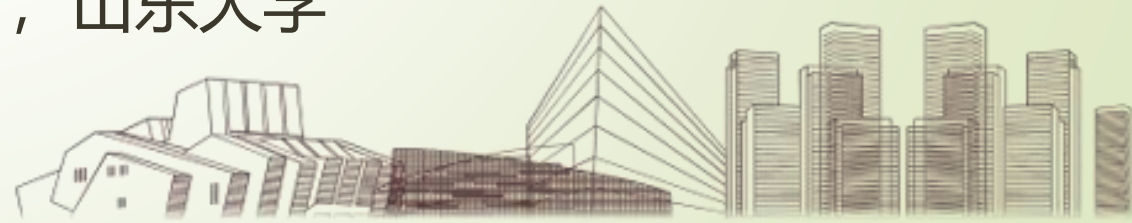


Extending the Predictive Power of Perturbative QCD Using the Principle of Maximum Conformality

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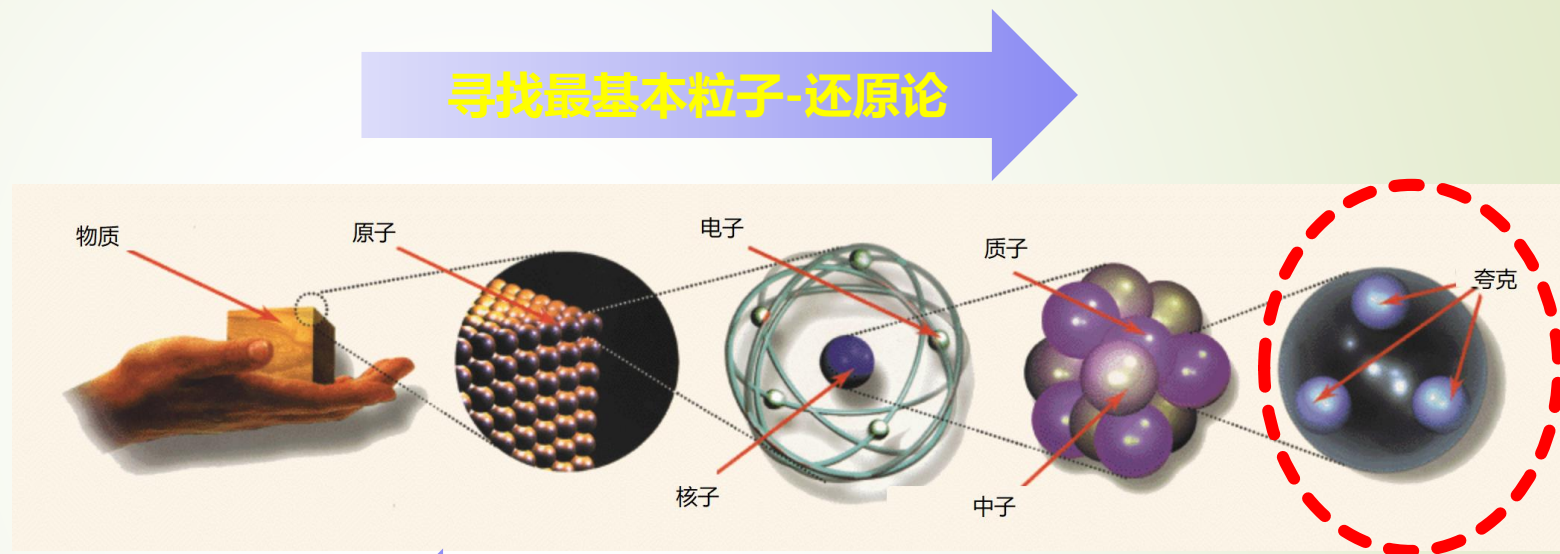
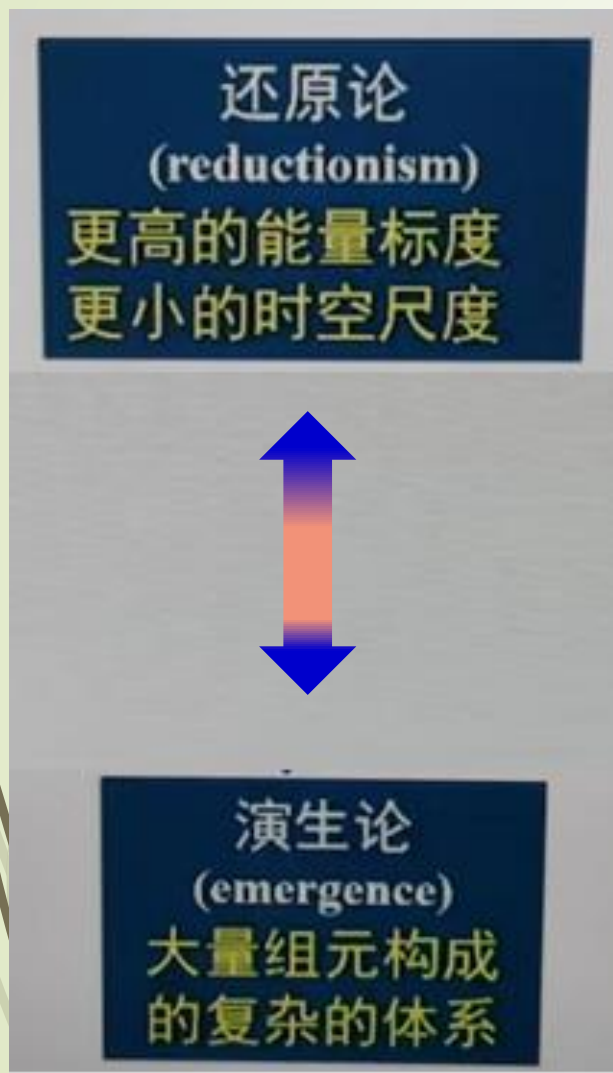
小结和展望

