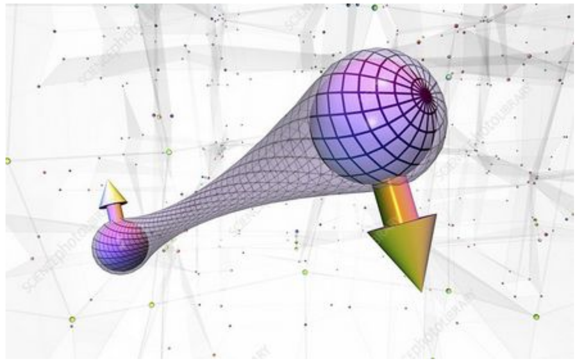
Bell inequalities and Quantum Entanglement in Weak Gauge Bosons production at Colliders



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The 2023 International Workshop on the High Energy Circular Electron Positron Collider
Nanjing University , Jiangsu Oct 23-27, 2023

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based on : Eur. Phys. J.C. 83 (2023) 9, 823, 2302.00683 [hep-ph]

- “Entanglement” between two systems is a pure quantum phenomena

- Expected to violates Bell inequalities → set of (classical) probability correlations

$$P(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+) \leq P(\hat{\mathbf{a}}+;\hat{\mathbf{c}}+) + P(\hat{\mathbf{c}}+;\hat{\mathbf{b}}+)$$

$\hat{\mathbf{a}},\ \hat{\mathbf{b}},\ \hat{\mathbf{c}}$

spin+ along **a** for particle 1 spin+ along **b** for particle 2

arbitrary directions

- Violations incompatible with classical physics based on causality and local realism (locality) (EPR paradox, hidden variables theories)

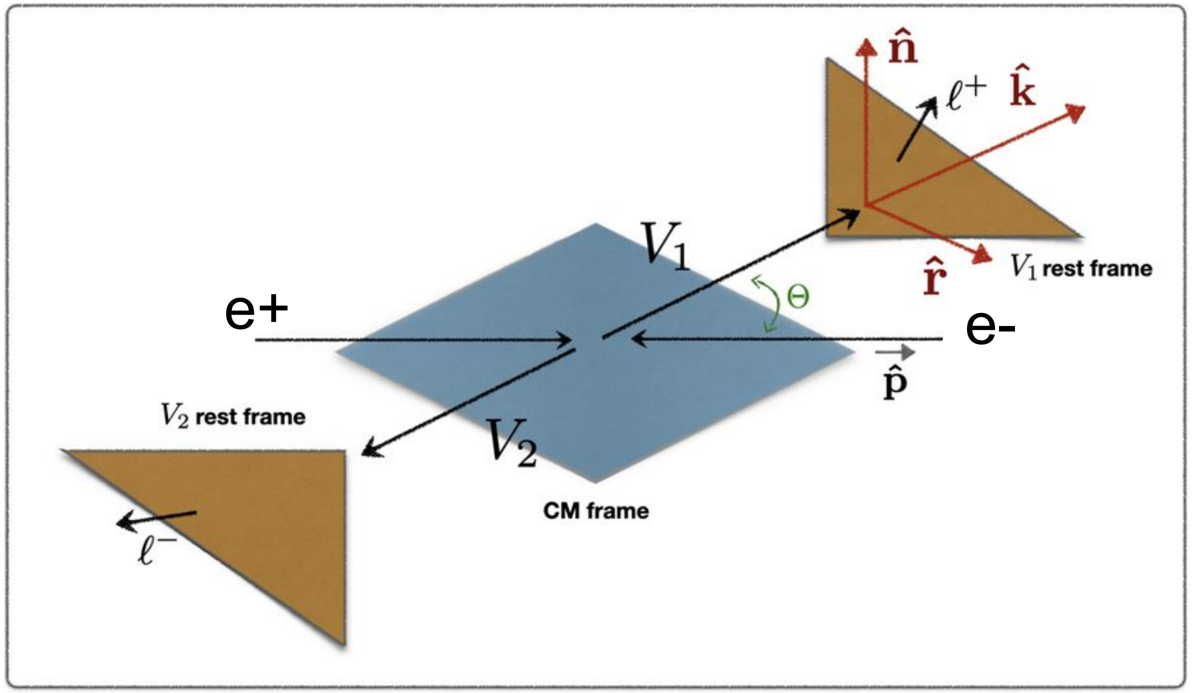
- pairs of two outcome measurements is at least required to test Bell inequalites

Entanglement & Bell inequalities in spin-1 systems (qutrits)

- Requires the knowledge of the **Polarization Density Matrix (PDM)** of spin-1 $V_1 V_2$ production

- PDM can be fully reconstructed from the angular distributions of the single V_1 V_2 **decay products**

PDM depends only on invariant mass of V_1V_2 system and on scattering angle θ in the V_1V_2 c.m. frame



- **but it can also be computed analytically**

- knowledge of the full polarization density matrix allows to quantify (where possible) entanglement and Bell inequality violations

Two-QUTRITS

System of two massive spin-1 particles (WW , ZZ , WZ)

Useful to decompose **density matrix** on the basis of tensor products of **Gell-Mann matrices**

$$\{\mathbb{1}\otimes\mathbb{1},\ \mathbb{1}\otimes T^a,\ T^a\otimes\mathbb{1},\ T^a\otimes T^b\}$$

$T^a \longrightarrow$ 3 x 3 Gell-Mann matrices

$$[A\otimes B]_{ii'jj'} = A_{ii'}B_{jj'}$$

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}[\mathbb{1}\otimes\mathbb{1}] + \sum_a f_a\,[\mathbb{1}\otimes T^a] + \sum_a g_a\,[T^a\otimes\mathbb{1}] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

