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The Very Early Universe:

Primordial Gravitational Waves & CMB Physics II

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From PGWs to the CMB

CMB Blackbody background

- CMB is a (nearly) perfect blackbody characterized by a phase space distribution function

$$f = \frac{1}{e^{E/T} - 1}$$

where the temperature $T(x, \hat{n}, t)$ is observed at our position x=0 and time t_0 to be nearly isotropic with a mean temperature of 2.725K

- Our observable is the temperature anisotropy

$$\Theta(\hat{\mathbf{n}}) \equiv \frac{T(0, \hat{\mathbf{n}}, t_0) - \bar{T}}{\bar{T}}$$

- Given that physical processes essentially put a band limit on this function it is useful to decompose it into a complete set of harmonic coefficients

PGWs induce temperature fluctuations & polarization in CMB

- The polarization is induced by Thomson scattering of this anisotropic radiation field. To account for the polarization, we must follow the time evolution of four distribution functions:

$$f_s(\boldsymbol{x}, \boldsymbol{q}; \eta)$$

q is photon momentum; s=(I,Q,U,V) are four Stokes parameters

- At unperturbed background:

$$\bar{f}_I(\boldsymbol{q}, \boldsymbol{x}; \eta) = \left[e^{h\nu/k_B T(\eta)} - 1\right]^{-1} \qquad \bar{f}_Q = \bar{f}_U = \bar{f}_V = 0$$

- Then we introduce the perturbations

$$\Delta_s e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = 4\delta f_s / (\partial \bar{f} / \partial \ln T)$$

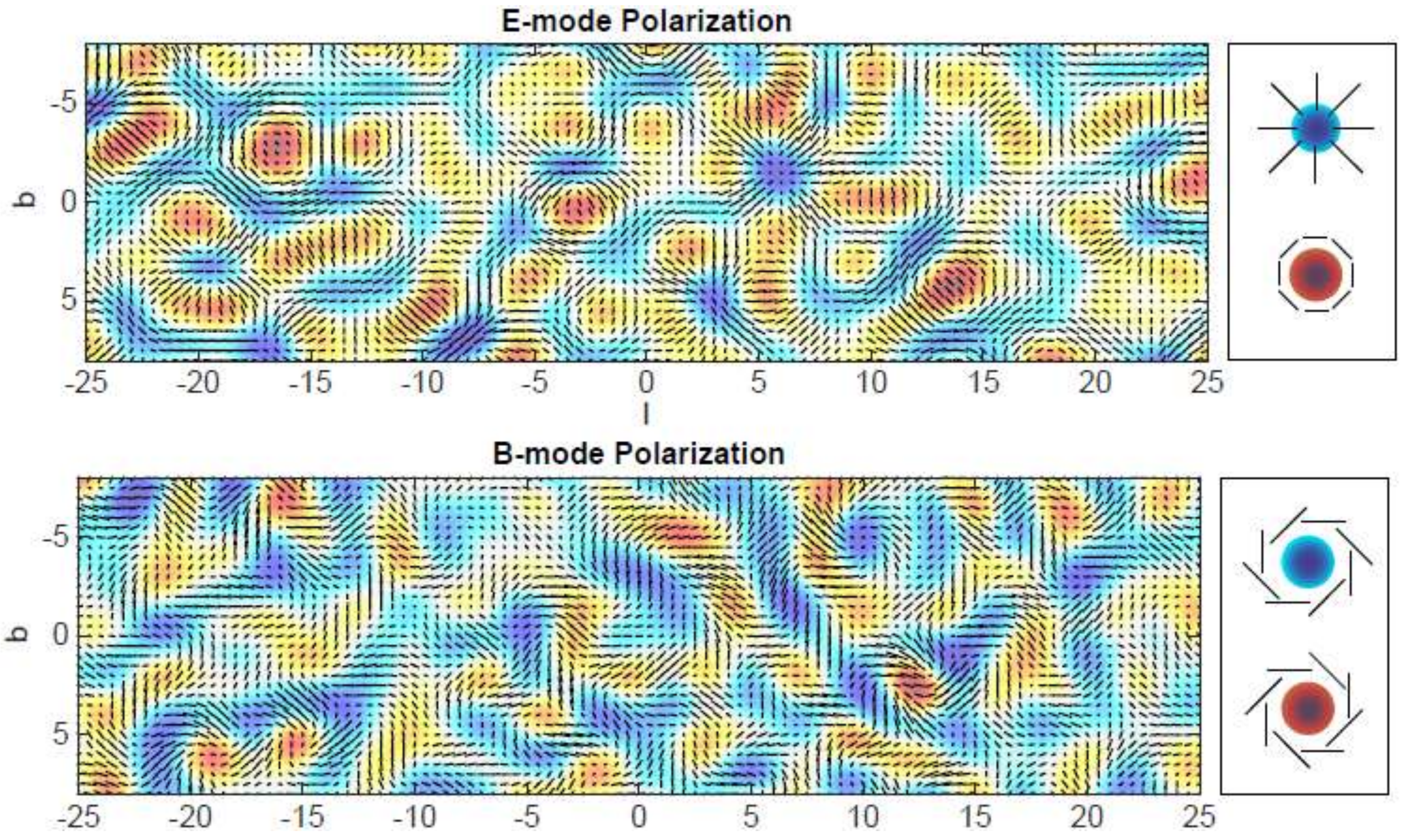
$$\Delta_I = \tilde{\Delta}_I (1-\mu)^2 \cos 2\phi, \qquad \Delta_Q = \tilde{\Delta}_Q (1+\mu)^2 \cos 2\phi, \qquad \Delta_U = \tilde{\Delta}_U 2\mu \sin 2\phi,$$

