quasi PDF 准部分子分布函数

LaMET 大动量有效理论

Reference: Ji PRL110, 262002 (2013) [1305.1539]

Science ..57, 1407–1412 [1404.6680]

Rev. Mod. Phys. 93, 035005 (2021) [2004.03543].

1. Introduction to PDF
2. composite operator renormalization ⟹ renormalization of PDF
3. quasi PDF. method of region
4. recent progress

I. introduction:

periodic table: proton, neutron } 99%

mass: $\hbar = c = 1$

$m_p = 938\,\mathrm{MeV}$

$m_n = 939.5\,\mathrm{MeV}$

$\mathcal{L} \sim m_q \bar{q} q$

$m_u = (2.16^{+0.49}_{-0.26})\,\mathrm{MeV}$

$m_d = (4.67^{+0.48}_{-0.17})\,\mathrm{MeV}$

$2m_u + m_d \sim 10\,\mathrm{MeV}$

spin: $S_p = \frac{1}{2}\hbar$, $(S_z)_p = \frac{1}{2}\hbar$

1988: $\langle S_z \rangle_u = 0.373 \pm 0.019 \pm 0.039$

$\langle S_z \rangle_d = -0.254 \pm 0.019 \pm \cdots$

$\langle S_z \rangle_s = -0.113 \pm \cdots$

$\langle S_z \rangle_{u+d+s} = \underline{\underline{0.006}} \pm \underline{0.058} \pm 0.017$ EMC PLB 206, 364 (1988)

<u>Charge radius</u>

Hydrogen

Muonic Hydrogen

ep. + Hydrogen spectroscopy + Muonic Hydrogen spectroscopy

$r_p = 0.84\ \mathrm{fm}$ $\not\leftrightarrow$ $r_p = 0.88\ \mathrm{fm}$

proton:

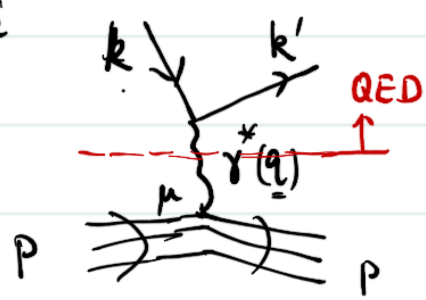
quark model

PLB 8, 214 (1964)

CERN-TH-401/412 (1964)

parton model: 1969

<u>DIS</u>



F_1 F_2

$q^2 = -Q^2$

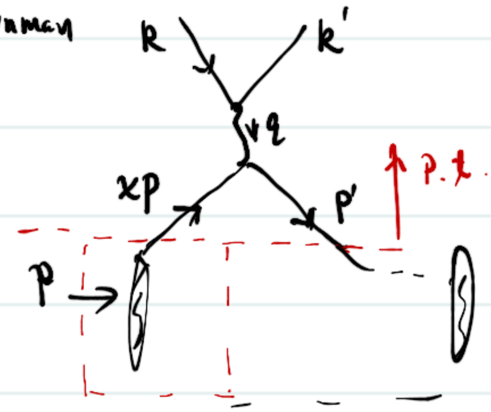
$2P\cdot q \to x = \dfrac{Q^2}{2P\cdot q}$

Borjorken scaling

$F_1(x, Q^2) \xrightarrow{Q^2\to\infty} F_1(x)$

$F_2(x, Q^2) \xrightarrow{Q^2\to\infty} F_2(x)$

1969. Feynman



$$x = \frac{Q^2}{2p\cdot q} = -\frac{q^2}{2p\cdot q}$$

$$p' = xp + q$$

$$p'^2 = \underbrace{x^2 p^2}_{0} + 2x\,p\cdot q + q^2 \sim 0$$

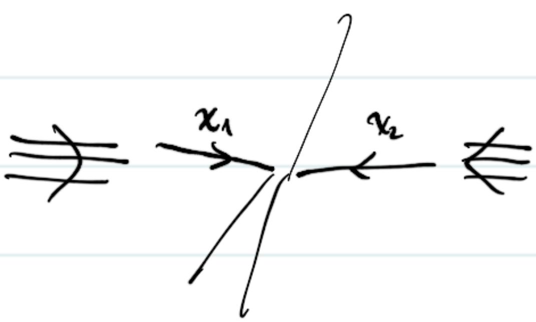
<u>Parton model</u>

$$\Rightarrow 2x\,p\cdot q + q^2 \sim 0$$

$$x = -\frac{q^2}{2p\cdot q}$$

<u>PDF</u>

Hadron + Hadron collider



$$d\sigma \sim \int dx_1\, dx_2\, \underbrace{f(x_1)}_{\text{PDF}}\, \underbrace{f(x_2)}_{\text{PDF}}\, \underline{d\hat{\sigma}(x_1, x_2, Q)}$$

