The Very Early Universe:

Primordial Gravitational Waves & CMB Physics I

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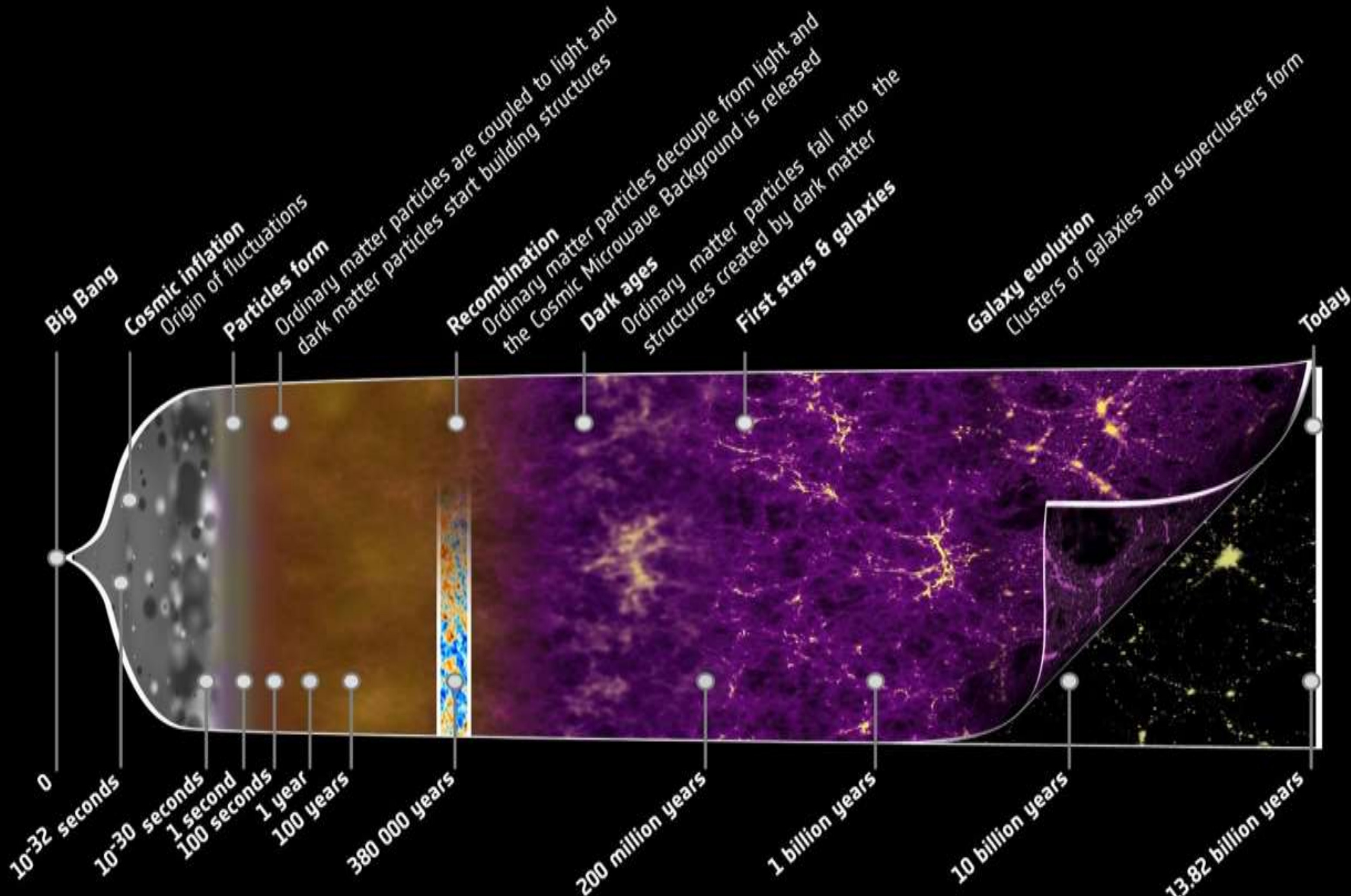
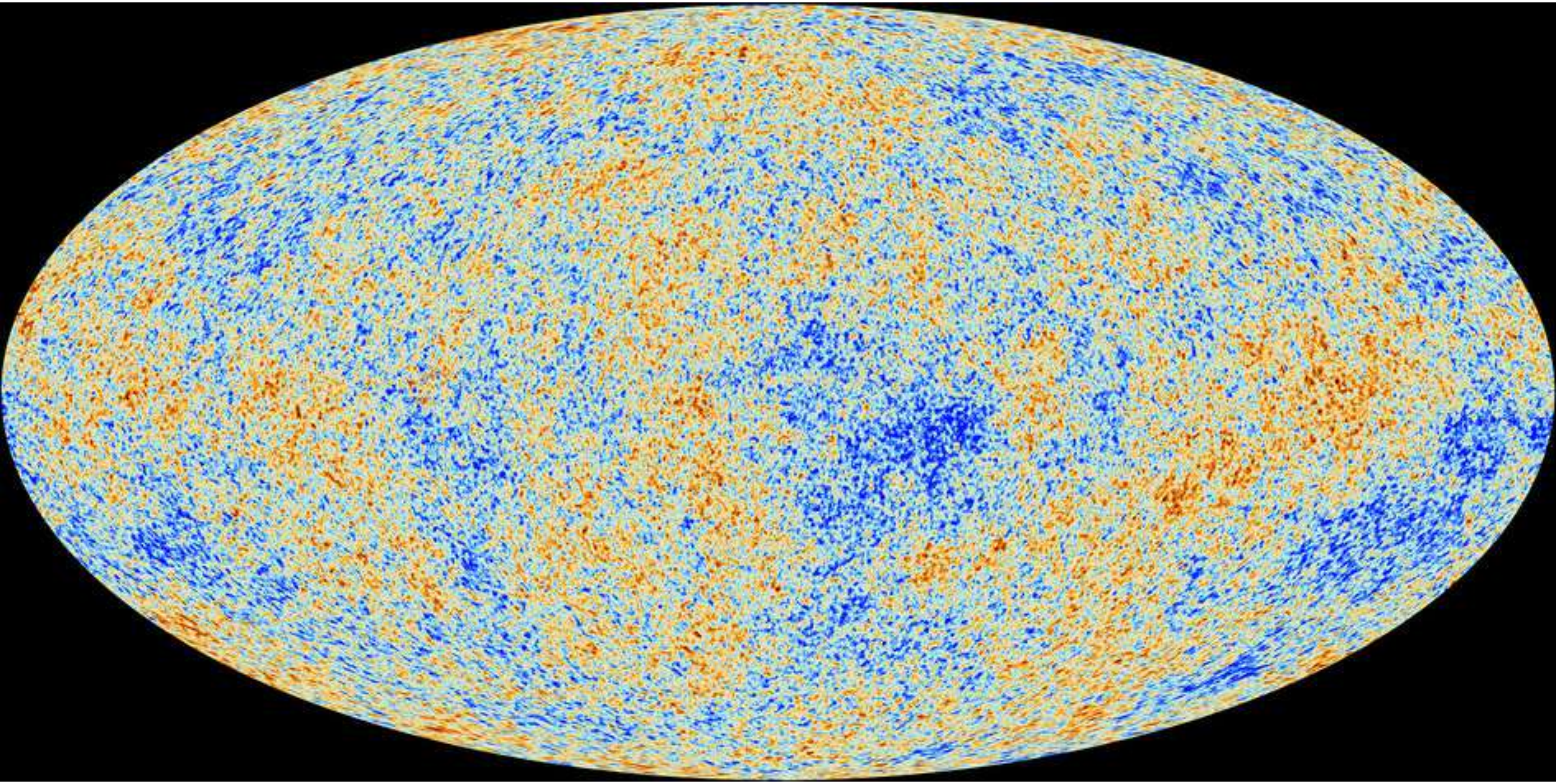
hot Big Bang & Inflation(Guth, Starobinsky, Sato, Fang, 1980s)

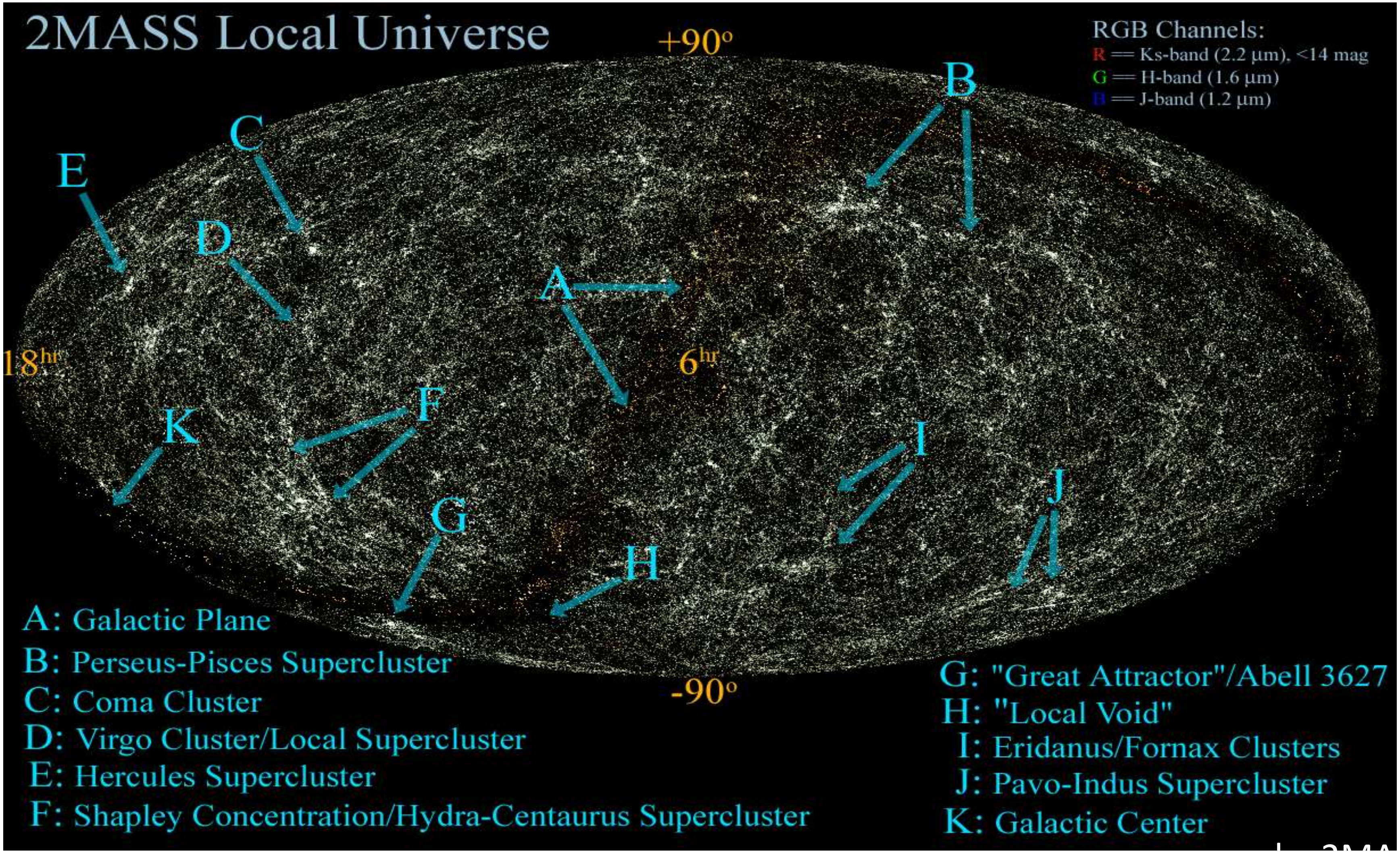


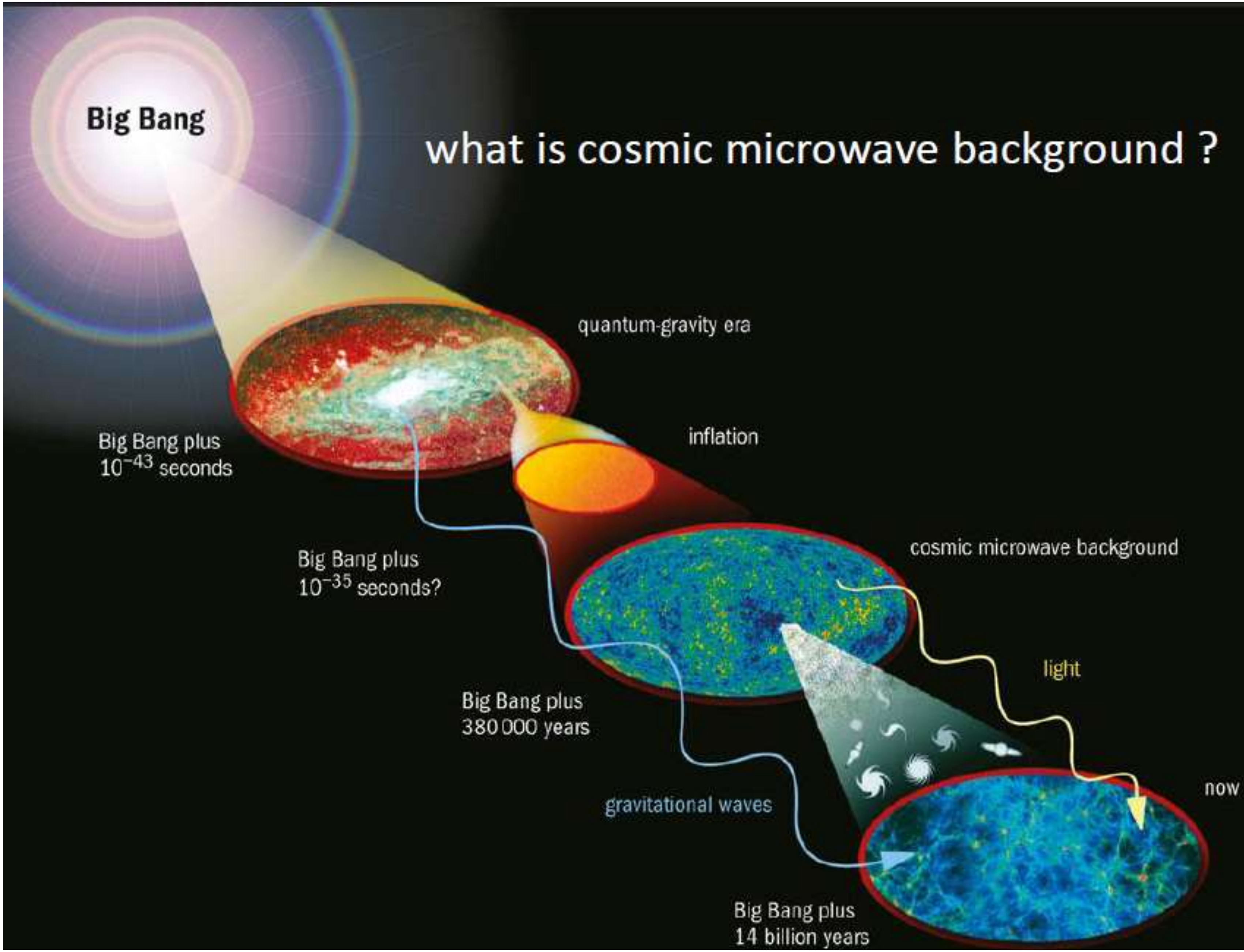
by Planck/ESA

Cosmic Microwave Background (CMB) Anisotropies



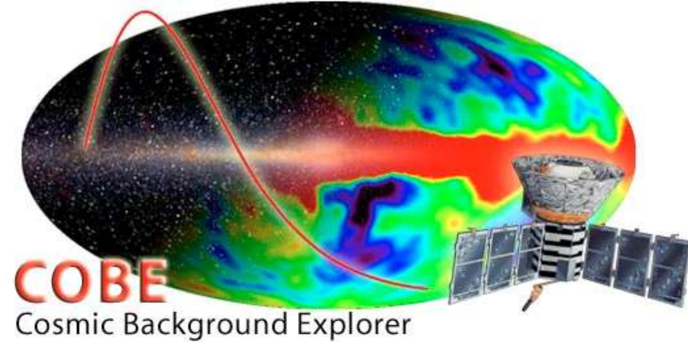
Large Scale Structure (LSS) Survey





what is cosmic microwave background ?
Big Bang
quantum-gravity era
Big Bang plus
10^{-43} seconds
inflation
Big Bang plus
10^{-35} seconds?
cosmic microwave background
light
Big Bang plus
380 000 years
gravitational waves
now
Big Bang plus
14 billion years