9x9 matrix

Spin polarization coefficients

Spin correlation coefficients

helicities

$$f_a = \frac{1}{6}\,\mathrm{Tr}\,[\rho\,(\mathbb{1}\otimes T^a)]\ ,\quad g_a = \frac{1}{6}\,\mathrm{Tr}\,[\rho\,(T^a\otimes\mathbb{1})]\ ,\quad h_{ab} = \frac{1}{4}\,\mathrm{Tr}\left[\rho\left(T^a\otimes T^b\right)\right]$$

- Scalar quantities that depend on VV invariant mass and scattering angle in c.m. frame
- we can also extract them from data, using the decay products of final Vector Bosons

at LHC for WW

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \mathrm{jets} + E_T^{\mathrm{miss}}$$

Alan Barr, PLB 285 (2022), 2106.01377 [hep-ph]
R. Ashby-Pickering, A. Barr, A. Wierzchucka
JHEP 05 107 (2023) 7, 2209.14033 [hep-ph]

Bell inequalities for Two-QUTRITS

- perform two measurements ⇒ $(A_1 \; , \; A_2) \quad (B_1 \; , \; B_2)$ each can take values $\rightarrow \{0, 1, 2\}$

  (i.e. corresponding to the spin-1 eigenvalues $S_z \rightarrow \{1,0,-1\}$)

- consider the following correlator $\mathcal{I}_3$ for probability measurements (CGLMP)

  Collins, Gisin, Linden, Massar, Popescu, PRL 88 (2002)

$$\begin{aligned}\mathcal{I}_3 \;\; = \;\; & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & -P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)\end{aligned}$$

$\mathcal{I}_3$ can be expressed as $\boxed{\mathcal{I}_3 = \mathrm{Tr}\left[\rho\, \mathcal{B}\right]}$ with $\mathcal{B}$ a suitable Bell operator (see backup slide)

Bell inequalities for two-qutrits

For deterministic local models $\boxed{\mathcal{I}_3 \leq 2}$ ⇒ **QM for qutrits can violate this inequality** (with upper bound = 4) falsifying all real-hidden variable local theories

in order to maximize the violation of Bell inequality

- still freedom to modify measured observables through unitary transformations on the Bell operator

$$\boxed{\mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)}$$

U,V are unitary 3x3 matrices (depend on the kinematic of the process)

Quantifying entanglement for Two-Qutrits

- CONCURRENCE $$\mathcal{C}[\rho] = \sqrt{1 - \mathrm{Tr}[(\rho_r)^2]}$$ $r = A \text{ or } B$

→ vanish for separable (not entangled) states

Trace performed in the subsystems *r = A or B*

Witness of Entanglement

- analytical solution exists for the lower bound $$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho]$$

Mintert, Buchleitner, PRL 98 (2007)

$$\mathscr{C}_2[\rho] = 2\max\left(0,\, \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_A)^2\right],\, \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_B)^2\right]\right)$$

- If non-vanishing unequivocally signal the presence of entanglement
- used as observable for entanglement in WW, ZZ and WZ productions

Testing Bell inequalities in Di-boson production

Fabbricesi, Floreanini, EG, Marzola
2302.00683 [hep-ph]

Via Drell-Yan mechanism of production

@ LHC $pp \to W^+W^- \quad pp \to ZZ \quad pp \to WZ$

- not sufficient N. of events to test Bell inequalities with large significance in the relevant kinematic region at the LHC
- pp → WW largely affected by systematic error induced by the missing neutrino momentum reconstruction
- pp → ZZ is the most promising channel, but difficult being less sensitive to Bell inequalites
- Resonant channel via Higgs → ZZ* : best place at LHC where to test Bell inequalites in qutrits

@ Lepton colliders (muon, FCC, CEPC) e+ e- → WW , ZZ

- e+e- colliders allow to test Bell inequalities in qutrits via **WW** or **ZZ**
- Via Higgs → τ τ (qubits) and WW*, ZZ* (qutrits) (where W* and Z* are off-shell states)

Correlation and polarization coefficients (for 2-qutrits)

At LHC (PDF do not factorize)

Parton luminosity

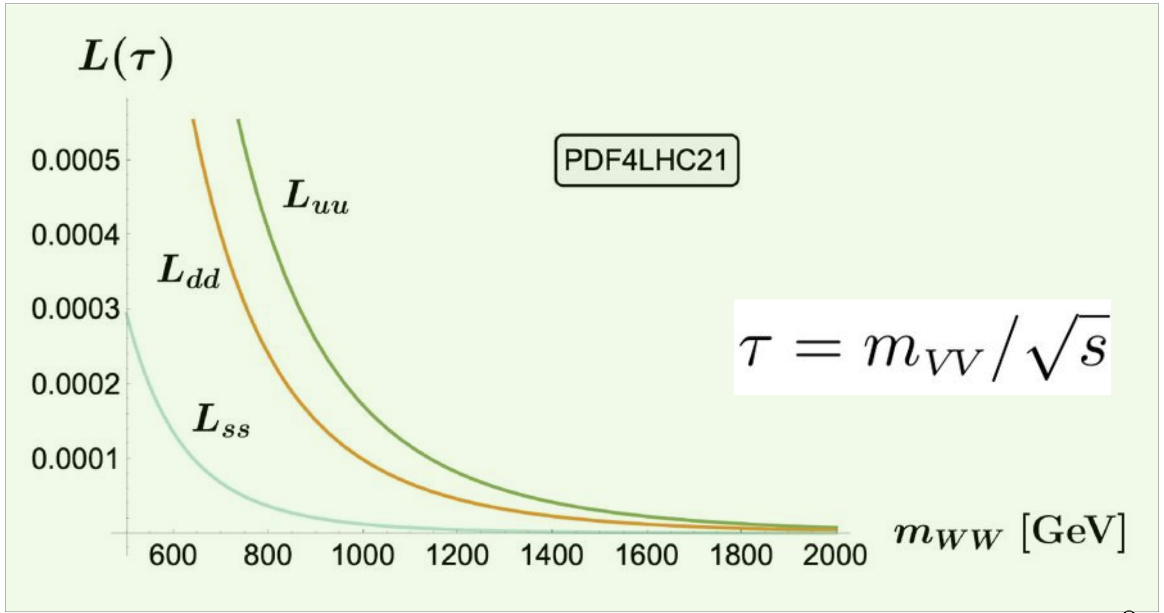
$$h_{ab}[m_{VV},\Theta] = \frac{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(\tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta] + \tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta+\pi]\right)}{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(A^{q\bar{q}}[m_{VV},\Theta] + A^{q\bar{q}}[m_{VV},\Theta+\pi]\right)}$$