which are variables as functions only of μ and time.

E and B modes from PGWs

- Note that the polarization is spin-2 field:

$$\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$



E and B modes from PGWs

- PGWs of k wave-number can induce the polarization tensor:

$$\mathcal{P}^{ab}_{k,+}(\theta,\phi) = \frac{T_0}{4\sqrt{2}} \sum_\ell (2\ell+1) P_\ell(\cos\theta) \tilde{\Delta}_{Q\ell} \begin{pmatrix} (1+\cos^2\theta)\cos 2\phi & 2\cot\theta \sin 2\phi \\ 2\cot\theta\sin 2\phi & -(1+\cos^2\theta)\csc^2\theta\cos 2\phi \end{pmatrix}$$

- It yields the E & B coefficients:

$$a^{\mathrm{E}\,k,+}_{\ell m} = \frac{\sqrt{\pi(2\ell+1)}}{4(\delta_{m,2}+\delta_{m,-2})^{-1}} \left[\frac{(\ell+2)(\ell+1)\tilde{\Delta}_{Q,\ell-2}}{(2\ell-1)(2\ell+1)} + \frac{6\ell(\ell+1)\tilde{\Delta}_{Q\ell}}{(2\ell+3)(2\ell-1)} + \frac{\ell(\ell-1)\tilde{\Delta}_{Q,\ell+2}}{(2\ell+3)(2\ell+1)} \right]$$

$$a^{\mathrm{B}\,k,+}_{\ell m} = \frac{-i}{2\sqrt{2}} \sqrt{\frac{2\pi}{(2\ell+1)}} (\delta_{m,2}-\delta_{m,-2}) \left[(\ell+2)\tilde{\Delta}_{Q,\ell-1} + (\ell-1)\tilde{\Delta}_{Q,\ell+1} \right]$$

- The angular power of B mode for fixing k takes:

$$C^{\mathrm{BB},\,k,+}_\ell = \frac{1}{2l+1} \sum_m |a^B_{\ell m}|^2 = \frac{\pi}{2} \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1} + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell+1} \right]^2$$

- Integrating out k, one gets the BB angular power spectrum

$$C^{\mathrm{BB}}_\ell = \frac{1}{2\pi} \int k^2\, dk \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) \right]^2$$

E and B modes from PGWs

- Comments:
  - Similar process applies for C_l^{EE}
  - The E-B cross correlation vanishes for the standard model
  - Taking the same analysis, it is easy to see that scalar perturbation only produces T and E, simply due to the fact that density perturbations do not produce a curl at linear level
  - But, B modes may still arise from density perturbations at nonlinear order
  - Question: How large are these foreground contaminations?

Lensing induced B modes

- The most relevant nonlinear effect is weak gravitational lensing induced by (scalar type) density perturbations between us and the CMB surface of last scatter.

astro-ph/9803150

- The Stokes parameters displace along a given direction:

$$\begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm obs.} (\theta) = \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta + \delta\theta) \simeq \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta) + \delta\theta \cdot \nabla \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta)$$

where $\delta\theta = \nabla\Phi$ is the lensing deflection along gravitational potential.