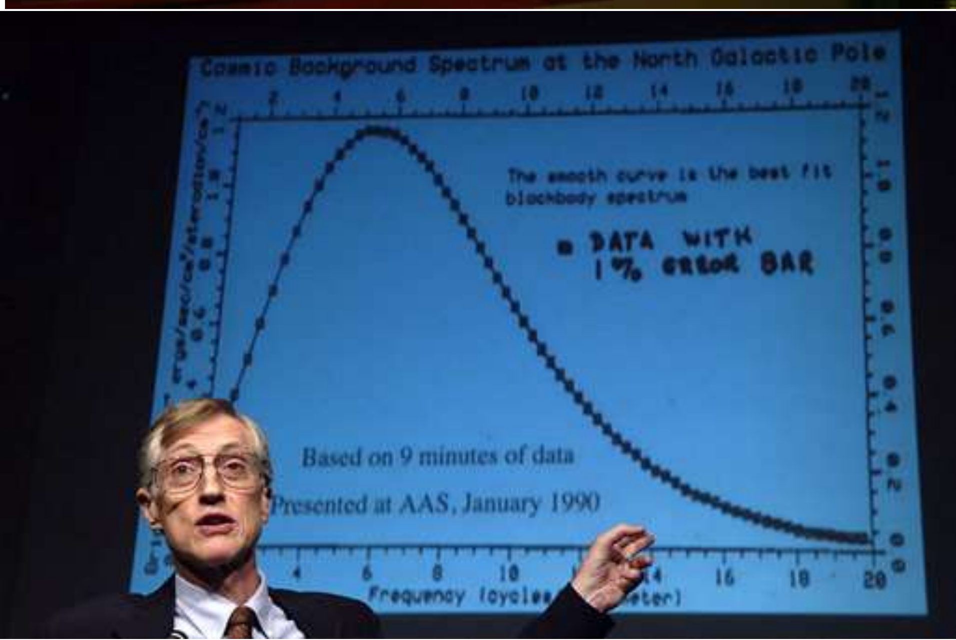
The Nobel Prize in Physics 2006

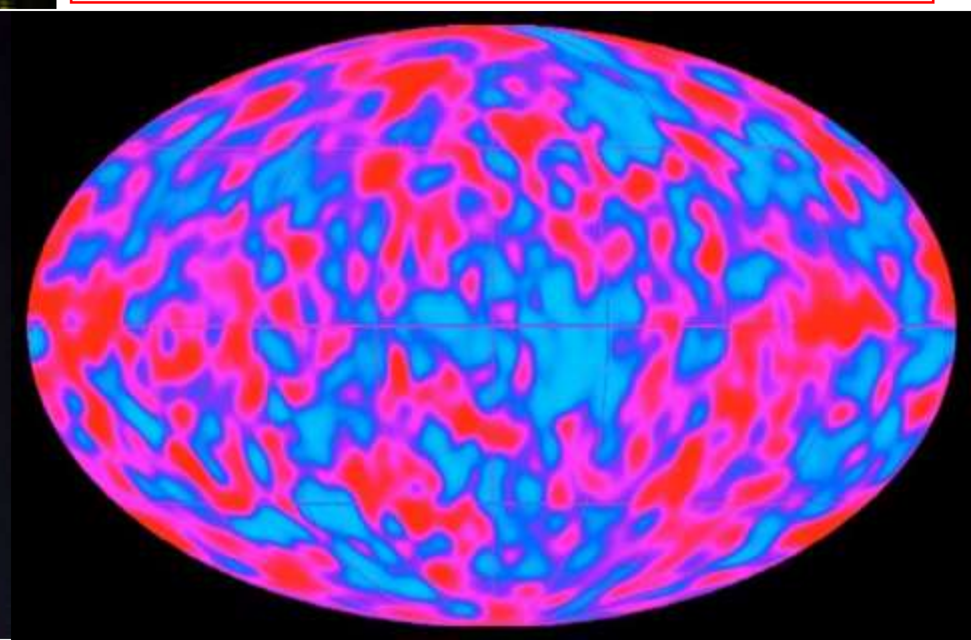


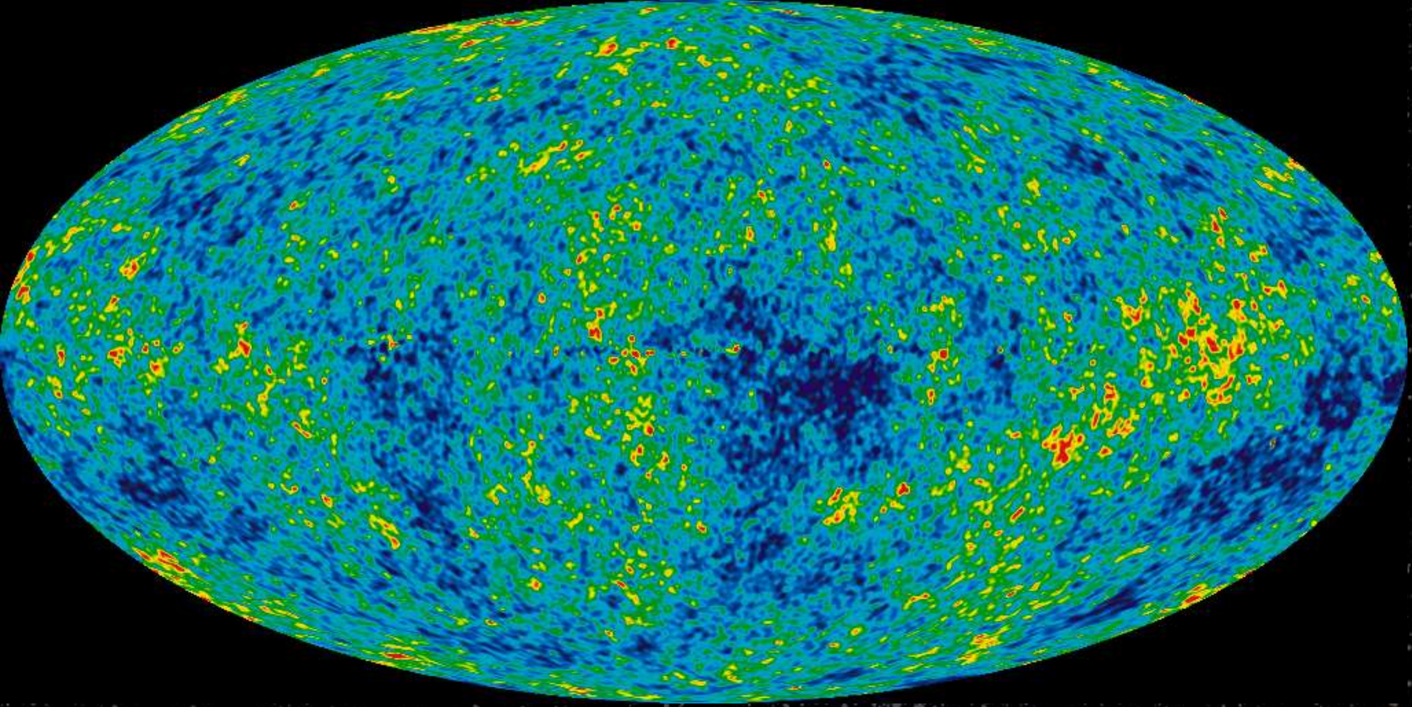
John C. Mather

George F. Smoot

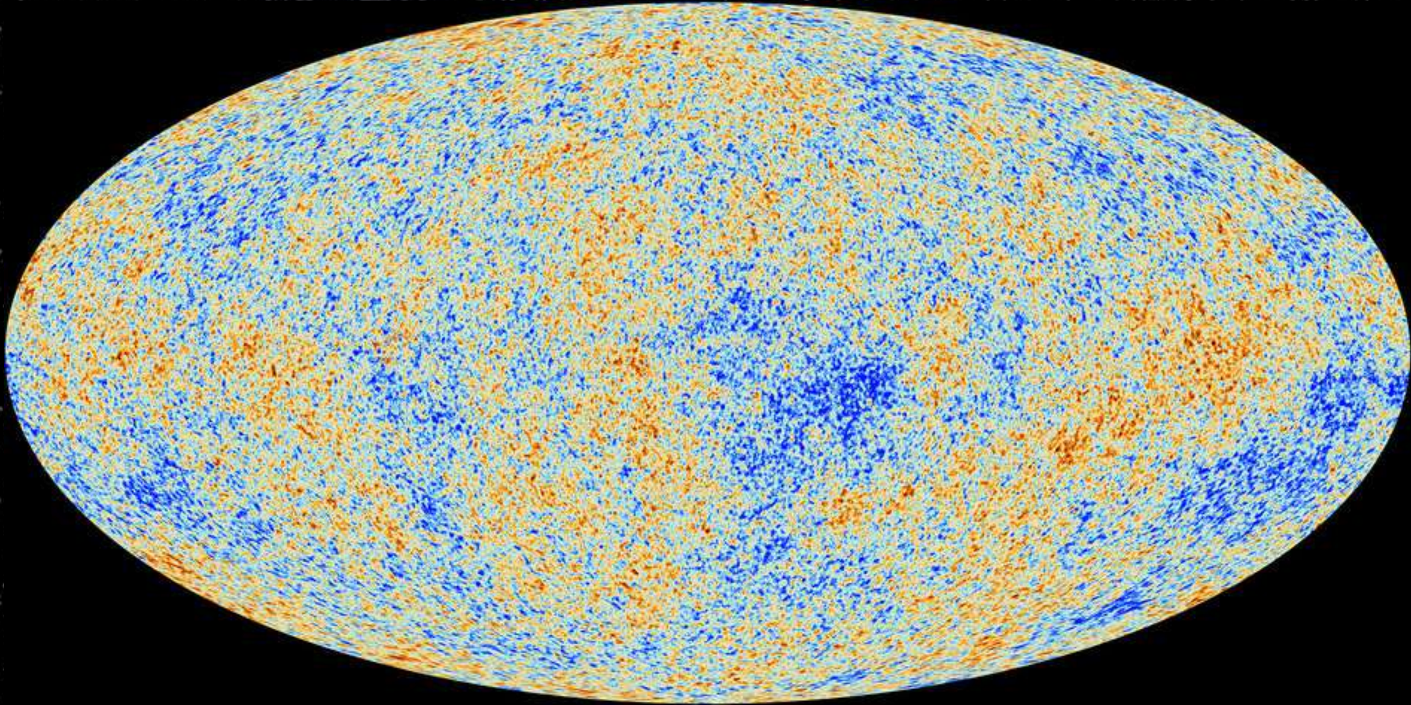
"for their discovery of the ***blackbody form*** and ***anisotropy*** of the cosmic microwave background radiation"