and analogously for the f_a and g_a polarization coefficients. where

$$A^{q\bar{q}} = |\overline{\mathcal{M}}^{q\bar{q}}_{WW}|^2$$ and $$\tilde{h}_{ab} = A^{q\bar{q}} h_{ab}$$

quark-antiquark PDF

$$L^{qq}(\tau) = \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{\mathrm{d}z}{z} q_q(\tau z) q_{\bar{q}}\left(\frac{\tau}{z}\right)$$



At e+ e- (no PDF uncertainty)

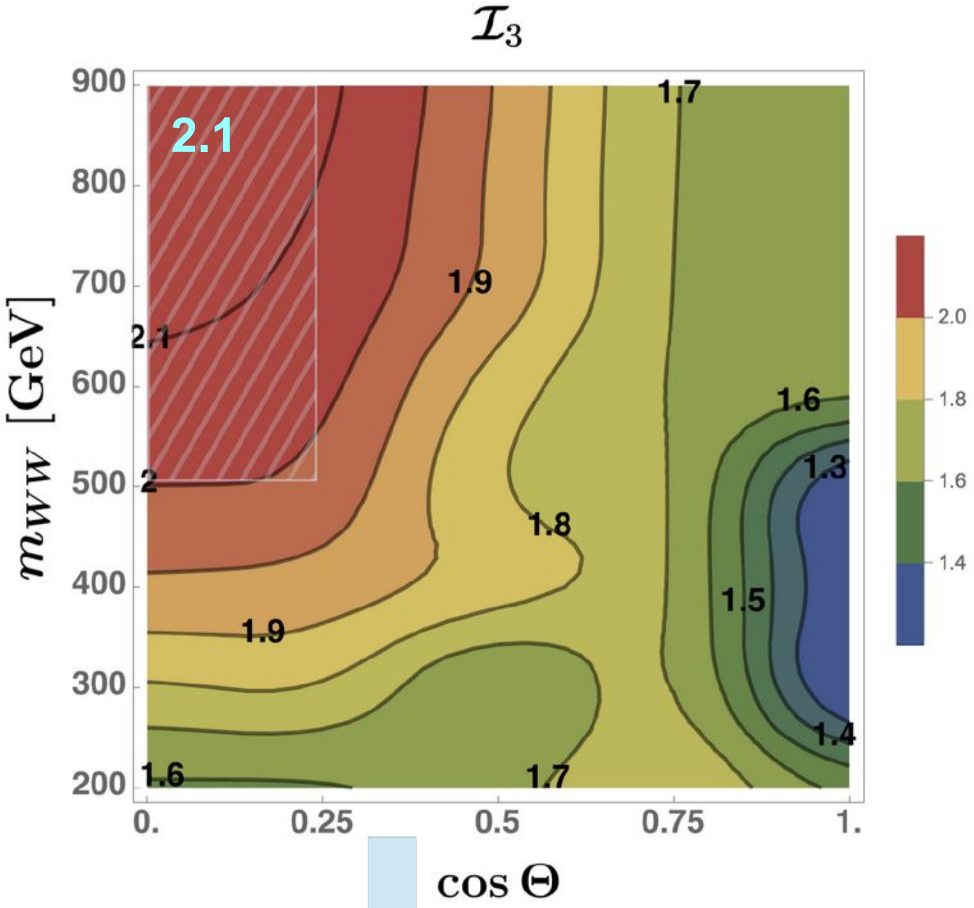
$$h_{ab}[m_{WW},\Theta] = \frac{\tilde{h}^{\ell\bar{\ell}}_{ab}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

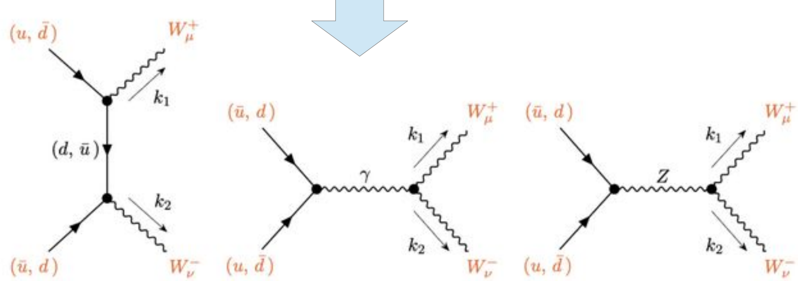
$$f_a[m_{WW},\Theta] = \frac{\tilde{f}^{\ell\bar{\ell}}_{a}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