<u>PDF definition:</u>

$$f(x) \sim \langle p|\, N_q \,|p\rangle \qquad q = xp$$

$$= \langle p|\, a_q^{+} a_q \,|p\rangle \sim \langle p|\, \bar{\psi}(z)\, \Gamma\, \psi(0)\, |p\rangle$$

$$f_{q/H}(x) = \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \underbrace{\langle P(p)|\, (\bar{q}\, W_c)(\xi)\, \frac{\gamma^+}{2}\, (W_c^{+} q)(0)\, |P(p)\rangle}_{\text{nonlocal}}$$

2. PDF property:

a. global fit . PRD4, 3418 (1971)

CT18 1912.10053

b. QCD: $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - g_s\bar{\psi}\not{A}\psi$

LQCD: discretized. imaginary

moments of PDF:

$\langle p|\bar{\psi}(0)\psi(0)|p\rangle$ $\quad a_0$

$\langle p|\bar{\psi}\overleftrightarrow{D}\psi(0)|p\rangle$ $\quad a_1$

$\langle p|\bar{\psi}\overleftrightarrow{D}\overleftrightarrow{D}\psi(0)|p\rangle$ $\quad a_2$

c light-front quantization.

d. composite operator renormalization: (Low energy)

$S = \bar{\psi}\psi$ $\qquad \boxed{\bar{\psi}\sigma^{\mu\nu}\psi}$

① $S^{(0)} = \bar{\psi}^{(0)}\psi^{(0)} = Z_S \cdot S$ $\qquad \psi^{(0)} = \sqrt{Z_\psi}\,\psi$

$S = \frac{1}{Z_S}S^{(0)} = \frac{1}{Z_S}\bar{\psi}^{(0)}\psi^{(0)} = \frac{Z_\psi}{Z_S}\bar{\psi}\psi$

$\langle S\rangle$ 有限的

$q(p) \to q(p)$: $\langle S\rangle$ 有限的 (UV发散)

$\langle q(p)|\, S\, |q(p)\rangle$ 有限的

g_s^0: $\underline{Z_\psi = Z_S = 1}$

$\underline{\bar u(p)\; u(p))}$

g_s^2: $S = \underline{\bar\psi\psi} + \underline{\left(\frac{Z_\psi}{Z_S}-1\right)}\, \underline{\bar\psi\psi}$

$+ \left(\frac{Z_\psi}{Z_S}-1\right)$

g_s^2

$\to \bar u u(p)$ → 有限的

$$(ig_s^0)^2 \mu^{2\varepsilon}\int \frac{d^dq}{(2\pi)^d}\,\gamma^\alpha\, \frac{i\not q}{q^2}\cdot\frac{i\not q}{q^2}\,\gamma^\beta\, \frac{-ig_{\alpha\beta}}{q^2} = -4ig_s^2 C_F\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}+\cdots$$

$$= \underline{\frac{8g_s^2C_F}{32\pi^2\varepsilon_{UV}}}+\cdots$$

$$Z_\psi = 1-2\,\frac{g_s^2 C_F}{32\pi^2\varepsilon_{UV}}$$

$$\to Z_S = 1+6\,\frac{g_s^2 C_F}{32\pi^2\varepsilon_{UV}}$$

$$\gamma_S = \mu\frac{d\ln Z_S}{d\mu} = -\frac{6g_s^2C_F}{16\pi^2}$$

$$g_s^0 = g_s\mu^\varepsilon$$

$$\downarrow\quad \frac{dg_s^0}{d\mu} = \frac{dg_s}{d\mu}\mu^\varepsilon + g_s\cdot\varepsilon\mu^{\varepsilon-1} = 0$$

e: PDF: (DGLAP)

