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Testing Bell inequalities in W boson pair production at Higgs factory

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Base on arXiv:2307.14895[hep-ph] in collaboration with
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An "Old" Physics

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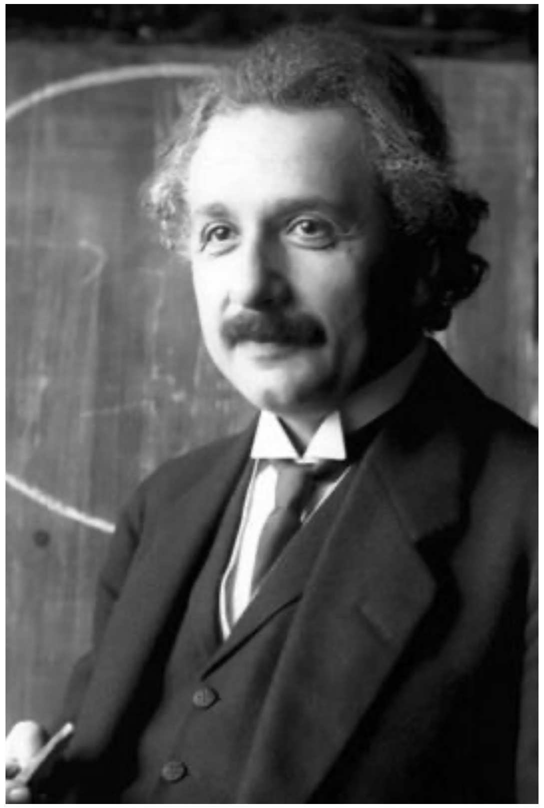
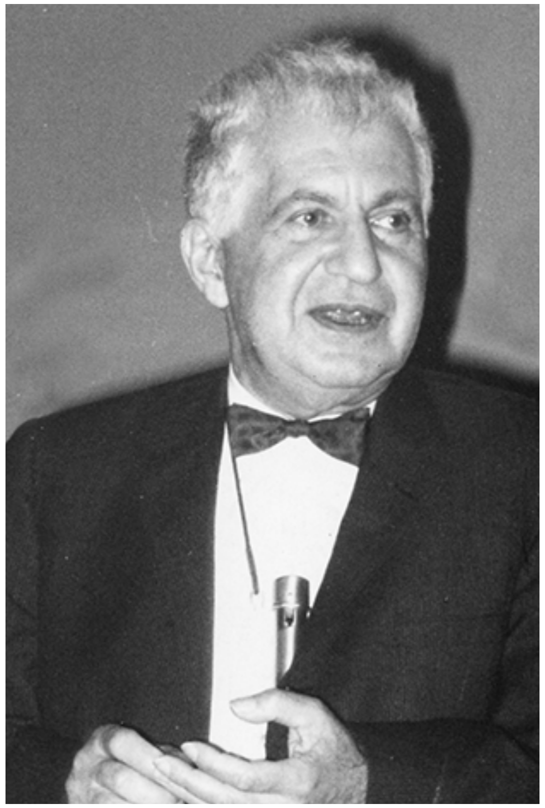
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



Albert Einstein
(1879/03/14-1955/04/18)

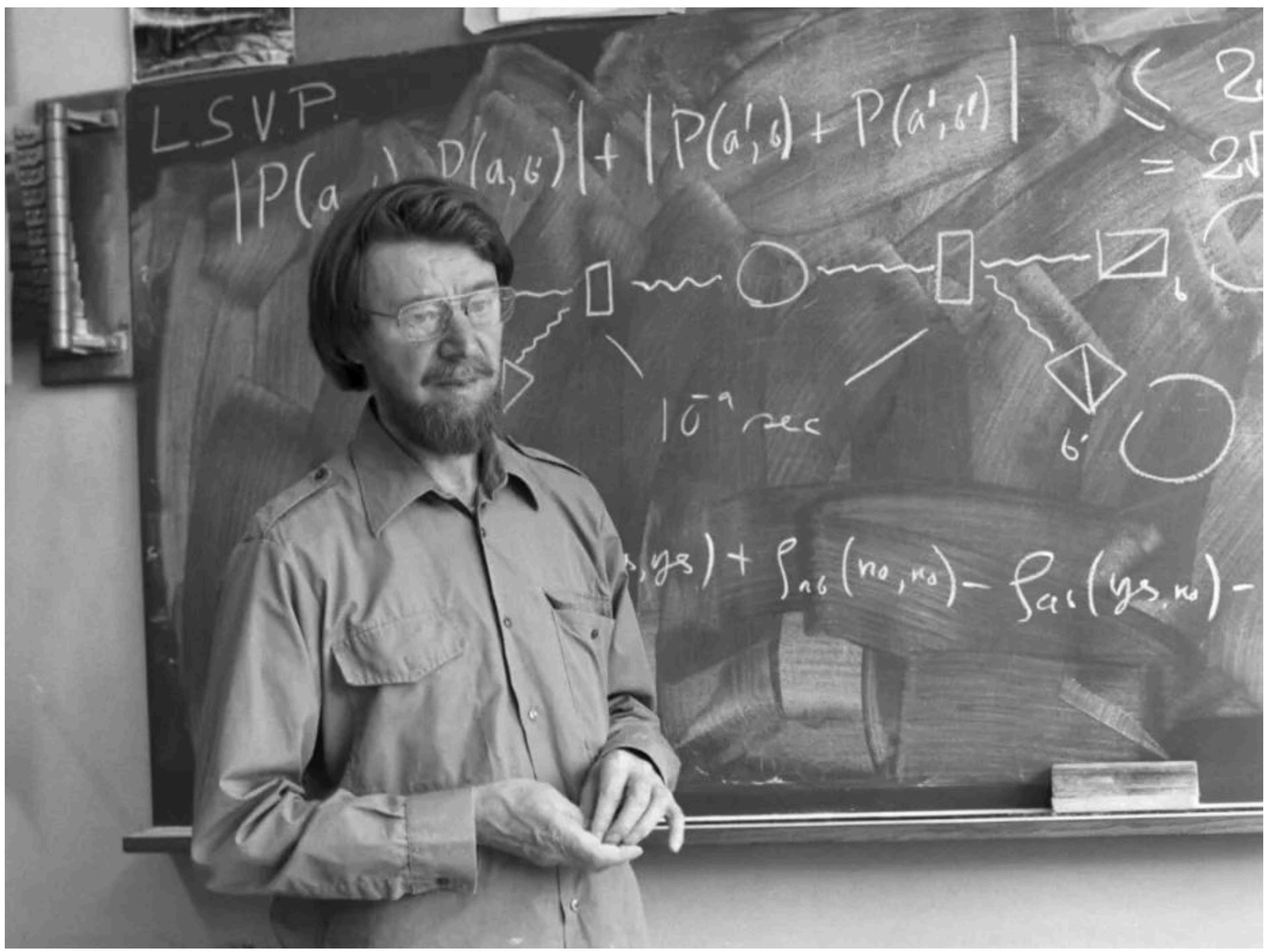


Boris Yakovlevich Podolsky
(1896/06/29-1966/11/28)

Nathan Rosen
(1909/03/22-1995/12/18)

An “Old” Physics

What is the essential different between a quantum theory and a theory based on determinism?



