The 2023 international workshop on the
Circular Electron Positron Collider (CEPC)

中國科學院高能物理研究所
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New Physics Investigations with Triangular Singularity

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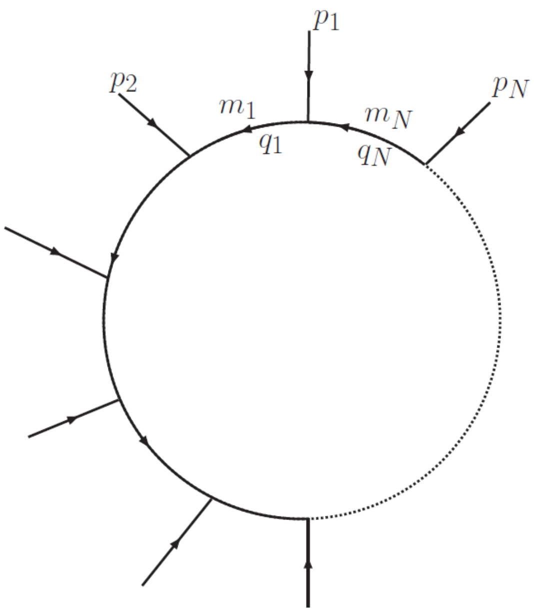
Based on work with Yu Gao, Yu Jia & Jia-Yue Zhang
arXiv: 2211.12920

Landau Singularity

- A situation when all internal particles go on shell inside a loop



One loop Feynman diagram with N external particles

The scalar N-point loop integral

$$\int \frac{d^D q}{(2\pi)^D i} \frac{1}{D_1 D_2 \cdots D_N} \qquad D_i = q_i^2 - m_i^2 + i\varepsilon$$

$$\int_0^\infty dx_1 \cdots dx_N \delta(\sum_{i=1}^{N} x_i - 1) \int \frac{d^D q}{(2\pi)^D i} \frac{1}{(x_1 D_1 + x_2 D_2 + \cdots x_N D_N)^N}$$

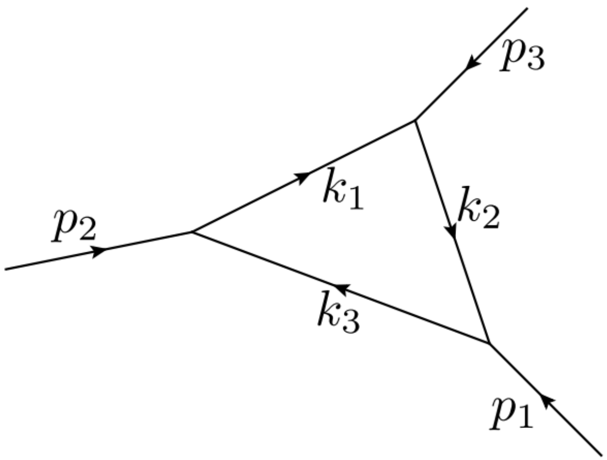
The **leading** Landau singularities are given by

$\sum_i x_i$=1 and x_i>0; $\qquad \sum_i x_i\, q_i^\mu = 0;$

$x_i(q_i^2 - m_i^2) = 0$ *L.D.Landau 1958'*

Such singularity is corresponding to kinematic pole of S-matrix, and its location is determined completely by kinematical variables.

N=3: Triangle Singularity (TS)



$$\begin{cases} \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \alpha_3 k_3^\mu = 0, \\ k_1^2 - m_1^2 = k_2^2 - m_2^2 = k_3^2 - m_3^2 = 0. \end{cases}$$

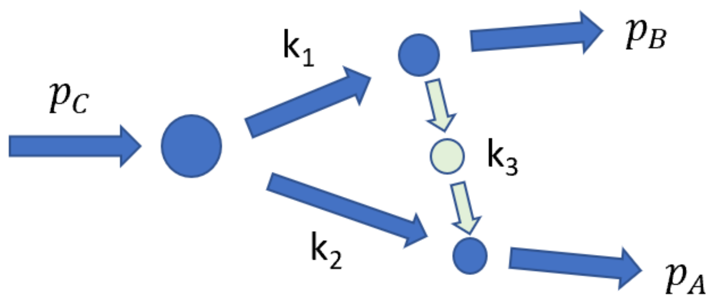
$$\Downarrow$$

$$\begin{cases} \beta_1 + \beta_2 y_{12} + \beta_3 y_{13} = 0 \\ \beta_1 y_{12} + \beta_2 + \beta_3 y_{23} = 0 \\ \beta_1 y_{13} + \beta_2 y_{23} + \beta_3 = 0 \end{cases}$$

$$y_{ij} = \frac{k_i \cdot k_j}{m_i m_j} = \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j} \quad (i \neq j \neq k) \qquad \beta_i = \alpha_i m_i$$

- The above equations build the mathematical relationship between internal mass and external momentum for triggering TS.
- However, only the singularity in physical region will emerge in amplitude.

