Top and bottom quark A_{FB} at NNLO QCD in (un)polarized electron positron collisions

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Outline

- **Motivation and Background**
- **Asymmetries for top quarks**
- **Asymmetries for bottom quarks**
- **Summary**

Motivation and Background

- e^+e^- collider offer clean experimental conditions for studying properties of heavy quark, e.g. masses, width, and electroweak couplings

- A_{FB}^{Q} can extract the form factor of the electroweak neutral current coupling

❖ $b\bar{b}$ pair production at Z-pole:

2.9σ deviations between the direct measured and its SM fit[SLD Group et al. 06]. The massive α_s^2 corrections to the b-quark diminish to 2.6σ[Chen et al. 17]. Massive+PMC, 2.1σ[Wang et al. 20]

❖ The measurement accuracy of A_{FB}^{b} and A_{FB}^{t} can reach per mille level at future e^+e^- collider. [Abramovicz et al. 03]

- e^+e^- collider offer polarized electron and positron beams

  enhances the sensitivity of the detector to electroweak parameters

Motivation and Background

- Concerning high order corrections:

NLO QCD[Jersak et al. 82; Arbuzov et al. 92; Djouadi et al. 95 etc],NLO EW[Beenakker et al. 91; Bohm et al. 89; Bardin et al. 01 etc],

Part of NNLO QCD[Gorishnii et al, 86; Chetyrkin et al. 96 etc]

Massless quark: Full NNLO QCD[Altarelli, Lampe 93; Ravindran, Neerven 98; Catani, Seymour 99, Weinzierl 07]

$e^+e^- \to t\bar{t}$ @ NNLO QCD [Gao, Zhu, PRL (2014); Chen et a.(16)]

$e^+e^- \to t\bar{t}_{\text{near - threshold}}$ @ NNNLO QCD [M.Beneke et al., PRL (2015)]

$e^+e^- \to t\bar{t}$ @ NNNLO QCD [Chen et al. 2022]

Our aim was to investigate the effect of beam polarization on A_{FB}^{Q} for $e^+e^- \to Q\bar{Q} + X$ at α_s^2, and compare with these results for an unpolarized results.
For $Q = t$, in the continuum above the $t\bar{t}$ threshold, for $Q = b$, at the Z pole.

Definition of A_{FB} and Benchmark Polarizations

- The polarized A_{FB}:

$$A_{FB} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{\sigma_A}{\sigma_S}$$

$\theta_t = \angle(e^-, t)$ scattering angle between t and e^-

- Four benchmark polarization configurations

e_L^-	e_R^+	P_v	P_a
-80%	$+30\%$	0.76	-0.5
$+80\%$	-30%	0.76	0.5
$+80\%$	$+30\%$	1.24	1.1
-80%	-30%	1.24	-1.1

$P_v = 1 + e_L^- e_R^+$,

$P_a = e_L^- + e_R^+$

For unpolarized beams:

P_v=1, P_a=0

Results: Asymmetries for top quark

- The **expanded** A_{FB}: $e^+e^- \to t\bar{t} + X$ @ NNLO QCD

$$A_{FB}^{NLO} = A_{FB}^{LO}\,[1 + A_1],$$

$$A_{FB}^{NNLO} = A_{FB}^{LO}\,[1 + A_1 + A_2]$$

$\sqrt{s} = 500$ GeV

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Beam polarization	LO		NLO		NNLO	
(e_L^-, e_R^+)	σ_S [pb]	A_{FB}^{LO}	σ_S [pb]	A_1 $[10^{-2}]$	σ_S [pb]	A_2 $[10^{-2}]$
(0, 0)	0.55084	0.4169	$0.62006^{-0.00489}_{+0.00571}$	$2.26^{+0.186}_{-0.159}$	$0.63038^{-0.00187}_{+0.00147}$	$1.16^{+0.004}_{-0.014}$
(−80%, +30%)	0.30850	0.4458	$0.34762^{-0.00276}_{+0.00323}$	$2.14^{+0.177}_{-0.151}$	$0.35362^{-0.00108}_{+0.00086}$	$1.10^{+0.003}_{-0.013}$
(+80%, −30%)	0.52877	0.4001	$0.59488^{-0.00467}_{+0.00545}$	$2.32^{+0.192}_{-0.164}$	$0.60456^{-0.00176}_{+0.00138}$	$1.20^{+0.004}_{-0.015}$
(+80%, +30%)	0.92534	0.3957	$1.04086^{-0.00816}_{+0.00953}$	$2.34^{+0.193}_{-0.165}$	$1.05771^{-0.00307}_{+0.00239}$	$1.21^{+0.004}_{-0.015}$
(−80%, −30%)	0.44074	0.4614	$0.49689^{-0.00397}_{+0.00463}$	$2.08^{+0.172}_{-0.147}$	$0.50564^{-0.00157}_{+0.00126}$	$1.07^{+0.003}_{-0.012}$

σ_S: symmetric cross section

Results: Asymmetries for top quark

$$A_{FB}^{NLO} = A_{FB}^{LO}\,[1 + A_1], \quad A_{FB}^{NNLO} = A_{FB}^{LO}\,[1 + A_1 + A_2]$$

$\sqrt{s} = 500$ GeV

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- The maximal spread R_i^{max} of $A_i (i = 1,2)$ are 12.3% and 13.3%.

$$R_i^{max} = \frac{(\max[A_i(e_L^-, e_R^+)] - \min[A_i(e_L^-, e_R^+)])}{A_i(0,0)}$$

- The effect of the beam polarization on A_1, A_2 (2.8 per mille, 1.3 per mille) are comparable to e^+e^- collider statistical uncertainty (5 per mille).

Results: Asymmetries for top quark

$\sqrt{s} = 380$ GeV

Beam polarization (e_L^-, e_R^+)	LO σ_S [pb]	LO A_{FB}^{LO}	NLO σ_S [pb]	NLO A_1 $[10^{-2}]$	NNLO σ_S [pb]	NNLO A_2 $[10^{-2}]$
(0, 0)	0.58477	0.2342	$0.78874^{-0.01484}_{+0.01741}$	$3.67^{+0.313}_{-0.267}$	$0.85037^{-0.01009}_{+0.01002}$	$2.92^{+0.188}_{-0.168}$
(−80%, +30%)	0.32039	0.2549	$0.43232^{-0.00814}_{+0.00955}$	$3.62^{+0.309}_{-0.263}$	$0.46633^{-0.00556}_{+0.00553}$	$2.86^{+0.183}_{-0.163}$
(+80%, −30%)	0.56846	0.2226	$0.76657^{-0.01441}_{+0.01691}$	$3.70^{+0.316}_{-0.270}$	$0.82623^{-0.00977}_{+0.00970}$	$2.95^{+0.191}_{-0.170}$
(+80%, +30%)	0.99800	0.2196	$1.34571^{-0.02530}_{+0.02968}$	$3.71^{+0.317}_{-0.270}$	$1.45035^{-0.01714}_{+0.01701}$	$2.96^{+0.192}_{-0.171}$
(−80%, −30%)	0.45224	0.2664	$0.61037^{-0.01151}_{+0.01350}$	$3.59^{+0.306}_{-0.261}$	$0.65856^{-0.00788}_{+0.00784}$	$2.82^{+0.180}_{-0.160}$

