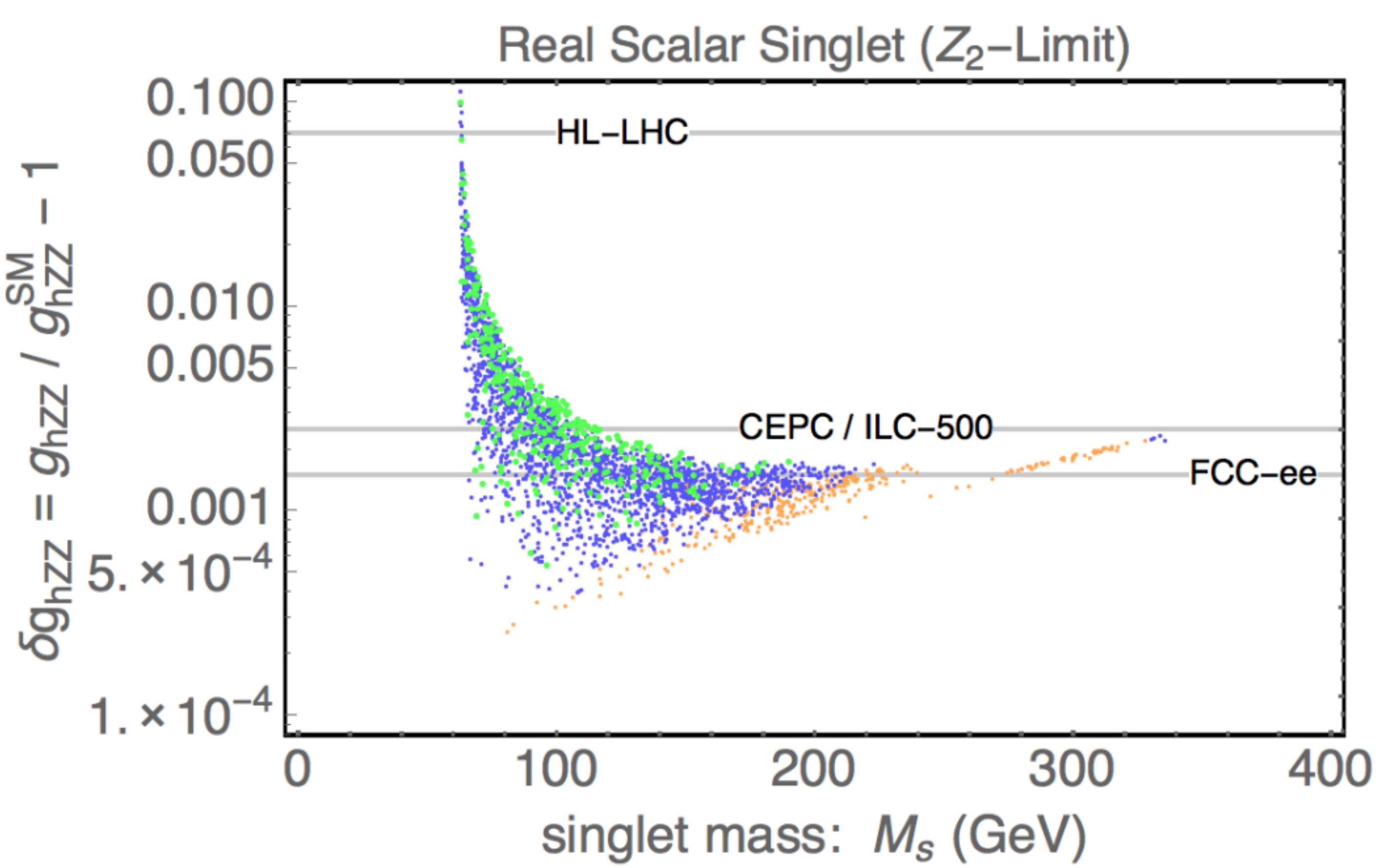
$$V_{\rm eff}(h,T) = V_0(h) + V_0^{CW}(h) + V_T(h,T) + V_r(h,T)$$



Curtin, Meade, Yu,1409.0005



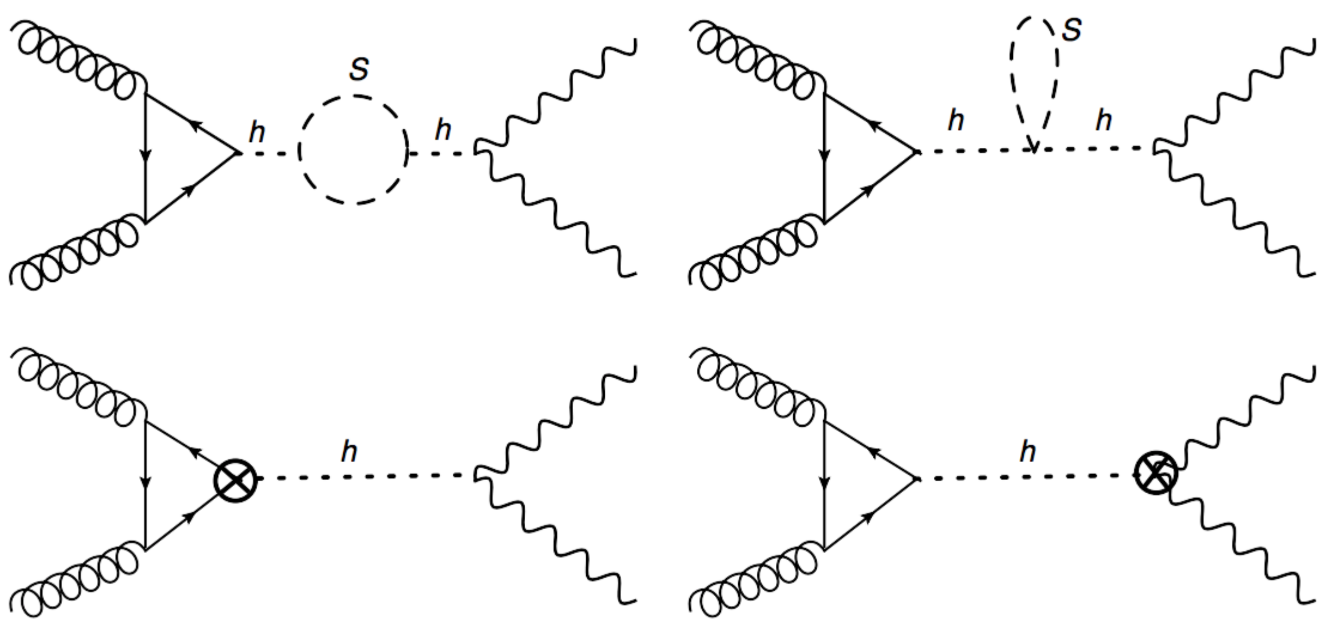
Craig,Englert, and McCullough,1305.5251

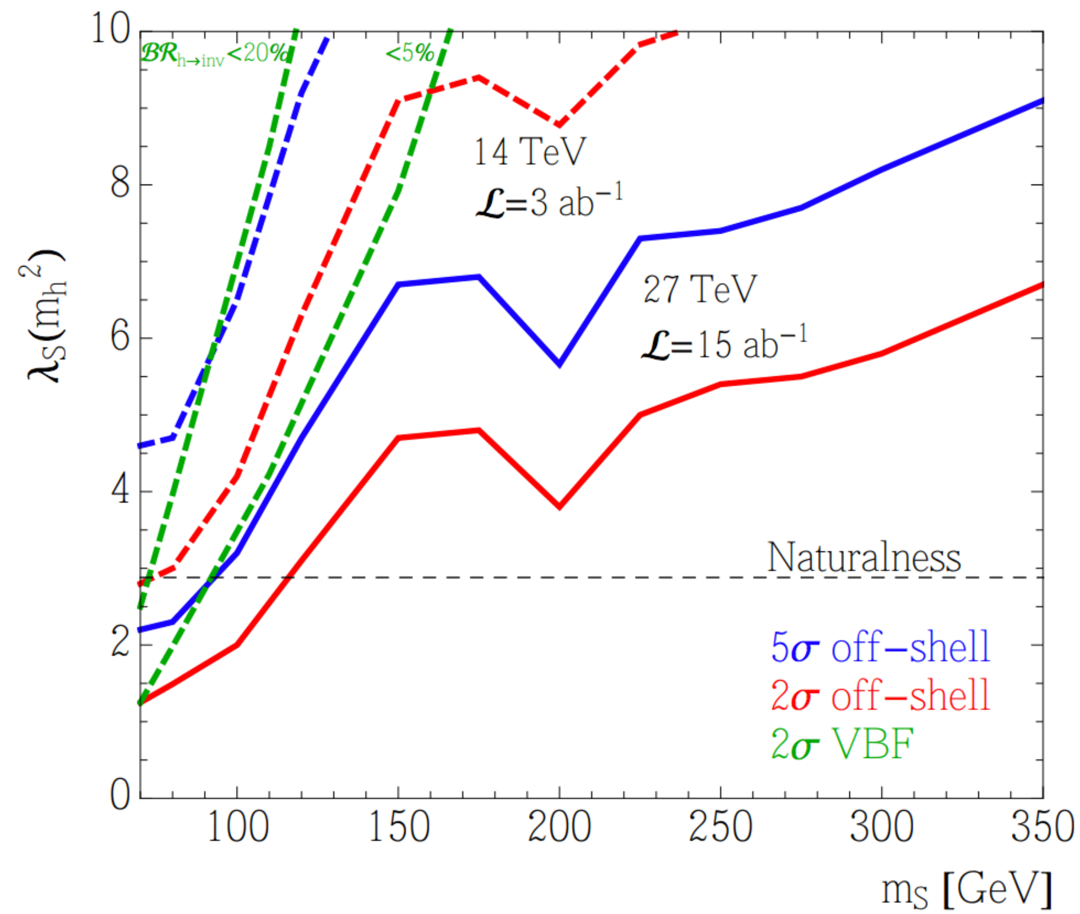


Huang, Long, and Wang,1608.06619

▸ Collider search for 2 step FOPT

◉ Off-shell Higgs@LHC





Goncalves,Han,and Mukhopadhyay, 1710.02149

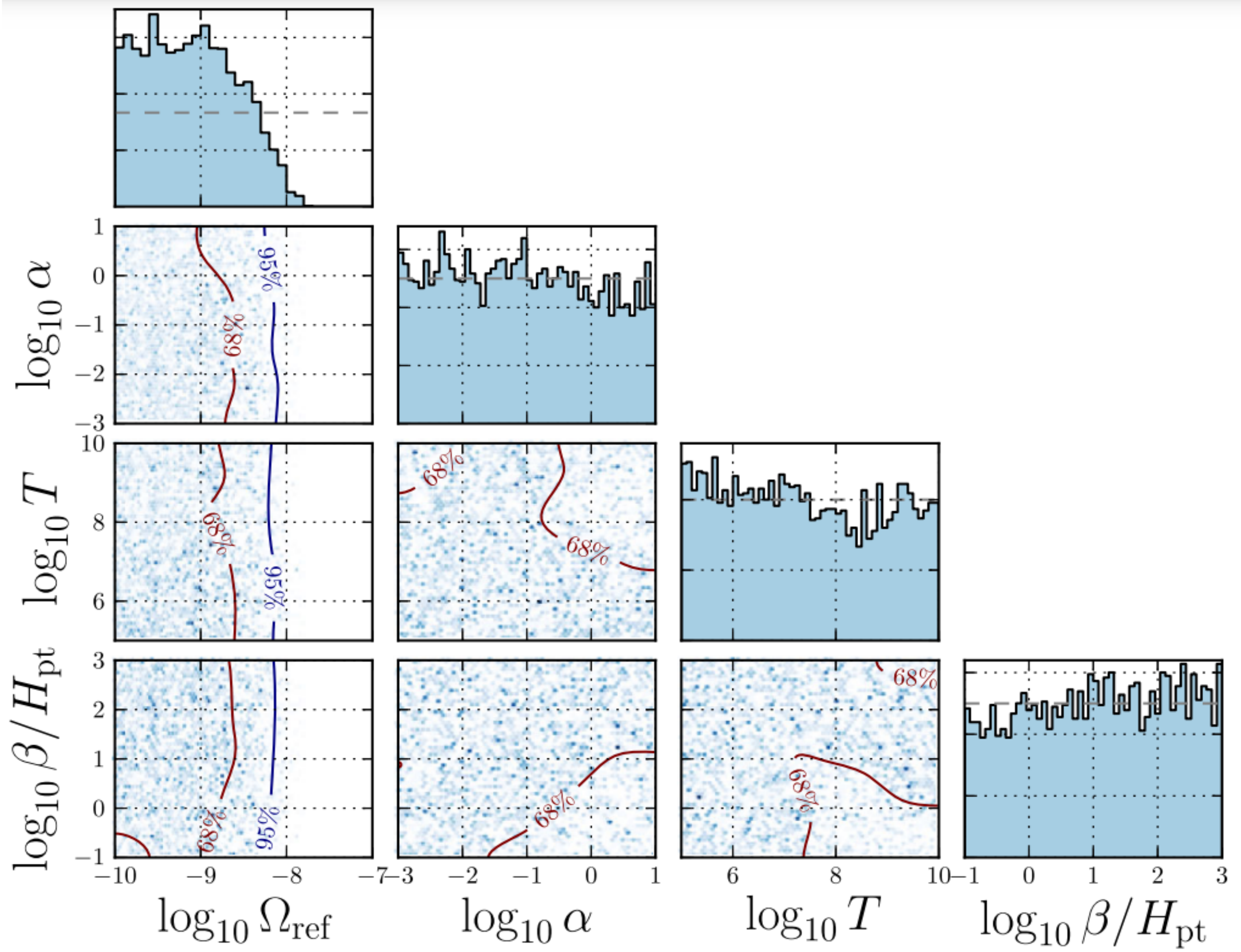
See also: Lee, Park,and Qian, 1812.02679

▸ LIGO-Virgo search for FOPT

High-scale PT

Romero, Martinovic,Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3



▸ PPTA search for FOPT

- **PPTA DR2 dataset constrain low-scale phase transition，dark sector and QCD scale FOPT**

PHYSICAL REVIEW LETTERS **127,** 251303 (2021)

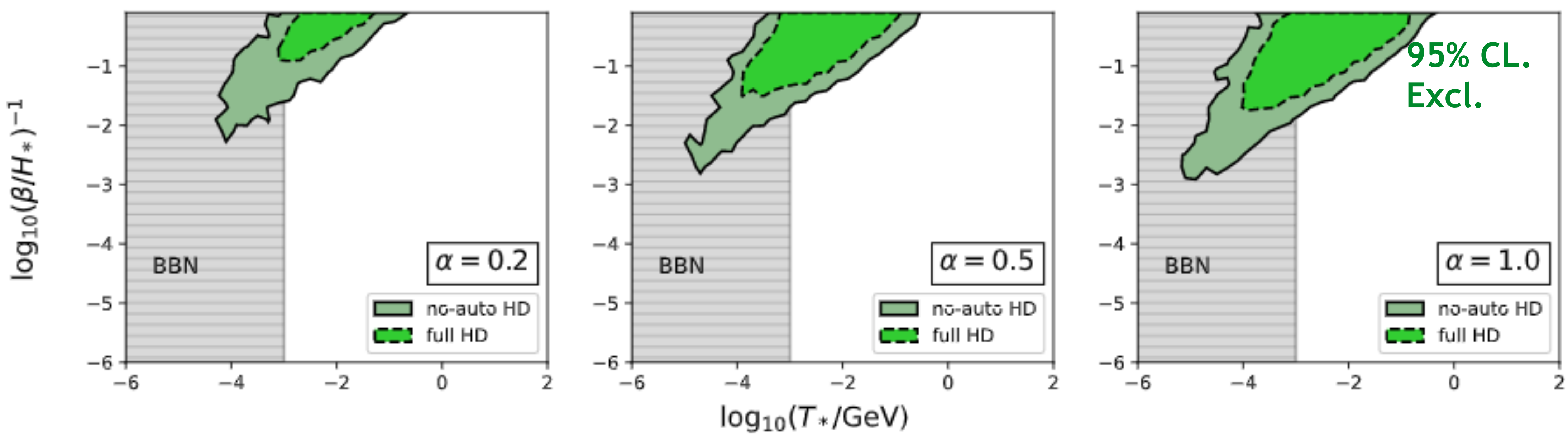
Editors' Suggestion | Featured in Physics

Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

Xiao Xue,[1,2,3] Ligong Bian,[4,5,*] Jing Shu,[1,2,6,7,8,†] Qiang Yuan,[9,10,7,‡] Xingjiang Zhu,[11,12,13,§] N. D. Ramesh Bhat,[14] Shi Dai,[15] Yi Feng,[16] Boris Goncharov,[11,12] George Hobbs,[17] Eric Howard,[17,18] Richard N. Manchester,[17] Christopher J. Russell,[19] Daniel J. Reardon,[12,20] Ryan M. Shannon,[12,20] Renée Spiewak,[21,20] Nithyanandan Thyagarajan,[22] and Jingbo Wang[23]

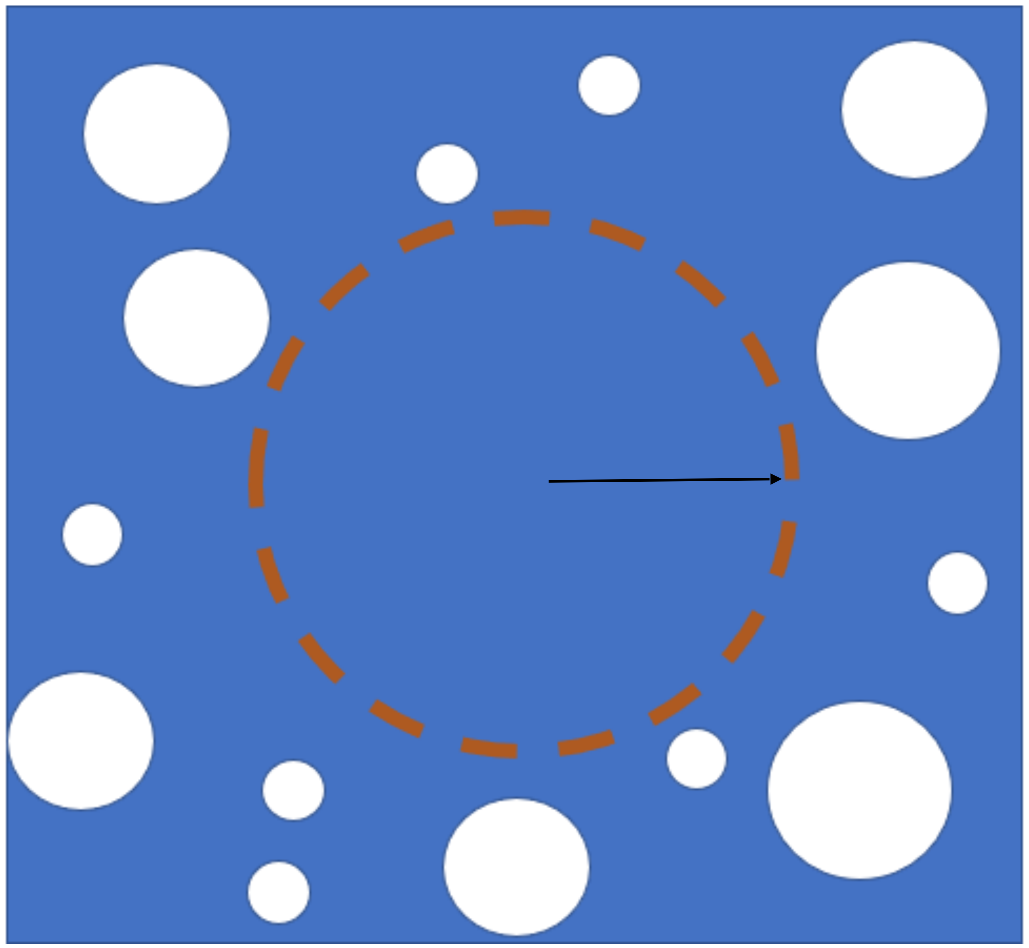
TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

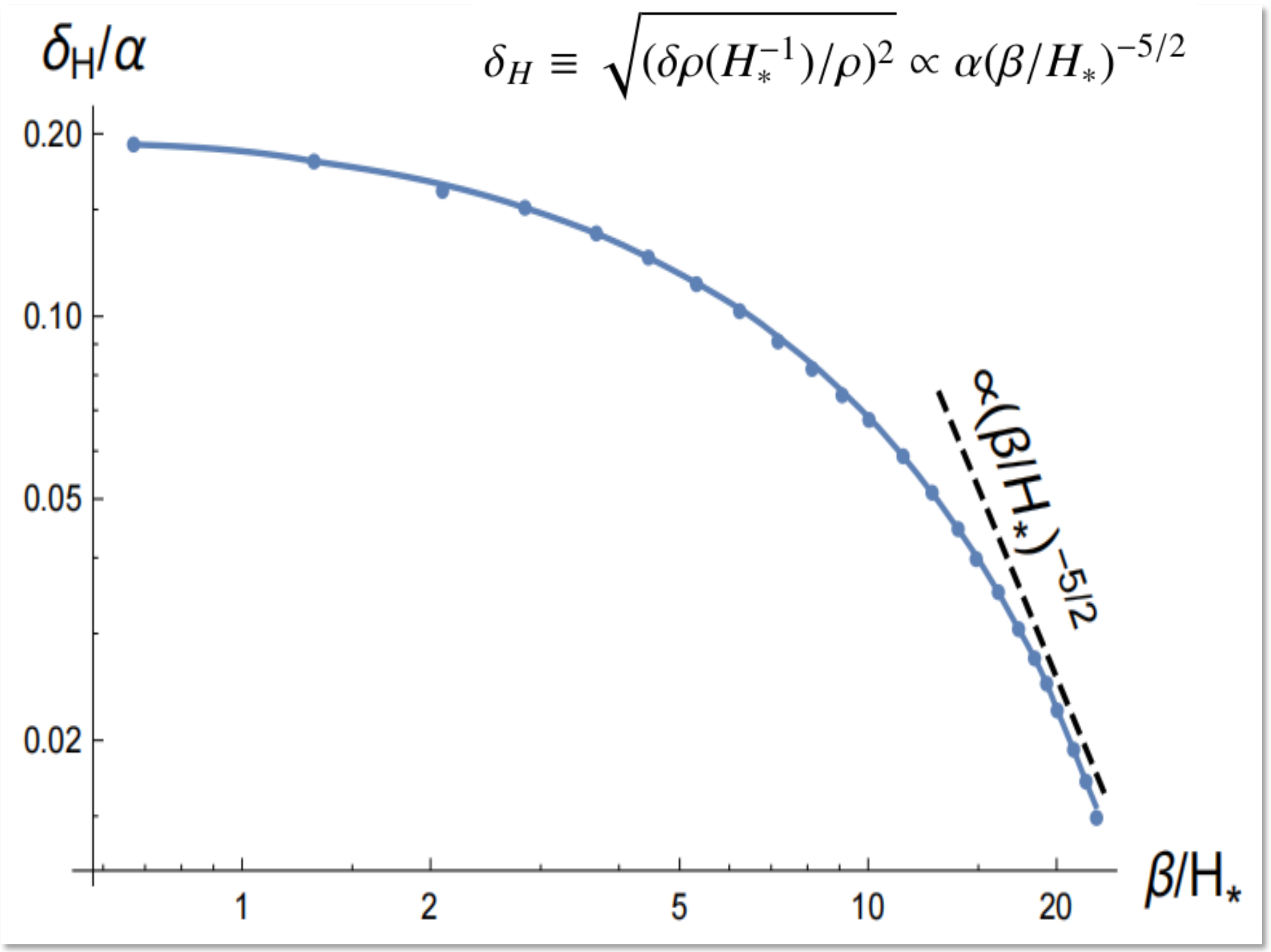
Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1-σ interval)	
					T_*/MeV, $\alpha \times 10^3$, β/H_*	$A_{\rm comred}$, $\gamma_{\rm comred}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}$, $3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}$, 271^{+165}_{-92}, $9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}$, $3.8^{+27.9}_{-3.4}$, 854^{+9622}_{-782}	$-14.51^{+0.64}_{-0.68}$, $3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}$, $3.2^{+19.9}_{-2.8}$, 1053^{+11256}_{-962}	$-14.45^{+0.62}_{-0.64}$, $3.27^{+1.37}_{-1.54}$



