

Top Decay at Next-to-Next-to-Next-to-Leading Order in QCD

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The 2023 International Workshop on the CEPC

2023年10月23-27日, 南京

Based on: [2309.01937 and work in progress]

In collaboration with Xiang Chen, Xin Guan and Yan-Qing Ma



Why Study the Top Quark

- Heaviest fundamental particle in SM

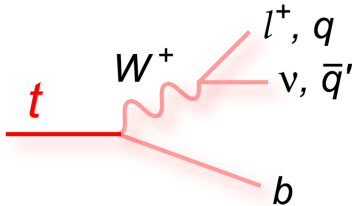
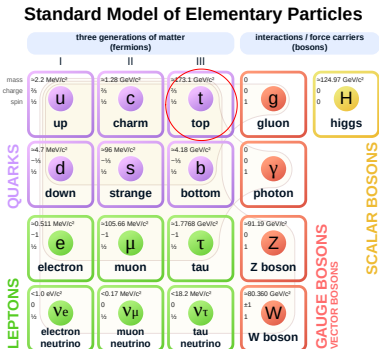
$$m_t = 172.69 \pm 0.30 \text{ GeV}$$

- Precision test of SM mechanism, and prods for possible BSM physics

- Decay exclusively to $b + W$ before hadronization:

$$\Gamma_t = 1.4 \text{ GeV} \gg \Lambda_{\text{QCD}}$$

- Convergence of the perturbative QCD series (e.g. renormalon issue)



Top Quark Mass m_t and Decay Width Γ_t

- PDG average for m_t :

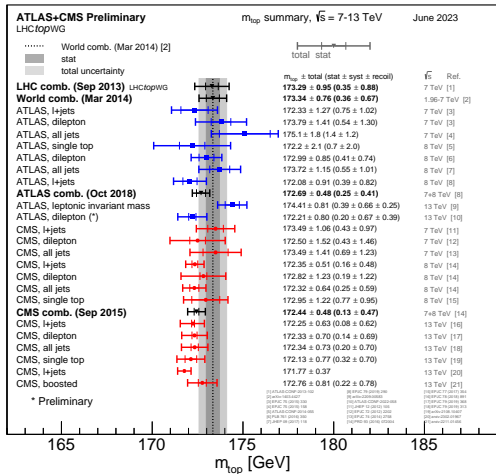
$172.69 \pm 0.30 \text{ GeV}$

- Current best measurement for Γ_t :

$1.36 \pm 0.02(\text{stat.})^{+0.14}_{-0.11}(\text{syst.}) \text{ GeV.}$

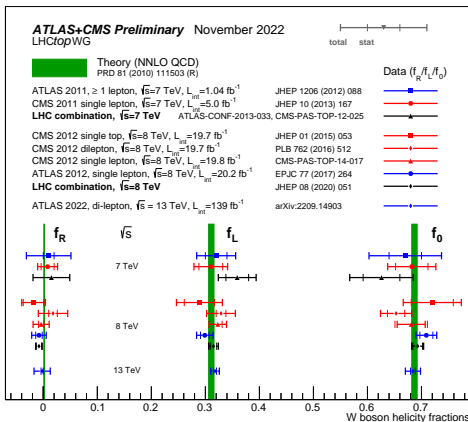
- Experimental uncertainties anticipated at future colliders:

$20 \sim 26 \text{ MeV}$



The W-helicities in Top Decay

W from $t \rightarrow b + W^+ + X_{\text{QCD}}$ is polarized even if the t -quark is unpolarized



- The current best measurements: $f_0 = 0.684 \pm 0.005$ (stat.) ± 0.014 (syst.),
 $f_L = 0.318 \pm 0.003$ (stat.) ± 0.008 (syst.) and $f_R = -0.002 \pm 0.002$ (stat.) ± 0.014 (syst.).
- Notoriously difficult to be predicted theoretically to high precision (via MC method)

Much Theoretical Work Done So Far

Given the key role played by the top-quark both in SM precision test and searching for BSM, there have been vast amount of works done in literature regarding $t \rightarrow b + W^+ + X_{\text{QCD}}$.