一个重要问题

如何获得可靠且精确的有限阶微扰预言

背景

我们对微观物质世界的认识越来越深刻



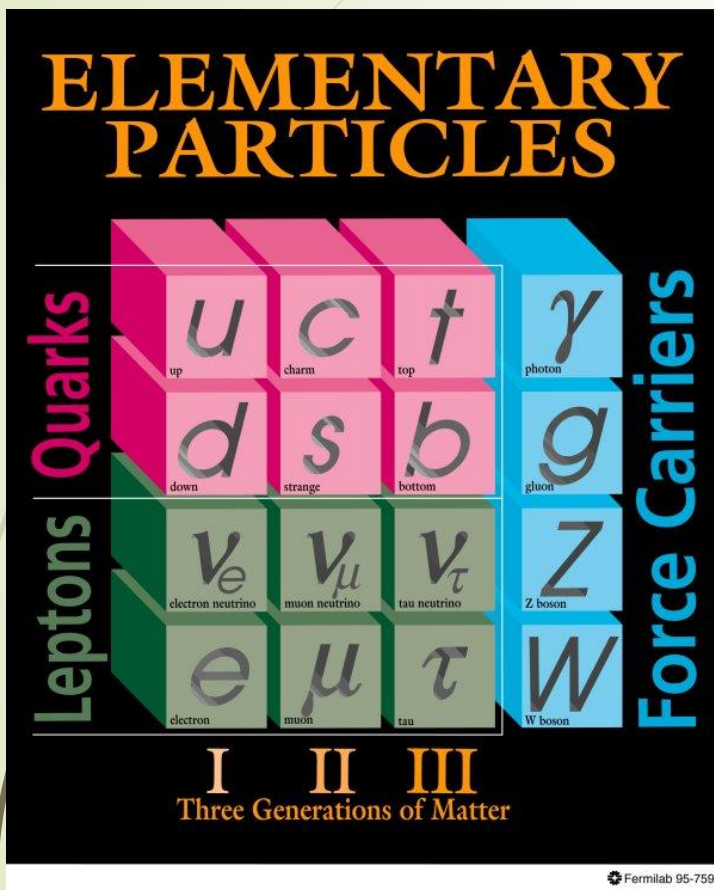
寻找最基本粒子-还原论

演生论-多粒子复杂系统

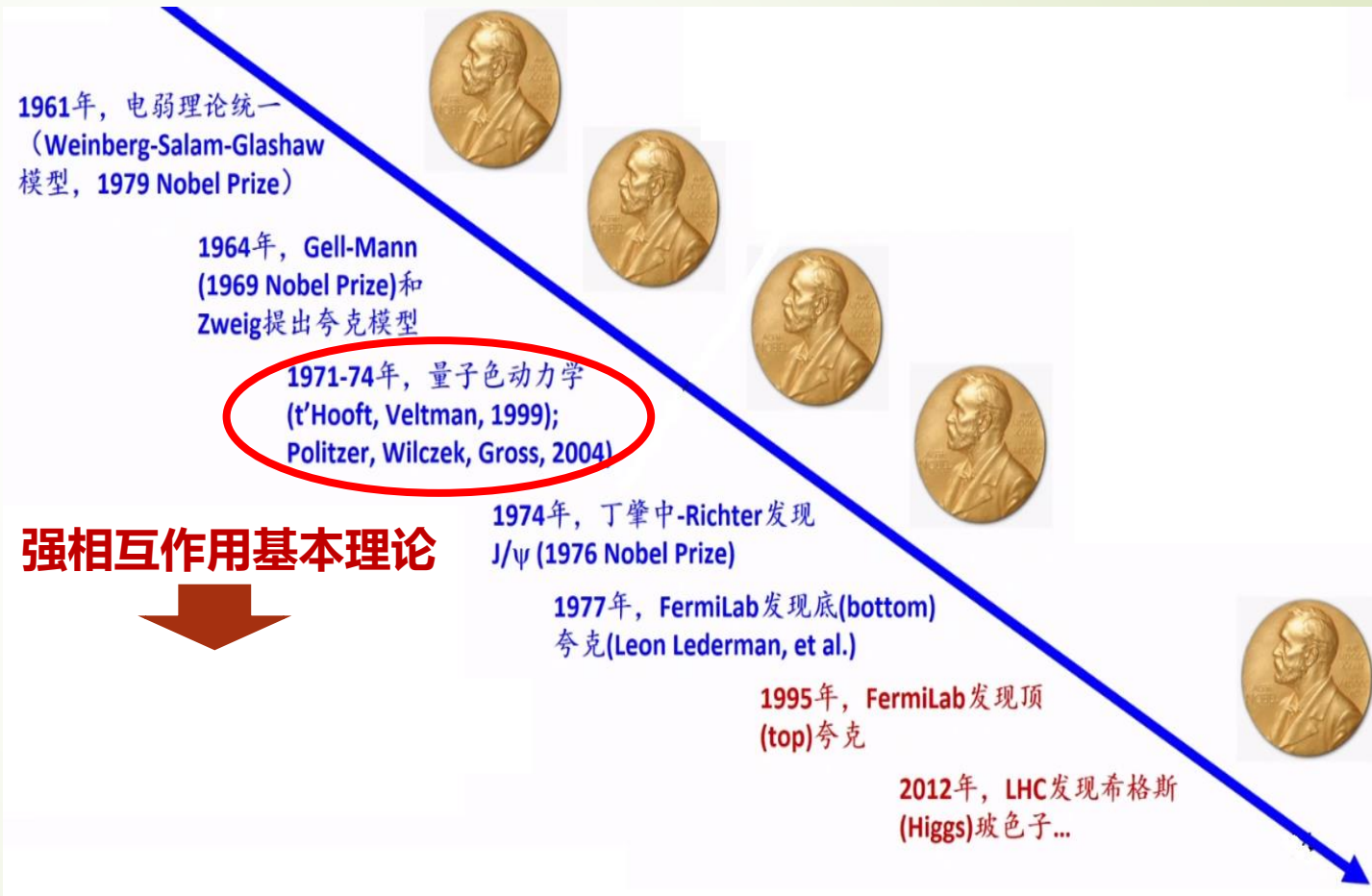
认识最基本物质
结构最根本

每个物质层次都有其独特现象及相应的“有效理论”
不同层次的研究方法可互通和相互借鉴

标准模型 --- 最成功的关于物质深层次结构的有效理论



强、弱、电磁相互作用

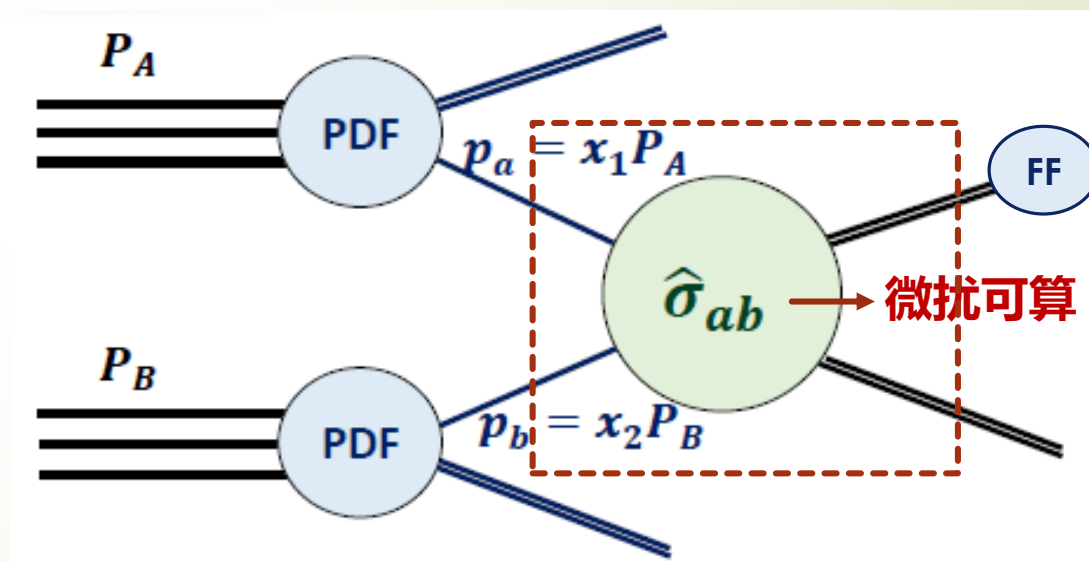
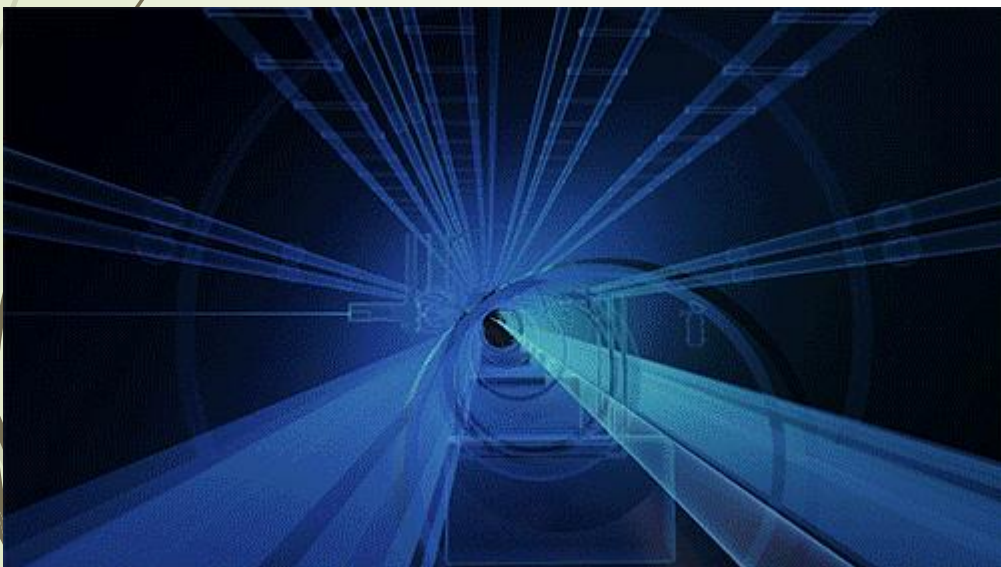


因子化理论

以能量标度为基础，将非微扰部分（普适、实验确定）逐步因子化出去，
直到分离出微扰可算的部分

引入一个尚未解决的问题：如何正确设定因子化能标（DGLAP可降低）？

典型因子化图像



因子化图像

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

Parton distribution functions

Perturbative partonic cross section (virtual & real radiation)

Fragmentation models

$$\hat{\sigma} = \sigma_0 [1 + \alpha_s + \alpha_s^2 + \dots]$$

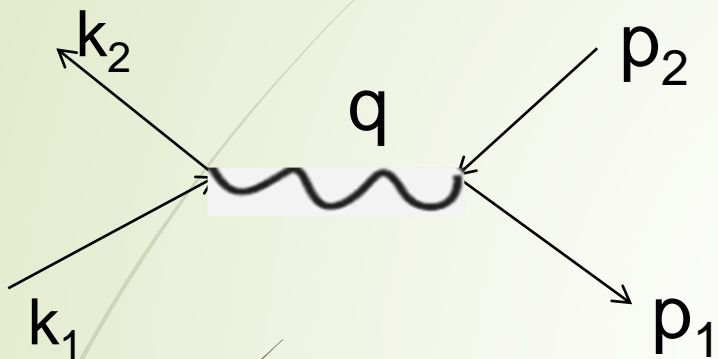
How to set the renormalization scale ?

Contains α_s^n of tree level process

NLO

NNLO current frontier

背景

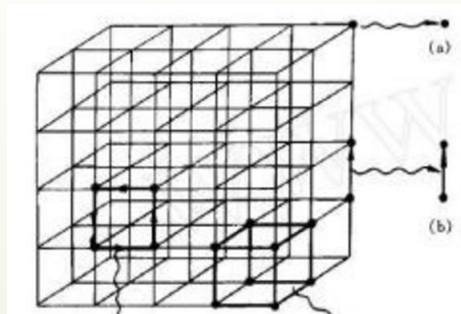


高能物理过程如何处理
能否计算得到有效预言

连续时空→离散晶格

微扰论
找准小量、确保小量、确定小量准确值

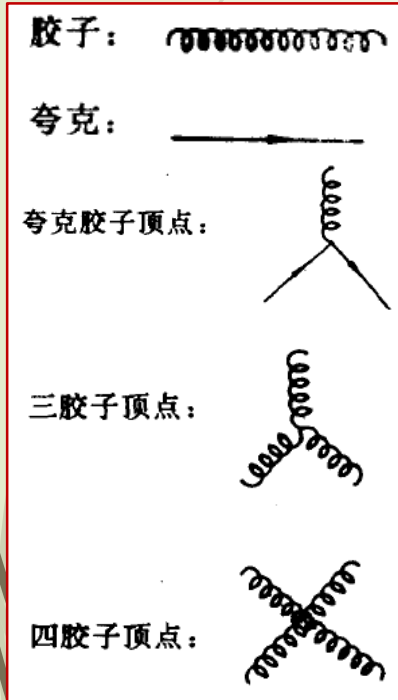
非微扰效应
QCD格点理论、QCD求和规则等



关联函数在全空间的
连续性

背景

强相互作用 量子色动力学 QCD



典型费曼规则

Quantum Chromodynamics (QCD)

Fields:

$\psi_i^f(x)$ Quark fields, Dirac fermions (like e^-)
 Color triplet: $i = 1, 2, 3 = N_C$
 Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$ Gluon fields, spin-1 vector field (like γ)
 Color octet: $a = 1, 2, \dots, 8 = N_C^2 - 1$

Lagrangian density:

$$\begin{aligned}
 L_{QCD}(\psi, A) = & \sum_f \bar{\psi}_i^f \left[\left(i\partial_\mu - g A_{\mu,a}(t_a)_{ij} \right) \gamma^\mu - m_f \right] \psi_i^f \\
 & - \frac{1}{4} \left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2 \\
 & + \text{gauge fixing} + \text{ghost terms} \\
 [t_a, t_b] = & i C_{abc} t_c
 \end{aligned}$$

Color matrix:

Gauge invariance:

$$\psi_i \rightarrow \psi'_j = U_{ji}(x) \psi_i$$

$$A_\mu \rightarrow A'_\mu = U(x) A_\mu U^{-1}(x) + \frac{i}{g} \left[\partial_\mu U(x) \right] U^{-1}(x)$$

where $A_\mu = A_{\mu,a} t_a$

局域性

强子
束缚态

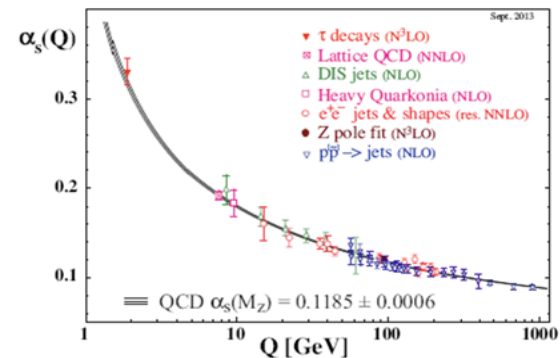
量子色动力学重要性质

- 夸克禁闭 (不存在自由夸克)

$$\Lambda_{QCD} \sim 1/R_{\text{had}} \sim 0.2 \text{ GeV}$$

- 渐近自由理论 (微扰可算)

$$\alpha_s(Q \gg \Lambda_{QCD}) < 1$$



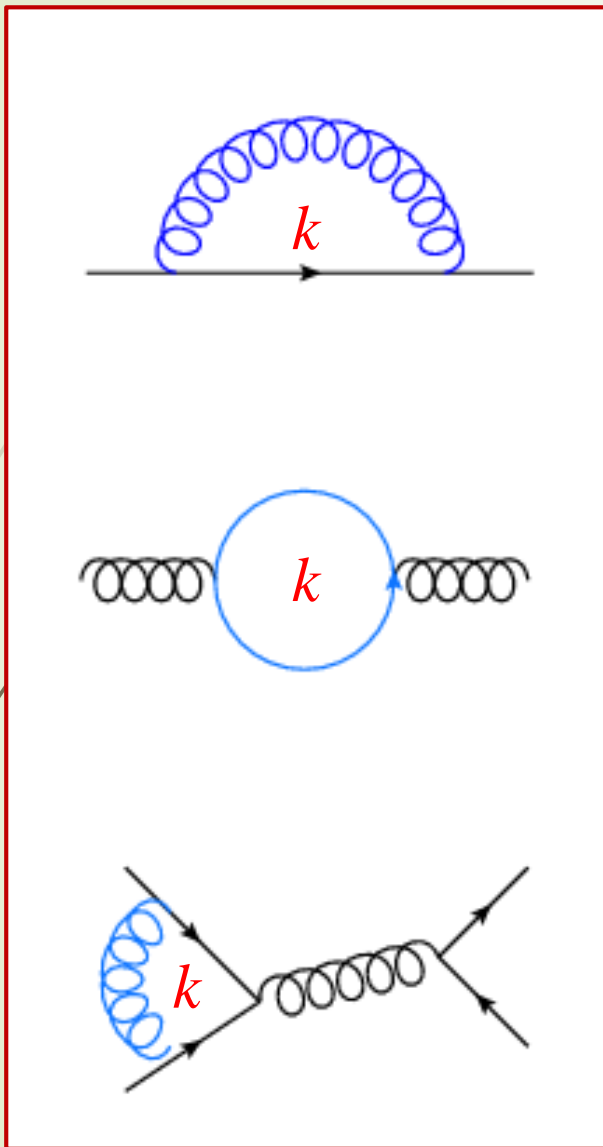
微扰QCD理论

$$\sigma_{\text{phy}}(Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu)}{2\pi} \right)^n$$



重整化、重整化能标和重整化方案

典型
单圈
修正



$$\int d^4 k$$

发散

微扰高阶圈动量积分引起
圈动量 $\rightarrow \infty$ ，导致紫外发散 (UV-发散)

如何得到有限结果？ 解决方案通常分两步

- 正规化 - 分离发散积分
- 重整化 - 划分发散 (重整化方案) $\alpha_s(M_Z)$
- 消除发散与实验值对应 (引入实验测量值作为基准点)

可重整理论，需且仅需引入少量参数，具备预言力

“理念--现实世界由可重整量子场论所描述”

- 1953年 Peterman 与 Stueckelberg – 提出重整化群不变性思想

$$\Gamma = g^{\text{ren}} - \Delta g + (g)^3 \int (\dots)$$

示意图
高阶需引入新抵消项

完整振幅 = 低价顶点 g^{ren} + 抵消项 Δg (吸收表观发散项) + ...

g^{ren} 与 Δg 之间的划分是任意的，完整振幅 (无穷阶) 不应该依赖于这种划分。**完整振幅与减除方式的无关性** —— 被称为重整化群不变性

用于定性分析，同时如何保证不变性？ ➡ 引入重整化群方程 ➡ 拓展重整化群方程

Quantum Electrodynamics at Small Distances

M. Gell-Mann and F. E. Low

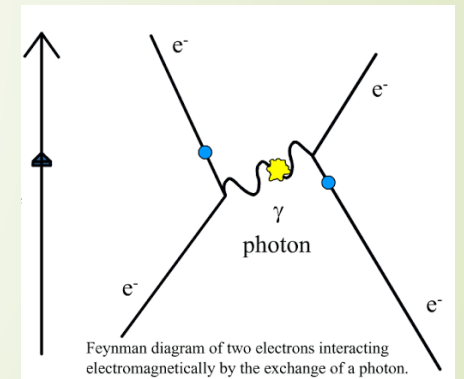
Phys. Rev. **95**, 1300 – Published 1 September 1954