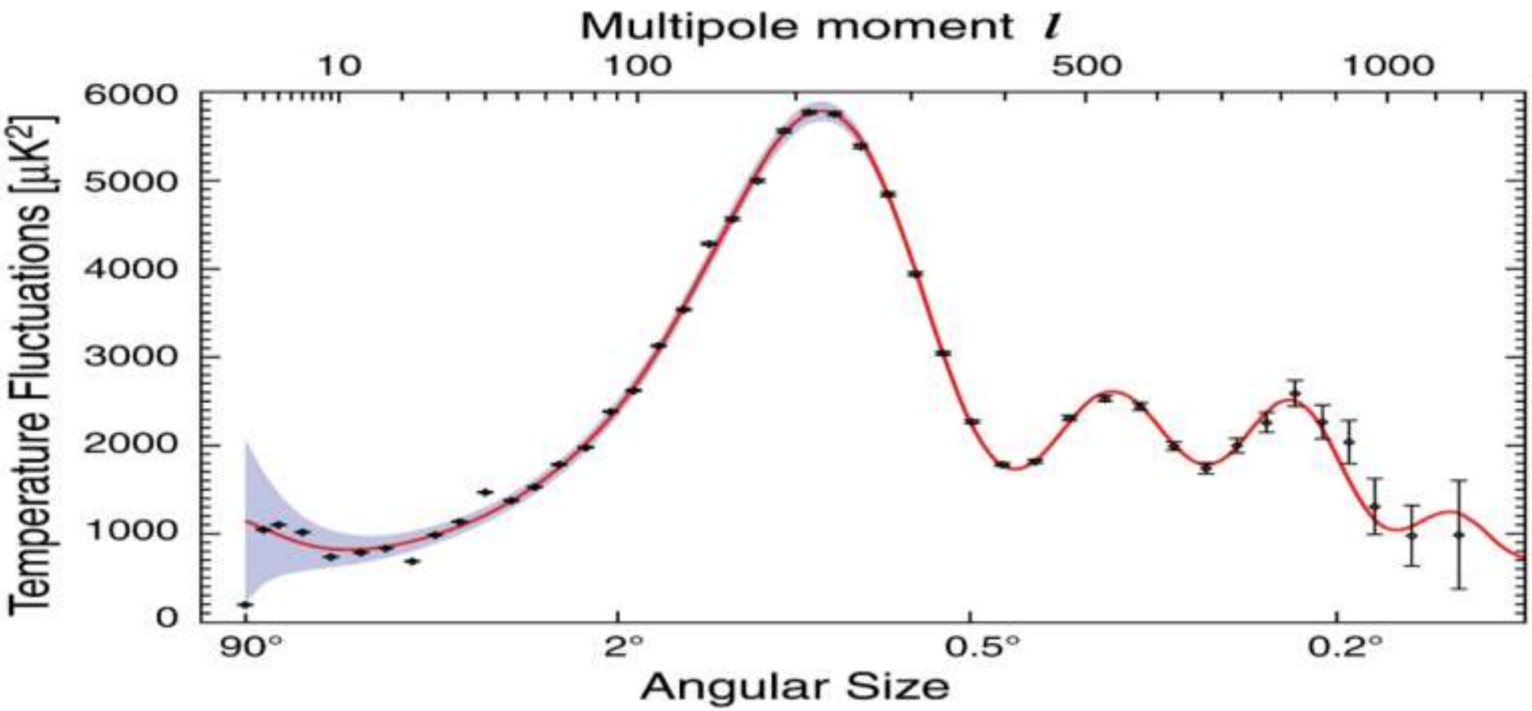


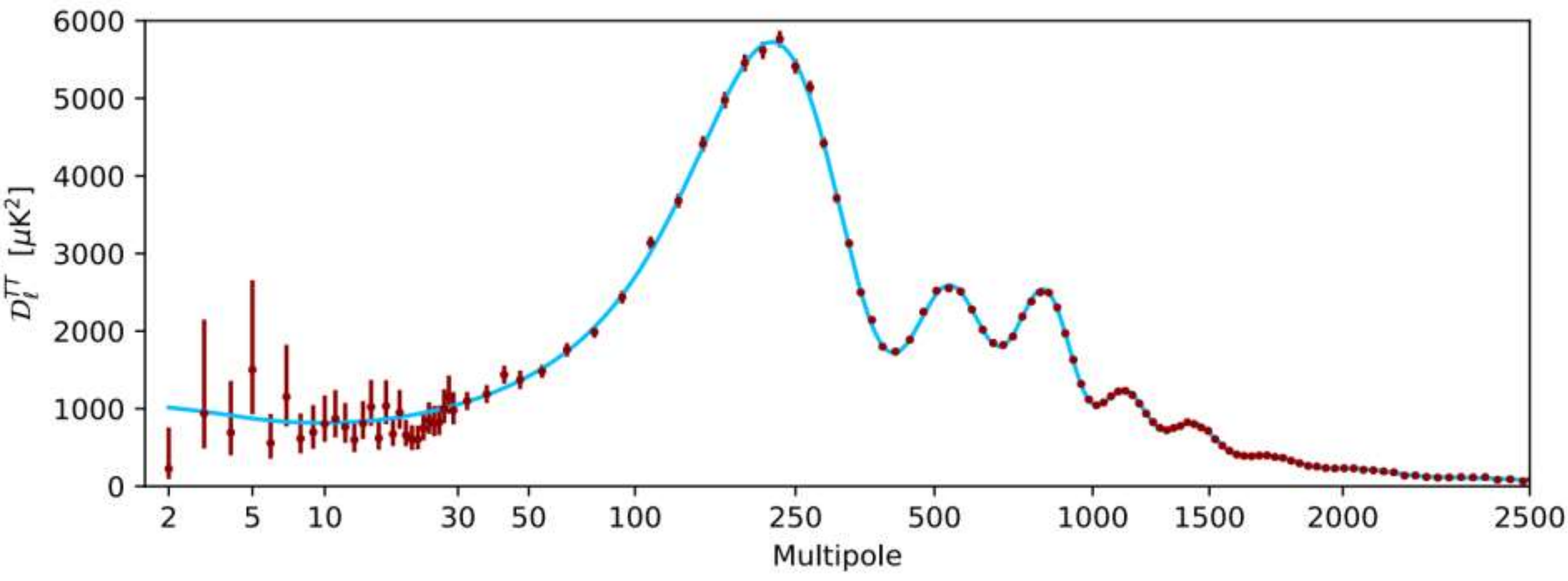


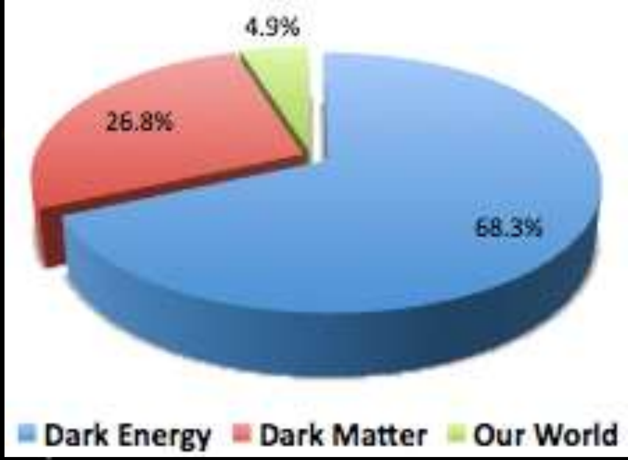
WMAP/NASA



Planck/ESA


Multipole moment l
10
100
500
1000
Temperature Fluctuations [μK²]
6000
5000
4000
3000
2000
1000
0
90°
2°
0.5°
0.2°
Angular Size


$\mathcal{D}_\ell^{TT}$ [μK^2]
6000
5000
4000
3000
2000
1000
0
2
5
10
30
50
100
250
500
1000
1500
2000
2500
Multipole



Observational Facts:

- Our universe has a thermal expanding history with 13.8 billion years

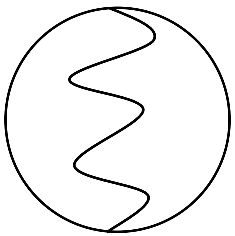
- The background looks the same at anywhere on sufficiently large scales

- Galaxies and clusters are basic blocks to form the LSS

CMB is super important to the study of cosmology

Primordial origin of temperature fluctuations

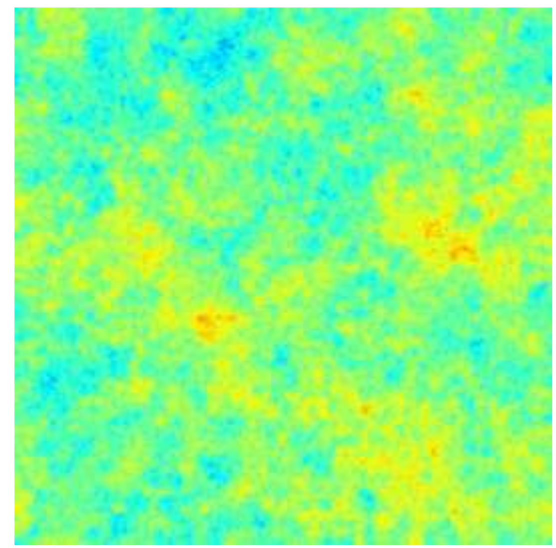
A first glance at perturbation theory in inflation



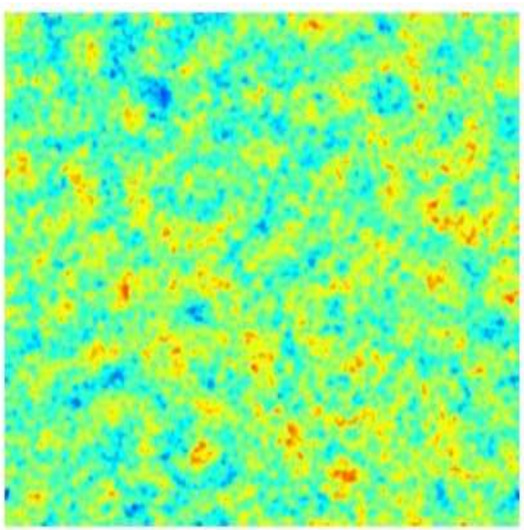
Quantum fluctuations
"waves in a pocket universe"

Inflation
$>10^{30}$ times bigger

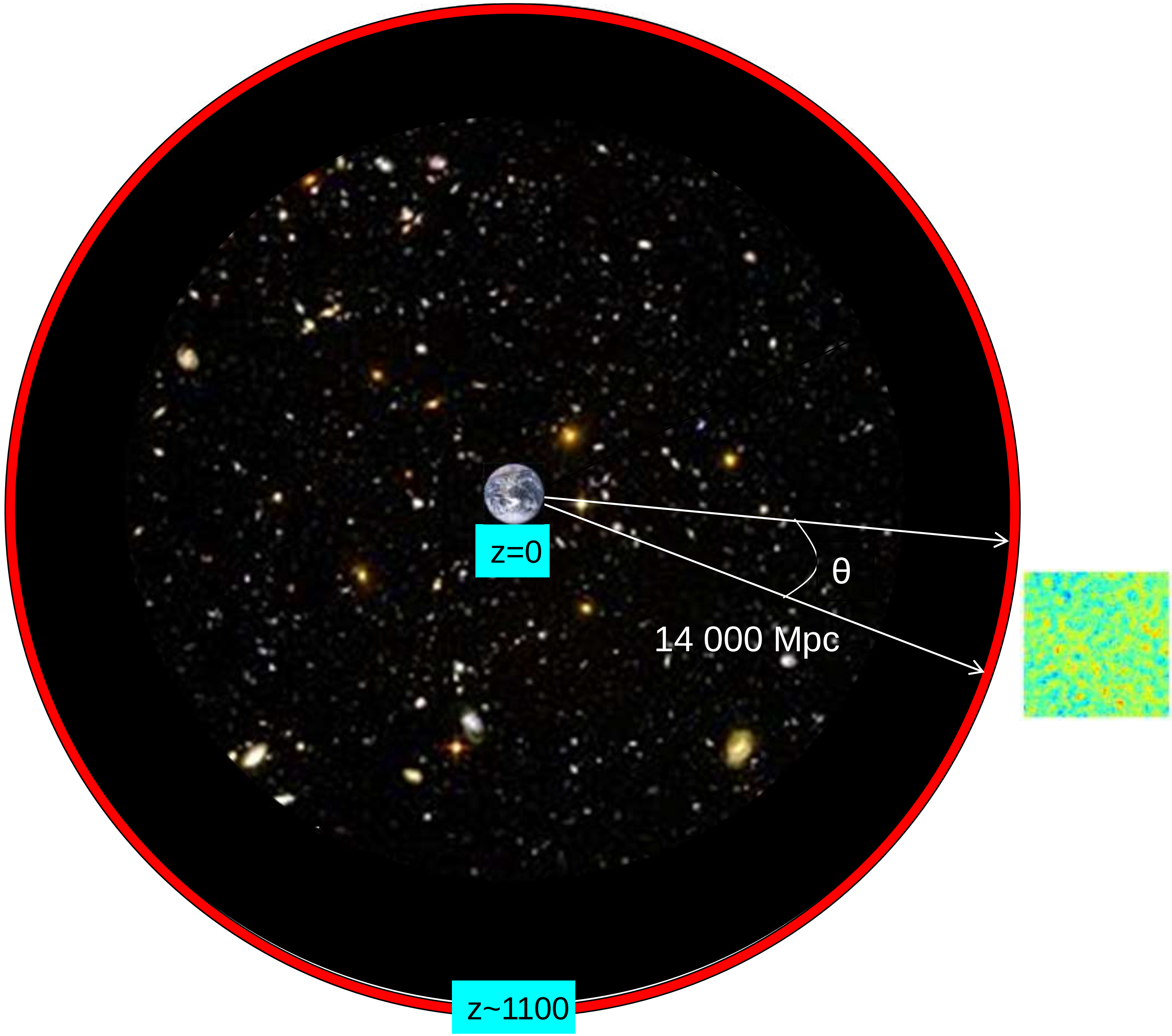
End of inflation
Huge size, amplitude $\sim 10^{-5}$



+ gravity
+ pressure
+ diffusion

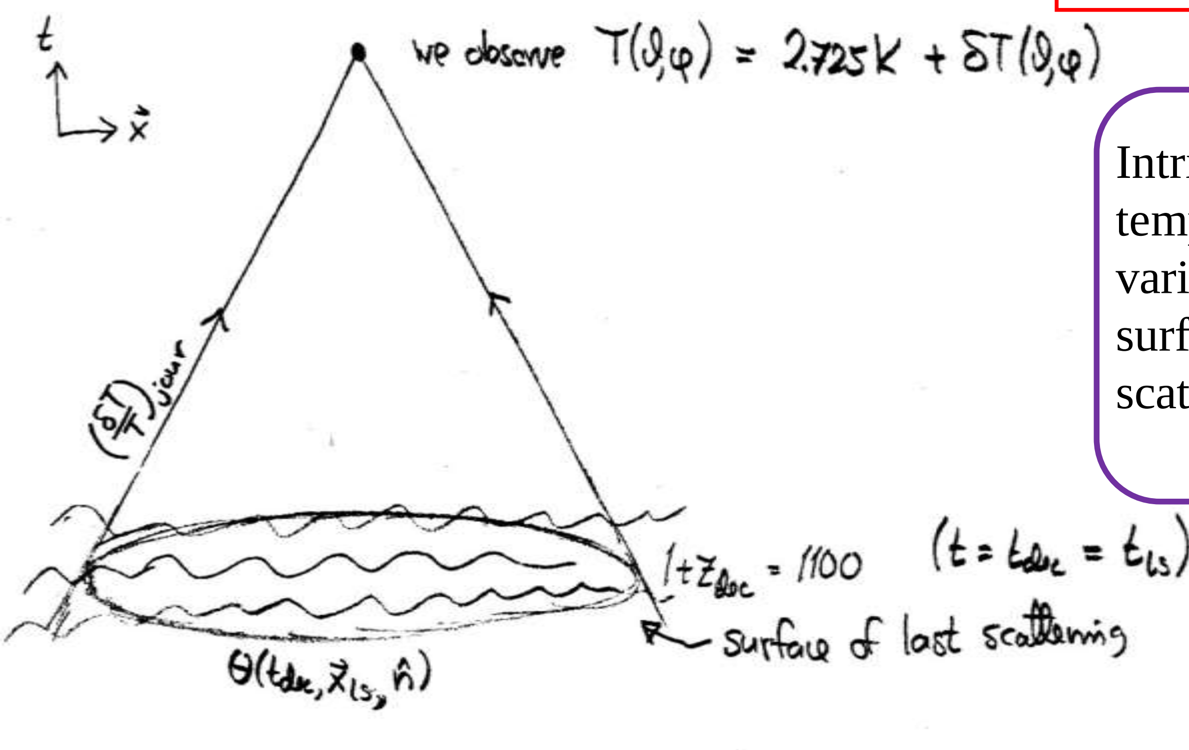


Last scattering surface


z=0
θ
14 000 Mpc
z~1100

Temperature Anisotropy

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \left(\frac{\delta T}{T}\right)_{\rm intr} + \left(\frac{\delta T}{T}\right)_{\rm jour}$$



Intrinsic temperature variation at the surface of last scattering

A variation in the redshift the photons have suffered during their "journey" to us