● The inclusive Γ_t

- ▶ Up to NNLO in QCD: [Jezabek etc 88; Czarnecki etc 90; Li etc 90; Czarnecki etc 98; Chetyrkin etc 99; Fischer etc 01; Blokland etc 04'05;.....; Czarnecki etc 10; Meng etc 22; Chen etc 22]

@NNLO in QCD: [LC, Chen, Guan, Ma 23; Chen, Li, Li, Wang, Wang, Wu 23]
[Datta, Rana, Ravindran, Sarkar 23 (only virtuals)]

- ▶ NLO Electroweak: [Denner Sack 91; Eilam, Mendel Migneron Soni 91]

● W-helicities $f_{L,R,0}$

- ▶ @NNLO in QCD: [Czarnecki, Korner, Piclum 10; Gao, Li, Zhu 12; Brucherseifer, Caola, Melnikov 13; Czarnecki, Groote Korner ,Piclum 18]

@NNLO in QCD: [LC, Chen, Guan, Ma 23]

- ▶ NLO Electroweak: [Do, Groote, Korner, Mauser 02]

● Differential results

- ▶ QCD:

@NLO [Fischer, Groote, Korner, Mauser 01; Brandenburg, Si, Uwer 02; Bernreuther, Gonzalez , Mellei 14; Kniehl, Nejad 21]

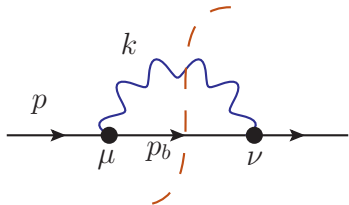
@NNLO [Gao, Li, Zhu 12; Brucherseifer, Caola, Melnikov 13; Campbell, Neumann, Sullivan 20]

@NNLO in QCD: [LC, Chen, Guan, Ma 23]

Cut Diagrams for Top Decay Width

Γ_t in terms of the **semi-inclusive** $\mathcal{W}_{tb}^{\mu\nu}$

$$\Gamma_t = \frac{1}{2m_t} \int \frac{d^{d-1}k}{(2\pi)^{d-1}2E} \mathcal{W}_{tb}^{\mu\nu} \sum_{\lambda}^{L,R,0} \varepsilon_{\mu}^*(k, \lambda) \varepsilon_{\nu}(k, \lambda),$$



$$\begin{aligned} \mathcal{W}_{tb}^{\mu\nu}(p, k) = & W_1 g^{\mu\nu} + W_2 p^{\mu} p^{\nu} + W_3 k^{\mu} k^{\nu} \\ & + W_4 (p^{\mu} k^{\nu} + k^{\mu} p^{\nu}) + W_5 i\epsilon^{\mu\nu\rho\sigma} p_{\rho} k_{\sigma}, \end{aligned}$$

Selection Criteria: the **cut diagrams** of t -quark self-energy function with exactly one (cut) **W propagator** interacting with the *external* t -quark plus (up to 3) QCD loops

Loop and Phase-space Integration

- Loop integrals are reduced using IBP [Chetyrkin 81] relations done with Blade [Guan, Liu, Ma 20], and the resulting *masters* are calculated using DE method [Kotikov 90; Remiddi 97] with AMFlow [Liu, Ma 22]

[→ See talk by Xiang Chen]

- The phase-space integrals, except for *W-momentum* k , are treated in the same manner as loop integrals by means of the reverse unitarity [Anastasiou, Melnikov 02]

- The **IR-divergent** phase-space integration of $\mathcal{W}_{tb}^{\mu\nu}$ over k are done “*manually*” using its power-log series representation (PSE) with ϵ assigned with **non-zero** numbers.

- ▶ **Level of Complexity:**

7×10^4 integrals reduced to **2988 master integrals**, for which PSE about 200 orders in k_0 are derived with the above method.