- 1954年, M.Gell-Mann 与 F.E.Low, 提出精细结构常数 α 重整化群变换
引入重整化群方程, 即 β - 函数

随能标跑动至少是二阶效应

$$\mu d\alpha/d\mu = O(\alpha^2) > 0$$

$$\alpha(Q) \rightarrow \alpha = 1/137.036 \dots \text{ as } Q \rightarrow 0$$



精细结构常数表示电子在第一玻尔轨道上的运动速度和真空中光速的比值
目前定义: 电磁相互作用的强度; 上述值仅作为零点值

$$\alpha(M_Z) \rightarrow 1/127$$

$$\mu da/d\mu = O(\alpha^2) > 0$$

争论

- 约1954年, Landau认为, QED中 β -函数的右边是**单增函数**, μ 足够大时, 出现奇点
——朗道奇点

结论: 重整化方法在数学上不完善

- Gell-Mann等人认为 β -函数会出现极限点, 耦合常数会终止于该点 (实验点) ——正好对应裸耦合常数 ——可通过 (未知) 非微扰方式确定。

注: 对QED来说, 上述讨论只具科学意义, the Landau scale $\sim 10^{286}$ eV

形成量子场论有缺陷的观念-迄今仍未严格解决

新可重整模型构造

- 1954年, C.N.Yang和R.Mills提出定域规范不变性, 构造简单拉氏量

$$\mathcal{L}^{\text{YM}} = -\frac{1}{4}G_{\mu\nu}G_{\mu\nu} - \bar{\psi}(\gamma D + m)\psi$$

引入无质量矢量粒子
——不现实对应现实粒子

- 1961年, S. Glashow添上质量项解释弱相互作用

$$\mathcal{L} = \mathcal{L}^{\text{YM}} - \frac{1}{2}M^2 A_\mu^2$$

手动放入质量
——不自然, 破坏定域规范不变性

整体连续对称性的自发破缺必定导致无质量的标量粒子

- 1961年- Goldstone连续对称性自发破缺会产生无质量Goldstone粒子
- 1964年, P.Higgs证明定域规范理论Goldstone粒子会被有质量粒子所取代; F.Englert和R.Brout证明矢量粒子将获得质量
- 1964年, A.Salam采用YM+Higgs机制构造弱相互作用模型
- 1967年, Weinberg将电磁与弱相互作用合而为一

需要解决的问题 - 证明模型的可重整性

- 1961年, 基于S. Glashow模型, Veltman证明其单圈可重整性; 但在继续证明所有圈的可重整性时遇到紫外困难
- 1967年, L.D.Faddeev和V.N.Popov规范场路径积分量子化加入规范约束条件, 保持局域规范不变性同时消除发散
- 1970年, tHooft加入Veltman课题组, 尝试分两步解决问题
 - I) 如何重整无质量纯 Yang-Mills 体系中的振幅? = 微扰论框架
 - II) 如何将这一切与质量项匹配起来? = 利用Higgs 理论

- 1971年，tHooff引入第五维实现无质量Yang-Mills场的单圈重整化；为维数重整化方法的前身。
- 1971-1972年，Taylor和Slavnov提出Slavnov-Taylor等式证明振幅的高阶可重整性。
- 1972年，tHooff和Veltman提出维数正规化/重整化-重整化多圈图有用工具：4- ϵ 维处理，消除对数发散、线性及平方发散将通过分部积分消除。振幅在复 ϵ 平面极点通过引入规范不变抵消项消除。

维数正规化理论可正确地描述观测到的相互作用 下一步，讨论重整化群效应

1964年，V. S. Vanyashin 与 M. V. Terentev 发现 $\beta(g) < 0$

1969年，Khriplovich 对 Yang-Mills 理论中的荷重整化做了正确的计算，结果同样得出不寻常的符号，但这一结果没有与渐进自由联系起来

- 1970年，Callan和Symanzik独立提出方程讨论重整化群效应——指出猜想——“所有可重整化理论的 β 函数与QED一致均大于零”
无法解释Bjorken于1969年发现的标度律——该标度律表明耦合常数在能标大时很小。
- 1973年，Politzer和Gross和Wilczek提出QCD理论的渐近自由性质，解释了标度律。

回到微扰QCD理论 – 按 α_s 展开可估算高能物理量

$$\rho = r_0 \alpha_s^p(\mu_R) + r_1 \alpha_s^{p+1}(\mu_R) + r_2 \alpha_s^{p+2}(\mu_R) + \dots$$

计算到无穷阶或足够高阶
对应物理量真实值

即使高能区，微扰论一直适用？如何保证
'tHooft, Can We Make Sense Out of QCD?

$k \rightarrow \infty$ ，导致紫外发散
重整化理论消发散



保证微扰可靠

往往忽略

正规化、重整化、**能标设定**

高精度预言三步走

准确预言具同等重要性

物理观测量

基本概念：微扰论预言应与人为引入的参数无关
重整化群不变性

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial R} \equiv 0$$

有没有可能获得**有限阶**下，与重整化能标和方案选择均无关的微扰论估算值？

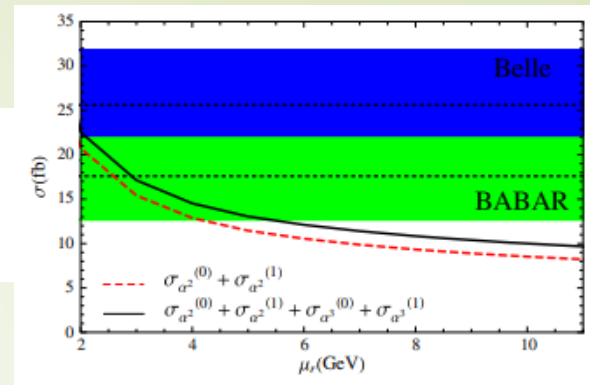
原始微扰序列 $\frac{\partial \rho_n}{\partial \mu_R} \neq 0$ n - 微扰阶数

传统观点：重整化能标依赖性不能避免，出路在于完成越来越高阶计算，压低不确定性；问题核心转变为确定“最优能标”【能解释实验最好】以及确定相应的误差。

该观点关键：能否找到方法适用于所有高能物理过程

$$\hat{\mu} = \frac{1}{w} \sum_{i=1}^n w_i x_i, \quad w_i = 1/\sigma_i^2 \text{ and } w = \sum_i w_i$$

The standard deviation of $\hat{\mu}$ is $1/\sqrt{w}$.



$e^+e^- \rightarrow J/\psi + \eta_c$

传统如何选择重整化能标

“猜”

- 1) 提高微扰收敛性
--消除发散Renormalon-项
- 2) 解释实验测量值

=> “Choose” its value (=Q) to eliminate large logs or the one to improve pQCD convergence

=> “Vary” its value within certain range such as $[1/2Q, 2Q]$ or $[1/3Q, 3Q]$ to discuss its uncertainty

重整化能标误差成为QCD理论重要系统误差之一

微扰收敛性破坏两种情况

Explicit breaking: α_s 值偏大

Implicit breaking: 系数发散 (如Renormalon-发散)

内禀

外禀

传统方案问题

1) 收敛性强烈依赖 α_s -压低;

消除log-项可提高收敛性, 能标误差区间任意且是**量级估算**;

微扰收敛性不同能标下会不一致, **无法确定序列内禀微扰属性**;

若出现不收敛, 无法判断这是内禀属性还是选择不恰当能标;

与实验不一致, **【无法高精度判断是否存在新物理以及微扰论是否适用】**;

2) 阶数增加也**无法获得每阶准确值**【深层次原因: 耦合常数值与系数不匹配】, 从而**无法获得准确微扰序列**;

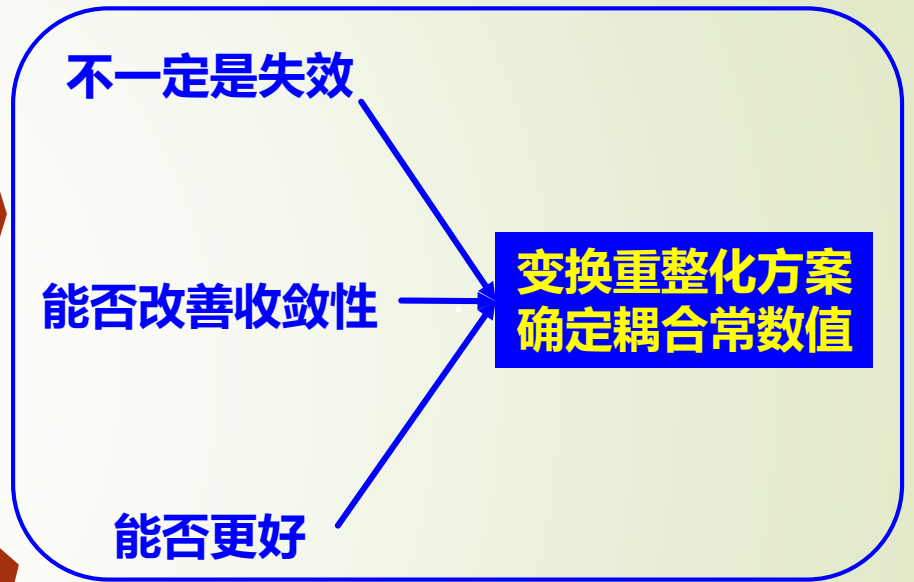
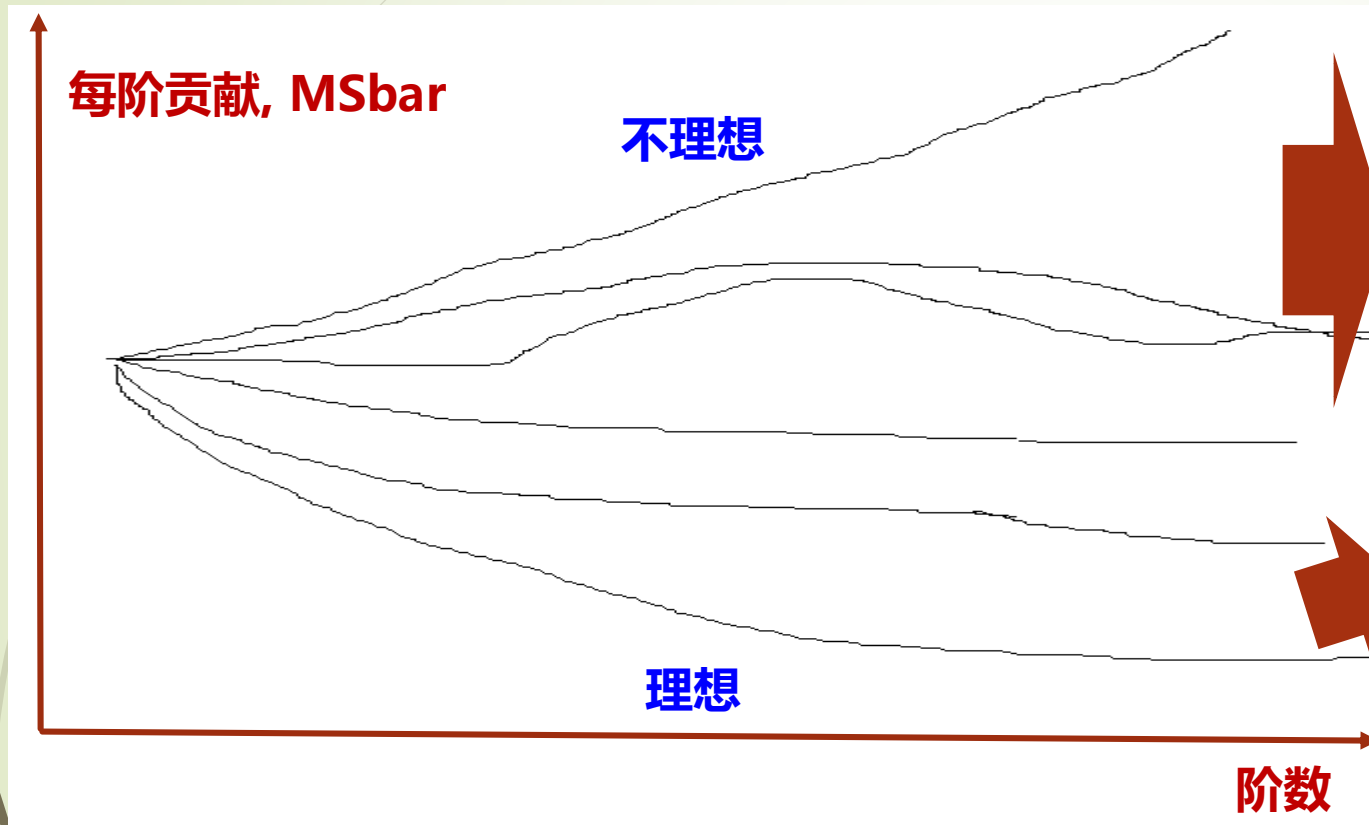
3) 理论预言**与重整化方案有关**;