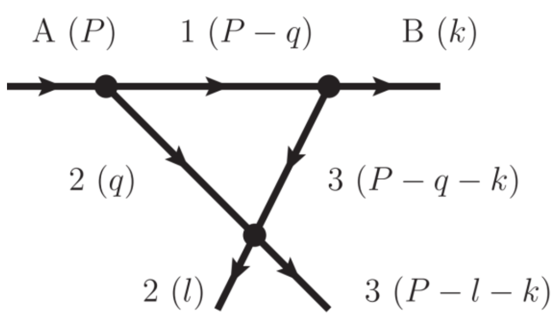
Physical picture for TS

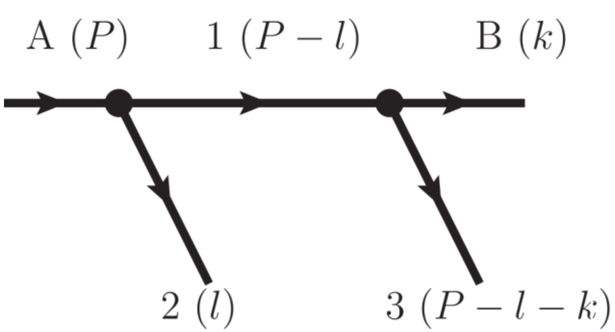


When 3 moves faster than 2, it can catch 2 and fuse to A.

In this case, singularity locate in the physical region.

- **Coleman Norton theorem**: the singularity is on the physical boundary if and only if the diagram can be interpreted as a classical process in spacetime.

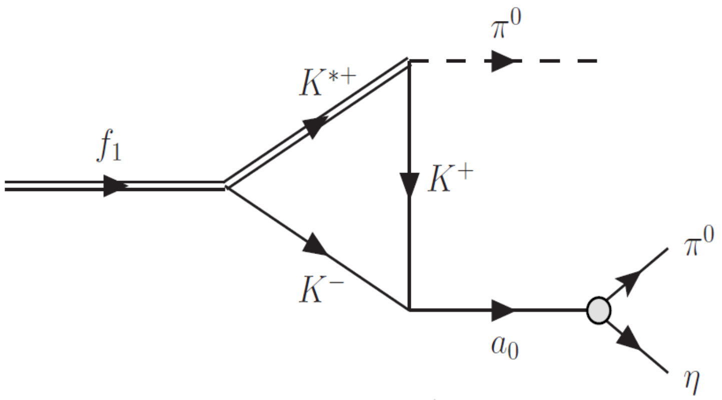




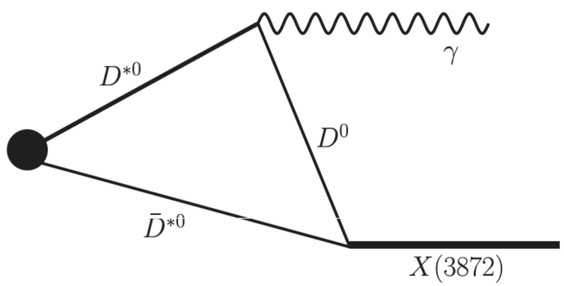
- **Schmid theorem**: $t_t^{(0)} + t_L = t_t^{(0)} e^{2i\delta}$

- When rescattering is **inelastic**, the Schmid theorem does not hold. The Singularity can be observed (due to the loop contribution) in the 2 + 3 invariant mass distribution.

Application in hadron physics



One TS diagram in f(1285) decay
Aceti, Dias, Oset, 1501.06505



X(3872) production with a TS diagram
F.K.Guo, 1902.11221

- Large number of hadronic states, make it easier to satisfy singularity conditions.
- Singularity may be mis-identified as new resonances. It can be also used to make precise measurements and enhance the production of hadronic molecules.
- For TS in hadron spectroscopy, see review Guo, Liu, Sakai, 1912.07030.