John Stewart Bell (1928/07/28-1990/10/01)

An “Old” Physics

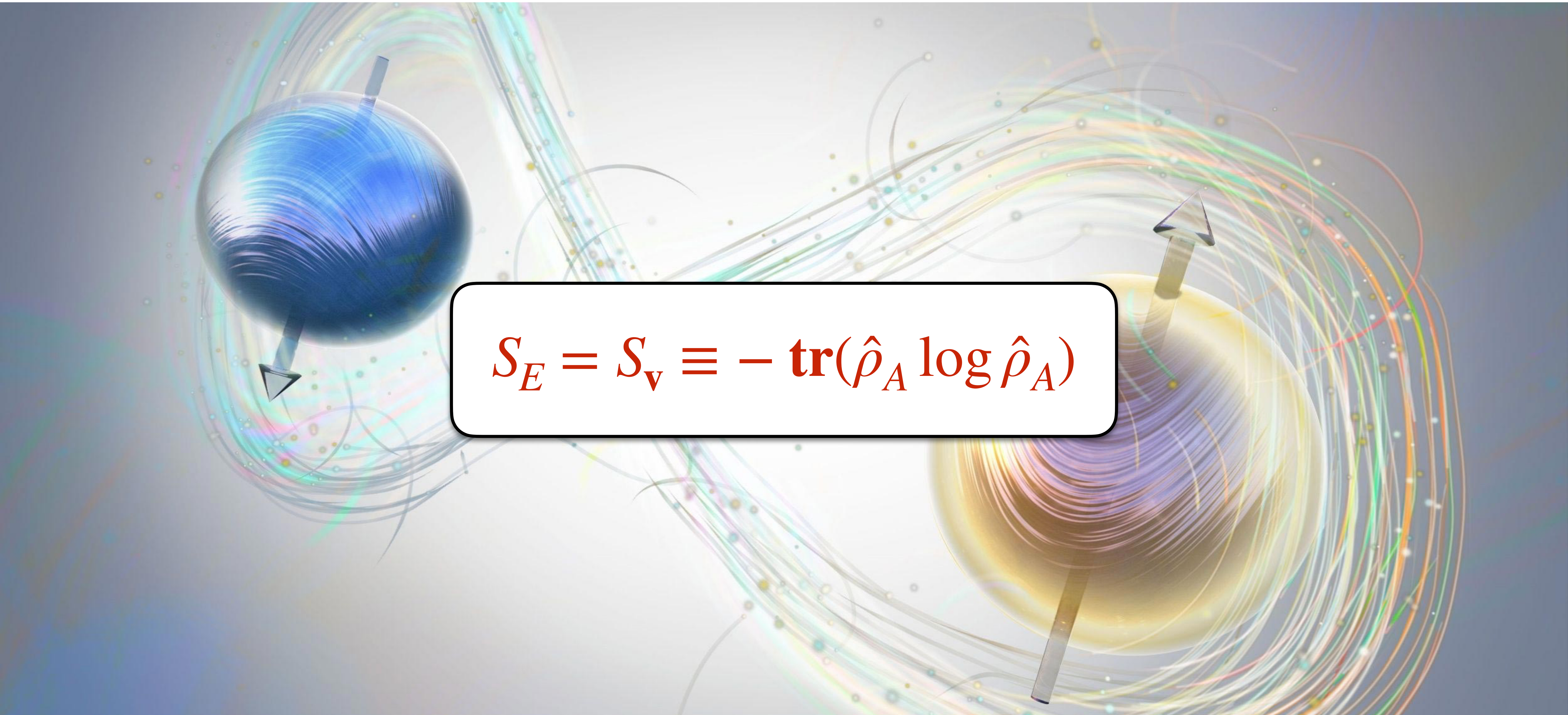
Entanglement: the more we know about the parts, the less we know about the whole system!

$$\mathbf{E}(QS) + \mathbf{E}(RS) + \mathbf{E}(RT) - \mathbf{E}(QT) \leq 2.$$

CHSH (John Clauser, Michael Horne, Abner Shimony, Richard Holt) inequality

An “Old” Physics

Entanglement entropy: a description of the degree of the entanglement between subsystems.



$$S_E = S_{\mathbf{v}} \equiv -\,\mathbf{tr}(\hat{\rho}_A \log \hat{\rho}_A)$$

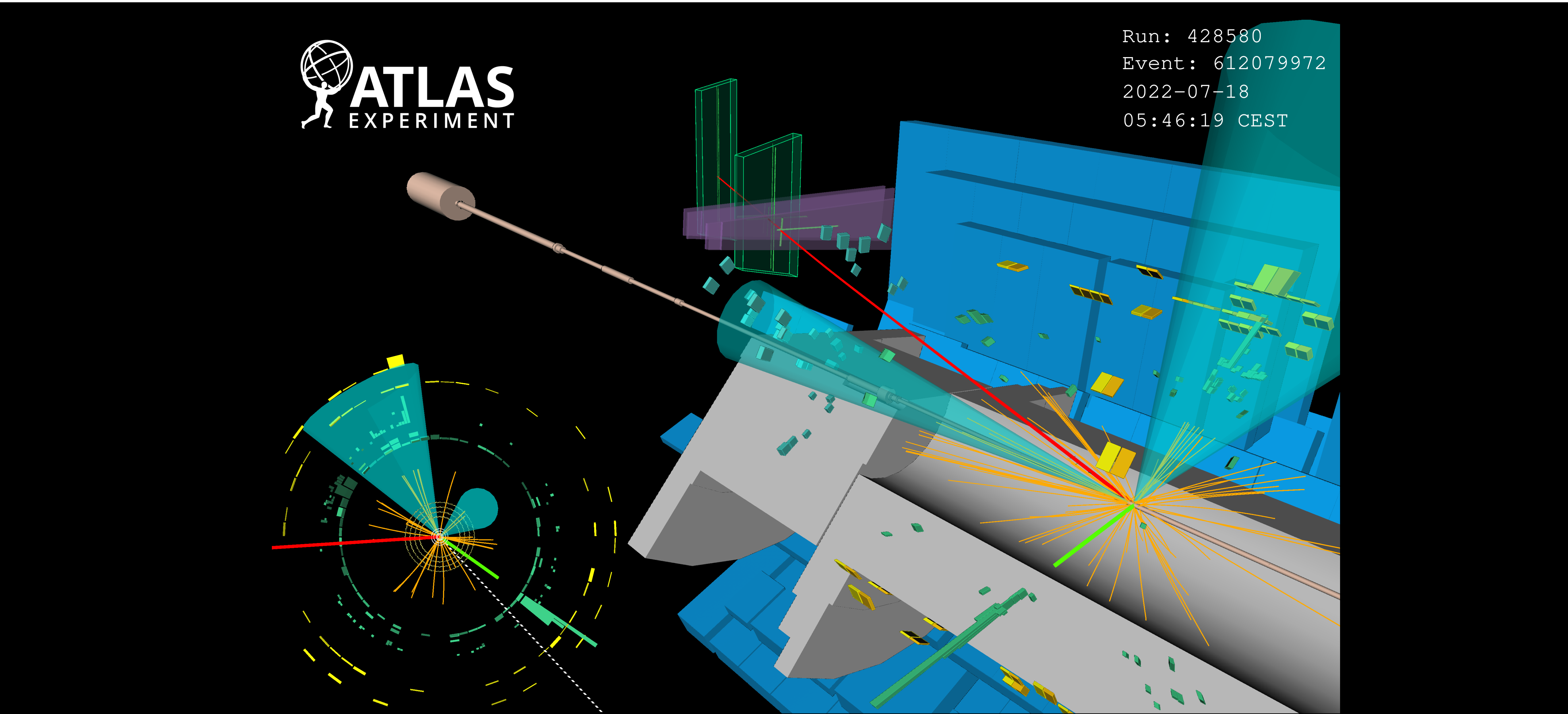
The Verification in particle physics

- Neutral pion system
- Neutral Kaon system
- Other hadron systems
- …
- Testing with higher energy?