真空延迟衰变与曲率扰动限制相变

Hubble-sized perturbations

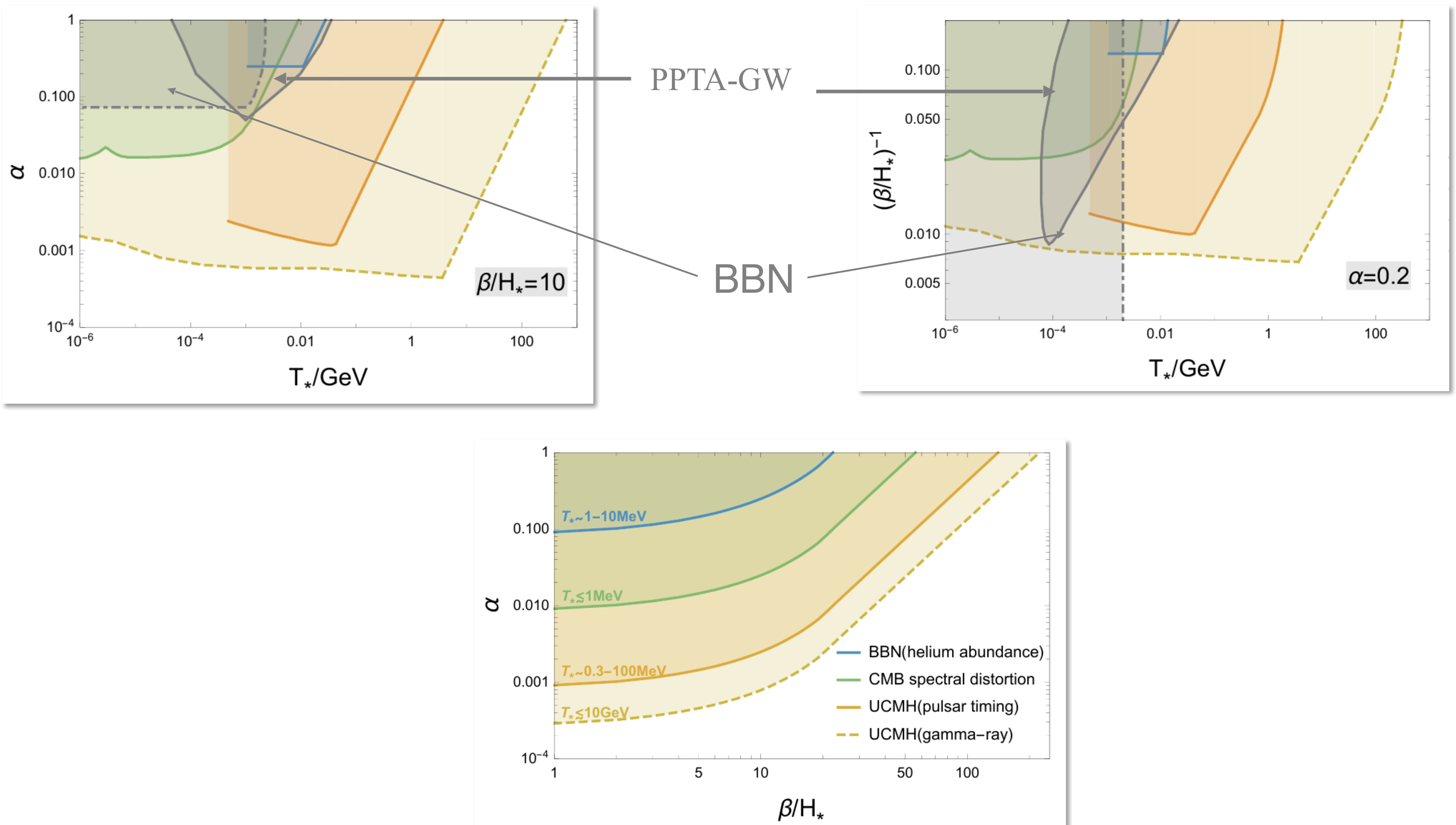




Liu, **Bian**, Cai, Guo, Wang, PRL130 (2023) 051001

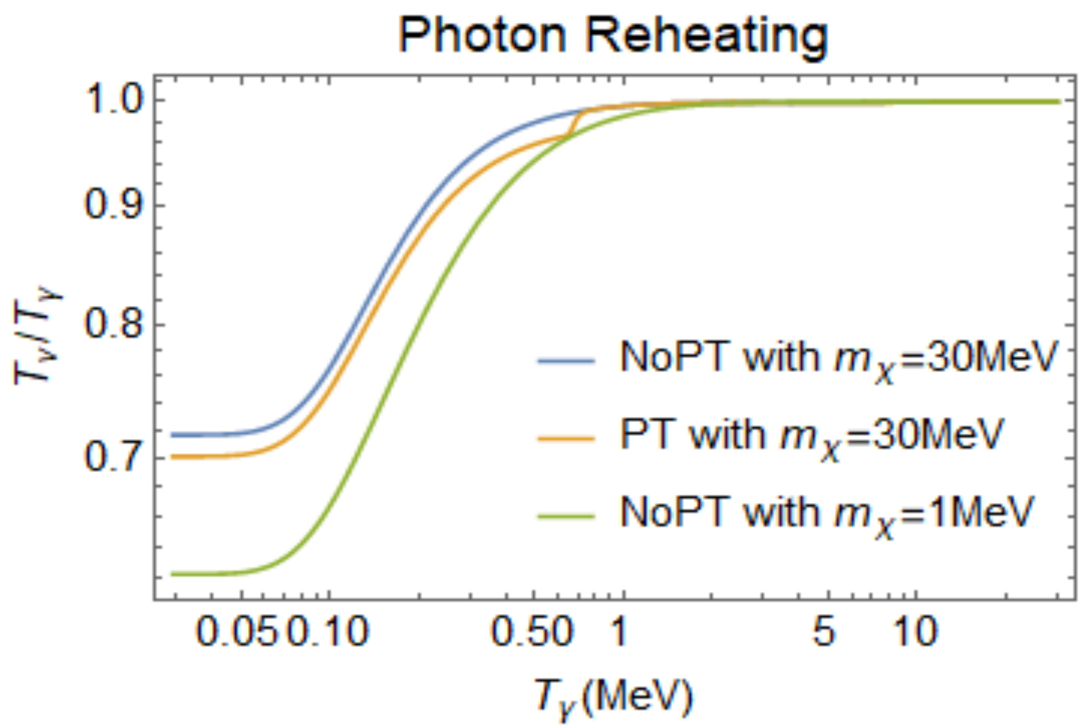
真空延迟衰变与曲率扰动限制相变

low-scale and slow 1st PTs motived for dark PT and BAU



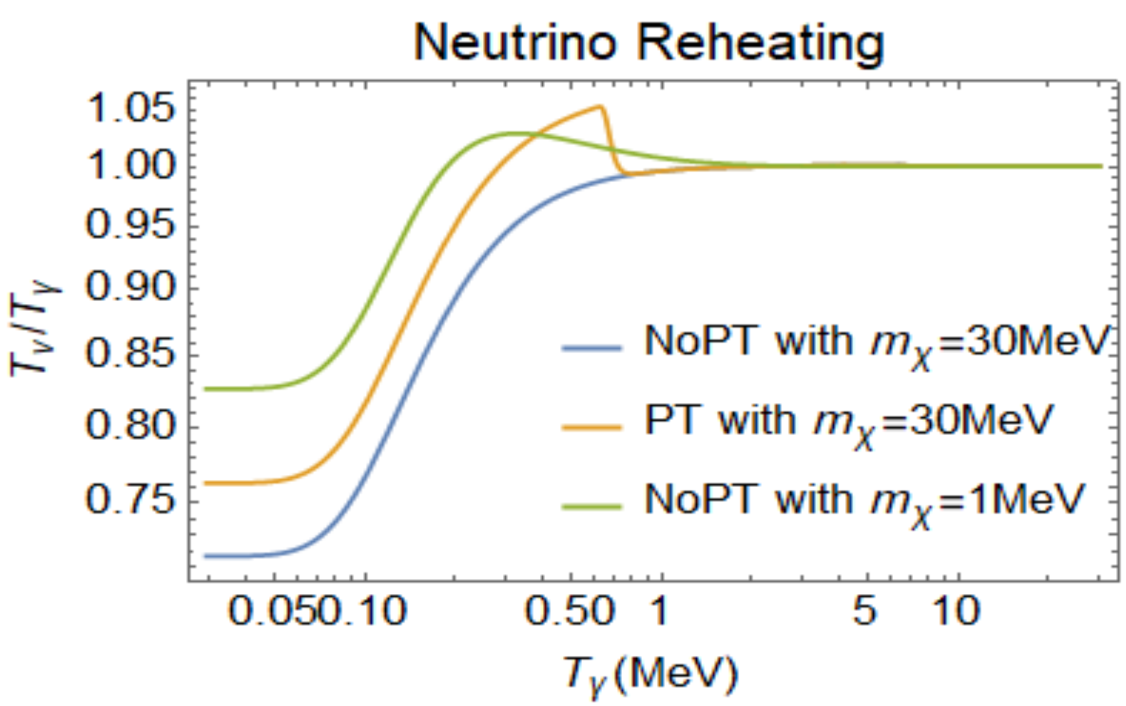
Liu, **Bian**, Cai, Guo, Wang, PRL130 (2023) 051001

宇宙学观测限制低能标一阶相变



$$\frac{dT_\gamma}{dt} = -\Big(4H\rho_\gamma + 3H\left(\rho_e + p_e\right) + 3H\left(\rho_\chi + p_\chi\right) + 3H\, T_\gamma \frac{dP_{\rm int}}{dT_\gamma} + \frac{\delta\rho_{\nu_e}}{\delta t} + 2\frac{\delta\rho_{\nu_\mu}}{\delta t} + \frac{d\rho_{\rm vac}}{dt}\Big)\Big/ f(T_\gamma)\,,$$

$$\frac{dT_\nu}{dt} = -\,(12H\rho_\nu - \frac{\delta\rho_{\nu_e}}{\delta t} - 2\frac{\delta\rho_{\nu_\mu}}{\delta t})/(3\,\frac{\partial\rho_\nu}{\partial T_\nu})\,.$$



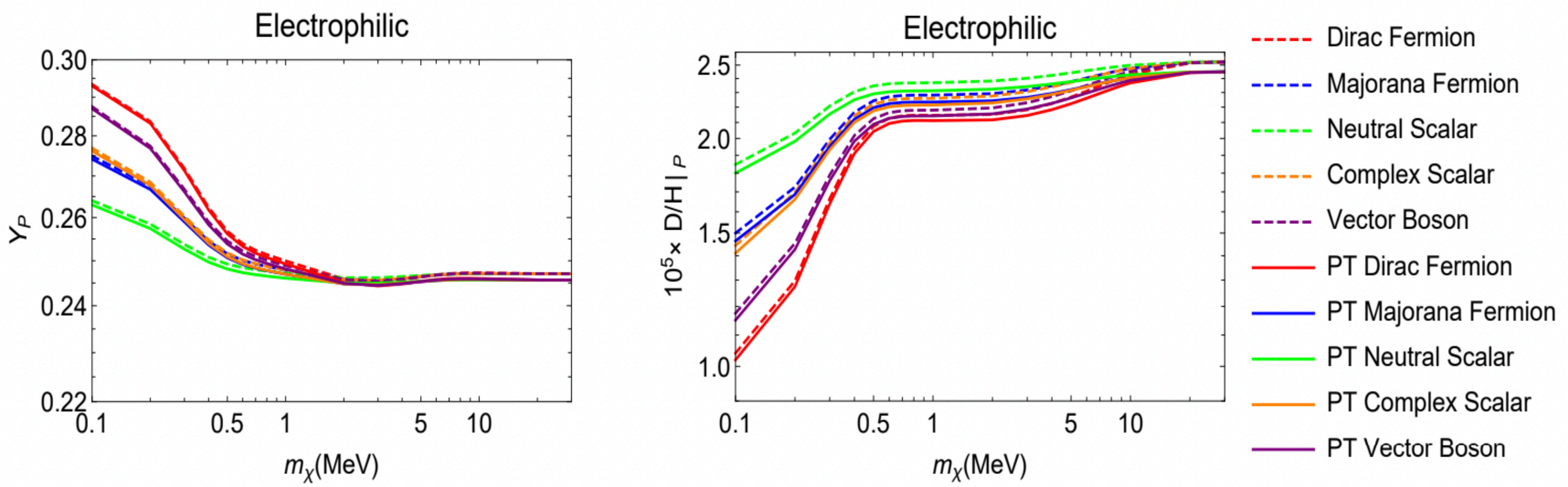
$$\frac{dT_\nu}{dt} = -\frac{12H\rho_\nu + 3H(\rho_\chi + p_\chi) - \frac{\delta\rho_{\nu_e}}{\delta t} - 2\frac{\delta\rho_{\nu_\mu}}{\delta t} + 3\,\frac{d\rho_{\rm vac}}{dt}}{3\,\frac{\partial\rho_\nu}{\partial T_\nu} + \frac{\partial\rho_\chi}{\partial T_\nu}}\,,$$

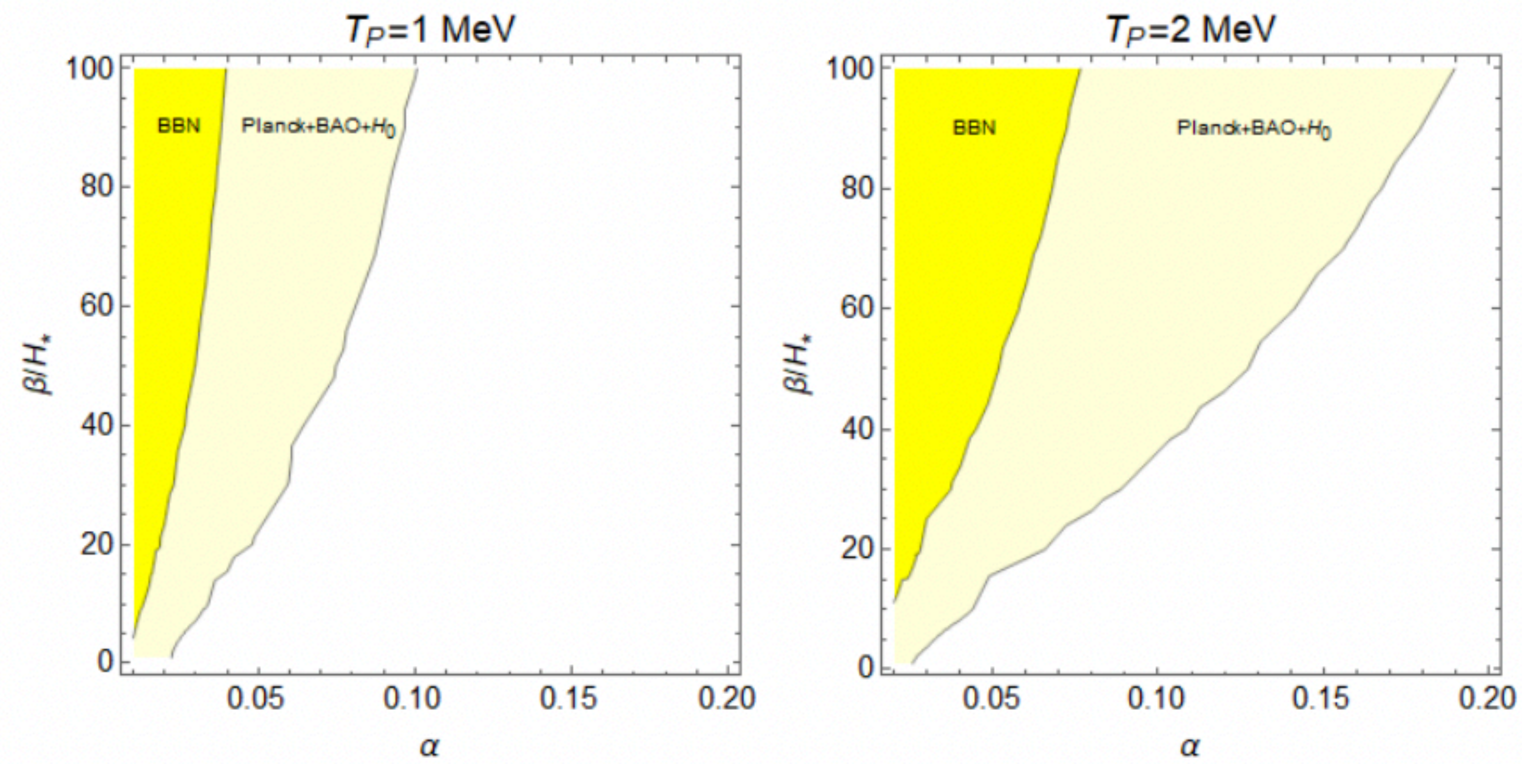
$$\frac{dT_\gamma}{dt} = -\frac{4H\rho_\gamma + 3H\left(\rho_e + p_e\right) + 3H\,T_\gamma\frac{dP_{\rm int}}{dT_\gamma} + \frac{\delta\rho_{\nu_e}}{\delta t} + 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{\frac{\partial\rho_\gamma}{\partial T_\gamma} + \frac{\partial\rho_e}{\partial T_\gamma} + T_\gamma\frac{d^2P_{\rm int}}{dT_\gamma^2}}$$

Deng, **Bian**,2304.06576

宇宙学观测限制低能标一阶相变

Primordial nucleosynthesis





CMB（$\Omega bh2$, Neff, YP）and BBN （YP, D/H|p）

Deng, **Bian**,2304.06576

一阶相变与Seesaw scale

Gravitational waves from first-order phase transitions in Majoron models of neutrino mass #9

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U. and HIAS, UCAS, Hangzhou and ICTP-AP, Beijing) (May 31, 2021)

Published in: *JHEP* 10 (2021) 193 • e-Print: 2106.00025 [hep-ph]

pdf DOI cite claim reference search 23 citations

Gravitational Waves from First-Order Phase Transitions: LIGO as a Window to Unexplored Seesaw Scales #1

Vedran Brdar (Heidelberg, Max Planck Inst.), Alexander J. Helmboldt (Heidelberg, Max Planck Inst.), Jisuke Kubo (Heidelberg, Max Planck Inst. and Toyama U.) (Oct 29, 2018)

Published in: *JCAP* 02 (2019) 021 • e-Print: 1810.12306 [hep-ph]

pdf DOI cite claim reference search 93 citations

Gravitational wave imprints of left-right symmetric model with minimal Higgs sector #1

Lukáš Gráf (Heidelberg, Max Planck Inst. and UC, Berkeley and UC, San Diego), Sudip Jana (Heidelberg, Max Planck Inst.), Ajay Kaladharan (Oklahoma State U.), Shaikh Saad (Basel U.) (Dec 22, 2021)

Published in: *JCAP* 05 (2022) 05, 003 • e-Print: 2112.12041 [hep-ph]

pdf DOI cite claim reference search 7 citations

Prospects of gravitational waves in the minimal left-right symmetric model #19

Mingqiu Li (Beijing, GUCAS), Qi-Shu Yan (Beijing, GUCAS and Beijing, Inst. High Energy Phys.), Yongchao Zhang (Southeast U., Nanjing and Washington U., St. Louis), Zhijie Zhao (Beijing, Inst. High Energy Phys.) (Dec 26, 2020)

Published in: *JHEP* 03 (2021) 267 • e-Print: 2012.13686 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational waves from neutrino mass and dark matter genesis #16

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U.) (Jan 21, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 095017 • e-Print: 2001.07637 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational wave pathway to testable leptogenesis #2

Arnab Dasgupta (Pittsburgh U.), P.S. Bhupal Dev (Washington U., St. Louis and McDonnell Ctr. Space Sci.), Anish Ghoshal (Warsaw U.), Anupam Mazumdar (U. Groningen, VSI) (Jun 14, 2022)

Published in: *Phys.Rev.D* 106 (2022) 7, 075027 • e-Print: 2206.07032 [hep-ph]

pdf DOI cite claim reference search 23 citations

Cosmological implications of a B − L charged hidden scalar: leptogenesis and gravitational waves #5

Ligong Bian (Chongqing U.), Wei Cheng (Beijing, Inst. Theor. Phys.), Huai-Ke Guo (Oklahoma U.), Yongchao Zhang (Washington U., St. Louis and Peking U., CHEP) (Jul 31, 2019)

Published in: *Chin.Phys.C* 45 (2021) 11, 113104 • e-Print: 1907.13589 [hep-ph]

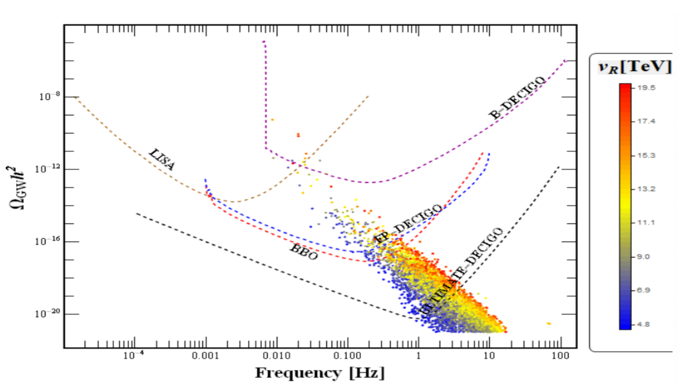
pdf DOI cite claim reference search 27 citations

Electroweak phase transition and gravitational waves in the type-II seesaw model #12

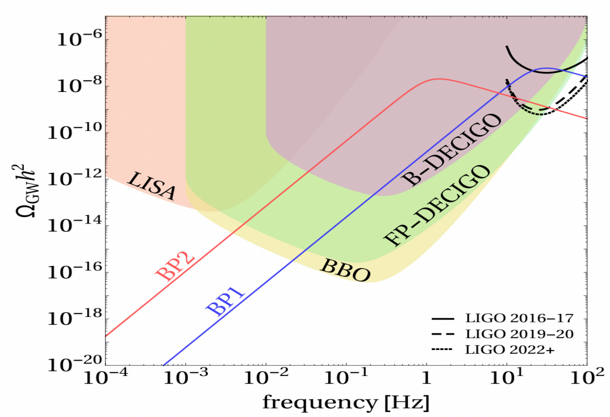
Ruiyu Zhou (CUPT, Chongqing), Ligong Bian (Chongqing U. and Peking U., CHEP), Yong Du (Beijing, Inst. Theor. Phys.) (Mar 3, 2022)

Published in: *JHEP* 08 (2022) 205 • e-Print: 2203.01561 [hep-ph]

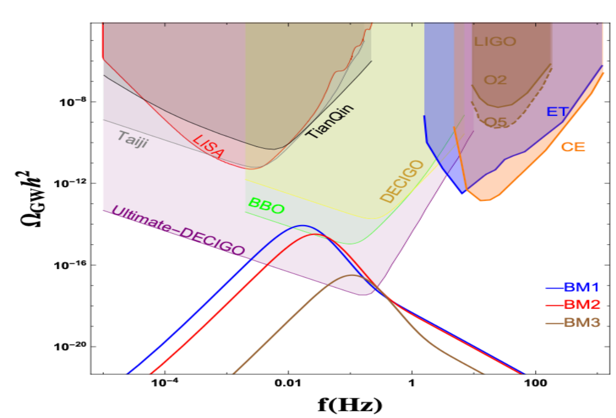
pdf DOI cite claim reference search 15 citations



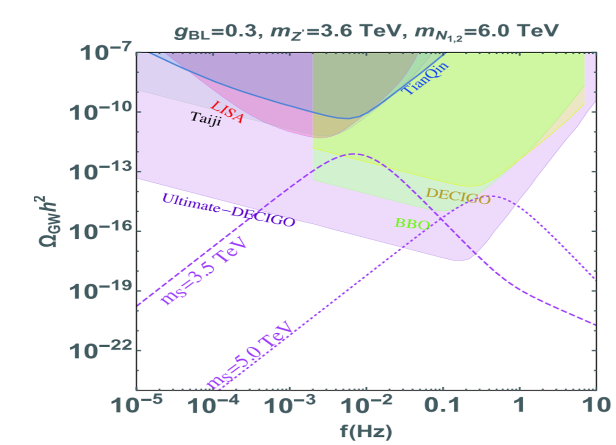
L-R



CSB

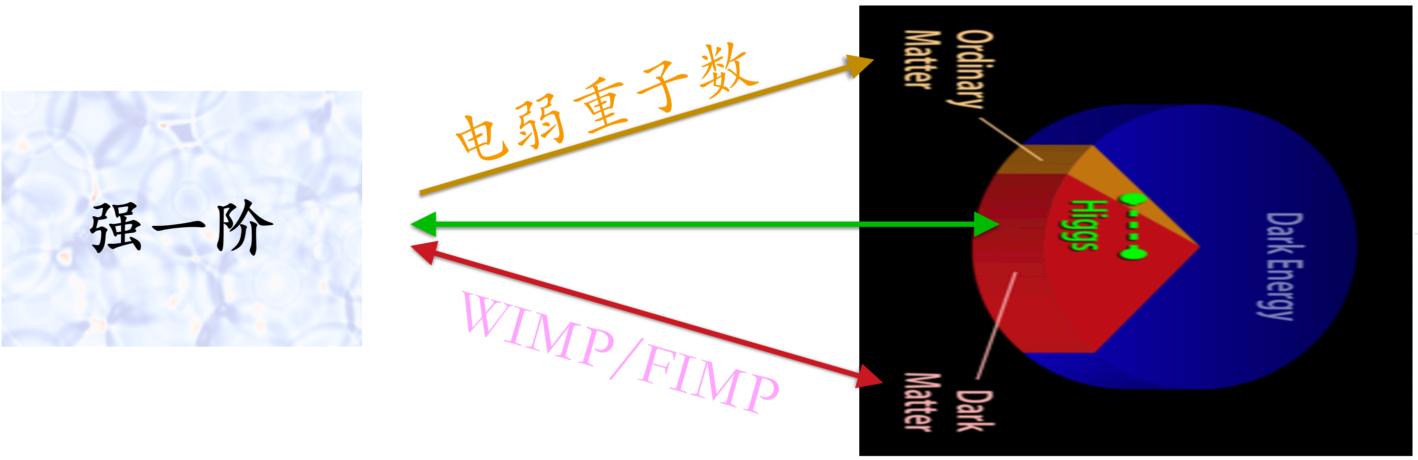


Type-II



Type-I

一阶相变效应



Impact of a complex singlet: Electroweak baryogenesis and dark matter #2

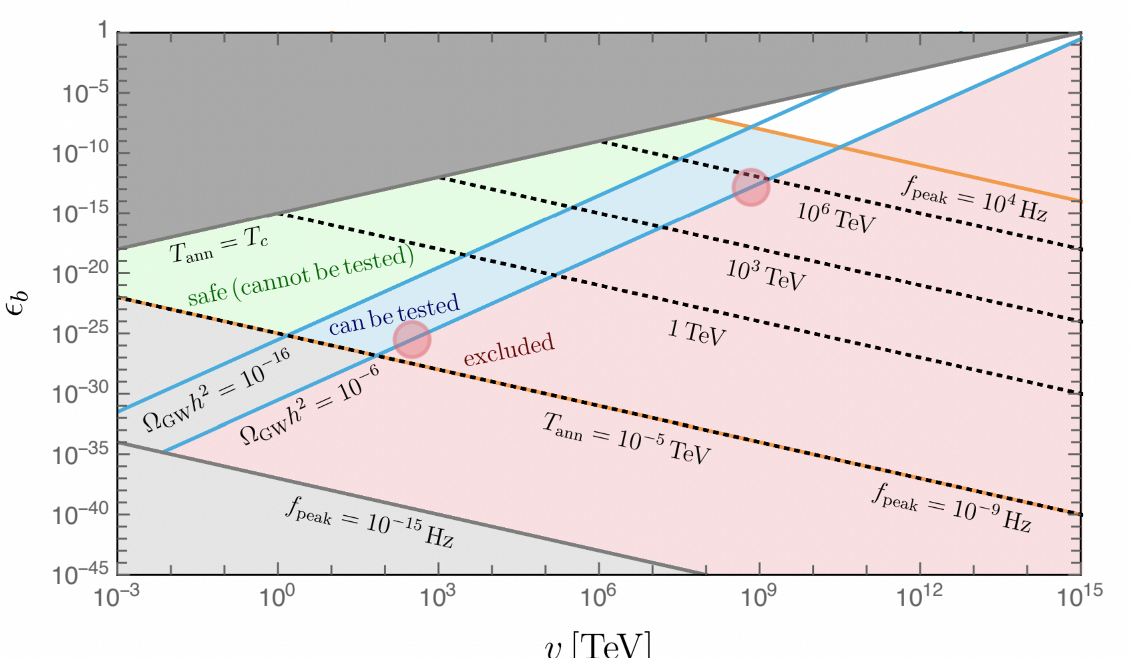
Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC)