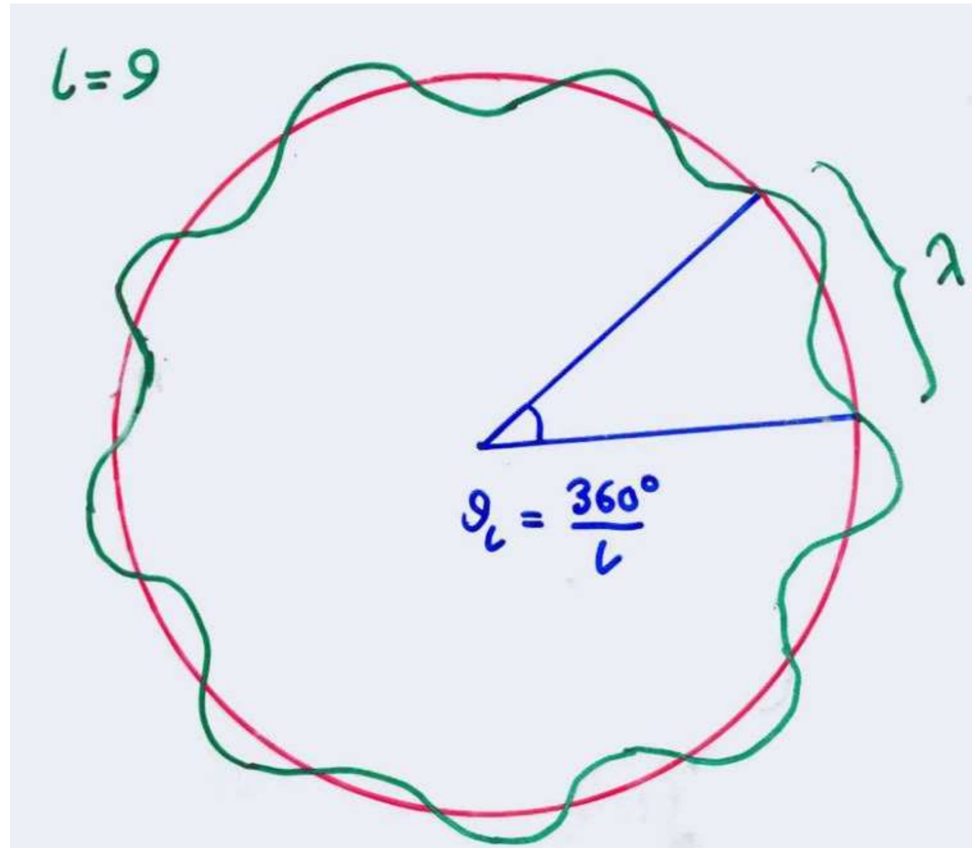
From: Hannu

Multipole Analysis and Angles

$$\frac{\delta T}{T_0}(\theta,\phi) = \sum a_{\ell m} Y_{\ell m}(\theta,\phi)$$

Angular power spectrum:

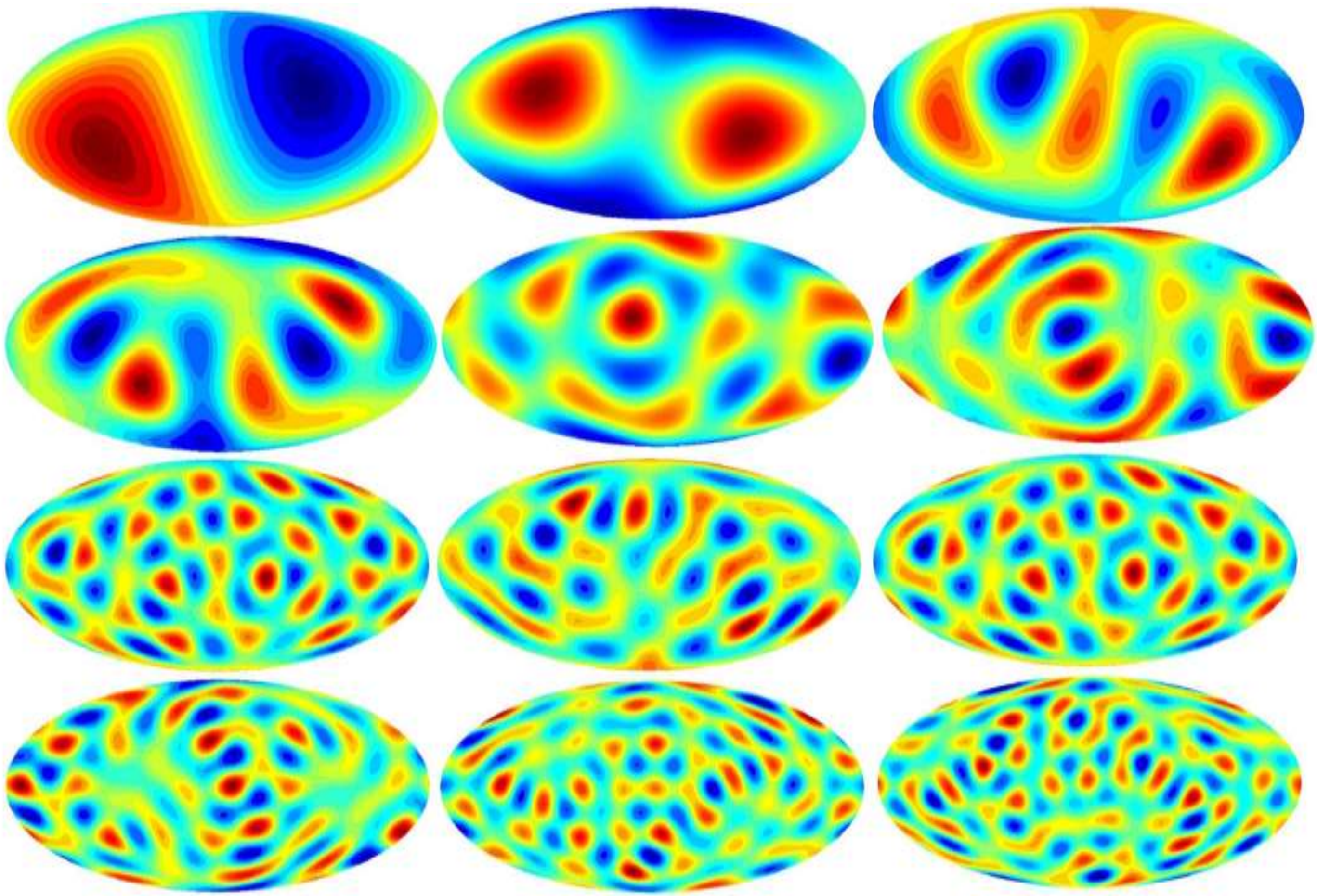
$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \frac{1}{2\ell+1}\sum_m \langle |a_{\ell m}|^2 \rangle$$



From: Hannu

Multipole Analysis and Angles

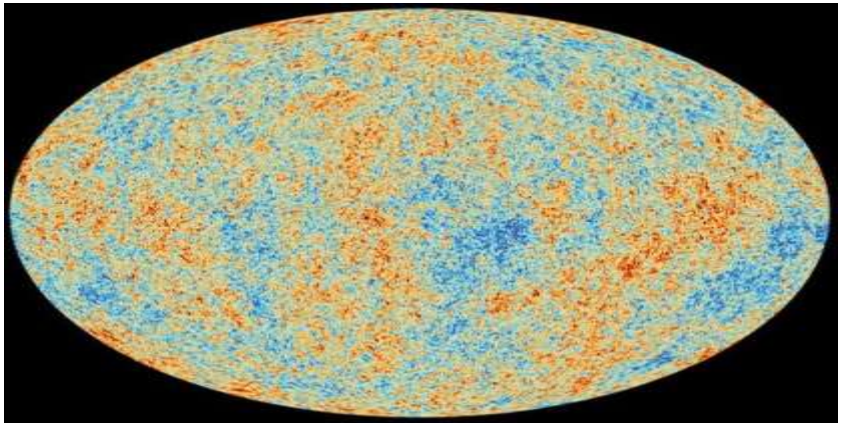
$l = 1 \sim 12.$

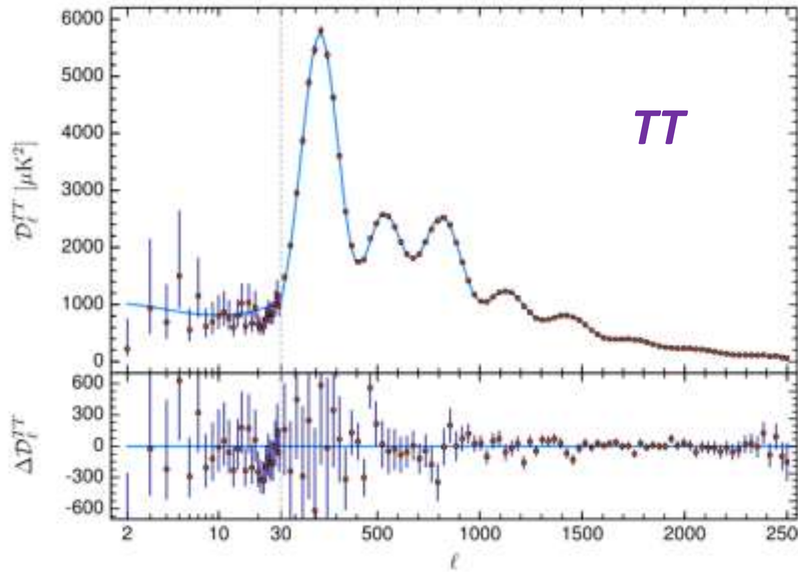


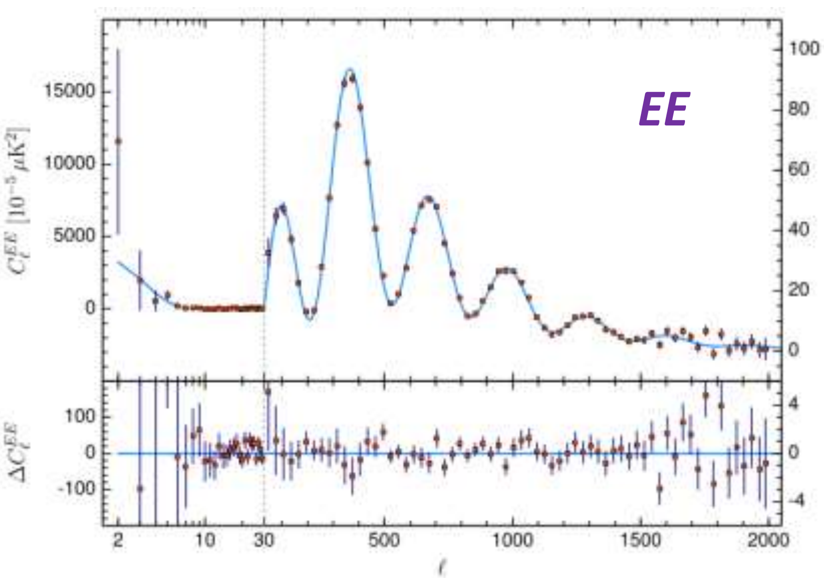
From: Hannu

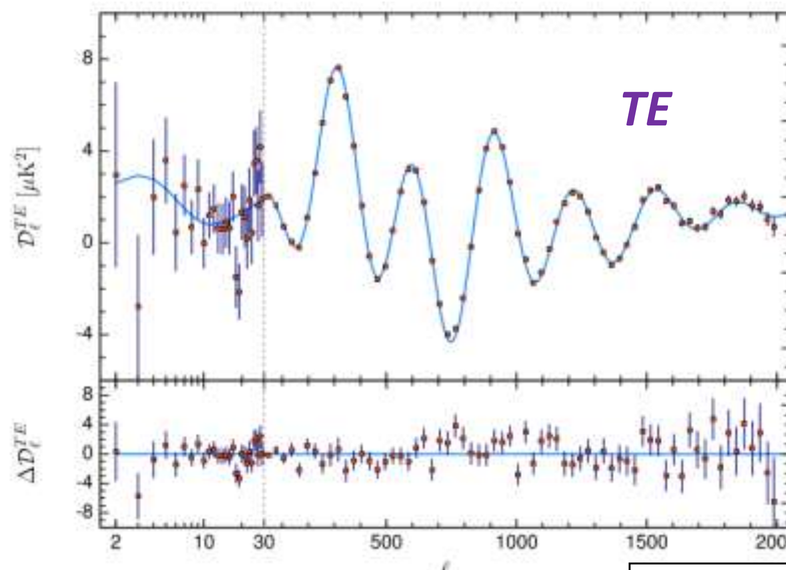
A novel lesson from Planck

CMB maps from Planck 2018





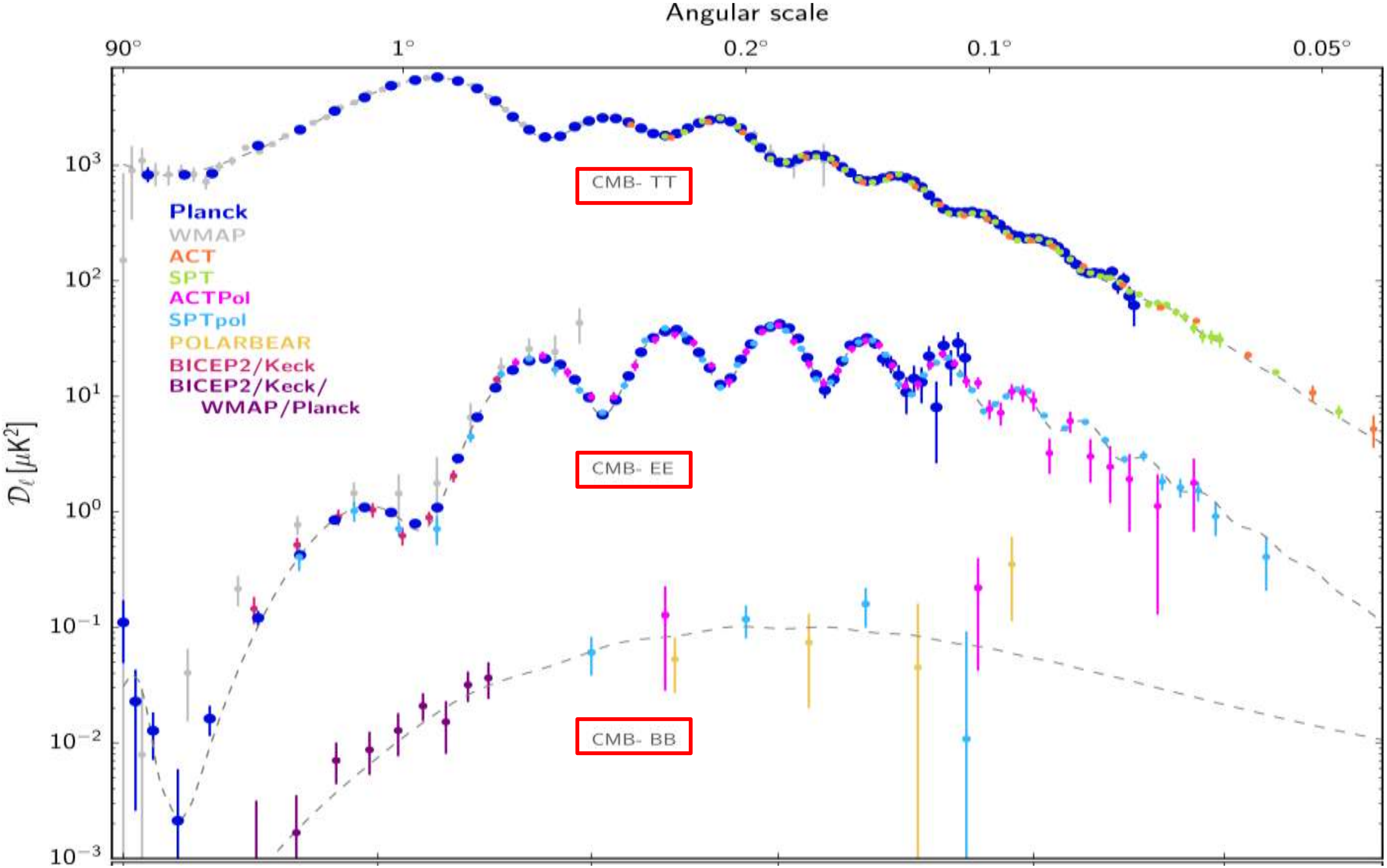




1807.06211

What can we learn from Planck 2018

Temperature, E and B mode power spectrum:

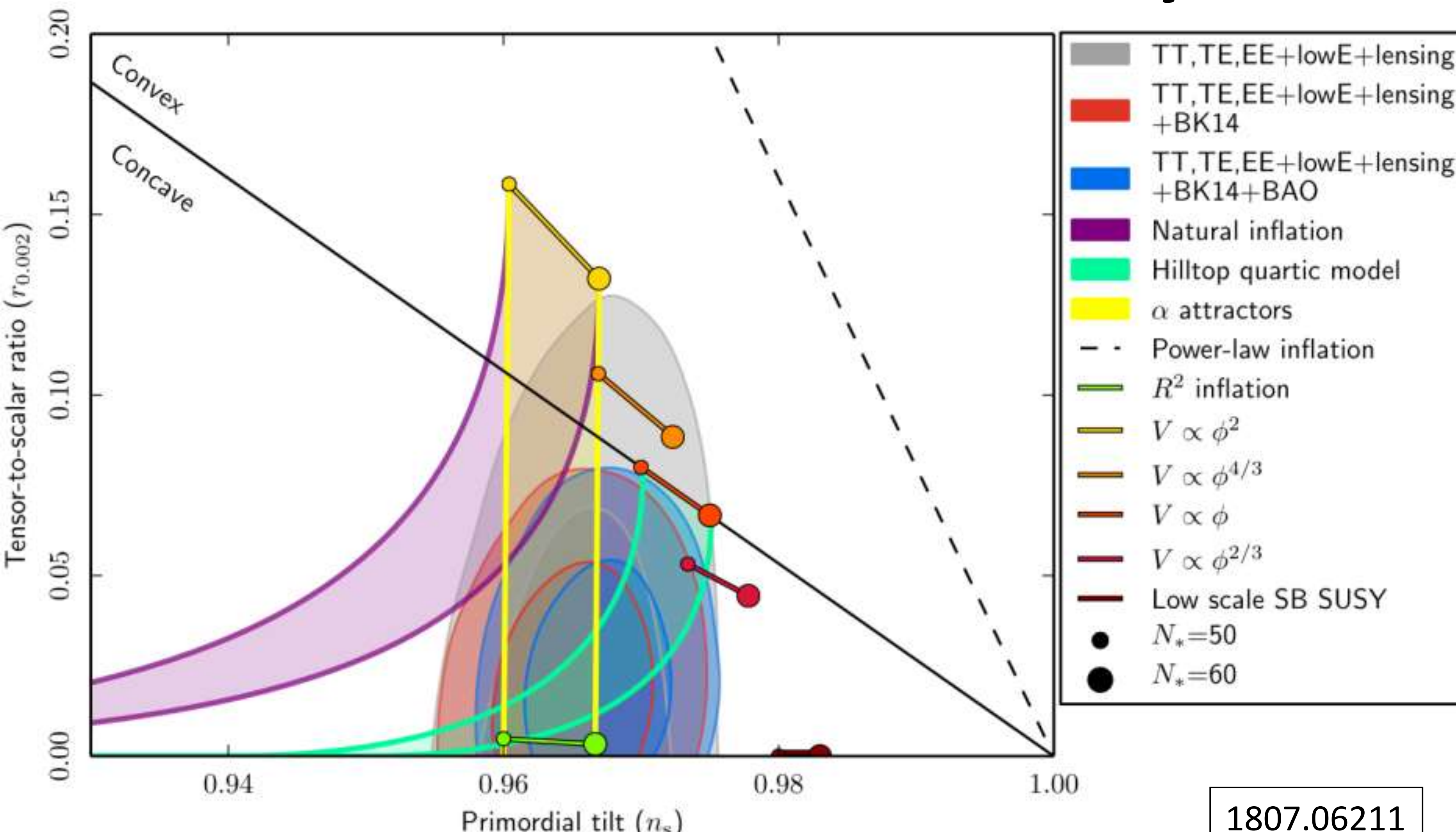


Concordance model: inflationary LCDM

Description	Symbol	Value
Physical baryon density parameter	$\Omega_b h^2$	0.02230 ± 0.00014
Physical dark matter density parameter	$\Omega_c h^2$	0.1188 ± 0.0010
Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
Scalar spectral index	n_s	0.9667 ± 0.0040
Curvature fluctuation amplitude, $k_0 = 0.002\ \mathrm{Mpc}^{-1}$	Δ_R^2	$2.441^{+0.088}_{-0.092} \times 10^{-9}$
Reionization optical depth	τ	0.066 ± 0.012

- 7 peaks in 2013, 19 peaks in 2015;
- LCDM is perfect in explaining three CMB maps from l =30 until l=2000;
- A nearly scale-invariant, adiabatic, Gaussian power spectrum of primordial fluctuations as predicted by inflation seems highly consistent with data.

Concordance model: inflationary LCDM



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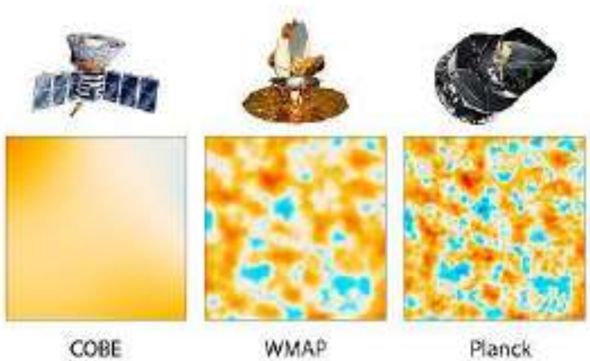
Planck 2018 data severely constrains the parameter space of inflationary cosmology.

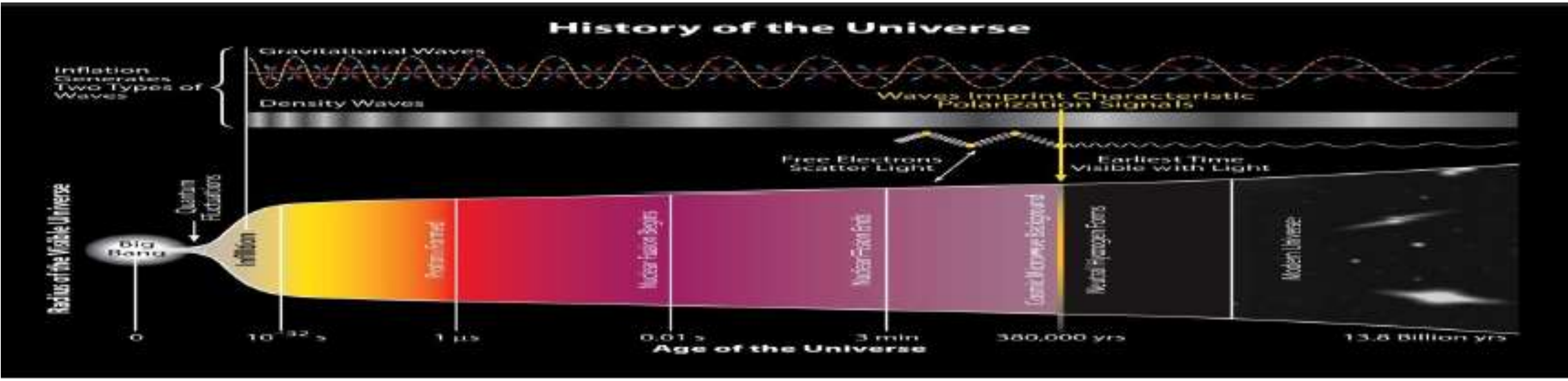
CMB leads to the precision cosmology

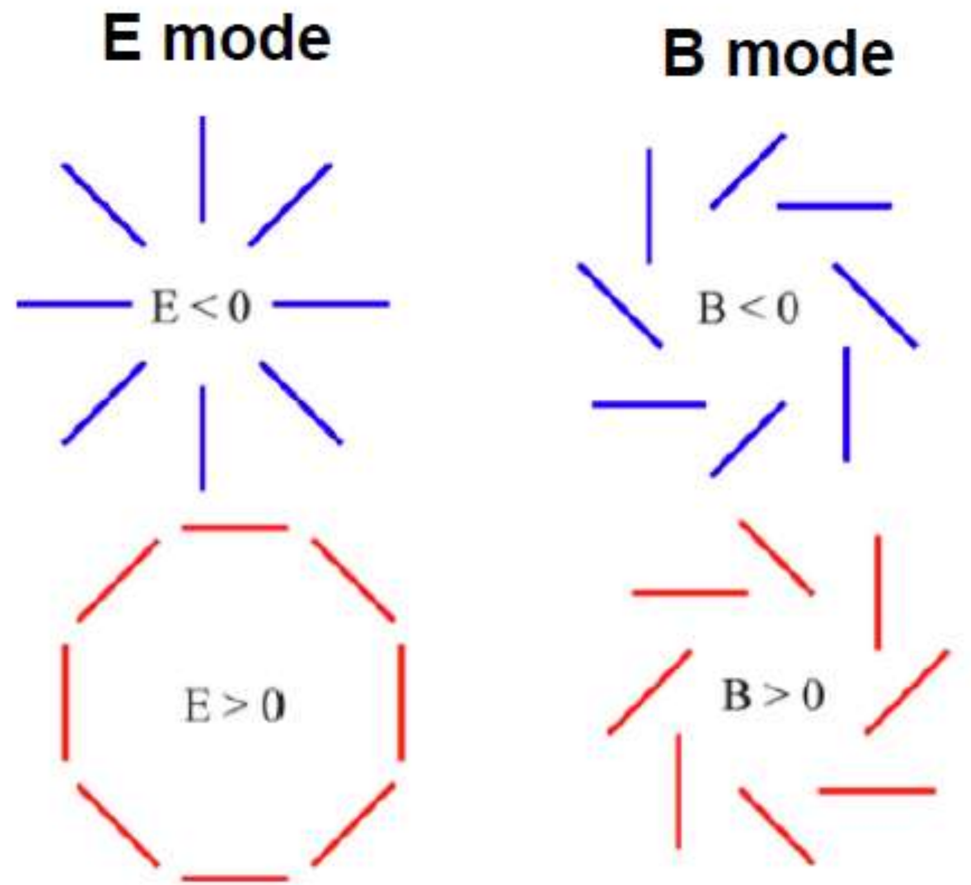


- 1998, cosmic acceleration: top ten breakthrough of 《science》
- 2003, CMB Involved in top ten breakthrough
- 1978 and 2006, Nobel prize
- 2011, cosmic acceleration win Nobel
- 2010 and 2012, WMAP win Shao's prize and Gruber prize