3) 无法获得令人信服的基于不准确微扰序列所估算出的未知高阶贡献

**传统做法极大地降低微扰论计算精度及预言能力
有没有可能解决或改善?**

微扰收敛性多种情形均出现，突显能标问题的重要性

微扰区域 --- $\alpha_s < 1$



α_s -幂次压低不足以抵消大的微扰系数
大系数源自那里？

Renormalon-发散

$$\alpha_s^n \beta_0^n n! \beta_i \approx \beta_0^{i+1}$$

随阶数变化的每阶贡献示意图—从理想到不理想
如何看待？

有限阶高精度微扰论预言的三要素

一、寻找高效微扰论高阶计算方法

== 能标问题的基础

目标：完成更高阶微扰计算，特定相空间处理处理
包括振幅计算、Feynman积分化简、多体相空间、自动化程序、重求和、FF等
逐步降低重整化能标不确定性

二、寻找满足不变性的重整化能标设定方案

== 能标问题的核心

目标：最大程度地获得有限阶下精确预言
与重整化能标及重整化方案无关的能标设定方案极其重要

三、寻找估算未知高阶项贡献可靠方法

== 能标问题的延拓

目标：基于已知微扰序列，估算未知阶贡献，更好判断有无新物理
已知序列若具备更好微扰收敛性，有助于压低未知高阶项所导致的理论不确定性

微扰论不确定性
输入参数 (m, α, \dots)
重整化、因子化能标
未知项贡献

pQCD理论提出不久，人们就已在考虑是否存在更好方案
想法：基于某种原理或某种设想来确定
最优重整化能标

**上世纪80年代，诞生BLM，PMS，
FAC等典型方案**

PMS-寻找数值稳定点

FAC-寻找与实验值接近点

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial R} \equiv 0$$

局域重整化群不变性

Optimized perturbation theory – minimize the higher-order contributions – **PMS**

How about directly set it to satisfy the RGI

Any observable \Leftrightarrow an effective coupling constant (idea useful)
Fastest Apparent Convergence - **FAC**

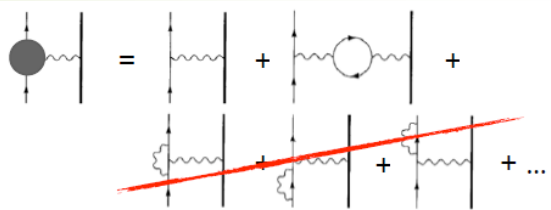
How about cut off all higher-order-terms ?

满足通常RGE

与实验结合

上世纪提出的三大解决方案

BLM



BLM \Rightarrow nf-term
QED极限 = GM-L方案
CSR \Rightarrow 解决方案不确定性

所有真空极化图贡献
相加确定能标

GM-L方案
QED不存在重整化能标
设定问题
“1/137-本身数值小”

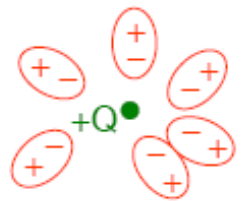
源自类比观察

检验能标设定方法：取QED极限能否回得GM-L方案

电磁相互作用 (QED) 和强相互作用 (QCD) 之相互借鉴

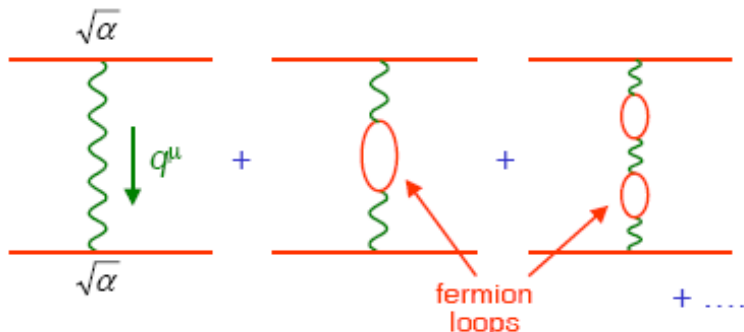
Running Couplings

◆ **QED:**

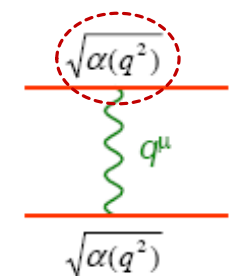


“Bare” electric charge is screened by “halo” of e^+e^- pairs

In terms of Feynman diagrams:



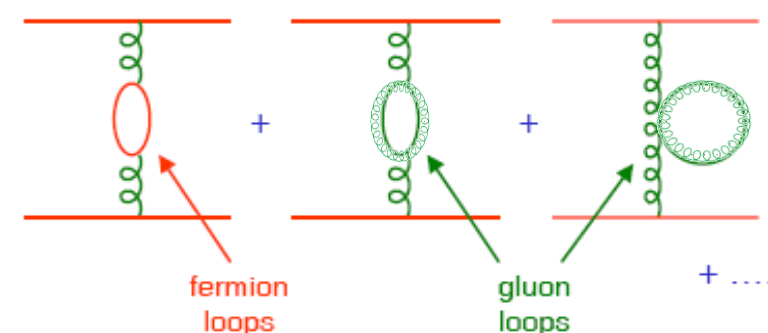
→ infinite series of diagrams which can be (approximately) summed:

$$\alpha(q^2) = \alpha(q_0^2) / \left[1 - \frac{\alpha(q_0^2)}{3\pi} \ln\left(\frac{q^2}{q_0^2}\right) \right]$$


→ replace by single diagram with running coupling

**q₀初始能标
恰当重求和可获
与q₀无关结果**

◆ **QCD:** similar, but now have:



Extra diagrams produce anti-screening

$$\alpha_s(q^2) = \alpha_s(q_0^2) / \left[1 + B\alpha_s(q_0^2) \ln\left(\frac{q^2}{q_0^2}\right) \right]$$

where $B = \frac{11N_c - 2N_f}{12\pi}$ with $N_c = 3$ $N_f = 6$

思考：正确方案究竟应该是什么样子

【不自动满足不变性】

- 选择任意重整化能标、任意重整化方案完成可能的高阶计算，获得初始微扰序列
- 基于某种理念或普适方法确定能标设定方法，寻找“有效”能标。
找到真正的微扰展开量，证明新微扰序列与能标选择无关
- 最后，证明新微扰论序列也与重整化方案无关

【重整化群不变性】

转化问题

符合重整化群不变性、正确的重整化能标设定方案究竟是什么？

PMS or BLM or FAC or 其它？

基于《重整化群方程》讨论能标方案自治性

PHYSICAL REVIEW D 86, 054018 (2012)

Self-consistency requirements of the renormalization group for setting the renormalization scale

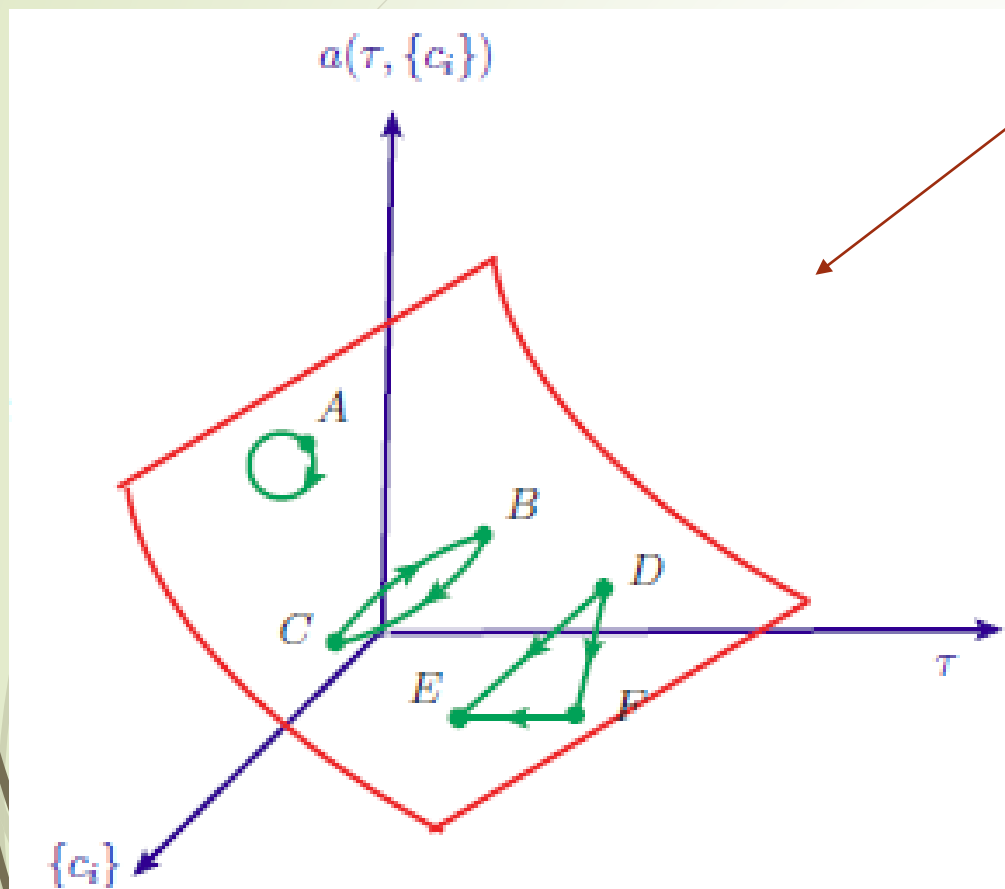
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判断理论预言是否满足**重整化群的基本性质**
自反性、对称性、传递性等



PMS--不满足传递性

IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. 78 (2015) 126201 (15pp)

doi:10.1088/0034-4885/78/12/126201

Review

基于重整化群不变性的探讨

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

Xing-Gang Wu¹, Yang Ma¹, Sheng-Quan Wang¹, Hai-Bing Fu¹,
Hong-Hao Ma¹, Stanley J Brodsky² and Matin Mojaza³

PMS存在非物理和微扰性问题 ---有效但非根本方案

PMS基本性质

压低Renormalon-发散项
不满足一般的微扰收敛性
低阶预言通常不可靠

贡献集中于低阶
说明数值稳定性是其最大目的

PMS只有总预言值才有意义

$R_{e^+e^-}$ four-loop level

	$n_f=3$	$n_f=4$	$n_f=5$
C_1	1.6401	1.5249	1.4097
C_2	-10.284	-11.6857	12.8047
C_3	-106.896	-92.9124	80.0075
C_1^{PMS}	-0.458	-0.1105	0.0479
C_2^{PMS}	-1.1361	0.2103	1.3075
C_3^{PMS}	-2.213	2.4988	16.4108

Factorial divergence suppressed!

R_τ (four loop)

	C'_1	C'_2	C'_3
Conv.	5.2023	26.3659	127.079
PMS	0.3906	1.2380	-6.1747

TABLE I. Coefficients for the perturbative expansion of $R_S(Q)$ before and after the PMS scale setting, where we have set $Q = 1.2$ GeV for $n_f=3$, $Q = 3$ GeV for $n_f=4$, and $Q = 31.6$ GeV for $n_f=5$.

TABLE V. Coefficients for the perturbative expansion of r_3^5 and after the PMS scale setting. $\mu_0 = M_\tau$.

$\Gamma(H \rightarrow b\bar{b})$ (four loop)

	C''_1	C''_2	C''_3
Conv.	29.145	41.765	-825.598
PMS	0.34376	21.2286	-142.849

TABLE IX. Coefficients for the perturbative expansion of \tilde{R}_3 before and after the PMS scale setting. $\mu_0 = M_H$.