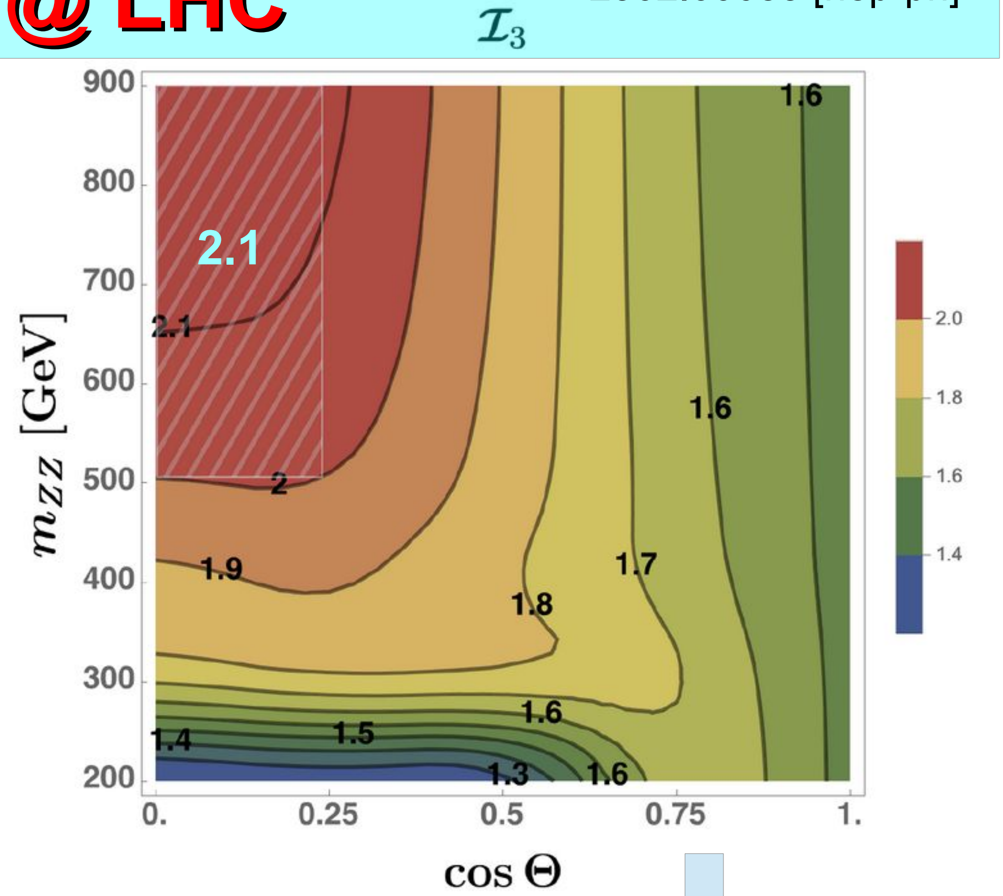
$$g_a[m_{WW},\Theta] = \frac{\tilde{g}^{\ell\bar{\ell}}_{a}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

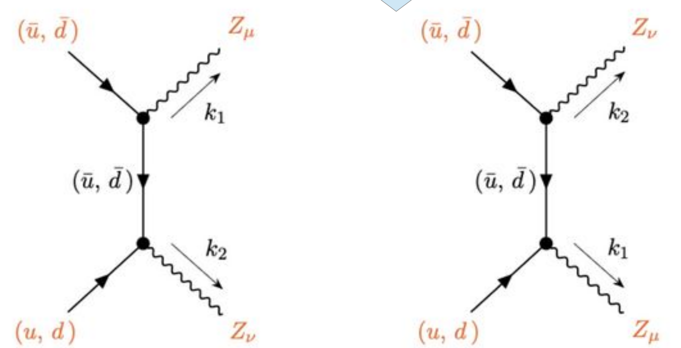
Bell inequalities @ LHC

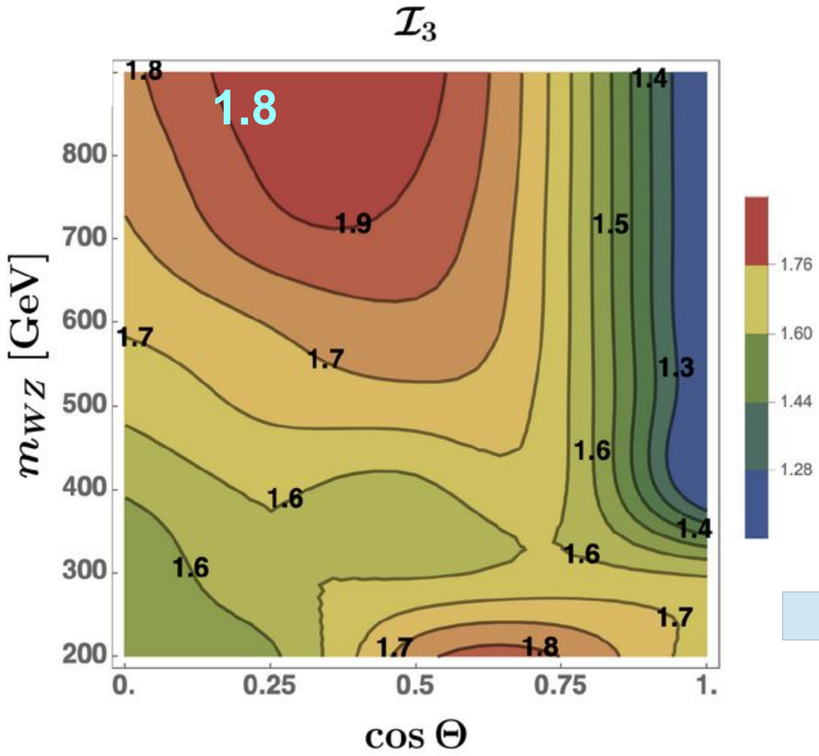
2302.00683 [hep-ph]