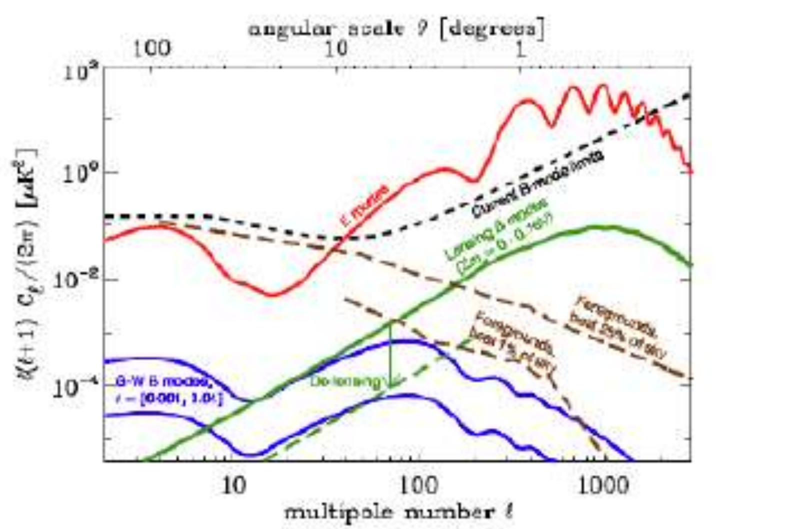
SPT BICEP/ keck Array ACT

CMB polarization is significant in next generation measurements

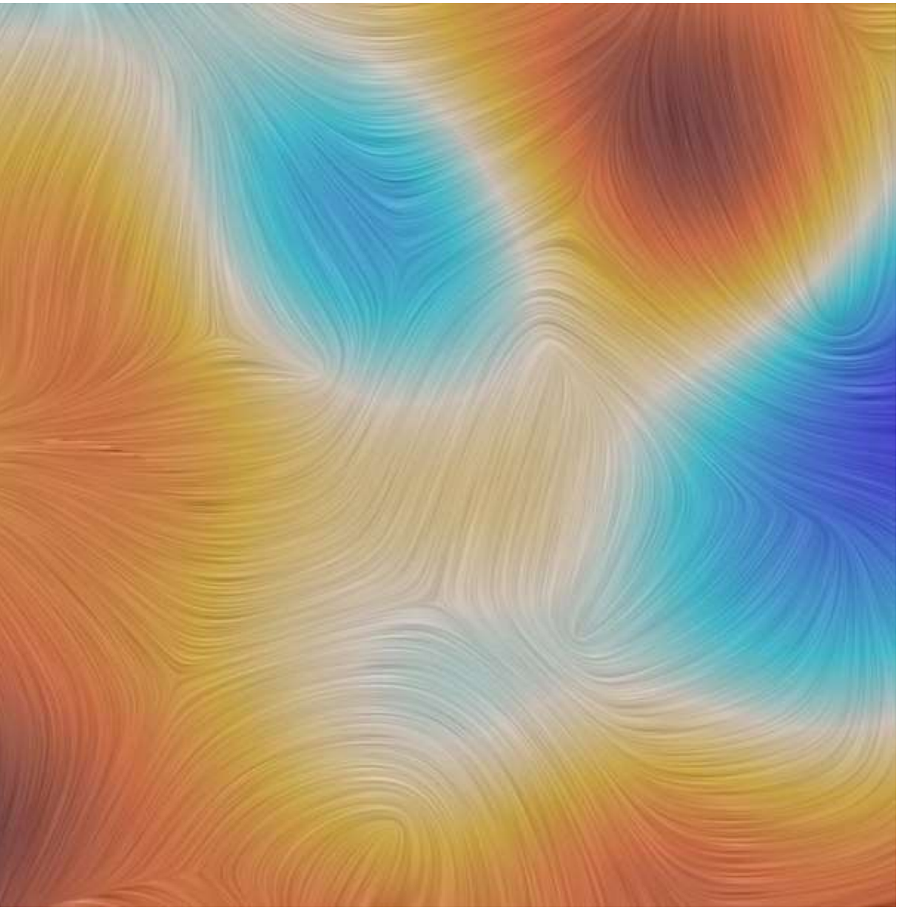


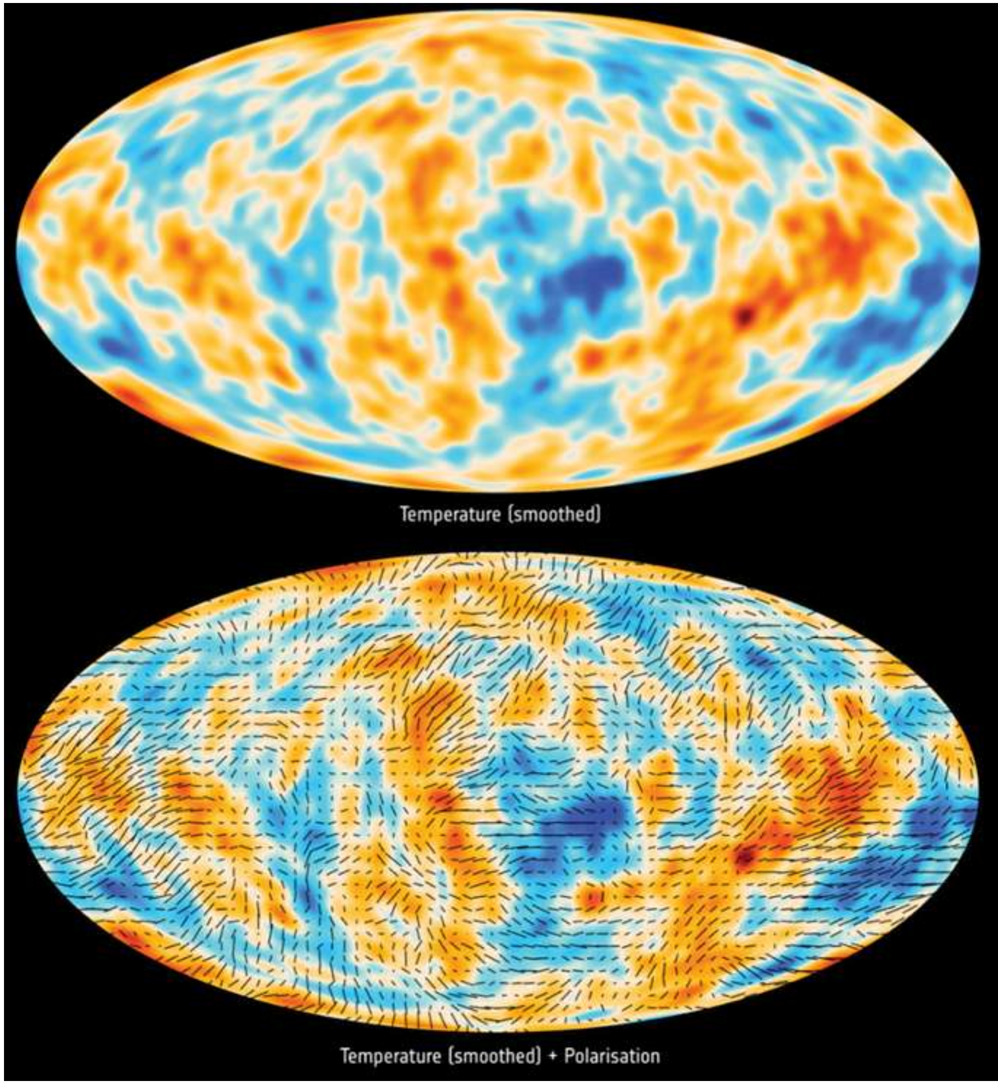


B mode survey can discriminate early universe models, namely, inflation; as well as testing the fundamental symmetries, such as CPT.



CMB Polarization





by Planck/ESA

Shocking (but old) news !

The Nobel Prize in Physics 2017
Rainer Weiss, Barry C. Barish, Kip S. Thorne

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The Nobel Prize in Physics 2017

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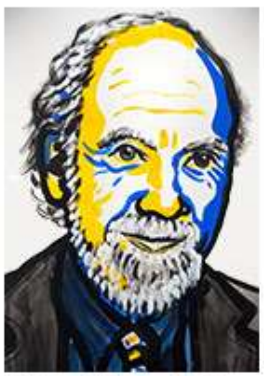
Rainer Weiss
Prize share: 1/2



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Barry C. Barish
Prize share: 1/4

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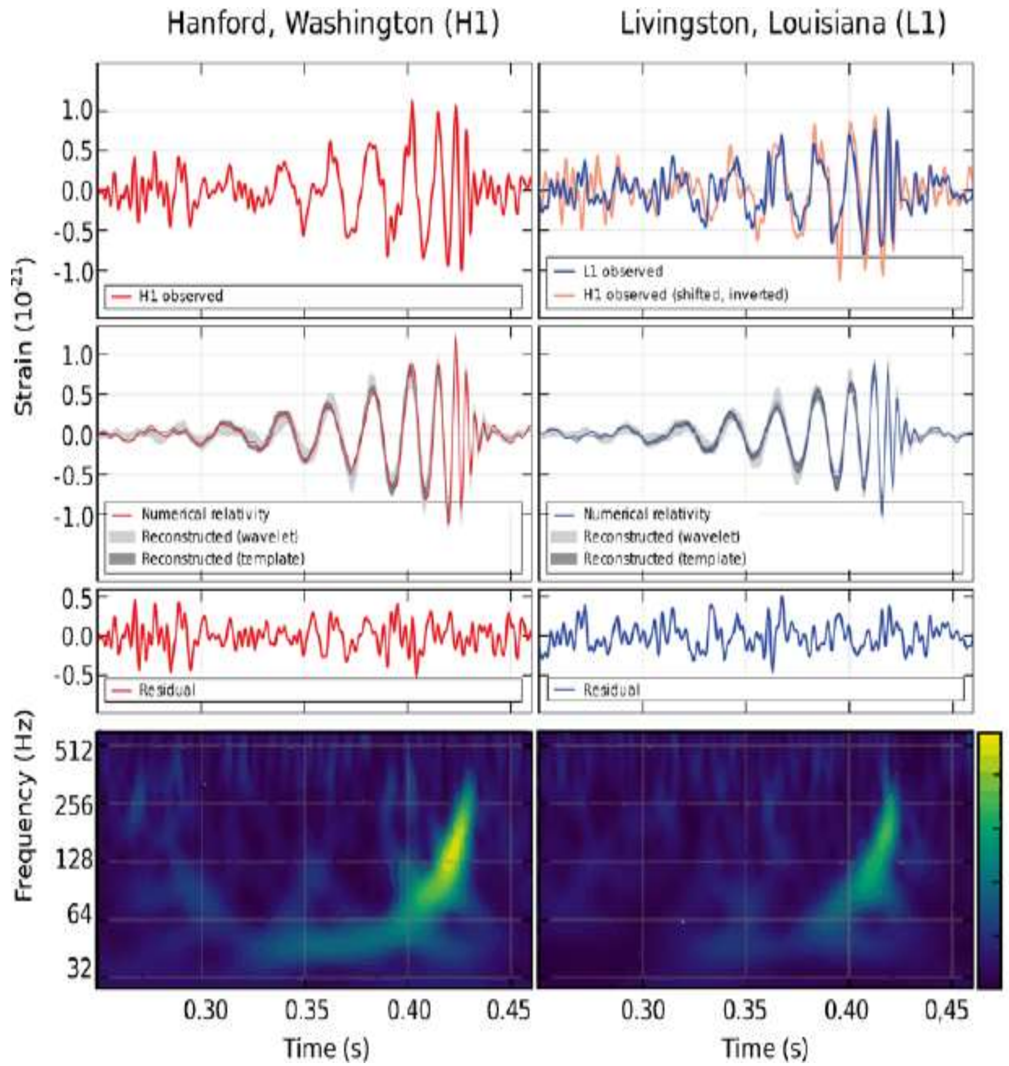
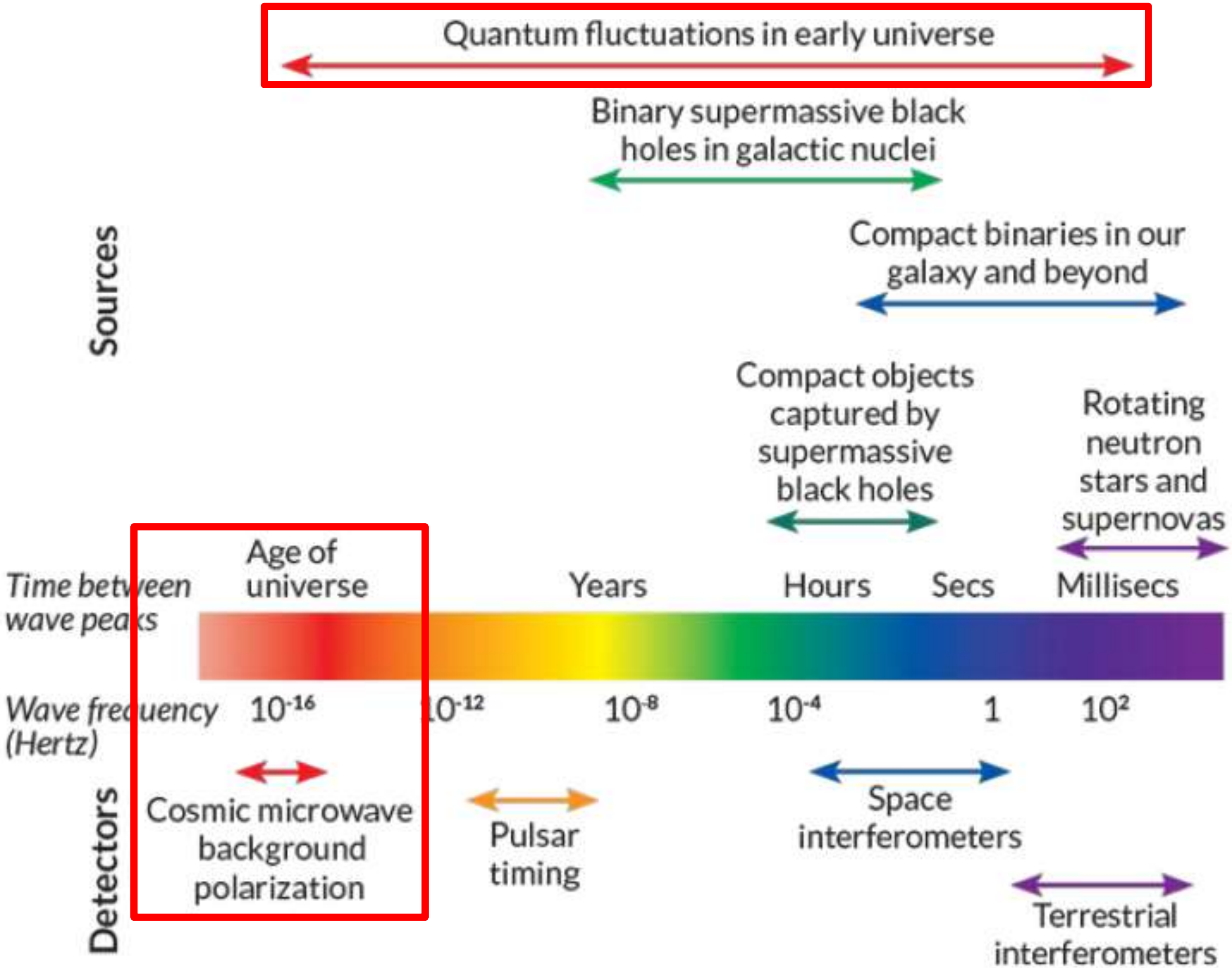
Kip S. Thorne
Prize share: 1/4

The Nobel Prize in Physics 2017 was divided, one half awarded to Rainer Weiss, the other half jointly to Barry C. Barish and Kip S. Thorne *"for decisive contributions to the LIGO detector and the observation of gravitational waves".*



Multi-messenger Astronomy!
The next story: **Primordial GWs**



Theory of Primordial Gravitational Waves

Classical Tensor Perturbations

- Consider a spatially flat Friedmann-Robertson-Walker background:
  - In cosmic time
$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$
  - In conformal time
$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j)$$
- Tensor perturbations are **traceless** & **transverse** :
$$\delta_{ij} \to \delta_{ij} + h_{ij} \qquad\qquad h_{ij} = h_{ji}\ ;\ h_{ii} = 0\ ;\ h_{ij,j} = 0$$
- Linearized Einstein equations (synchronous gauge)

Anisotropic stress

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \boxed{(T^i_j - P\delta_{ij})}$$
- Classical evolution is similar to the massless scalar field case, despite of the spacetime index.