(Feb 26, 2015)
Published in: *Phys.Rev.D* 93 (2016) 6, 065032 • e-Print: 1502.07574 [hep-ph]

Thermally modified sterile neutrino portal dark matter and gravitational waves from phase transition: The Freeze-in case #3

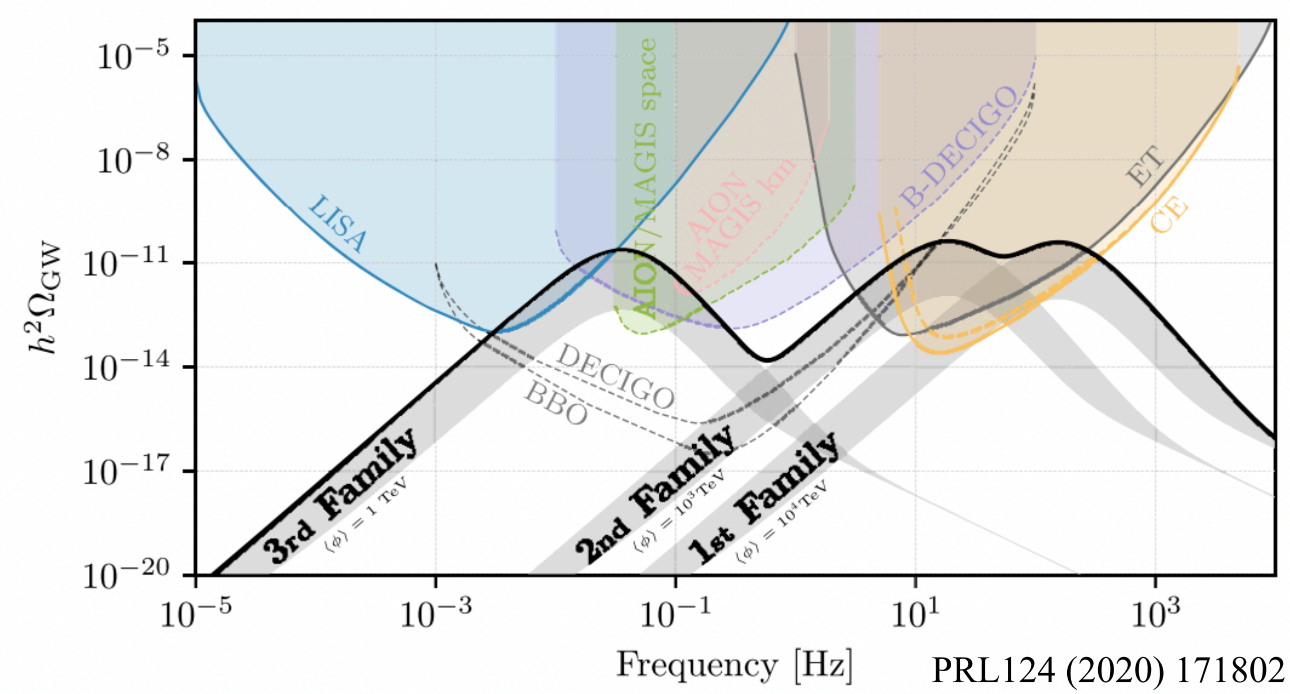
Ligong Bian (Chongqing U. and Chung-Ang U.), Yi-Lei Tang (Korea Inst. Advanced Study, Seoul) (Oct 7, 2018)
Published in: *JHEP* 12 (2018) 006 • e-Print: 1810.03172 [hep-ph]

离散的味对称性A4 与畴壁（DW）



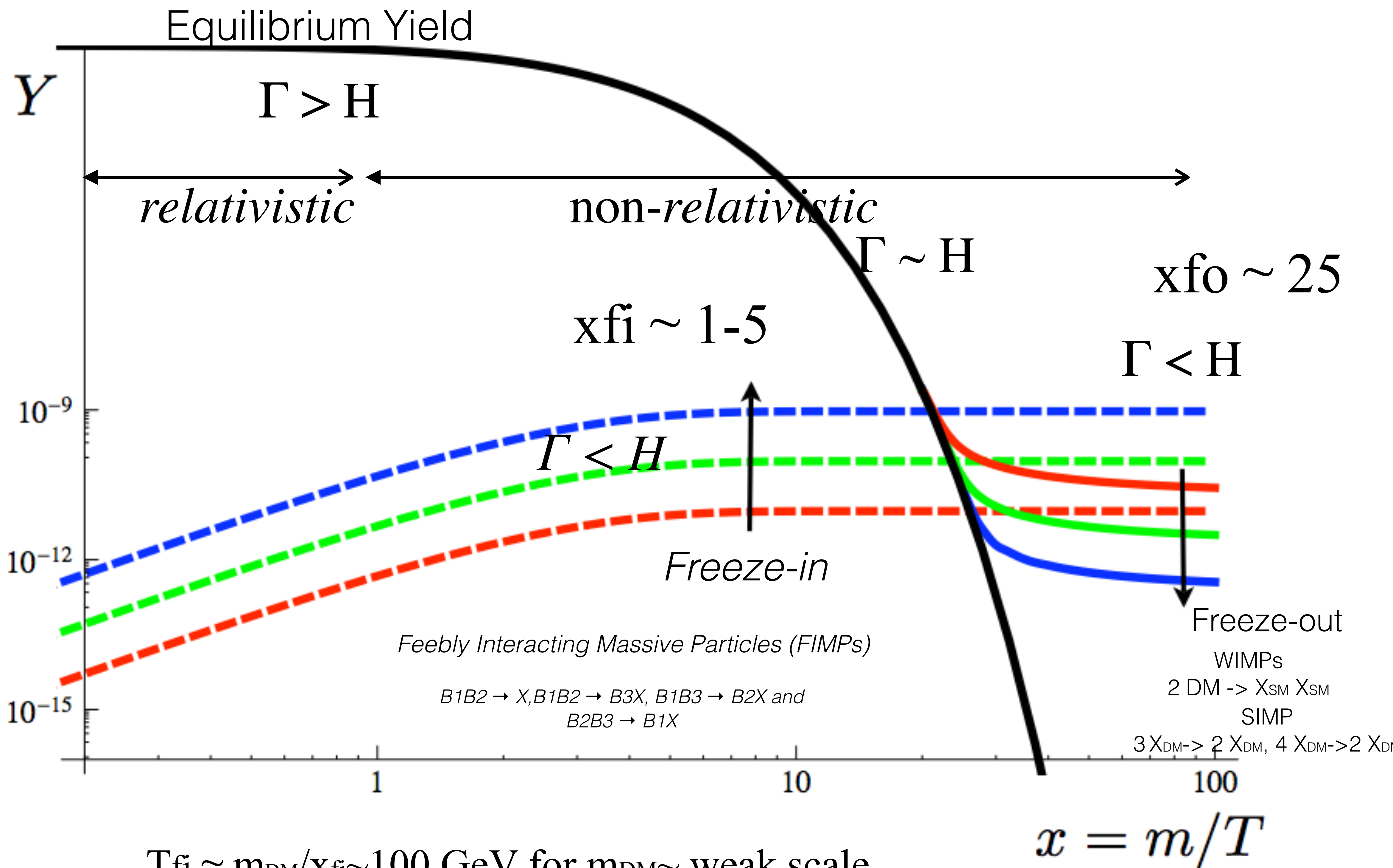
JCAP 02 (2021) 032

三代夸克和轻子质量等级问题与FOPT



PRL124 (2020) 171802

Freeze-in and Freeze-out



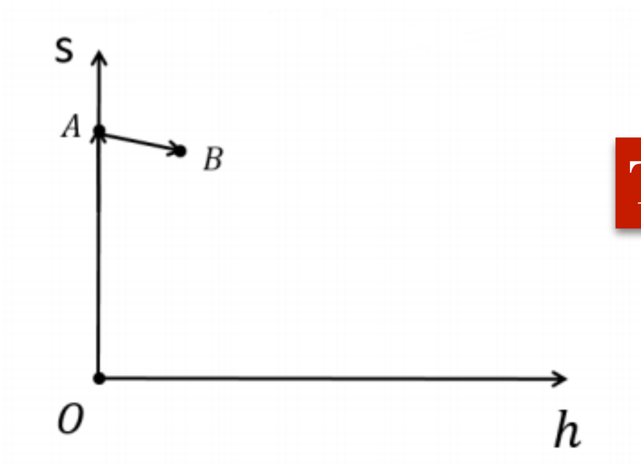
$T_{fi} \sim m_{DM}/x_{fi} \sim 100$ GeV for $m_{DM} \sim$ weak scale

$T_{fo} \sim m_{DM}/x_{fo} \sim O(1)$ GeV for $m_{DM} \sim$ weak scale

2-step FOPT DM & EWBG

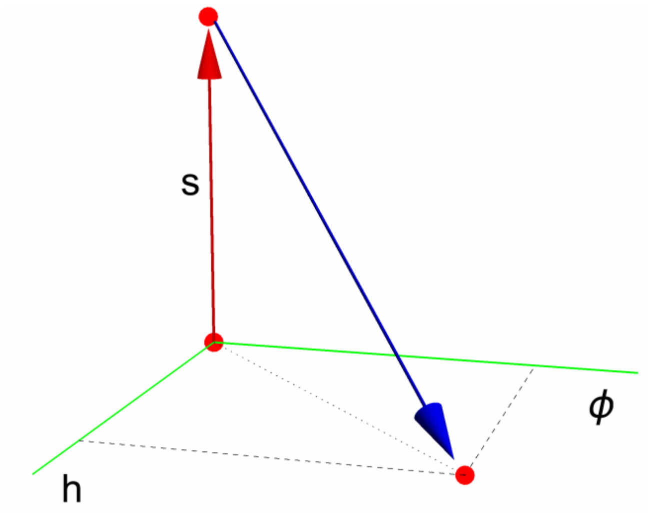
WIMP DM+EWBG

CxSM Jiang, **Bian***,Huang,Shu 16, Chiang, Ramsey-Musolf, Senaha 18,…

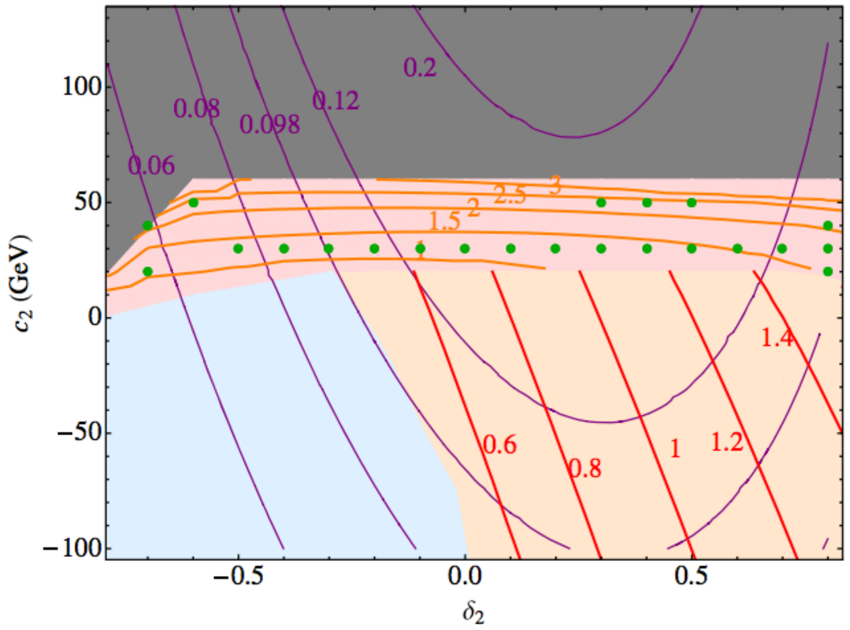


Two-step FOPT

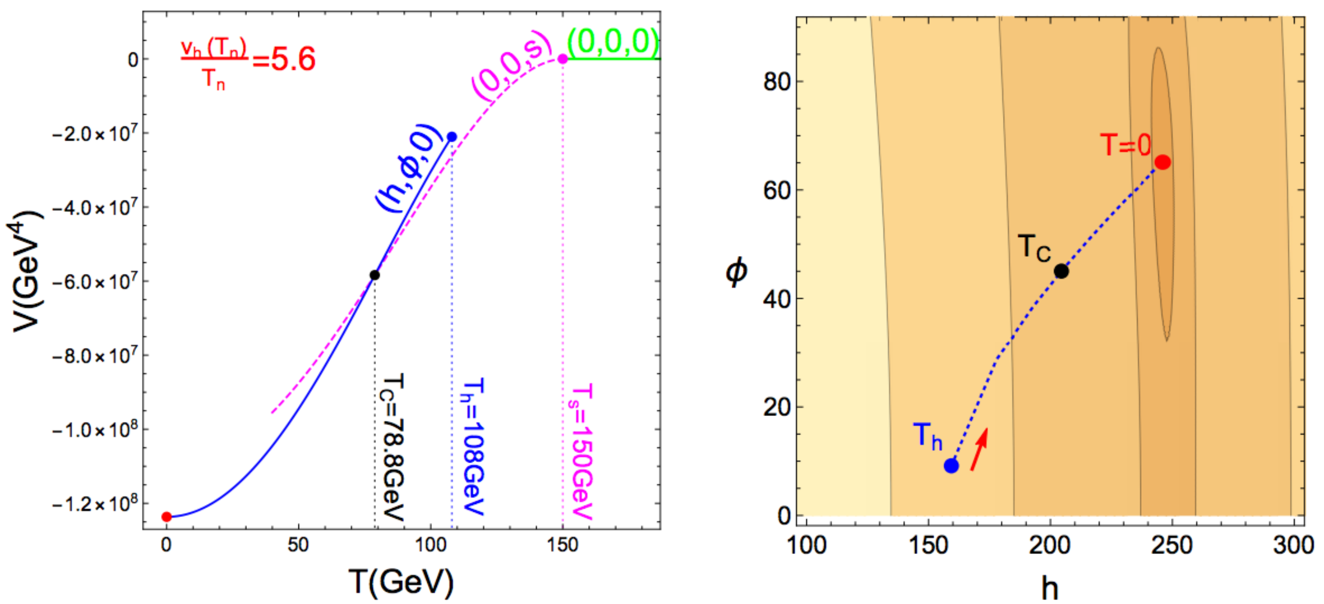
WIMP DM+FOPT



DM
one-step
two-step
BAU

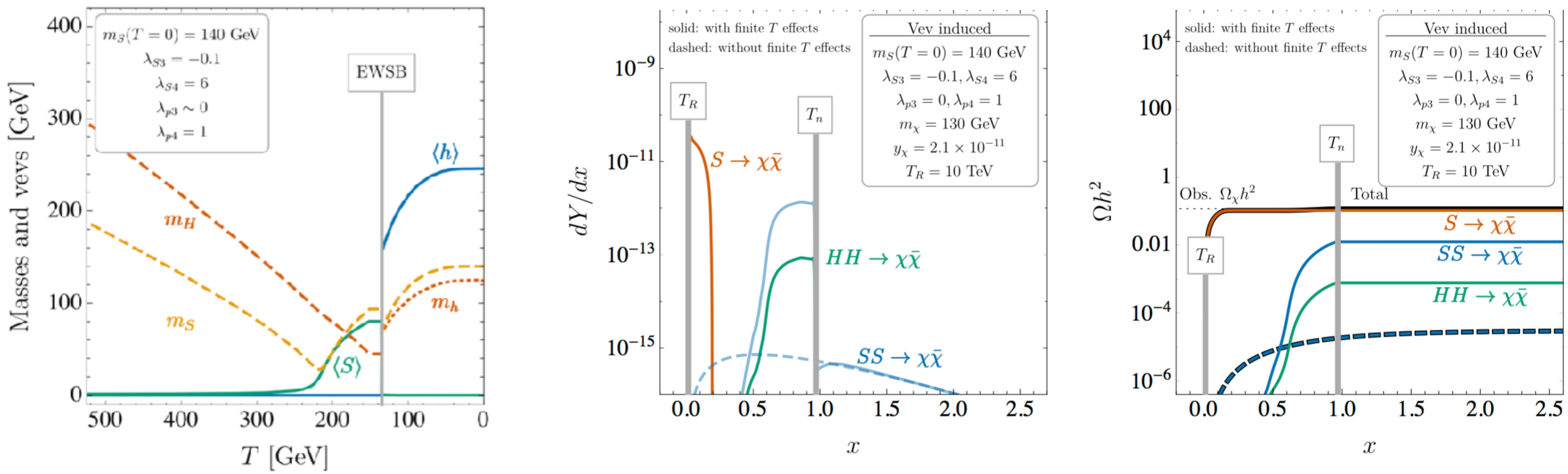


Jiang, **Bian***,Huang,Shu 16

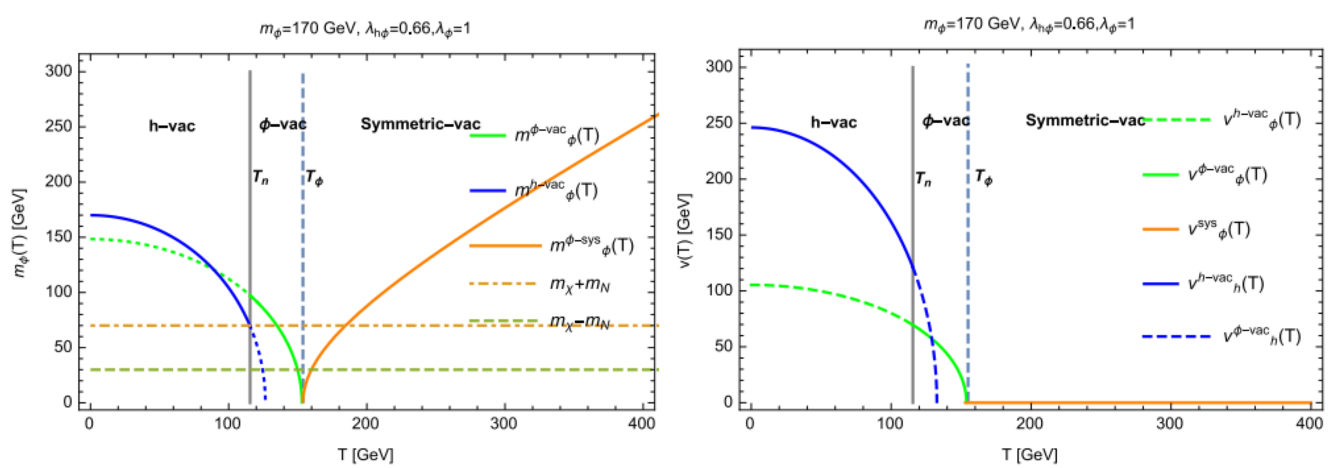


SM+2 real scalars **Chao, Guo, Shu 17,…**

FIMP DM and Two-step FOPT

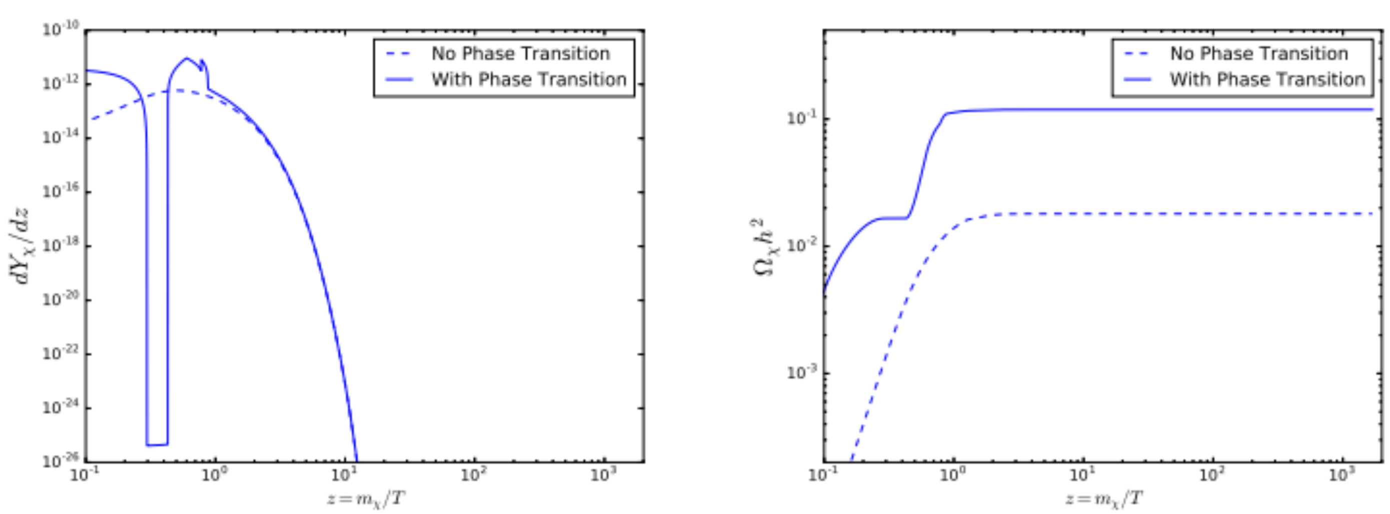


1712.03962,Michael J. Baker et al.