- If no PGWs, there is only E mode at LSS with:

$$\tilde{Q}(\ell) = 2\tilde{E}(\ell)\cos 2\varphi_\ell \qquad U(\ell) = -2E(\ell)\sin 2\varphi_\ell$$

Lensing induced B modes

- Gravitational deflection leads to

$$B(\boldsymbol{\ell}) = \frac{1}{2}[\sin 2\varphi_{\boldsymbol{\ell}}\, \delta Q(\boldsymbol{\ell}) - \cos 2\varphi_{\boldsymbol{\ell}}\, \delta U(\boldsymbol{\ell})] = \int \frac{d^2 \boldsymbol{l}_1}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)] E(\boldsymbol{\ell}_1) \Phi(\boldsymbol{\ell} - \boldsymbol{\ell}_1) \sin 2\varphi_{\boldsymbol{\ell}_1}$$

- The angular power spectrum of lensing B modes takes

$$C_\ell^{\rm BB} = \int \frac{d^2 \boldsymbol{l}_1}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)]^2 \sin^2 2\varphi_{\boldsymbol{\ell}_1} C^{\Phi\Phi}_{|\boldsymbol{\ell} - \boldsymbol{\ell}_1|} C^{\rm EE}_{\ell_1}$$

- It was detected by SPT in 2013.

1307.5830

Foreground contributions to B modes

- Galactic foregrounds:
  - Synchrotron: Galactic synchrotron emission is dominant at frequencies below 100 GHz, and both WMAP and Planck have observed its polarization signature at frequencies from 30 to 90 GHz;
  - Dust: Above 100 GHz, thermal emission from asymmetric dust in the interstellar medium, which align themselves with the Galactic magnetic field, induces a strong polarization signal;
  - …
- One must use techniques of de-lensing and non-Gaussian diagnosis to eliminate foreground contaminations to extract signals of PGWs.

So far where we are …

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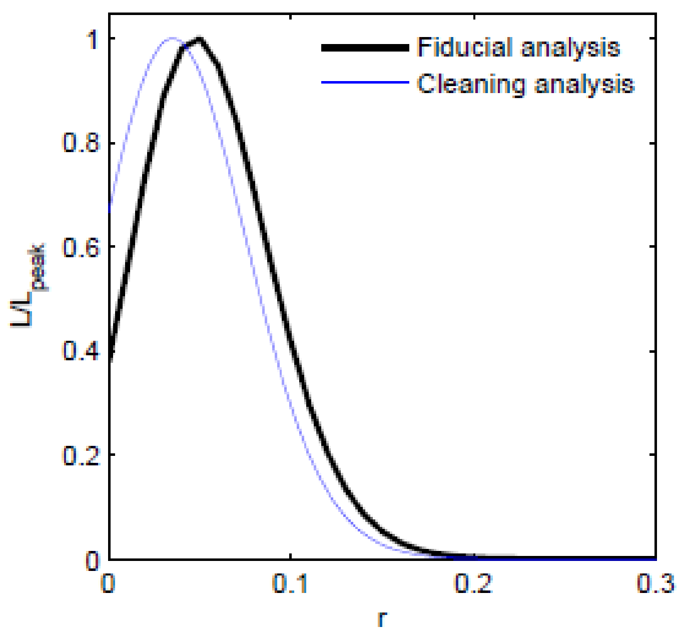
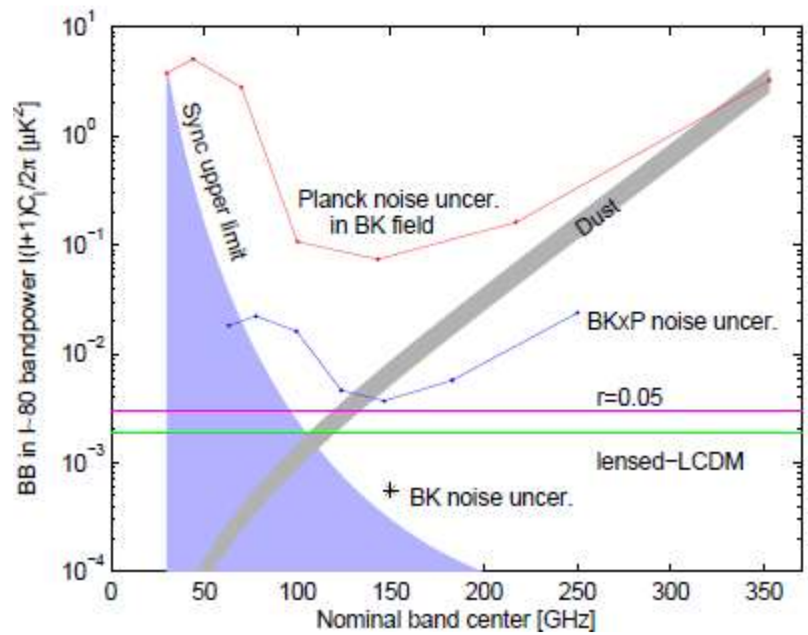
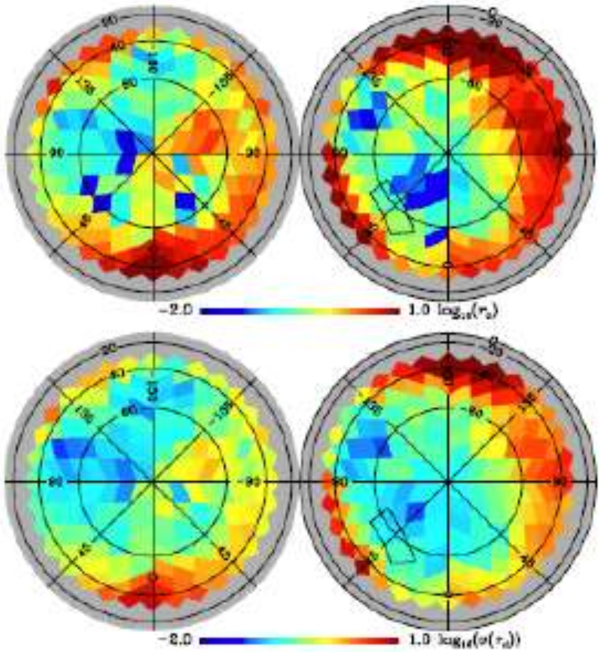