Classical Tensor Perturbations

- Fourier expansion:

$$h_{ij}(x) = \sqrt{16\pi G}\sum_r \int \frac{d^3k}{(2\pi)^3}\epsilon^r_{ij}(\mathbf{k})h^r_{\mathbf{k}}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}$$

where the symmetric polarization tensor is traceless & transverse, and is normalized as

$$\sum_{ij}\epsilon^r_{ij}(\mathbf{k})\epsilon^s_{ij}(\mathbf{k})^* = 2\delta^{rs}$$

- Equation of motion in Fourier space (no source term):

$$h^{r\,\prime\prime}_{\mathbf{k}} + 2\frac{a'}{a}h^{r\,\prime}_{\mathbf{k}} + k^2 h^r_{\mathbf{k}} = 0$$

- Assuming a power law expansion:

$$a \propto \tau^n \quad n = \frac{2}{1+3w} \quad w = P/\rho$$

namely, w=1/3 for radiation; w=0 for dust matter; w~-1 for inflation

Classical Tensor Perturbations

- Equation of motion with a constant background equation of state

$$h_{\mathbf{k}}^{r\prime\prime} + 2\frac{n}{\tau}h_{\mathbf{k}}^{r\prime} + k^2 h_{\mathbf{k}}^r = 0$$

- General solutions in terms of Bessel functions:

$$h_k(\tau) = \tau^{1-n}\left(Aj_{\nu-1/2}(k\tau) + By_{\nu-1/2}(k\tau)\right)$$

$$\nu^2 = n(n-1) + \frac{1}{4}$$

- A little mathematical properties:

For large z:

$$J_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\cos\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

$$Y_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\sin\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

For small & negative z:

$$J_\alpha(z) \sim \frac{(-1)^\alpha}{(-\alpha)!}\left(\frac{2}{z}\right)^\alpha$$

$$Y_\alpha(z) \sim -\frac{(-1)^\alpha\Gamma(-\alpha)}{\pi}\left(\frac{z}{2}\right)^\alpha$$

- Thus, general solutions have two asymptotical forms:
  - When |k\tau| >> 1, oscillatory behaviors
  - When |k\tau| << 1, constant and time-varying modes

Quantization and Power Spectrum

- In the linear theory, one can take the analogy with the massless scalar field case to build the quantum theory associated with the tensor modes in a curved spacetime.

$$h_{ij}(\mathbf{x},\tau) = \sum_r \sqrt{16\pi G} \int \frac{d^3k}{(2\pi)^3} \left[\epsilon^r_{ij}(\mathbf{k}) \underbrace{h_k(\tau) a^r_{\mathbf{k}}}_{\text{physical time-dependent operator}} e^{i\mathbf{k}\cdot\mathbf{x}} + \epsilon^r_{ij}(\mathbf{k})^* h_k(\tau)^* a^{r\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

- The conjugate momenta are expressed as

$$\pi_{ij} = \frac{\delta^{(2)} S}{\delta h'_{ij}} = \frac{1}{2} a^2 h'_{ij}$$

- Quantized Poisson brackets

$$\begin{aligned}
[h_{ij}(\tau,\mathbf{k}), h_{kl}(\tau,\mathbf{k}')] &= 0 \, , \\
[p_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= 0 \, , \\
[h_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\
&\times \; \delta^{(3)}(\mathbf{k}-\mathbf{k}') \, .
\end{aligned}$$

$\Rightarrow$

Wronskian normalization condition

$$\boxed{h_k \dot{h}^*_k - h^*_k \dot{h}_k = \frac{i}{a^2}}$$

Quantization and Power Spectrum

- Power spectrum is defined by

$$\langle 0|h_{ij}(\mathbf{x},\tau)h_{ij}(\mathbf{y},\tau)|0\rangle \equiv \int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\Delta_T^2(k,\tau)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

- Or, it is simply expressed as

$$\boxed{\Delta_T^2(k,\tau) = 64\pi G\frac{k^3}{2\pi^2}|h_k(\tau)|^2}$$

- Energy spectrum is associated with the energy density via:

$$T^{\mu\nu}_{GW} = -\frac{2}{\sqrt{\bar{g}}}\frac{\delta S_{GW}}{\delta \bar{g}_{\mu\nu}} \qquad \rho_{GW} = T_{GW}{}^0{}_0 = \bar{g}_{00}T^{00}_{GW}$$

$$\Downarrow$$

$$\Omega_{GW}(k,\tau) \equiv \frac{1}{\rho_c(\tau)}\frac{d\langle 0|\rho_{GW}|0\rangle}{d(\ln k)} = \frac{8\pi G}{3H(\tau)^2}\frac{k^3}{2\pi^2 a^2(\tau)}\left(|\dot{h}_k(\tau)|^2 + k^2|h_k|^2\right)$$

Quantization and Power Spectrum

- Comments:
  - Primordial GWs could be originated from quantum fluctuations of the spacetime of the baby universe such that they can be accommodated with the quantum theory;
  - These quantum mechanically generated spacetime ripples need to be squeezed into classical perturbations so that the power and energy spectra can be probed observationally;
  - The above two theoretical expectations happen to be in agreement with two asymptotical solutions for primordial GWs in forms of the Bessel functions
- Question: Is there any causal mechanism that can connect two asymptotical solutions of primordial GWs dynamically?

Recall Inflationary Cosmology

Consider a Lagrangian of a free massive scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2$$

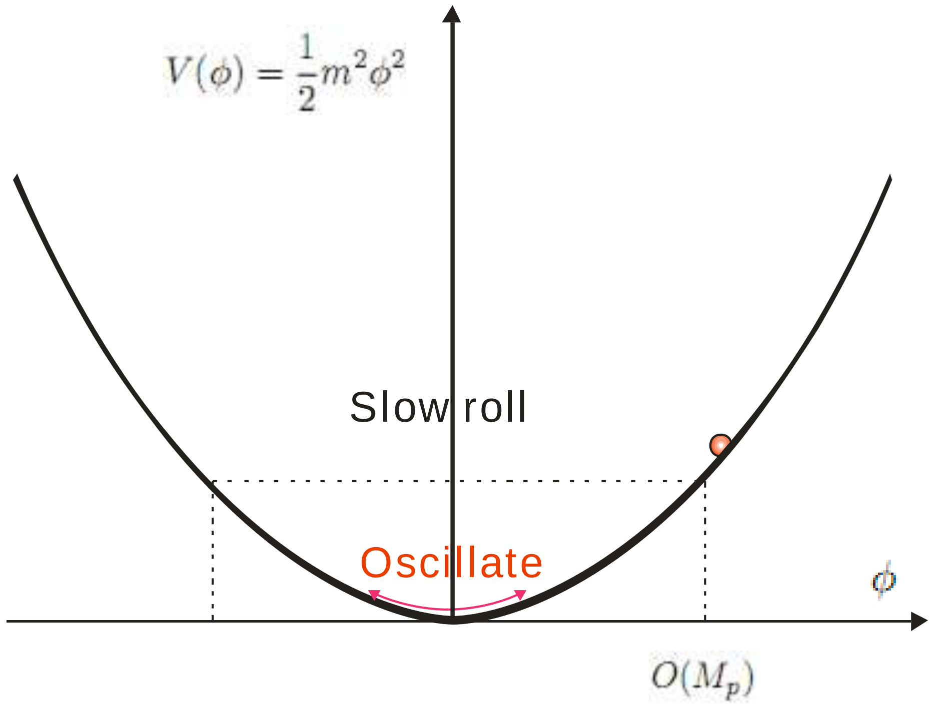
Its cosmological evolution follows the Friedmann equation and the Klein-Gordon equation:

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

This model yields an accelerating phase at high energy scale, when the amplitude of the scalar is larger than the Planck mass.

Inflation



Guth '81; Linde '82;
Albrecht & Steinhardt, '82;
Starobinsky '80; Sato '81; Fang '81; …

PGWs of Inflationary Cosmology

- Slow roll inflation yields a phase of nearly exponential expansion (\tau < 0):

$$a \propto e^{Ht} \sim -\frac{1}{H\tau} \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Slow roll parameters are introduced by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta \equiv M_p^2 \frac{V_{,\phi\phi}}{V}$$

and are much less than unity.

- Mukhanov-Sasaki variable and the EoM:

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Applying the vacuum initial condition, the solution for PGWs during inflation is given by

$$\boxed{h_k(\tau) = -\frac{H}{\sqrt{2k}}\tau\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} = H\sqrt{\frac{k}{2}}\tau^2 h_1^{(2)}(k\tau)}$$

PGWs of Inflationary Cosmology

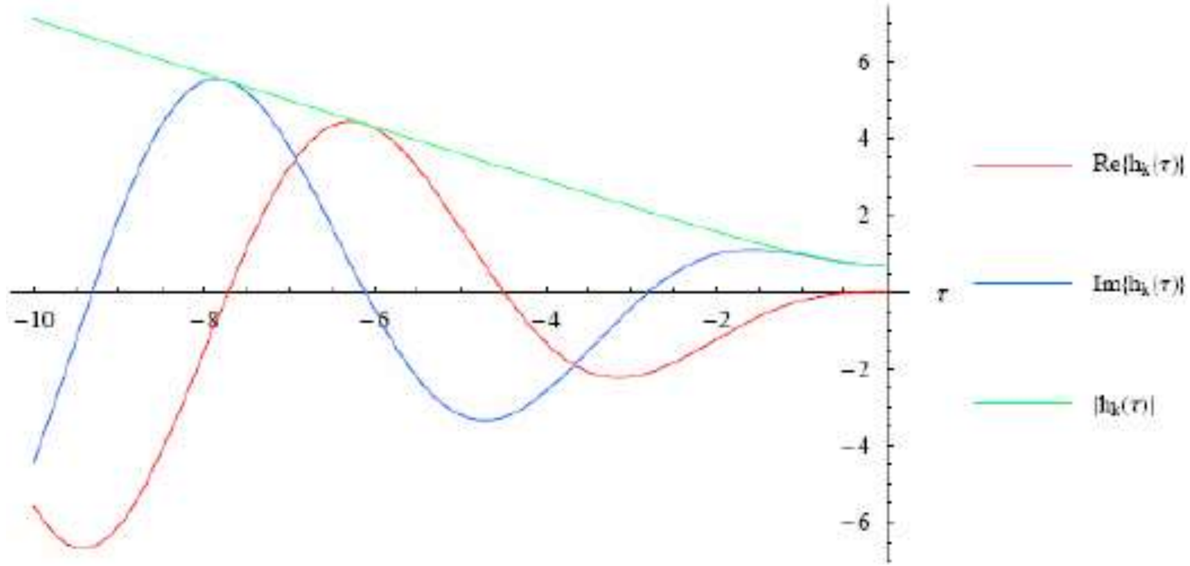
- When $|k\tau| >> 1$, the sub-Hubble solution connects with the vacuum fluctuations

$$h_{\mathbf{k}}^{\mathrm{UV}} \to -\frac{H\tau}{\sqrt{2k}}e^{-ik\tau}$$

- When $|k\tau| << 1$, the super-Hubble solution get frozen to become constant mode

$$h_{\mathbf{k}}^{\mathrm{IR}} \to \frac{iH}{\sqrt{2k^3}}$$

- Hubble crossing condition: k = aH or $|k\tau| = 1$
- For a k=1 mode, the solution is numerically given in the following



PGWs of Inflationary Cosmology

- At the end of inflation ($\epsilon = 1$), for the tensor modes outside the Hubble radius:
  - Power spectrum: $$P_T \equiv \Delta_T^2(k, \tau_e) = \frac{2H_*^2}{\pi^2 M_p^2}$$
  - Spectral index: $$n_T \equiv \frac{d \ln P_T}{d \ln k} = -2\epsilon_*$$
  - Tensor-to-Scalar ratio: $$r \equiv \frac{P_T}{P_\zeta} = 16\epsilon_* \quad \Rightarrow \quad \boxed{r = -8n_T}$$

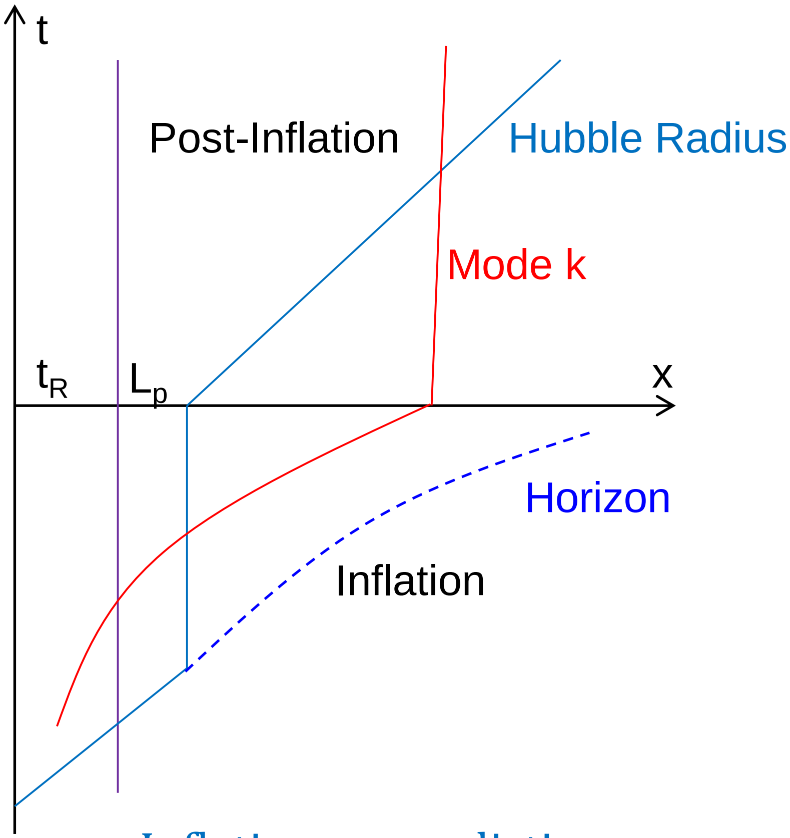
    Consistency relation for the model of single field slow roll inflation
  - Energy spectrum: $$\Omega_{GW}(k, \tau_e) = \frac{H^2}{6\pi^2 M_p^2} = \frac{1}{12}\Delta_T^2(k, \tau_e)$$ $$\Omega_{GW}(k, \tau_{\text{today}}) = T(k; \tau_{\text{today}}, \tau_e) \times \Omega_{GW}(k, \tau_e)$$
- Today's energy spectrum can be related to the primordial energy spectrum via the transfer function.