$R_{e^+e^-}$ four-loop level

	LO	NLO	N ² LO	N ³ LO	total
Conv.	0.04499	0.00285	-0.00117	-0.00033	0.04635
PMS	0.04608	0.00010	0.00013	0.00007	0.04638

Accidental series convergence

R_τ (four loop)

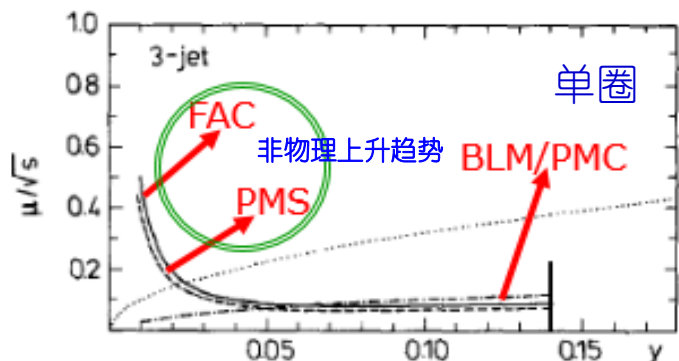
	LO	NLO	N ² LO	N ³ LO	total
Conv.	0.10320	0.05541	0.02898	0.01441	0.20200
PMS	0.19935	0.01552	0.00981	-0.00975	0.21493

$q_{3,PMS}^{LO} \gg q_{3,PMS}^{NLO} \sim q_{3,PMS}^{N^2LO} \sim q_{3,PMS}^{N^3LO}$

	LO	NLO	N ² LO	N ³ LO	total
Conv.	0.20371	0.03767	0.00194	-0.00138	0.24194
PMS	0.23967	0.00061	0.00161	-0.00046	0.24144

$q_{3,PMS}^{NLO} < q_{3,PMS}^{N^2LO}$

$\Gamma(H \rightarrow b\bar{b})$ (four loop)



G. Kramer and B. Lampe, Z. Phys. C 39, 101 (1988).

利用不变量 实现不同方案间表达式转换

Those RG invariants are also helpful for transforming the pQCD approximant ϱ_n under the \mathcal{R} scheme to any other scheme (e.g. the \mathcal{S} scheme). This can be achieved by applying two transformations simultaneously,

$$a_s^{\mathcal{R}} \rightarrow a_s^{\mathcal{S}} \text{ and } C_i^{\mathcal{R}} \rightarrow C_i^{\mathcal{S}}. \quad (9)$$

The coupling constant $a_s^{\mathcal{S}}$ can be derived from $a_s^{\mathcal{R}}$ by using the extended RG equations, in which the $\beta_{i>2}^{\mathcal{S}}$ -terms that determine the scale running behavior of $a_s^{\mathcal{S}}$ can be achieved by using the relation,

$$\beta^{\mathcal{S}}(a_s^{\mathcal{S}}) = (\partial a_s^{\mathcal{S}} / \partial a_s^{\mathcal{R}}) \beta^{\mathcal{R}}(a_s^{\mathcal{R}}). \quad (10)$$

The coefficients $C_i^{\mathcal{S}}$ can be obtained from the coefficients $C_i^{\mathcal{R}}$ by using the RG-invariants ρ_i , e.g. up to N³LO level, we have

$$C_1^{\mathcal{S}} = C_1^{\mathcal{R}}, \quad (11)$$

$$C_2^{\mathcal{S}} = C_2^{\mathcal{R}} + \frac{p}{16\beta_0} (\beta_2^{\mathcal{R}} - \beta_2^{\mathcal{S}}), \quad (12)$$

$$C_3^{\mathcal{S}} = C_3^{\mathcal{R}} + \frac{p+1}{16\beta_0} C_1^{\mathcal{R}} (\beta_2^{\mathcal{R}} - \beta_2^{\mathcal{S}}) + \frac{p}{128} (\beta_3^{\mathcal{R}} - \beta_3^{\mathcal{S}}). \quad (13)$$

It is noted that the above method for scheme transformation is useful, since at present, most of the calculation are done within the $\overline{\text{MS}}$ -scheme.

$$R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

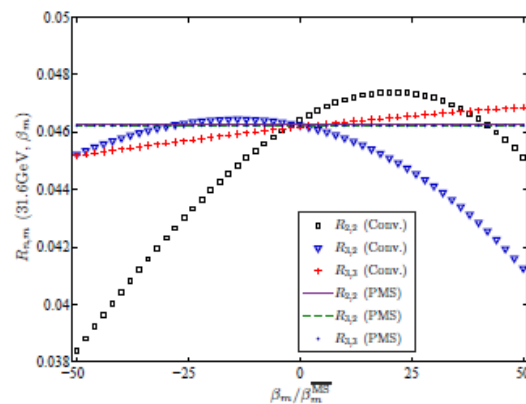


FIG. 2. The pQCD predictions for $R_{n,m}(Q, \beta_m)$ versus the $\beta_{m>2}$ -term at the N²LO ($n=2$) and N³LO ($n=3$) levels respectively. $Q=31.6$ GeV. As a comparison, both the results for conventional scale setting and PMS are presented.

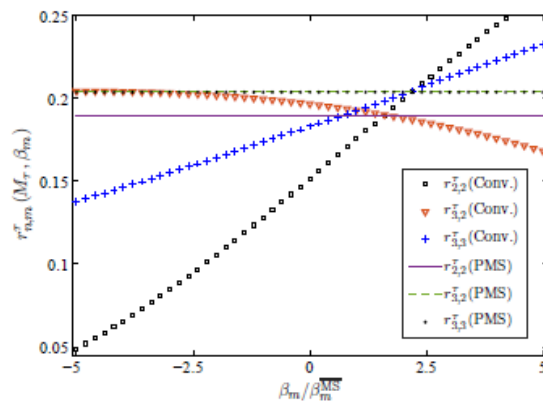


FIG. 5. The pQCD predictions for $r_{n,m}^{\tau}(M_{\tau}, \beta_m)$ versus the $\beta_{m>2}$ -term at the N²LO ($n=2$) and N³LO ($n=3$) levels respectively. As a comparison, both the results for conventional scale setting and PMS are presented.

$$R_{\tau}(M_{\tau}) = \frac{\Gamma(\tau \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma(\tau \rightarrow \nu_{\tau} + e^{-}\bar{\nu}_e)}$$

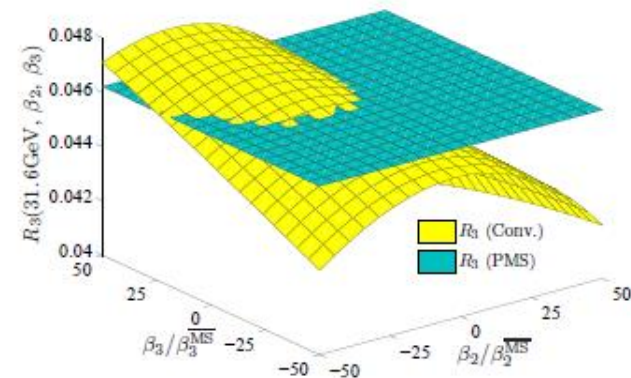


FIG. 3. Comparison of the combined $\{\beta_2, \beta_3\}$ -dependence for the N³LO prediction R_3 under the conventional (Conv.) scale setting and PMS, respectively. $Q=31.6$ GeV.

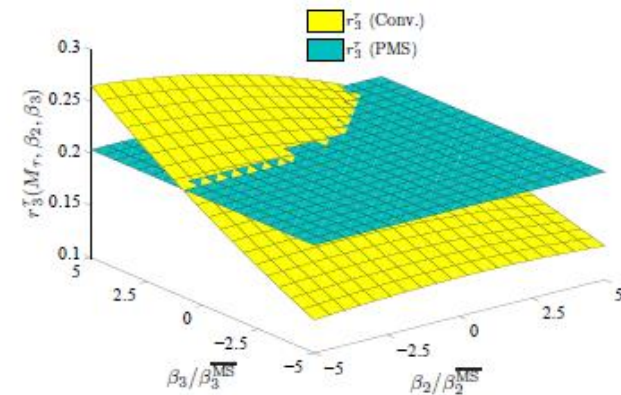


FIG. 6. Comparison of the combined $\{\beta_2, \beta_3\}$ -dependence for the N³LO prediction r_3^{τ} under the conventional (Conv.) scale setting and PMS, respectively.

BLM取得一定成功

基于nf-项处理浮于表象，且很难处理高阶项
如何延拓到单圈之外，seBLM是有效方法但是不成功尝试

seBLM方案

JHEP 06 (2007) 009

$$a_s(\mu) = \frac{a_s(\mu_0)}{1 - \beta(a_s(\mu_0))/a_s(\mu_0) \ln \frac{\mu^2}{\mu_0^2}}$$

seBLM – 直接基于大 β_0 近似，目的变成提高QCD微扰收敛性；
(随后改进，seBLM+双重展开，仍然基于近似RGE解)

Phys.Lett.B770(2017)494

$$a_s(\mu) = \frac{a_s(\mu_0)}{1 - \beta(a_s(\mu_0))/a_s(\mu_0) \ln \frac{\mu^2}{\mu_0^2}}$$

即能发挥BLM优点，又可解决高阶问题
正确的出路在那？

单圈成功、突出 β_0 -》Renormalon

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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G. Peter Lepage

Institute for Advanced Study, Princeton, New Jersey 08540

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Paul B. Mackenzie

Fermilab, Batavia, Illinois 60510

(Received 23 November 1982)

BLM

$$\rho_n(Q) = a_s(\mu) \left(1 + \sum_{i=1}^n C_i(\mu, Q) a_s^i(\mu) \right), \quad (29)$$

where Q is the scale at which ρ is measured and $a_s = \alpha_s/\pi$.
The local RGI indicates that

$$\left(\frac{\partial \rho_n}{\partial \tau} \right) = \left(\frac{\partial}{\partial \tau} \Big|_{a_s} + \beta(a_s) \frac{\partial}{\partial (a_s/4)} \right) \rho = 0, \quad (30)$$

$$\left(\frac{\partial \rho_n}{\partial \beta_m} \right) = \left(\frac{\partial}{\partial \beta_m} \Big|_{a_s} - \beta(a_s) \int_0^{a_s/4} d\left(\frac{d_s}{4}\right) \frac{(d_s/4)^{m+2}}{[\beta(d_s)]^2} \frac{\partial}{\partial (a_s/4)} \right) \rho_n = 0, \quad (m = 2, 3, \dots) \quad (31)$$

谁更成功

引用高不一定正确

也许，次要点更重要

需要创新和发散思维
需要多尝试

PMS

引入拓展重整化群方程

RESOLUTION OF THE RENORMALISATION-SCHEME AMBIGUITY IN PERTURBATIVE QCD

P.M. STEVENSON

Physics Department, University of Wisconsin-Madison, Madison, WI 53706, USA

Received 24 November 1980

Next-to-leading-order QCD predictions depend on the arbitrary choice of renormalisation scheme. I resolve this ambiguity and show how to find the unique, optimum prediction, given the result in some arbitrary, initial scheme.

FAC

引入有效强耦合常数概念


RENORMALIZATION GROUP IMPROVED PERTURBATIVE QCD

G. GRUNBERG ¹

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

Received 5 June 1980


综述：系统对比典型方案



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Progress in Particle and Nuclear Physics

Journal homepage: www.elsevier.com/locate/ppnp




Review

The renormalization scale-setting problem in QCD

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- I) “猜”是自然而简单的选择
- II) 寻找最优能标是一种进步，但并非解决问题，只是掩盖问题
- III) 有没有可能存在深层次解决方案？ --- 或许就是PMC

$$\alpha_s(q^2) = \alpha_s(q_0^2) / \left[1 + B \alpha_s(q_0^2) \ln \left(\frac{q^2}{q_0^2} \right) \right]$$

观察：β-项比nf-项更为基本

二 最大共形原理

重整化群方程确定微扰展开式中强耦合常数值

- 利用重整化群方程确定每一阶相应的beta-项
- 将同类型已知beta-项重求和，确定强耦合常数准确值
- 进一步，确定每一阶有效能标的准确值
- 证明新的微扰序列与重整化能标选择无关
- 证明新的微扰序列与重整化方案选择无关

确定相应的PMC能标，早期称为最优能标
现称为有效能标

$$\beta = - \sum_{i=0}^{\infty} \beta_i \cdot \left(\frac{\alpha_s(\mu_r)}{4\pi} \right)^{i+2}$$

PMC发展历程

- 1981年, Brodsky等提出BLM机制 (单圈)
- 1992年, 提出 (错误) 延拓方案认为BLM机制存在问题 – 包括Stevenson, Grunberg等
- 1995年, 提出自洽能标对应关系CSR, 拓展BLM机制到两圈
- 1997年, 提出采用 β -函数替换BLM中 n_f -项想法, 但一直未明确如何实现该想法
- 2007年, 拓展seBLM到高圈, 但基于大 β_0 近似, 目标变成提高微扰收敛性 (走偏了)
- 2011年, 明确 β -函数更基础的想法, 命名PMC (单圈重新表述BLM)
-
- 2011年, 提出PMC – BLM对应原理, 实现PMC
真正实现BLM思想到任意阶 (PMC首篇正式发表论文)
- 2012年, 将PMC用于Top对截面及正反不对称性分析, 获得与实验一致结果
- 2014年, 给出对应原理解释, 提出PMC新的公式体系
- 2015年, 证明简并关系普适性以及PMC两种公式体系的微扰等价性
基于重整化群不变性完成PMC与PMS深入对比
- 2016年, 提出PMC单能标方案, 证明不同PMC实现方案的微扰等价性
- 2017年, 基于PMC单能标方案将自洽能标对应关系CSR推到任意高阶
尝试将PMC应用于低能区的可能性
- 2018年, 证明PMC对于任意重整化方案均有方案无关性
提出基于PMC共形序列以及Pade估算未知高阶项方法
-
- 2020年, 提出新的有效方案 PMC_∞ -基于Intrinsic Conformality
- 2021年, 证明 PMC_∞ 与 PMC_a 等价, 可以通过消beta-项使每阶系数与能标无关
- 2022年, 基于PMC单能标方案获得强耦合常数在整个物理能区内的光滑衔接
- 2023年, 提出估算未知高阶项贡献的新思路 (Bayes分析、线性回归)
完整的微扰表达式不确定性分析、高圈CSR关系。。。