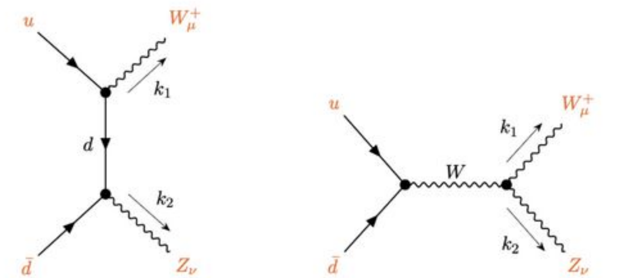






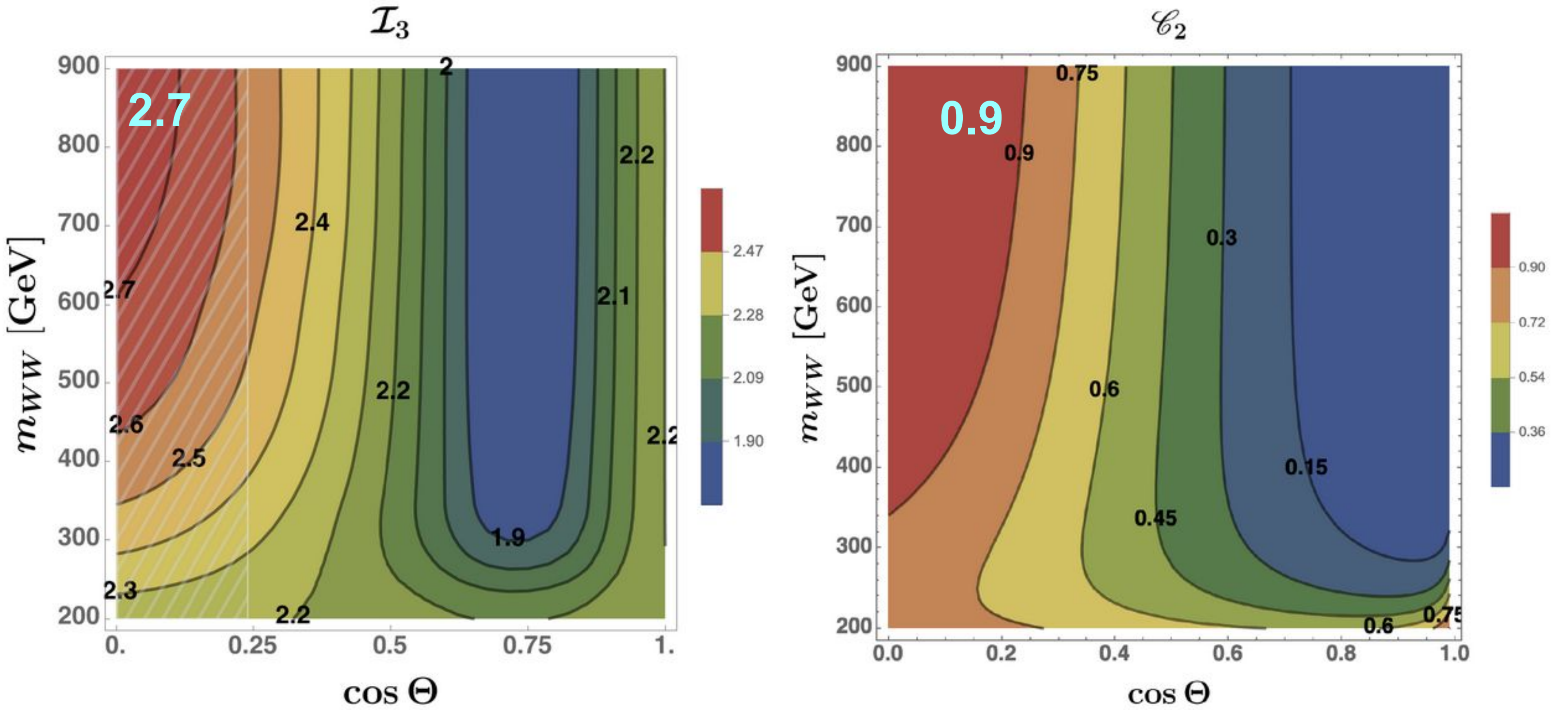






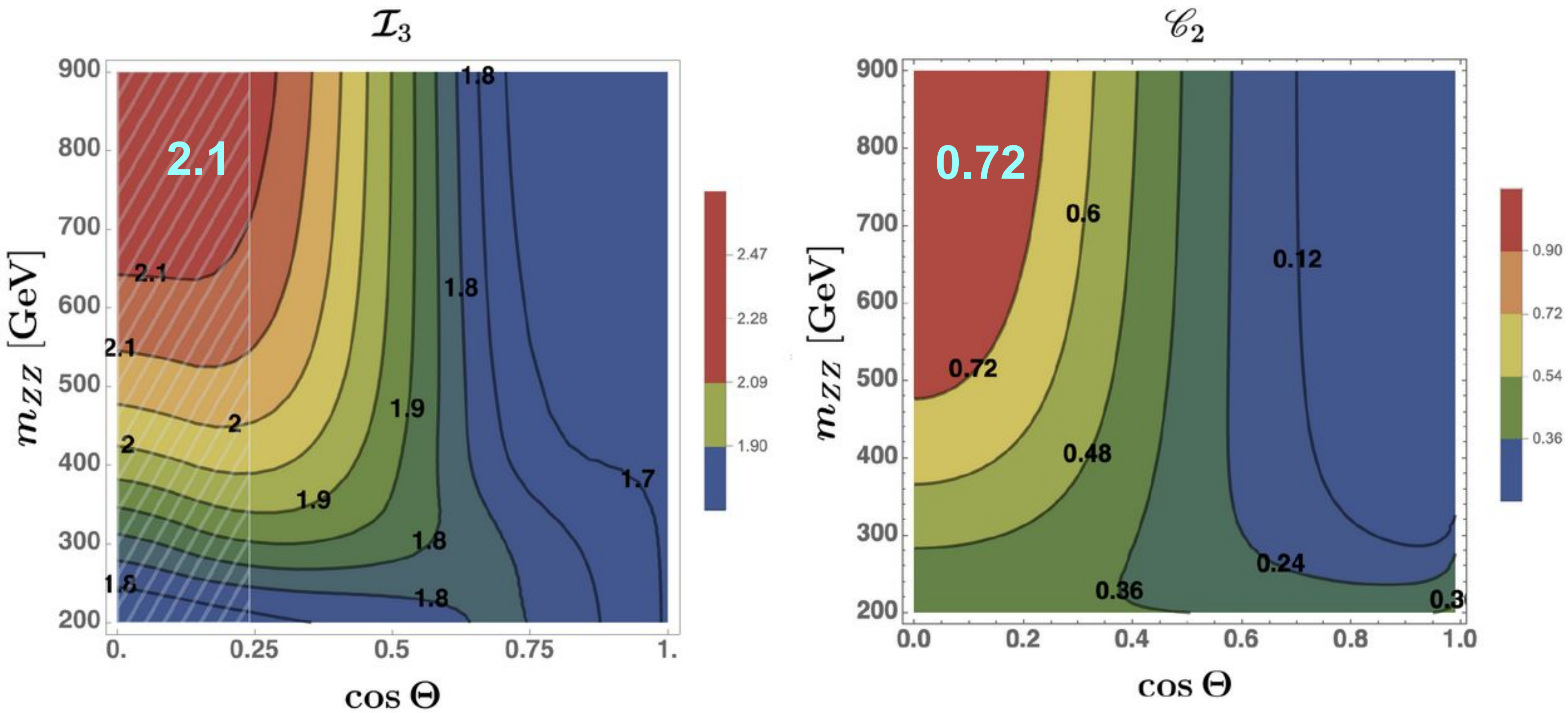
Bell inequalities and Entanglement at FCC, CEPC