Amplified effects

larger thermal masses before PT



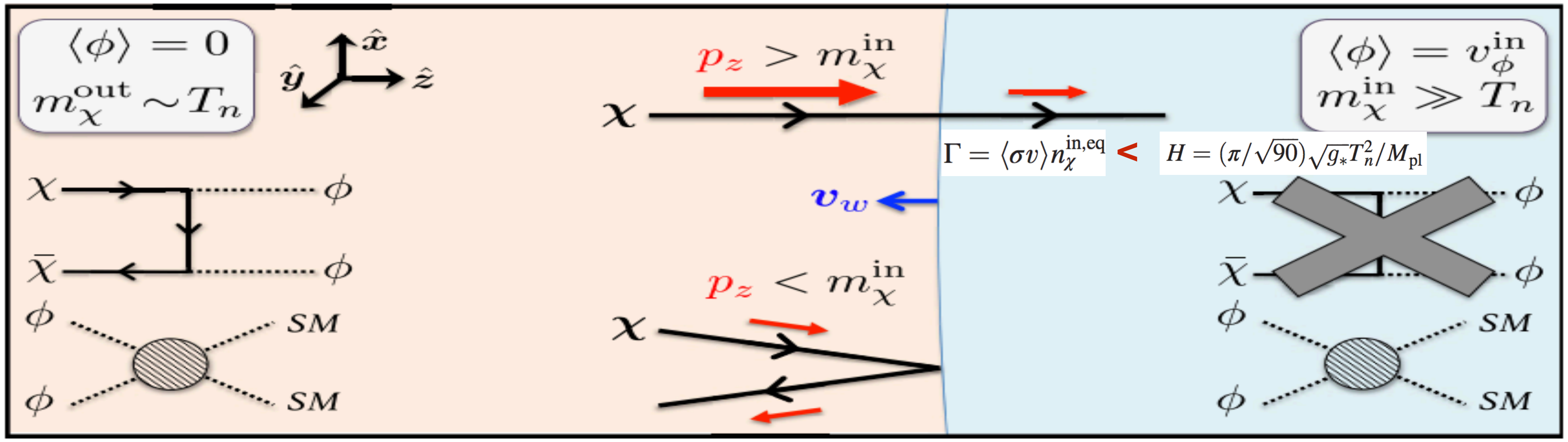
Temporarily open of decay channel

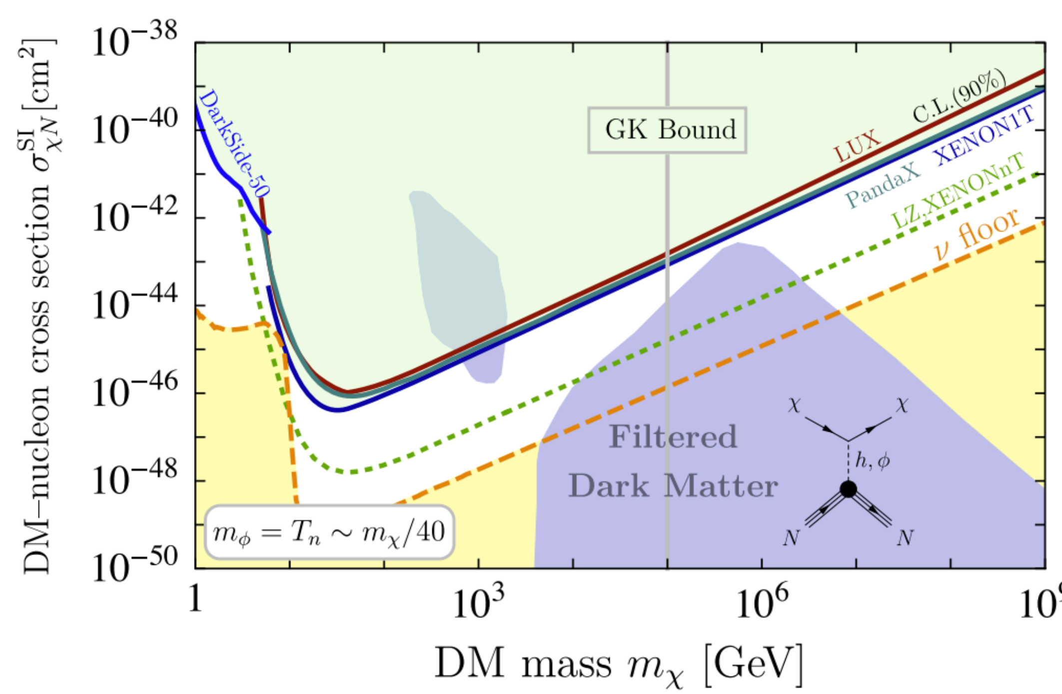
Bian, Tang,18

WIMP 暗物质与强一阶相变

$$\mathcal{L} \supset -V(\phi) - y_\chi \phi \bar{\chi}\chi - \beta \phi^2 H^\dagger H$$

过滤暗物质





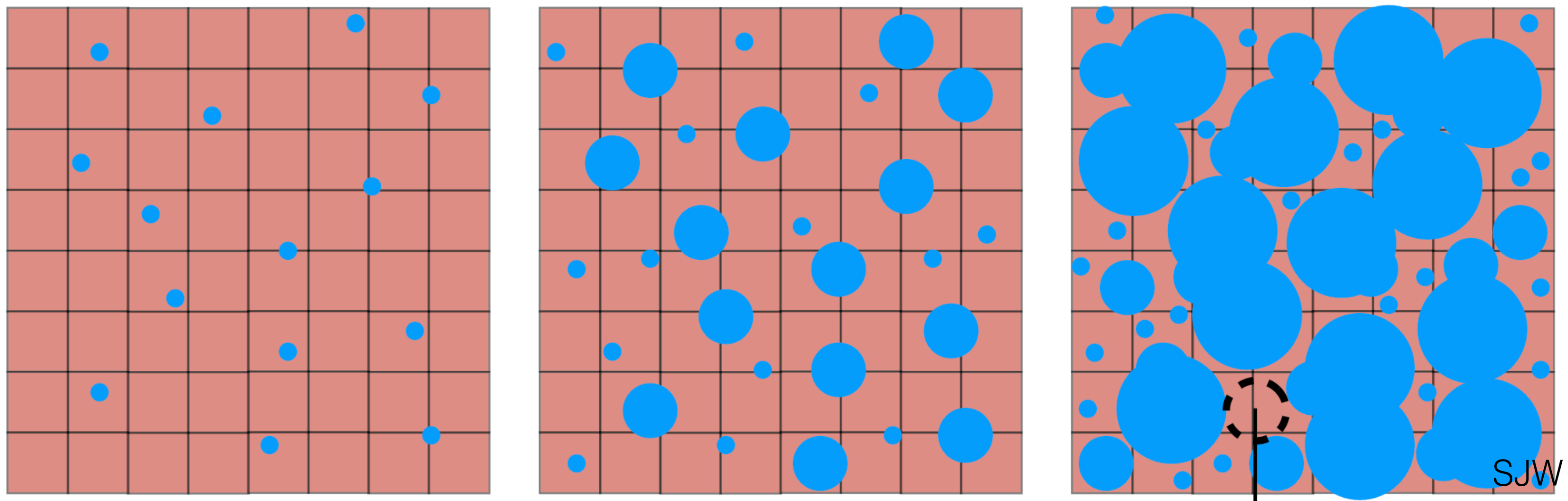
暗区一阶相变

Baker , Kopp, and Long, Phys.Rev.Lett. 125 (2020) 15, 151102

see also: Chao, Li, Wang, JCAP 06 (2021) 038

PBH 暗物质和一阶相变

PBH from postponed vacuum decay



Postponed Hubble volume V_H

↓

Un-diluted false vacuum energy

↓

PBH

Probability for a Hubble volume not to decay until time t_n

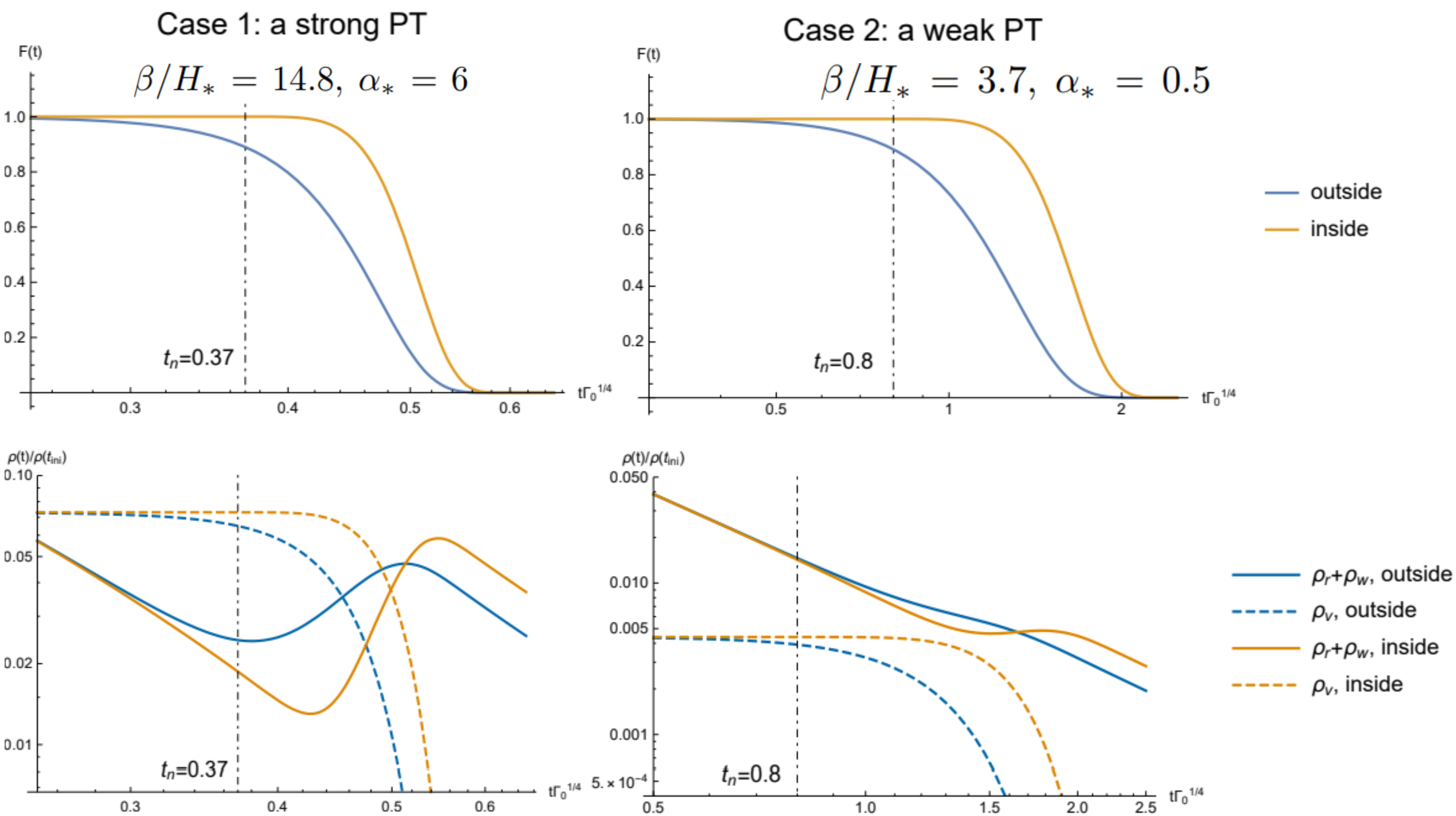
$$V_H(t) = \frac{4}{3}\pi H(t_{\rm PBH})^{-3}\frac{a(t)^3}{a(t_{\rm PBH})^3}$$

$$P(t_n) = \exp\left[-\frac{4\pi}{3}\int_{t_i}^{t_n}\frac{a^3(t)}{a^3(t_{\rm PBH})}H^{-3}(t_{\rm PBH})\Gamma(t)dt\right]$$

PBH abundance $\Omega_{\rm PBH}^{\rm form} = P(t_n)$

Collapse of the Hubble horizon

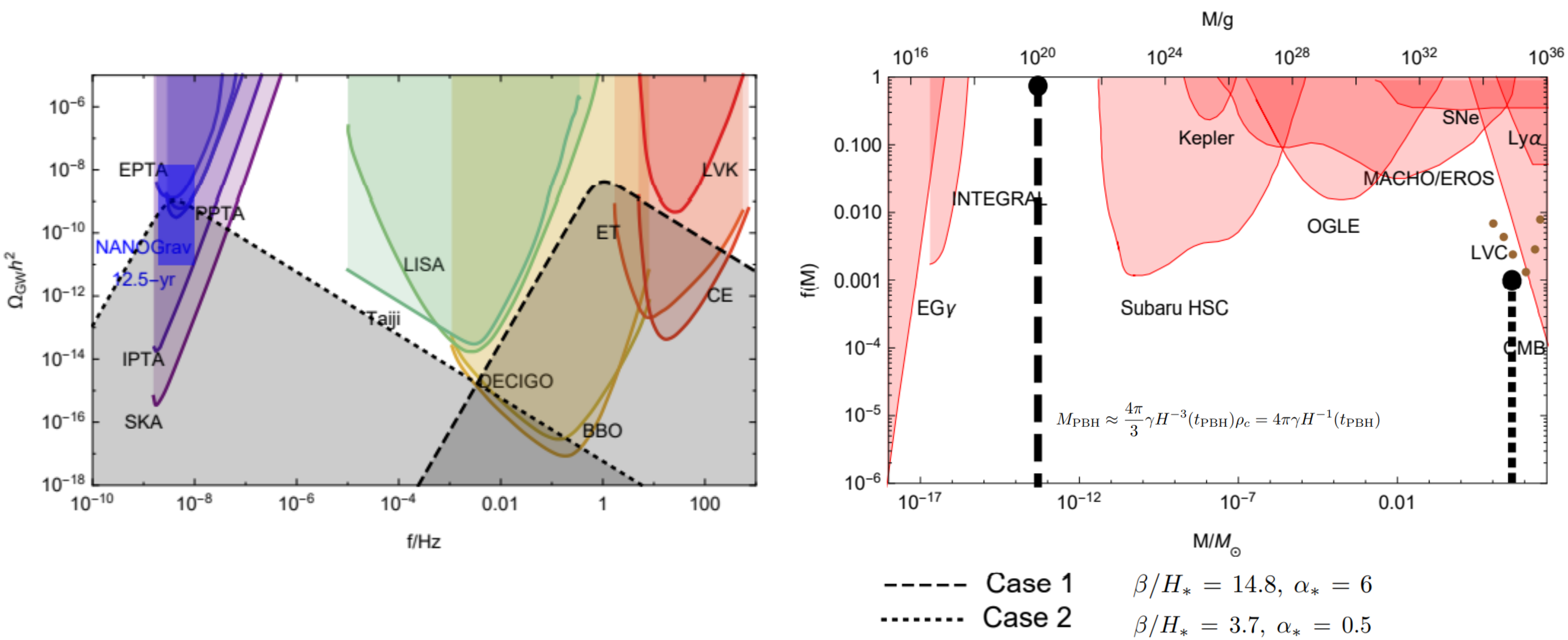
PBH 暗物质和一阶相变



$$\delta(t_{\rm PBH}) = \frac{\rho_v(t_{\rm PBH}; t_n) + \rho_r(t_{\rm PBH}; t_n)}{\rho_v(t_{\rm PBH}; t_i) + \rho_r(t_{\rm PBH}; t_i)} - 1 \geq \delta_c \Rightarrow t_{\rm PBH}$$

PBH 暗物质和一阶相变

PBH is more abundant in strong and slow first-order PTs.



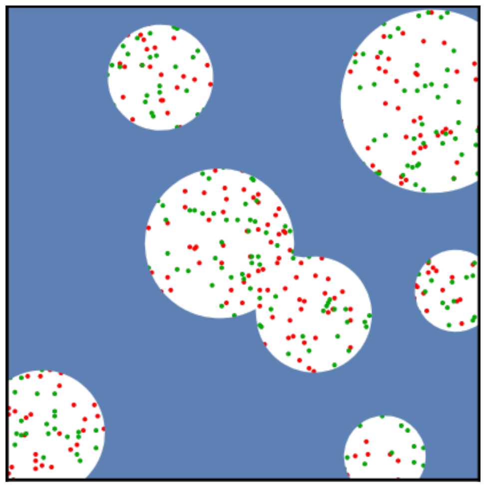
➢ Case 1: PBHs constitute all dark matter, Ω_{GW} to be probed CE,ET

➢ Case 2: GWs explain the CPL observed by NANOGrav, PBHs explain the coalescence events observed by the LIGO-Virgo collaboration

Liu, **Bian**, Cai, Guo, Wang, PRD **105** (**2022**) L021303

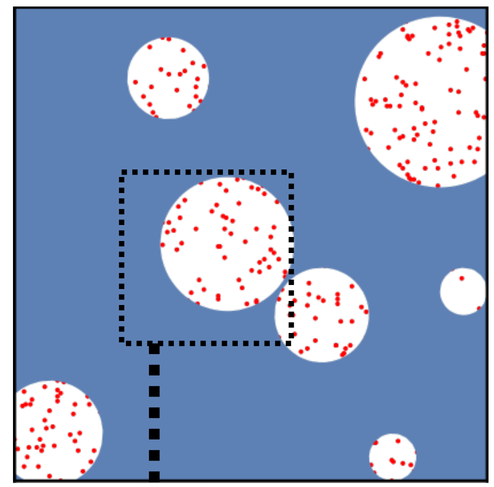
Formation of Fermi-ball solitons

Trapping fermions in the false vacuum by mass gap $\Delta m_\chi \gg T_*$

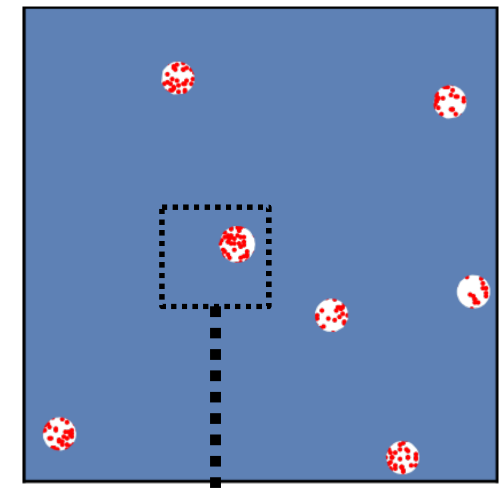


$\chi\bar{\chi} \to \phi\phi$

$n_\chi > n_{\bar{\chi}}$



Pauli principle



Charge trapped: $Q_{\text{FB}} \approx \eta_\chi s_* V_*$

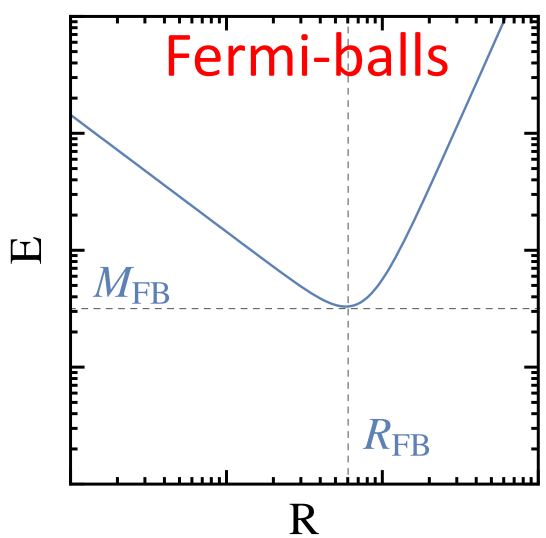
Remnant volume

Fermion asymmetry $\eta_\chi = \dfrac{n_\chi - n_{\bar{\chi}}}{s}$

$$E = \frac{3\pi}{4}\left(\frac{3}{2\pi}\right)^{2/3}\frac{Q_{\text{FB}}^{4/3}}{R} + \frac{4\pi}{3}R^3U_0$$

Fermi-gas energy

Volume energy



$$M_{\text{FB}} = Q_{\text{FB}}\left(12\pi^2U_0\right)^{1/4};$$

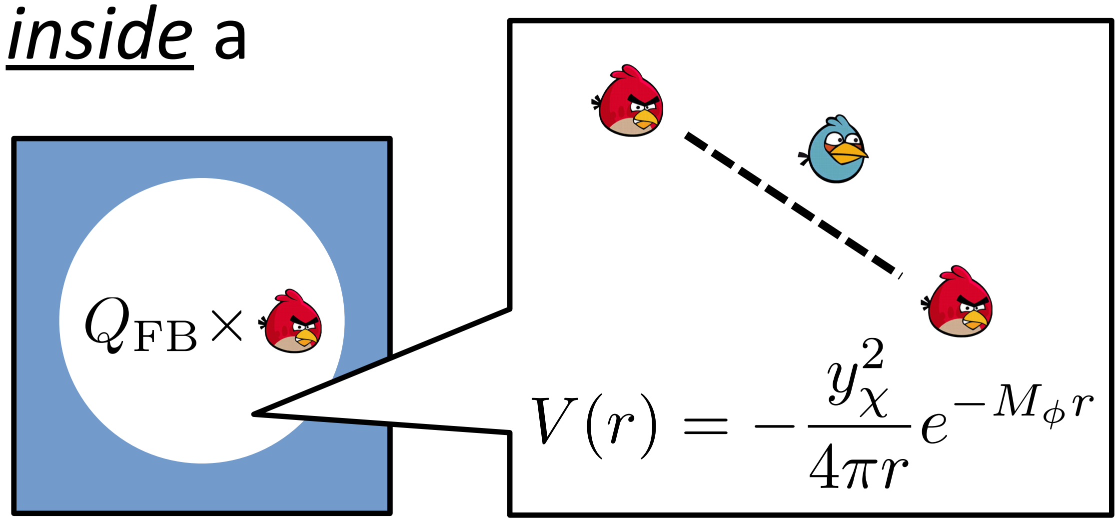
$$R_{\text{FB}} = Q_{\text{FB}}^{1/3}\left[\frac{3}{16}\left(\frac{3}{2\pi}\right)^{2/3}\frac{1}{U_0}\right]^{1/4}$$

Could be macroscopic DM

Hong, Jung and Xie, PRD 102 (2020) 7, 075028

Fermi-balls collapse to black holes

Yukawa force *inside* a Fermi-ball



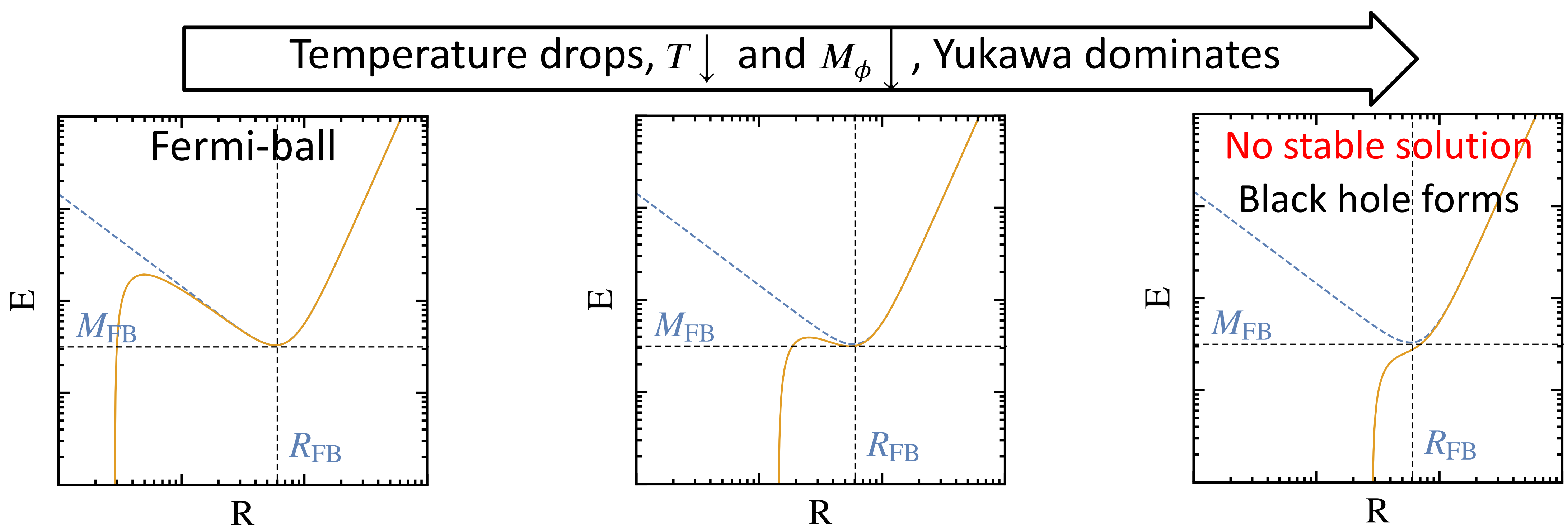
Originates from

$\mathscr{L} \supset -y_\chi \phi \bar{\chi} \chi$

Range of force

$L_\phi = \frac{1}{M_\phi}$

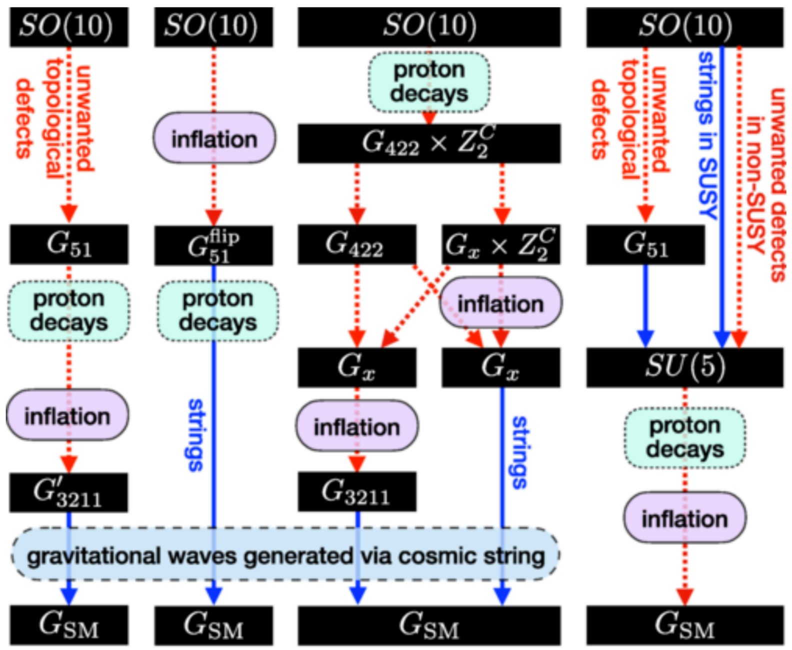
$$E \approx \frac{3\pi}{4}\left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{\rm FB}^{4/3}}{R} + \frac{4\pi}{3} R^3 U_0 - \frac{15 y_\chi^2}{40\pi} \frac{Q_{\rm FB}^2}{R} \left(\frac{1}{R M_\phi}\right)^2$$