萌芽

发展

邀请综述

2013 PPNP

2015 RPP

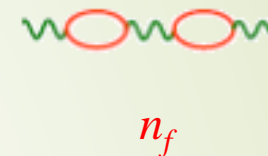
2019 PPNP

2023 PPNP (投)

进一步发展

基础方案：逐阶完成 — PMC多能标方案

$$\rho(Q) = \sum_{i=1}^n \left(\sum_{j=0}^{i-1} c_{i,j} n_f^j \right) a_s^{p+i-1}(\mu_r) + \mathcal{O}(a_s^{n+1}),$$



利用重整化群方程，将原始nf-序列重组或转换为β-函数序列

$$\begin{aligned} \rho(Q) = & r_{1,0} a_s(\mu_r) + (r_{2,0} + \beta_0 r_{2,1}) a_s^2(\mu_r) \\ & + (r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}) a_s^3(\mu_r) \\ & + (r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2} \beta_1 \beta_0 r_{3,2} \\ & + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}) a_s^4(\mu_r) + \mathcal{O}(a_s^5) \end{aligned}$$

简并关系

逐阶消去β-函数，可获得标准的多能标PMC方案表达式

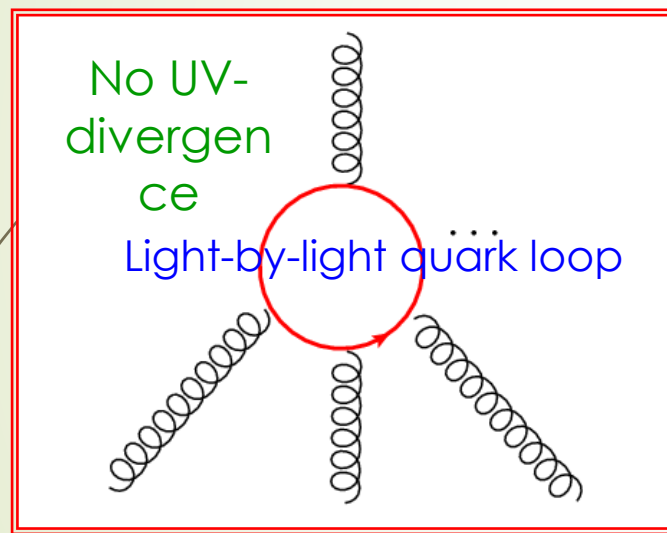
$$\rho(Q) = \hat{r}_{1,0} a_s(Q_1) + \hat{r}_{2,0} a_s^2(Q_2) + \hat{r}_{3,0} a_s^3(Q_3) + \hat{r}_{4,0} a_s^4(Q_4) + \mathcal{O}(a_s^5),$$

$$\ln \frac{Q_k^2}{Q^2} = \frac{-r_{k+1,1}/r_{k,0}}{1 + \frac{1}{2} \left[\frac{\partial \beta}{\partial a} + (k-1) \frac{\beta}{a} \right] \ln \frac{Q_k^2}{Q^2} + \dots}$$

自洽：展开到对应阶数
微扰展开

证明简并关系对于非阿理论具有普遍性
Phys.Lett.B 748 (2015) 13-18

**一、源自重整化群方程的简并关系的应用极为重要，
导致 n_f 与 β 系数一一对应，无需引入额外假设**



**二、区分 n_f 是否源自RGE非常重要
确定正确的强耦合常数值**

Those n_f -terms, which are irrelevant to the ultra-violet cutoff and have no relation to the β -terms, should be identified and kept separately after the PMC scale setting

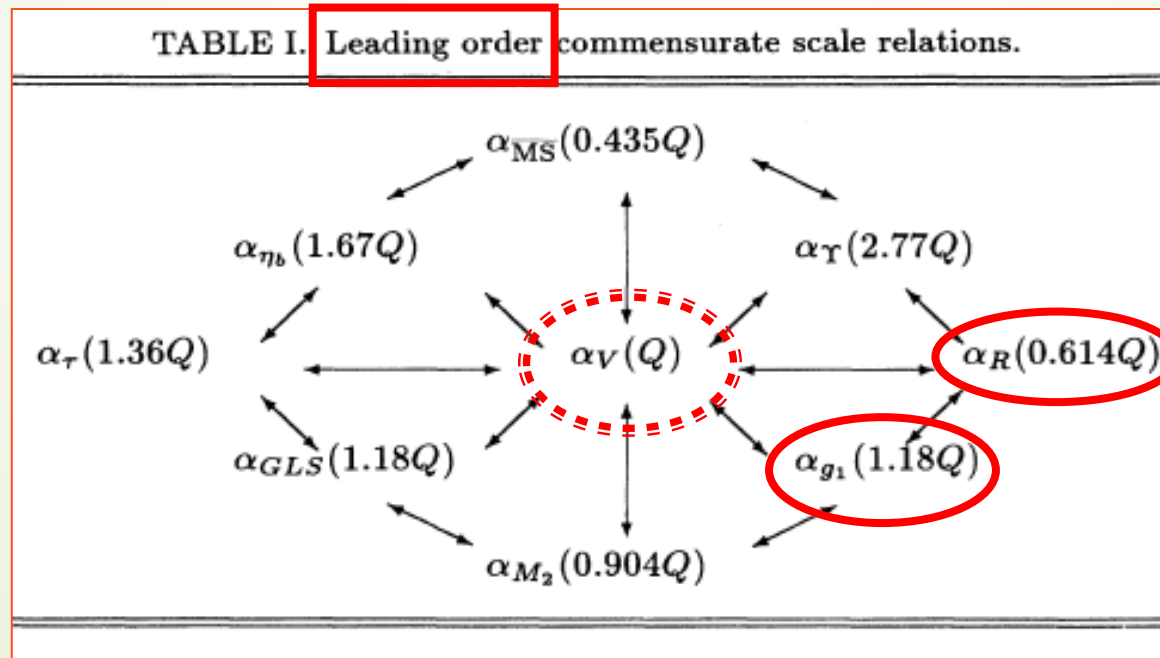
**区分表达式中与 α_s 跑动无关
但与质量跑动相关的 n_f -项**

$$\frac{\partial \rho_\delta}{\partial \delta} = -\beta(a_s) \frac{\partial \rho_\delta}{\partial a_s} - \overline{m} \gamma_m(a_s) \frac{\partial \rho_\delta}{\partial \overline{m}}$$

2209.10777

PMC多能标方案下很难给出重整化方案无关性的严格证明
举例证明 --- 自治能标关系 (CSR) 保证不同方案下结果一致

S.J. Brodsky and H.J. Lu, Phys.Rev. D51, 3652(1995)



更高阶CSR的举例, 正在准备当中

简化方案：PMC单能标方案

未知高阶项导致残留能标及方案不确定性

- 残留的能标依赖性** – 原因：PMC能标为微扰展开
越高阶项的PMC能标，越不确定
- 思路：引入单一有效强耦合常数，单一PMC能标
 - 解决：**PMC单能标方案 (2017)**
- 残留的方案依赖性** – 原因：重整化群方程自身与方案有关
- 思路：引入方案无关的重整化群方程，简化处理
 - 解决：**单参数C-方案强耦合常数 (2018)**

PMC单能标方案

$$\begin{aligned} \rho(Q) = & r_{1,0}\alpha(\mu)^p + [r_{2,0} + p\beta_0 r_{2,1}] \alpha(\mu)^{p+1} + \left[r_{3,0} + p\beta_1 r_{2,1} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2} \beta_0^2 r_{3,2} \right] \alpha(\mu)^{p+2} \\ & + \left[r_{4,0} + p\beta_2 r_{2,1} + (p+1)\beta_1 r_{3,1} + \frac{p(3+2p)}{2} \beta_1 \beta_0 r_{3,2} + (p+2)\beta_0 r_{4,1} + \frac{(p+1)(p+2)}{2} \beta_0^2 r_{4,2} \right. \\ & \left. + \frac{p(p+1)(p+2)}{3!} \beta_0^3 r_{4,3} \right] \alpha(\mu)^{p+3} + \dots, \end{aligned}$$

$$\rho(Q) = \sum_{n \geq 1} r_{n,0} \alpha(\mu)^{n+p-1} + \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2} \beta] \sum_{j \geq 1} (-1)^j \Delta_n^{(j-1)} r_{n+j,j}$$

根据重整化群不变性,

$$r_{i,j} = \sum_{k=0}^j C_j^k \tilde{r}_{i-k,j-k} L^k$$

其中, $\tilde{r}_{i,j} = r_{i,j}|_{\mu=Q}$ $L = \ln \mu^2 / Q^2$

$$\begin{aligned} \rho(Q) = & \sum_{n \geq 1} \tilde{r}_{n,0} \alpha(\mu)^{n+p-1} + \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2} \beta] \sum_{j \geq 1} (-1)^j \Delta_n^{(j-1)} \tilde{r}_{n+j,j} \\ & + \sum_{k \geq 1} L^k \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2} \beta] \sum_{j \geq k} (-1)^j C_j^k \Delta_n^{(j-1)} \tilde{r}_{n+j-k,j-k}, \end{aligned}$$

$$\rho(Q) = \sum_{n \geq 1} \tilde{r}_{n,0} \alpha(Q_*)^{n+p-1}$$

$$\ln \frac{Q_*^2}{Q^2} = T_0 + T_1 a_s(Q) + T_2 a_s^2(Q) + \mathcal{O}(a_s^3),$$

吸收 β 项, 确定单个有效PMC能标

C-方案强耦合常数

存在残留方案依赖性原因，实际在于RGE本身是依赖于方案的；
为最大程度的降低方案依赖性，有没有可能RGE是不依赖于方案的？

重新定义强耦合常数有可能实现目标

$$\frac{1}{a_\mu} + \frac{\beta_1}{\beta_0} \ln a_\mu = \beta_0 \left(\ln \frac{\mu^2}{\Lambda^2} - \int_0^{a_\mu} \frac{da}{\tilde{\beta}(a)} \right).$$

As suggested by Ref.(Boito *et al.*, 2016), one can define a new coupling $\hat{a}_\mu = \hat{a}_s(\mu)/\pi$ in the following way:

$$\frac{1}{\hat{a}_\mu} + \frac{\beta_1}{\beta_0} \ln \hat{a}_\mu = \beta_0 \left(\ln \frac{\mu^2}{\Lambda^2} + C \right),$$

$$\hat{\beta}(\hat{a}_\mu) = \mu^2 \frac{\partial \hat{a}_\mu}{\partial \mu^2} = -\frac{\beta_0 \hat{a}_\mu^2}{1 - \frac{\beta_1}{\beta_0} \hat{a}_\mu} = -\beta_0 \hat{a}_\mu^2 \sum_{i=0}^{\infty} (\beta_1/\beta_0)^i \hat{a}_\mu^i.$$

$$\frac{\partial \hat{a}_\mu}{\partial C} = \hat{\beta}(\hat{a}_\mu). \quad \text{随能标及方案跑动RGE}$$

$$\begin{aligned} \hat{a}_\mu = & a_\mu - C \beta_0 a_\mu^2 + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} + \beta_0^2 C^2 - \beta_1 C \right) a_\mu^3 \\ & + \left[\frac{\beta_1^3}{2\beta_0^3} - \frac{\beta_3}{2\beta_0} + \left(2\beta_2 - \frac{3\beta_1^2}{\beta_0} \right) C + \frac{5}{2} \beta_0 \beta_1 C^2 - \beta_0^3 C^3 \right] a_\mu^4 + \mathcal{O}(a_\mu^5). \end{aligned}$$

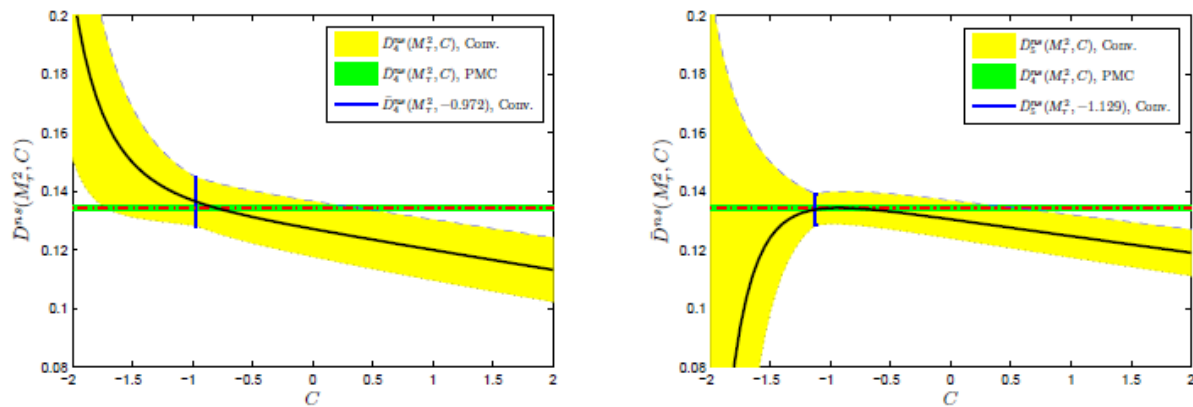


FIG. 4 (color online) $\bar{D}^{ns}(M_\tau^2, C)$ for the Adler function as a function of the parameter C . The solid line is the prediction using conventional scale setting, the lighter-shaded band is the uncertainty for a four-loop prediction $\Delta = \pm |\hat{c}_4(\mu/M_\tau)\hat{a}_\mu^4|_{\text{MAX}}$ (Left) and for an approximate five-loop prediction $\Delta = \pm |\hat{c}_5(\mu/M_\tau)\hat{a}_\mu^5|_{\text{MAX}}$ (Right), where MAX is the maximum value for $\mu \in [M_\tau, 4M_\tau]$. When $C = -0.972$ (Left) and $C = -1.129$ (Right), the error bar as shown by a vertical solid line is the minimum. The dash-dot line represents the four-loop PMC prediction, and the darker shaded band is for $\Delta = \pm |\hat{r}_{4,0}\hat{a}_{Q_*}^4|$. The independence of the PMC prediction on the parameter C demonstrates its scheme-independence.