Many references…

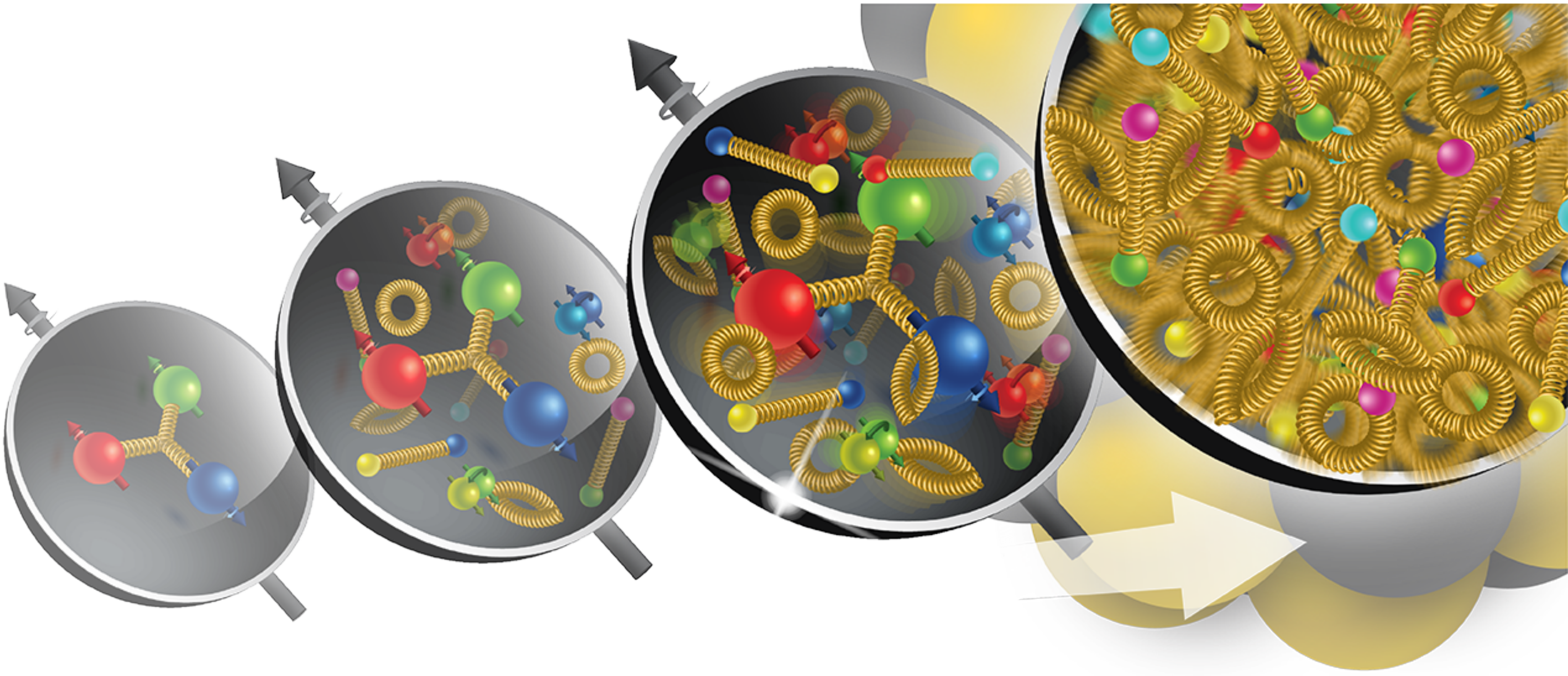
The Verification in EW scale

- The most popular topic: $t\bar{t}$ production at the LHC.
- Why?



The Verification in EW scale

- It is not easy, why?



The Verification in EW scale

A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique!

$$\sigma = p_1 \rho_{A1} \otimes \rho_{B1} + p_2 \rho_{A2} \otimes \rho_{B2} + \ldots \textbf{?}$$

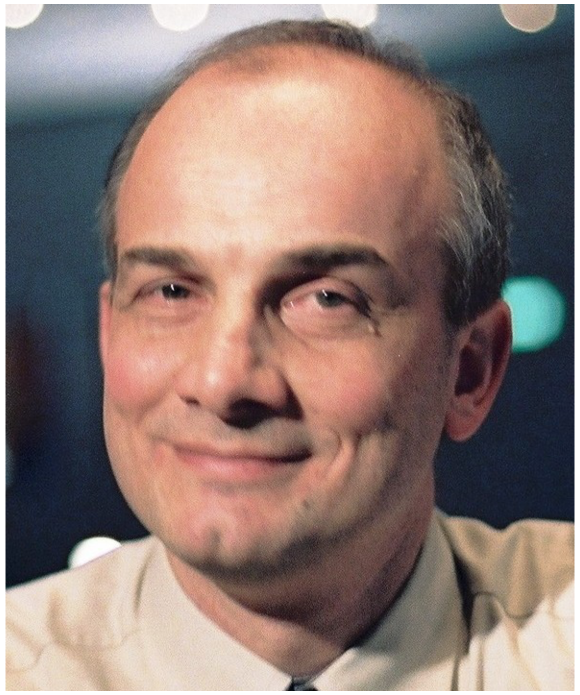
The Verification in EW scale

A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique!

$$\sigma = p_1 \rho_{A1} \otimes \rho_{B1} + p_2 \rho_{A2} \otimes \rho_{B2} + \ldots \; \textbf{?}$$

"Finally , we prove that the weak membership problem for the convex set of separable bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits



★ That is the reason why people introduce some extremely complicate quantity such as CONCURRENCE.

WW production at Higgs factory

- The initial state is a mixed state

➡ (Generalized) Bell inequality

$$\begin{aligned}\mathcal{I}_3 \equiv +&\big[P\left(A_1=B_1\right)+P\left(B_1=A_2+1\right)\\&+P\left(A_2=B_2\right)+P\left(B_2=A_1\right)\big]\\-&\big[P\left(A_1=B_1-1\right)+P\left(B_1=A_2\right)\\&+P\left(A_2=B_2-1\right)+P\left(B_2=A_1-1\right)\big]\end{aligned}$$

$$\max_{\hat{A}_1,\hat{A}_2,\hat{B}_1,\hat{B}_2} \mathcal{I}_3(\hat{A}_1,\hat{A}_2;\hat{B}_1,\hat{B}_2) > 2$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The initial state is a mixed state
  - ➡ (Generalized) Bell inequality
- 9-dim but not 4-dim Hilbert space.

3x3>2x2

- QuNit vs. qubit?
  - ➡ “the results for large N are shown to be more resistant to noise with a suitable choice of the observables”

D. Kaszlikowski, P. Gnaciński, M. Żukowski, W. Miklaszewski, A. Zeilinger, Phys. Rev. Lett. **85**, 4418 (2000);
T. Durt, D. Kaszlikowski, M. Żukowski, Phys. Rev. A **64**, 024101 (2001);
J.-L. Chen, D. Kaszlikowski, L. C. Kwek, C. H. Oh, M. Żukowski, Phys. Rev. A **64**, 052109 (2001);
D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The density matrix (some technical details…)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i\hat{S}_i + q^{ij}\hat{S}_{\{ij\}},\ \ i,j = 1,2,3$$

$$\begin{aligned}\hat{\rho}_{WW} \;=\; & \frac{1}{9}\hat{I}_9 + \frac{1}{3}d^i_+\hat{S}^+_i \otimes \hat{I}_3 + \frac{1}{3}d^i_-\hat{I}_3 \otimes \hat{S}^-_i \\ & + \frac{1}{3}q^{ij}_+\hat{S}^+_{\{ij\}} \otimes \hat{I}_3 + \frac{1}{3}q^{ij}_-\hat{I}_3 \otimes \hat{S}^-_{\{ij\}} \\ & + C^{ij}_d\hat{S}^+_i \otimes \hat{S}^-_j + C^{i,jk}_{d,q}\hat{S}^+_i \otimes \hat{S}^-_{\{jk\}} \\ & + C^{ij,k}_{q,d}\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_k + C^{ij,k\ell}_q\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_{\{k\ell\}}\end{aligned}$$

WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate

$$
\begin{aligned}
& d_+^2 = d_-^2 = q_+^{12} = q_+^{23} = q_-^{12} = q_-^{23} = C_d^{12} = C_d^{21} \\
& = C_d^{23} = C_d^{32} = C_{d,q}^{1,12} = C_{d,q}^{1,23} = C_{d,q}^{2,31} = C_{d,q}^{2,11} = C_{d,q}^{2,22} \\
& = C_{d,q}^{2,33} = C_{d,q}^{3,12} = C_{d,q}^{3,23} = C_{q,d}^{12,1} = C_{q,d}^{23,1} = C_{q,d}^{31,2} \\
& = C_{q,d}^{11,2} = C_{q,d}^{22,2} = C_{q,d}^{33,2} = C_{q,d}^{12,3} = C_{q,d}^{23,3} = C_q^{12,31} \\
& = C_q^{12,11} = C_q^{12,22} = C_q^{12,33} = C_q^{23,31} = C_q^{23,11} = C_q^{23,22} \\
& = C_q^{23,33} = C_q^{31,12} = C_q^{31,23} = C_q^{11,12} = C_q^{11,23} = C_q^{11,22} \\
& = C_q^{22,12} = C_q^{22,23} = C_q^{22,11} = C_q^{33,12} = C_q^{33,23} = C_q^{33,31} \\
& = 0.
\end{aligned}
\tag{46}
$$