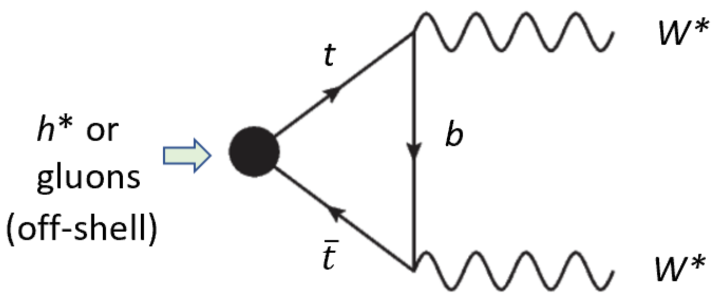
TS in hadron physics

Structures	Processes	Loops	I/F	Refs.
$\rho(1480)$ [78, 79]	$\pi^- p \to \phi\pi^0 n$	$K^*\bar{K}K$	I	[80, 81]
$\eta(1405/1475)$ [82–86]	$\eta(1405/1475) \to \pi f_0$	$K^*\bar{K}K$	I	[87–91][a,b]
$f_1(1420)$ [92]	$f_1(1285) \to \pi a_0/\pi f_0$	$K^*\bar{K}K$	I	[89, 93–95][b]
$a_1(1420)$ [96, 97]	$a_1(1260) \to f_0\pi \to 3\pi$	$K^*\bar{K}K$	I	[97–99]
1.4 GeV [100]	$J/\psi \to \phi\pi^0\eta/\phi\pi^0\pi^0$	$K^*\bar{K}K$	I	[101][b]
1.42 GeV	$B^- \to D^{*0}\pi^- f_0(a_0), \tau \to \nu_\tau\pi^- f_0(a_0)$	$K^*\bar{K}K$	I	[102, 103]
	$D_s^+ \to \pi^+\pi^0 f_0(a_0), \bar{B}_s^0 \to J/\psi\pi^0 f_0(a_0)$	$K^*\bar{K}K$	I	[104, 105]
$f_2(1810)$ [10]	$f_2(1640) \to \pi\pi\rho$	$K^*\bar{K}^*K$	I	[106]
1.65 GeV	$\tau \to \nu_\tau\pi^- f_1(1285)$	$K^*\bar{K}^*K$	I	[107]
1515 MeV	$J/\psi \to K^+K^- f_0\,(a_0)$	$\phi\bar{K}K$	I	[108]
2.85 GeV, 3.0 GeV	$B^- \to K^-\pi^- D_{s0}^*/K^-\pi^- D_{s1}$	$K^{*0}D^{(*)0}K^+$	I	[109, 110]
5.78 GeV	$B_c^+ \to \pi^0\pi^+ B_s^0$	$\bar{K}^{*0}B^+\bar{K}$	F	[111]
[4.01, 4.02] GeV	$[\bar{D}^{*0}D^{*0}] \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[112]
4015 MeV	$e^+e^- \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[113, 114]
4015 MeV	$B \to KX\pi$, $pp/p\bar{p} \to X\pi$+anything	$D^{*0}\bar{D}^{*0}D^0$	I	[115, 116]
$\Upsilon(11020)$ [117, 118]	$e^+e^- \to Z_b\pi$	$B_1(5721)\bar{B}B^*$	I	[119, 120]
3.73 GeV	$X \to \pi^0\pi^+\pi^-$	$D^{*0}\bar{D}^0D^0$	F	[121]
[4.22, 4.24] GeV	$e^+e^- \to \gamma J/\psi\phi/\pi^0 J/\psi\eta$	$D^*_{s0(s1)}\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
[4.08, 4.09] GeV	$e^+e^- \to \pi^0 J/\psi\eta$	$D^*_{s0(s1)}\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
$Z_c(3900)$ [31, 32]	$e^+e^- \to J/\psi\pi^+\pi^-$	$D_1\bar{D}D^*$	F	[119, 123–127][c]
		$D_0^*(2400)\bar{D}^*D$	F	[128, 129]
$Z_c(4020, 4030)$ [33, 130]	$e^+e^- \to \pi^+\pi^- h_c(\psi')$	$D_{1(2)}\bar{D}^{(*)}D^{(*)}$	F	[125]
$X(4700)$ [131, 132]	$B^+ \to K^+J/\psi\phi$	$K_1(1650)\psi'\phi$	F	[133]
$Z_c(4430)$ [30, 134]	$\bar{B}^0 \to K^-\pi^+J/\psi$	$\bar{K}^{*0}\psi(4260)\pi^+$	F	[135]
$Z_c(4200)$ [136, 137]	$\bar{B}^0 \to K^-\pi^+\psi(2S)$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[135]
	$\Lambda_b^0 \to p\pi^- J/\psi$	$N^*\psi(3770)\pi^-$	F	[135]
$X(4050)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}^{*0}X\pi^+$	F	[139]
$X(4250)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[139]
$Z_b(10610)$ [34]	$e^+e^- \to \Upsilon(1S)\pi^+\pi^-$	$B_J^*\bar{B}^*B$	F	[128]