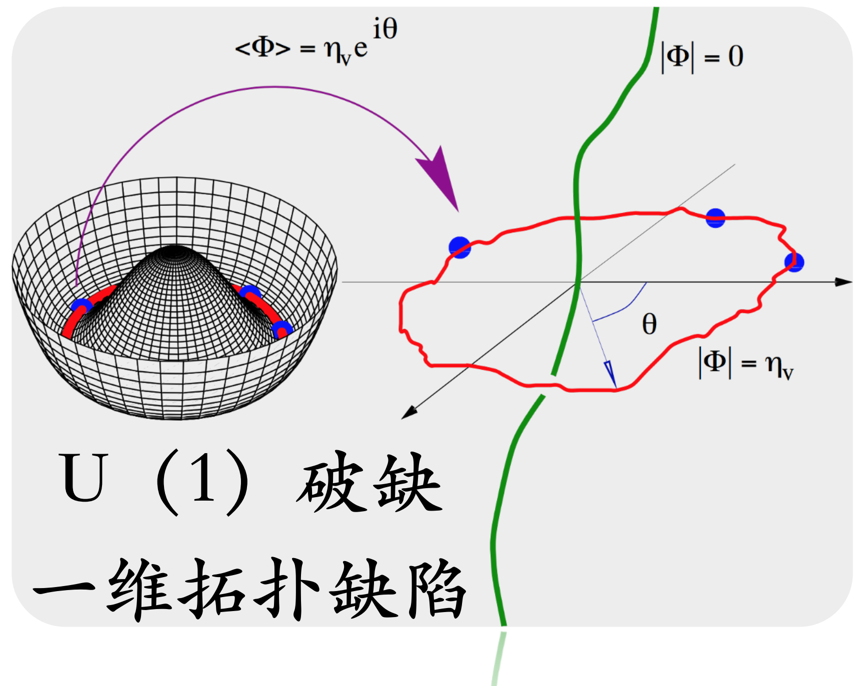
Kawana and Xie, PLB 824 (2022) 136791

Cosmic string

通常形成于GUTs



Phys. Rev. Lett. **126**, 021802

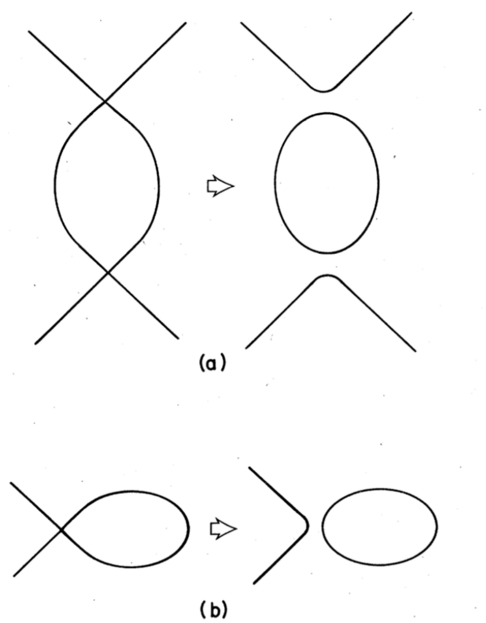


On CPUs

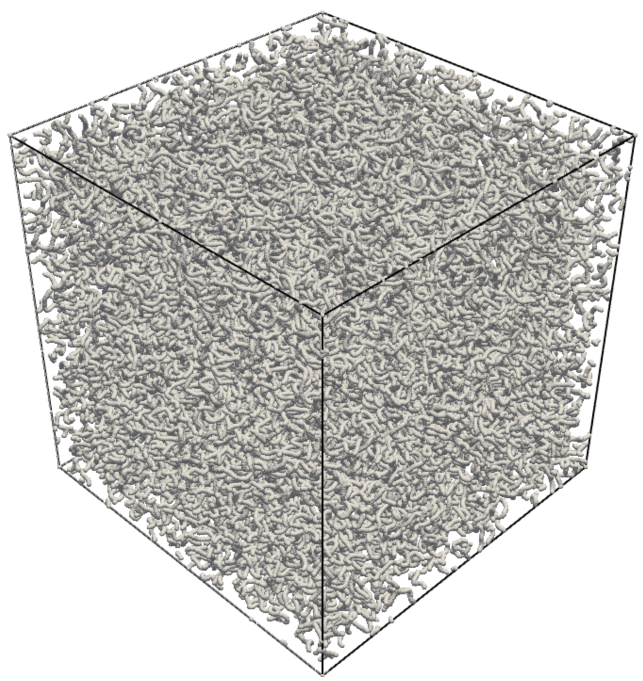
1024^3

T. W. B. Kibble

宇宙弦成圈

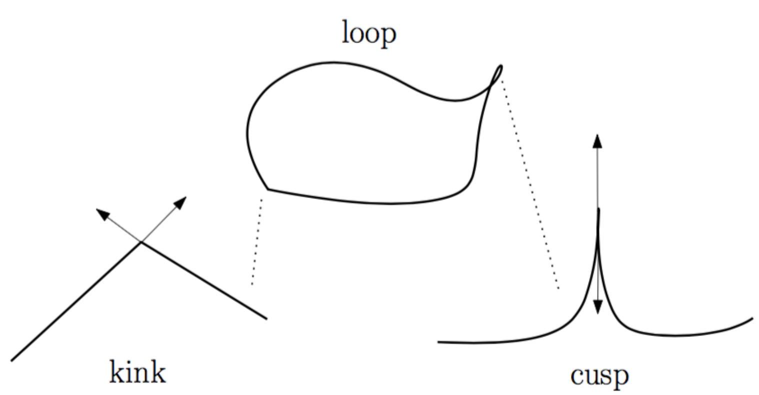


Phys.Rev.D 30 (1984) 2036



我们的宇宙弦模拟结果

引力波辐射

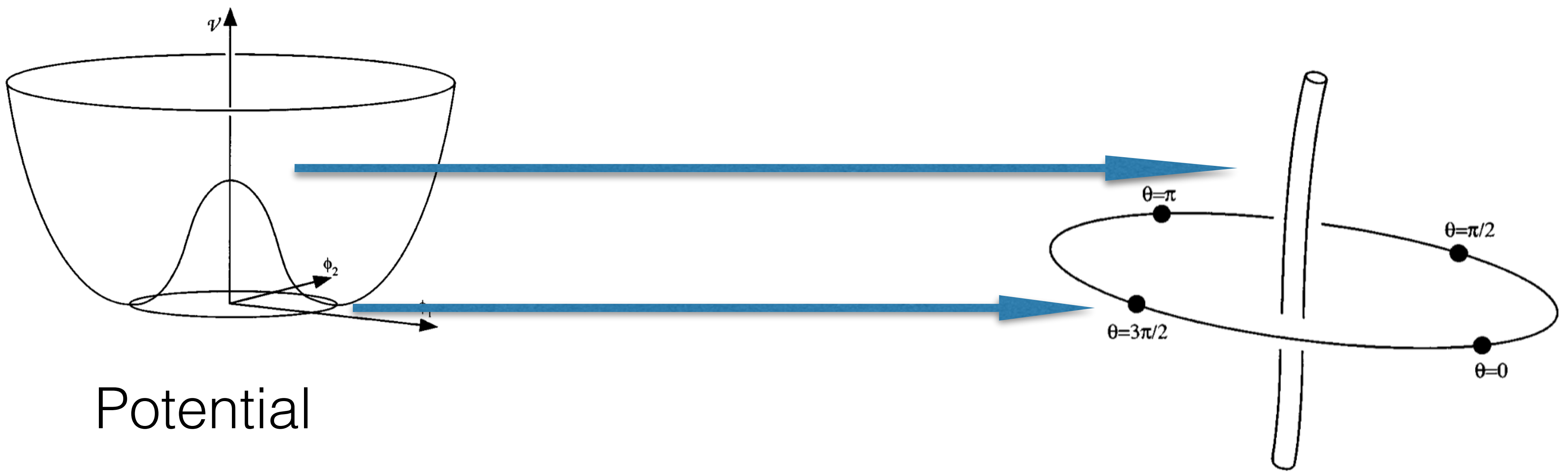


Yann Gouttenoire *et al* JCAP07(2020)032

▸ Cosmic string simulation FOPT

CS: SSB of U(1) symmetry

The one-dimension topological defects: cosmic string



T. W. B. Kibble

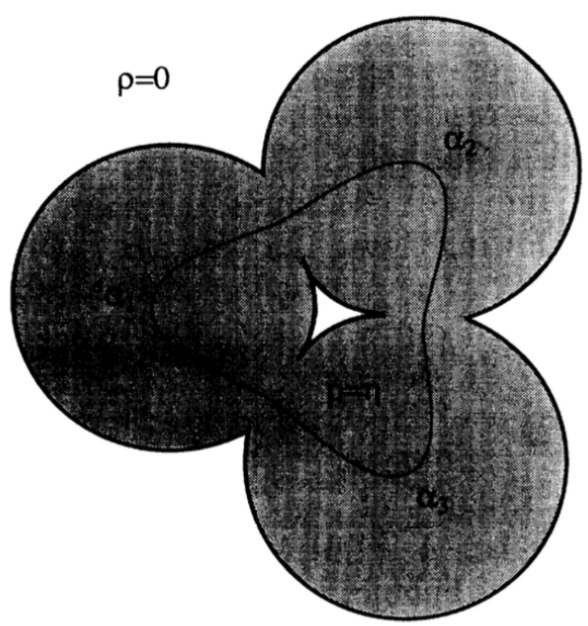


FIG. 1. Three bubbles of the broken symmetry phase ($\rho=\eta$) colliding. If the phase change of the scalar field around the loop γ is $\pm 2\pi$, a string (or antistring) is formed. If the phases α_i are ordered, then the requirement for a string is $\alpha_1+\pi<\alpha_3<\alpha_2+\pi$.

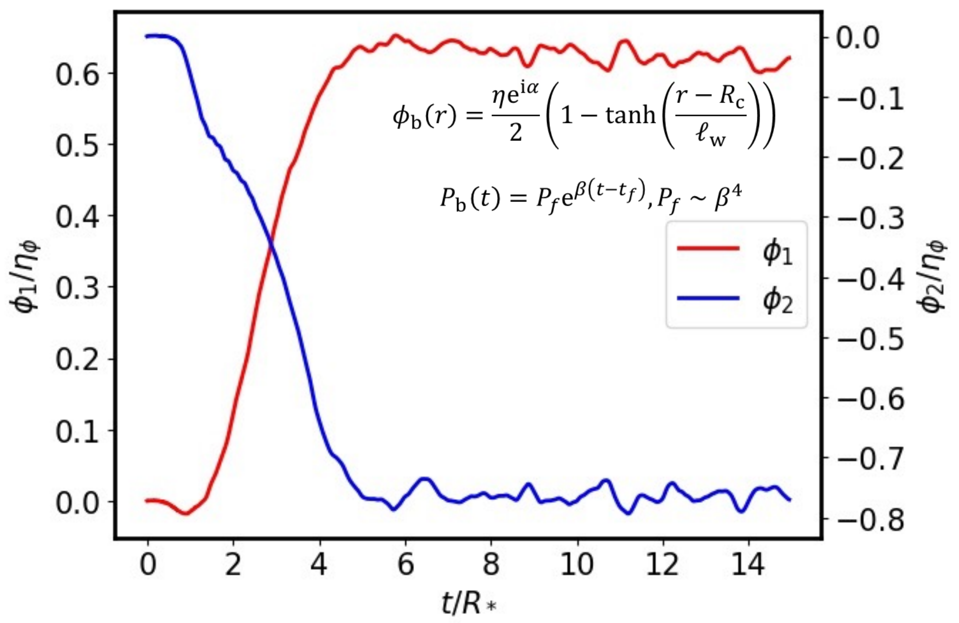
ξstr of the string network is essentially the typical bubble diameter for SFOPT???

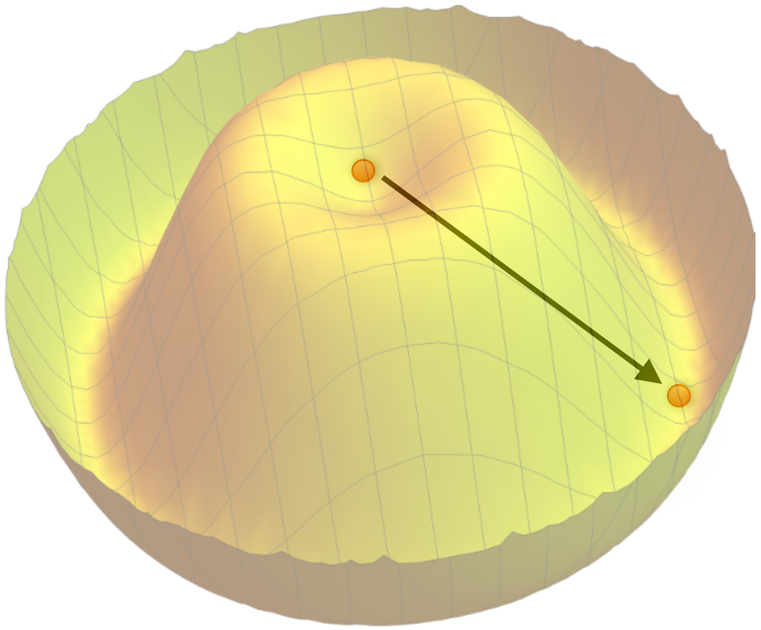
Aust. J. Phys., 1997, 50, 697–722

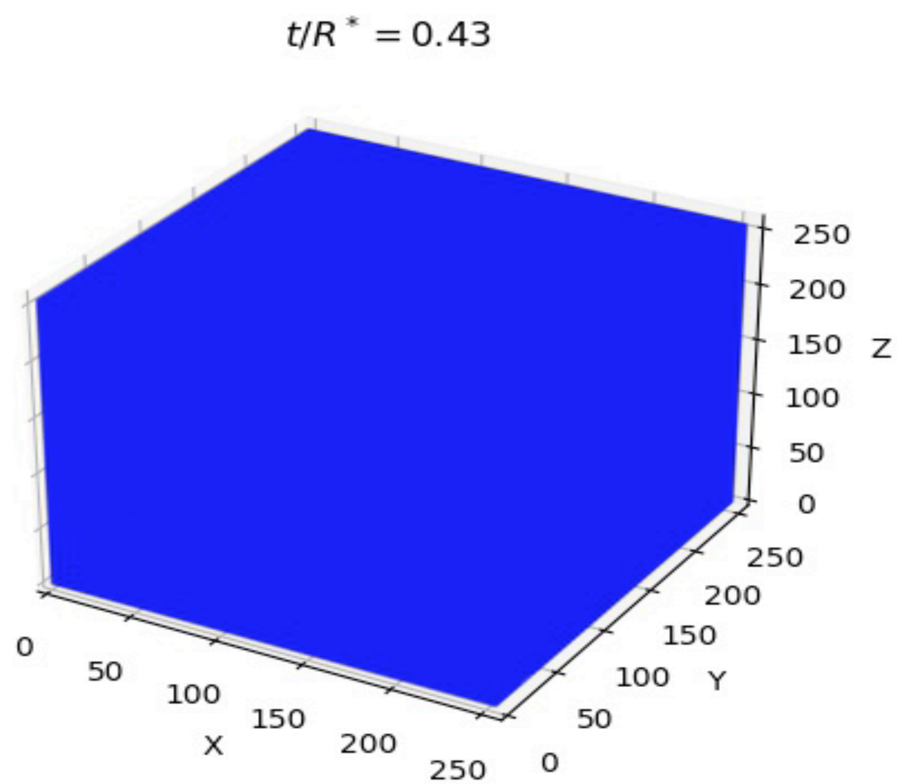
Phys.Rev.D 49 (1994) 1944-1950

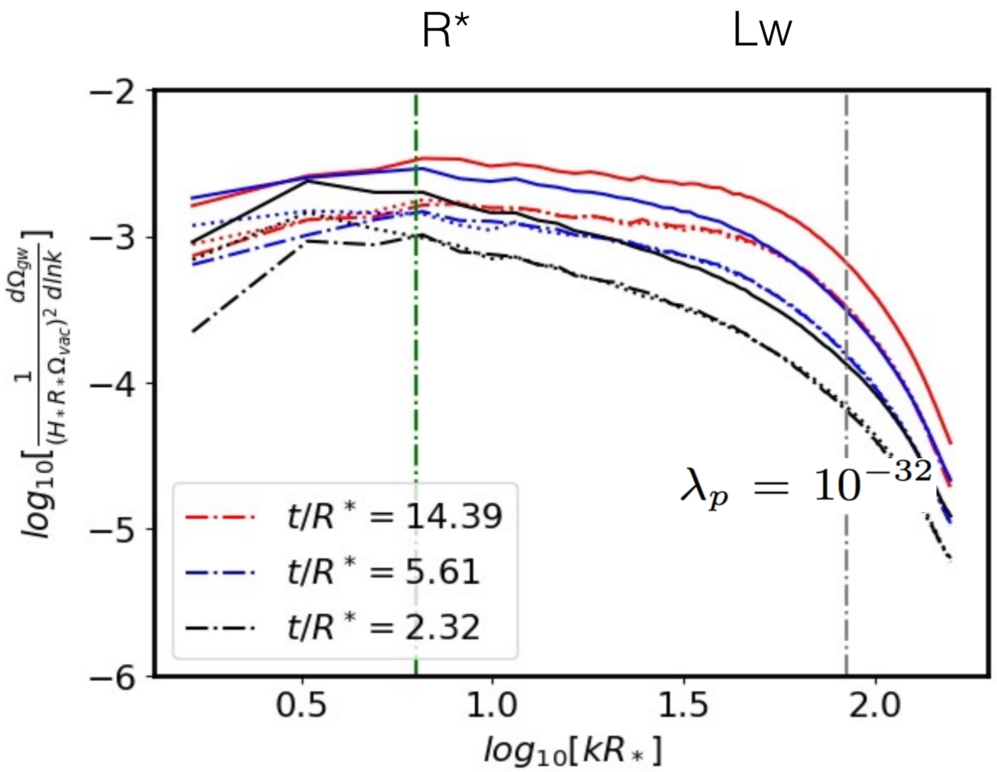
With Global U(1)

Motivated for strong CP and axion DM







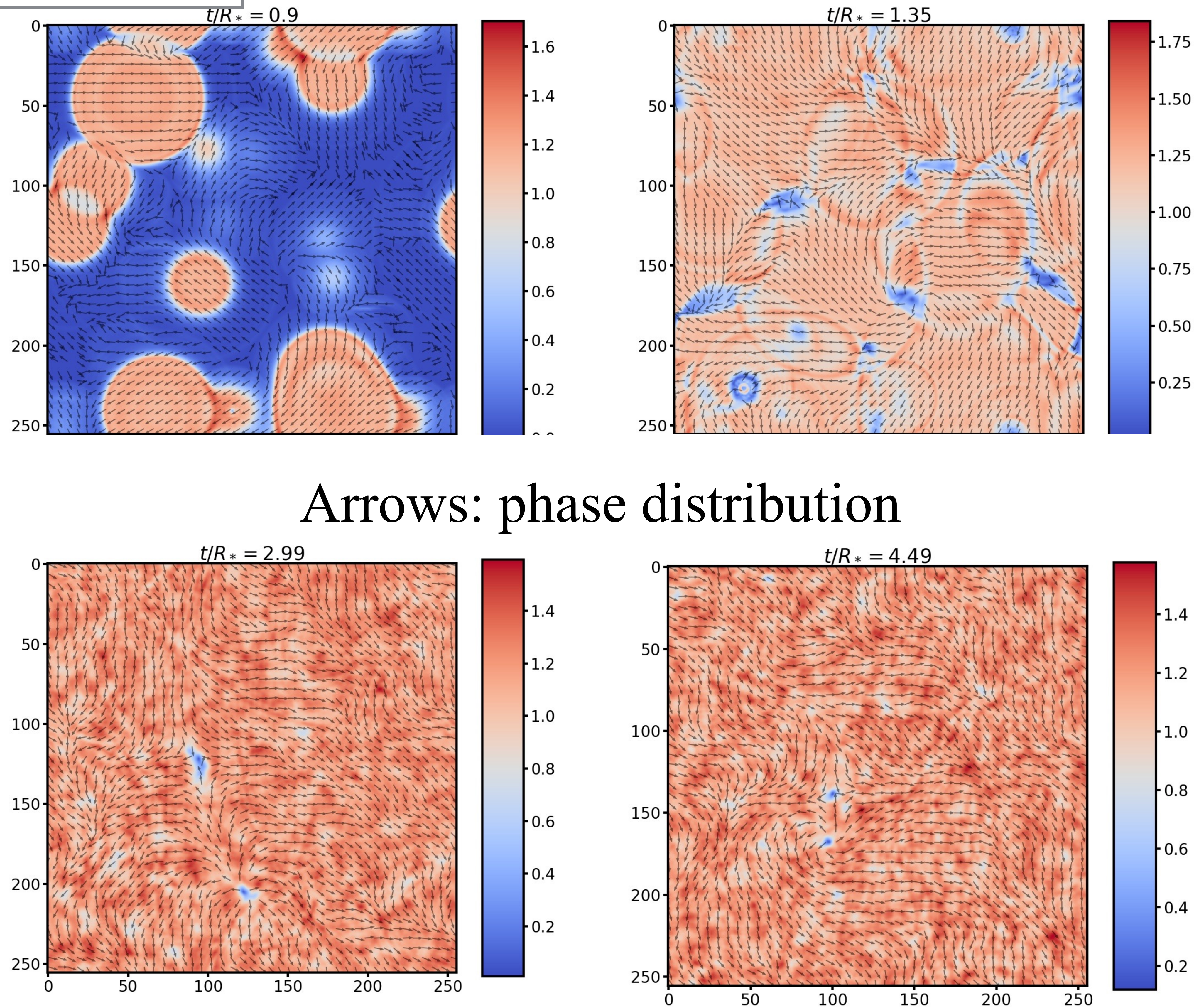


Zhao, Di, **Bian***, Cai*,2204.04427

▸ Bubbles and vortex&anti-vortex

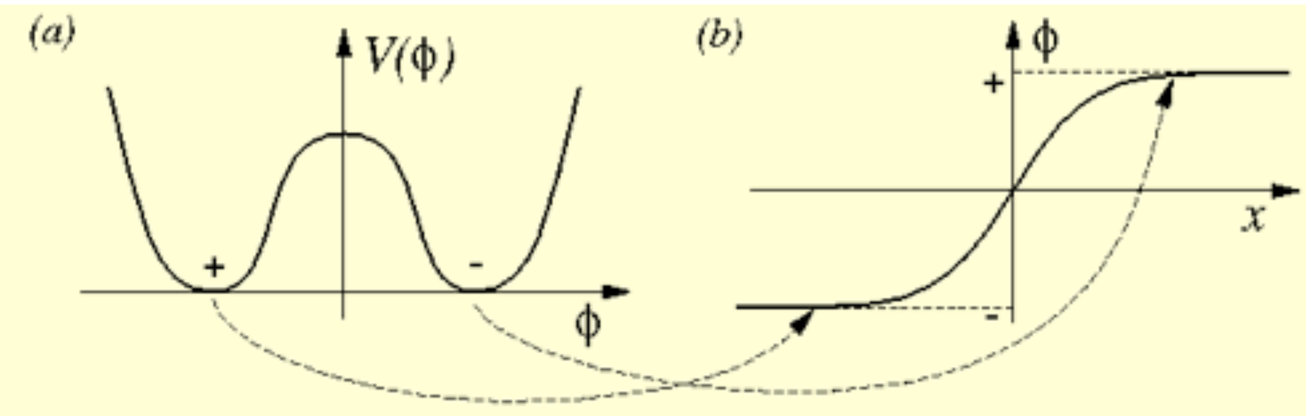
Type-b

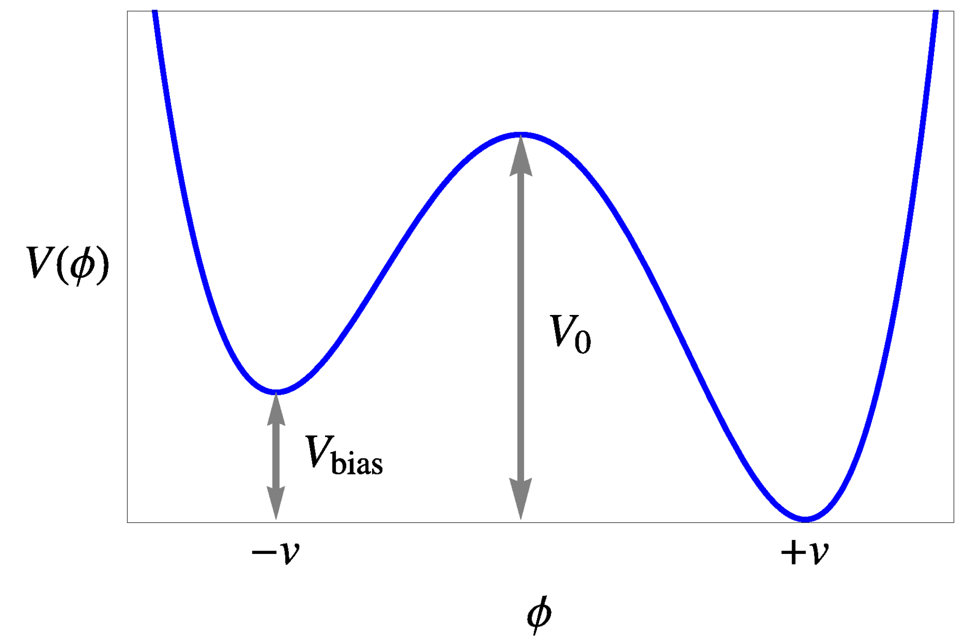
With Global U(1)