WW production at Higgs factory

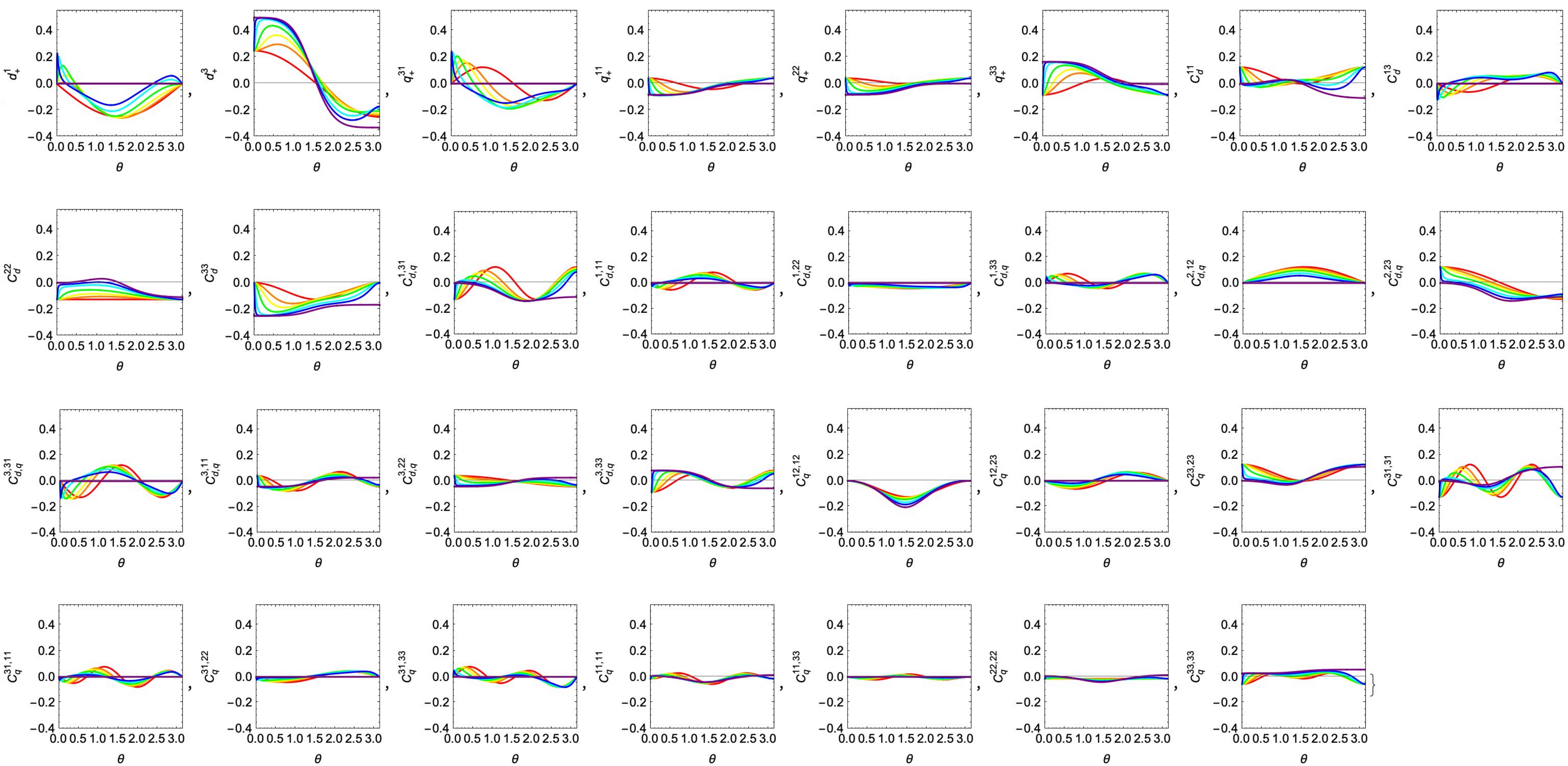
- The density matrix (some technical details…) is easy to calculate

$$
\begin{aligned}
& d_+^1 = d_-^1,\ d_+^3 = -d_-^3,\ q_+^{31} = -q_-^{31},\ q_+^{11} = q_-^{11},\ q_+^{22} = q_-^{22}, \\
& q_+^{33} = q_-^{33},\ C_d^{13} = -C_d^{31},\ C_{d,q}^{1,31} = -C_{q,d}^{31,1},
\end{aligned}
$$

$$
\begin{aligned}
& C_{d,q}^{1,11} = C_{q,d}^{11,1},\ C_{d,q}^{1,22} = C_{q,d}^{22,1},\ C_{d,q}^{1,33} = C_{q,d}^{33,1}, \\
& C_{d,q}^{2,12} = C_{q,d}^{12,2},\ C_{d,q}^{2,23} = -C_{q,d}^{23,2},\ C_{d,q}^{3,31} = C_{q,d}^{31,3}, \\
& C_{d,q}^{3,11} = -C_{q,d}^{11,3},\ C_{d,q}^{3,22} = -C_{q,d}^{22,3},\ C_{d,q}^{3,33} = -C_{q,d}^{33,3}, \\
& C_q^{12,23} = -C_q^{23,12},\ C_q^{31,11} = -C_q^{11,31},\ C_q^{31,22} = -C_q^{22,31}, \\
& C_q^{11,33} = C_q^{33,11} = -C_q^{22,33} = -C_q^{33,22}.
\end{aligned} \tag{47}
$$

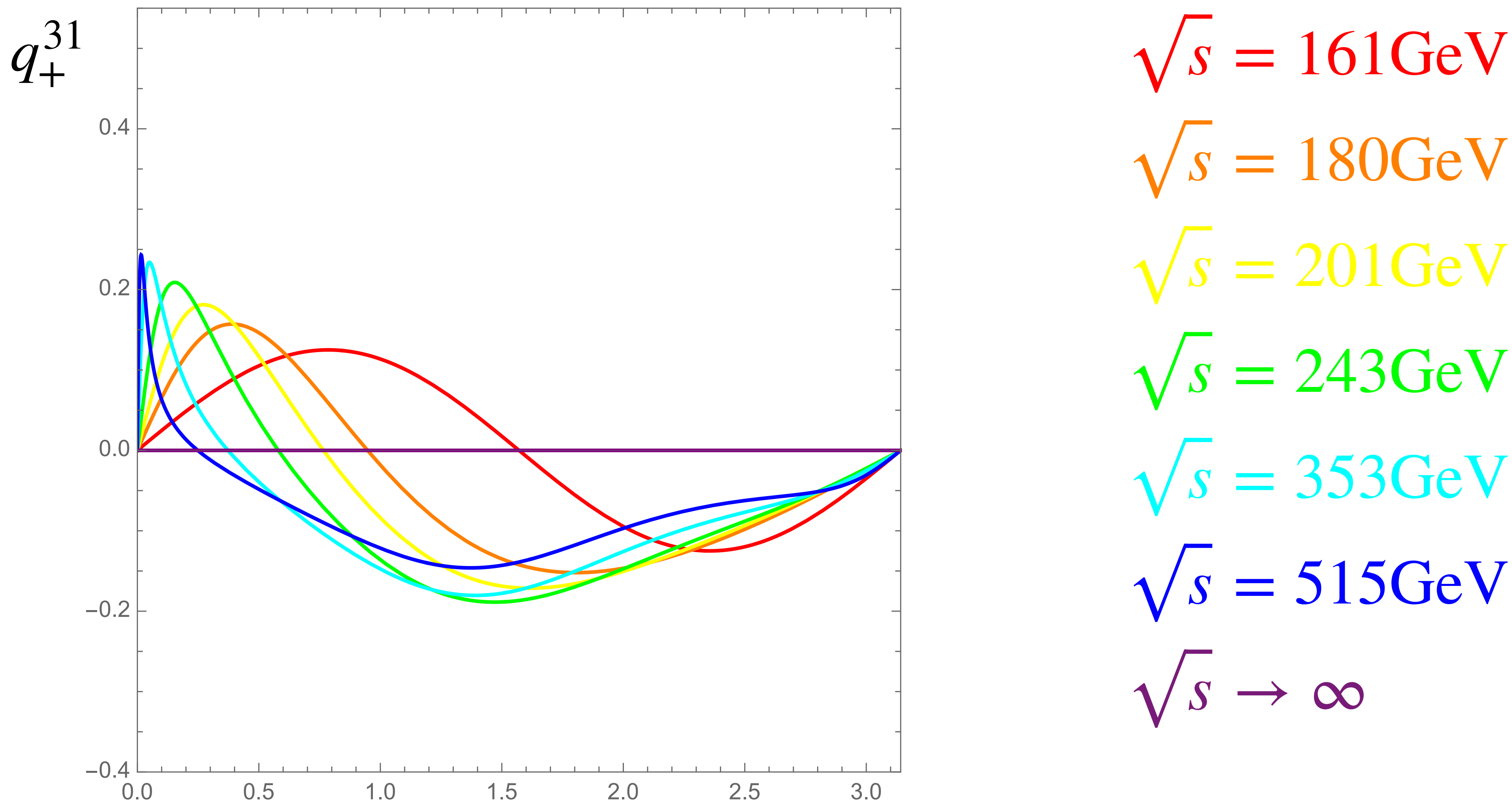
WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate



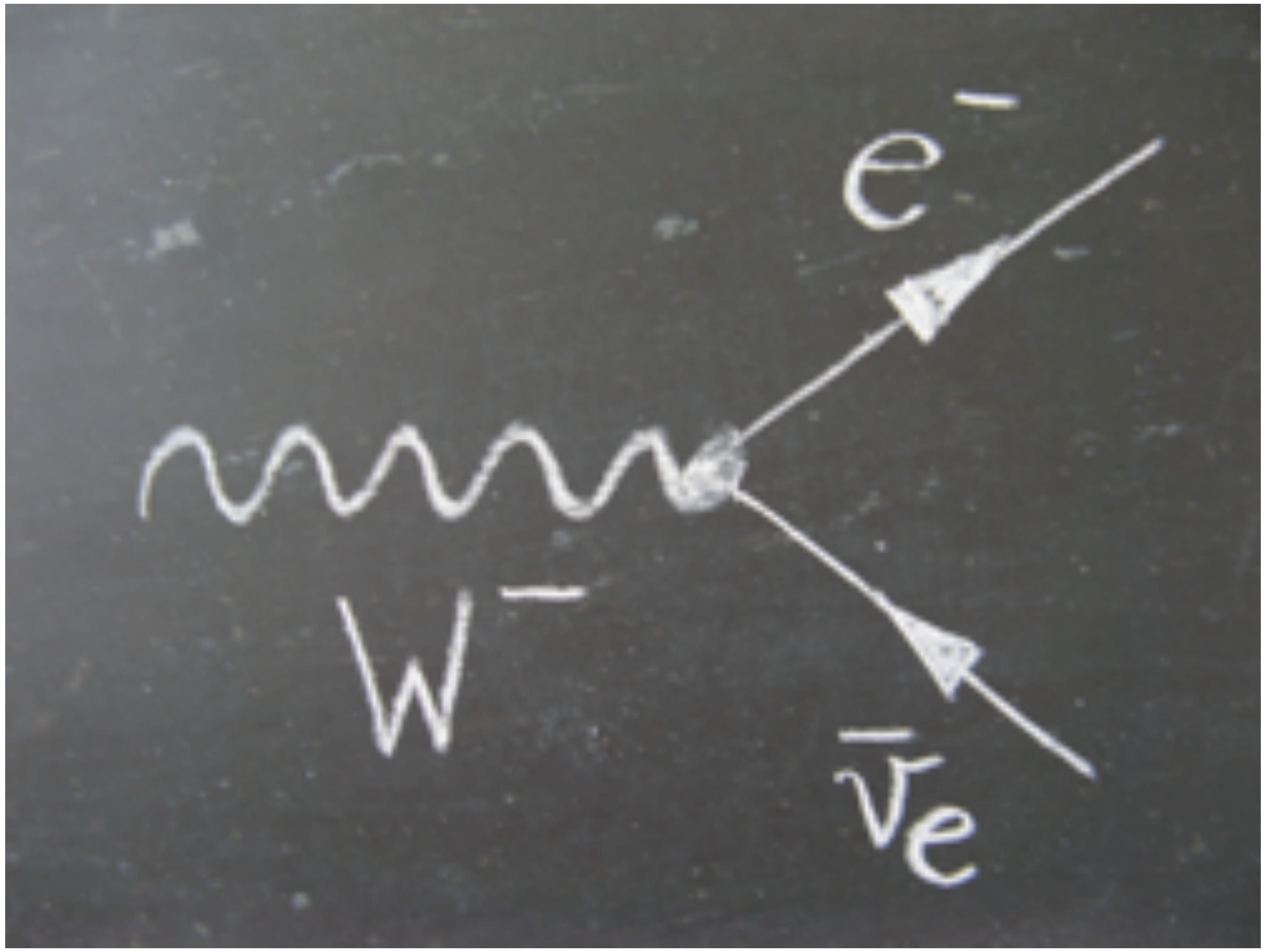
WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate
- $\beta \to 0, \infty$ are not very good approximations.



WW production at Higgs factory

- How to measure it at Higgs factory???
- “Measuring” the polarization direction of the W boson.



WW production at Higgs factory

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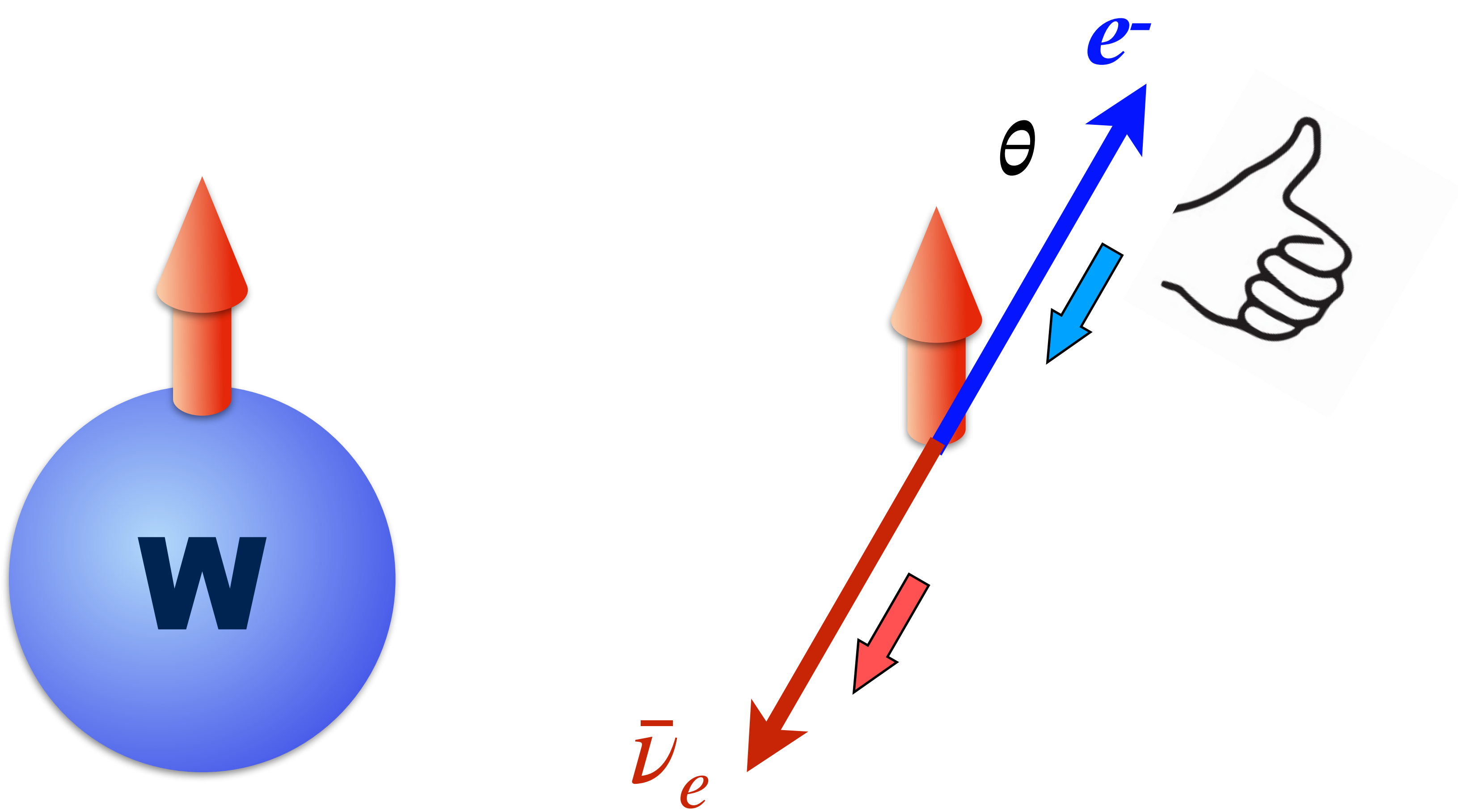