Guo, Liu, Sakai, 1912.07030

LS at electroweak energy scale

- Known example in the SM: h*-> ttb -> W*W*

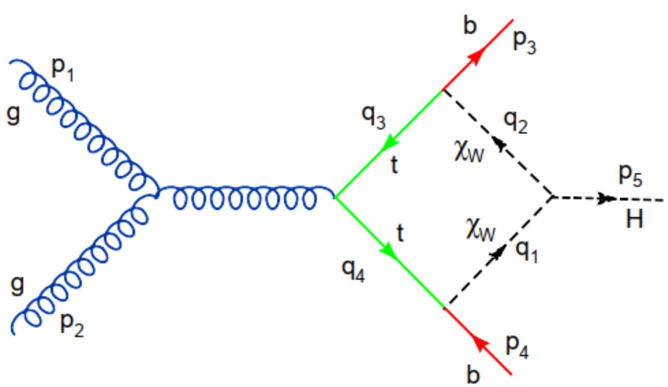


Virtual final-state W bosons:
Can trigger T.S. for $350 < \sqrt{s} < 750$ GeV
No physical solution for two on-shell Ws.

Leads to an anomalous threshold:
finite correction in cross-section.

- N=4 (box) Landau singularity in gg-> h bb

Boudjema and Duc Ninh, 0806.1498

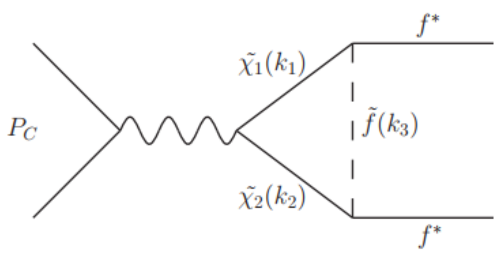


Virtual final-state H bosons:
Can trigger L.S. for $\sqrt{s} > 2m_t$ and
$\sqrt{p_5^2} > 2m_W$

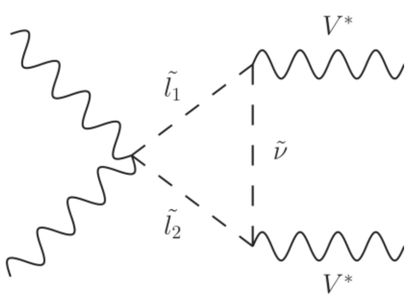
All the four particles in the loop can be simultaneously on-shell.

TS in BSM

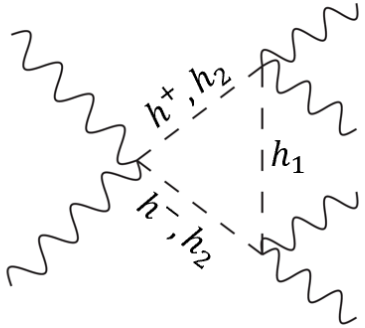
- A kaleidoscope of new particles in BSM, also make it easy to satisfy Landau singularity conditions.
- BSM particles fill in the internal lines and couple to the SM, which can produce a **purely visible SM final states** that carry BSM scale energies.
- Extra bosons in BSM to provide four-particles vertices to **evade large virtuality suppression**, which does **not realize** in the electroweak-scale SM.



Drell-Yan like
(gaugino+sfermion loop, MSSM)

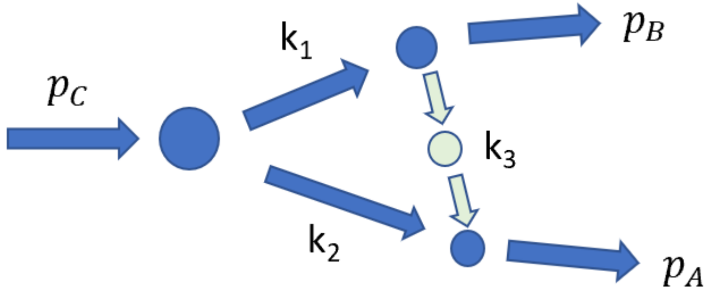


VBF diagram
(slepton loop, MSSM)



VBF – all 4-boson coupling
(2HDM)

Kinematic region of TS



Assume k_3 direction:
$B \to A$

$$m_1^2 \in \left[\frac{m_c^2 m_3 + m_B^2 m_2}{m_2 + m_3} - m_2 m_3, (m_c - m_2)^2\right]$$

In term of the invariant mass of 2+3 / 1+2 system

$$p_C^2 \in \left[(m_1 + m_2)^2, m_1^2 + m_2^2 + m_2 m_3 + \frac{m_2}{m_3}\left(m_1^2 - p_B^2\right)\right]$$

$$p_A^2 \in \left[(m_2 + m_3)^2, m_2^2 + m_3^2 + m_1 m_2 + \frac{m_2}{m_1}\left(m_3^2 - p_B^2\right)\right]$$