- No signal of PGWs: r < 0.07 at 2σ under a joint analysis of data from BICEP2/Keck Array & Planck 2015.
- No signal of PGWs: r < 0.036 at 2σ under a joint analysis of data from BICEP3/Keck Array & Planck 2018.

Polarization foreground from galaxy

full sky coveraged is required !

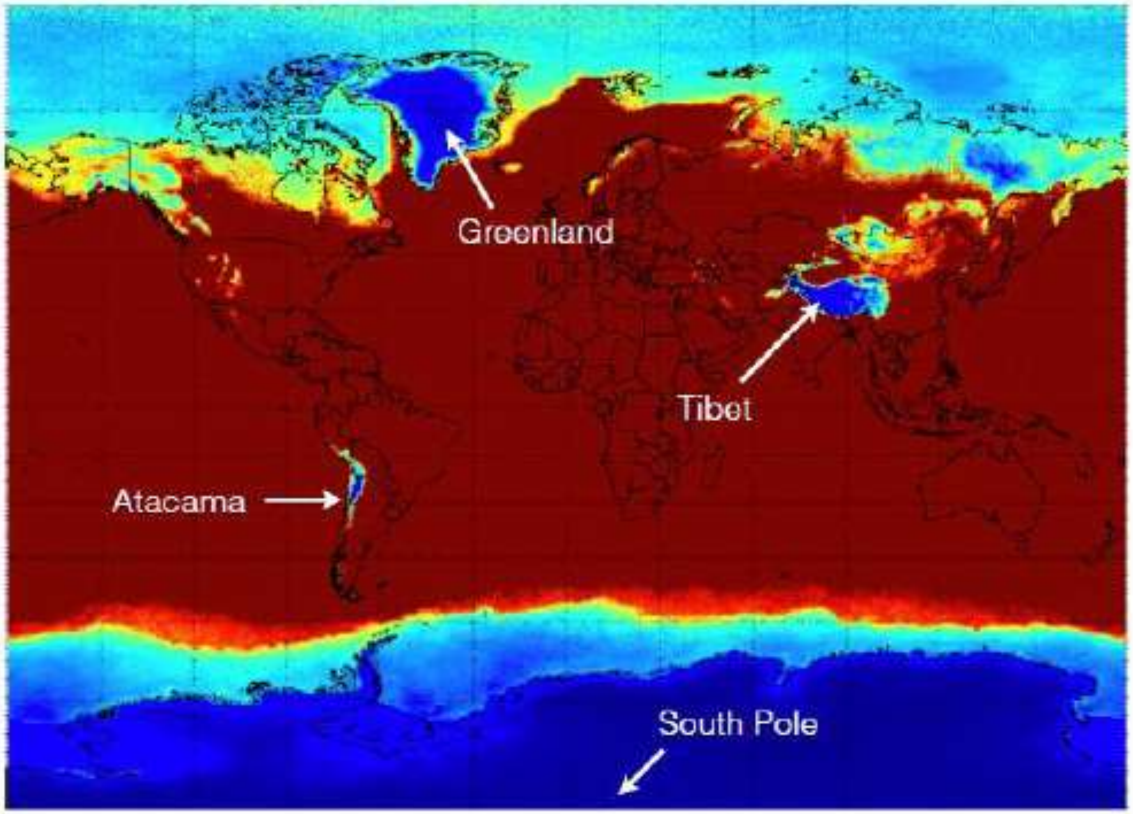