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$\sqrt{s} = 700$ GeV

Beam polarization (e_L^-, e_R^+)	LO σ_S [pb]	LO A_{FB}^{LO}	NLO σ_S [pb]	NLO A_1 $[10^{-2}]$	NNLO σ_S [pb]	NNLO A_2 $[10^{-3}]$
(0, 0)	0.32344	0.5144	$0.34560^{-0.00151}_{+0.00176}$	$0.90^{+0.071}_{-0.061}$	$0.34759^{-0.00040}_{+0.00024}$	$4.6^{+0.009}_{-0.048}$
(−80%, +30%)	0.18367	0.5440	$0.19644^{-0.00087}_{+0.00101}$	$0.79^{+0.063}_{-0.054}$	$0.19766^{-0.00024}_{+0.00015}$	$4.2^{+0.027}_{-0.058}$
(+80%, −30%)	0.30796	0.4968	$0.32887^{-0.00143}_{+0.00166}$	$0.96^{+0.076}_{-0.065}$	$0.33068^{-0.00037}_{+0.00021}$	$4.8^{-0.001}_{-0.043}$
(+80%, +30%)	0.53779	0.4922	$0.57422^{-0.00249}_{+0.00289}$	$0.97^{+0.077}_{-0.066}$	$0.57734^{-0.00064}_{+0.00037}$	$4.9^{-0.004}_{-0.041}$
(−80%, −30%)	0.26435	0.5597	$0.28288^{-0.00126}_{+0.00147}$	$0.74^{+0.059}_{-0.050}$	$0.28469^{-0.00035}_{+0.00022}$	$4.0^{+0.037}_{-0.063}$

Results: Asymmetries for top quark

The **unexpanded** A_{FB}:

$$A_{FB}(\alpha_s) = A_{FB}^{LO}\, C_1, \quad A_{FB}(\alpha_s^2) = A_{FB}^{LO}\, C_2, \quad (A_{FB}^{LO} = \frac{\sigma_A^0}{\sigma_S^0})$$

Beam polarization (e_L^-, e_R^+)		(0, 0)	(−80%, +30%)	(+80%, −30%)	(+80%, +30%)	(−80%, −30%)
380 GeV	$C_1 - 1$ $[10^{-2}]$	$2.72^{+0.170}_{-0.149}$	$2.68^{+0.169}_{-0.148}$	$2.75^{+0.168}_{-0.152}$	$2.75^{+0.169}_{-0.152}$	$2.66^{+0.161}_{-0.145}$
	$C_2 - 1$ $[10^{-2}]$	$5.41^{+0.435}_{-0.360}$	$5.32^{+0.429}_{-0.355}$	$5.47^{+0.437}_{-0.367}$	$5.48^{+0.440}_{-0.368}$	$5.26^{+0.418}_{-0.352}$
400 GeV	$C_1 - 1$ $[10^{-2}]$	$2.69^{+0.180}_{-0.155}$	$2.63^{+0.177}_{-0.149}$	$2.72^{+0.179}_{-0.160}$	$2.73^{+0.180}_{-0.158}$	$2.59^{+0.174}_{-0.151}$
	$C_2 - 1$ $[10^{-2}]$	$4.96^{+0.374}_{-0.316}$	$4.84^{+0.364}_{-0.305}$	$5.03^{+0.379}_{-0.323}$	$5.05^{+0.381}_{-0.322}$	$4.77^{+0.356}_{-0.304}$
500 GeV	$C_1 - 1$ $[10^{-2}]$	$2.01^{+0.146}_{-0.129}$	$1.90^{+0.136}_{-0.121}$	$2.06^{+0.152}_{-0.133}$	$2.08^{+0.152}_{-0.133}$	$1.85^{+0.136}_{-0.116}$
	$C_2 - 1$ $[10^{-2}]$	$3.23^{+0.200}_{-0.178}$	$3.07^{+0.193}_{-0.168}$	$3.33^{+0.206}_{-0.183}$	$3.36^{+0.208}_{-0.183}$	$2.98^{+0.184}_{-0.161}$
700 GeV	$C_1 - 1$ $[10^{-2}]$	$0.84^{+0.060}_{-0.055}$	$0.74^{+0.053}_{-0.050}$	$0.90^{+0.066}_{-0.057}$	$0.91^{+0.065}_{-0.058}$	$0.69^{+0.051}_{-0.044}$
	$C_2 - 1$ $[10^{-2}]$	$1.32^{+0.075}_{-0.068}$	$1.17^{+0.072}_{-0.063}$	$1.40^{+0.082}_{-0.069}$	$1.42^{+0.079}_{-0.071}$	$1.10^{+0.066}_{-0.058}$

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The numbers in superscript (subscript) refer to the changes if renormalization scale μ is set to $2\sqrt{s}$ ($\sqrt{s}/2$) compared to the central value at $\mu = \sqrt{s}$.