2302.00683 [hep-ph]



$$e^+ \, e^- \to W^+ \, W^-$$

absence of PDF increases entanglement and Bell violations



$$e^+ \, e^- \to Z \, Z$$

ZZ channel not much different than LHC → due to the fact that PDF almost factorize (t,u channels)

Events and sensitivity

N. events			$\ell^+\nu_\ell jj$	$\ell^-\ell^+\ell^-\ell^+$
$\sqrt{s} = 1$ TeV	muon	$(\mathcal{L} = 1\ \mathrm{ab}^{-1})$	5.7×10^3	44
$\sqrt{s} = 368$ GeV	FCC	$(\mathcal{L} = 1.5\ \mathrm{ab}^{-1})$	9.2×10^4	748

2302.00683 [hep-ph]

$m_{WW} > 200$ GeV and $\cos\Theta < 0.25$

10^4 **pseudo experiments**

- $e^+ e^- \to W^+ W^-$

Significance ~ **2** to reject the null hypothesis both @ muon collider and FCC, CEPC

mean value I_3 = 2.6

- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
  → n. of events reduced accordingly
- systematic error due to missing neutrino in WW mass reconstruction largely dominates against statistical one
- no background considered

- $e^+ e^- \to Z Z$

Significance to reject null hypthesis
~ **2** @ muon collider and more than **4** @ FCC, CEPC

mean value I_3 = 2.17

- Lepton reconstruction efficiency as above
- only statistic error assumed
- no background considered

FCC significance is larger than muon collider due to larger n. of events at FCC (statistical error reduced)

Other processes where to test Bell inequalites @ FCC, CEPC

via resonant Higgs decays

Altakach, Priyanka, Maltoni, Mawatari, Sakurai, PRD 107 (2023) 9, 2211.10513 [hep-ph]

$$e^+e^- \to Zh \to Z\tau^+\tau^-$$

Kai Ma, Tong Li, 2309.08103 [hep-ph]

- **Qubits final states (tau lepton pairs)** → difficult at LHC but promising at FCC, CEPC

(use Clauser-Horne-Shimony-Holt (CHSH) inequality)

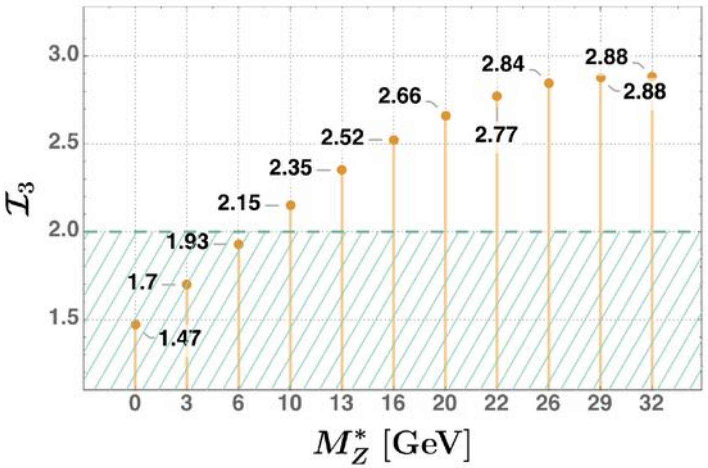
Bell inequality can be tested at FCC up to 3σ level of significance

- **Qutrits final states ($h \to ZZ^*$)** @ LHC

consistent with results via simulation of events

Alan Barr, PLB 285 (2022), 2106.01377 [hep-ph]
R. Ashby-Pickering, A. Barr, A. Wierzchucka JHEP 05 107 (2023) 7, 2209.14033 [hep-ph]
Aguilar-Saavedra, Bernal, Casas, Moreno, 2209.13441 [hep-ph]
Aguilar-Saavedra, 2209.14033 [hep-ph]

analitical predictions 2302.00683 [hep-ph]



- @ LHC Bell inequality can be tested up 4σ level
- @ FCC, CEPC via e+e- → Z h → ZZZ*
  Z h → Z WW*

  not yet analyzed

via s-channel production

$$e^+ e^- \to \tau^+ \tau^-$$

→ promising channel at FCC

- **Qubits final states**

P. Privitera PLB 275 (1992) 172

Summary

- Bell inequalities in weak gauge boson production can be already tested at LHC via resonant Higgs decay $H \to ZZ^*$ (sensitivity up to $4\,\sigma$ level of signif. @ LHC-HL)

- FCC, CEPC can test Bell inequalities in qutrit systems of (**on-shell**) WW and ZZ production with more than 4σ level of significance
  $\rightarrow$ while via Drell-Yan production it is disfavored at LHC

- FCC, CEPC can also test Bell inequalities in qubit systems via tau-pair production