$$O^{(0)} = Z_O\, O$$

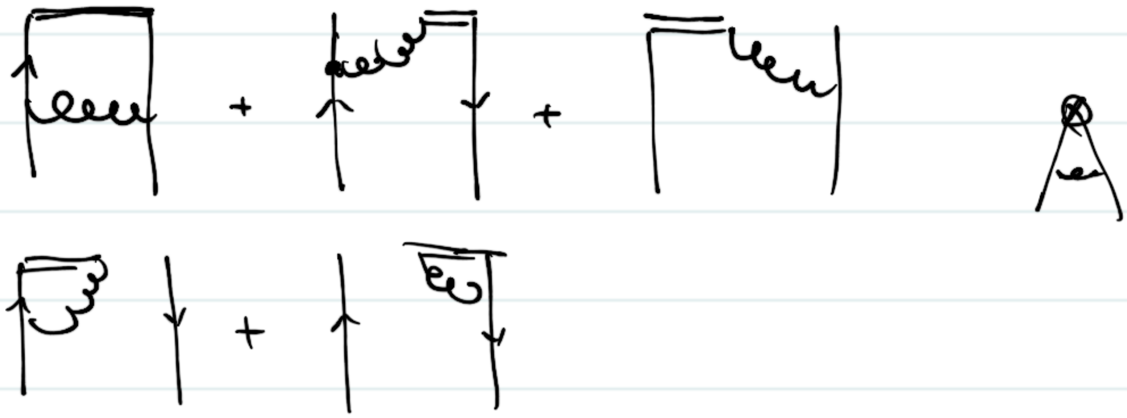
$$O = \frac{O^{(0)}}{Z_O} = \frac{(\bar\psi^{(0)}\, W\, \psi^{(0)})}{Z_O} = \frac{Z_\psi}{Z_O}\,(\bar\psi W\psi)$$

$$f(x)=\int\frac{d\xi^-}{2\pi}\, e^{-ixp^+\xi^-}\langle P(p)|\,\bar q\, W_c(\xi)\,\frac{\not n_+}{2}\,W_c\, q_c(0)\,|P(p)\rangle$$

$$W_c(\xi)=P\, e^{-ig_s\int_0^\infty ds\, n_+\cdot A^a(\xi+sn_+)\,T^a}$$

tree level:

1-loop:



$$f_{q/q_0}(x)=\frac{g_s^2 C_F}{8\pi^2}\left[\frac{1+x^2}{1-x}\left(\frac{1}{\varepsilon}-\ln\frac{-p^2x(1-x)}{\mu^2}\right)+(x-2)\right]_+$$

PDF (微扰) DGLAP

a. 光锥坐标系

$$n_{+\mu}=\frac{1}{\sqrt2}(1,0,0,1)\qquad n_{-\mu}=\frac{1}{\sqrt2}(1,0,0,-1)$$

$$n_+\cdot n_-=1\qquad n_\pm^2=0$$

$$a_\mu=n_-a\,n_{+\mu}+a_{\perp\mu}+n_+a\,n_{-\mu}$$

$$=a^-n_{+\mu}+a_{\perp\mu}+a^+n_{-\mu}$$

$$a^+=\frac{a^0+a^3}{\sqrt2}\qquad a^-=\frac{a^0-a^3}{\sqrt2}$$

$$d^4k = dk^- dk^- d^2k_\perp \qquad \mu^{4-d} d^dk = \mu^{4-d} dk^+ dk^- d^{d-2}k_\perp$$

b. PDF:

$$f_{q/H}(x) = \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \langle p(p)| (\bar q W_c)(\xi) \frac{\not n_+}{2} (W_c^+ q)(0) |p(p)\rangle$$

$$W_c(\xi) = P\exp\left[-ig_s\int_0^\infty ds\, n_+ A^a(\xi+sn_+) T^a\right]$$

$$\hat O_H = T\left[\hat O_I e^{i\int d^4x H_I(x)}\right] \qquad H_I(x) = \int d^4x\, g_s \bar\psi \not A \psi(x)$$

c. perturbative property: $|p(p)\rangle \to |q(p)\rangle$

树图: $W_c = 1$ $\quad \hat O_H = \hat O_I$

$$f_{q/H}(x) = \int\frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-}\langle q(p)|\bar q(\xi)\frac{\not n_+}{2} q(0)|q(p)\rangle$$

$$= \frac{1}{2}\sum_s\int\frac{d\xi^-}{2\pi} e^{ip^+\xi^- - ixp^+\xi^-}\,\bar u \frac{\not n_+}{2}u(p) \qquad \boxed{\sum_s u\bar u = \not p}$$