north hemisphere south hemisphere

- Planck can provide us the full sky coverage, but the S/N is very limited;
- After Planck, there is so far no further space-based projects;
- The ground-based CMB polarization projects will be the key developments in the next decade.

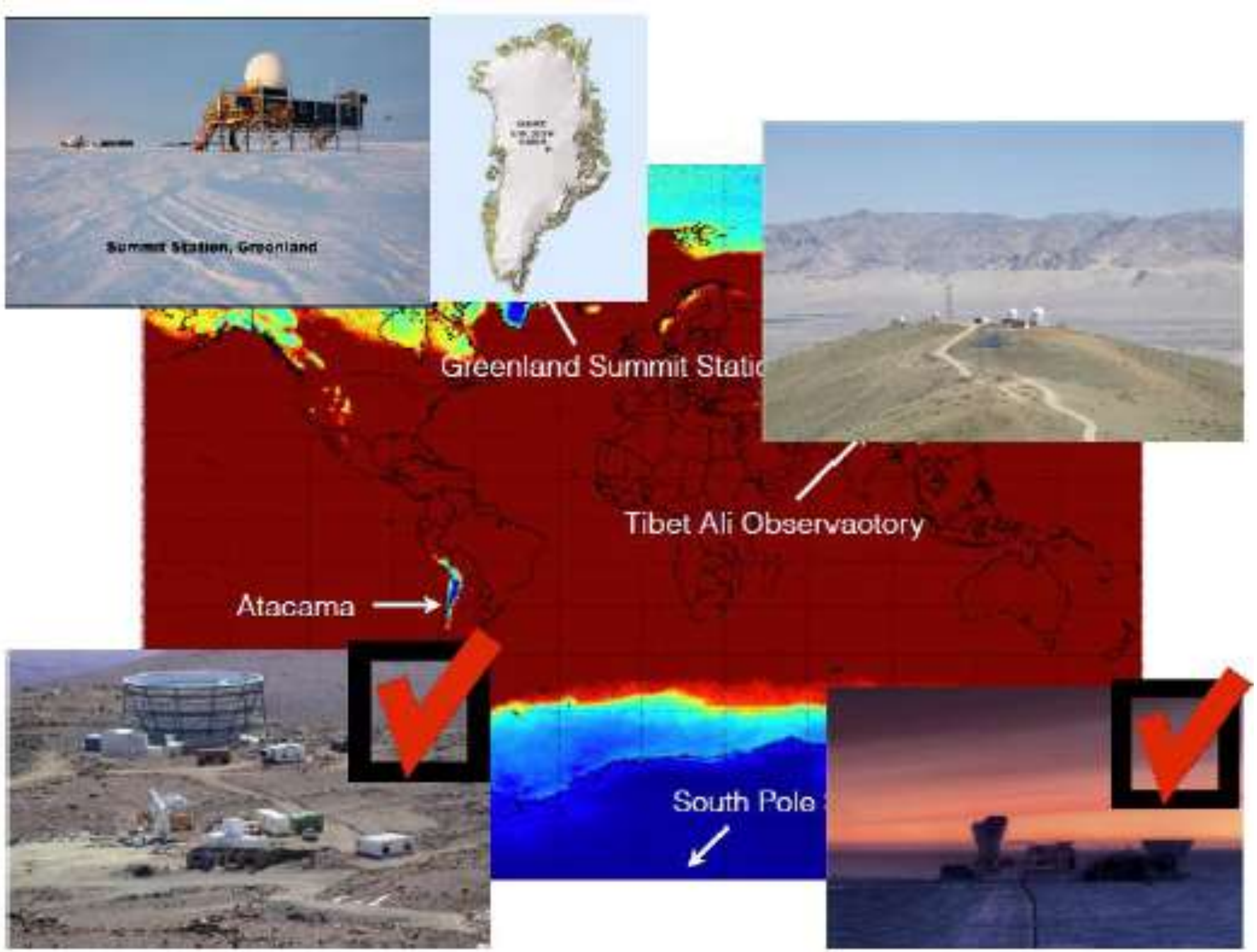
A full sky coverage is needed!