- in both resonant Higgs decay $e^+e^- \to Zh \to Z\tau^+\tau^-$

- and non-resonant s-channel production $e^+ e^- \to \tau^+ \tau^-$

Backup slides

W W

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+\, d\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1 V_2}\left(\Gamma_+ \otimes \Gamma_-\right)\right]$$

depend on the invariant mass m_{VV} (or velocity β) and scattering angle Θ in the V_1V_2 cm frame

Rahaman, Singh, NPB 984 (2022), 2109.09345 [hep-ph]

$$d\Omega^\pm = \sin\theta^\pm d\theta^\pm\, d\phi^\pm$$

solid angle of $\ell^\pm$ polar angle azimuthal angle

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1 V_2}$ = density matrix of V_1V_2

$\Gamma_\pm$ → Density matrices that describe the polarization of the two decaying W into final leptons (the charged ones assumed to be massless)

these are projectors in the case of the W-bosons because of their chiral couplings to leptons can be computed by rotating to an arbitrary polar axis the spin ± 1 states of gauge bosons taken in the z-direction

$$\Gamma_{\pm} = \frac{1}{3}\mathbb{1} + \sum_{i=1}^{8} \mathfrak{q}_{\pm}^{a}\, T^{a}$$

Density matrices for W-bosons

the functions $\mathfrak{q}_{\pm}^{a}$ can be written in terms of the respective spherical coordinates

some polynomial of spherical coordindates

$$\begin{aligned}
h_{ab} &= \frac{1}{\sigma}\int\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{+}\,\mathrm{d}\Omega^{-}}\,\mathfrak{p}_{+}^{a}\,\mathfrak{p}_{-}^{b}\,\mathrm{d}\Omega^{+}\mathrm{d}\Omega^{-}\\
f_{a} &= \frac{1}{\sigma}\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{+}}\,\mathfrak{p}_{+}^{a}\,\mathrm{d}\Omega^{+}\\
g_{a} &= \frac{1}{\sigma}\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{-}}\,\mathfrak{p}_{-}^{a}\,\mathrm{d}\Omega^{-}
\end{aligned}$$

$\mathfrak{p}_{\pm}^{n}$ a particular set of orthogonal functions $\Rightarrow$ $\left(\frac{3}{4\pi}\right)\int \mathfrak{p}_{\pm}^{n}\,\mathfrak{q}_{\pm}^{m}\,\mathrm{d}\Omega^{\pm} = \delta^{nm}$

For the ZZ production the density matrices $\Gamma_\pm$ are not projector due to the Z boson coupling

$$\mathcal{L} \supset -i\frac{g}{\cos\theta_W}\left[g_L(1-\gamma^5)\gamma_\mu + g_R(1+\gamma^5)\gamma_\mu\right] Z^\mu$$

$$\tilde{\mathfrak{q}}^n = \frac{1}{g_R^2+g_L^2}\left[g_R^2\,\mathfrak{q}_+^n + g_L^2\,\mathfrak{q}_-^n\right]$$

$$\tilde{\mathfrak{p}}^n = \sum_m \mathfrak{a}_m^n \mathfrak{p}_+^m$$

$$\mathfrak{a}_m^n = \frac{1}{g_L^2-g_R^2}\begin{pmatrix} g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 & 0 \\ 0 & g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 \\ 0 & 0 & g_R^2-\frac{1}{2}\,g_L^2 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2}\,g_L^2 \\ 0 & 0 & 0 & g_R^2-g_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2-g_L^2 & 0 & 0 & 0 \\ g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 & 0 \\ 0 & g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2}\,g_L^2 & 0 & 0 & 0 & 0 & \frac{1}{2}\,g_L^2-g_R^2 \end{pmatrix}.$$

Wigner's *Q symbols*

$$
\begin{aligned}
\mathfrak{q}^1_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(\cos\theta^\pm \pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{q}^2_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(\cos\theta^\pm \pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{q}^3_\pm &= \frac{1}{8}\Big(1 \pm 4\cos\theta^\pm + 3\cos 2\theta^\pm\Big)\,,\\
\mathfrak{q}^4_\pm &= \frac{1}{2}\sin^2\theta^\pm\cos 2\,\phi^\pm\,,\\
\mathfrak{q}^5_\pm &= \frac{1}{2}\sin^2\theta^\pm\sin 2\,\phi^\pm\,,\\
\mathfrak{q}^6_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(-\cos\theta^\pm \pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{q}^7_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(-\cos\theta^\pm \pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{q}^8_\pm &= \frac{1}{8\sqrt{3}}\Big(-1 \pm 12\cos\theta^\pm - 3\cos 2\theta^\pm\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{p}^1_\pm &= \sqrt{2}\sin\theta^\pm\Big(5\cos\theta^\pm \pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{p}^2_\pm &= \sqrt{2}\sin\theta^\pm\Big(5\cos\theta^\pm \pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{p}^3_\pm &= \frac{1}{4}\Big(5 \pm 4\cos\theta^\pm + 15\cos 2\theta^\pm\Big)\,,\\
\mathfrak{p}^4_\pm &= 5\sin^2\theta^\pm\cos 2\,\phi^\pm\,,\\
\mathfrak{p}^5_\pm &= 5\sin^2\theta^\pm\sin 2\,\phi^\pm\,,\\
\mathfrak{p}^6_\pm &= \sqrt{2}\sin\theta^\pm\Big(-5\cos\theta^\pm \pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{p}^7_\pm &= \sqrt{2}\sin\theta^\pm\Big(-5\cos\theta^\pm \pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{p}^8_\pm &= \frac{1}{4\sqrt{3}}\Big(-5 \pm 12\cos\theta^\pm - 15\cos 2\theta^\pm\Big)\,.
\end{aligned}
$$

problem of finding optimal choice of Bell violation has been found

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right]$$

for the maximal entangled state

$$|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} |i\rangle \otimes |i\rangle$$