- ▶ **Consistency Check:**

A perfect agreement in the result for the **inclusive** Γ_t with $\frac{d^{d-1}k}{(2\pi)^{d-1}2E}$ calculated in this way and by directly applying the reverse unitarity [Anastasiou, Melnikov 02]

Results for the Inclusive Γ_t

The QCD effects on Γ_t in SM can be parameterized as

$$\Gamma_t = \Gamma_0 \left[\mathbf{c}_0 + \frac{\alpha_s}{\pi} \mathbf{c}_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \mathbf{c}_2 + \left(\frac{\alpha_s}{\pi} \right)^3 \mathbf{c}_3 + \mathcal{O}(\alpha_s^4) \right],$$

$$\text{with } \Gamma_0 \equiv \frac{G_F m_W^2 m_t |V_{tb}|^2}{12\sqrt{2}}.$$

We choose $\mu = m_t/2$, motivated by the **kinetic energy** $m_t - m_W - m_b$ of the QCD radiations, at which our N3LO result reads: [LC, Chen, Guan, Ma 23]

$$\begin{aligned} \Gamma_t &= 1.48642 - 0.140877 - 0.023306 - 0.007240 \text{ GeV} \\ &= \mathbf{1.31500} \text{ GeV} \end{aligned}$$

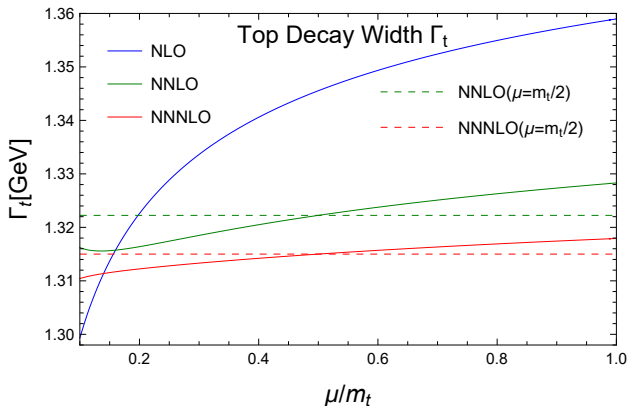
SM Inputs:

$$\begin{aligned} G_F &= 1.166379 \times 10^{-5} \text{ GeV}^{-2}, \quad m_t = 172.69 \text{ GeV}, \\ m_W &= 80.377 \text{ GeV}, \quad \alpha_s(m_t/2) \approx 0.1189. \end{aligned}$$

The **leading-color** part of Γ_t agrees with a parallel computation [Chen, Li, Li, Wang, Wang, Wu 23]

The QCD Scale Uncertainty of Γ_t

The scale dependence of the fixed-order results for Γ_t in $\mu/m_t \in [0.1, 1]$



- **NNLO** scale-variation **never** cover the **NNLO** result at any scales less than $\mu/m_t = 0.6$.
- **Pure $\mathcal{O}(\alpha_s^3)$** correction decreases Γ_t by $\sim 0.8\%$ of the **NNLO** result at $\mu = m_t$ roughly 10 MeV(exceeding NNLO scale-hand)

The Offshell W and Finite m_b Effects

- With $\frac{1}{k^2 - m_W^2 + i\epsilon} \rightarrow \frac{1}{k^2 - m_W^2 + im_W\Gamma_W}$,

$$\begin{aligned}\Gamma_t(m_W) &\rightarrow \tilde{\Gamma}_t = \int_0^{m_t^2} \frac{dk^2}{2\pi} \frac{2m_W\Gamma_W}{(k^2 - m_W^2)^2 + (m_W\Gamma_W)^2} \Gamma_t(m_W^2 \rightarrow k^2) \\ &= \tilde{\Gamma}_0 \left[\tilde{c}_0 + \frac{\alpha_s}{\pi} \mathbf{c}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{c}_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \mathbf{c}_3 + \mathcal{O}(\alpha_s^4) \right],\end{aligned}$$

we find: $\frac{\tilde{c}_i - c_i}{c_i}$ takes -1.54% , -1.53% , -1.39% , -1.23% for $i = 0, 1, 2, 3$.