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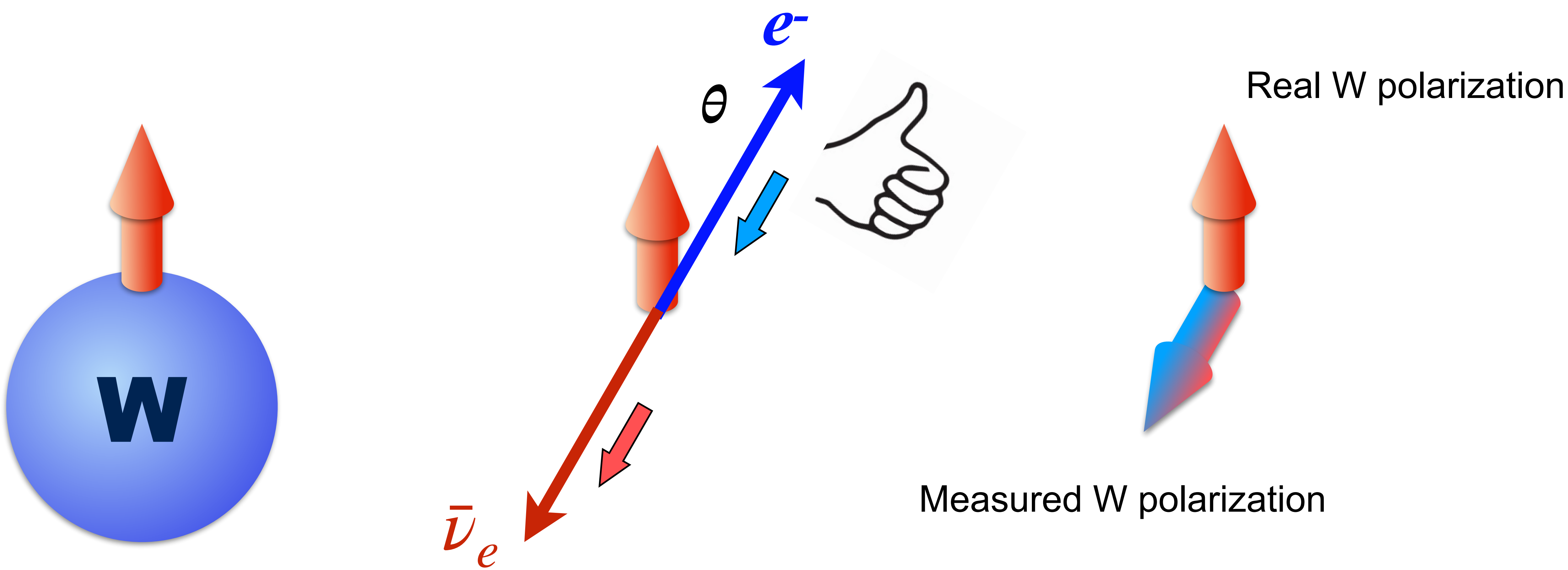


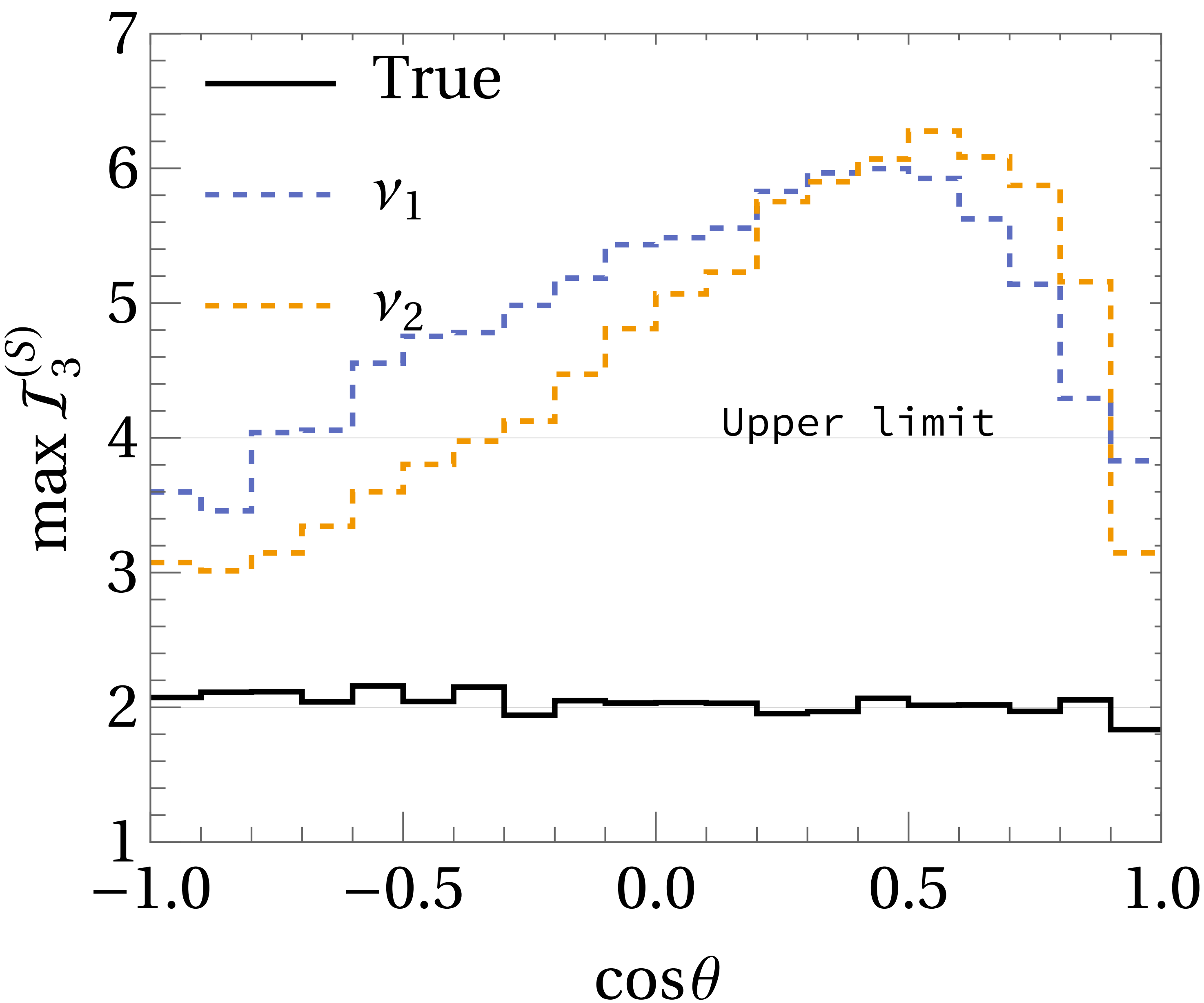
WW production at Higgs factory

- How to measure it at Higgs factory???
- "Measuring" the polarization direction of the W boson.
- Projection operators of the spin eigenstates:

$$\hat{\Pi}_{\mathbf{n}} = \frac{1}{2}(\hat{S}_{\mathbf{n}} + \hat{S}_{\mathbf{n}}^2)$$