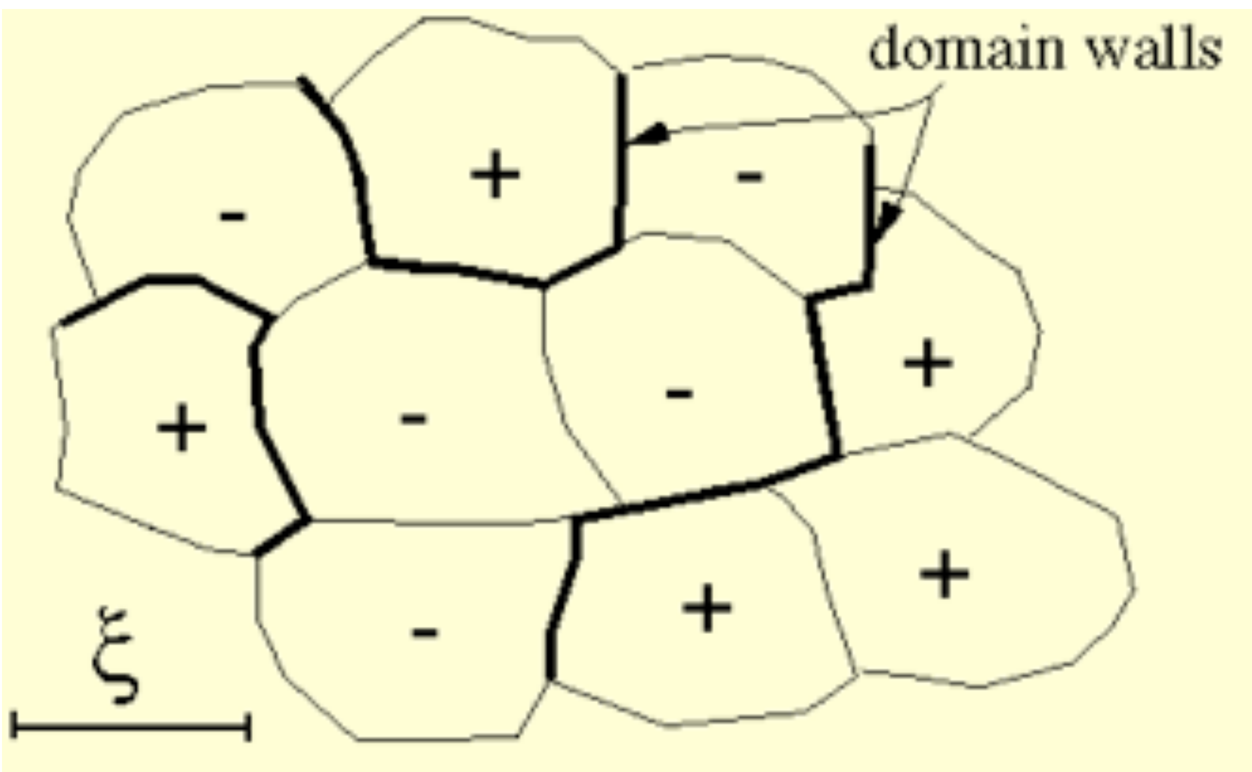


Arrows: phase distribution

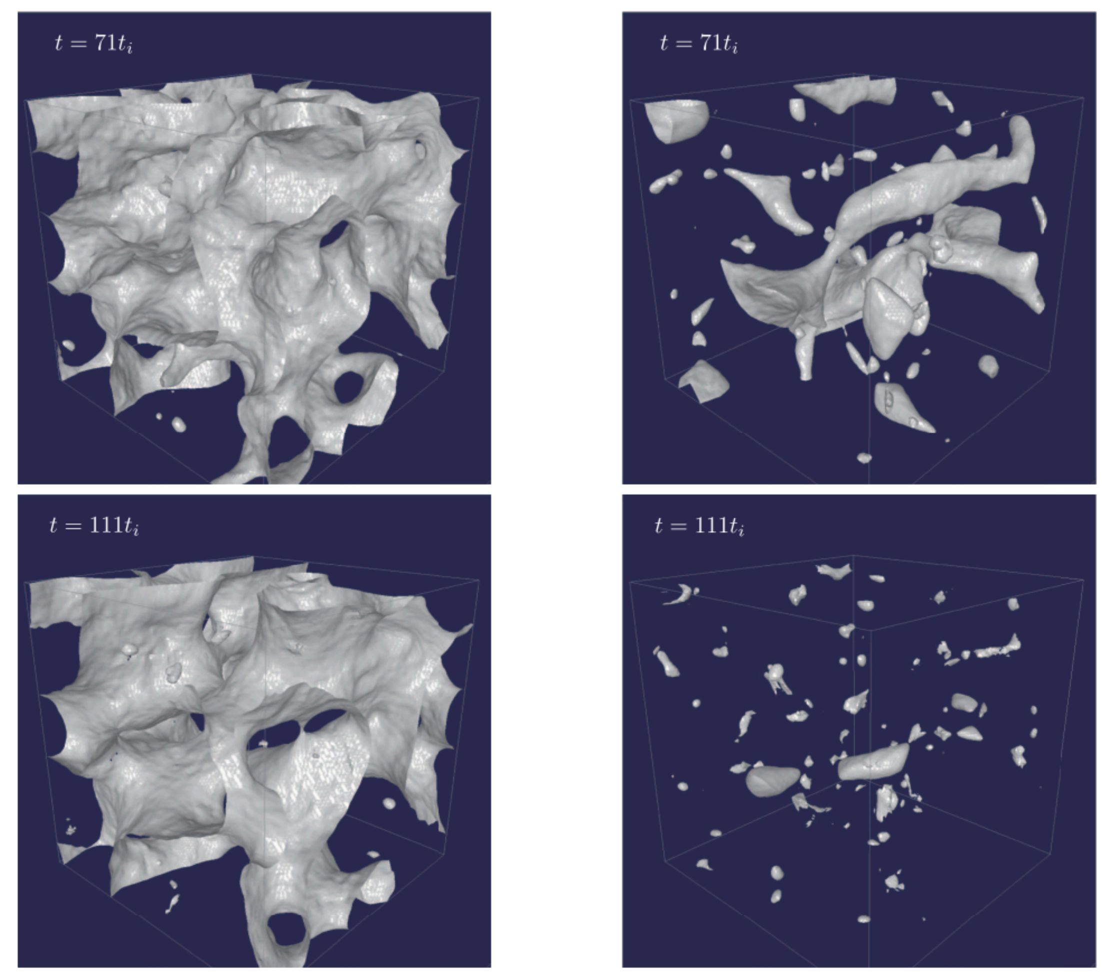
DW & Discrete symmetry







Kibble mechanism



1002.1555

DW & GW

Amplitude: $$\Omega_{\rm GW}^{dw} h^2 (t_0)_{\rm peak} \simeq 5.20 \times 10^{-20} \times \tilde{\epsilon}_{\rm gw} \mathcal{A}^4 \left(\frac{10.75}{g_*}\right)^{1/3} \left(\frac{\sigma_{\rm wall}}{1{\rm TeV}^3}\right)^4 \left(\frac{1{\rm MeV}^4}{\Delta V}\right)^2$$

Area parameter

peak frequency: $$f^{dw}(t_0)_{\rm peak} = \frac{a(t_{\rm dec})}{a(t_0)} H(t_{\rm dec}) \simeq 3.99 \times 10^{-9}{\rm Hz}\mathcal{A}^{-1/2} \left(\frac{1{\rm TeV}^3}{\sigma_{\rm wall}}\right)^{1/2} \left(\frac{\Delta V}{1{\rm MeV}^4}\right)^{1/2}$$

Slope: $\Omega_{GW}^{dw} h^2 \propto f^3$ when $f < f_{peak}$, and $\Omega_{GW}^{dw} h^2 \propto f^{-1}$ when $f \geqslant f_{peak}$

Domain wall decay before they overclose Universe

$$\sigma_{wall} < 2.93 \times 10^4 {\rm TeV}^3 \mathcal{A}^{-1} (\frac{0.1sec}{t_{dec}})$$

Domain wall decay before the BBN

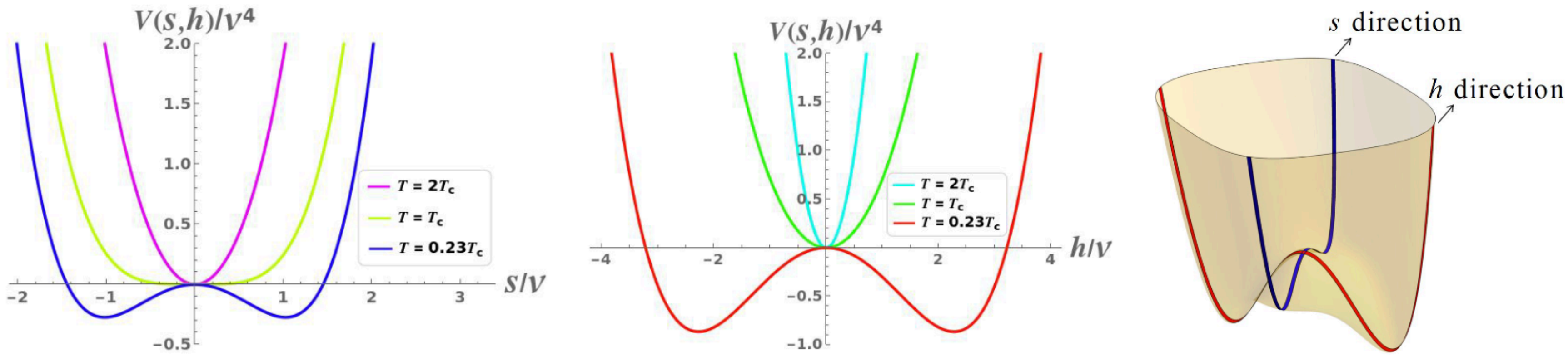
$$t_{dec} \approx \mathcal{A}\sigma_{wall}/(\Delta V)$$

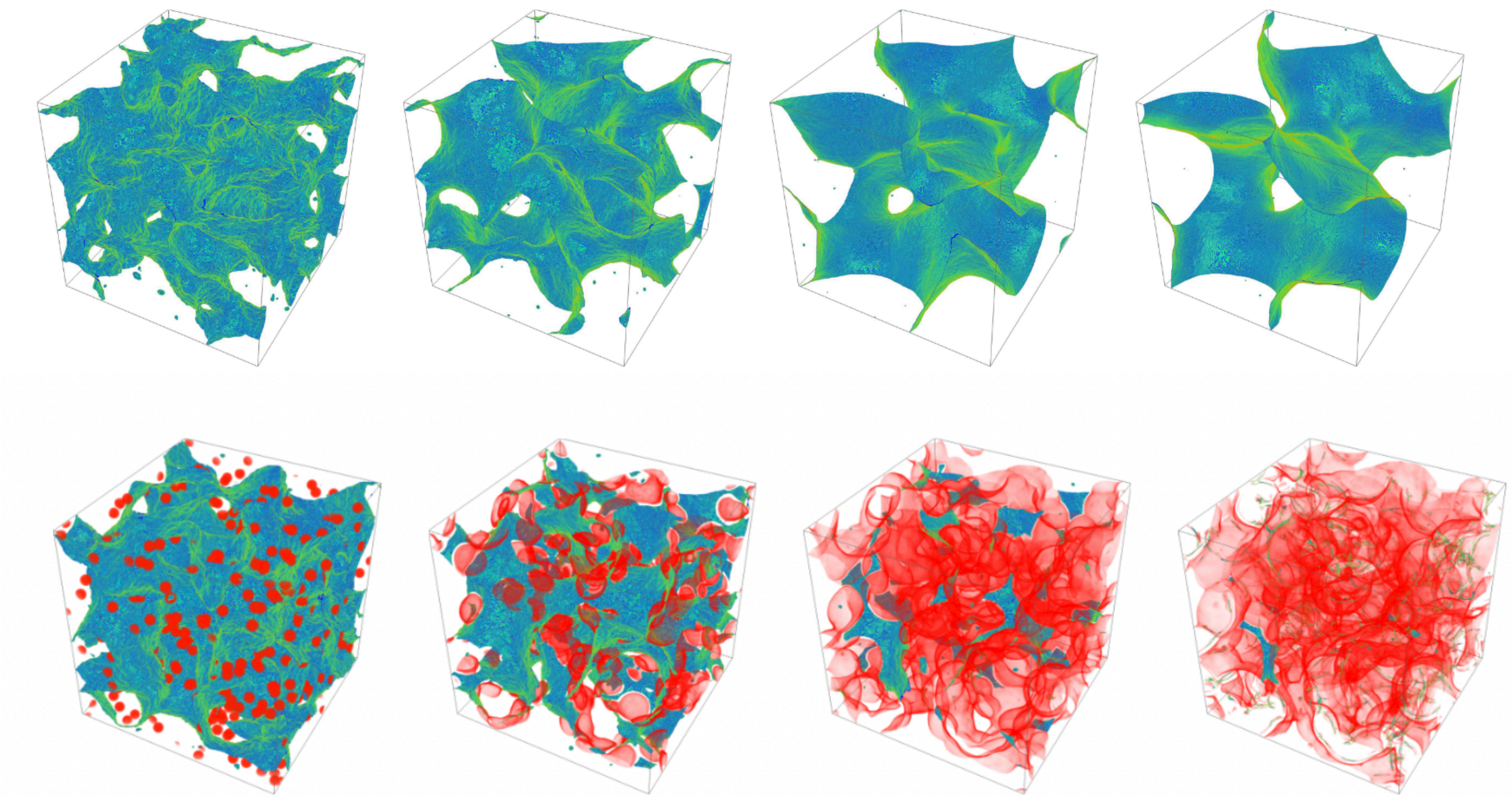
$$\Delta V \gtrsim 6.6 \times 10^{-2} {\rm MeV}^4 \mathcal{A} \left(\frac{\sigma_{wall}}{1{\rm TeV}^3}\right)$$