- Similarly, keeping $m_b = 4.78$ GeV, we find: $\frac{c_1^{m_b} - c_1}{c_1} \approx \frac{c_2^{m_b} - c_2}{c_2} \approx -1.47\%$.
- The NLO electroweak K-factor is re-evaluated to be $K_{EW}^{NLO} = 1.0168$.

Taking these misc-effects into account, we finally obtain the **to-date most-precise high-precision theoretical prediction**:

$$\Gamma_t = 1.3148_{-0.005}^{+0.003} \times |V_{tb}|^2 + 0.027 (m_t - 172.69) \text{ GeV}$$

the error of which meets the request by future colliders.

Results for W-helicity Fractions

Top decay width with **polarized W**:

$$\Gamma_\lambda = \frac{1}{2m_t} \int \frac{d^{d-1}k}{(2\pi)^{d-1}2E} \mathcal{W}_{tb}^{\mu\nu} \varepsilon_\mu^*(k, \lambda) \varepsilon_\nu(k, \lambda)$$

The **W-helicity fractions** $f_\lambda^{[n]} = \frac{\sum_{i=0}^n \Gamma_\lambda^{[n]}}{\sum_{i=0}^n \Gamma_t^{[n]}}$ truncated to $\mathcal{O}(\alpha_s^3)$ in massless QCD:

$$\begin{aligned} f_0^{[3]} &= 0.697706 - 0.008401 - 0.001954 - 0.000613, \\ &= 0.686737, \end{aligned}$$

$$\begin{aligned} f_L^{[3]} &= 0.302294 + 0.007254 + 0.001799 + 0.000586, \\ &= 0.311933, \end{aligned}$$

$$\begin{aligned} f_R^{[3]} &= 0. + 0.001147 + 0.000155 + 0.000027, \\ &= 0.001330, \end{aligned}$$

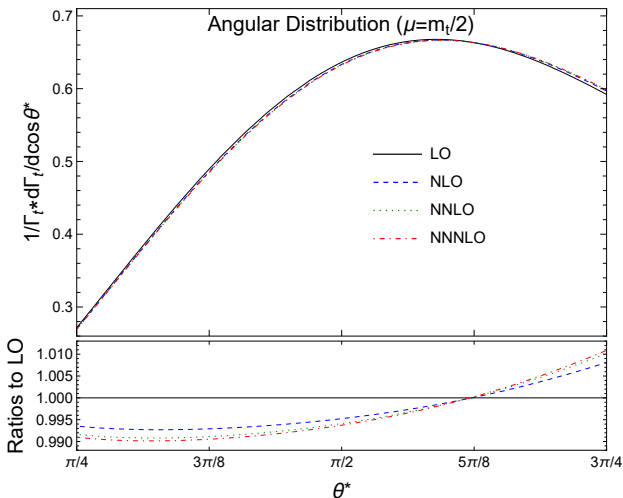
(The above results evaluated at $\mu = m_t$ agree with [\[Czarnecki, Korner, Piclum 10\]](#) up to NNLO)

With NLO EW-correction and m_b effects included, our **final results** read:

$$\boxed{f_0^{[3]} = 0.686_{-0.003}^{+0.002}, \quad f_L^{[3]} = 0.312_{-0.002}^{+0.001}, \quad f_R^{[3]} = 0.00157_{-0.00002}^{+0.00002}.}$$