How many places suitable for CMB?



- Blue areas indicate high atmospheric transmission rate, which are suitable for CMB observations!
- Four best places on Earth: Greenland, Tibet, Atacama desert, Antarctica

Ground-based CMB experiments



Full-sky coverage expects the CMB experiments in the north part of the earth

A future lesson from CMB experiments

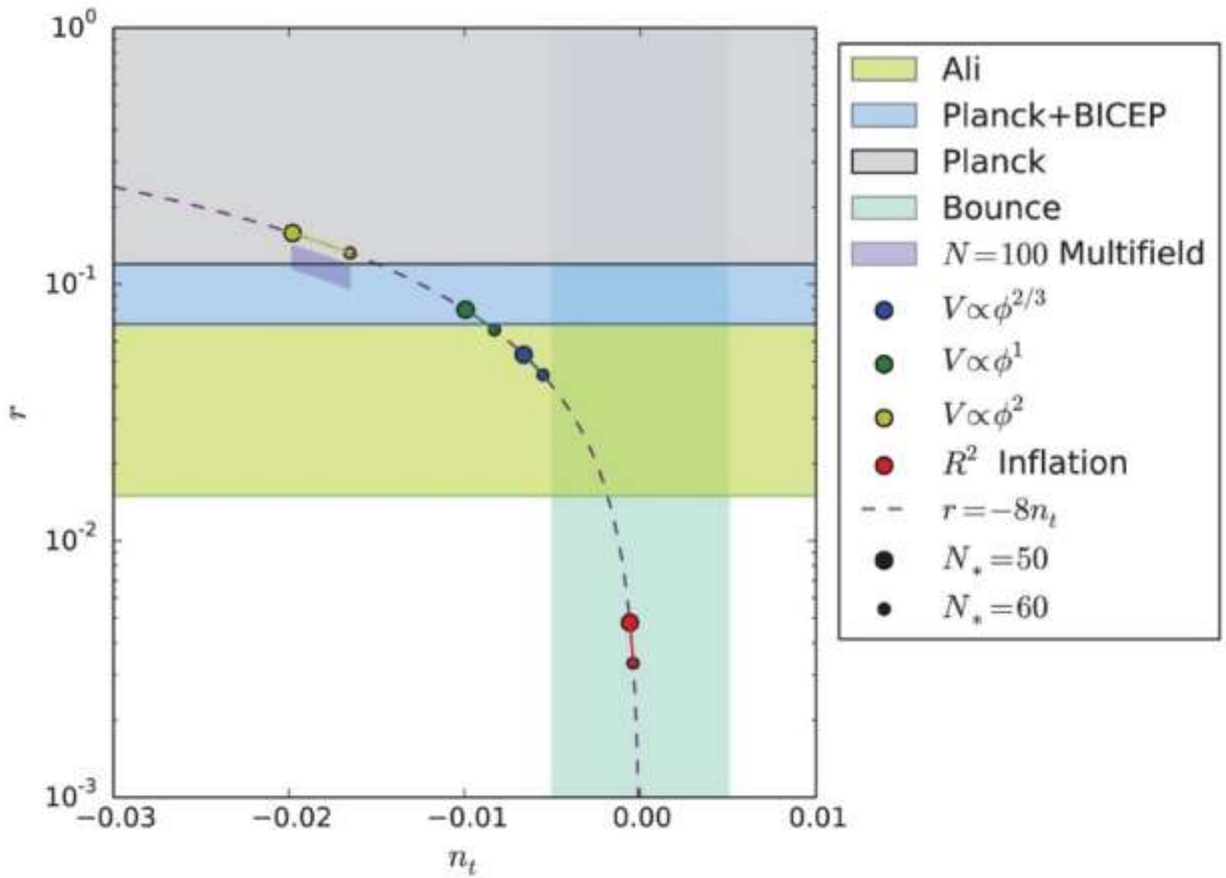
Overview: predictions of very early universe models