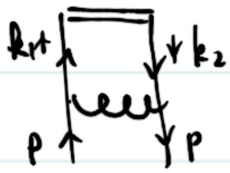
$$= \frac{1}{4}\delta(p^+ - xp^+)\, tr[\not n_+ \not p]$$

$$= \delta(1-x)$$ ①

单圈(g_s^2):

消失 $n_+^2=0$

DR + p^2 offshell: $\frac{1}{\epsilon}$ UV 发散 : $\log(p^2)$ 红外发散



(a)

$$f^{(a)} = \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \langle q(p)|\, i g_s \int d^4\eta_2\, \bar q(\eta_2) \not{A}^a(\eta_2) T^a q(\eta_2)$$

$$\times\ \bar q(\xi^-) \frac{\gamma^+}{2} q(0)$$

$$\times\ i g_s \int d^4\eta_1\, \bar q(\eta_1) \not{A}^b(\eta_1) T^b q(\eta_1)\, |q(p)\rangle$$

$$= -\frac{g_s^2 C_F}{2}\sum_s \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \int d^4\eta_1 \int d^4\eta_2$$

$$\times\ \bar u(p) e^{ip\eta_2} \gamma^\mu \int \frac{d^4k_2}{(2\pi)^4} \frac{i\not{k}_2}{k_2^2} e^{-i(\eta_2-\xi)k_2} \frac{\gamma^+}{2}$$

$$\times \int \frac{d^4k_1}{(2\pi)^4} \frac{i\not{k}_1}{k_1^2} e^{-i(0-\eta_1)k_1} \gamma^\nu \int \frac{d^4q}{(2\pi)^4} \frac{-ig_{\mu\nu}}{q^2} e^{-i(\eta_2-\eta_1)q} u(p) e^{-ip\eta_1}$$

$$= -ig_s^2 C_F \frac{1}{2}\sum \int \frac{d^4q}{(2\pi)^4} \frac{\bar u(p)\gamma^\mu(\not{p}-\not{q})\gamma_+(\not{p}-\not{q})\gamma_\mu u(p)}{2[(p-q)^2]^2 q^2}\, \delta(p^+ - xp^+ - q^+)$$

$$\frac{1}{2}\sum_s u(p,s)\bar u(p,s) = \not{p}$$

$$= \frac{g_s^2 C_F}{8\pi^2}(1-x)\left[\frac{1}{\epsilon} - \ln\frac{-p^2x(1-x)}{\mu^2} + \frac{2-x}{x-1}\right] \quad ③$$

$\frac{1}{\epsilon}$: $\frac{1}{\epsilon} - \gamma + \ln 4\pi$ 紫外发散

$\ln(-p^2)$: 红外发散

其余+自能图:

$$f(x) = \frac{g_s^2 C_F}{8\pi^2}\left[\frac{1+x^2}{1-x}\left(\frac{1}{\epsilon} - \ln\frac{-p^2x(1-x)}{\mu^2}\right) + (x-2)\right]_+$$

$$[g(x)]_+ = g(x) - \delta(1-x)\int_0^1 dy\, g(y)$$

引入复合算符重整化:

$$f^{r,1}(x) = \frac{g_s^2 C_F}{8\pi^2}\left[-\frac{1+x^2}{1-x}\ln\left(\frac{-p^2x(1-x)}{\mu^2}\right) + (x-2)\right]_+ \quad ②$$

DGLAP: $\mu \frac{d}{d\mu} f^r(x,\mu) = \frac{\alpha_s}{\pi}\int \frac{dy}{y} P\left(\frac{x}{y}\right) \cdot f^r(y,\mu)$

