Jet Clustering performance based on Quantum Approximate Optimization Algorithm (QAOA)

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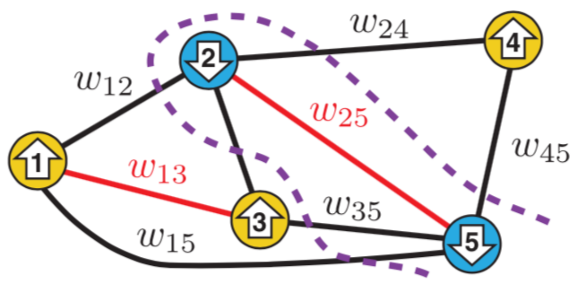
Motivation:

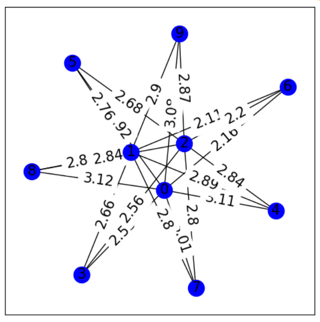
- Finding useful applications for near-term quantum machines is interesting in the quantum era.
- Quantum Approximate Optimization Algorithm (QAOA) has a high potential to showcase the advantages of quantum computing in the NISQ era.
- Jet clustering, which is actually a combinatorial problem, can be explored with QAOA after mapping a collision event into a graph.

Introduction:

MaxCut:

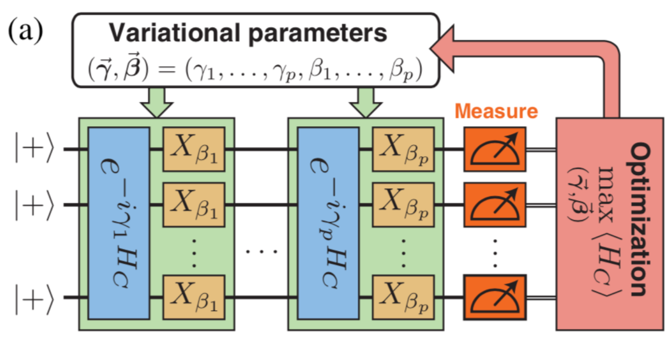
- Graph : Set of vertices or nodes connected by weighted edges (W_{ij})
- cut : Partition of vertices into two disjoint subsets
- Goal : Letting the weighted sum of edges with two nodes located in two subsets as large as possible





QAOA for MaxCut:

1. Define problem Hamiltonian $\hat{H}_C = \frac{1}{2}\sum_{(i,j)\in E} w_{ij}(I - Z_i Z_j)$, and mixer Hamiltonian $\hat{H}_M = \sum_{j\in V} X_j$
2. Initialize the quantum circuit in the highest energy state of the mixer Hamiltonian $|s\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{x\in(0,1)^n}|x\rangle$,
3. Define the unitaries, $\hat{U}_C(\gamma) = e^{-i\gamma\hat{H}_C}$ and $\hat{U}_M(\beta) = e^{-i\beta\hat{H}_M}$, where γ and β are variational parameters of the circuit.
4. Initialize the 2P variational parameters and the final state output by the circuit $|\psi_P(\gamma,\beta)\rangle = \prod_{j=1}^{P} U(\beta_j, \hat{H}_M)U(\gamma_j, \hat{H}_C)|s\rangle$.
5. By repeated measurements, the expectation value of the $\hat{H}_C$ with respect to the $|\psi_P(\gamma,\beta)\rangle$ is $F_P(\gamma,\beta) = \langle\psi(\gamma,\beta)|\hat{H}_C|\psi_P(\gamma,\beta)\rangle$
6. The variational parameters are optimized by a classical optimizer $(\gamma^*,\beta^*) = \arg\max_{\gamma,\beta} F_P(\gamma,\beta)$, and the approximation ratio α is defined as $\alpha = \frac{F_P(\gamma^*,\beta^*)}{C_{max}}$, where C_{max} corresponds to the best value of $F_P(\gamma,\beta)$

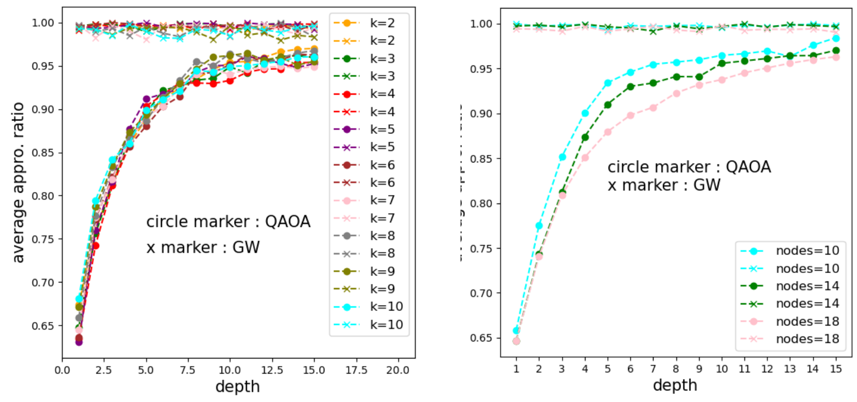


Mapping a collision event into a graph:

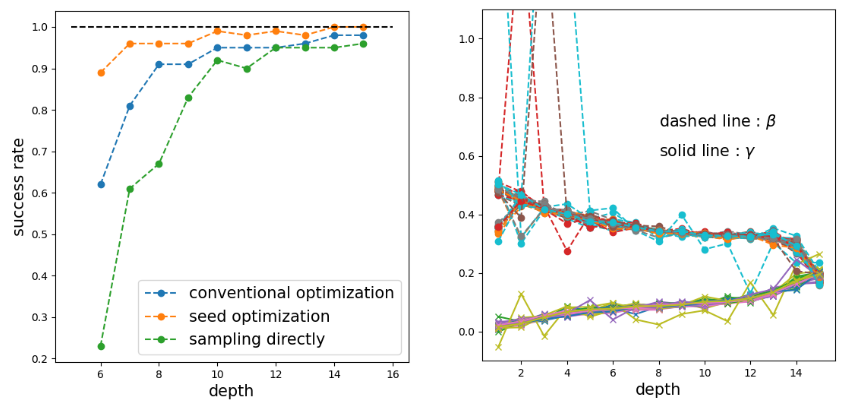
- The particles are represented with nodes and the weight of the edge is calculated as the angle between two particles.
- For an event with n particles, each particle can have edges with other n-1 particles, but we only keep the k edges with the largest weight.
- A graph with nodes=10 and k = 3.

Performance analysis:

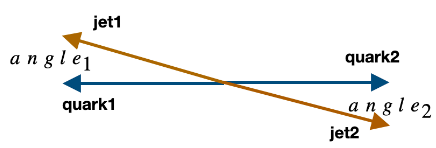
- **The dependence of QAOA performance** on the value of k, depth, and nodes within the graph. GW is a classical algorithm used to solve MaxCut.

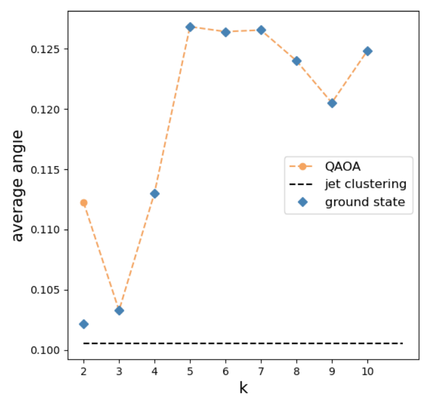


- **Parameter transferability:** The hard problem of the QAOA is variational parameters optimization. Abstract optimized parameters from 100 graphs, and reuse these optimized parameters to similar graphs to sample directly, which can conserve computing resources with a performance decrease of less than 2% compared to regular optimization procedure, as initial parameters to optimize in a further step, which can improve the QAOA performance. The further optimized parameters are more concentrated, which illustrates the parameter transferability in another aspect. The success rate is defined as the ratio of graphs with an approximation ratio larger than 0.96.



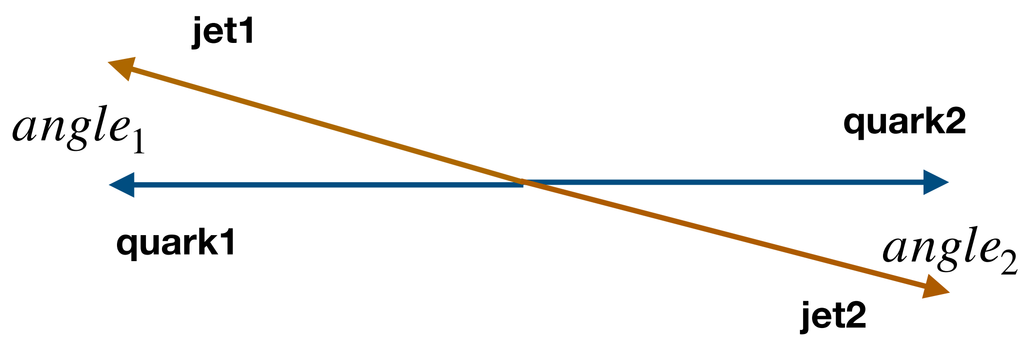
- **Jet clustering performance** can be evaluated with the angle between the reconstructed jet and the corresponding quark.





Summary:

- Based on the jet clustering problem, the QAOA performance would be better with increasing depth of circuit, and worse with increasing number of nodes within the graph, independent of the graph's connectivity.
- The optimized parameters can be reused on similar graphs to sample directly with a performance decrease of less than 2% but conserve computing resources. When seeing optimized parameters as initial parameters and optimizing in a further step, the QAOA performance would be better.
- A well-modeling method and quantum algorithm are needed for jet clustering in the quantum era.


jet1
$angle_1$
quark2
quark1
$angle_2$
jet2