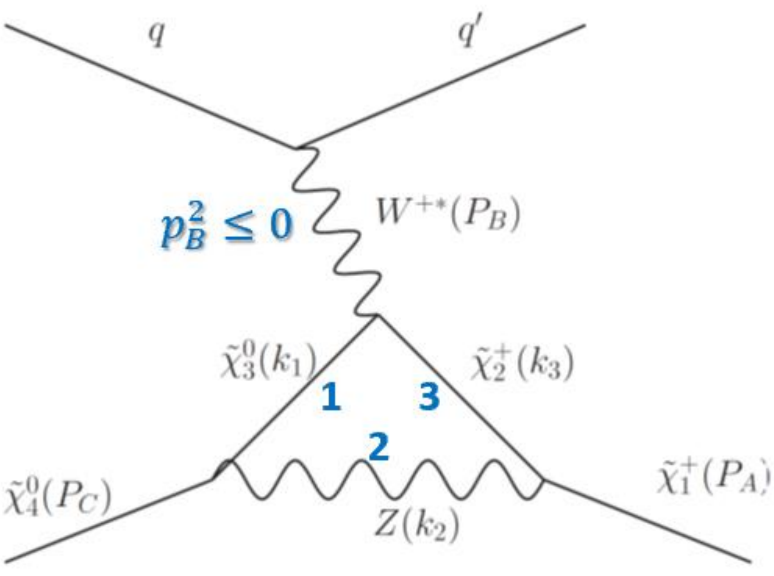
From above range we know, the external invariant momentum-square p_A^2, p_C^2 must be positive and p_B^2 is free. This leads to **two physical scenarios**:

- All three invariant momentum-squares are positive, which typically corresponds to a decay process or an s-channel collision process into two final-state momentum systems.

- One negative invariant momentum-square, i.e. $p_B^2 < 0$, that can occur in a *t*-channel scattering process with p_B as a virtual momentum exchange.

TS in t-channel

A massive system C exchanges momentum during a collision process and converts to particle system A.

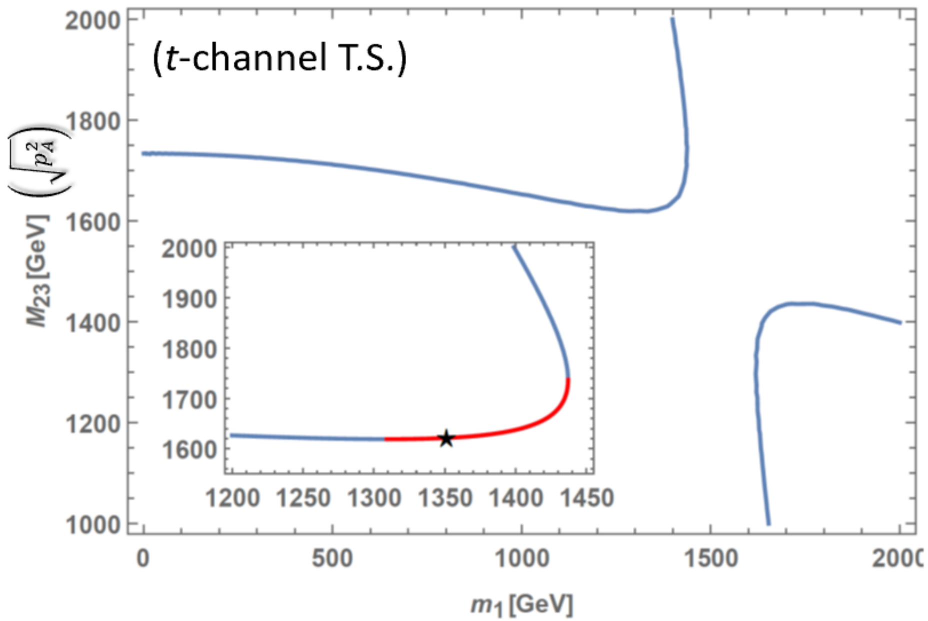


The t-channel process refers to an incited conversion with momentum transfer from the environment.

- We choose $m_3 > m_1$ so that spontaneous decay would not occur and particle 1 must receive external momentum to realize 1+B ->3 process.
- Initial state can not be the lightest stable state of a decay-able particle spectrum (like a LSP dark matter) .
- A negative p_B^2 can be extended to (soft) $\sqrt{|p_B^2|} \ll m_{BSM}$ region.

$$\begin{cases} m_1 \to m_3 \\ p_C^2 \to p_A^2 \end{cases} \quad \text{for } p_B^2 \to 0$$