$$\rho = |\Psi_+\rangle\langle\Psi_+|$$

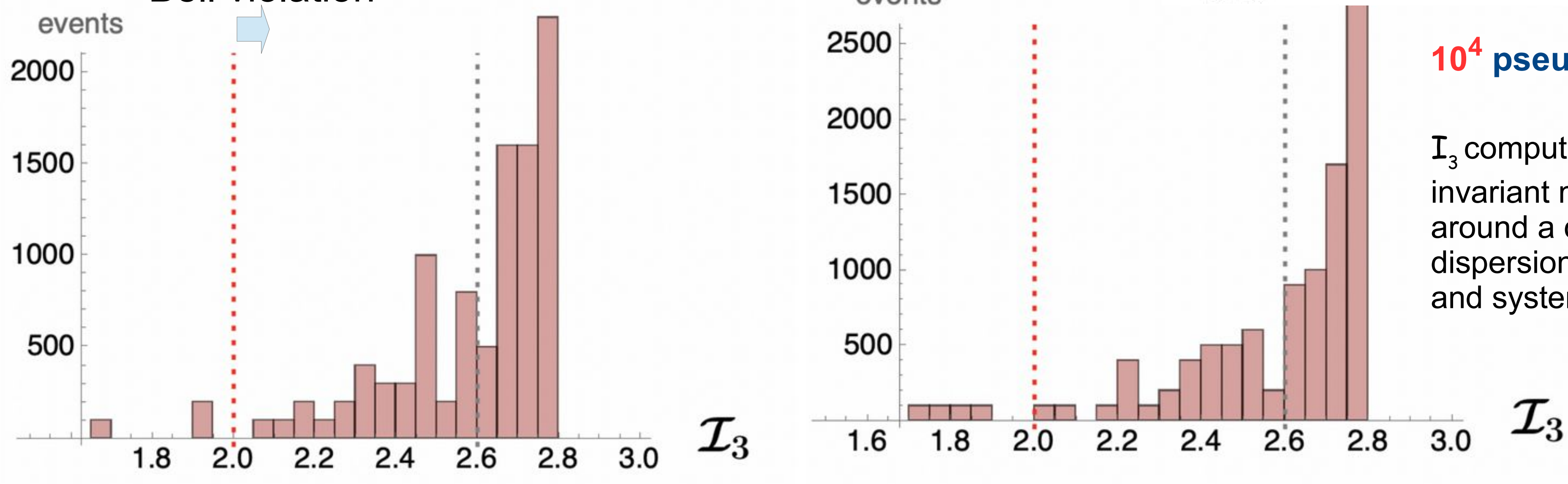
it is given by the Bell operator

written on the basis of spin-operators, where

$$\mathcal{B} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\
0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\
0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

S_3 spin operator $\longrightarrow$ diag$\{1, 0, -1\}$

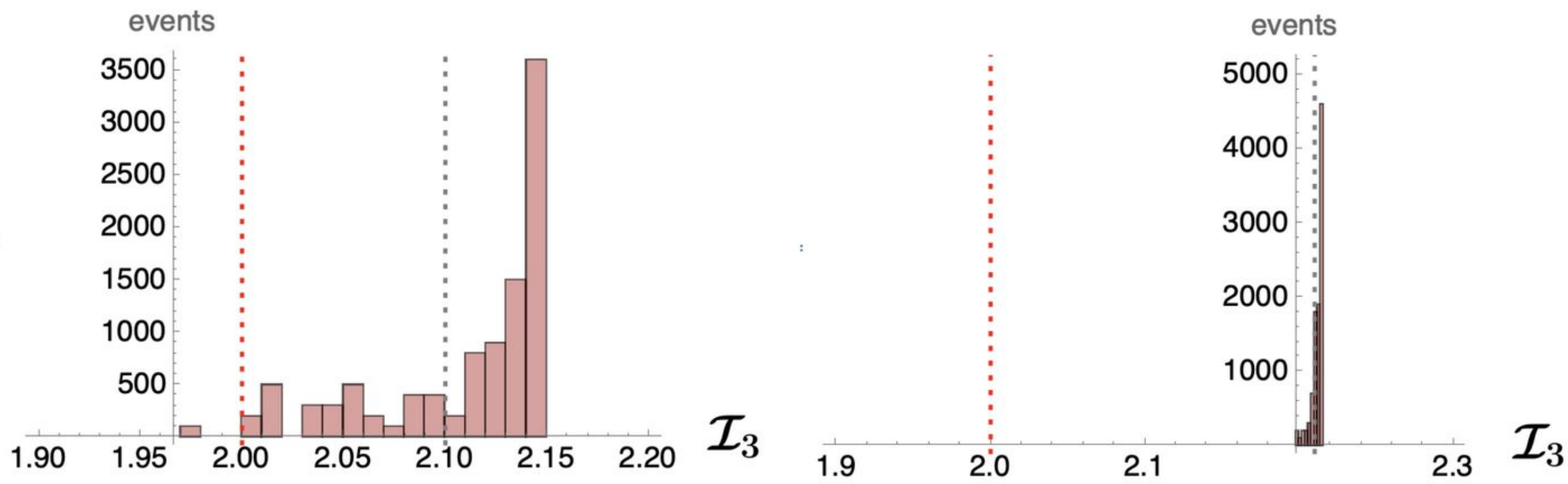
$m_{WW} > 200$ GeV and $\cos\Theta < 0.25$



10^4 pseudo experiments

I_3 computed varying invariant mass and angle around a central value with dispersion given by statistical and systematic errors

WW : Distribution of the events (muon collider, left, FCC, right) in the W^+W^- process. The events have mean value $\mathcal{I}_3 = 2.6$. The threshold value of 2 for Bell inequality violation is shown as a dashed red line.



ZZ Distribution of the events (muon collider, left, FCC, right) in the ZZ process. The events have mean value $\mathcal{I}_3 = 2.17$. The threshold value of 2 for Bell inequality violation is shown as a dashed red line.