Models	Predictions			
Inflation	$r = 16\,\varepsilon$	$n_t = -2\,\varepsilon$	$\alpha_t < 0$	$f_{nl}=5(1-n_s)/12$
Bounce	$r \leq O(1)$	$\|n_t\|<O(1)$	$\alpha_t > 0$	$f_{nl} \approx -5/2$
Ekpyrosis	$r<<O(1)$	$n_t = 2$	$\alpha_t < 0$	$f_{nl} > 1$
String gas	$r \leq O(1)$	$n_t \approx 1-n_s$	$\alpha_t > 0$	$f_{nl} << O(1)$

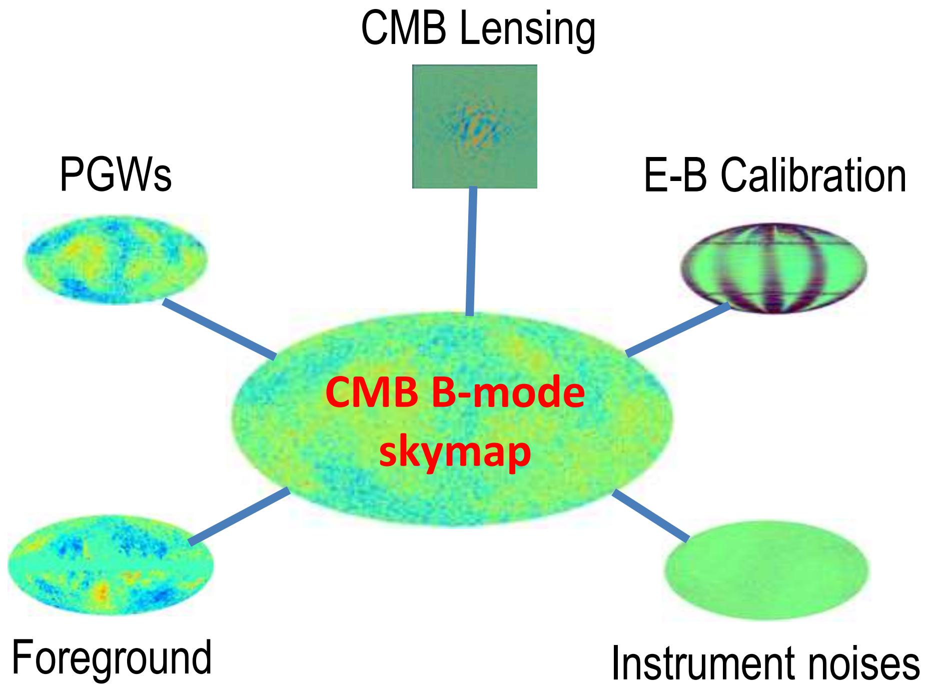
Constraining the very early universe models

Q: there are too many models of the very early universe, namely,

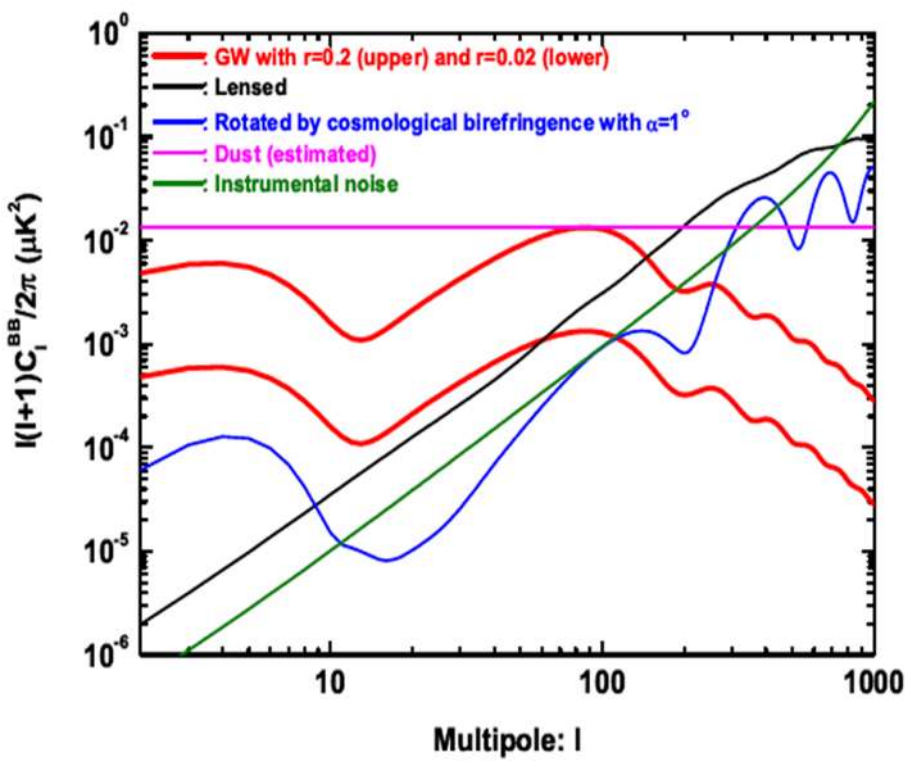
- Inflationary models
- Nonsingular bounce models
- Models of emergent universe