Dalitz plot in t-channel



Conventional choice:

fix m_2, m_3 and p_B^2, p_C^2

m_1 and $m_{23} = \sqrt{p_A^2}$ as variables

blue: trajectory of det$|y_{ij}|$=0 (*t*-channel)

red: physical solutions

asterisk: MSSM benchmark

Landau equation equiv. as

$$\beta_i + \sum_j^{j\neq i} \beta_j y_{ij} = 0,$$

$$\text{where} \quad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_k^2}{2m_i m_j},$$

$$\text{and } \beta_i \equiv \alpha_i m_i$$

Solutions require

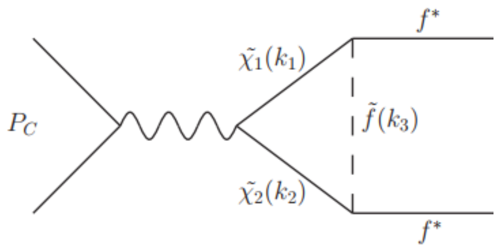
$$det \begin{vmatrix} 1 & y_{12} & y_{13} \\ y_{12} & 1 & y_{23} \\ y_{13} & y_{23} & 1 \end{vmatrix} = 0$$

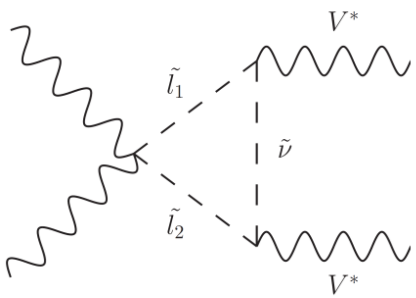
$$1+2y_{12}y_{23}y_{13}-y_{12}^2-y_{23}^2-y_{13}^2 = 0$$

containing 6 kinematic parameters.

$\alpha_i > 0$ select a small section (red) of physical solutions.

TS in s-channel

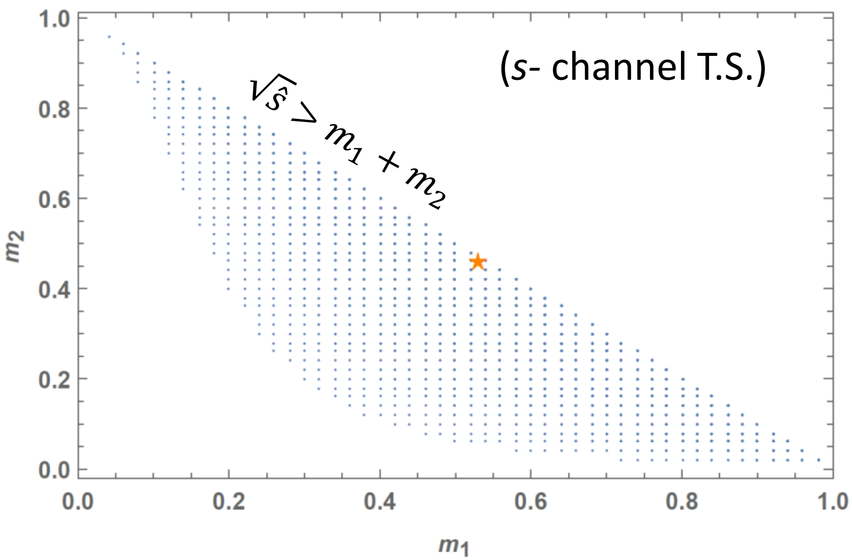




For physical solutions, the external invariant momentum-square p_C^2, p_A^2 satisfy

$$p_C^2 \in \left[(m_1+m_2)^2, m_1^2+m_2^2+m_2m_3+\frac{m_2}{m_3}\left(m_1^2-p_B^2\right) \right]$$

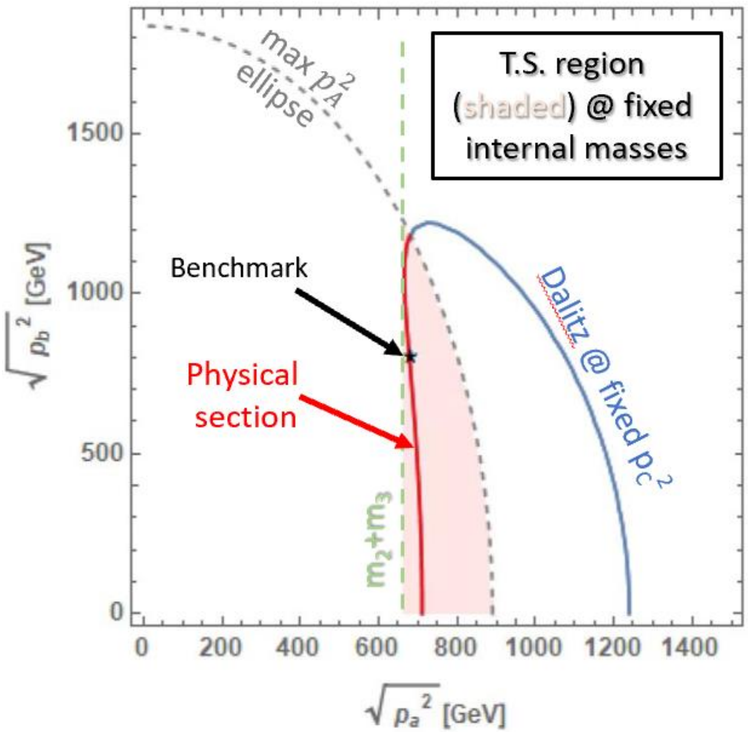
$$p_A^2 \in \left[(m_2+m_3)^2, m_2^2+m_3^2+m_1m_2+\frac{m_2}{m_1}\left(m_3^2-p_B^2\right) \right]$$