Discussion: The dependence on beam polarization

- The dependence for Flavor non-singlet need:

  The coherent contribution of the Z and γ exchange

  The final-state quark is massive

- ✓ $t\bar{t}$ production is completely dominated by the flavor non-singlet corrections

- Flavor singlet contributions are small($\sim 10^{-4}$ to $\mathrm{A}_{\mathrm{FB}}^{\mathrm{LO}}$)

Results: Asymmetries for b quark at the Z peak

For the thrust axis

Beam polarization e_L^-, e_R^+	LO σ_S [pb]	LO A_{FB}^{LO}	NLO σ_S [pb]	NLO A_1 $[10^{-2}]$	NNLO σ_S [pb]	NNLO A_2 $[10^{-2}]$
(0, 0)	8747.4	0.1512	9122.8	$-2.88^{+0.273}_{-0.338}$	9164.8	$-1.11^{+0.085}_{-0.037}$
(−80%, +30%)	5710.7	-0.3644	5955.8	$-2.88^{+0.273}_{-0.338}$	5990.6	$-1.19^{+0.064}_{-0.022}$
(+80%, −30%)	7585.3	0.5394	7910.9	$-2.88^{+0.273}_{-0.338}$	7939.9	$-0.99^{+0.115}_{-0.058}$
(+80%, +30%)	12908.8	0.6531	13462.8	$-2.88^{+0.273}_{-0.338}$	13508.5	$-0.96^{+0.122}_{-0.064}$
(−80%, −30%)	8784.7	-0.5862	9161.7	$-2.88^{+0.273}_{-0.338}$	9220.1	$-1.25^{+0.049}_{-0.338}$

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- A_1 is independent of the beam polarization
- Non-singlet corrections A_2^{ns} amounts to about 65% of the total A_2
- The maximal spread R_2^{max} of A_2 is 27% arise from the flavor singlet contributions

(3 per mille)

Results: Asymmetries for b quark at the Z peak

$$y_{ij}^{X} = (1 - cos\theta_{ij})\frac{2r_X}{s} \qquad r_F = \begin{cases} max(E_i^2, E_j^2), & \text{If softer of i, j is flavored} \\ min(E_i^2, E_j^2), & \text{If softer of i, j is flavorless} \end{cases}$$

$$r_D = min(E_i^2, E_j^2)$$

For the b-jet axis

Jet algorithms (y_{cut})	LO σ_S [pb]	LO A_{FB}^{LO}	NLO σ_S [pb]	NLO A_1 [10^{-2}]	NNLO σ_S [pb]	NNLO A_2 [10^{-2}]
Flavor k_T, 0.01	8747.4	0.1512	9120.7	$-2.88^{+0.037}_{+0.021}$	9150.4	$-1.09^{+0.216}_{-0.303}$
Flavor k_T, 0.05	8747.4	0.1512	9111.4	$-2.67^{+0.034}_{+0.020}$	9121.4	$-0.83^{+0.167}_{-0.236}$
Flavor k_T, 0.10	8747.4	0.1512	9098.2	$-2.42^{+0.031}_{+0.018}$	9097.7	$-0.68^{+0.138}_{-0.195}$
Flavor k_T, 0.15	8747.4	0.1512	9082.8	$-2.18^{+0.028}_{+0.016}$	9075.3	$-0.59^{+0.120}_{-0.169}$
Durham, 0.01	8747.4	0.1512	9100.2	$-2.58^{+0.033}_{+0.019}$	9112.1	$-0.87^{+0.174}_{-0.244}$
Durham, 0.05	8747.4	0.1512	9050.4	$-1.84^{+0.024}_{+0.013}$	9035.8	$-0.52^{+0.105}_{-0.147}$
Durham, 0.10	8747.4	0.1512	9018.2	$-1.46^{+0.019}_{+0.011}$	8992.4	$-0.38^{+0.076}_{-0.107}$
Durham, 0.15	8747.4	0.1512	8996.7	$-1.26^{+0.016}_{+0.009}$	8964.4	$-0.33^{+0.065}_{-0.091}$