WW production at Higgs factory

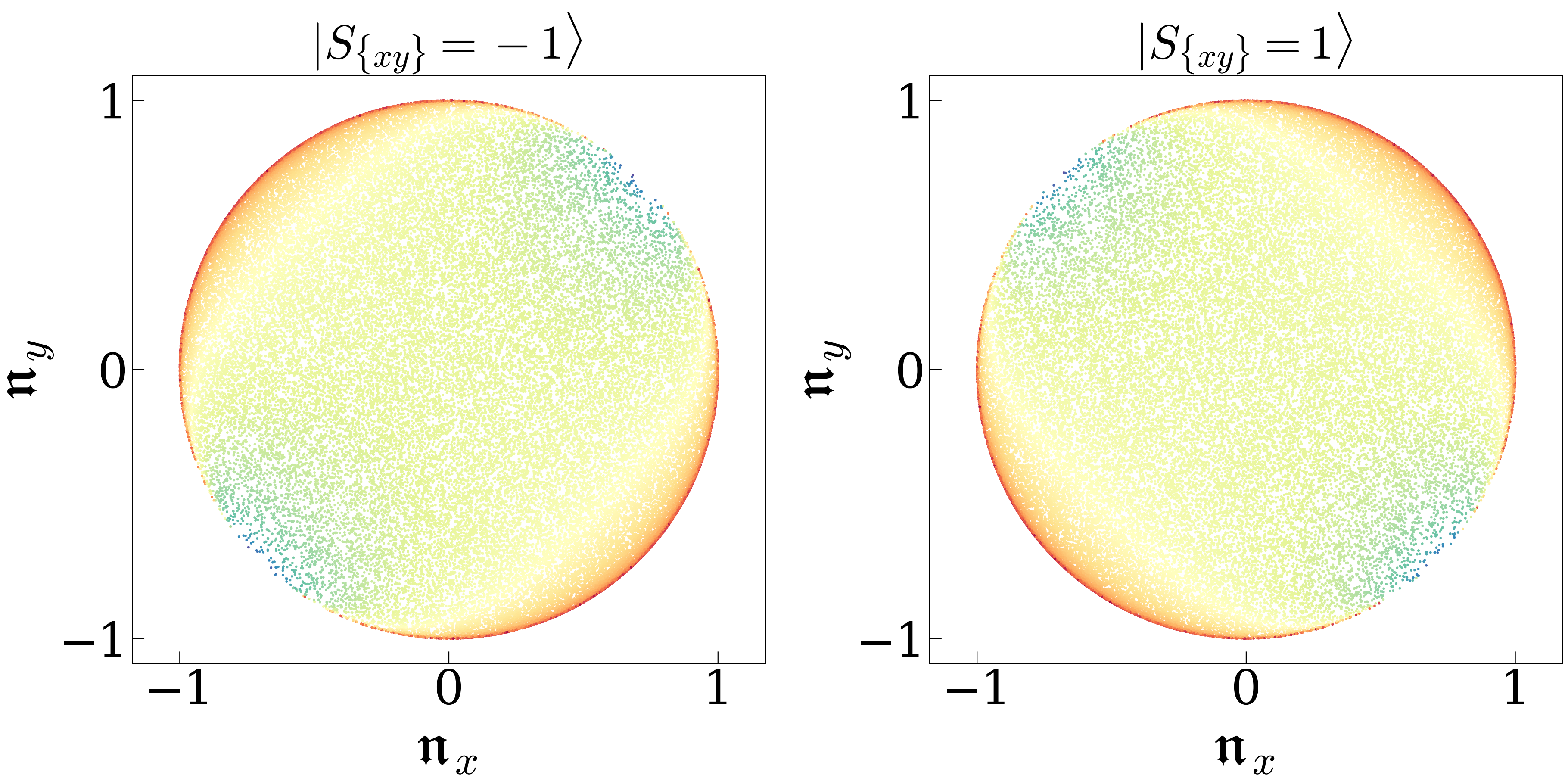
- Collider phenomenology



WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.

$$\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}_{\mathbf{n}}^2$$



WW production at Higgs factory

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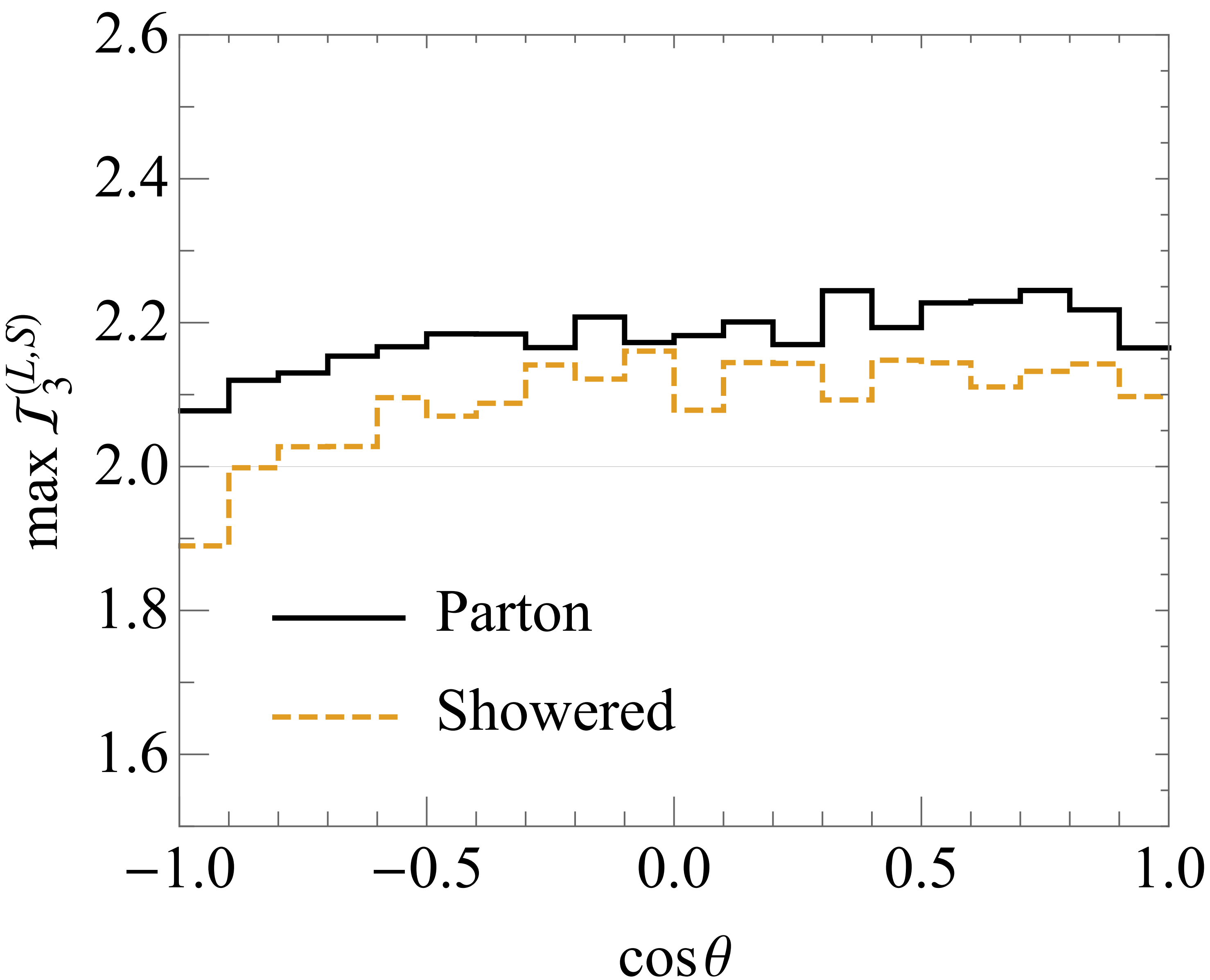