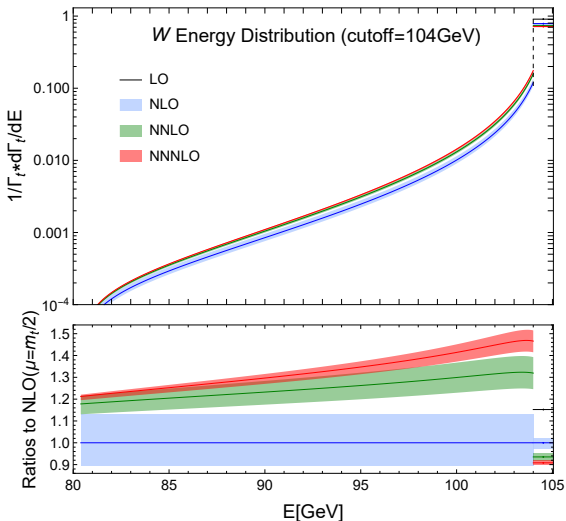
Results for $\cos \theta^*$ Angular Distribution

$$\frac{1}{\Gamma_t} \frac{d\Gamma_t}{d\cos\theta^*} = \frac{3}{4}(\sin^2 \theta^*) f_0 + \frac{3}{8}(1 - \cos \theta^*)^2 f_L + \frac{3}{8}(1 + \cos \theta^*)^2 f_R$$



where θ^* is the angle between the charged-lepton momentum from the W -decay in W -rest frame and the W -momentum in t -rest frame.

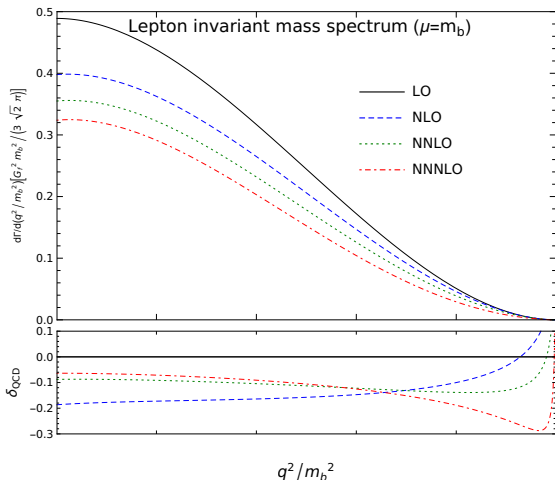
Results for W-energy Distribution



- ▶ **In the bulk:** QCD corrections are **positive and quite sizable**: pure $\mathcal{O}(\alpha_s^3)$ correction modifies the lowest order by $7 \sim 14\%$ for $E \in [94, 104]$ GeV.
- ▶ **In the rightmost 1 GeV-bin:** QCD corrections up to $\mathcal{O}(\alpha_s^3)$ **decrease** the Born-level result.

Bonus: lepton-pair invariant-mass spectrum in B-decay

$$\Gamma(b \rightarrow ul\bar{\nu}_l) = \Gamma_0 \left(1 - 2.4131 \left(\frac{\alpha_s}{\pi} \right) - 21.27 \left(\frac{\alpha_s}{\pi} \right)^2 - 270.7 \left(\frac{\alpha_s}{\pi} \right)^3 \right).$$



- ▶ The **leading-color** part agrees with a parallel computation [Chen, Li, Li, Wang, Wang, Wu 23].
- ▶ The $\mathcal{O}(\alpha_s^3)$ corrections agrees well with a recent approximation [Fael, Uskovitsch 23].

Summary and Outlook

- ☑ We have provided the **to-date most-precise high-precision theoretical prediction** for top-quark decay width:

$$\Gamma_t = 1.3148^{+0.003}_{-0.005} \times |V_{tb}|^2 + 0.027 (m_t - 172.69) \text{ GeV}$$

the error of which meets the request by future colliders.

- ☑ By a novel approach to complete IR-divergent phase-space integration over W -momentum, we determined, in addition, W -helicity fractions, $\cos\theta^*$ distribution and W -energy distribution at α_s^3 for the first time.
- ☑ Furthermore, the lepton invariant-mass distribution in the B-meson decay $b \rightarrow ul\bar{\nu}_l$ is derived up to α_s^3 .
- ☑ The approach can be readily applied to the decay of polarized t -quarks at α_s^3 , as well as the mixed QCD-electroweak corrections.

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谢谢 (Thank you)!