Unpolarized beams, and excluding b-flavor number zero events

Results: Two-jet asymmetry of b quark

The QCD correction terms A_1^{2j} and A_2^{2j} for the two-jet A_{FB}:

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y_{cut}	Flavor k_T			Durham		
	σ_S [pb]	A_1^{2j} $[10^{-2}]$	A_2^{2j} $[10^{-2}]$	σ_S [pb]	A_1^{2j} $[10^{-2}]$	A_2^{2j} $[10^{-2}]$
0.01	5781.3	$-0.151^{+0.002}_{+0.001}$	$0.029^{+0.017}_{-0.011}$	5984.9	$-0.168^{+0.002}_{+0.001}$	$0.032^{+0.018}_{-0.012}$
0.05	7666.2	$-0.390^{+0.005}_{+0.003}$	$-0.015^{+0.005}_{-0.002}$	8011.6	$-0.467^{+0.006}_{+0.003}$	$-0.090^{+0.012}_{-0.016}$
0.10	8231.5	$-0.571^{+0.007}_{+0.004}$	$-0.061^{+0.008}_{-0.010}$	8548.1	$-0.696^{+0.009}_{+0.005}$	$-0.160^{+0.028}_{-0.038}$
0.15	8491.6	$-0.705^{+0.009}_{+0.005}$	$-0.109^{+0.018}_{-0.025}$	8754.7	$-0.851^{+0.011}_{+0.006}$	$-0.222^{+0.041}_{-0.057}$

※ QCD corrections to A_1^{2j} and A_2^{2j} are small

※ Not monotonic due to flavor singlet contributions

Summary

- A complete fully differential NNLO QCD calculation of massive $Q\bar{Q}$ productions at order α_s^2 in **polarized** e^+e^- collisions was done.

- Application to A_{FB} for top quark in the continuum to order α_s^2
  - ✓ Beam polarizations change A_{FB}^t by the order of few per mille
  - ✓ Predictions for the $\cos\theta_t$ distributions at order α_s^2

- Application to A_{FB} for to bottom at Z pole to order α_s^2
  - ✓ Only NNLO QCD are effected by beam polarization (per mille level)
  - ✓ Explore inclusive b-jet asymmetry and two-jet asymmetry
  - ✓ The two-jet asymmetry is a precision observable