1309.5001

一阶相变解决畴壁问题（宇宙学上的 Domain wall 问题）

3D snapshots of the
pure DW networks

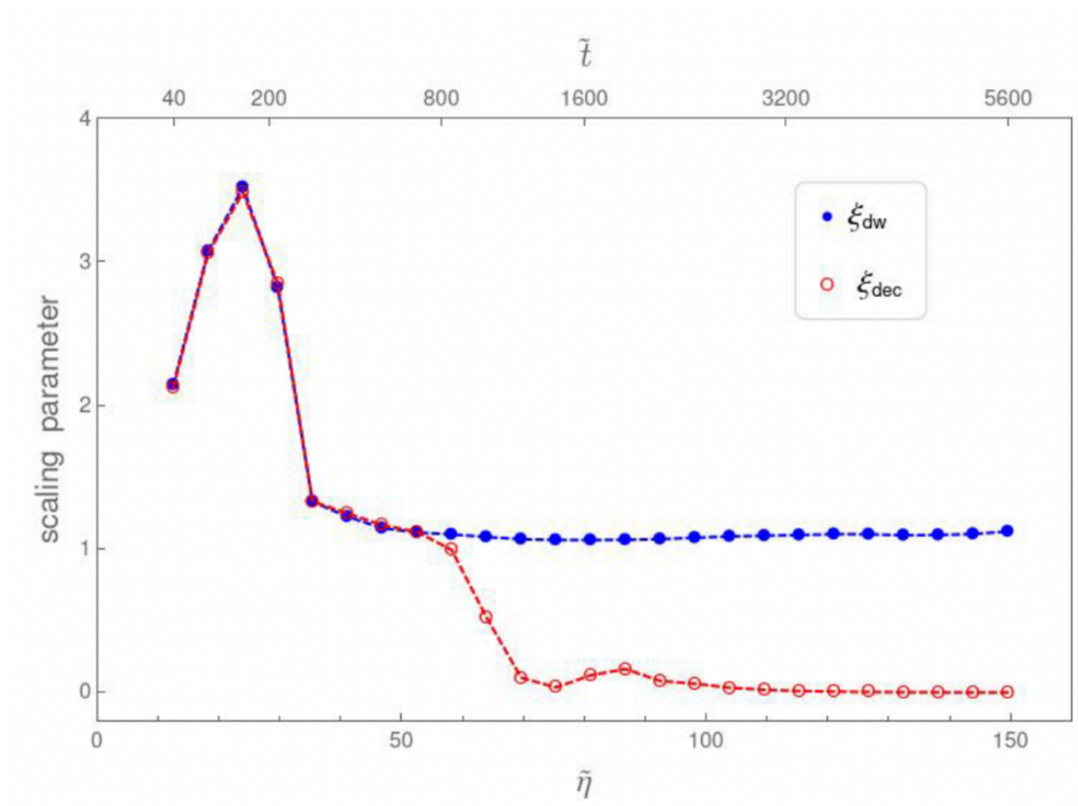


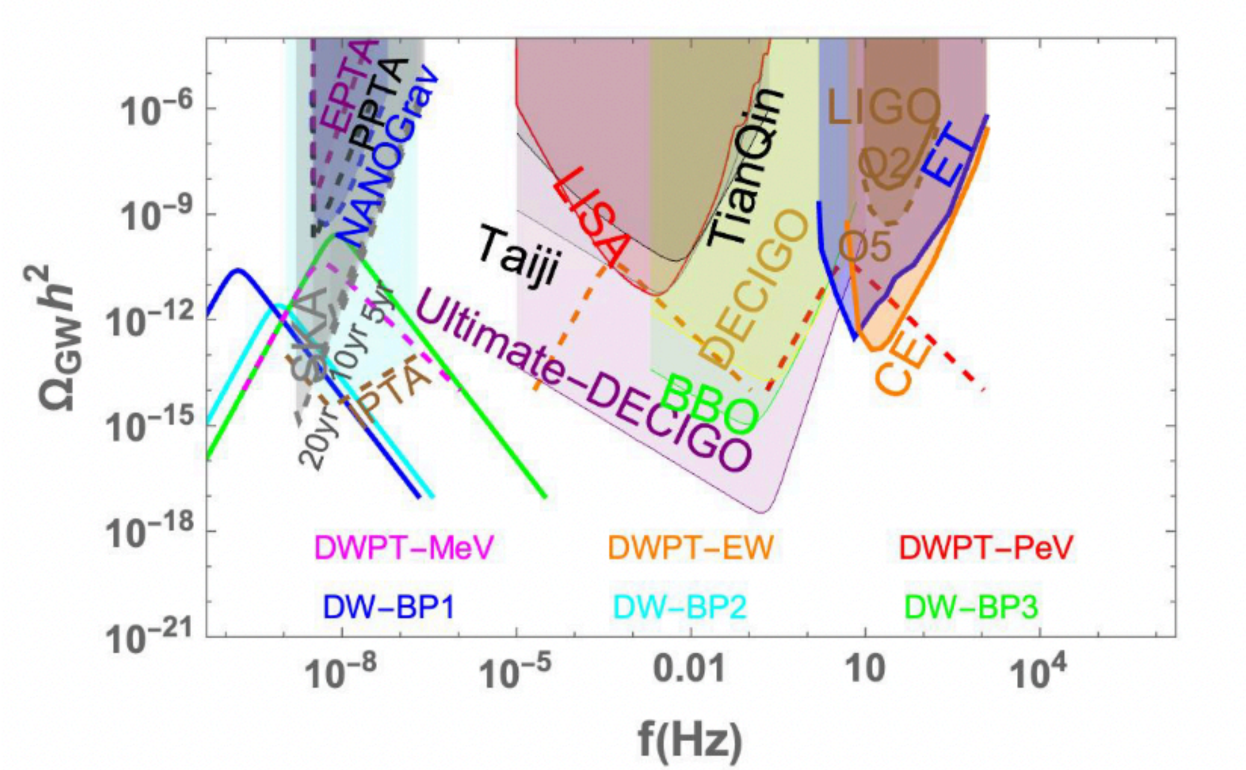


一阶相变解决畴壁问题（宇宙学上的 Domain wall 问题）

Second order PT followed by a first-order PT that leads to EWSB

量子隧穿耗散经典的畴壁



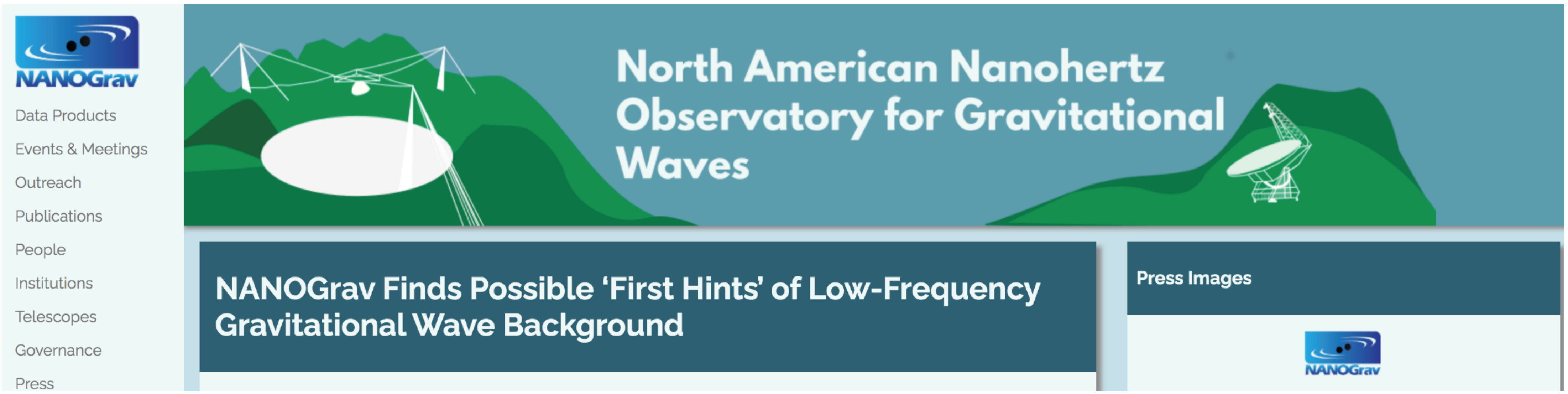


On CPUs

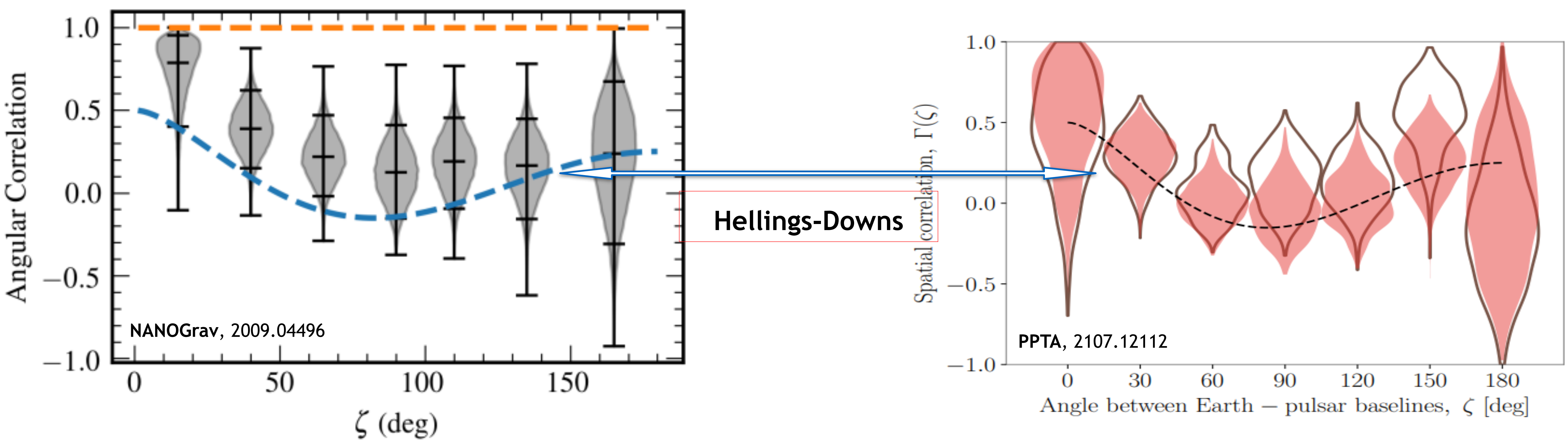
格子大小 1024^3

Bian, Jia, Li, 2304.05220

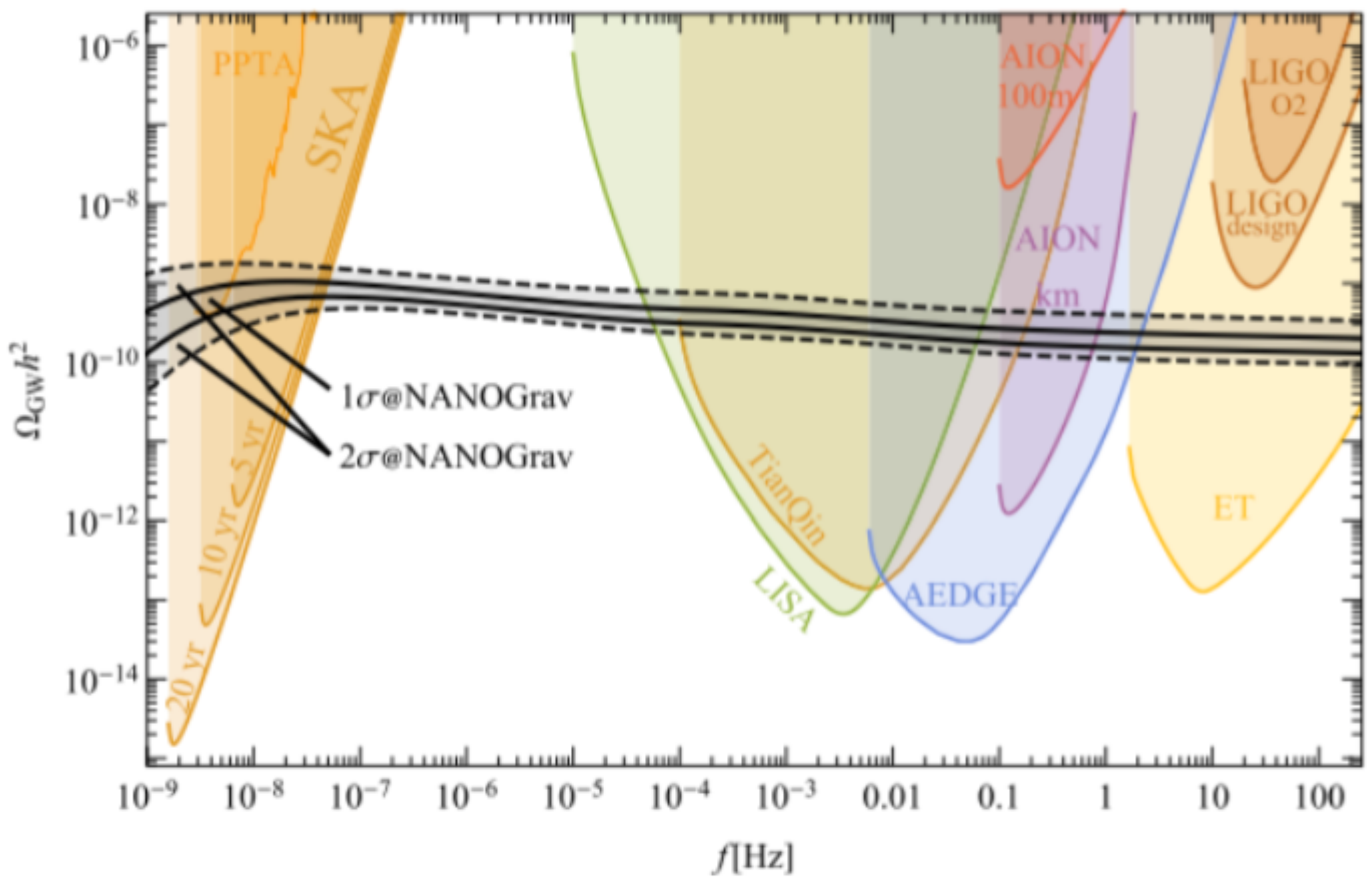
脉冲星计时阵列实验与随机引力波



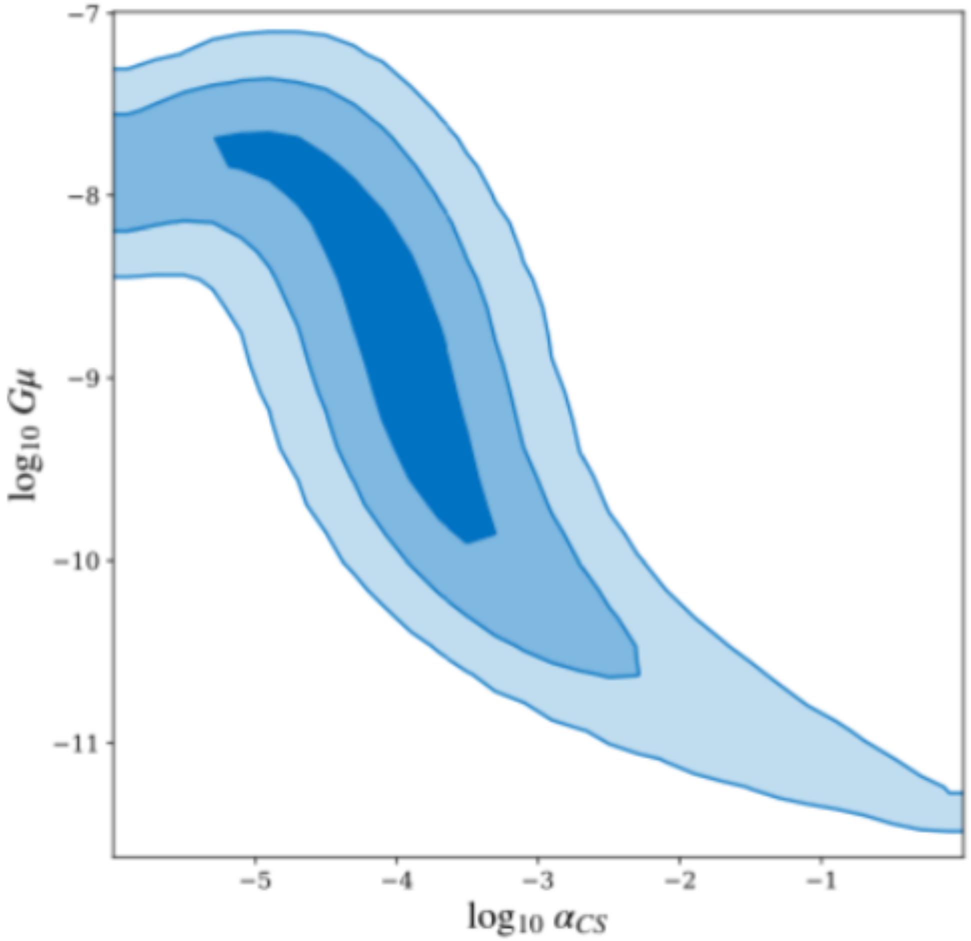
Some CPL signals, SGWB？？？



NANOGrav 12.5-yr dataset & cosmic string



John Ellis etal, Phys.Rev.Lett.126.041304(2021)



Bian, Cai, Liu, Zhou,Phys.Rev.D 103 (2021) 8, L081301

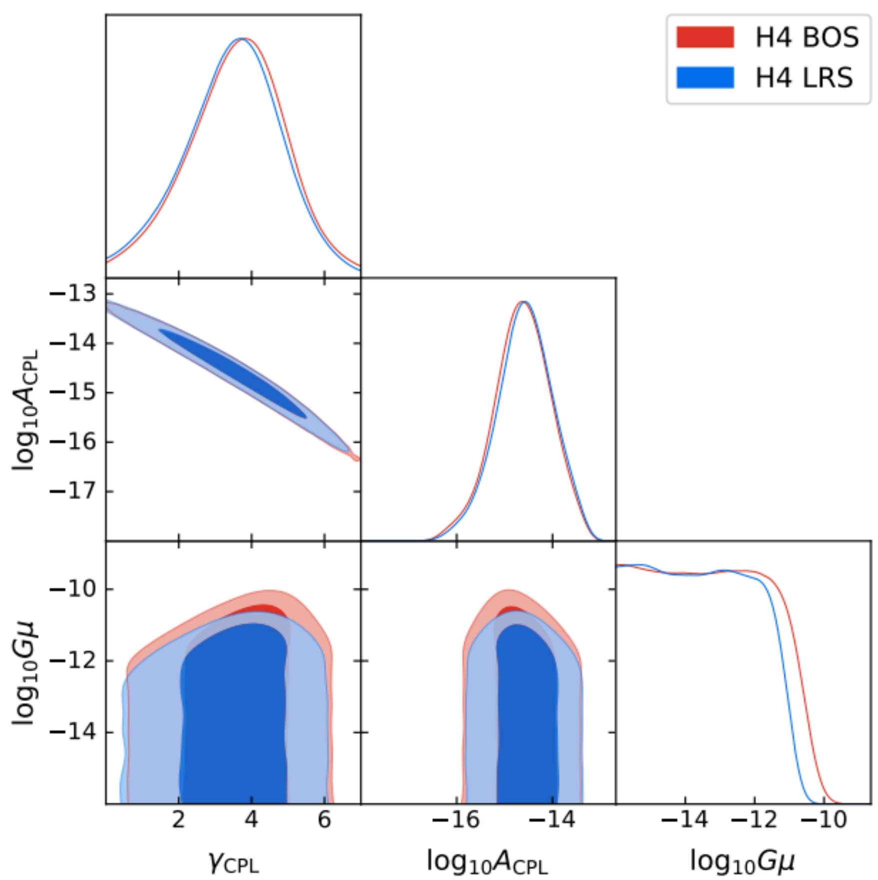
▸ PPTA dataset & cosmic string

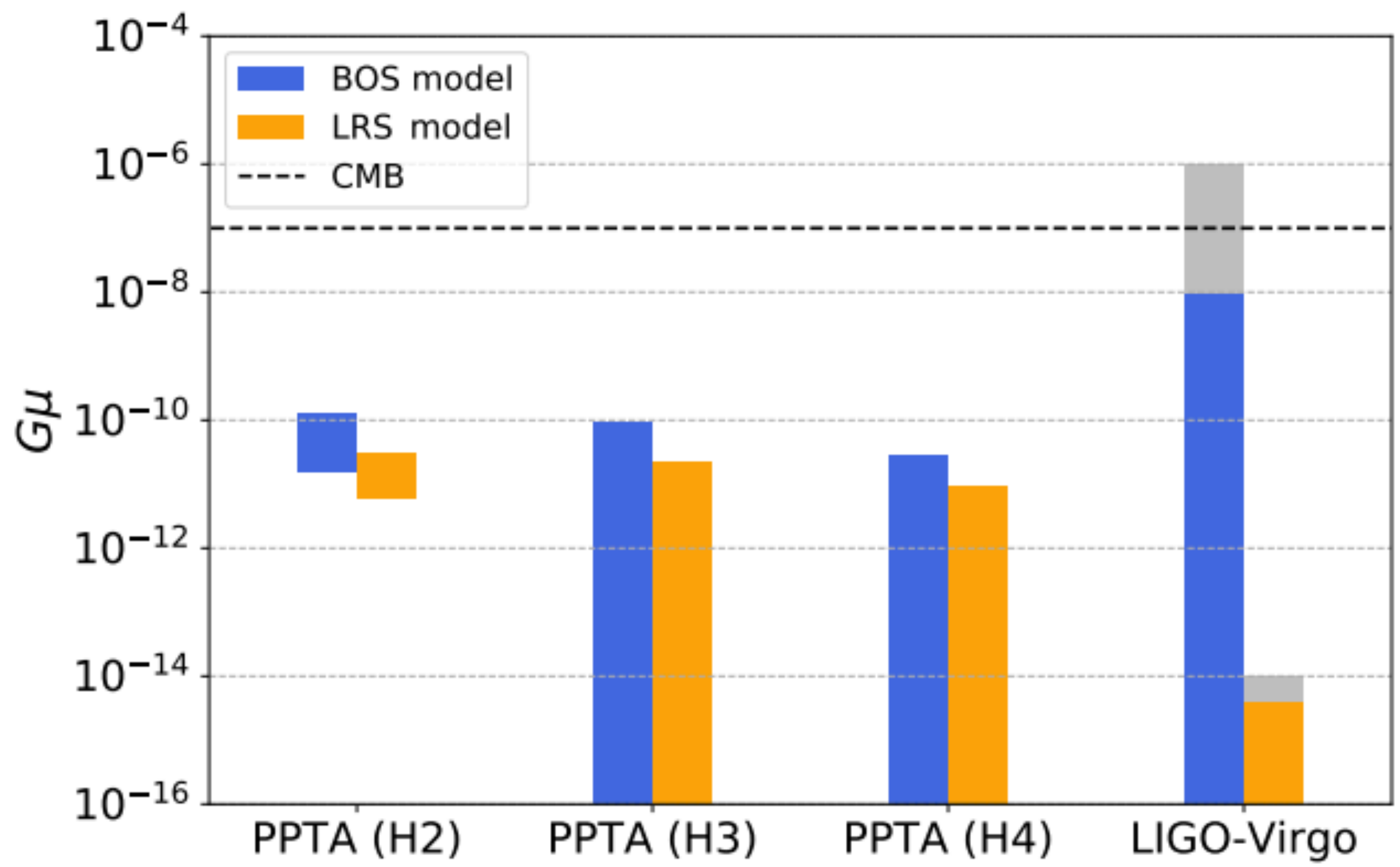
TABLE I: Hypotheses, Bayes factors, and estimated model parameters for the BOS model.

Hypothesis	Pulsar Noise	CPL Process	HD process CS spectrum	Bayes Factors	Parameter Estimation (1σ interval)	
					$\log_{10} G\mu$	$\log_{10} A_{\rm CPL}, \gamma_{\rm CPL}$
H0:Pulsar Noise	✓					
H1:CPL	✓	✓		$10^{3.2}$ (/H0)		$-14.48^{+0.62}_{-0.64}, 3.34^{+1.37}_{-1.53}$
H2:CS	✓		✓(full HD)	$10^{3.1}$ (/H0)	$-10.38^{+0.21}_{-0.21}$	
H3:CS1	✓	✓	✓(full HD)	1.96 (/H1)	< -10.02 (95% C.L.)	$-15.58^{+1.21}_{-1.64}, 3.11^{+1.95}_{-2.02}$
H4:CS2	✓	✓	✓(no-auto HD)	0.60 (/H1)	< -10.54 (95% C.L.)	$-14.61^{+0.58}_{-0.59}, 3.63^{+1.24}_{-1.40}$

TABLE II: Hypotheses, Bayes factors, and estimated model parameters for the LRS model.

Hypothesis	Pulsar Noise	CPL process	HD process CS spectrum	Bayes Factors	Parameter Estimation (1σ interval)	
					$\log_{10} G\mu$	$\log_{10} A_{\rm CPL}, \gamma_{\rm CPL}$
H0:Pulsar Noise	✓					
H1:CPL	✓	✓		$10^{3.2}$ (/H0)		$-14.48^{+0.62}_{-0.64}, 3.34^{+1.37}_{-1.53}$
H2:CS	✓		✓(full HD)	$10^{3.3}$ (/H0)	$-10.89^{+0.14}_{-0.17}$	
H3:CS1	✓	✓	✓(full HD)	1.62 (/H1)	< -10.64 (95% C.L.)	$-15.44^{+1.18}_{-1.74}, 3.08^{+1.94}_{-1.99}$
H4:CS2	✓	✓	✓(no-auto HD)	0.55 (/H1)	< -11.04 (95% C.L.)	$-14.57^{+0.58}_{-0.59}, 3.54^{+1.24}_{-1.41}$

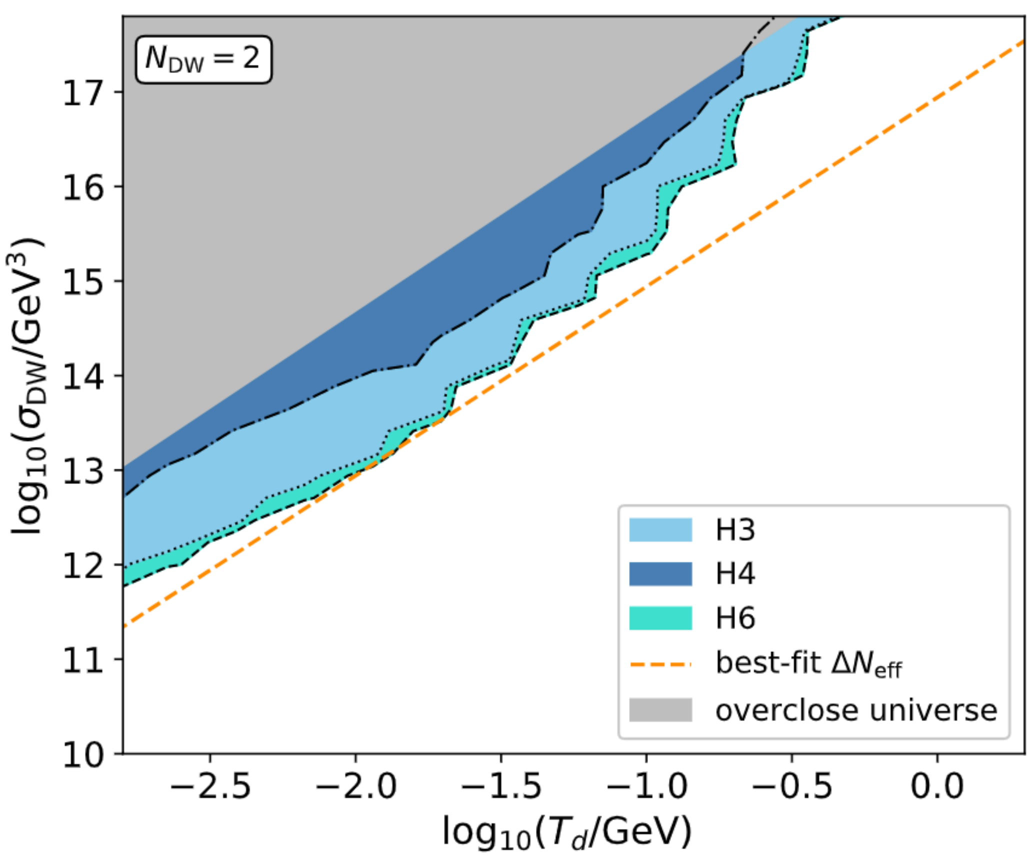


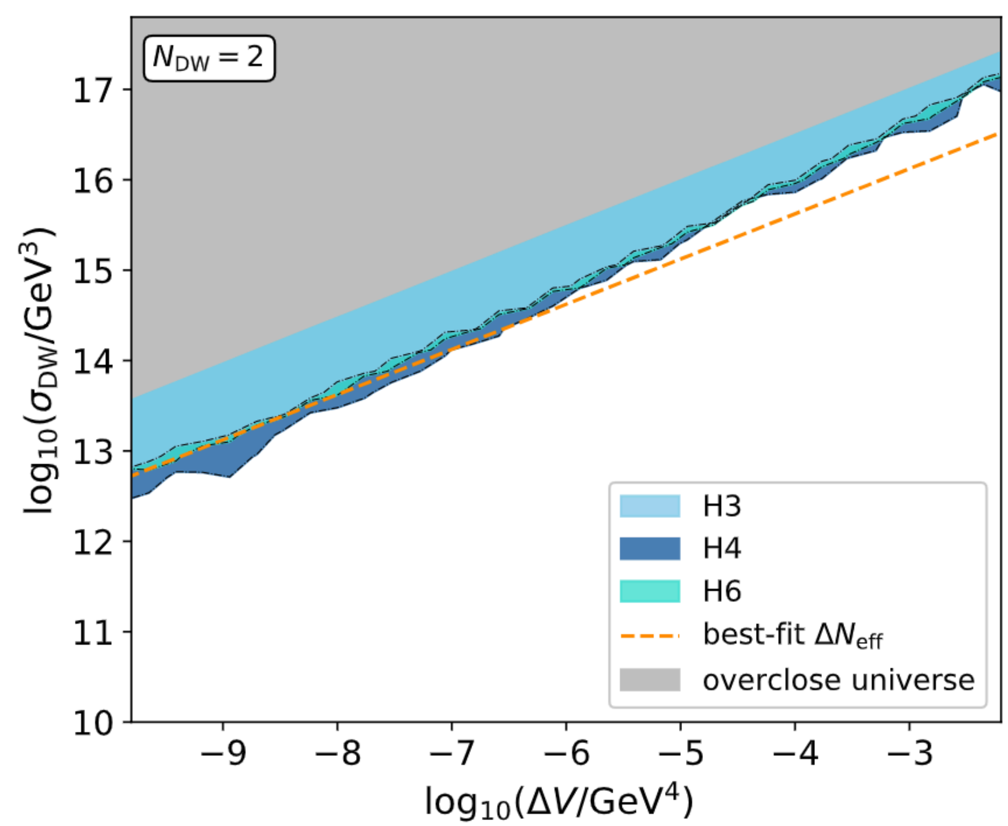


Bian*, Shu*, Wang, Yuan*, Zong,2205.07293

PPTA 限制畴壁引力波

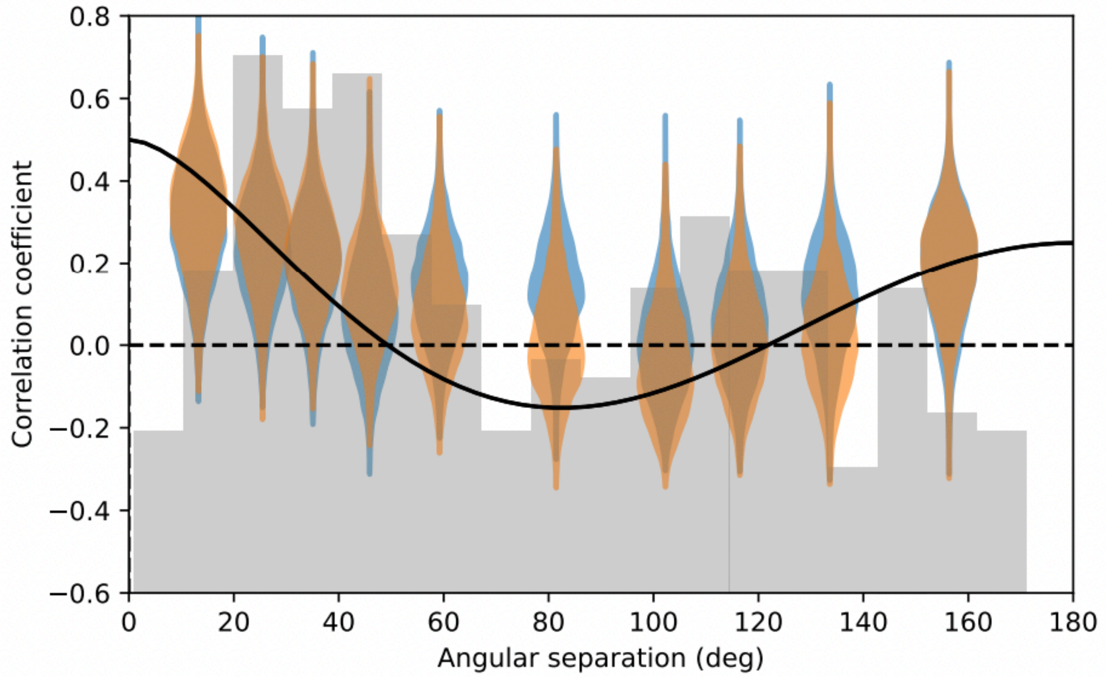
Hypothesis	Pulsar Noise	CPL	SMBHB	DW Spectra	Parameter Estimation (68% C.L.)		
					$\log_{10}\Delta V/\mathrm{GeV}^4$, $\log_{10}\sigma_{\mathrm{DW}}/\mathrm{GeV}^3$ $\log_{10}T_d/\mathrm{GeV}$, $\log_{10}\sigma_{\mathrm{DW}}/\mathrm{GeV}^3$	$A_{\mathrm{CPL}}, \gamma_{\mathrm{CPL}}, A_{\mathrm{SMBHB}}$ (ΔV) $A_{\mathrm{CPL}}, \gamma_{\mathrm{CPL}}, A_{\mathrm{SMBHB}}$ (T_d)	Bayes Factors (ΔV) Bayes Factors (T_d)
H0	✓				—	—	—
H1	✓	✓			—	$-14.48^{+0.62}_{-0.64}$, $3.34^{+1.37}_{-1.53}$, —	$10^{3.2}$ (/H0)
H2	✓			✓(full HD)	$-6.21^{+2.59}_{-1.36}$, $14.42^{+1.61}_{-0.76}$ $-1.10^{+0.66}_{-0.49}$, $14.60^{+2.25}_{-1.20}$	—	$10^{2.5}$ (/H0) $10^{2.7}$ (/H0)
H3	✓	✓		✓(full HD)	> -9.59, < 16.64 (95% C.L.) > -2.48, < 17.73 (95% C.L.)	$-14.71^{+0.76}_{-1.51}$, $3.16^{+1.63}_{-1.78}$, — $-14.67^{+0.65}_{-0.98}$, $3.42^{+1.47}_{-1.65}$, —	0.86 (/H1) 0.98 (/H1)
H4	✓	✓		✓(no-auto HD)	> -9.62, < 16.32 (95% C.L.) > -2.54, < 17.75 (95% C.L.)	$-12.53^{+0.14}_{-0.13}$, $1.28^{+0.36}_{-0.36}$, — $-13.62^{+0.14}_{-0.14}$, $4.67^{+0.33}_{-0.32}$, —	0.58 (/H1) 0.85 (/H1)
H5	✓		✓		—	—, —, $-14.89^{+0.10}_{-0.12}$	$10^{3.3}$ (/H0)
H6	✓		✓	✓(full HD)	> -7.89, < 14.85 > -0.94, < 16.45	—, —, $-14.94^{+0.13}_{-0.35}$ —, —, $-14.92^{+0.11}_{-0.16}$	0.70 (/H5) 0.90 (/H5)