$$\frac{\partial \hat{\rho}_n}{\partial C} = -\frac{\partial \hat{a}_\mu}{\partial C} \frac{\partial \hat{\rho}_n}{\partial \hat{a}_\mu} = -\mu^2 \frac{\partial \hat{a}_\mu}{\partial \mu^2} \frac{\partial \hat{\rho}_n}{\partial \hat{a}_\mu} = -\hat{\beta}(\hat{a}_\mu) \frac{\partial \hat{\rho}_n}{\partial \hat{a}_\mu},$$

消除 β -项，即消除方案C依赖

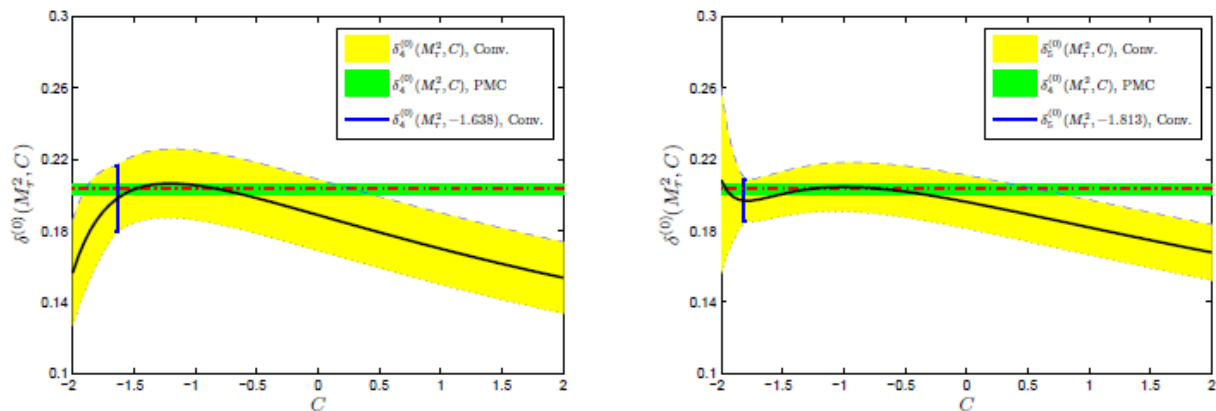



FIG. 8 (color online) $\delta^{(0)}(M_\tau^2, C)$ for τ decay as a function of the parameter C . The solid line is the prediction using conventional scale setting, the lighter-shaded band is the uncertainty for a four-loop prediction $\Delta = \pm |\hat{c}_4(\mu/M_\tau)\hat{a}_\mu^4|_{\text{MAX}}$ (Left) and for an approximate five-loop prediction $\Delta = \pm |\hat{c}_5(\mu/M_\tau)\hat{a}_\mu^5|_{\text{MAX}}$ (Right), where MAX is the maximum value for $\mu \in [M_\tau, 4M_\tau]$. When $C = -1.638$ (Left) and $C = -1.813$ (Right), the error bar as shown by a vertical solid line is the minimum. The dash-dot line represents the four-loop PMC prediction, and the darker shaded band is for $\Delta = \pm |\hat{r}_{4,0}\hat{a}_{Q_*}^4|$. The independence of the PMC prediction on the parameter C demonstrates its scheme-independence.

综述：系统阐述解决思路



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Review

The QCD renormalization group equation and the elimination of fixed-order scheme-and-scale ambiguities using the principle of maximum conformality

Xing-Gang Wu ^{a,*}, Jian-Ming Shen ^{a,b}, Bo-Lun Du ^a, Xu-Dong Huang ^a, Sheng-Quan Wang ^c, Stanley J. Brodsky ^{c,*}

同时给出：PMC预言与重整化能标和重整化方案无关的一个证明

最大共形原理的一些应用实例

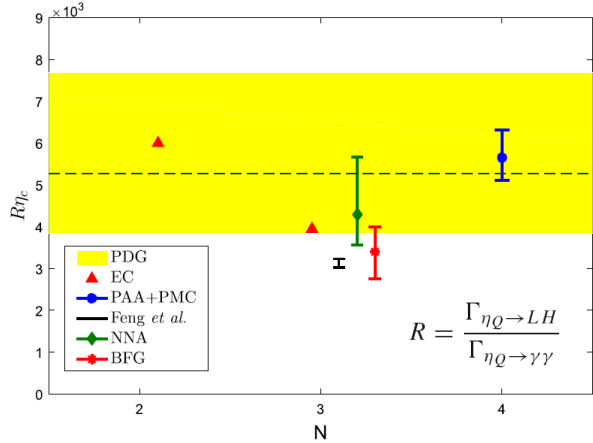


Fig. 3 The R_{η_c} -ratio under various approaches. “EC” is the exact prediction by using the known NLO or NNLO PMC series, and “PAA+PMC” is the PAA prediction by using the PMC NNLO series. The error of “PAA+PMC” at the NNNLO level ($N=4$) is caused by $\Delta\alpha_s(M_Z^2) = \pm 0.0011$. As comparisons, the PDG value, the NNLO predictions ($N=3$) of Feng et al. [10], NNA [11] and BFG [11] are also presented

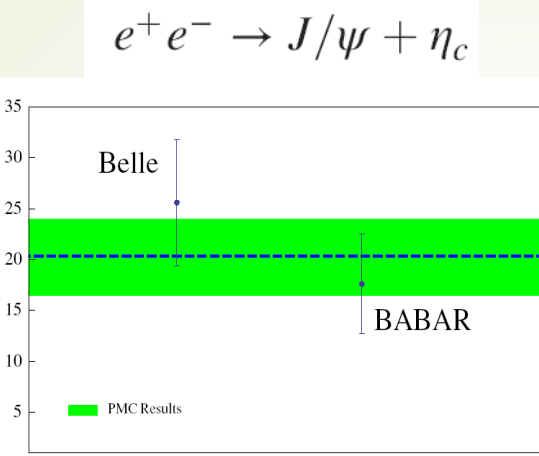
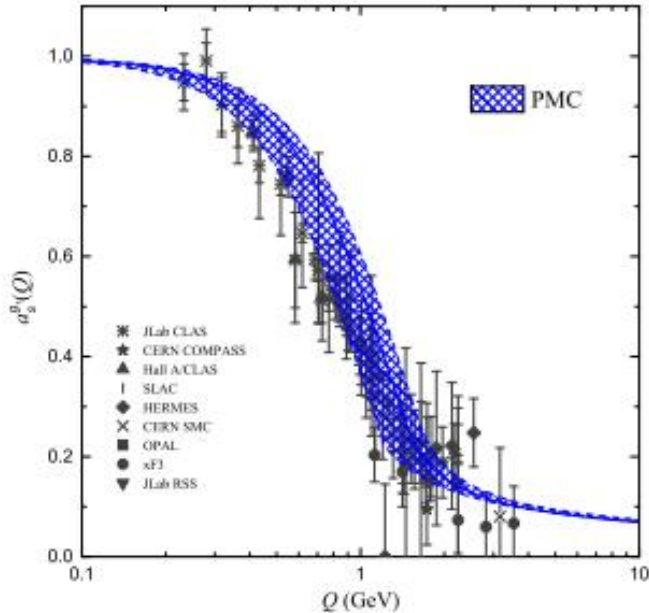


FIG. 3. Our final PMC prediction for the total cross section of $e^+e^- \rightarrow J/\psi + \eta_c$. The shaded band denotes the squared average errors of the uncertainties from $m_c = 1.5 \pm 0.1$ GeV and the wave functions at the zero $|R_s^{\eta_c}(0)|^2 = |R_s^{J/\psi}(0)|^2 = 0.978 \pm 0.04$ GeV³, whose central value is for $m_c = 1.5$ GeV and $|R_s^{\eta_c}(0)|^2 = |R_s^{J/\psi}(0)|^2 = 0.978$ GeV³. The Belle [2] and BABAR [3] measurements are presented as a comparison.

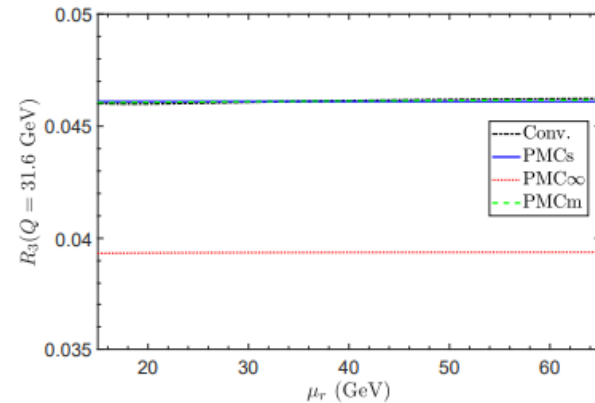


FIG. 1. The renormalization scale dependence of the four-loop prediction $R_3(Q = 31.6$ GeV) using the conventional PMCm, PMCs and PMC_∞ scale-setting procedures.

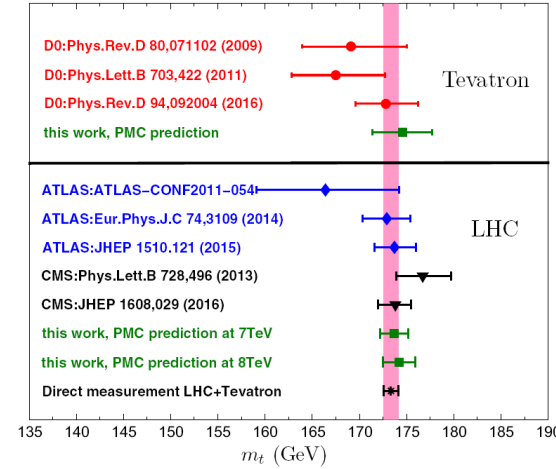


FIG. 6 A summary of the top-quark pole mass determined indirectly from the top-quark pair production channels at the Tevatron and LHC. As a reference, the combination of Tevatron and LHC direct measurements of the top-quark mass is presented as a shaded band. It gives $m_t = 173.34 \pm 0.76$ GeV [4]

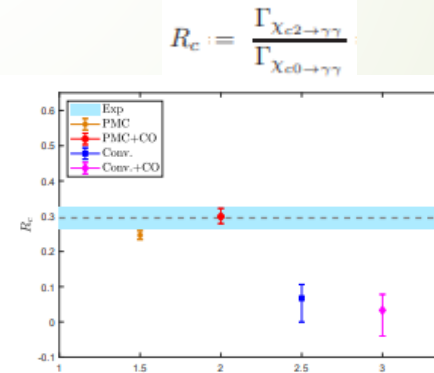


FIG. 4: The NNLO R_c -ratios with or without color-octet (CO) contributions under conventional and PMC scale-setting approaches. The error bars are for $\Delta\alpha_s(M_r) = \pm 0.016$ and $\mu_r \in [1\text{GeV}, 2m_c]$. The dashed line is the central value of BESIII and the shaded band represents its error [4].

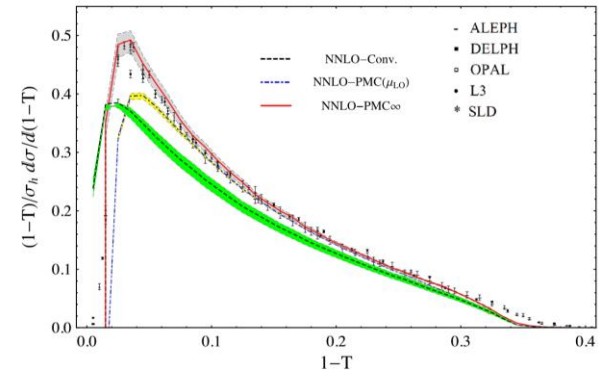
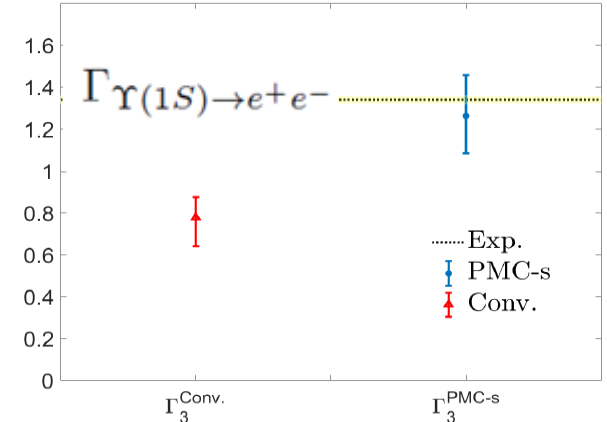


FIG. 3. The thrust distribution at NNLO under the conventional (dashed black), the PMC(μ_{L0}) (dot-dashed blue), and the PMC_∞ (solid red). The experimental data points are taken from the ALEPH, DELPHI, OPAL, L3, and SLD experiments [33–37]. The shaded areas show theoretical error predictions at NNLO, and they have been calculated by varying the remaining initial scale value in the range $\sqrt{s}/2 \leq \mu_0 \leq 2\sqrt{s}$.