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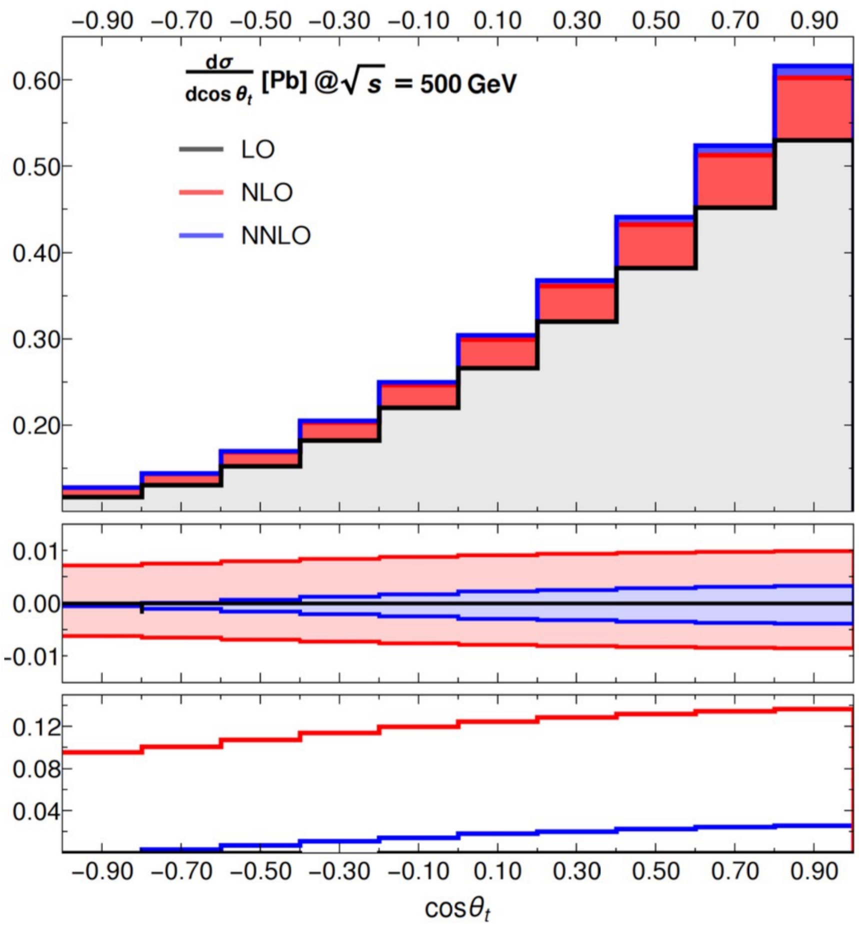
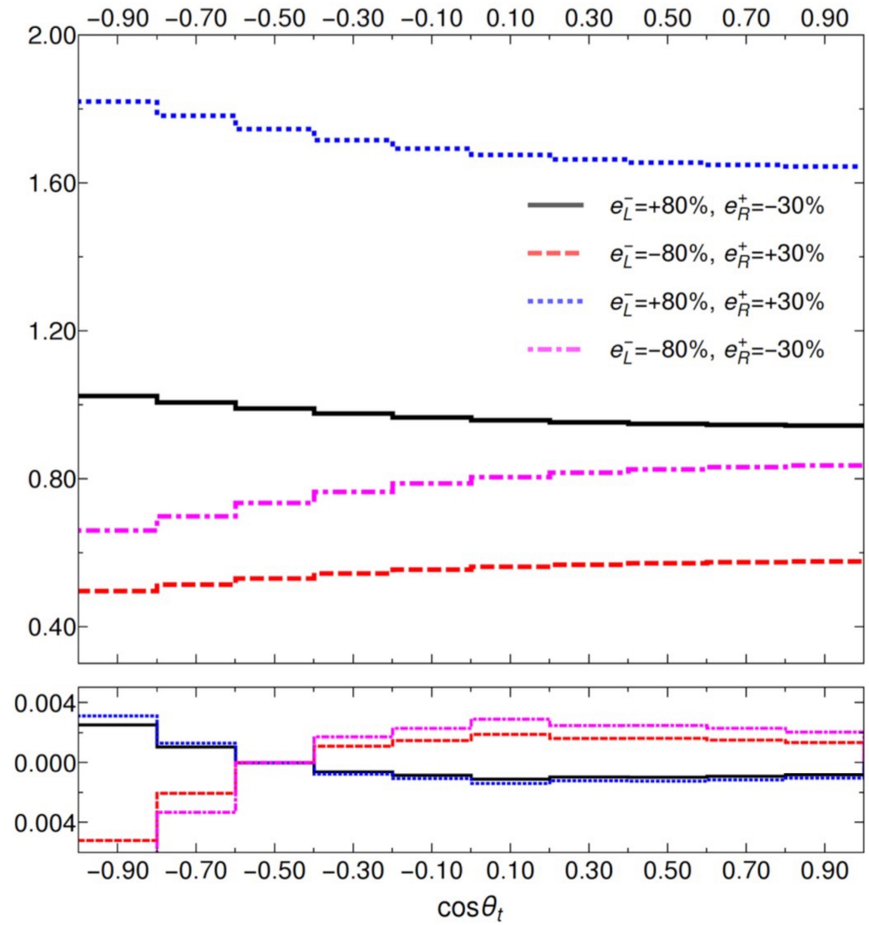
谢谢大家！

Appendix

- $\theta_t = \angle(e^-, t)$ scattering angle between t and e^-





Appendix

$$T = \max_{\boldsymbol{n}_T} \frac{\sum_{i=1}^{n} |\boldsymbol{k}_i \cdot \boldsymbol{n}_T|}{\sum_{i=1}^{n} |\boldsymbol{k}_i|}, \qquad |\boldsymbol{n}_T| = 1.$$

The orientation of the thrust axis is fixed by requiring $\boldsymbol{n}_T \cdot \boldsymbol{k}_1 > 0$.