将①与②代入到 DGLAP evolution,

$$P(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$
$$= C_F\left[\frac{1+z^2}{1-z}\right]_+$$

$$\left(\frac{1+z^2}{1-z}\right)_+ = \frac{1+z^2}{1-z} - \delta(1-z)\int dy\, \frac{1+y^2}{1-y} \quad \downarrow$$
$$\frac{1+z^2}{(1-z)_+} = \frac{1+z^2}{1-z} - \delta(1-z)\int dy\, \frac{2}{1-y}$$

$$\int dy\, \frac{1}{1-y}(1+y^2-2) = \int dy\, \frac{(-1)(1+y)(1-y)}{1-y} = (-1)\cdot\int_0^1 dy\,(1+y) = -\frac{3}{2}$$

quasi-PDF: (LaMET)

qPDF def: $n_{0\mu} = (1,0,0,0)$

$n_{3\mu} = (0,0,0,1)$

$$\tilde{f}(x) = \int \frac{d\xi^3}{2\pi} e^{ixP^3\xi^3} \langle P(p)|(\bar{q} W_c)(\xi)\left\{\frac{\gamma_3}{2}, \frac{\gamma_0}{2}\right\} W_c^\dagger q(0)|P(p)\rangle$$

- time-independent
- Spatial separated (z)
- $\frac{\gamma_3}{2}, \frac{\gamma_0}{2}$ ✓ Lattice QCD 离散化

树图: $\tilde{f}(x) = \frac{p^0}{p^3}\delta(x-1)$

单圈(a)图: $\tilde{f} = \frac{g_s^2 C_F}{8\pi^2}\begin{cases} (x-1)\ln\frac{x-1}{x} + 1 & x>1 \\ (x-1)\left(\ln\frac{-p^2/p_3^2}{4} + 2\right) & x\in(0,1) \\ (1-x)\ln\frac{x-1}{x} - 1 & x<0 \end{cases}$

- $\ln(-p^2)$ 红外发散:
- No UV divergence
- 红外发散: qPDF = PDF

- 因子化: $\tilde{f}_{q/q_0}(x, \mu^2, P_z) = \int_0^1 \frac{dy}{y}\, C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) f_{q/q_0}(y, \mu^2)$

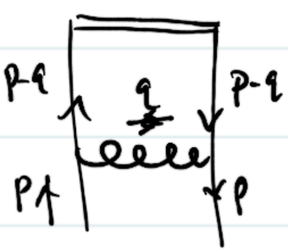
  $\tilde{f}_{q/q_0}$: quasi PDF; C: UV; f_{q/q_0}: PDF

comment: $x>1$ → 存在其它部分子 $x<0$ → 1966. Weinberg suppressed

$x=1$ → $x=0$ zero-mode

$x\in(0,1)$ ✓

method of region



qPDF: leading power contibution

$$\tilde{f} = (D-2)\, i g_s^2 C_F \mu^{2\epsilon} \frac{1}{2} \int \frac{d\xi^3}{2\pi} e^{i(xp^3-p^3+q^3)} \int \frac{d^dq}{(2\pi)^d} \frac{\mathrm{tr}[\not p(\not p-\not q)\gamma_0(\not p-\not q)]}{2[(p-q)^2]^2 q^2}$$

leading region: hard

collinear

soft ✗

collinear mode: ∘ δ function: $\int \frac{d\xi^3}{2\pi} e^{i(xp^3-p^3+q^3)\xi^3} = \delta(q^3 + xp^3 - p^3)$

$$= \delta\left(\frac{q^+-q^-}{\sqrt2} + x\frac{p^+-p^-}{\sqrt2} - \frac{p^+-p^-}{\sqrt2}\right)$$

$\underline{\text{hard } q^+ \sim q^-}$

$$\underset{p^- \ll p^+}{\overset{q^- \ll q^+}{=}} \sqrt{2}\,\delta(q^+ + x p^+ - p^+)$$

$$\circ\ \mathrm{tr}\left[\not{p}\,(\not{p}-\not{k})\,\gamma_0\,(\not{p}-\not{k})\right]$$

$$= \mathrm{tr}\left[\underline{\not{p}}\,(\not{p}-\not{k})\,\frac{\gamma_+ + \gamma_-}{\sqrt{2}}\,(\not{p}-\not{k})\right]$$

$$= \underline{\frac{1}{\sqrt{2}}}\,\mathrm{tr}\left[\not{p}\,(\not{p}-\not{k})\,\gamma_+\,(\not{p}-\not{k})\right]$$

$$\underline{\text{quasi-PDF}}\ \tilde{f} = \underset{\underline{\text{hard}}}{\tilde{f}} + \underset{\underline{\text{collinear}}}{\tilde{f}} = \underset{\text{hard} \equiv C}{\tilde{f}} + \underline{f\ (\text{PDF})}$$

$$= \underline{C \otimes f}$$