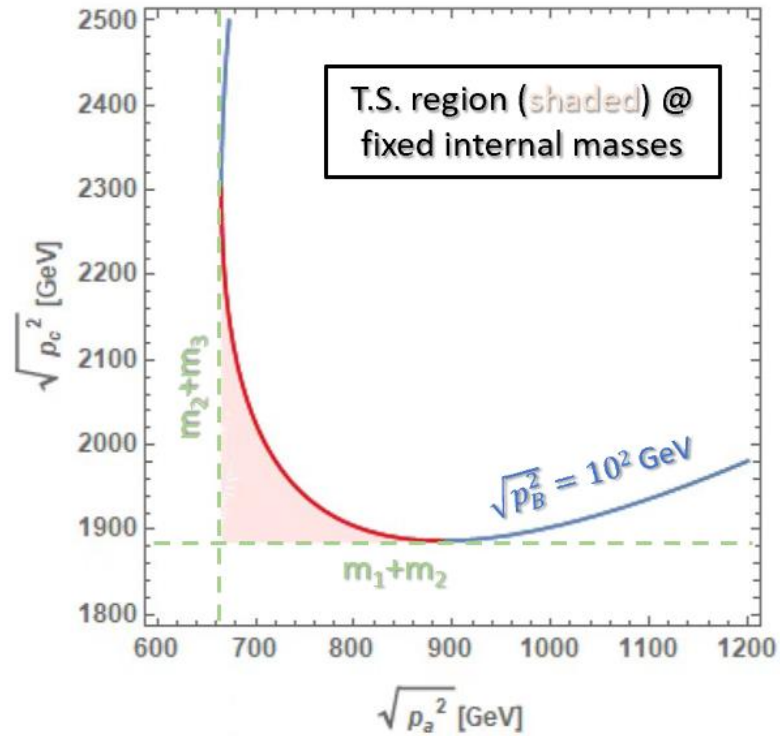


T.S. region on $\{m_1, m_2\}$ plane at given $\sqrt{p_C^2}$, and $p_A^2, p_B^2 > 0$

(1) above pair-production threshold
(2) Satisfy physical boundary ($\alpha_i > 0$)