WW production at Higgs factory

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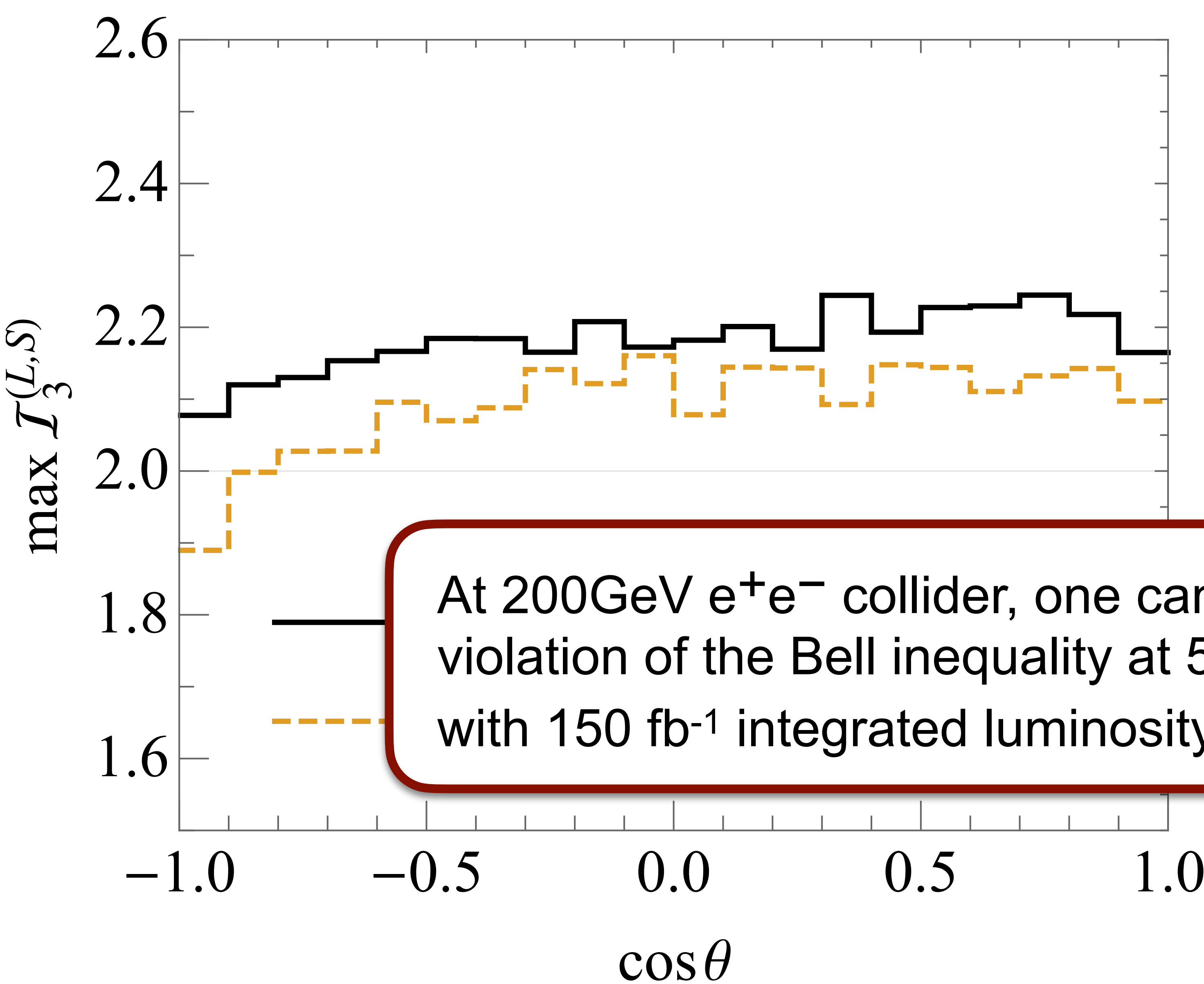


At 200GeV e^+e^- collider, one can verify the violation of the Bell inequality at 5.0σ significance with 150 fb^{-1} integrated luminosity.

Conclusion and Discussion

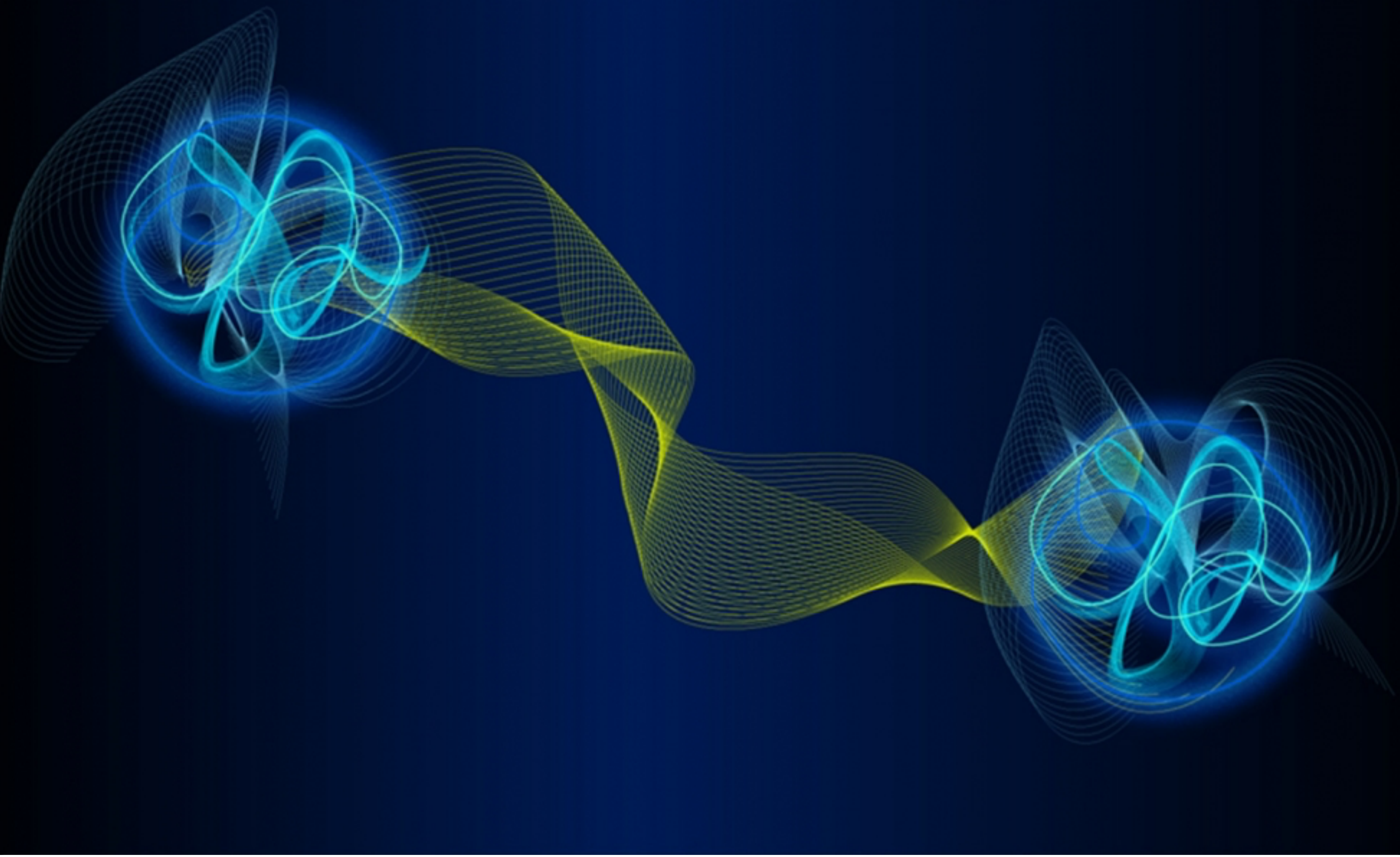
- We provide a realistic approach to test Bell inequalities in W pair systems using a new set of Bell observables based on measuring the linear polarization of W bosons.
- Our observables depend on only part of the density matrix that can be correctly measured in the semi-leptonic decay mode of W.
- To our best knowledge, this is the first attempt of testing Bell inequality in a basic qu3it system (**beyond qubit**).

*　　　　*　　　　*　　　　*　　　　*

- At the threshold, because the s-channel contribution of the WW production is higher suppressed, the initial state is nearly pure. In this limit, the final state W is still a mixed state and the entanglement entropy is S_E=log2.

Conclusion and Discussion

- Though one could argue that all of the information has been included in the spin correlations, it is the Bell inequalities which can show the quantum property of system clearly.
- It would be an quite interesting question that whether some specific new physics effect can change the violation of the Bell inequalities or increase or decrease the entanglement entropy (e.g., increase the entanglement entropy from log2 to log3).
- Since the origin of the entanglement between the final state particles is the high energy scattering process itself, one could probably classify the interactions (both SM and NP) by their ability of generating entanglement between the final state particles.
- It is still an open question to testing the entanglement in an essential QFT system (beyond quantum mechanism).



Thank you!