微扰论未知高阶项贡献的估算

**解决能标设定问题之后，
能否将未知高阶效应所导致的不确定性压到最低？**

**降低微扰论未知高阶项所带来的不确定性本质上与能标设定是两个问题
微扰论中未知高阶项导致的理论不确定性也是微扰论的魅力所在**

$$\rho_n^{[N/M]}(Q) = a^p \times \frac{b_0 + b_1 a + \dots + b_N a^N}{1 + c_1 a + \dots + c_M a^M}$$

$$= \sum_{i=1}^n C_i a^{p+i-1} + \underbrace{C_{n+1} a^{p+n}}_{\text{circled}} + \dots$$

估算未知高阶项贡献几种方法

- 传统方法：能标变化一定倍数—粗略估算与耦合常数相关的非共形项贡献
- 保守估算：利用微扰性质，认为下一阶不会大于上一阶的值
- 重求和方法：寻找母函数，分数型-Pade近似
- 概率分析：Bayesian分析

$$P(A \cap B) = P(A) * P(B|A) = P(B) * P(A|B)$$

估算未知高阶项具有某个量值的概率

相比于传统微扰序列，PMC序列具有收敛性更好+更精确+去掉RGE关联项等性质

有潜力及优势获得更可靠高阶项的预言

且能更快的获得有限阶下更准确预言 — 在足够高阶情形下，“传统序列效果也可以”

Bayesian分析方法 — 初步

基本假设

所有系数有界: $|c_i| \leq \bar{c}$ ($\bar{c} > 0$), $\forall i$. [JHEP09(2011)039]

- ▶ \bar{c} 的数量级对所有可能取值等可能. 即 $\ln \bar{c}$ 在 $[-|\ln \epsilon|, |\ln \epsilon|]$ 内均匀分布 ($\epsilon \rightarrow 0$),

$$g_0(\bar{c}) = \frac{1}{2|\ln \epsilon|} \frac{1}{\bar{c}} \theta\left(\frac{1}{\epsilon} - \bar{c}\right) \theta(\bar{c} - \epsilon), \quad (3)$$

- ▶ 系数在 $[-\bar{c}, \bar{c}]$ 内均匀分布

$$h_0(c_i|\bar{c}) = \frac{1}{2\bar{c}} \theta(\bar{c} - |c_i|), \quad \forall i, \quad (4)$$

- ▶ 所有系数相互独立

$$h(c_j, c_k|\bar{c}) = h_0(c_j|\bar{c})h_0(c_k|\bar{c}), \quad \forall j, k, j \neq k. \quad (5)$$

PMC使第三个假设更合理

隐含使用假设-微扰收敛性已经呈现

定义自洽临界置信度, $p_c\%$,

p_c 为满足如下不等式组的最小 p 值,

$$\rho_{i-1} + c_i^{(p)} \alpha_s^i \geq \rho_i + c_{i+1}^{(p)} \alpha_s^{i+1}, \quad (i = l+1, \dots, k),$$

$$\rho_{i-1} - c_i^{(p)} \alpha_s^i \leq \rho_i - c_{i+1}^{(p)} \alpha_s^{i+1}, \quad (i = l+1, \dots, k).$$

用如下区间作为 ρ_k 的理论误差区间,

$$[\rho_k - c_{k+1}^{(p_c)} \alpha_s^{k+1}, \rho_k + c_{k+1}^{(p_c)} \alpha_s^{k+1}],$$

假设既是简化、也限定了适用范围; 虽有局限, 但是迈出第一步

Bayesian分析方法 — 初探

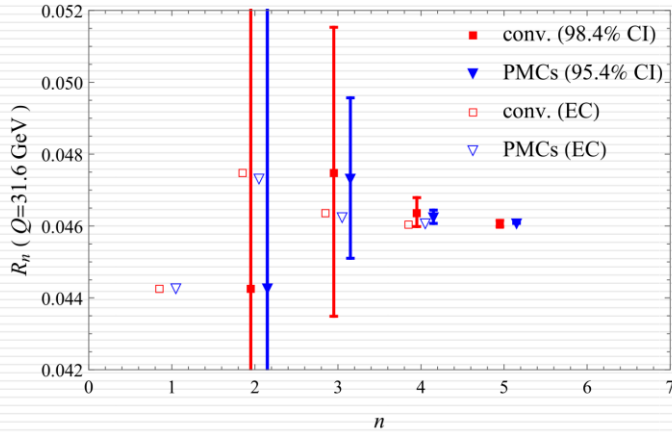


Figure: 根据 $R_n(Q)(n = 1, 2, 3, 4)$ 预测 $R_{n+1}(Q)$ 的 $p_c\%$ -CI.

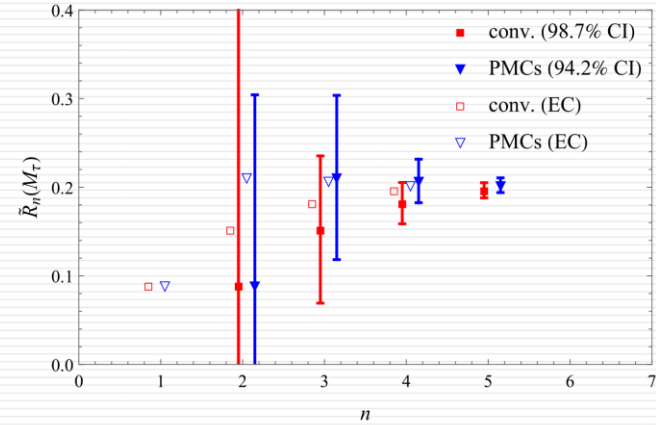


Figure: 根据 $\tilde{R}_n(M_\tau)(n = 1, 2, 3, 4)$ 预测 $\tilde{R}_{n+1}(M_\tau)$ 的 $p_c\%$ -CI.

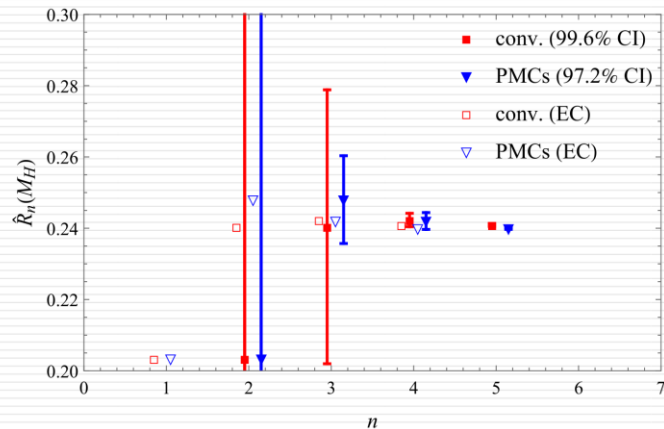


Figure: 根据 $\hat{R}_n(M_H)(n = 1, 2, 3, 4)$ 预测 $\hat{R}_{n+1}(M_H)$ 的 $p_c\%$ -CI.

注：传统方案下的结果仅考虑能标取为中心值的情况：

$$R_{e^+e^-}(Q)$$

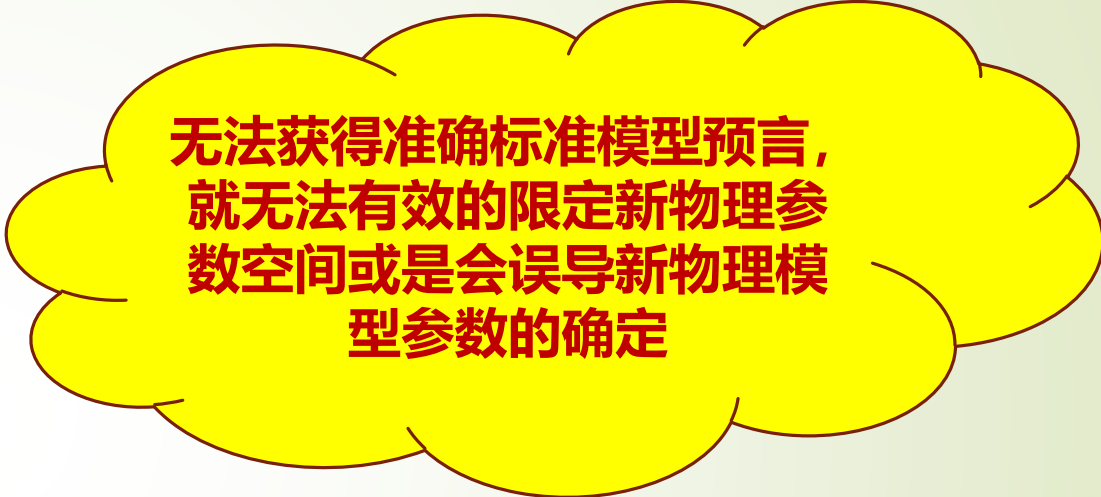
$$R_\tau(M_\tau)$$

$$\Gamma(H \rightarrow b\bar{b})$$



四

小结和展望

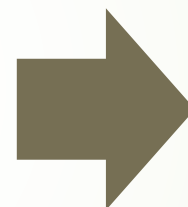
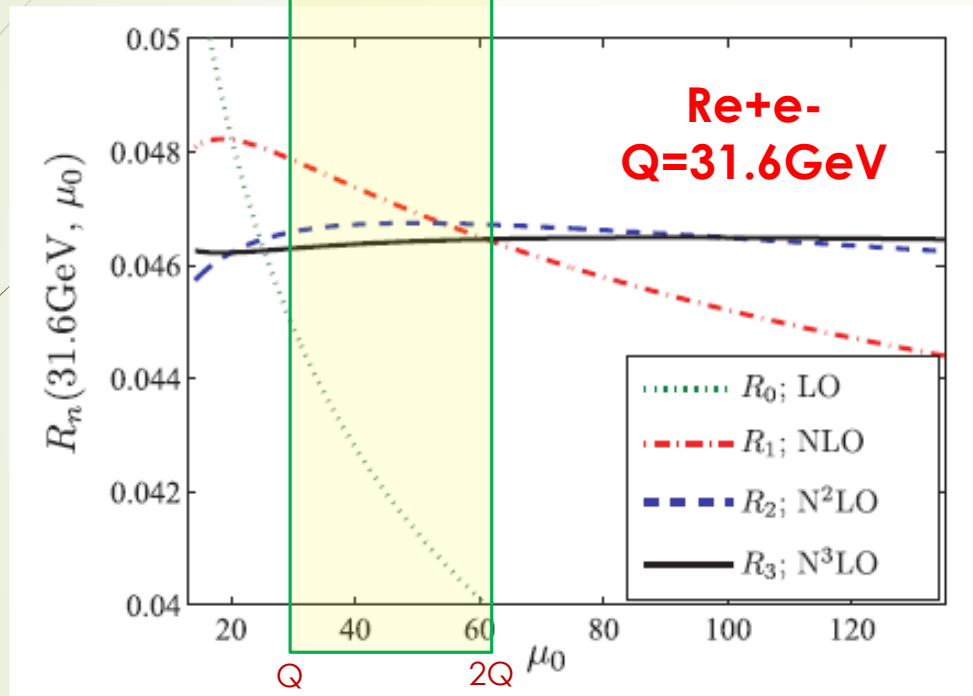


无法获得准确标准模型预言,
就无法有效的限定新物理参
数空间或是会误导新物理模
型参数的确定

传统“猜”存在本质缺陷

最核心的就是降低QCD理论预言能力和准确性

传统理念
典型过程



逐步更精确，但

- 微扰收敛性非内禀属性，各阶能标依赖仍然大；
- 不是每个高阶例子都显现相消后，误差减少；
- 计算能力仅在有限低阶

**需敢于挑战传统
更显能标方案的重要性**

**PMC不是选择“特殊/有效-能标”
而是基于-重整化群方程以及基本重整化群不变性 —
通过系统确定高能物理过程的强耦合常数值，从而解
决传统方案下的重整化能标和重整化方案依赖问题**



PMC仍然在路上

“路虽远行则将至，事虽难做则必成”