Dalitz plot in s-channel

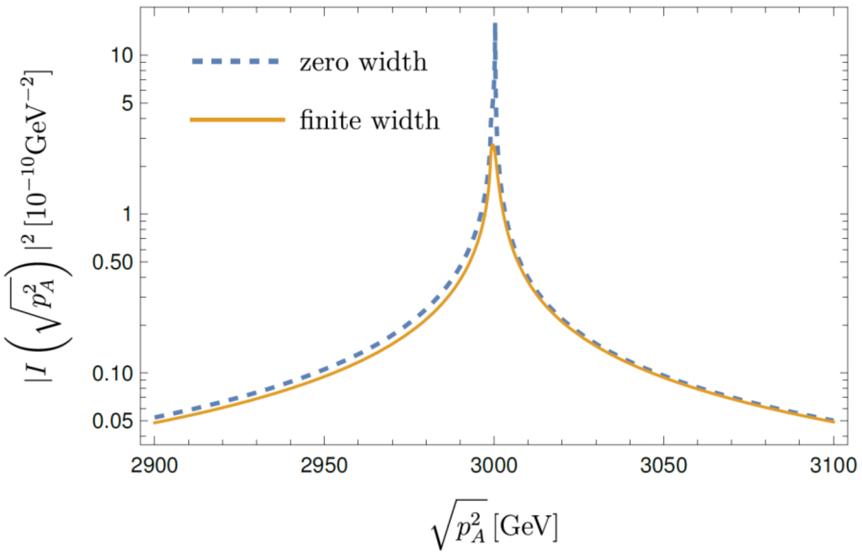




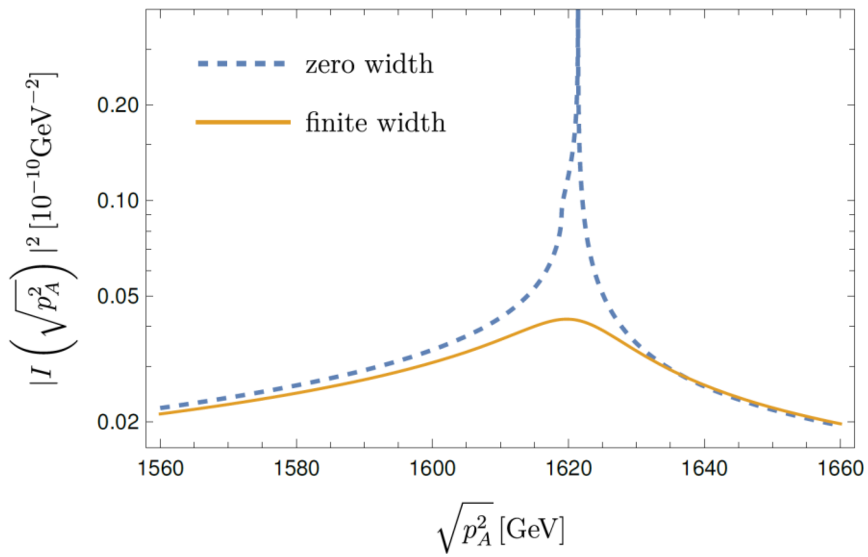
- Fixing all three internal BSM masses and one external momentum, the relation between the two remaining external invariant momenta is a Dalitz curve.
- Don’t fix external momentum and only fix internal mass, the physical Dalitz curve sweep across the parameter plane and covers a ‘total’ shaded region.

Peak in physical region

Singularity encoded in $$I\left(\sqrt{p_A^2}\right)=\int\frac{\mathrm{d}^4 l}{i\pi^2}\left[\frac{1}{l^2-m_3^2}\cdot\frac{1}{(l+p_A)^2-m_2^2}\cdot\frac{1}{(l+p_A+p_C)^2-m_1^2}\right]$$



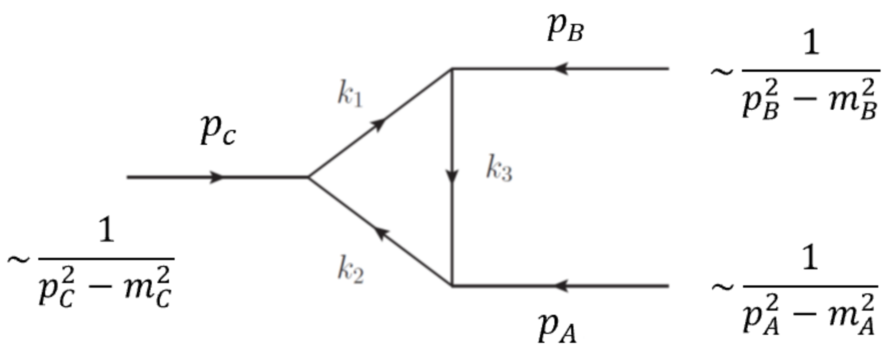
(a) WW fusion



(b) t-channel

Plots: VBF & t-channel at MSSM benchmark scenarios, with p_C^2 fixed.
Finite width of internal particles gives a small Im part.
Singularity -> a finite peak (broaden with the particle widths) .

Question: visibility



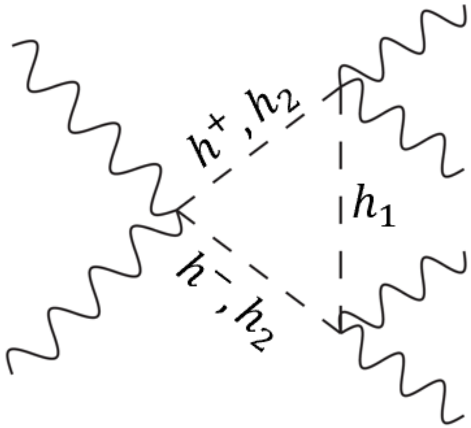
A not small virtuality suppression as

$$\sqrt{p_C^2} > m_1 + m_2$$

$$\sqrt{p_A^2} > m_2 + m_3$$

when BSM particles are heavy.

Less suppressed scenario



IS: 4-particle vertex allows for TS in a *boson-fusion* process, avoids a large 1/s.

FS: 4-particle vertices replaces a highly virtual propagator with two *more identifiable,* on-shell particles.

Four-particle vertices can play a special role in high energy TS diagrams.

Summary

- The diverse particle spectrum in BSM theories can , and *often*, provide candidate particles to fill in a triangle loop diagram and satisfy triangular singularity at a high energy collider.
- TS with BSM loops can lead to a fully identifiable SM final state that carry BSM scale energies which is helpful to reconstruct and identify and offer a unique opportunity to search for new physics at colliders.
- A t-channel scattering also triggers TS with virtual momentum exchange, different from traditional s-channel decay processes, and potentially extends to a soft-collision regime.
- BSM four-point vertices can play a significant role of evading large virtuality suppression which is unlikely to realize in the Standard Model. The complete calculation (xsec and bkg comparison) should be pursued deeply in future research.