Statistics of all possible B-mode components



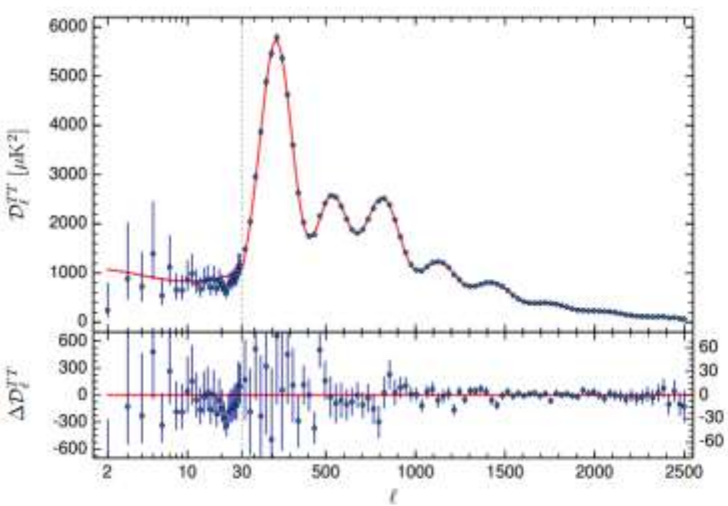
Q: How can we identify all the components that can give rise to CMB B-mode?



Plan:

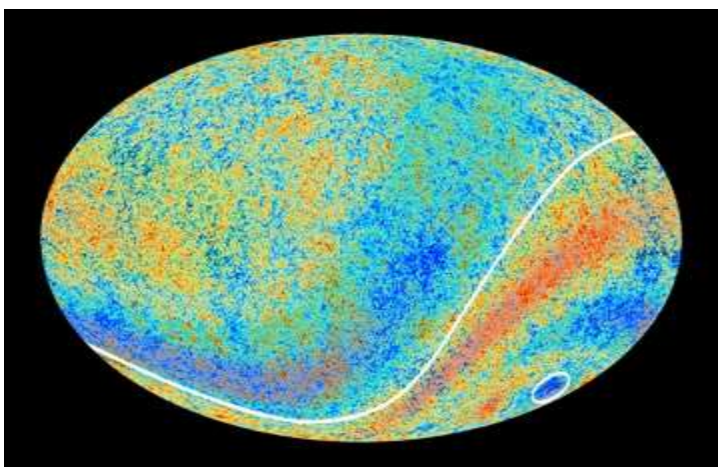
Different components exhibit different statistical properties. These can be used to exact the signals from PGWs.

CMB large scale anomalies



- Large scale suppression
- Cold spot
- Hemispherical power asymmetry
- Power deficit near l=30

Q: primordial origin, or, observational contamination?



Plan:

✓ Combine together AliCPT in North sphere and BICEP, PolarBear in South sphere

✓ Build theoretical models to explain associated phenomena

Summary & Outlook

Today

- The detection of CMB fluctuations can be regarded as the starting point for cosmology as a precision science
- The paradigm of early universe has been greatly developed
- The Big Bang has became the Standard Model in cosmology
- Inflationary cosmology obtained a large amount of initial achievements
- Bounce cosmology is ambitious on solving big bang singularity
- The GW Astronomy has initiated

In Near Future

- The probe of the very early universe is crucial for exploring fundamental physics

- Multi-messenger provides a novel means of cosmological research

- It becomes possible to observationally probe accurate physics near the Big Bang: CMB B-modes

A new era has begun…


Thanks!