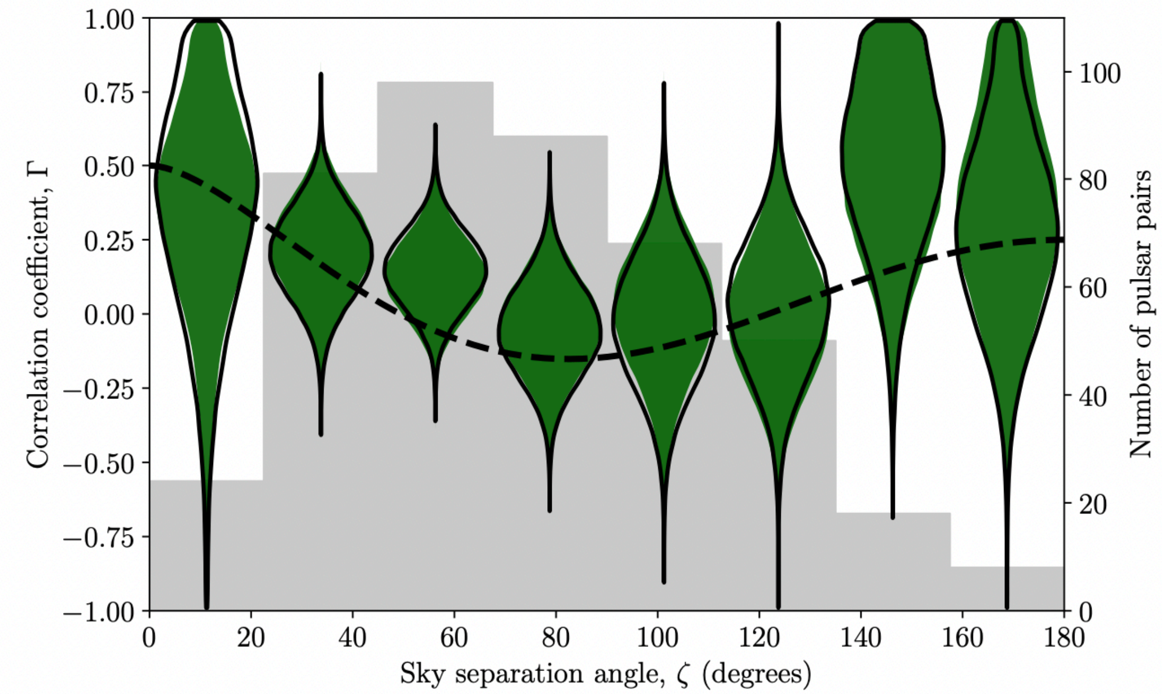


Bian, Ge, Li, Shu, Zong, 2212.07871

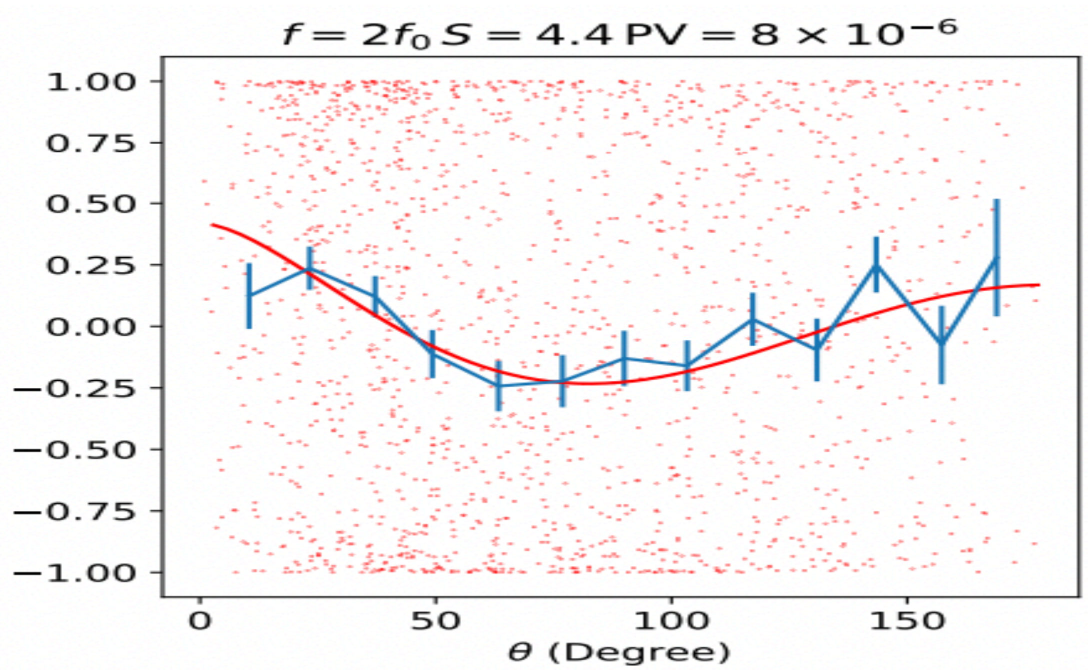
New dataset from PTAs



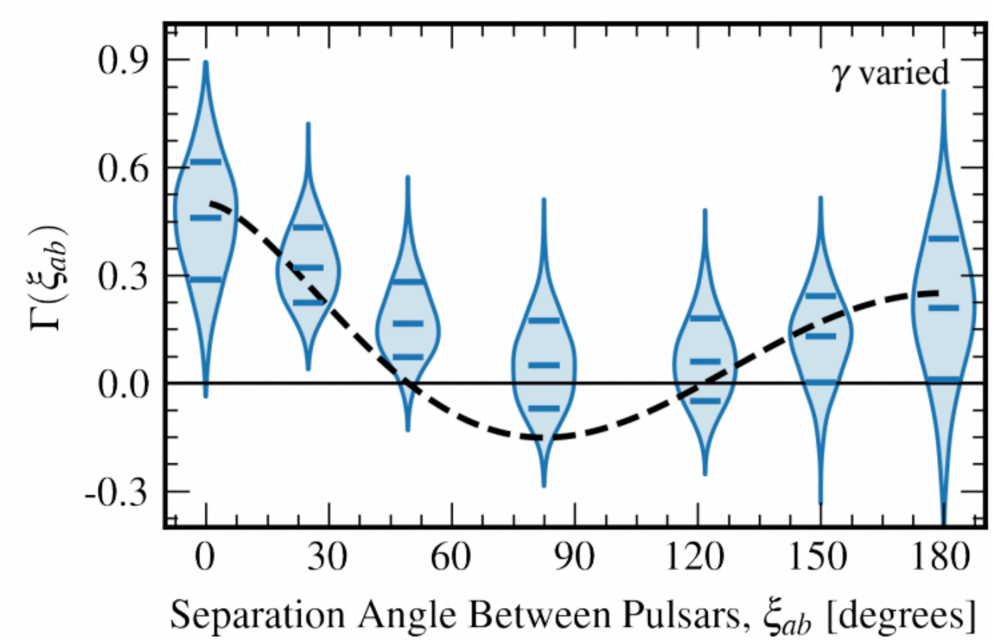
EPTA,2306.16214



PPTA,2306.16215

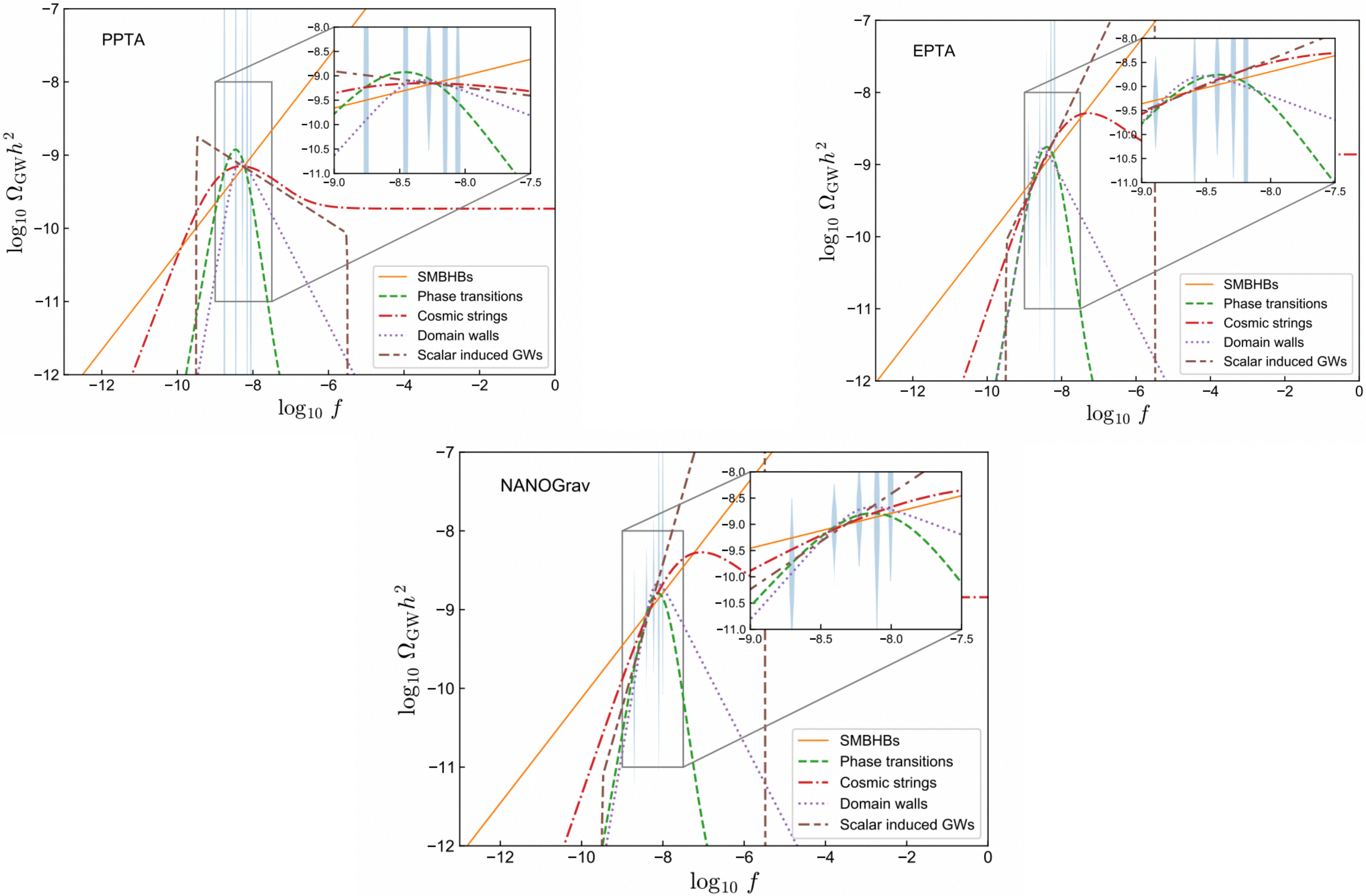


CPTA ,2306.16216



NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays

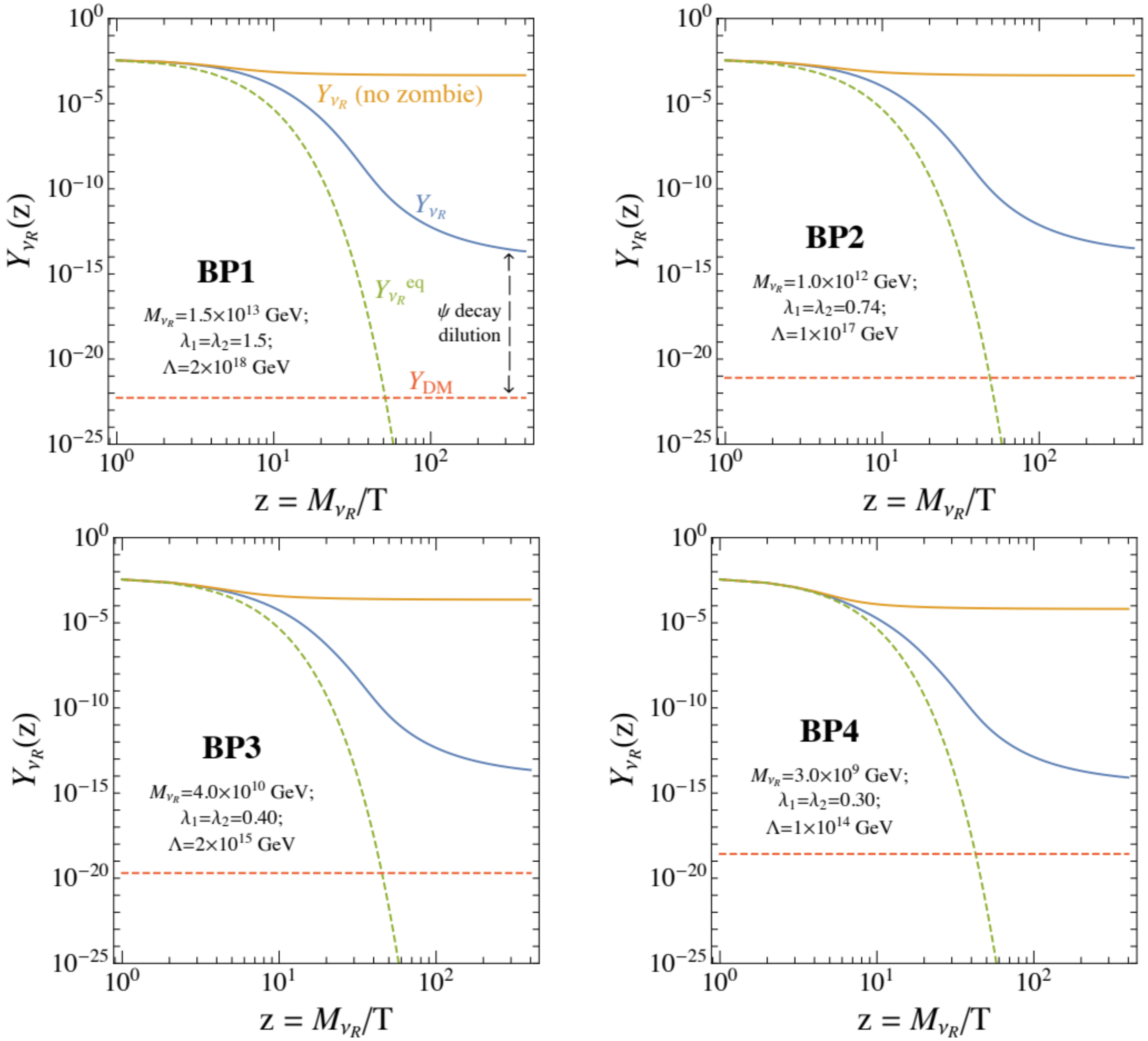


Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

Zombie DM and CS GW

$$\mathcal{L}_{B-L} = \sum_i \bar{\nu}_R^i i \not{D} \nu_R^i - \frac{1}{2}\sum_{i,j}\left(\lambda_R^{ij}\bar{\nu}_R^{i,c}\Phi\nu_R^j + \text{h.c.}\right) - \sum_{i,j}\left(\lambda_D^{ij}\bar{\ell}_L^i\tilde{H}\nu_R^j + \text{h.c.}\right)$$
$$+ D_\mu\Phi^\dagger D^\mu\Phi - \lambda_\phi\left(|\Phi|^2 - \frac{v_\phi^2}{2}\right)^2 - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu},$$
$$\mathcal{L}_{\rm DM} = \bar{\psi}\left(i\not{D} - M_\psi\right)\psi + \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{1}{2}M_S^2 S^2 + \left(\lambda_1 S\bar{\psi}\nu_R + \text{h.c.}\right) + \lambda_2 S\bar{\psi}\psi,$$

	$\nu_R^{1,2}$	ν_R^3	Φ	ψ	S
$B-L$	-1	-1	2	-1	0
$\mathbb{Z}_2$	1	-1	1	-1	1



zombie collision

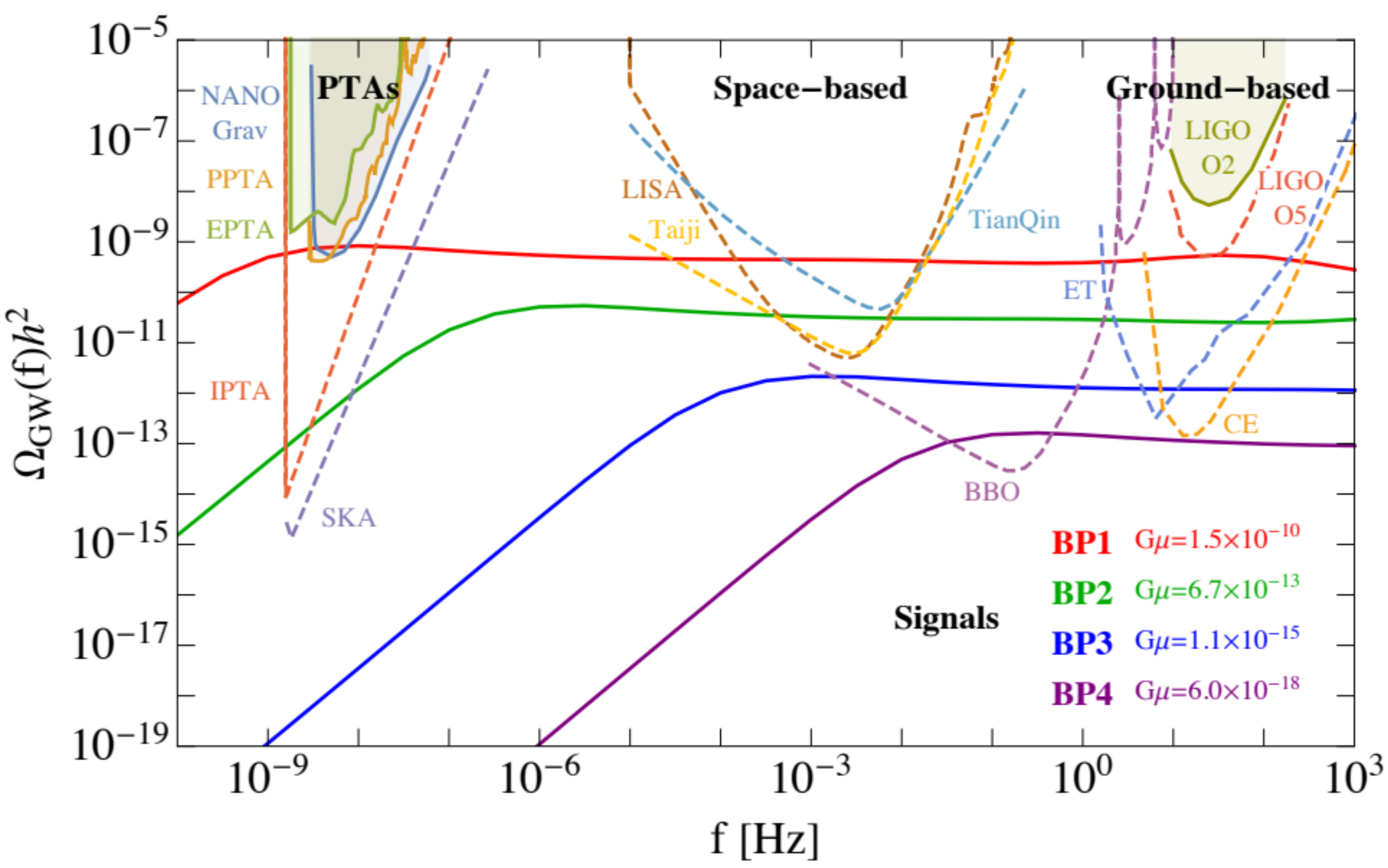
$$\nu_R + \psi \to \psi + \psi, \quad \nu_R + \psi \to \psi + \bar{\psi},$$

Entropy dilution

$$\mathcal{L}_6 = \frac{1}{\Lambda^2}\sum_{i,j,j'}(\bar{\psi}^c u_R^i)(\bar{d}_R^{j,c} d_R^{j'}) + \text{h.c.}$$

Bian,Liu,Xie,21

Zombie DM and CS GW



$$\Omega_{\rm GW}(f)h^2 = \frac{8\pi h^2}{3M_{\rm Pl}^2 H_0^2} G\mu^2 f \sum_{n=1}^{\infty} C_n(f) P_n$$

$$G\mu \sim \frac{v_\phi^2}{M_{\rm Pl}^2} \sim 10^{-10} \times \left(\frac{M_{\nu_R}/\lambda_R}{10^{14}\,{\rm GeV}}\right)^2$$

$$C_n = \frac{2n}{f^2}\int_0^\infty \frac{dz}{H(z)(1+z)^6} n_{\rm CS}\left(\frac{2n}{(1+z)f}, t(z)\right)$$

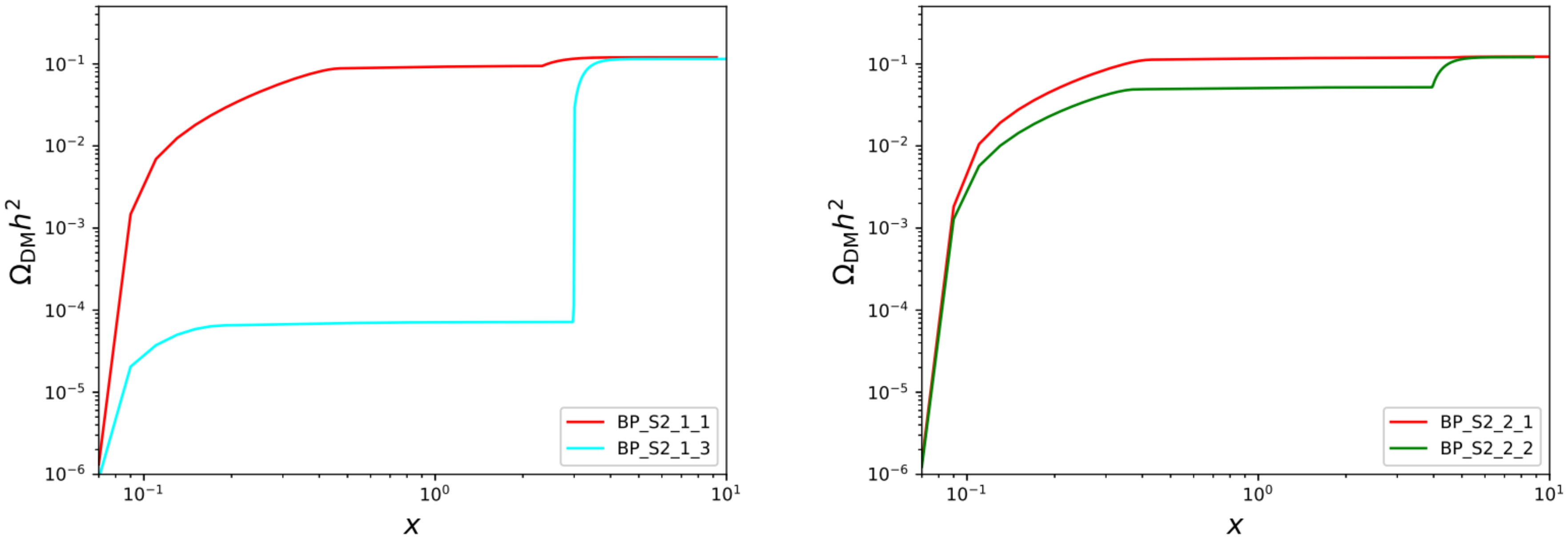
$$n_{\rm CS}^{r}(\ell,t) = \frac{0.18}{t^{3/2}(\ell+\Gamma G\mu t)^{5/2}}, \quad (\ell \leqslant 0.1\,t);$$

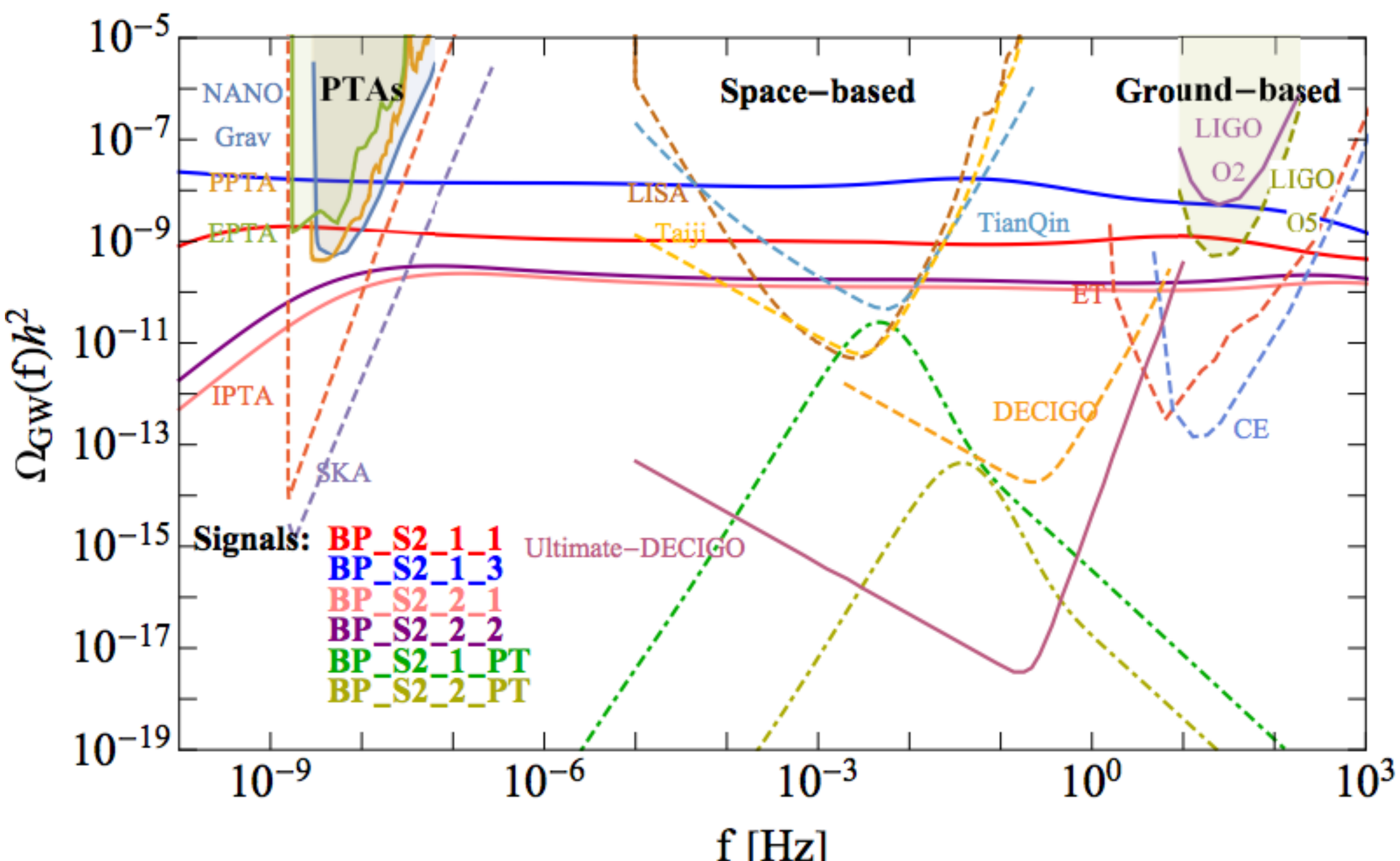
$$n_{\rm CS}^{r,m}(\ell,t) = \frac{0.18\, t_{\rm eq}^{1/2}}{t^2(\ell+\Gamma G\mu t)^{5/2}}, \quad (\ell \leqslant 0.09\, t_{\rm eq} - \Gamma G\mu t);$$

$$n_{\rm CS}^{m}(\ell,t) = \frac{0.27 - 0.45(\ell/t)^{0.31}}{t^2(\ell+\Gamma G\mu t)^2}, \quad (\ell \leqslant 0.18\, t)$$

Bian,Liu,Xie, JHEP 11 (2021) 175

FIMP DM and CS GW





Bian,Tang,Zhou,21

▸ Related topics

❖Lattice simulation

- **PT GW simulation, Electroweak sphaleron, PT dynamics**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**

❖Pheno

1. EWSB and GW from FOPT

- **Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily**

2. BAU and GW from FOPT

- **Sphaleron process, bubble dynamics**

3. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

谢谢！