Sketch Plot for Inflation and Bounce

Crucial facts:

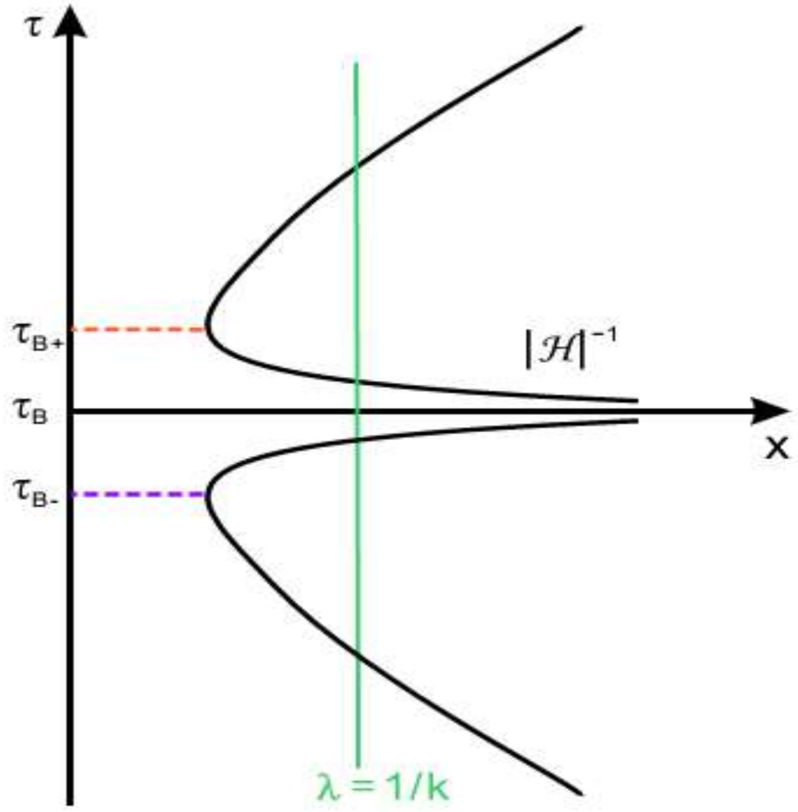
- Fluctuations originate on sub-Hubble scales
- Fluctuations propagate for a long time on super-Hubble scales
- Trans-Planckian problem: Inflation; Bounce

Brandenberger, hep-ph/9910410



Inflationary prediction:
a scale-invariant spectrum

CYF, Qiu, Brandenberger & Zhang, 0810.4677



Can we obtain a scale-invariant spectrum as in inflation?

PGWs of Matter Bounce Cosmology

- Matter bounce requires a phase of matter-dominated contraction (w=0, $\tau <\ 0$):

$$a \sim \tau^2 \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Similar to the slow roll, there is

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w)$$

- Inserting to the EoM of Mukhanov-Sasaki variable

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Using the vacuum initial condition, the solution for PGWs in matter contraction takes

$$v = (\eta - \tilde{\eta}_{B-})^{1/2}\{A_k^T J_{-(3/2)}[k(\eta - \tilde{\eta}_{B-})] + B_k^T J_{3/2}[k(\eta - \tilde{\eta}_{B-})]\},$$

$$A_k^T = i\frac{\sqrt{\pi}}{2} \quad \text{and} \quad B_k^T = -\frac{\sqrt{\pi}}{2}$$

PGWs of Matter Bounce Cosmology

- Therefore, the asymptotic form of the solution in matter contracting phase is

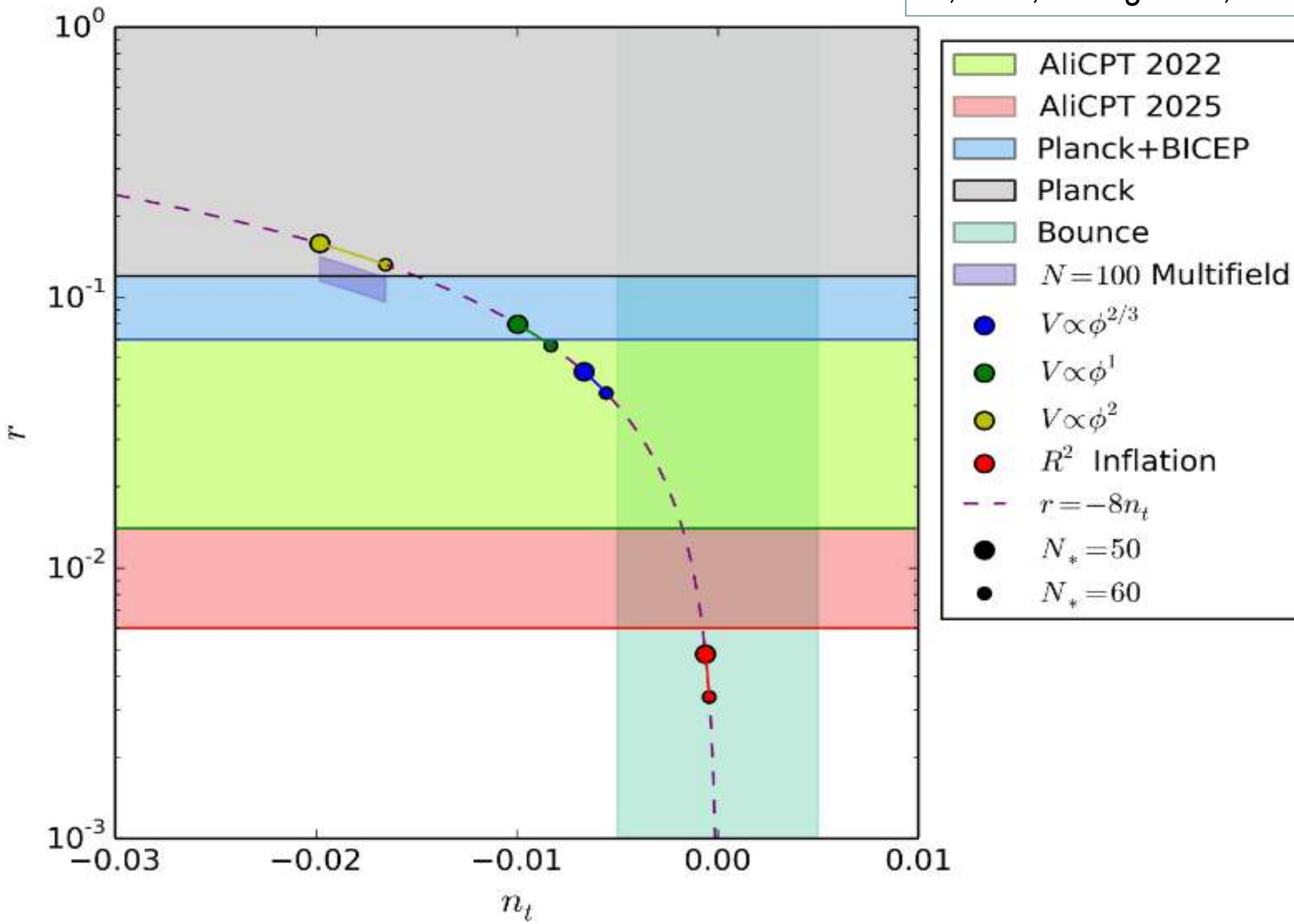
$$v(k,\eta) = \begin{cases} -\frac{i}{\sqrt{2}} k^{-(3/2)} (\eta - \tilde{\eta}_{B-})^{-1}, & \text{outside Hubble radius} \\ \frac{1}{\sqrt{2k}} e^{-ik(\eta - \tilde{\eta}_{B-})}, & \text{inside Hubble radius} \end{cases}$$

- After evolving PGWs through the bouncing phase to connect with thermal expansion,
  - Power spectrum: $$P_T(k) = G\frac{32k^3}{\pi}\left|\frac{v^f}{a}\right|^2 = \frac{2\rho_{B+}}{27\pi^2 M_p^4}$$
  - Spectral index: $$n_T \equiv \frac{d\ln P_T}{d\ln k} = 0 \pm (\text{bg error})$$
  - Tensor-to-Scalar ratio: $$r \equiv \frac{P_T}{P_\zeta} \lesssim O(1)$$

Consistency relation is broken for matter bounce

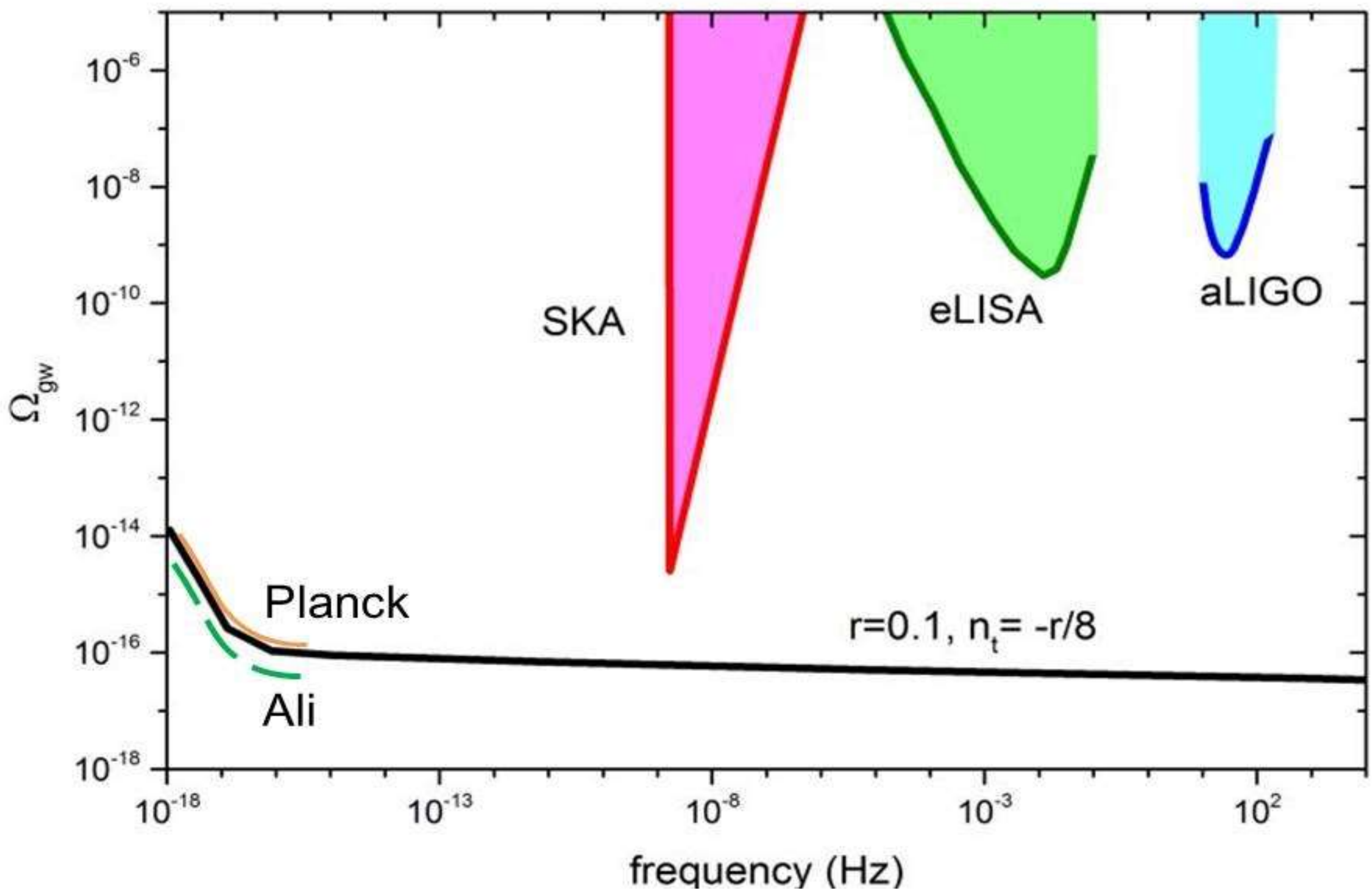
Consistency relation of inflationary PGWs and comparison with matter bounce and experiments

CYF & Zhang, 1605.01840;
Li, *CYF*, Zhang et.al., 1710.03047



Energy spectrum of inflationary PGWs and comparison with experimental sensitivities

CYF & Zhang, 1605.01840



Evolutions after primordial era

- Based on the big bang theory, assuming a CDM model:

  Radiation + Matter + instantaneous transition

$$a(\tau) = \begin{cases} H_0\sqrt{\Omega_{r0}}\tau, & 0 \le \tau \le \tau_{eq} \\ a_{eq}\left(\frac{\tau}{\tau_{eq}}\right)^2, & \tau_{eq} < \tau \le \tau_0 \end{cases}$$

- Solutions:
  - Radiation expansion

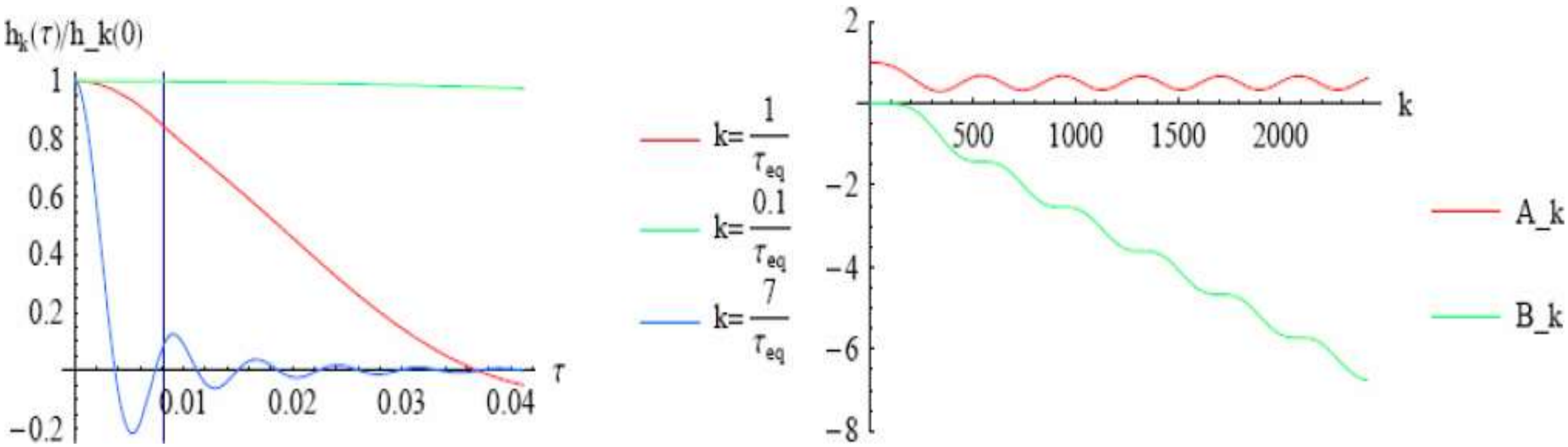
$$h_k(\tau) = Aj_0(k\tau) + By_0(k\tau) \longrightarrow A = h_k(0), \;\; B = 0$$

  - Matter expansion

$$h_k(\tau) = A_k\left(\frac{3j_1(k\tau)}{k\tau}\right) + B_k\left(\frac{3y_1(k\tau)}{k\tau}\right)$$

Evolutions after primordial era

- Modes reenter the Hubble radius during radiation and matter phases:



- The transfer coefficients would be more complicated if more elements are taken into account, namely, the dark energy era, smooth transitions, anisotropic stress (cosmic neutrinos), etc.

Evolutions after primordial era

- Comments:
  - The generation of PGWs are associated with the environment of the primordial phase;
  - Evolutions after the primordial era, i.e. transfer functions, are associated with phases of radiation, matter, dark energy, and phase transitions that the universe has experienced, as well as the relativistic d.o.f., such as free-streaming neutrinos;
  - The study of the cosmic background of PGWs can reveal important information about the evolution of the universe throughout the whole history.
- Question: How can we probe them in cosmological observations?

To be continued …

From PGWs to the CMB